IIT – JEE: 2026 **TW TEST (3 YRS.)** **DATE: 30/07/23**

TOPIC: VECTORS

CALCULUS IN PHYISCS

ANSWER KEY

1.	(C)	2.	(D)	3.	(A)	4.	(A)	5.	(C)
6.	(C)	7.	(A)	8.	(A)	9.	(C)	10.	(A)
11.	(B)	12.	(C)	13.	(B)	14.	(A)	15.	(A)
16.	(B)	17.	(C)	18.	(B)	19.	(A)	20.	(D)

SOLUTIONS

1. (C)

$$\frac{d\left(x^n\right)}{dx} = nx^{x-1}$$

2. (D)

$$\frac{d\left(\cos x - x\ln x + x\right)}{dx} = -\sin x - \left(\ln x + x\cdot\frac{1}{x}\right) + 1$$

3. (A)

$$\frac{d\left(\frac{1+x}{1-x}\right)}{dx} = \frac{1(1-x) - -1(1+x)}{(1-x)^2}$$

4. (A)

$$a = \frac{dv}{dt} = 6t + 2 = 14 \text{ at } t = 2$$

5. (C)

$$\frac{dy}{dx} = 1 - \frac{1}{x^2}; \frac{d^2y}{dx^2} = +\frac{2}{x^3}$$

For minimum, $\frac{dy}{dx} = 0 \Rightarrow x = \pm 1$

At $x = 1, \frac{d^2y}{dx^2} = \frac{+2}{1^3} > 0$

So, y is min at $x = 1$.

$$y_{\min} = 1 + \frac{1}{1} = 2$$

6. (C)

$$\int\left(\cos x-\frac{1}{x}+1\right)dx=\sin x-\ln x+x+C$$

7. (A)

Let $Z=\sec x \Rightarrow \frac{dz}{dx}=\sec x+\tan x$

$\Rightarrow dx=\sec x\tan x\,dx$

So, integral is $\int \sec x.\sec x\tan x\,dx$

$$\int z\,dx=\frac{z^2}{2}+C=\frac{\sec^2 x}{2}+C$$

8. (A)

$$\int\frac{3x^2+2}{x^3+2x+8}dx$$

Let, $P=x^3+2x+8$

$$\frac{dP}{dx}=3x^2+2$$

$$\Rightarrow \quad \frac{3x^2+2}{x^3+2x+8}dx=\int\frac{dP}{P}=\ln P+C$$

$$=\ln\left(x^3+2x+8\right)+C$$

9. (C)

$y=\ln P^2=2\ln P$

$$\Rightarrow \quad \frac{dy}{dP}=\frac{2}{P}$$

$$x=\frac{P^3}{3} \Rightarrow \frac{dx}{dP}=P^2$$

$$\frac{dy}{dx}=\frac{dy}{dP}\times\frac{dP}{dx}=\frac{2}{P}\times\frac{1}{P^2}=\frac{2}{P^3}$$

10. (A)

$v=3t^2-2t+5$

$$\Rightarrow \quad dS=3\int t^2dt-2\int tdt+5\int dt$$

$$\left[S\right]_0^S=3\left[\frac{t^3}{3}\right]_2^3-2\left[\frac{t^2}{2}\right]_2^3+5\left[t\right]_2^3$$

$S=(27-8)-(9-4)+5(3-2)$

$S=19-5+5=19$

11. (B)

$R=\sqrt{3^2+4^2+2*3*4\cos 60}=\sqrt{37}$

12. (C)

$\left|\overrightarrow{AB}\right|=\sqrt{6^2+2^2}=\sqrt{40}$

13. (B)

14. (A)

$$\hat{A}=\frac{\vec{A}}{|\vec{A}|}$$

15. (A)

$$\left|\frac{\hat{i}}{\sqrt{2}}+\frac{\hat{j}}{\sqrt{2}}\right|=\sqrt{\frac{1}{2}+\frac{1}{2}}=1$$

16. (B)

$$\vec{P}\cdot\vec{Q}=a^2-2a-3=0$$

$$a=3$$

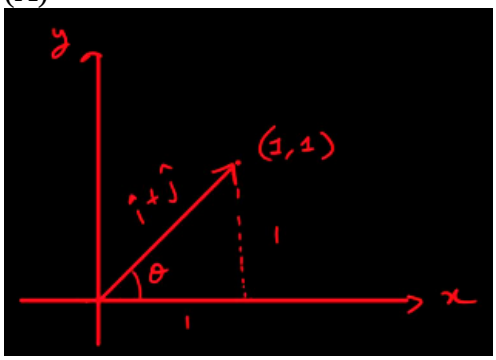
17. (C)

$$\hat{A}=\frac{\vec{A}}{A}=\frac{2\hat{i}-\hat{j}+\hat{k}}{\sqrt{2^2+1^2+1^2}}=\frac{2\hat{i}-\hat{j}+\hat{k}}{\sqrt{6}}$$

18. (B)

Both are opposite. Hence angle between then π.

19. (A)



$$\tan\theta=\frac{1}{1}=1$$

$$\theta=45^{\circ}$$

20. (D)

$$\vec{A}=\hat{i}-2\hat{j}+\hat{k}$$

$$\vec{B}=3\hat{i}+\hat{j}-2\hat{k}$$

$$\vec{A}\times\vec{B}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & -2 & 1\\ 2 & 1 & -2\end{vmatrix}$$

$$=\hat{i}(4-1)-\hat{j}(-2-3)+\hat{k}(1+6)$$

$$=3\hat{i}+5\hat{j}+7\hat{k}$$

$$\left|\vec{A}\times\vec{B}\right|=\sqrt{3^2+5^2+7^2}=\sqrt{83}$$

So, $\dfrac{\vec{A}\times\vec{B}}{\left|\vec{A}\times\vec{B}\right|}=\dfrac{3\hat{i}+5\hat{j}+7\hat{k}}{\sqrt{83}}$

Answer Key

21.	(A)	22.	(A)	23.	(B)	24.	(C)	25.	(D)
26.	(B)	27.	(C)	28.	(C)	29.	(B)	30.	(B)
31.	(B)	32.	(B)	33.	(B)	34.	(D)	35.	(C)
36.	(A)	37.	(D)	38.	(B)	39.	(B)	40.	(D)

Solution

21. (A)

Atomic mass in gram = Mass of 6.022×10^{23} atoms

$$=\frac{1.15\times6.022\times10^{23}}{3.011\times10^{22}}=23$$

22. (A)

$$\%N=\frac{\text{Mass of nitrogen}}{\text{Molecular mass}}\times100\%$$

$$20=\frac{14}{m}\times100\%$$

$$\therefore m=70$$

23. (B)

$$\text{Initiated moles}=\frac{0.098}{98}=10^{-3}$$

$$\text{Removed moles of } H_2SO_4=\frac{3.01\times10^{20}}{6.02\times10^{23}}=0.5\times10^{-3}$$

$$\text{Remaining moles of } H_2SO_4=10^{-3}-0.5\times10^{-3}=0.5\times10^{-3}$$

24. (C)

$$\text{Mass of NaCl in 10 g salt}=\frac{10\times95}{100}=95\text{ g}$$

$$\therefore \text{ Number of molecules of NaCl}=\frac{95}{58.5}\times6.023\times10^{23}=9.78\times10^{22}\simeq10^{23}$$

25. (D)

$Na_2SO_4 \cdot nH_2O$

Molar mass = (142 + 18n)

$\therefore$ Mass of water $= \frac{12.6}{26.8} \times (142 + 18n)$

$18n = \frac{12.6}{26.8} \times (142 + 18n)$

$\therefore\ n = 7$

26. (B)

Element	%	Atomic ratio	Simplest ratio
X	50	$\frac{50}{10} = 5$	$\frac{5}{2.5} = 2$
Y	50	$\frac{50}{20} = 2.5$	$\frac{2.5}{2.5} = 1$

Formula = X_2Y

27. (C)

$4PH_3(s) \rightarrow P_4(g) + 6H_2(g)$

4 mL $\rightarrow$ 1 mL 6 mL

100 mL $\rightarrow \frac{100}{4}$ $\frac{6}{4} \times 100$

100 mL $\rightarrow$ 25 mL 150 mL

Volume increase by 75 mL

28. (C)

Let *x* L liquid contain 4 mole in it

Number of moles $= \frac{\text{Mass}}{\text{Molar mass}}$

$4 = \frac{x \times 100 \times 1.4}{280}$

$\therefore\ x = 0.8\,\text{L}$

29. (B)

No. of moles of Na^+ = 3 × No. of moles of Na_3PO_4

$= 3 \times \frac{0.4 \times 20}{1000} = 0.024$

30. (B)

No. of moles of sucrose $= \frac{25.6\,\text{g}}{342.3\,\text{g}} = 0.075$

No. of moles of hydrogen atom $= 0.075 \times 22$

No. of atoms of $H = 0.075 \times 22 \times 6.023 \times 10^{23} = 9.9 \times 10^{23}$

31. (B)

$n_{H_2} = \frac{10}{2} 5, \quad n_{O_2} = \frac{64}{32} = 2$

$2H_2 + O_2 \rightarrow 2H_2O$

32. (B)

No. of moles of $H_2O = \frac{0.72}{18} = 0.04$ $\qquad n_H = 2 \times n_{H_2O} = 2 \times 0.04 = 0.08$

No. of moles of $CO_2 = \frac{3.08}{44} = 0.07$ $\qquad n_C = n_{CO_2} = 0.07$

C : H = 7 : 8

33. (B)

Mass = 18 g

No. of molecules in 18 g $H_2O = 6.023 \times 10^{23}$

No. of electrons in 18 g water = $6.023 \times 10^{23} \times 10 = 6.023 \times 10^{24}$

34. (D)

Charge on 1 mol Al^{3+} = 3e N_A coulomb

35. (C)

Volume of 1 drop $= \frac{2}{35}$ mL

Number of moles in 1 drop $= \frac{2 \times 1.2}{35 \times 70} = \frac{1.2}{(35)^2}$

Number of molecules in one drop $= \frac{1.2}{(35)^2} \times N_A$

36. (A)

$0.2 = \frac{n_{\text{glucose}}}{n_{\text{glucose}} + n_{H_2O}}$

Let $n_{\text{glucose}} = x$

$\Rightarrow \quad n_{H_2O} = 4x$

Molality $= \frac{n_{\text{glucose}}}{W_{H_2O}(kg)} = \frac{x}{4x \times \frac{18}{1000}} = \frac{250}{18} = 13.8$

37. (D)

For minimum MW, number of oxygen atoms per molecule = 1

So $3.2 = \frac{16 \times 1}{MW_{\min}} \times 100$

$\Rightarrow MW_{\min} = \frac{1600}{3.2} = 500$

38. (B)

$x+1+2\times2=y\times1.5$

$\Rightarrow\ x+4=1.5y$

Since final molarity is average of both molarities, the volume of both solution has to be equal

$\Rightarrow\ x=2$

$\Rightarrow\ y=4$

39. (B)

$n_{Cl^-}=2\times1\times2+2\times1\times1+1\times1\times3$

$=4+2+3=9$

$[Cl^-]=\frac{9}{5}=1.8\ M$

40. (D)

$n_{OH^-}=V\times\frac{1}{10}\times2$

$n_{H^+}=0.5\times2\times2$

$V\times\frac{1}{10}\times2=0.5\times2\times2$

$V=10\ L$

ANSWER KEY

41.	(C)	42.	(B)	43.	(B)	44.	(C)	45.	(C)
46.	(D)	47.	(A)	48.	(D)	49.	(B)	50.	(A)
51.	(A)	52.	(B)	53.	(C)	54.	(C)	55.	(B)
56.	(C)	57.	(D)	58.	(B)	59.	(B)	60.	(A)

SOLUTIONS

41. (C)

$\cos 15^\circ - \sin 15^\circ$

$= \dfrac{\sqrt{3}+1}{2\sqrt{2}} - \left(\dfrac{\sqrt{3}-1}{2\sqrt{2}}\right) = \dfrac{1}{\sqrt{2}}$

42. (B)

Since $\operatorname{cosec}^2\theta = 1+\cot^2\theta = 1+\dfrac{9}{16} = \dfrac{25}{16}$ $\left(\because \tan\theta = -\dfrac{4}{3}\right)$

$\sin^2\theta = \dfrac{1}{\operatorname{cosec}^2\theta} = \dfrac{16}{25} \Rightarrow \sin\theta = \pm\dfrac{4}{5},$

Both the values are acceptable, since $\tan\theta = -\dfrac{4}{3}$

i.e., θ lies in 2nd or 4th quadrant.

43. (B)

$\therefore \cos x + \cos\left(120^\circ - x\right) + \cos\left(120^\circ + x\right) = 2\cdot\cos 120^\circ \cdot \cos x + \cos x$

$= 2\cdot\cos\left(90^\circ + 30^\circ\right)\cdot\cos x + \cos x = 20\left(-\sin 30^\circ\right)\cdot\cos x + \cos x$

$= \left(2 - \dfrac{1}{2}\right)\cdot\cos x + \cos x = -\cos x + \cos x = 0$

44. (C)

$\cos\dfrac{\pi}{6} = \sqrt{3}/2$

$\cos\dfrac{5\pi}{6} = -\sqrt{3}/2$

$\cos 7\pi/6 = -\sqrt{3}/2$

$\cos 19\pi/6 = -\sqrt{3}/2$

45. (C)

$\sin\theta = -\frac{1}{\sqrt{2}}$ and $\tan\theta = 1$

$\Rightarrow \sin\theta = \sin 225° \Rightarrow \theta = 225°$

Since $\sin\theta$ is –ve and $\tan\theta$ is $+ve$ in third quadrant.

46. (D)

$3\tan A + 4 = 0 \Rightarrow \tan A = -\frac{4}{3}$

$\Rightarrow \sin A = \pm\frac{\tan A}{\sqrt{1+\tan^2 A}} = \pm\frac{-4/3}{\sqrt{1+16/9}} = \frac{4}{5}$ ($\because A$ is in 2nd quadrant)

and $\cos A = -\frac{3}{5}$. Thus, $2\cot A - 5\cos A + \sin A = 2\left(-\frac{3}{4}\right) - 5\left(-\frac{3}{5}\right) + \frac{4}{5} = \frac{23}{10}$.

47. (A)

$\tan(90° - \theta) = \cot\theta, \cot(90° - \theta) = \tan\theta.$

48. (D)

We have $\sin A = \frac{4}{5}$ and $\cos B = -\frac{12}{13}$

Now, $\cos(A+B) = \cos A\cos B - \sin A\sin B$

$= \sqrt{1-\frac{16}{25}}\left(-\frac{12}{13}\right) - \frac{4}{5}\sqrt{1-\frac{144}{169}}$

$= -\frac{3}{5}\times\frac{12}{13} - \frac{4}{5}\left(-\frac{5}{13}\right) = -\frac{16}{65}$

(Since A lies in first quadrant and B lies in third quadrant).

49. (B)

$\tan 20^o + 2\tan 50^o - \tan 70^o$

$= \frac{\sin 20^o}{\cos 20^o} - \frac{\sin 70^o}{\cos 70^o} + 2\tan 50^o$

$= \frac{\sin 20^o \cos 70^o - \cos 20^o \sin 70^o}{\cos 20^o \cos 70^o} + 2\tan 50^o$

$= \frac{\sin(20^o - 70^o)}{\frac{1}{2}[\cos(70^o + 20^o) + \cos(70^o - 20^o)]} + 2\tan 50^o$

$= \frac{2\sin(-50^o)}{\cos 90^o + \cos 50^o} + 2\tan 50^o$

$= \frac{-2\sin 50^o}{0 + \cos 50^o} + 2\tan 50^o$

$= -2\tan 50^o + 2\tan 50^o = 0.$

50. (A)

$\tan 75^\circ - \cot 75^\circ = \cot 15^\circ - \cot 75^\circ$

$= (2+\sqrt{3}) - (2-\sqrt{3}) = 2\sqrt{3}$.

51. (A)

$\sin 330^\circ = -1/2$

$\tan 225^\circ - +1$

$\cos 210^\circ = -\sqrt{3}/2$

Ans. (A)

52. (B)

$3A = 2A + A$

$\tan 3A = \tan(2A + A)$

$= \dfrac{\tan 2A + \tan A}{1 - \tan A \tan 2A}$

$\therefore \tan 3A - \tan 3A \tan A \tan 2A = \tan 2A + \tan A$

$\therefore \tan 3A \tan 2A \tan A = \tan 3A - \tan 2A - \tan A$

53. (C)

$(1+\tan 11^\circ)(1+\tan 22^\circ)(1+\tan 34^\circ)(1+\tan 23^\circ)$

$= (1+\tan 11^\circ)(1+\tan 34^\circ)(1+\tan 22^\circ)(1+\tan 23^\circ)$

$= 2(2) = 4$

54. (C)

Substitute the corresponding values.

55. (B)

$3\left[\cos^4\alpha + \sin^4\alpha\right] - 2\left[\cos^6\alpha + \sin^6\alpha\right] = 3\left[1 - 2\sin^2\alpha\cos^2\alpha\right] - 2\left[1 - 3\sin^2\alpha\cos^2\alpha\right] = 1$

56. (C)

If $(A+B) = \dfrac{\pi}{2}$ then $\left(\sin^2 A + \sin^2 B\right) = 1$

57. (D)

$$\frac{\cos(\theta - 90^\circ)\sec\theta\tan(180^\circ - \theta)}{\sec(360^\circ - \theta)\sin(\theta - 180^\circ)\tan(360^\circ - \theta)}$$

$$= \frac{\sin\theta(\sec\theta)(-\tan\theta)}{\sec\theta(-\sin\theta)(-\tan\theta)} = -1$$

58. (B)

$1358^0 = 8(180^0) + 82^0,\ 3608^0 = 20(180^0) + 8$

59. (B)

If $(A+B)=90^0$ then $(\sin^2 A+\sin^2 B)=1$

60. (A)