Polynomials. Ch-4.

Ex-1 Level-I

1. Degree is 3 as highest power is 3.

2. $1 = 1 \cdot x^0$ so degree is 0.

3. $x^3 + 0 \cdot x^2 + 3x - 1 \Rightarrow 1, 0, 3, -1$: Coefficient form

4. $p(x) = x^3 - 3x^2 + 4x - 5$

$p(1) = 1^3 - 3(1)^2 + 4(1) - 5 = 1 - 3 + 4 - 5 = -3$

5. $-2x^3 + 3x^2 - 5x + 6$

6. A polynomial having only one term is monomial.

7. $2x + y = 5$ (Multiply by 2)

$4x + 2y = 10$

8. $(x+a)(x+b) = x(x+b) + a(x+b)$

$= x^2 + bx + ax + ab$

$= x^2 + (a+b)x + ab$

9. $150 \times 98 = 150 \times (100-2)$

$= 15000 - 300 = 14700$

10. $p(x) = 3x - 6 = 0$

$3x = 6$

$x = 2$

11. $(9p - 5q)^2 + 180pq$

$(9p)^2 + (5q)^2 - 2 \times 9p \times 5q + 180pq$

$(9p)^2 + (5q)^2 - 90pq + 180pq$

$(9p)^2 + (5q)^2 + 90pq = (9p)^2 + (5q)^2 + 2 \times (9p) \times 5q$

$= (9p + 5q)^2$

12. $x^2 - x - 42 = (x + K)(x + 6)$

$= x^2 + (K+6)x + 6K$

$K + 6 = -1$

$K = -1 - 6 = -7$

13. $p(x) = 4x^4 - 3x^3 - 3x^2 + x + 7$ $\quad g(x) = 1 - 2x = 0$

$\Rightarrow x = 1/2$

So remainder: $p(1/2) = 4\left(\frac{1}{2}\right)^4 - 3\left(\frac{1}{2}\right)^3 - 3\left(\frac{1}{2}\right)^2 + \frac{1}{2} + 7$

$= \frac{4}{16} - \frac{3}{8} - \frac{3}{4} + \frac{1}{2} + 7$

$= \frac{1}{4} - \frac{3}{8} - \frac{3}{4} + \frac{1}{2} + 7$

$= 7 - \frac{3}{8} = \frac{53}{8}$

14. $q(x) = 3x^2 - 2x + 1$, $g(x) = x + 2$ $r = 2x - 5$

dividend = divisor × quotient + remainder

$= (3x^2 - 2x + 1)(x + 2) + 2x - 5$

$= 3x^3 + 4x^2 - 3x + 2 + 2x - 5$

$= 3x^3 + 4x^2 - x - 3$

15. $25x^2 + 16y^2 + 40xy = (5x + 4y)^2$ at $x = 1$ & $y = -1$

$= [5 \times 1 + 4 \times (-1)]^2$

$= [5 - 4]^2 = 1$

16. $3.5 \times 3.5 - 2.5 \times 2.5 = (3.5 + 2.5)(3.5 - 2.5)$

$= 6 \times 1 = 6$

17. $(x + y - z)^2 = x^2 + y^2 + z^2 + 2(xy - yz - zx)$

18. $(x + 2y + 2z)^2 + (x - 2y - 2z)^2$

$\Rightarrow x^2 + 4y^2 + 4z^2 + 4xy + 8yz + 4xz + x^2 + 4y^2 + 4z^2 - 4xy + 8yz - 4xz$

$\Rightarrow 2x^2 + 8y^2 + 8z^2 + 16yz$

19. $x^2 - 7x + 12 = x^2 - 3x - 4x + 12$

$= x(x - 3) - 4(x - 3)$

$= (x - 3)(x - 4)$

20. $6a^2 - 2a = 2a(3a - 1)$ → Distributive property

Level-2

1. Degree = 3 $[x^2y' \to 2+1=3]$

2. $(-x+y+z)^2 = x^2+y^2+z^2-2xy+2yz-2xz$

3. $a+b+c=12$; $ab+bc+ca=$ ~~67~~

$(a+b+c)^2 = 12^2$

$a^2+b^2+c^2+2(ab+bc+ca)=144$

$50+2(ab+bc+ca)=144$

$ab+bc+ca=\frac{144-50}{2}=$ ~~37~~ 47

4. $a+b+c=13$, $ab+bc+ca=50$

$(a+b+c)^2=13^2$

$a^2+b^2+c^2+2(ab+bc+ca)=169$

$a^2+b^2+c^2=169-2\times50=69$

5. $x-\frac{1}{x}=\sqrt{6}$

$\left(x-\frac{1}{x}\right)^2=6 \Rightarrow x^2+\frac{1}{x^2}-2=6$

$x^2+\frac{1}{x^2}=8$

6. $(a+b)^3-(a-b)^3$

$(a^3+b^3+3ab^2+3a^2b)-(a^3-b^3-3a^2b+3ab^2)$

$\Rightarrow 2b^3+6a^2b$ ~~$\Rightarrow 2b(a^2+3b$~~

$2b(b^2+3a^2)$

7. $(x-a)(x-b)(x-c)=x^3-(a+b+c)x^2+(ab+bc+ca)x-abc$

$=x^3-7x^2+0\cdot x+36=x^3-7x^2+36$

8. $5 + 8x - 4x^2$

$\Rightarrow 5 + 10x - 2x - 4x^2$

$\Rightarrow 5(1+2x) - 2x(1+2x)$

$\Rightarrow (1+2x)(5-2x)$

9. $2x^2 + xy - 3y^2 + x + ay - 10 = (2x+3y+b)(x-y-2)$

$(2x+3y+b)(x-y-2) = 2x^2 - 2xy - 4x + 3xy - 3y^2 - 6y + bx - by - 2b$

$= 2x^2 + xy - 3y^2 + (b-4)x - (b+6)y - 2b$

on comparing them

$b - 4 = 1 \Rightarrow b = 5$

$-(b+6) = a \Rightarrow a = -11$

10. $HCF = x \quad LCM = x^3 - 9x$

$HCF \times LCM = x \times (x^3 - 9x) = x \times x(x^2 - 9)$

$= x \times x(x-3)(x+3)$

$HCF \times LCM$ = product of given Expression

$x \times x \times (x-3)(x+3) = (x^2+3x) \times E(x)$ { $E(x)$: other Expression }

$E(x) = \frac{x^2(x-3)(x+3)}{x(x+3)}$

$= x(x-3) = x^2 - 3x$

11. $GCD = HCF = x - 6$

$P_1(x) = x^2 - ax - 6 \Rightarrow P_1(6) = 0 \Rightarrow 6^2 - a \times 6 - 6 = 0 \Rightarrow a = 5$

$P_2(x) = x^2 - 2x + b$

$P_2(6) = 0 = 6^2 - 2\times 6 + b$

$b = -24$

12. $(x+y+z)=1 \quad ; \quad xy+yz+xz=-1 \quad ; xyz=-1$

$(x+y+z)^2 = 1^2$

$x^2+y^2+z^2+2(xy+yz+xz) = 1$

$x^2+y^2+z^2 = 1+2 = 3$

$x^3+y^3+z^3-3xyz = (x+y+z)(x^2+y^2+z^2-xy-yz-xz)$

$x^3+y^3+z^3-3\times(-1) = 1\times[3-(-1)]$

$x^3+y^3+z^3 = 4-3 = 1$

13. $HCF = x-6 \qquad LCM = (x-6)(x-1)(x+6)$

$P_1(x) = x^2-7x+6 = (x-1)(x-6)$

So $P_2(x) = (x-6)(x+6)$

14. $p(x) = 4x^3-12x^2+14x-3 \qquad g(x): x-\frac{1}{2}=0$

$x=\frac{1}{2}$

remainder: $P(\frac{1}{2}) = 4(\frac{1}{2})^3-12(\frac{1}{2})^2+14(\frac{1}{2})-3$

$= \frac{4}{8} - \frac{12}{4} + 7 - 3$

$= \frac{1}{2} - 3 + 4 = 1+\frac{1}{2} = \frac{3}{2}$

Ex-2.

Level-I

1. $x^2+5x+6+A = x^3-x^2+3x-2$

$A = x^3-x^2+3x-2-(x^2+5x+6)$

$= x^3-2x^2-2x-8$

2. $x^4+2x^2-3x+7-5 = x^3+x^2+x-1$

$(x^4+2x^2-3x+7)-(x^3+x^2+x-1) = 5$

$x^4-x^3+x^2-4x+8 = 5$

3. $25x^2+16y^2+40xy = (5x+4y)^2$ at $x=1, y=-1$

$= [5x1+4x(-1)]^2$

$= [5-4]^2 = 1$

4. degree $= 2$

5. $p(x) = 2x+5 = 6$

$x = \frac{-5}{2}$

6. degree $= 3$ $(x^2y^1 \Rightarrow 2+1=3)$

7. $3^3+5^3-(8)^3 \Rightarrow$ $[a^3+b^3+c^3 = 3abc$ if $a+b+c=0]$

$3^3+5^3-8^3 = 3\times3\times5\times(-8)$

$= -360$

8. $p(x,y) = x^2-y^2-xy$ $\quad q(x,y) = -x^2+y^2+3xy$

$4p(x,y)-5q(x,y) = 4(x^2-y^2-xy)-5(-x^2+y^2+3xy)$

$= 4x^2-4y^2-4xy+5x^2-5y^2-15xy$

$= 9x^2-9y^2-19xy$

9. $8a^3 - 2a^2b - 15ab^2$

$a(8a^2 - 2ab - 15b^2)$

$a(8a^2 - 12ab + 10ab - 15b^2) = a[4a(2a-3b) + 5b(2a-3b)]$

$= a(2a-3b)(4a+5b)$

10. $x^{12} - y^{12} = (x^6)^2 - (y^6)^2 = (x^6 - y^6)(x^6 + y^6)$

$= (x^3 - y^3)(x^3 + y^3)((x^2)^3 + (y^2)^3)$

$= (x-y)(x^2 + y^2 + xy)(x+y)(x^2 + y^2 - xy)$ ✕

$(x^2 + y^2)(x^4 + y^4 \neq x^2y^2)$

11. $1 - x + x^2 - x^3$

$1(1-x) + x^2(1-x) = (1-x)(1+x^2)$

12. $3\sqrt{3}x^3 + y^3 = (\sqrt{3}x)^3 + y^3 = (\sqrt{3}x + y)(3x^2 + y^2 - \sqrt{3}xy)$

13. $5.63 \times 5.63 + 11.26 \times 2.37 + 2.37 \times 2.37$

$= (5.63)^2 + 2 \times 5.63 \times 2.37 + (2.37)^2 = [5.63 + 2.37]^2$

$= 64$

14. $x + y = 3$; $x^2 + y^2 = 5$

$(x+y)^2 = 9$

$x^2 + y^2 + 2xy = 9 \Rightarrow 2xy = 9 - 5 = 4$

$xy = 2$

15. $x^2 + x - 20 = x^2 + 5x - 4x - 20$

$= x(x+5) - 4(x+5)$

$= (x+5)(x-4)$ so $x-4$ is other factor.

16. $p = 2-a$

$a+p = 2$

$(a+p)^3 = 2^3 \Rightarrow a^3+p^3+3a^2p+3ap^2 = 8$

$a^3+p^3+3ap(a+p) = 8$

$a^3+p^3+6ap = 8$

$a^3+p^3+6ap-8 = 0$

17. $x-\frac{1}{x} = \sqrt{5}$

$\left(x-\frac{1}{x}\right)^2 = (\sqrt{5})^2$

$x^2+\frac{1}{x^2}-2 = 5 \Rightarrow x^2+\frac{1}{x^2} = 7$

18. $2x^2-6 \;\big|\; 6x^5-28x^3+3x^2+30x-9 \;\big|\; 3x^3-5x+\frac{3}{2}$

$-6x^5 \mp 18x^3$

$-10x^3+3x^2-30x$

$-10x^3 \pm 30x$

$3x^2-9$

$3x^2-9$

0

19. $(x^3+y^3)(x^3-y^3) = x^6-y^6$

20. $\frac{x^2-xy}{x+2y} \div \frac{x^3-xy^2}{x^2+3xy+2y^2}$

$\frac{x(x-y)}{x+2y} \times \frac{x^2+3xy+2y^2}{x(x^2-y^2)} = \frac{x(x-y)}{x+2y} \times \frac{(x+2y)(x+y)}{x(x-y)(x+y)}$

$= 1$

Level-2

1. (i) $a^2x^2 + (ax^2+1)x + a$

$a^2x^2 + a + (ax^2+1)x$.

$a(ax^2+1) + x(ax^2+1) = (ax^2+1)(a+x)$

(ii) $6ab - b^2 + 12ac - 2bc$

$b(6a-b) + 2c(6a-b) = (6a-b)(b+2c)$

(iii) $\underbrace{(a-b+c)}_{X}^2 + \underbrace{(b-c+a)}_{Y}^2 + 2(a-b+c)(b-c+a)$

$X^2 + Y^2 + 2XY = (X+Y)^2 = (a-b+c+b-c+a)^2$

$= 4a^2$

(iv) $xy^9 - yx^9 = xy(y^8 - x^8) = xy(x^4-y^4)(x^4+y^4)$

$= xy(x^2-y^2)(x^2+y^2)(x^4+y^4)$

$= xy(x-y)(x+y)(x^2+y^2)(x^4+y^4)$

(v) $x^2 + \frac{12}{35}x + \frac{1}{35} = x^2 + \frac{7}{35}x + \frac{5}{35}x + \frac{1}{35}$

$= x(x+\frac{1}{5}) + \frac{1}{7}(x+\frac{1}{5}) = (x+\frac{1}{5})(x+\frac{1}{7})$

(vi) $9(2a-b)^2 - 4(2a-b) - 13 = 9(2a-b)^2 + 9(2a-b) - 13(2a-b) - 13$

$= 9(2a-b)[2a-b+1] - 13[2a-b+1]$

$= [2a-b+1][9(2a-b) - 13]$

(vii) $7(x-2y)^2 - 25xy(x-2y) + 12$

$7(x-2y)^2 - 21(x-2y) - 4(x-2y) + 12$

$\Rightarrow 7(x-2y)[x-2y-3]-4[x-2y-3]$

$\Rightarrow [x-2y-3][7(x-2y)-4]$

2. $\frac{x^2-9}{x^2-5x+6} = \frac{(x-3)(x+3)}{(x-3)(x-2)} = \frac{x+3}{x-2}$

3. $3p(x)+7q(x)+r(x) = 3(4x^3-3x^2+2x+1)+7(x^3-x^2+x+1)+(x^2-2x+1)$

$= 19x^3-15x^2+11x+11$

4. $a+b=10 \quad a^2+b^2=58$

$(a+b)^2=10^2$

$a^2+b^2+2ab=100 \Rightarrow ab=\frac{100-58}{2}=21$

$(a+b)^3=10^3$

$a^3+b^3+3ab(a+b)=1000$

$a^3+b^3+3\times21\times10=1000$

$a^3+b^3=1000-630=270$

5. $\underbrace{(a+b)}_{X}^3+\underbrace{(a-b)}_{Y}^3+6a(a^2-b^2)$

$x^3+y^3+3xy(x+y)=(x+y)^3=8a^3$

6. $P(x)=g(x)q(x)+r(x)$

$P(x)-r(x)=g(x)q(x)$

$g(x)q(x) = x^3 - 3x^2 - x - 2 - [-2x - 4]$

$g(x)(x-2) = x^3 - 3x^2 + x + 2$

$$
\begin{array}{r|l}
x^3 - 3x^2 + x + 2 & x^2 - x - 1 \\
\hline
-x^3 + 2x^2 & \\
\hline
-x^2 + x & \\
-x^2 + 2x & \\
\hline
-x + 2 & \\
-x + 2 & \\
\hline
0 &
\end{array}
$$

$g(x) = x^2 - x - 1$

7. $(3x - 5y - 4)(9x^2 + 25y^2 + 15xy + 12x - 20y + 16)$

use identity $(a+b+c)(a^2+b^2+c^2-ab-bc-ca) = a^3+b^3+c^3-3abc$

so $\Rightarrow (3x)^3 + (-5y)^3 + (-4)^3 - 3 \times 3x \times (-5y) \times (-4)$

$\Rightarrow 27x^3 - 125y^3 - 64 - 120xy$

8. $\left(x^2 + \frac{1}{x^2} + 2\right) - 2x - \frac{2}{x}$

$\left(x + \frac{1}{x}\right)^2 - 2\left(x + \frac{1}{x}\right) = \left(x + \frac{1}{x}\right)\left(x + \frac{1}{x} - 2\right)$

9. $2x^2 + y^2 + 8z^2 - 2\sqrt{2}xy + 4\sqrt{2}yz - 8xz$

$(\sqrt{2}x)^2 + y^2 + (2\sqrt{2}z)^2 + 2 \times (-\sqrt{2}x)y + 2 \times y \times (2\sqrt{2}z) + 2 \times (-\sqrt{2}x) \times (2\sqrt{2}z)$

$\Rightarrow \left(-\sqrt{2}x + y + 2\sqrt{2}z\right)^2$

10. $(\sqrt{2}a)^3 + (2b)^3 + (-3c)^3 - 3 \times (\sqrt{2}a) \times (2b) \times (-3c) =$

$(\sqrt{2}a + 2b - 3c)(2a^2 + 4b^2 + 9c^2 - 2\sqrt{2}ab + 6bc + 3\sqrt{2}ac)$

11. $x = \frac{a-b}{a+b}$ or $\frac{1}{x} = \frac{a+b}{a-b}$

apply C & D

$$\frac{1+x}{1-x} = \frac{a+b+a-b}{a+b-(a-b)} = \frac{2a}{2b} = \frac{a}{b}$$

Similarly $\frac{1+y}{1-y} = \frac{b}{c}$ & $\frac{1+z}{1-z} = \frac{c}{a}$

So their Multiplication is $\frac{a}{b} \times \frac{b}{c} \times \frac{c}{a} = 1$

12. $2x\left(3x - \frac{1}{2}\right) = 0$

$2x = 0$ or $3x - \frac{1}{2} = 0$

$x = 0$ or $x = \frac{1}{6}$

13. $p(x) = x^{11} + 1$ $\quad g(x) = x + 1 = 0$

$x = -1$

remainder $= p(-1) = (-1)^{11} + 1 = 0$

14. $\left(\frac{x}{3}\right)^3 + \left(\frac{y}{5}\right)^3 + 3 \times \left(\frac{x}{3}\right)^2 \times \frac{y}{5} + 3 \times \left(\frac{x}{3}\right) \times \left(\frac{y}{5}\right)^2 = \left[\frac{x}{3} + \frac{y}{5}\right]^3$

15. $9x^2 + Kxy + 16y^2 = [(3x)^2 + (4y)^2 + 2 \times 3x \times 4y] - 2 \times 3x \times 4y + Kxy$

$= [3x + 4y]^2 + (K - 24)xy$

$K - 24 = 0 \Rightarrow K = 24$

16. $2x + 2$; $3x^2 - 12$; $4x^2 + 12x + 8$

$2(x+1)$; $3(x-2)(x+2)$; $4(x+2)(x+1)$

as no common factor so HCF = 1

17. LCM = $2(x+1) \times 3(x-2)(x+2) \times 2$

$= 12(x+1)(x-2)(x+2)$

17. (i) $x^3 - x^5$ & $x^4 - x^7$

$x^3(1-x)(1+x)$; $x^4(1-x)(1+x^2+x)$

HCF $= x^3(1-x)$

(ii) $30(x^2-3x+2)$; $50(x^2-2x+1)$

$3 \times 10(x-2)(x-1)$; $5 \times 10(x-1)(x+1)$

HCF $= 10(x-1)$

18. (i) $x^3 + y^3$; $x^2 - y^2$

$(x+y)(x^2+y^2-xy)$; $(x-y)(x+y)$

LCM $= (x+y)(x-y)(x^2+y^2-xy)$

(ii) $x^4 + x^2y^2 + y^4$; $x^2 + xy + y^2$

$(x^2+y^2+xy)(x^2+y^2-xy)$; x^2+y^2+xy

LCM $= (x^2+y^2+xy)(x^2+y^2-xy) = x^4+y^4+x^2y^2$

19. $\frac{2}{a-1} - \frac{2}{a+1} - \frac{4}{a^2+1} - \frac{8}{a^4+1}$

$\frac{2a+2-2a+2}{a^2-1} - \frac{4}{a^2+1} - \frac{8}{a^4+1}$

$$\frac{4}{a^2-1} - \frac{4}{a^2+1} - \frac{8}{a^4+1}$$

$$\frac{4a^2+4-4a^2+4}{a^4-1} - \frac{8}{a^4+1}$$

$$\frac{8}{a^4-1} - \frac{8}{a^4+1} = \frac{8a^4+8-8a^4+8}{a^8-1}$$

$$= \frac{16}{a^8-1}$$

20.- $m^2+n^2-mn = \left(\frac{x+1}{x-1}\right)^2 + \left(\frac{x-1}{x+1}\right)^2 - \left(\frac{x+1}{x-1}\right)\left(\frac{x-1}{x+1}\right)$

$$= \frac{x^2+2x+1}{(x-1)^2} + \frac{x^2-2x+1}{(x+1)^2} - 1$$

$$= \frac{(x+1)^2(x+1)^2 + (x-1)^2(x-1)^2 - (x-1)^2(x+1)^2}{(x-1)^2(x+1)^2}$$

$$= \frac{x^4+4x^3+6x^2+4x+1+x^4-4x^3+6x^2-4x+1-x^4-1+2x^2}{(x^2-1)^2}$$

$$= \frac{x^4+14x^2+1}{x^4-2x^2+1}$$