X- 2.KINEMATICS

Exercise Solutions

Objective Questions

LEVEL 1

1. (a)

2. (d)

3. (a)
$G = Fr^2/m^2 = [MLT^{-2}][L^2]/[M^{-2}] = [M^{-1}L^3T^{-2}]$

4. (a)
Let speed of the boat be ‘v’
Distance travelled downstream = (v + 5)t
Distance travelled upstream = (v - 5)t
Here, (v + 5)t = 3(v - 5)t
$\Rightarrow$ v = 10 km/h

5. (a)
Let total distance travelled be ‘D’
Time taken to cover 60% of distance = $3D/5v_1$
Time taken to cover rest 40% of distance = $2D/5v_2$
Total time taken = $3D/5v_1 + 2D/5v_2$
Average Velocity = Total Distance / Total time = $D/(3D/5v_1 + 2D/5v_2) = 5v_1v_2/(2v_1 + 3v_2)$

6. (b)
We know that v = u + at
Here, $v = 5*10^3 + 10^3t$
Here, $v = 2u = 10*10^3 = 5*10^3 + 10^3t$
t = 5 sec

7. (a)

8. (d)
We know that, $s_n = u + a(n- ½)$
$s_5 = 25 + 50/9*(5-½) = 50$ m

9. (a)
We know that, $v^2 = u^2 + 2as$
At maximum height i.e. s = h: final velocity (v) = 0, initial velocity (u) is given as v, acceleration(a) = -g
$\Rightarrow v = \sqrt{(2gh)}$
At above equation again to find s_x and s_y
At X, velocity = v/4 $\Rightarrow (v/4)^2 = v^2 + 2(-g)s_x$
$\Rightarrow 15/16\ v^2 = 2gs_x \Rightarrow s_x = 15h/16$
At Y, velocity = v/6 => $(v/6)^2 = v^2 + 2(-g)s_y$
$\Rightarrow 35/36\ v^2 = 2gs_x$ => $s_y = 35h/36$
$s_y - s_x = 35h/36 - 15h/16 = 5h/144$

10. (c)
Let they meet after time 't'
Distance travelled by first ball (h) = $\frac{1}{2} gt^2$ (Using second equation of motion) = $5t^2$
Distance travelled by second ball (s) = $25t - \frac{1}{2} gt^2 = 100 - h$
$\Rightarrow 25t - 5t^2 = 100 - 5t^2 \Rightarrow t = 4$ sec

11. (a)
Displacement in horizontal direction = 4m
Displacement in vertical direction = 3m
Net displacement = $\sqrt{(4^2 + 3^2)} = 5$ m

12. (a)
Total distance travelled = $v_1t_1 + v_2t_2$ +
Total time taken = $t_1 + t_2$
Average velocity = Total distance / Total time = $(v_1t_1 + v_2t_2)/(t_1 + t_2)$

13. (a)
Since v = u + at
Here, u = 0 => a = v/t = 3.2/2 = 1.6 m/s^2
v = at = a*5 = 8 m/s

14. (c)
Using second equation, we get h = $\frac{1}{2} gT^2$
At t = T/3, s = $(\frac{1}{2} gT^2)/9 = h/9$
Distance from the ground = h – h/9 = 8h/9

15. (a)
We know that s = ut + $\frac{1}{2} gt^2$
Here time (t) is independent of mass and only depends on s, u and g. Thus it is same for both the bodies.

16. (b)
Displacement = Distance between point A and B = $\sqrt{(r^2 + r^2)} = \sqrt{(2r)}$
Total distance travelled = ¾*Circumference of circle = $3\pi r/2$

17. (d)
Total distance travelled = Area under graph = 100 + 80 + 80 = 260m

18. (b)
Acceleration = slope between t = 0 and t = 5 sec = 40/5 = 8 m/s^2

19. (a)
Retardation = slope between t = 7 and t = 11 sec = -40/4 = -10 m/s^2

20. (a)
Average velocity = Total distance/Total time = 260/11 m/s

LEVEL 2

1. (b)
Let time taken be 't'
Distance covered in last second = $\frac{1}{2} g(2t-1)$ ($s_n = u + a(n- \frac{1}{2})$)
Distance covered in first 3 seconds => s = $\frac{1}{2} gt^2 = \frac{1}{2} g(3)^2 = 9g/2$

Equating both, we get t = 5s

2. (c)

Total time of flight = 2u/g = 20s

At t = 10s, the first arrow will be at its maximum height and second arrow will not be at its maximum height.

Speed of first at time 20 sec = u = 98 m/s in downward direction

Speed of second arrow at t = 20 sec => v = u + at

Here u = 98 m/s, g = -10 m/s^2 and t = 15 sec (Second arrow was released at t = 5s)

$\Rightarrow$ v = 98 – 9.8*15 = -49 m/s = 49m/s in downward direction

Ratio of speeds = 2:1

3. (c)

Distance travelled in first 10 sec, s = ½ g $(10)^2$ = 500m (Using second equation of motion)

Velocity acquired = 10g = 100m/s (Using first equation of motion)

Distance remaining = 1995 m

Let speed acquired be v

$v^2 = u^2 + 2as \Rightarrow v^2 = (100)^2 - 2(2.5)1995 = 25 \Rightarrow v = 5$ m/s

4. (a)

Let the time of acceleration and retardation be t_1 and t_2 respectively

Maximum speed (v) = $u + at_1 = \alpha t_1$

For car to stop, final velocity should be zero.

$\Rightarrow v = u + at_2 = \alpha t_1 - \beta t_2 = 0 \Rightarrow t_2 = \alpha t_1/\beta$

Also, $t_1 + t_2 = t \Rightarrow t_1 + \alpha t_1/\beta = t$

$\Rightarrow t_1 = t\beta/(\alpha+\beta)$

$\Rightarrow$ Maximum speed = $\alpha t_1 = t\alpha\beta/(\alpha+\beta)$

5. (d)

Vertical speed = ucos(60) = u/2 = 10 m/s

Time taken to reach ground can be found by using second equation of motion

$\Rightarrow 40 = -10t + \frac{1}{2} g t^2 = -10t + 5t^2$

$\Rightarrow t^2 - 2t - 8 = 0$

$\Rightarrow$ t = 4s (Negative value of t can be ignored)

6. (c)

Along the incline, acceleration due to gravity = -gsin(30) = -g/2

Using $v^2 = u^2 + 2as$ at highest point along the incline i.e. v = 0 => $u^2 = 40g$

Using this in the formula for Range for projectile motion, we get R =u^2sin(60)/g = 20√3 m

7. (d)

Let initial velocity of the body be 'u'

At the highest point, only horizontal component of the velocity will remain i.e. ucos60 = u/2

Initially, K = ½ mu^2

Kinetic energy at highest point = ½ mv^2 = ½ $mu^2/4$ = K/4

8. (b)

Range is proportional to square of u => If range is doubled, u' = u√2

Time of flight is directly proportional to u => T' = T√2

9. (b)

At the highest point, Potential energy, PE = mgh = $mg(u^2\sin^2\theta)/2g$ = ½ $mu^2\sin^2\theta$

Kinetic energy will be only due to horizontal component of velocity $\Rightarrow$ KE = ½ $mu^2\cos^2\theta$

PE/KE = $\tan^2\theta$

10. (d)

R is same for angle of projection θ and $90-\theta$

$h_1 = u^2\sin^2\theta/2g$ and $h_2 = u^2\sin^2(90-\theta)/2g = u^2\cos^2\theta/2g$

$\sqrt{(h_1h_2)} = u^2\sin\theta\cos\theta/2g = u^2\sin2\theta/4g = R/4$

$R = 4\sqrt{(h_1h_2)}$

Subjective Questions

1. Let the speed of bus be 'v'

Relative speed to bus in the direction of motion of cycle = v – 20

Relative speed to bus in the direction opposite to motion of cycle = v + 20

Distance between two buses = vT

Thus, vT/(v-20) = 18 min = 0.3 h

And vT/(v+20) = 6 min = 0.1 h

Dividing the two equations and solving, we get v = 40 km/h

And T = 9 min i.e. in either direction, a bus leaves every 9 mins.

2. Let the speed of car be 'v'

Relative speed to bus w.r.t. car = v + 30

Time interval of meeting the two cars = 5/(v+30) = 4 mins = 1/15 h. Thus, v = 45 km/h

3. Let the speeds be v_1 and v_2

In opposite direction, relative speed = $v_1 + v_2$

Relative speed = Distance/time = 4 m/s $\Rightarrow v_1 + v_2 = 4$

In same direction, relative speed = $v_1 - v_2$

Relative speed = Distance/time = 0.4 m/s $\Rightarrow v_1 - v_2 = 0.4$

On solving the two equations, we get $v_1 = 2.2$ m/s and $v_2 = 1.8$ m/s

4. Let the retardation offered by plank be '-a' and its width be 'd'

We know that $v^2 = u^2 + 2as$

Here v = u – u/20 = 19u/20

$(19u/20)^2 = u^2 - 2ad \Rightarrow 2ad = 39u^2/400$

To completely, stop the bullet, final velocity must be 0

$0 = u^2 - 2adn$ where n is the number of planks required

$\Rightarrow n = u^2/2ad = 400/39 = 10.2$

Number of planks is a natural number so least number of planks required is 11

5. Let the acceleration be 'a' and AC be 3x

Thus, AB = x and BC = 2x

Now, using $v^2 = u^2 + 2as$ between A and C

$\Rightarrow 25^2 = 5^2 + 2a(3x) \Rightarrow ax = 100$

Speed after reaching B => $v^2 = 5^2 + 2ax = 25 + 200 = 225$

Speed at B, v = 15 m/s

Using first equation of motion i.e. v = u + at between AB, we get $15 = 5 + at_1$

$t_1 = 10/a$, where t_1 is time taken to reach B from A

Now, using the equation again between B and C

$v = u + at_2 \Rightarrow 25 = 15 + at_2 \Rightarrow t_2 = 10/a$

Thus, $t_1 = t_2$ and ratio is 1:1

6. Just when ball is about to hit the ground, its velocity, $v = -\sqrt{(2gh)} = -\sqrt{(20g)}$ (using $v^2 = u^2 + 2as$)

Just after rebound, velocity is given by $\sqrt{(2gh')} = \sqrt{(5g)}$ (Again using $v^2 = u^2 + 2as$)

Change in velocity = $\sqrt{(5g)} - (-\sqrt{(20g)}) = \sqrt{(5g)} + \sqrt{(20g)}$
Acceleration = Change in speed/Time = $(\sqrt{(5g)} + \sqrt{(20g)})/0.01$
$= 100^* (\sqrt{(5g)} + \sqrt{(20g)}) = 100^*(7 + 14) = 2100$ N

7. To calculate time take by stone to reach the well, we use second equation of motion
$78.4 = ½ gt^2 \Rightarrow t = 4$ s
Time taken by sound = 0.23 s
Speed of sound = Distance/Time = 78.4/0.23 = 340.87 m/s

8. Time taken to fall 50 m $\Rightarrow 50 = ½ gt^2$ (Using second equation of motion)
$\Rightarrow t = \sqrt{10}$ s
Velocity acquired = gt = $10\sqrt{10}$ m/s (Using first equation of motion)
Now we analyze the motion with parachute:
$v^2 = u^2 + 2as \Rightarrow 9 = 1000 - 2(2)h \Rightarrow h = 991/4 = 247.75$ m
Time taken $t = (v - u)/a = (3-10\sqrt{10})/(-2) = 14.3$ s
(a) Total time spend in air = $14.3 + \sqrt{10} = 17.47$ s
(b) Height at which he bailed out = h = 247.75 m

9. Let the height be 'h' and time after the first stone is thrown be 't'
For first stone, $h = -30t + ½ gt^2$ (Using second equation of motion)
For second stone, $h = ½ g(t-4)^2 = ½ gt^2 + 8g - 4gt$
Comparing the two equations, we get, t = 8 s
Putting it in second equation, h = 80 m

10. Acceleration due to gravity on the planet = 19.6 m/s^2 = 2g
Let the safe height be 'h'
The final velocity after falling h on this planet should be same as that on Earth after falling 2m
Using third equation, $v^2 = u^2 + 2as$ and putting u = 0
2 as should be equal on both planets i.e. $2g(2) = 2(2g)h \Rightarrow h = 1$ m

11. Let height be 'h' and time taken by first stone be 't'
Using second equation, $h = ½ gt^2$
For second stone, $h - 20 = ½ g(t-1)^2 \Rightarrow h = 20 + ½ gt^2 + g/2 - gt = 25 + ½ gt^2 - 10t$
Comparing the two equations, we get t = 2.5 s
Putting value of t in equation 1, we get h = 31.25 m

12. Initial speed, $u = \sqrt{2gh} = 20\sqrt{5}$ m/s (Using third equation of motion)
Let the stones meet after time 't' at distance 'h' from ground
For first stone, $h = 20\sqrt{5}t - ½ gt^2$ (Using second equation of motion)
For first stone, $h = 20\sqrt{5}(t-2) - ½ g(t-2)^2 = 20\sqrt{5}t - 40\sqrt{5} - ½ gt^2 + 2gt - 2g$
Comparing the two equations, we get $t = 1 + 2\sqrt{5}$ s
Putting it in equation 1, we get h = 95 m

13. For first ball, $200 = -10t + ½ gt^2$ (Using second equation of motion)
$t^2 - 2t - 40 = 0 \Rightarrow t = 1 + \sqrt{41}$
For second ball, $200 = 10t + ½ gt^2$ (
$t^2 + 2t - 40 = 0 \Rightarrow t = -1 + \sqrt{41}$
Time difference = 2 sec

14. $50 = -15t + ½ gt^2$ (Using second equation of motion)
$t^2 - 3t - 10 = 0 \Rightarrow t = 5$ sec

15. (d)
Velocity is given by slope ($\tan\theta$) of s-t graph

V_A = tan30 and V_B = tan60
V_A/V_B = tan30/tan60 = 1/3

16. Let acceleration be ‘a’, constant speed be ‘v’ and retardation be ‘-b’
v = u + at ⇒> v = 100a
Distance travelled while accelerating, $s_1 = ½\ a(100)^2$ (Using second equation of motion)
Distance travelled at constant speed s_2 = vt = 100a*300
After deaccelerating, 0 = 100a – b*150 (Using first equation of motion)
⇒ b = 2a/3
Distance travelled while deaccelerating, $s_3 = 100a*150 - ½\ b(150)^2$
$s_3 = 100a*150 - ½*(2a/3)*(150)^2 = 50a*150$
It is given that $s_1 + s_2 + s_3 = 4250$
5000a + 30000a + 7500a = 4250 ⇒ a = 0.1 m/s^2
(a) Constant speed, v = 100a = 10 m/s
(b) Acceleration, a = 0.1 m/s^2
(c) Deacceleration, b = 2a/3 = 0.066 m/s^2

17. V = 72 km/h = 20 m/s
Acceleration, a = v/t = 20/30 = 0.66 m/s^2 (Using first equation of motion)
Distance travelled while accelerating, $s_1 = ½\ a(30)^2$ (Using second equation of motion)
⇒ s_1 = 300 m
We know that, $v^2 = u^2 + 2as$
After retardation, v = 0, u = 20 m/s and s = 50 m
Thus, retardation, a = -4 m/s^2
After deaccelerating, 0 = 20 – 4*t (Using first equation of motion)
Time taken in retarded motion, t = 5 s
Distance travelled at constant speed = 950 - s_1 – 50 = 600 m
Time duration while speed is constant = 600/20 = 30s
Total time taken = 30 + 5 + 30 = 65 s

18. Let maximum speed be ‘v’
v = $6t_1$ (Using first equation of motion)
After deaccelerating, $0 = 6t_1 – 4*t_2$ (Again using first equation of motion) ⇒ $t_2 = 1.5t_1$
It’s given that $t_1 + t_2 = 6$ ⇒ t_1 = 2.4 s
Thus, v = 6*2.4 = 14.4 m/s
While accelerating, $s_1 = v^2/2a = v^2/2(6)$ = 17.28 m (Using third equation of motion)
Similarly, while deaccelerating, $s_2 = v^2/2(4)$ = 25.92 m
Net displacement = 17.28 + 25.92 = 43.2 m

19. Let the time of acceleration and retardation be t_1 and t_2 respectively
Maximum speed (v) = $u + at_1 = \alpha t_1$
For the car to stop, final velocity should be zero.
⇒ $v = u + at_2 = \alpha t_1 - \beta t_2 = 0$ => $t_2 = \alpha t_1/\beta$
Also, $t_1 + t_2 = t$ => $t_1 + \alpha t_1/\beta = t$
⇒ $t_1 = t\beta/(\alpha+\beta)$
⇒ Maximum speed, $v = \alpha t_1 = \alpha\beta t/(\alpha+\beta)$
Displacement while accelerating, $s_1 = v^2/2\alpha = \alpha\beta^2t^2/2(\alpha+\beta)^2$ (Using third equation of motion)
Similarly, while deaccelerating, $s_2 = v^2/2\beta = \alpha^2\beta t^2/2(\alpha+\beta)^2$
Total displacement = $s_1 + s_2 = \alpha\beta t^2/2(\alpha+\beta)$

20. (a) T = 2*39.2*sin60/9.8 = 6.93 s
$H = (39.2)^2*\sin^2 60/2g$ = 58.8 m

$R = (39.2)^2 * \sin120/g = 135.79$ m

(b) Max Range = u^2/g ($\theta = 45$)

Height = $u^2 * \sin^2 45/2g = u^2/4g = R/4$

(c) $v^2 * \sin(2\theta)/g = 2 * v^2 * \sin^2\theta/2g \Rightarrow \sin(2\theta) = \sin^2\theta \Rightarrow \tan\theta = 2$

$\Rightarrow \sin\theta = 2/\sqrt{5}$ and $\cos\theta = 1/\sqrt{5}$

$\Rightarrow \sin(2\theta) = 4/5$

$\Rightarrow R = 4u^2/5g$

(d) $u^2 * \sin(2\theta)/g = u^2 * \sin^2\theta/2g \Rightarrow \sin(2\theta) = \sin^2\theta/2 \Rightarrow \tan\theta = 4 \Rightarrow \theta = \tan^{-}(4)$

(e) H = R/4 = 100 m

(f) Time of flight is doubled $\Rightarrow$ u is doubled

Range is proportional to $u^2 \Rightarrow$ Range becomes four times

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