

(ch-2)

9th

Ratio & Proportion.

level-1

1. $a:b = 5:4$ $b:c = 16:25$

we need b same in both ratios

$$a:b = 5 \times 4 : 4 \times 4 \\ = 20 : 16$$

So $a:b:c$ is $20:16:25$.

2. $a:b = 3:4$

$$7a-4b : 3a+b = \frac{7a-4b}{3a+b}$$

(keep $a=3x$, $b=4x$)

$$= \frac{21x - 16x}{9x + 4x} = \frac{5x}{13x} = \frac{5}{13}$$

3. $\frac{7x-4y}{3x+y} = \frac{5}{13}$

(Cross Multiply)

$$91x - 52y = 15x + 5y$$

$$76x = 57y$$

$$\frac{x}{y} = \frac{57}{76} = \frac{3}{4} \Rightarrow x:y = 3:4$$

$$4. \quad a:b = 2:3; \quad b:c = 4:5 \quad c:d = 6:7$$

as b is common the connected ratios so we need them to be same.

$$c:d = 30:35 \quad (6 \times 5 : 7 \times 5)$$

$$b:c = 24:30 \quad (4 \times 6 : 5 \times 6)$$

$$a:b = 16:24 \quad (2 \times 8 : 3 \times 8)$$

$$\text{so } a:b:c:d = 16:24:30:35$$

5. For proportionality $ad = bc$

$$(x-4)(x+10) = (x-2)(x+2)$$

$$x^2 + 6x - 40 = x^2 - 4$$

$$6x = 40 - 4 = 36$$

$$x = 6$$

6. For proportionality $ad = bc$

$$(3+x)(16+x) = (7+x)(8+x)$$

[x is number to be added]

$$x^2 + 19x + 48 = x^2 + 15x + 56$$

$$4x = 56 - 48 = 8$$

$$x = 2$$

7. a, b, c are in proportion.

$$\text{so } b^2 = ac$$

$$a^2 = b^2 = a^2 : ac = a : c$$

$$8. \quad \frac{1}{4} : \frac{1}{5} : \frac{1}{6} \quad (\text{LCM of } (4, 5, 6) = 60)$$

$$\frac{60}{4} : \frac{60}{5} : \frac{60}{6} \Rightarrow 15 : 12 : 10$$

$$\text{15's ratio part} = \frac{15}{15+12+10} \times 2220 = 900$$

$$\text{12's ratio part} = \frac{12}{15+12+10} \times 2220 = 720$$

$$\text{10's ratio part} = \frac{10}{15+12+10} \times 2220 = 600$$

$$9. \quad x, y, z \text{ are in proportion so } y^2 = xz.$$

$$\begin{aligned} (x+y+z)(x-y+z) &= (x+z)^2 - y^2 \\ &= x^2 + z^2 + 2xz - y^2 \\ &= x^2 + z^2 + 2y^2 - y^2 \\ &= x^2 + y^2 + z^2 \end{aligned}$$

$$10. \quad \begin{aligned} \text{Cost of 570 bags is Rs. 17100} \\ \text{Cost of 1 bag is } \frac{\text{Rs. } 17100}{570} = \text{Rs. } 30 \end{aligned}$$

$$\text{So for Rs. } 19200 \text{ bags bought is } = \frac{19200}{30} = 640$$

$$11. \quad \text{Let Sameer's age be } 5x \text{ \& Anand's age be } 4x$$

$$\text{ratio of age after 3 years is } \frac{5x+3}{4x+3} = \frac{11}{9}$$

$$45x + 27 = 44x + 33$$

$$x = 6$$

$$\text{Anand's age} = 4x = 24 \text{ years}$$

12.

$$A_I + B_I + C_I = 80,000$$

Income = Saving + Expenditure.

$$A_E = 8000, B_E = 12,000, C_E = 15,000$$

$$A_S = 2x, B_S = 3x, C_S = 4x$$

$$A_E + A_S = A_I, B_E + B_S = B_I, C_E + C_S = C_I$$

$$\text{So } (8000 + 2x) + (12,000 + 3x) + (15,000 + 4x) = 80,000$$

$$35,000 + 9x = 80,000$$

$$9x = 45,000$$

$$x = 5000$$

$$B_I = B_E + B_S = 12,000 + 3 \times 5000 = 27,000$$

13. $x : y = 2 : 3$ keep $x = 2K, y = 3K$

$$\frac{3x + 2y}{2x + 5y} = \frac{6K + 6K}{4K + 15K} = \frac{12K}{19K} = \frac{12}{19}$$

14. $a : b = 7 : 8$ keep $a = 7K, b = 8K$

$$(4a - \frac{b}{2}) : (4a + \frac{b}{2}) = \frac{4 \times 7K - \frac{8K}{2}}{4 \times 7K + \frac{8K}{2}} = \frac{24K}{32K} = \frac{3}{4}$$

15. $\frac{2x + 3y}{3x + 4y} = \frac{13}{18}$

$$36x + 54y = 39x + 52y \Rightarrow 3x = 2y \Rightarrow \frac{x}{y} = \frac{2}{3} \text{ or } 2:3$$

16. Let numbers be $2x$ & $3x$.

$$(3x)^2 - (2x)^2 = 320$$

$$5x^2 = 320$$

$$x^2 = 64 \Rightarrow x = 8$$

Numbers will be 16, 24.

17. Let x be the number to be added.

$$\frac{9+x}{11+x} = \frac{4}{5} \Rightarrow 45 + 5x = 44x + 44$$
$$\boxed{x = -1}$$

18. $12a^2 + 35b^2 - 43ab = 0$

• Divide by b^2

$$12\left(\frac{a}{b}\right)^2 + 35 - 43\left(\frac{a}{b}\right) = 0 \quad \text{let } \frac{a}{b} = a:b = r$$

$$12r^2 - 43r + 35 = 0$$

$$12r^2 - 75r - 28r + 35 = 0$$

$$3r(ur-5) - 7(ur-5) = 0$$

$$(3r-7)(ur-5) = 0$$

$$r = \frac{7}{3} \text{ or } \frac{5}{4}$$

19. $a:b = 6 \cdot 5 : 4 \cdot 5$, $b:c = 8 \cdot 5 : 4 \cdot 5$
 $= 13 : 9$ $= 17 : 9$

we need b value same

$$\text{So } a:b = 13 \times 17 : 9 \times 17, \quad b:c = 17 \times 9 : 9 \times 9$$
$$= 221 : 153 \quad \quad \quad = 153 : 81$$

$$a : c = 221 : 61$$

Level-2

1. Let P & Q's age 7 years ago are $7x$ & $6x$.
their age's after 6 years are $7x+13$ & $6x+13$
Check by options \rightarrow ages should be negative.

$$(a) \quad \frac{7x+13}{6x+13} = \frac{13}{11} \quad \Rightarrow \quad 1 + \frac{x}{6x+13} = 1 + \frac{2}{11}$$

$$\frac{x}{6x+13} = \frac{2}{11}$$

$$11x = 12x + 26$$

$$-x = 26 \quad \times$$

2. Let Ram & Shayam's monthly income be $3x$ & $4x$
& their expenditures be $4y$ & $5y$ respectively.

$$\text{Income} = \text{Saving} + \text{Expenditure}$$

$$\rightarrow \text{For Shayam} \quad 4x = 5y + 400$$

$$x = \frac{5y}{4} + 100$$

$$\text{For Ram} \quad 3x = 4y + \text{Saving}$$

$$3\left(\frac{5y}{4} + 100\right) = 4y + \text{Saving}$$

$$\frac{15y}{4} + 300 - 4y = \text{Saving}$$

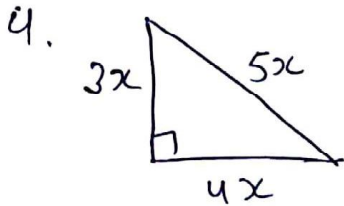
$$\text{Saving} = 300 - \frac{y}{4}$$

- 8 So Ram's saving will be less than 300. So ~~can't~~
310 (a option) can't be Ram's Saving.

3. $2x : y = 4 : 3$ let $x = 4k$ & $y = 3k$

$$(2y - x) : (2x - y) = \frac{2 \times 3k - 4k}{2 \times 4k - 3k} = \frac{2k}{5k} = \frac{2}{5}$$

triplicate of this ratio is $\left(\frac{2}{5}\right)^3 = \frac{8}{125}$



$$P = 3x + 4x + 5x = 24$$

$$12x = 24$$

$$x = 2$$

so triangle sides are 6, 8, 10.

5. Subduplicate ratio of $16a^4x^6 : 9b^6x^4$

$$\Rightarrow \sqrt{16a^4x^6} : \sqrt{9b^6x^4}$$

$$4a^2x^3 : 3b^3x^2$$

6. Triplicate ratio of $(a+b)^{7/3} : (a-b)^{7/3}$

$$\Rightarrow \left[(a+b)^{7/3}\right]^3 : \left[(a-b)^{7/3}\right]^3$$

$$\Rightarrow (a+b)^7 : (a-b)^7$$

7. Subtriplicate ratio of $464x^3 : 27y^6$

$$\Rightarrow [464x^3]^{1/3} : [27y^6]^{1/3}$$

$$\Rightarrow 4x : 3y^2$$

8. Sub-triplicate^{ratio} of subduplicate ratio of $4096x^6 : 792y^{12}$

$$\Rightarrow (4096x^6)^{1/6} : (792y^{12})^{1/6}$$

$$\Rightarrow 4x : 3y^2$$

$$9. \quad \frac{3}{x} = \frac{12}{20}$$

$$60 = 12x \Rightarrow x = \frac{60}{12} = 5$$

10. let third proportion be x

$$(a^2 - b^2) \times x = (a+b)^2$$

$$x = \frac{(a+b)^2}{(a+b)(a-b)} = \frac{a+b}{a-b}$$

11. let third proportion be x .

$$27 \times x = 9^2$$

$$x = \frac{81}{27} = 3$$

$$12. \quad \frac{y}{x-2} = \frac{y+x}{2} = \frac{x}{y} \quad \left[\text{use property } \frac{a}{b} = \frac{c}{d} = \frac{e}{f} = \frac{a+c+e}{b+d+f} \right]$$

$$\frac{y}{x-2} = \frac{y+x}{2} = \frac{x}{y} = \frac{2x+2y}{x+y} = 2$$

$$\text{So } x : y = 2 : 1 \quad [\text{let } x = 2k \quad y = k]$$

$$\frac{y+x}{2} = 2 \quad \frac{3k}{2} = 2 \quad z = \frac{3k}{2}$$

$$x : y : z = 2k : k : \frac{3k}{2}$$

$$\text{or } 4k : 2k : 3k$$

$$4 : 2 : 3$$

$$13. \quad 2x^2 - y^2 = \frac{7}{15}xy$$

$$\text{So } 2x^2 - 15y^2 = 7xy \quad [\text{divide by } y^2]$$

$$\text{So } \left(\frac{x}{y}\right)^2 - 15 - 7\left(\frac{x}{y}\right) = 0$$

$$\text{let } \frac{x}{y} = r$$

$$30x^2 - 7y - 15 = 0$$

$$30y^2 - 25x + 18y - 15 = 0$$

$$5x(6x-5) + 3(6x-5) = 0$$

$$(6x-5)(5x+3) = 0$$

$$x = \frac{5}{6} \text{ or } -\frac{3}{5}$$

$x \neq -\frac{3}{5}$ as x & y are positive numbers.

$$\text{so } x : y = 5 : 6.$$

14. $\frac{P}{a} = \frac{Q}{R} = \frac{R}{S} = \frac{2}{3}$

let P's share be K .

then a's share is $\frac{3}{2}K$.

R's share is $\frac{9}{4}K$.

S's share is $\frac{27}{8}K$.

$$K + \frac{3}{2}K + \frac{9}{4}K + \frac{27}{8}K = 1300$$

$$\frac{8K + 12K + 18K + 27K}{8} = 1300$$

$$65K = 1300 \times 8$$

$$K = 160 = \text{P's share}$$

15. let Ram & Shyam's age be $3x$ & $2x$.

ratio of ages ~~after~~ 20 years ago is $\frac{3x-20}{2x-20} = 1 + \frac{x}{2x-20}$

check by options s.t. $2x-20$ & $3x-20$ are positive.

16. $\frac{a+b}{xa+yb} = \frac{b+c}{xb+yc} = \frac{c+a}{xc+ya}$ (use property $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = \frac{a+c+e}{d+f+k}$)

so $\frac{a+b}{xa+yb} = \frac{b+c}{xb+yc} = \frac{c+a}{xc+ya} = \frac{2(a+b+c)}{x(a+b+c) + y(a+b+c)} = \frac{2}{x+y}$

Let A, B, C's share be $3x$, $4x$ and $7x$ respectively
and

$$3x + 4x + 7x = 5600$$

$$14x = 5600$$

$$x = 400$$

$$B's \text{ share} = 4x = 1600$$

18. Find ratio of sugar in each bakery

$$A \rightarrow \text{Sugar ratio} = \frac{5}{5+4} = \frac{5}{9} = 0.555$$

$$B \rightarrow \text{Sugar ratio} = \frac{13}{13+12} = 0.52$$

$$C \rightarrow \text{Sugar ratio} = \frac{29}{29+24} = 0.547$$

So A's biscuits are sweetest.

19. Milk in solution is $30 \times \frac{70}{100} = 21L$

$$\text{water} = 9L$$

Let water added be xL so Total volume of water $= (9+x)L$

$$\frac{21}{9+x} = \frac{3}{2} \Rightarrow 42 = 27 + 3x$$

$$3x = 15$$

$$x = 5L$$

20. $a \propto x^2$

$$a = Kx^2$$

$$\text{for } x=2 \quad a=20 \quad \text{gives } 20 = K(2)^2 \Rightarrow K=5$$

$$\text{so for } x=8 \quad a = 5 \times (8)^2 = 320$$

Ex-2.

1. Let Monthly income of Mr Raju & Mr Mehta be $9x$ & $10x$,
& their saving be $9y$ & $10y$ respectively.

As Income = Saving + Expenditure

$$10x = 10y + 15000$$

$$10(x-y) = 15000 \Rightarrow x-y = 1500$$

Let Monthly expenditure of Mr Raju be K .

$$9x = 9y + K$$

$$K = 9(x-y) = 9 \times 1500 = 13500$$

2. Let the term to be added by K .

$$\frac{x+K}{y+K} = \frac{y}{x} \Rightarrow x^2 + xK = y^2 + yK$$

$$xK - yK = y^2 - x^2$$

$$K(x-y) = -(x-y)(x+y)$$

$$K = -(x+y)$$

3. C's part for correct ratio = $585 \times \frac{6}{3+4+6} = 270$

C's part for mistaken ratio = $585 \times \frac{1}{\frac{1}{3} + \frac{1}{4} + \frac{1}{6}} = 260$

$$\text{Loss incurred} = 270 - 260 = 10$$

4. $4x^2 + 25y^2 = 20xy$ (divide by y^2)

$$4\left(\frac{x}{y}\right)^2 + 25 = 20\left(\frac{x}{y}\right)$$

$$\text{Let } \frac{x}{y} = r$$

$$4r^2 - 20r + 25 = 0$$

$$4x^2 - 25(2x-5)^2 = 0$$

$$2x = 5$$

$$x = \frac{5}{2}$$

$$\text{so } x : y = 5 : 2.$$

5. Let ratio of amounts with Mr Umar & Mr. Gumar be 3:2.

if so
$$\frac{3x+5}{4x-5} = \frac{4}{3}$$

$$9x + 15 = 16x - 20$$

$$7x = 35$$

$$x = 5$$

If Mr Umar gives 5 to Mr Gumar then

$$\frac{3x-5}{4x+5} = \frac{15-5}{20+5} = \frac{10}{25} = \left(\frac{2}{5}\right)$$

6.

$$F = ma$$

$$10 = m \times 2 \Rightarrow m = 5 \text{ kg.}$$

$$\text{for } a = 4, F = ?$$

$$F = m \times 5 \times 4 = 20 \text{ N.}$$

7.

$$P \propto \frac{1}{\sqrt{y}} \Rightarrow P = \frac{K}{\sqrt{y}}$$

$$\text{for } y = 2, P = 40 \Rightarrow K = 40\sqrt{2}$$

$$\text{for } P = 20, y = ? \quad 20 = \frac{K}{\sqrt{y}} = \frac{40\sqrt{2}}{\sqrt{y}}$$

$$\sqrt{y} = 2\sqrt{2} \Rightarrow y = 8.$$

$$8. \frac{x}{b-c} = \frac{y}{c-a} = \frac{z}{a-b} = K.$$

$$x = K(b-c) ; y = K(c-a) ; z = K(a-b)$$

$$(i) x+y+z = K(b-c) + K(c-a) + K(a-b) \\ = K(b-c+c-a+a-b) \\ = K \times 0 = 0$$

$$(ii) ax+by+cz = aK(b-c) + bK(c-a) + cK(a-b) \\ = K(ab-ac+bc-ab+ca-abc) \\ = K \times 0 = 0$$

$$9. a = \frac{b+c}{2} ; c = \frac{a+b}{2} \quad \& \quad b^2 = ac$$

$$\begin{aligned} 2a &= b+c \\ c &= 2a-b \\ \frac{1}{a} + \frac{1}{c} &= \frac{2}{b+c} + \frac{2}{a+b} \\ 2c &= a+b \Rightarrow 2(2a-b) = a+b \\ &= \frac{2a+2b+2b+2c}{(b+c)(a+b)} \\ &= \frac{2a+4b+2c}{ab+b^2+ac+bc} \end{aligned}$$

$$b^2 = \frac{b+c}{2} \times \frac{a+b}{2}$$

$$4b^2 = ab + b^2 + ac + bc$$

$$4b^2 = ab + 2b^2 + bc$$

$$2b^2 = ab + bc \quad (\text{divide by } abc \text{ on both sides})$$

$$\frac{2b^2}{abc} = \frac{aba}{abc} + \frac{bc}{abc}$$

$$\frac{2b}{ac} = \frac{1}{c} + \frac{1}{a}$$

$$\frac{2}{b} = \frac{1}{c} + \frac{1}{a}$$

$$\therefore b^2 = ac$$

10. let other 2 numbers be x & y

$$xy = (16)^2 = 256 = 128 \times 2$$

$$x + y = 130$$

$$\text{So } x = 128 \text{ \& } y = 2.$$

$$11. (a+b+c)(a-b+c) = a^2 + b^2 + c^2$$

$$(a+b+c)(a+c-b) = a^2 + b^2 + c^2$$

$$(a+c)^2 - b^2 = a^2 + b^2 + c^2$$

$$a^2 + c^2 + 2ac - b^2 = a^2 + b^2 + c^2$$

$$2ac = 2b^2 \Rightarrow b^2 = ac$$

So a, b, c are in continued proportion.

12.

$$\frac{ua + gb}{uc + gd} = \frac{ua - gb}{uc - gd}$$

Apply Alternando.

$$\frac{ua + gb}{ua - gb} = \frac{uc + gd}{uc - gd}$$

Apply Componendo & dividendo

$$\frac{(ua + gb) + (ua - gb)}{(ua + gb) - (ua - gb)} = \frac{(uc + gd) + (uc - gd)}{(uc + gd) - (uc - gd)}$$

$$\frac{2a}{2b} = \frac{2c}{2d} \Rightarrow \frac{a}{b} = \frac{c}{d}$$

$$\text{or } a : b = c : d.$$