X- Progression

Exercise Detailed Solutions

Exercise-1

$Sol1.$ (i) $a=1$ $d=3$

$$General\ term = t_n = a+(n-1)d = 1+(n-1)3 = 3n-3+1$$

$$t_n = 3n-2$$

(ii) $Sequence: a,\ a+d,\ a+2d, a+3d, \ldots\ldots\ldots\ldots$

$1,\ 4,\ 7,\ 10,\ \ldots\ldots\ldots\ldots$

(iii) $t_{12} = a+11d = 1+11\times 3 = 1+33 = 34$

$$(iv)\ s_n = \frac{n[2a+(n-1)d]}{2} = \frac{n[2\times 1+(n-1)3]}{2} = \frac{n[3n-1]}{2}$$

$$s_{50} = \frac{50[3\times 50-1]}{2} = 25\times 149 = 3725$$

$Sol2.$ (i) $a=1$ $r=1.2,$ $general\ term = t_n = a\times r^{n-1} = 1\times 1.2^{n-1} = 1.2^{n-1}$

(ii) $Sequence\ 1,\ 1.2, 1.44,\ 1.728, \ldots\ldots\ldots\ldots$

(iii) $t_{12} = a\times r^{11} = 1\times 1.2^{11} = 7.43$

$$(iv)\ S_n = \frac{a(r^n-1)}{r-1} = \frac{1(1.2^n-1)}{1.2-1} = 5(1.2^n-1)$$

$$S_{10} = 5\left(1.2^{10}-1\right) = 5\times 6.192 = 30.96$$

$Sol3.$ $Let\ nth\ term\ be\ -108.$ $t_n = -108,$ $a+(n-1)d = -108$

$100+(n-1)\times(-16) = -108$

$16(n-1) = 208,$ $4n-4 = 52,$ $n = 14$ $Hence\ 14th\ term\ will\ -108.$

$Sol4.$ $Let\ nth\ term\ be\ 64.$ $t_n = 64,$ $a\times r^{n-1} = 64,$ $2\times\left(\sqrt{2}\right)^{n-1} = 64,$ $2^{\frac{n+1}{2}} = 2^6,$ $\frac{n+1}{2} = 6$

$n+1 = 12,$ $n = 11,$ $Hence\ 11th\ term\ would\ be\ 64.$

$$Sum = S_{11} = \frac{2\left[\left(\sqrt{2}\right)^{11}-1\right]}{\sqrt{2}-1} = 2\left[(2)^{11/2}-1\right]\times\left[\sqrt{2}+1\right] = 2^7 + 2^{13/2} - 2^{3/2} - 2$$

$Sol5.$ $Let\ n\ terms\ are\ needed.\ So\ S_n = 120,$ $\frac{a(r^n-1)}{r-1} = 120,$ $\frac{3\times(3^n-1)}{3-1} = 120$

$3^{n+1}-3 = 240,$ $3^{n+1} = 243 = 3^5,$ $n = 4.$ $Hence\ sum\ of\ first\ 4\ terms\ be\ 120.$

$Last\ term = 4th\ term = a\times r^3 = 3\times 3^3 = 81$

$Sol6.\ s_3 = 27$, *Let the terms are* $a-d,\ a,\ a+d.\quad a-d+a+a+d=27,\ 3a=27,\ a=9$

$$(9-d)^2+9^2+(9+d)^2=275,$$
$$2d^2+243=275,\ d=\pm 4$$
terms are $9-4, 9, 9+4$ *that is* $5, 9, 13$

$Sol7.\ (a-b),\ (b-c)\ \&\ (c-a)$ *are in AP*

$$(b-c)-(a-b)=(c-a)-(b-c)$$
$$b-c-a+b=c-a-b+c$$
$$2b+b=2c-a+c+a$$
$$3b=3c$$
$$b=c\quad proved$$

$Sol8.$ *Sequence would be* $13, 17, 21, \ldots\ldots\ldots\ldots 97,\ 97=13+(n-1)4, n=22$

$$S_{22}=\frac{22[13+97]}{2}=11\times 110=1210$$

$Sol9.\ \dfrac{a+b}{2}=5,\ ab=24,\ a+b=10,\ a+\dfrac{24}{a}=10,\ a^2-10a+24=0$

$(a-6)(a-4)=0,\ a=4$ *or* $a=6$ *numbers are* $4\ \&\ 6$

$Sol10.$ *Same as question 1 solution*

$Sol11.$ *Sequence* $10, 17, 24, \ldots\ldots\ldots\ldots$

$(i)\ a=10,\ d=7$

(ii) *General term* $=t_n=a+(n-1)d=10+(n-1)7=7n+3$

$$t_8=7\times 8+3=59$$

$$(iii)\ S_n=\frac{n(2a+(n-1)d)}{2}=\frac{n(2\times 10+(n-1)\times 7)}{2}=\frac{n(7n+13)}{2}$$

$$S_{10}=\frac{10(7\times 10+13)}{2}=5\times 83=415$$

Sol12 Same as question 3

$Sol13.\ a=1\ d=4,\quad$ *let* $t_n=10091,\ a+(n-1)\times d=10091, 1+(n-1)4=10091$

$$(n-1)\times 4=10090$$
$$n-1=2522.5$$
$n=2523.5$, n *must be an integer hence* 10091 *is not the term of sequence*

$Sol14.$ *Let n terms be taken.* $S_n=0,\ \dfrac{n(2a+(n-1)d)}{2}=0,\ n(2\times 21+(n-1)(-3))=0$

$$3(n-1)=42$$
$$n-1=14$$
$n=15$ *Hence 15 terms.*

Let $t_n=0,\ a+(n-1)d=0,\ 21+(n-1)(-3)=0,\ n=8$ *Hence 8th term will be zero*

$Sol15.\ k+2,\ 4k-6\ \&\ 3k-2\ are\ in\ AP$

$$(4k-6)-(k+2)=(3k-2)-(4k-6)$$
$$3k-8=-k+4$$
$$4k=12$$
$$k=3$$

$Sol16.\ a=-2,\ t_{10}=a+9d=16,\ -2+9d=16,\ d=2,\ t_{15}=a+14d=-2+14\times 2=26$

$Sol17.\ -5,\ A_1,A_2,A_3,A_4,A_5,A_6,A_7,A_8,\ 13$

$$t_{10}=13=a+9d=-5+9d,\ d=2$$
$$A_1=t_2=a+d=-5+2=-3$$
$$A_2=-1,\ A_3=1,\ A_4=3, A_5=5,\ A_6=7,\ A_7=9,\ A_8=11$$

Exercise-2

LEVEL-1

$sol1(b)$ $\quad 2+5+8+\ldots\ldots+\ n\ terms\ =60100$

$$\frac{n[2\times 2+(n-1)\times 3]}{2}=60100$$
$$n[4+3n-3]=120200$$
$$n[3n+1]=120200$$

$Use\ options\ n=200\ satisfies$

$sol2.(a)\quad 1+3+5+\ldots\ldots\ldots+\ n\ terms$

$$\frac{n(2\times 1+(n-1)2)}{2}=\frac{n(2\times 1+2n-2)}{2}=\frac{n(2n)}{2}$$

n^2

$sol\ 3(d)$ $\quad t_5=ar^4=2$

$$t_1\times t_2\times t_3\times t_4\times t_5\times\times t_6\times t_7\times t_8\times t_9$$
$$a\times ar^1\times ar^2\times ar^3\times ar^4\times ar^5\times ar^6\times ar^7\times ar^8$$
$$a^9r^{36}=\left(ar^4\right)^9=2^9$$

$Sol4.(d)$ $\quad t_3=a+2d=12,\ t_7=a+6d=24$

$Solving\ these\ two\ d=3\ \&\ a=6$

$$t_{10}=a+9d=6+9\times 3=6+27=33$$

$Sol5.(a)$ $\quad t_4=a+3d=2$

$$t_1+t_2+t_3+t_4+t_5+t_6+t_7$$
$$a+a+d+a+2d+a+3d+a+4d+a+5d+a+6d$$
$$7a+21d=7(a+3d)=7\times 2=14$$

$sol6.(d)\quad t_2 = ar = 2,\ r = \frac{2}{a}$

$$\frac{a}{1-r} = 8$$

$$\frac{a}{1-\frac{2}{a}} = 8,\ \frac{a^2}{a-2} = 8,\ a^2 - 8a + 16 = 0, a = 4$$

$Sol7.(c)\ 1, G_1, G_2, 64$

property of GP product of terms equidistant from start & last are same

$1 \times 64 = G_1 \times G_2$ *use options*

$Sol8.(d)\ 3 + 7 + 11 + 15 + \ldots\ldots + n \text{ terms } = 406$

$$\frac{n\left[2\times 3 + (n-1)\times 4\right]}{2} = 406$$

$$n\left[6 + 4n - 4\right] = 812$$

$$n\left[4n+2\right] = 812,\ 2n\left[2n+1\right] = 812,\ n\left[2n+1\right] = 406,\ n = 14 \text{ satisfies}$$

$Sol9.(c)\ t_n = 2n + 5,\ t_3 = 2\times 3 + 5 = 6 + 5 = 11$

$Sol10.(b)\ t_{n+1} = a + (n + 1 - 1)d = a + nd$

$Sol11.(c)\ t_n = 3n + 8,\ t_{n-1} = 3(n-1) + 8 = 3n + 5$

$Sol12.(b)\ GM = (5\times 10\times 20)^{1/3} = \left(10^3\right)^{1/3} = 10$

$Sol13.(d)\ \frac{1}{100}, \frac{1}{10000}, \frac{1}{1000000}, \ldots\ldots\ldots\ldots$

$$a = \frac{1}{100},\ r = \frac{1}{100},\ nth\ term = t_n = a\, r^{n-1} = 10^{-2} \times \left(10^{-2}\right)^{n-1} = 10^{-2+2-2n}$$

$$t_n = 10^{-2n} = 100^{-n}$$

$Sol14.(a)\ \text{we know } s_n = \frac{a(r^n - 1)}{r-1} = a\,(r+1)$

$$r^n - 1 = r^2 - 1$$

$$n = 2$$

$Sol15.(a)\ t_7 = a\, r^6 = x,\ t_9 = a\, r^8 = y$

$$\frac{t_9}{t_7} = \frac{a\, r^8}{a\, r^6} = \frac{y}{x}$$

$$r^2 = \frac{y}{x}$$

$$r = \pm\sqrt{\frac{y}{x}}$$

$Sol16.(c)$ $t_p = a+(p-1)d = x,\ t_q = a+(q-1)d = y.\ t_r = a+(r-1)d = z$

$t_p = bR^{p-1} = x,\ t_q = bR^{q-1} = y,\ t_r = bR^{r-1} = z$

$x^{y-z} \times y^{z-x} \times z^{x-y}$

$\left[bR^{p-1}\right]^{y-z} \times \left[bR^{q-1}\right]^{z-x} \times \left[bR^{r-1}\right]^{x-y}$

$b^{y-z+z-x+x-y} \times (R)^{(p-1)(y-z)+(q-1)(z-x)+(r-1)(x-y)}$

$b^0 \times (R)^{(p-1)(r-q)d+(q-1)(r-p)+(r-1)(p-q)} = 1$

$Sol17.(c)$ $q-p = r-q,\ y^2 = xz$

$x^{q-r} \times y^{r-p} \times z^{p-q}$

$x^{q-r} \times y^{r-p} \times z^{q-r}$

$(xz)^{q-r} \times y^{r-p} = \left(y^2\right)^{q-r} \times y^{r-p} = y^{2q-2r+r-p} = y^{q-p+q-r} = y^{r-q+q-r}$

$y^0 = 1$

$Sol18.(c)$ *Let the roots are a and b then* $\dfrac{a+b}{2} = 8,\ (ab)^{1/2} = 5,\ ab = 25$

Required equation, $x^2 - sum \times x + Product = 0$

$x^2 - 16x + 25 = 0$

$Sol19.(c)$ $b-a = c-b = d-c = e-d = f-e$

$e-c = 2d-c-c = 2(d-c)$

$Sol20.(a)$ $3,\ 10,\ 17, \ldots\ldots\ldots\ t_n = 3+(n-1)7 = 7n-4$

$63,\ 65,\ 67,\ \ldots\ldots\ldots t_n = 63+(n-1)2 = 2n+61$

$7n-4 = 2n+61$

$5n = 65,\ n = 13$

LEVEL-2

$Sol1.(c)$ $t_m = \dfrac{1}{n},\ t_n = \dfrac{1}{m},\ a+(m-1)d = \dfrac{1}{n}$

$a+(n-1)d = \dfrac{1}{m},\ d = \dfrac{1}{mn},\ a + \dfrac{(m-1)}{mn} = \dfrac{1}{n}$

$a = \dfrac{m-(m-1)}{mn} = \dfrac{1}{mn}$

$t_{mn} = a+(mn-1)d = \dfrac{1+mn-1}{mn} = 1$

$Sol2.(c)$ $a = 1\ ,\ r = y,\ t_\infty = \dfrac{a}{1-r} = \dfrac{1}{1-y} = x,\ 1-y = \dfrac{1}{x},\ y = \dfrac{x-1}{x}$

$Sol3.(b)$ $\quad Log_3 2, Log_3\ (2^x - 5)\ \&\ Log_3\left(2^x - \frac{7}{2}\right)$ *are in AP*

$$2\times Log_3\ (2^x-5) = Log_3 2 + Log_3\left(2^x - \frac{7}{2}\right)$$

$$(2^x-5)^2 = 2\times\left(2^x-\frac{7}{2}\right),\ (2^x-5)^2 = 2^x\times 2 - 7,$$

use options at $x=2$ log *of negative not defined so rejected. So* $x=3$

$Sol4.(c)$ $\quad 2,5,8,\ldots\ldots\ldots\ldots\ldots\ S_{2n} = \dfrac{2n(2a+(2n-1)d)}{2} = n(4+(2n-1)3) = n(6n+1)$

$$57,59,61,\ldots\ldots\ldots\ldots S_n = \frac{n(2a+(n-1)d)}{2} = \frac{n(114+(n-1)2)}{2} = n(n+56)$$

$$S_{2n} = S_n,\ n(6n+1) = n(n+56),\ n=11$$

$Sol5.(c)$ $\quad t_p = a+(p-1)d = q$

$$t_q = a+(q-1)d = p,\ d=-1\ \&\ a=p+q-1$$

$$t_r = a+(r-1)d = p+q-1-r+1 = p+q-r$$

$Sol6.(a)$ *Let the GP be* $a, ar^1, ar^2, ar^3, ar^4, ar^5, ar^6, \ldots\ldots\ldots\ldots$

Odd term GP $\quad a, ar^2, ar^4, ar^6, \ldots\ldots\ldots\ S_{odd} = \dfrac{a(r^{2n}-1)}{r^2-1}$

Even term GP $\quad ar, ar^3, ar^5, ar^7, \ldots\ldots\ldots\ S_{even} = \dfrac{ar(r^{2n}-1)}{r^2-1}$

$$\frac{S_{odd}}{S_{even}} = \frac{1}{r}$$

$Sol7.(b)$ $\quad \dfrac{1}{c+a} - \dfrac{1}{b+c} = \dfrac{1}{a+b} - \dfrac{1}{c+a}$

$\dfrac{b-a}{b+c} = \dfrac{c-b}{a+b},\ b^2-a^2 = c^2-b^2,\ b^2 = \dfrac{a^2+c^2}{2}$, *Hence* $a^2, b^2\ \&\ c^2$ *are in AP*

$Sol8.(d)$ $\quad a=-5,\ d=7,\ t_n = a+(n-1)d = -5+(n-1)7 = 7n-12$, n *must be Integer*

use options $7n-12=345,\ n=51$

$Sol9.(c)$ *Let the intergers are* $a-2d,\ a-d, a, a+d\ \&\ a+2d$ *be 5 distinct positive integers*

$$a-2d+a-d+a+a+d+a+2d = 10020,\ 5a=10020,\ a=2004$$

since *all terms are positive integers* & d *positive. So* $d=1$, *last term* $=2006$

$Sol10.(c)$ *Let the side be* $a-d, a\ \&\ a+d$.

$$(a+d)^2 = a^2+(a-d)^2,\ 4ad = a^2,\ a=4d,$$

$3d, 4d\ \&\ 5d$. *Smallest* $=3d=$ *multiple of* 3 *use options*

$Sol 11.(c)$ $3, 7, 11, 15, \ldots\ldots\ldots\ldots\ldots\ldots\ t_{125} = a + (n-1)d = 3 + 124 \times 4 = 499$

$4, 7, 10, 13, \ldots\ldots\ldots\ldots\ldots\ldots t_{125} = a + 124d = 4 + 124 \times 3 = 376$

$7, 19, 31, \ldots\ldots\ldots\ldots t_n < 376$

$7 + (n-1)12 < 376,\ n-1 < 30.75,\ n < 31.75,\ n \text{ is an interger},\ n = 31$

$Sol\ 12.(d)$ $204,\ 210, \ldots\ldots\ldots\ldots\ldots\ldots\ldots, 1098.$

$$t_n = a + (n-1)d = 1098$$

$$204 + (n-1)6 = 1098,\ n = 150$$

$$S_n = \frac{150}{2}(204 + 1098) = 75 \times 1302 = 97650$$

$Sol 13.(d)$ $$8^{\frac{1}{1-\sin x}} = 8^2,\ \frac{1}{1-\sin x} = 2,\ 1 - \sin x = \frac{1}{2},\ \sin x = \frac{1}{2},\ x = 30$$

$Sol 14(d)$ $$\sum_{n=1}^{\infty} 1 + \left(\frac{3}{5}\right)^n = 1 + \frac{3/5}{1 - \frac{3}{5}} = 1 + \frac{3}{2} = \frac{5}{2}$$

Exercise-3

Level-1

Sol.1. $T_2 = ar = 18$ $\quad T_6 = ar^5 = 1458$

$$\frac{T_6}{T_2} = \frac{ar^5}{ar} = \frac{1458}{18} = \frac{486}{6} = 81$$

i) ar= 18

a = 18

ii) $r^4 = 3^4$

r = 3

iii) a, ar, ar^2, …..

6, 18, 54, …..

Sol.2 a, b, c $\rightarrow$ A.P.

$$\frac{a}{abc}, \frac{b}{abc}, \frac{c}{abc} \rightarrow \text{A.P.}$$

$$\frac{1}{ba}, \frac{1}{ac}, \frac{1}{ab} \rightarrow \text{A.P.}$$

Sol.3 $$\frac{1}{a}, \frac{1}{b}, \frac{1}{c} \rightarrow \text{A.P.}$$

$$\frac{a+b+c}{a}, \frac{a+b+c}{b}, \frac{a+b+c}{c} \rightarrow \text{A.P.}$$

$$\frac{1+b+c}{a}, \frac{1+a+c}{b}, \frac{1+a+b}{c} \rightarrow \text{A.P.}$$

$$\frac{b+c}{a}, \frac{a+c}{b}, \frac{a+b}{c} \rightarrow \text{A.P.}$$

Sol.4 a, b, c $\rightarrow$ G. P.

$a^2, b^2, c^2 \rightarrow$ G. P.

$$\frac{1}{a^2}, \frac{1}{b^2}, \frac{1}{c^2} \rightarrow \text{G. P.}$$

Sol.5

$a^2, b^2, c^2 \rightarrow$ G.P.

$a, b, c \rightarrow$ G.P.

$$\frac{a}{abc}, \frac{b}{abc}, \frac{c}{abc} \rightarrow \text{G.P.}$$

$$\frac{1}{bc}, \frac{1}{ac}, \frac{1}{ab} \rightarrow \text{G.P.}$$

$bc, ac, ab \rightarrow$ G.P.

Sol.6

Let the number are $\frac{a}{r}, a, ar$

$a^3 = 216$

$a = 6$

$\frac{6}{r}, 6, 6r$

$$\frac{6}{r} + 6r = 20$$

$$6r^2 - 20r + 6 = 0$$

$$6r^2 - 18r - 2r + 6 = 0$$

$$6r(r-3) - 2(r-3) = 0$$

$r = 3, \quad r = \frac{1}{3}$

$\frac{6}{r}, 6, 6r$

$$\frac{36}{r^2} + 36 + 36r^2 = 156$$

$$\frac{36}{r^2} + 36r^2 = 120$$

$\frac{6}{3}, 6, 6 \times 3$

$\boxed{2, 6, 18}$ Ans

Sol.7

$a-3d,\ a-d,\ a+d,\ a+3d$

$4a=20\ ;\ a=5$

$5-3d,\ 5-d,\ 5+d,\ 5+3d$

$(5-3d)^2+(5-d)^2+(5+d)^2+(5+3d)^2=120$

$2(25+9d^2)+2(25+d^2)=120$

$20d^2=20$

$d^2=1$

$d=\pm 1$

$5-3,\ 5-1,\ 5+1,\ 5+3$

$\boxed{2,4,6,8}$ Ans.

Sol.8

$b-a=c-b \Rightarrow 2b=a+c$

L.H.S. $\frac{1}{a-b}+\frac{1}{b-a}+\frac{1}{a+c}$

$=-\frac{1}{b-a}+\frac{1}{b-a}+\frac{1}{a+c}$

$\frac{1}{a+c}=\boxed{\frac{1}{2b}}=$ R.H.S.

Proved.

Sol.9

$16,\ 12,\ 9,\ ---$

$a=16 \quad r=\frac{12}{16}=\frac{3}{4}$

$S_\infty=\frac{a}{1-r}=\frac{16}{1-\frac{3}{4}}=\frac{16}{1/4}=\boxed{64\text{ cm}}$

Ans

Sol.10

$$7t_7 = 11t_{11} \qquad t_{18} = 0$$

$$7(a+6d) = 11(a+10d)$$

$$7a + 42d = 11a + 110d$$

$$-4a = 68d$$

$$a = -17d$$

$$a + 17d = 0$$

$$t_{18} = 0 \quad \text{Proved.}$$

Sol.11

$$t_n = (n-1)(n-2)(n-3)$$

$$t_1 = (1-1)(1-2)(1-3) = 0\times(-1)\times(-2) = 0$$

$$t_2 = 0$$

$$t_3 = 0$$

$$t_4 = 3\times2\times1 = 6$$

Sol.12

$$t_1 = 1-8+13 = 14-8 = 6 \quad \text{Yes}$$

Sol.13

$$\frac{n(n+1)}{2} = 66$$

$$n^2+n-132 = 0$$

$$n^2+12n-11n-132 = 0$$

$$n(n+12)-11(n+12) = 0$$

$$n = -12 \quad \boxed{n = 11} \text{ Ans.}$$

Sol.14

$S_n = 3n^2 + 5n \qquad T_1 = S_1 = 3+5 = 8$

$T_n = S_{n+1} - S_n$

$= 3[(n+1)^2 - n^2] + 5[n+1-n]$

$= 3[2n+1] + 5 = \boxed{6n+8}$ Ans

$T_2 = 6+8 = 14$

$d = 14-8 = \boxed{6}$ Ans

Sol.15 Same as question 7

Sol.16 Same as question 2

Sol.17

$(a-d)^2 + a^2 + (a+d)^2 = 275$

$2d^2 + 243 = 275$

$2d^2 = 32$

$d^2 = 16$

$d = \pm 4$

$3a = 27$

$a = 9$

$\boxed{5, 9, 13}$ Ans

Sol.18 Same as question 8

Sol.19

Ans. $= \dfrac{\frac{x+a}{x} + \frac{x-a}{x}}{2} = \dfrac{2}{2} = \boxed{1}$

Sol.20

$a+d+a+4d-a-2d = 10 \qquad a+d+a+8d = 17$

$a+3d = 10 \quad - (1)$

$2a+9d = 17$

$3a+9d = 30$

$-a = -13$

$\boxed{a = 13}$

$13+3d = 10$

$\boxed{d = -1}$ Ans

Sol.21

$S_{10} = S_1 = ar^{9}$

$S_{20} = S_1 + S_2 = ar^{19}$

$\frac{S_1+S_2}{S_1} = \frac{ar^{19}}{ar^{9}} = r^{10}$

$\boxed{r = \left(1+\frac{S_2}{S_1}\right)^{1/10}}$

Sol.22

$y^2 = xz$ given

L.H.S. $(x+2y+2z)(x+2z-2y)$

$(x+2z)^2 - 4y^2$

$x^2+4z^2+4xz-4xz$

x^2+4z^2

$=$ R.H.S. Proved.

Sol.23

$(b-c)^2 = (a-b)(c-a) = ac-a^2-bc+ab$

$b^2+c^2-2bc = ac-a^2-bc+ab$

$a^2+b^2+c^2 = bc+ab+ac$

$(a+b+c)^2 = a^2+b^2+c^2+2(ab+bc+ca)$

$= bc+ab+ac+2(ab+bc+ca)$

$\boxed{= 3(ab+bc+ca)}$ Ans.

Sol.24

$\frac{a}{r}, a, ar$ $\qquad \frac{a}{r}+a+ar=70$

$\frac{4a}{r}, 5a, 4ar$ $\qquad 10a = 4ar + \frac{4a}{r}$

$10 = 4r + \frac{4}{r}$

$4r^2 - 10r + 4 = 0$

$4r^2 - 8r - 2r + 4 = 0$

$4r(r-2) - 2(r-2) = 0$

$(4r-2)(r-2) = 0$

$r = \frac{1}{2}$ or $r = 2$

$\frac{a}{2} + a + 2a = 70$

$\frac{a}{2} + 3a = 70$

$\frac{7a}{2} = 70$

$a = 20$

$\frac{20}{2}, 20, 20 \times 2$

$\boxed{10, 20, 40}$ Ans.

Sol.25

$\frac{a}{r}, a, ar$

$\frac{a}{r}, 2a, ar$

$4a = \frac{a}{r} + ar$

$4 = \frac{1}{r} + r$

$r^2 - 4r + 1 = 0$

$r = \frac{4 \pm \sqrt{16-4}}{2} = \frac{4 \pm 2\sqrt{3}}{2}$

$r = 2 \pm \sqrt{3}$

$\boxed{r = 2 + \sqrt{3}}$ Ans.

Sol.26 Same as question 3

Sol.27

$\frac{S_n}{S'_n} = \frac{3n+8}{7n+15}$

$\frac{\frac{n}{2}[2a + (n-1)d]}{\frac{n}{2}[2b + (n-1)d']} = \frac{3n+8}{7n+15}$

$\frac{a + \left(\frac{n-1}{2}\right)d}{b + \left(\frac{n-1}{2}\right)d'} = \frac{3n+8}{7n+15}$

$\frac{n-1}{2} = 11$

$n = 23$

$\frac{a + 11d}{b + 11d'} = \frac{3 \times 23 + 8}{7 \times 23 + 15} = \frac{69+8}{161+15} = \boxed{\frac{77}{176}}$ Ans

Sol.28

$T_p = x + (p-1)d = a$ — (1) $x \rightarrow$ 1st term

$T_q = x + (q-1)d = b$ — (2)

$T_r = x + (r-1)d = c$ — (3)

(2) − (3)

$(q-r)d = b - c$

$(q-r) = \frac{b-c}{d}$

(3) − (1)

$(r-p)d = c - a$

$r - p = \frac{c-a}{d}$

(1) − (2)

$(p-q)d = a-b$

$p-q = \frac{a-b}{d}$

$(q-r)a + (r-p)b + (p-q)c$

$= \frac{ab-ac+bc-ab+ac-bc}{d} = 0$

Proved

Sol.29

$\frac{p}{2}[2a+(p-1)d] = \frac{q}{2}[2a+(q-1)d]$

$2pa - 2qa + [p(p-1) - q(q-1)]d = 0$

$2a(p-q) + (p^2-q^2-(p-q))d = 0$

$(p-q)[2a+(p+q-1)d] = 0$

$S_{p+q} = \frac{p+q}{2}[2a+(p+q-1)d]$

$= \frac{p+q}{2} \times 0 = \boxed{0}$ Ans

Level-2

Sol1.(d) $t_n = \frac{1}{\sqrt{2n-1}+\sqrt{2n+1}} = \frac{\sqrt{2n+1}-\sqrt{2n-1}}{2}$

$$S_n = \frac{\left(\sqrt{2n+1}-\sqrt{2\times 1-1}\right)}{2} = \frac{\left(\sqrt{2n+1}-1\right)}{2}$$

Sol.2 (a)

$$\frac{1}{a_1a_2}+\frac{1}{a_2a_3}+.......+\frac{1}{a_na_{n+1}}$$

$a_2 - a_1 = a_3 - a_2 = \ldots\ldots = a_{n+1} - a_n = d$

$$\frac{1}{d}\left(\frac{1}{a_1}-\frac{1}{a_2}\right)+\frac{1}{d}\left(\frac{1}{a_2}-\frac{1}{a_3}\right)+.........+\frac{1}{d}\left(\frac{1}{a_n}-\frac{1}{a_{n+1}}\right)$$

$$=\frac{1}{d}\left(\frac{1}{a_1}-\frac{1}{a_2}+\frac{1}{a_2}-\frac{1}{a_3}+.........+\frac{1}{a_n}-\frac{1}{a_{n+1}}\right)$$

$$=\frac{1}{d}\left(\frac{1}{a_1}-\frac{1}{a_{n+1}}\right)=\frac{a_{n+1}-a_1}{d\left(a_1a_{n+1}\right)}=\frac{nd}{d\,a_1a_{n+1}}=\frac{n}{a_1a_{n+1}}$$

$t_a = a_{n+1} = a_1 + (n+1-1)d$

$a_{n+1} - a_1 = nd$

Sol. 3. (b)

$$1^2 = \left(\sqrt{2}+1\right)\left(\sqrt{2}-1\right) = 2-1 = 1$$

G.P.

Sol.4. (c)

$$1+\left(1+x\right)+\left(1+x+x^2\right)+\left(1+x+x^2+x^3\right)+......$$

$$1+\frac{1-x^2}{1-x}+\frac{1-x^3}{1-x}+\frac{1-x^4}{1-x}+\frac{1-x^5}{1-x}+........$$

$$\frac{1}{1-x}\left((1-x)+(1-x^2)+(1-x^3)+(1-x^4)+........\right)$$

$$\frac{1}{1-x}\left(n-\frac{x\left(x^n-1\right)}{x-1}\right)$$

$$\frac{1}{1-x}\left(n-\frac{x\left(1-x^n\right)}{1-x}\right)$$

$$\frac{n\left(1-x\right)-x\left(1-x^n\right)}{\left(1-x\right)^2}$$

Sol.5 (d) 3, 7, 11, ….

$T_n = 3 + (n - 1)\, 4 - 4n - 1$

7, 11, 15, ….

$T_n = 7 + (n - 1)\, 4 = 4n + 3$

$$T_n = \frac{1}{(4n-1)(4n+3)} = \left(\frac{1}{4n-1} - \frac{1}{4n+3}\right)\frac{1}{4}$$

$$S_n = \left(\frac{1}{4-1} - \frac{1}{4n+3}\right)\frac{1}{4} = \left(\frac{1}{3} - \frac{1}{4n+3}\right)\frac{1}{4}$$

$$S_\infty = \left(\frac{1}{3} - \frac{1}{\infty}\right)\frac{1}{4} = \boxed{\frac{1}{12}}$$ Ans.

Sol.6. (b) 1, 2, 3, …… $T_n = n$

2, 3, 4, …… $T_n = n + 1$

$$T_n = \frac{1}{n(n+1)} = \frac{1}{n} - \frac{1}{n+1}$$

$$S_n = 1 - \frac{1}{n+1}$$ $$\boxed{S_\infty = 1}$$

Sol.7. (a)

$$T_n = n.(2n+1)^2 = n(4n^2+4n+1)$$

$$T_n = 4n^3+4n^2+n$$

$$S_n = 4\sum n^3 + 4\sum n^2 + \sum n$$

$$S_n = 4.\left(\frac{n(n+1)}{2}\right)^2 + 4\,\frac{n(n+1)(2n+1)}{6} + \frac{n(n+1)}{2}$$

$$S_{20} = 4\left(\frac{20\times 21}{2}\right)^2 + \frac{4\times 20\times 21\times 41}{6} + \frac{20\times 21}{2}$$

$$= 176400 + 11480 + 210$$

$$= \boxed{88090} \text{ Ans.}$$

Sol.8. (b) $0.7 + 0.77 + 0.777 + \ldots..$

$$=\frac{7}{9}(0.9+0.99+0.999+....)$$

$$=\frac{7}{9}\left(\left(1-\frac{1}{10}\right)+\left(1-\frac{1}{100}\right)+\left(1-\frac{1}{1000}\right)+....\right)$$

$$=\frac{7}{9}\left(10-\frac{\frac{1}{10}\left(1-\left(\frac{1}{10}\right)10\right)}{1-\frac{1}{10}}\right)$$

$$=\frac{7}{81}\left(89+\frac{1}{10}10\right)$$

Sol.9. (c)

$$(1^3+3^3+\_\_+9^3)-(2^3+4^3+6^3+8^3)$$

$$(1^3+2^3+3^3+\_\_+9^3)-2.(2^3+4^3+6^3+8^3)$$

$$\left(\frac{9(9+1)}{2}\right)^2 - 2.2^3(1^3+2^3+3^3+4^3)$$

$$(9\times 5)^2 - 2.2^3\left(\frac{4(4+1)}{2}\right)^2$$

$$45^2 - 16\times 100$$

$$45^2 - 40^2$$

$$5\times 85$$

$$\boxed{425} \text{ Ans.}$$

Sol.10. (a)

$$x = \log_{0.4}\left(\frac{1/3}{1-1/3}\right) = \log_{0.4}\left(\frac{1}{2}\right) = \log_{0.4}(0.5)$$

$$= \frac{2}{2}\log_{0.4} 0.5 = \log_{(0.4)^2}(0.5)^2 = \log_{0.16} 0.25$$

$$(0.16)^x = (0.16)^{\log_{0.16} 0.25} = 0.25 = \boxed{\frac{1}{4}}$$ Ans.