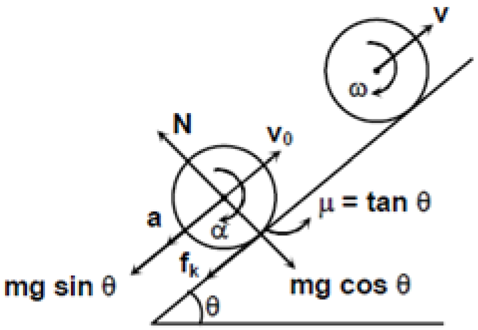
PART (A) : PHYSICS

ANSWER KEY

1.	(B)	2.	(A)	3.	(A)	4.	(B)	5.	(C)
6.	(B)	7.	(D)	8.	(B)	9.	(B)	10.	(D)
11.	(B)	12.	(A)	13.	(D)	14.	(C)	15.	(B)
16.	(B)	17.	(A)	18.	(C)	19.	(C)	20.	(A)
21.	(7)	22.	(2)	23.	(6)	24.	(4)	25.	(6)
26.	(2)	27.	(6)	28.	(4)	29.	(4)	30.	(2)

SOLUTIONS

1. (B)



$$a = \frac{mg\sin\theta + f_k}{m} = \frac{mg\sin\theta + mg\sin\theta}{m}$$

$$a = 2g\sin\theta$$

$$\tau_{CM} = I_{CM}\alpha$$

$$f_k R = \frac{2}{5}mR^2\alpha$$

$$\alpha = \frac{5g\sin\theta}{2R}$$

When pure rolling motion starts

$$v = \omega R$$

$$v_0 - at = \alpha t R$$

$$v_0 - 2g\sin\theta t = \frac{5g\sin\theta t}{2}$$

$$v_0 = \frac{9g\sin\theta t}{2}$$

$$t = \frac{2v_0}{9g\sin\theta}$$

2. (A)

$$\int \tau.dt = I\omega$$

$$\int 2qE\ell dt = I\omega$$

$$\int 2q\ell \frac{R^2}{2\ell}\frac{dB}{dt}.dt = \left(\frac{4m\ell^2}{12} + m\ell^2 + m\ell^2\right)\omega$$

$$qR^2B = \frac{7m\ell^2}{3}\omega$$

$$\omega = \left(\frac{3qBR^2}{7m\ell^2}\right)$$

3. (A)

$$\phi = \frac{\sqrt{3}}{4}\ell^2 B\sin\omega t$$

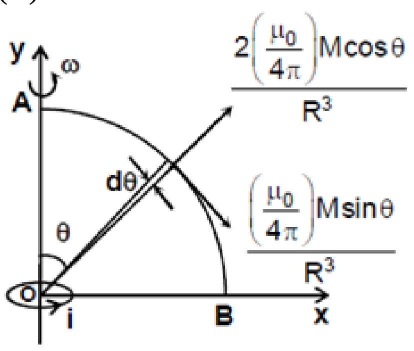
$$|\varepsilon| = \frac{\sqrt{3}}{4}\ell^2\omega B\cos\omega t$$

$$I = \frac{\sqrt{3}\ell^2\omega B}{4R}\cos\omega t$$

$$P_{av} = \frac{\int_0^{2\pi/\omega} I^2 R dt}{\int_0^{2\pi/\omega} dt} = \left(\frac{\sqrt{3}\ell^2 B\omega}{4R}\right)^2 R\frac{\int_0^{\frac{2\pi}{\omega}} \cos^2\omega t dt}{\int_0^{2\pi/\omega} dt}$$

$$P_{av} = \left(\frac{\sqrt{3}\ell^2 B\omega}{4R}\right)^2 R\frac{1}{2} = \frac{3B^2\omega^2\ell^4}{32R}$$

4. (B)



$$v = R\sin\theta.\omega$$

$$\int_0^{\pi/2} \frac{2\mu_0}{4\pi R^3} M\cos\theta.R\sin\theta.\omega R d\theta$$

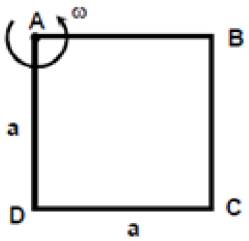
$$= \frac{\mu_0 M\omega}{2\pi\times 2R}\int_0^{\pi/2}\sin 2\theta.d\theta = \frac{\mu_0 M\omega}{4\pi R}\left(-\frac{\cos 2\theta}{2}\right)_0^{\pi/2}$$

$$\frac{\mu_0 M\omega}{4\pi R}\left(\frac{1}{2}+\frac{1}{2}\right)=\frac{\mu_0 M\omega}{4\pi R}=\frac{\mu_0 \omega i\pi a^2}{4\pi R}=\frac{\mu_0 a^2\omega i}{4R}$$

5. (C)

$$F = M\left|\frac{dB}{dr}\right| = \frac{\mu_0}{4\pi}\frac{6M^2}{d^4} = I\pi R^2\left(\frac{3\mu_0 IR^2}{2d^4}\right)$$

6. (B)

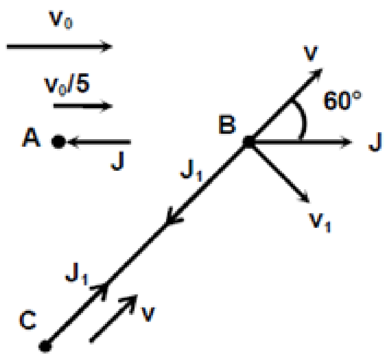


Using conservation of angular momentum about the vertical axis passing through vertex 'A'.

$$mv\frac{a}{2}+mv\frac{a}{2}=\frac{2}{3}ma^2\omega$$

$$\Rightarrow mva=\frac{2}{3}ma^2\omega \Rightarrow \omega=\frac{3v}{2a}$$

7. (D)



$$J = m\left(v_0-\frac{v_0}{5}\right)$$

$$J=\frac{4mv_0}{5}$$

$$J\cos 60^{o} = 2mv$$

$$\frac{2mv_0}{5}=2mv$$

$$v=\frac{v_0}{5}$$

Impulse due to string on the ball 'C' during collision is

$$J_1 = mv$$

$$\Rightarrow J_1=\frac{mv_0}{5}$$

8. (B)

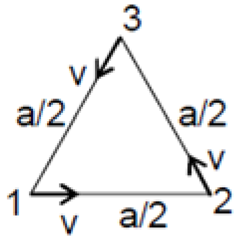
The wavelength of sound received by the observer is

$$\lambda = \frac{4\left(v - \frac{v}{5}\right)}{f} = \frac{16v}{5f}$$

9. (B)

$$F = \int_{h/2}^{h} (P_0 + \rho g y) 2R dy = \left(P_0 + \rho g \frac{3h}{4}\right) Rh$$

10. (D)



Acceleration of each particle

$$a_0 = v\omega = v\frac{v\sin 60^o}{a/2} = \frac{\sqrt{3}v^2}{a}$$

11. (B)

$$80 = 10\log\frac{I_1}{I_0} \quad \text{.......... (i)}$$

$$40 = 10\log\frac{I_2}{I_0} \quad \text{............. (ii)}$$

From (i) and (ii)

$$40 = 10\log\frac{I_1}{I_2}$$

$$4 = \log\left(\frac{r_2}{r_1}\right)^2$$

$$\frac{r_2}{r_1} = 10^2$$

$$r_2 = 10 \times 10^2 \text{cm} = 10\,\text{m}$$

12. (A)

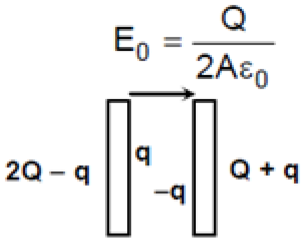
Mobility will remain same.

13. (D)

Frequency of the sonometer wire, $f = \frac{2}{2\ell}\sqrt{\frac{F}{\mu}} = \frac{1}{0.17}\sqrt{\frac{289}{1}} = 100\,\text{Hz}$

Frequency of the tuning fork is $f_0 = f + 5 = 105\,\text{Hz}$

14. (C)



$$\left(\frac{2Q-q}{2\varepsilon_0 A}\right)+\left(\frac{Q}{2\varepsilon_0 A}\right)=\left(\frac{Q+q}{2\varepsilon_0 A}\right)$$

$$(2Q-q)+Q=Q+q$$

$$q=Q$$

15. (B)

$$I=\frac{I_0}{2}\cos^2 37^\circ \times \cos^2\theta=\frac{I_0 8}{100}$$

$$\frac{I_0}{2}\times\frac{16}{25}\times\cos^2\theta=\frac{I_0\times 4}{50}$$

$$\cos^2\theta=\frac{1}{4}$$

$$\cos\theta=\frac{1}{2}$$

$$\theta=60^\circ$$

16. (B)

$$\sin 60^\circ=\mu\sin r$$

$$\Rightarrow \frac{\sqrt{3}}{2}=\sqrt{3}\sin r$$

$$r=30^\circ$$

The net deviation produced in the ray

$\delta=30^\circ+100^\circ+30^\circ=160^\circ$ clockwise

17. (A)

L.S.B. $=(1000-5)\,\text{kHz}=995\,\text{kHz}$

U.S.B. $=(1000+5)\,\text{kHz}=1005\,\text{kHz}$

18. (C)

$$m=\rho\pi r^2\ell$$

$$100\times\frac{\Delta m}{m}=\left(\frac{\Delta\rho}{\rho}+2\frac{\Delta r}{r}+\frac{\Delta\ell}{\ell}\right)\times100=\left(\frac{0.004}{0.4}+2\times\frac{0.006}{0.6}+\frac{0.05}{5}\right)\times100$$

Maximum % error in the mass $=(0.01+2\times0.01+0.01)\times100=4\%$

19. (C)

$$(\mu_1-\mu_2)t=3\lambda$$

$$t=\frac{3\lambda}{(\mu_1-\mu_2)}=\frac{3\times5\times10^{-7}}{(1.8-1.5)}=5\,\mu m$$

20. (A)

The equation of process is $P=\alpha\sqrt{V}$

$PV^{-1/2}=\text{constant}$

This is a polytropic process with a polytropic constant, $x=-(1/2)$

$\therefore$ Molar heat capacity of the gas

$$C=C_V+\frac{R}{(1-x)}=\frac{5R}{2}+\frac{R}{\left(1+\frac{1}{2}\right)}=\frac{5R}{2}+\frac{2R}{3}=\frac{19R}{6}$$

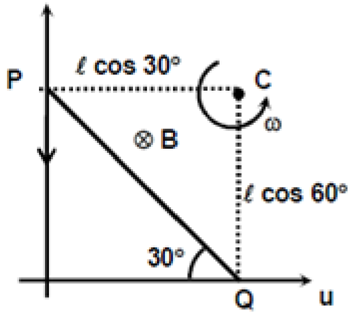
21. (7)

$$\frac{4}{3}\sin37^\circ=1\times\sin r$$

$r=53^\circ$

So, $x=4\tan37^\circ+3\cot37^\circ=7\,m$

22. (2)



Angular velocity of the rod is

$$\omega=\frac{u}{\ell\cos60^\circ}=\frac{2u}{\ell}$$

Potential difference between P and Q

$$=\frac{1}{2}B\omega\ell^2\left(\cos^2 30^\circ-\cos^2 60^\circ\right)=\frac{Bu\ell}{2}$$

$\therefore n=2$

23. (6)

Pitch of the screw gauge, $P = 1\text{mm}$

Least count, $L.C. = \frac{P}{N} = \frac{1}{100} = 0.01\text{mm}$

Diameter of the wire, $d = 3\text{mm} + 25 \times L.C. = 3\text{mm} + 25 \times 0.01\text{mm}$

$\therefore d = 3.25\text{mm}$

24. (4)

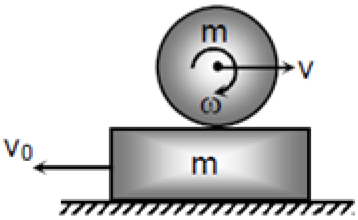
$I = \frac{6}{300} = 0.02$ ampere = 20 milliampere

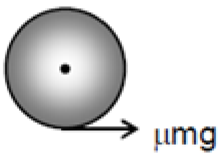
25. (6)

$$T = 2\pi\sqrt{\frac{\frac{mR^2}{2} + mx^2}{mgx}}$$

T will be minimum for $x = \frac{R}{\sqrt{2}} = \frac{40}{\sqrt{2}} = \frac{40}{\sqrt{2}} = 20\sqrt{2} = 28.2\text{cm}$

26. (2)





$\tau_{CM} = I_{CM}\alpha$

$\mu mgR = \frac{mR^2}{2}\alpha$

$\alpha = \frac{2\mu g}{R}$

When pure rolling motion starts

$\omega R - v = v_0$

$(\omega_0 - \alpha t)R - \mu gt = \mu gt$

$\omega_0 R - 2\mu gt - \mu gt = \mu gt$

$\omega_0 R = 4\mu gt$

$\Rightarrow t = \frac{\omega_0 R}{4\mu g} = \frac{40 \times 0.4}{4 \times 0.2 \times 10} = 2\,\text{sec}$

27. (6)

The equation of process AB is

$$P=\left(\frac{P_0}{V_0}\right)V \Rightarrow PV^{-1}=\frac{P_0}{V_0}=\text{constant}$$

$\therefore$ This is a polytropic process with a polytropic constant, $x=-1$

$\therefore$ Molar heat capacity, $C=C_V+\frac{R}{(1-x)}=\frac{5R}{2}+\frac{R}{2}=3R$

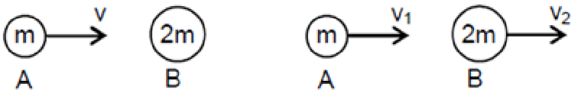
28. (4)

We know $E \propto \sqrt{P_{av}}$

$$\frac{E_2}{E_1}=\sqrt{\frac{P_2}{P_1}}=\sqrt{\frac{100}{200}}=\frac{1}{\sqrt{2}}$$

$\Rightarrow \quad E_2=\frac{E}{\sqrt{2}}$

29. (4)



From conservation of momentum

$mv+0=mv_1+2mv_2$

$v=v_1+2v_2$ .... (i)

Coefficient of restitution;

$e=\frac{v_2-v_1}{v}$

$v_2-v_1=v$ .... (ii)

From (i) and (ii);

$v_2=\frac{2v}{3}, \quad v_1=-\frac{v}{3}$

$$\frac{\lambda_A}{\lambda_B}=\frac{2mv_2}{mv_1}=\frac{2v_2}{v_1}=4$$

30. (2)

When $y=\frac{\beta}{4}=\frac{D\lambda}{4d}$

Path difference, $\Delta r=d\sin\theta=d\left(\frac{y}{D}\right)=d\left(\frac{\lambda}{4d}\right)=\frac{\lambda}{4}$ (for small 'θ')

$\therefore$ Phase difference, $\delta=\frac{2\pi}{\lambda}\times\Delta r=\frac{2\pi}{\lambda}\times\frac{\lambda}{4}=\frac{\pi}{2}$

$$I=I_0\cos^2\left(\frac{\delta}{2}\right)=I_0\cos^2\left(\frac{\pi}{4}\right)=\frac{I_0}{2}$$

PART (B) : CHEMISTRY

ANSWER KEY

31.	(D)	32.	(D)	33.	(A)	34.	(B)	35.	(C)
36.	(A)	37.	(C)	38.	(C)	39.	(D)	40.	(B)
41.	(B)	42.	(B)	43.	(B)	44.	(D)	45.	(B)
46.	(B)	47.	(A)	48.	(A)	49.	(D)	50.	(B)
51.	(18)	52.	(6)	53.	(6)	54.	(4)	55.	(9)
56.	(5)	57.	(5)	58.	(10)	59.	(6)	60.	(2)

SOLUTIONS

31. (D)
Lower the positive charge, larger the ion.

32. (D)
Bond length $\propto \frac{1}{\text{Bond order}}$

33. (A)

34. (B)
Entropy is a state function hence
$\Delta S_{A\to B} = \Delta S_{A\to C} + \Delta S_{C\to D} + \Delta S_{D\to B}$
= [50 + 30 + (-20)]eu
= 60eu

35. (C)

36. (A)
NO_2 is a brown coloured gas, because molecular orbital electronic configuration of NO_2 confirms the presence of unpaired electrons, which can easily undergo transition form ground state energy level to excited state level by absorbing light of suitable wavelength.
N_2O_3 is blue coloured liquid because low energy difference between occupied and unoccupied energy levels of electrons among molecules of N_2O_3, it absorbs a part of visible light spectrum which cause colour in N_2O_3.

37. (C)
$m = \frac{13.44}{134.1} = 0.1$
i = 3
$\Delta T_b = iK_b m = 3 \times 0.52 \times 0.1 = 0.156 \simeq 0.16$

38. (C)

39. (D)

40. (B)

41. (B)

42. (B)

43. (B)

$4HNO_3 \rightarrow 2H_2O + 2NO_2 + 3O_2$

44. (D)

45. (B)

46. (B)

47. (A)

$CH_3CH_2CHO + H_2NHN - CO - NH_2 \rightarrow CH_3 - CH_2 - CH = N - NH - CONH_2$

48. (A)

49. (D)

$$\underset{(A)}{CH_3CH_2\,COOH} \xrightarrow{NH_3} CH_3CH_2COONH_4 \xrightarrow{\Delta} CH_3 - CH_2 - \overset{O}{\overset{||}{C}} - NH_2$$

$$CH_3 - CH_2 - \overset{O}{\overset{||}{C}} - NH_2 \xrightarrow{Br_2,KKOH} CH_3CH_2\text{-}NH_2$$

50. (B)

51. (18)

$t_{1/2} = \dfrac{[A_0]}{2k}$

$\Rightarrow 6 = \dfrac{0.2}{2k} \quad \therefore k = \dfrac{0.2}{2\times 6} = \dfrac{1}{60}$

$[A_0] - [A_t] = kt$

$0.5 - 0.2 = \dfrac{1}{60}\times t \qquad \therefore t = 0.3\times 60 = 18h$

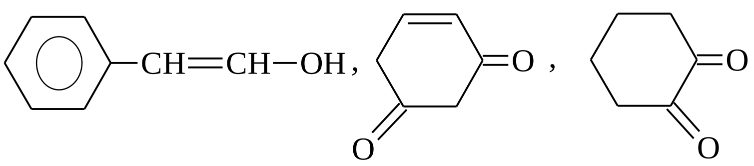
52. (6)
$MgCl_2.6H_2O$

53. (6)
H_2SO_4, H_3PO_3, H_2CO_3, $H_2S_2O_7$, H_2CrO_4 and H_2SO_3

54. (4)

55. (9)
9 sigma bonds, $\therefore x = 9$
1 pi bond $\therefore y = 1$
2 lone pairs $\therefore z = 2$
$x - 2y + z = 9 - 2\times1 + 2 = 9$

56. (5)



$C_6H_5-\overset{O}{\overset{\|}{C}}-CH_3$, $C_6H_5-\overset{O}{\overset{\|}{C}}-CH_2-\overset{O}{\overset{\|}{C}}-CH_3$

57. (5)

$$R-NH_2 + CH_3-\overset{O}{\overset{\|}{C}}-Cl \xrightarrow[-HCl]{} R-NH-\overset{O}{\overset{\|}{C}}-CH_3$$

Since, each –$COCH_3$ group displacer one H-atom, r x n of 1 mole of $CH_3-\overset{O}{\overset{\|}{C}}-Cl$ with one –NH_2 group, the molecular mass increases with 42 units.

$\therefore$ No. of NH_2 groups $= \frac{210}{42} = 5$

58. (10)
4 moles of NaOH and 1 mole of Br_2 required

59. (6)
$8MnO_4^- + 3S_2O_3^{2-} \rightarrow 6SO_4^{2-} + 8MnO_2$

60. (2)

	A	+	B	$\rightleftharpoons$	C	+	D
At eq^n	1- x		1 – x		1 + x		1 + x

$$\Rightarrow 100 = \left(\frac{1+x}{1-x}\right)^2$$

$\therefore x = 0.818$

[D] = 1 + x = 1 + 0.818 = 1.818

PART (C) : MATHEMATICS

ANSWER KEY

61.	(A)	62.	(C)	63.	(D)	64.	(D)	65.	(B)
66.	(B)	67.	(B)	68.	(B)	69.	(C)	70.	(C)
71.	(B)	72.	(C)	73.	(D)	74.	(A)	75.	(B)
76.	(B)	77.	(A)	78.	(A)	79.	(B)	80.	(D)
81.	(4)	82.	(601)	83.	(16)	84.	(1)	85.	(7)
86.	(19)	87.	(88)	88.	(28)	89.	(35)	90.	(12)

SOLUTIONS

61. (A)

Equaiton of tangent at (4, 2) is $yy_1 = \frac{1}{2}(x+x_1) \Rightarrow 2y = \frac{1}{2}(x+4) \Rightarrow 4y = x+4 \Rightarrow y = \frac{x}{4}+1$

So, slope of tangent $= \frac{1}{4}$

$\therefore$ Slope of normal $= -4$

62. (C)

$$\frac{x+1}{1} = \frac{y+3}{3} = \lambda = \frac{z-2}{-2}$$

$\Rightarrow$ any point on this line is $P(\lambda-1, 3\lambda-3, -2\lambda+2)$.

If this point lies in the plane $3x+4y+5z-5=0$

Then $3\lambda-3+12\lambda-12-10\lambda+10-5=0$

$\Rightarrow$ $5\lambda-10=0 \Rightarrow \lambda=2$.

$\therefore$ Point of intersection of the line and the plane is $A(1, 3, -2)$.

It is given that $AP = \sqrt{14}$.

$\therefore$ $(\lambda-2)^2 + (3\lambda-6)^2 + (2\lambda-4)^2 = 14$

$\Rightarrow$ $14\lambda^2 - 56\lambda + 56 = 14$

$\Rightarrow$ $\lambda^2 - 4\lambda + 3 = 0 \Rightarrow \lambda = 1, 3$.

63. (D)

Probability of happing of atleast one of them

= 1 – (probability of not happening of all of them)

$= 1-(1-p_1)(1-p_2)(1-p_3)$

64. (D)

In set of integers Z 0 does not divide 0, So R is not reflexive.

2 R 4 but 2 R 4 but 4 $\not R$ 4

So R is not symmetric.

$aRb, bRc \Rightarrow a$ divides b, b divides c

$\Rightarrow b \neq 0, b = pa, c = qb$

$\Rightarrow c = (pq)a,\ pq \in \mathbf{Z}$

$\Rightarrow a$ divides c

$\Rightarrow a\,R\,c$

$\therefore$ R is transitive.

65. (B)

$p \to (p \vee q) \equiv \sim p \vee (p \vee q)$

$\equiv (\sim p \vee q) \vee q \equiv T \vee q = T$

$\therefore$ $p \to (p \vee q)$ is also a tautology.

Thus, $p \to (q \to p)$ is equivalent to $p \to (p \vee q)$

66. (B)

Let (h, k) be the mid-point of a chord of the circle $x^2 + y^2 = 16$.

Then, equation of the chord is

$hx + ky - 16 = h^2 + k^2 - 16$

$\Rightarrow$ $hx + ky = h^2 + k^2$

If this is parallel to $4x - 2y + 5 = 0$.

Then, $-\frac{h}{k} = 2 \Rightarrow -h = 2k$

$\Rightarrow h + 2k = 0$

Hence, the locus of (h, k) is $x + 2y = 0$.

67. (B)

If $(a, b) \in R$, then $(b, a) \in R^{-1} = R$

So, R is symmetric.

68. (B)

Equation of the straight line having (x_1, y_1) as its mid point is of the form

$\frac{x}{x_1} + \frac{y}{y_1} = 2 = \frac{x}{6} + \frac{y}{8} = 2 \Rightarrow 4x + 3y = 24$

69. (C)

Let $y=\dfrac{x^2+x+2}{x^2+x+1}=1+\dfrac{1}{x^2+x+1}=1+\dfrac{1}{\left(x+\frac{1}{2}\right)^2+\frac{3}{4}}>1$

$\Rightarrow \quad \dfrac{1}{y-1}=x^2+x+1=\left(x+\dfrac{1}{2}\right)^2+\dfrac{3}{4}\geq\dfrac{3}{4}$

Thus, $y>1$ and $y-1\leq\dfrac{4}{3}\Rightarrow \quad y\leq\dfrac{7}{3}$

$\therefore \quad 1<y\leq\dfrac{7}{3}$

70. (C)

Let O be the centre of the balloon of radius r which subtends an angle α at E, the eye of the observer. If EA and EB are the tangents to the balloon, then

$\angle OEA=\angle OEB=\dfrac{\alpha}{2}$

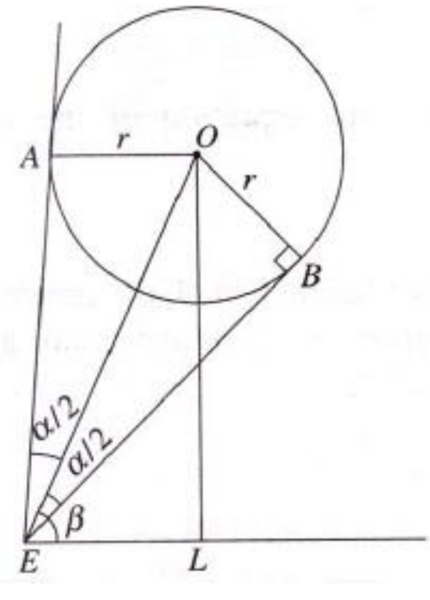
In ΔOEA, $\sin\left(\dfrac{\alpha}{2}\right)=\dfrac{OA}{OE}\Rightarrow OE=r\,\text{cosec}\left(\dfrac{\alpha}{2}\right)$

Let $OL=h$ be the height of the centre of the balloon, then from ΔOLE

$h=OE\sin\beta$

$=r\,\text{cosec}\left(\dfrac{\alpha}{2}\right)\sin\beta$

$=\dfrac{r\sin\beta}{\sin\left(\dfrac{\alpha}{2}\right)}$



71. (B)

Let $f(x)=x^{1/x} \Rightarrow \log f(x)=\left(\frac{1}{x}\right)\log x$.

Differentiating both the sides, we have $f'(x)=f(x)\left[\frac{1-\log x}{x^2}\right]$

So, $f'(x)=0 \Leftrightarrow x=e$.

Also $f'(x)>0$ for $0<x<e$ and $f'(x)<0$ for $e<x<\infty$.

Thus, $f(x)$ has a maximum at $x=e$ and max $f(x)=e^{1/e}$.

72. (C)

Let $p=3h+2$ and $q=k$ or $h=\frac{p-2}{3}$ and $k=q$

Since, (h,k) lies on $x^2+y^2=1$, we have $h^2+k^2=1$ or $\left(\frac{p-2}{3}\right)^2+q^2=1$

The locus is $\left(\frac{x-2}{3}\right)^2+y^2=1$ which has eccentricity $e=\sqrt{1-\frac{1}{9}}=\frac{2\sqrt{2}}{3}$

73. (D)

Tangent to the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ is

$y=mx\pm\sqrt{a^2m^2-b^2}$

Given that $y=ax+\beta$ is the tangent of hyperbola

$\Rightarrow \quad m=\alpha$ and $\therefore \quad a^2m^2-b^2=\beta^2$

Locus is $a^2x^2-y^2=b^2$ which is hyperbola.

74. (A)

Let $I=\int\frac{x^2-x}{x^3-x^2+x-1}dx$

$=\int\frac{x(x-1)}{x^2(x-1)+(x-1)}dx=\int\frac{x\,dx}{x^2+1}$

$=\frac{1}{2}\int\frac{2x\,dx}{(x^2+1)}$

Let $x^2+1=t \Rightarrow 2x\,dx=dt$

$\therefore \quad I=\frac{1}{2}\int\frac{dt}{t}=\frac{1}{2}\log t+c=\frac{1}{2}\log\left(x^2+1\right)+c$

Where $'c'$ is the constant of integration.

75. (B)

We know that $\left(x_1+x_2+...+x_n\right)^3$

$=\sum \frac{6}{\lambda_1!\lambda_2!....\lambda_n!}x_1^{\lambda_1}x_2^{\lambda_2}........x_n^{\lambda_n}$

Where $\lambda_1+\lambda_2+...+\lambda_n=3$ and $\lambda_i\ge 0$

Thus number of distinct terms in the expansion of $\left(x_1+x_2+....+x_n\right)^3$ is equal to the number of non negative integral solutions of $\lambda_1+\lambda_2+....+\lambda_n=3$

= number of ways of distributing 3 identical object among *n* persons

$={}^{3+n-1}C_3={}^{n+2}C_3$

76. (B)

Given $f\left(x\right)$ is continuous at $x=0$. Therefore,

$\lim_{x\to 0} f\left(x\right)=f\left(0\right)$

or $\lim_{x\to 0}\frac{\left(3^x-1\right)^2}{\sin x\ln\left(1+x\right)}=f\left(0\right)$

or $f\left(0\right)=\lim_{x\to 0}\frac{\left(\frac{3^x-1}{x}\right)^2}{\left(\frac{\sin x}{x}\right)\left(\frac{\ln\left(1+x\right)}{x}\right)}=\left(\ln 3\right)^2$

77. (A)

Use, $\int_a^b f\left(x\right)dx=\int_a^b f\left(a+b-x\right)dx$

78. (A)

$\left|z-14-6i\right|=\left|\left(z-2-i\right)-\left(12+5i\right)\right|\le\left|z-2-i\right|+\left|12+5i\right|$

$\Rightarrow\ \left|z-14-6i\right|\le 5+13=18$

$\therefore$ Option (A) is correct.

The complete solution can be obtained geometrically

79. (B)

$\left(\frac{1}{1}-\frac{1}{2}+\frac{1}{2}-\frac{1}{3}+....+\frac{1}{n}-\frac{1}{n+1}\right)^2$

$\frac{n^2}{\left(n+1\right)^2}$

80. (D)

$$\left|\frac{2\sqrt{(a+b)^2-ab}}{a+b}\right|=1$$

$\Rightarrow \quad (a+b)^2=4\left(a^2+b^2+ab\right)$

$\Rightarrow \quad 3a^2+3b^2+2ab=0$

81. (4)

Let D be the origin of reference and $\overline{DA}=\bar{a}, \overline{DB}=\bar{b}, \overline{DC}=\bar{c}$

$\left|\overline{AB}\times\overline{CD}+\overline{BC}\times\overline{AD}+\overline{CA}\times\overline{BD}\right|$

$=\left|\left(\bar{b}-\bar{a}\right)\times\left(-\bar{c}\right)+\left(\bar{c}-\bar{b}\right)\times\left(-\bar{a}\right)+\left(\bar{a}-\bar{c}\right)\times\left(-\bar{b}\right)\right|$

$=2\left|\bar{a}\times\bar{b}+\bar{b}\times\bar{c}+\bar{c}\times\bar{a}\right|$

$=2$ (2 area of ΔABC) = 4 area of ΔABC.

Hence, $\lambda=4$.

82. (601)

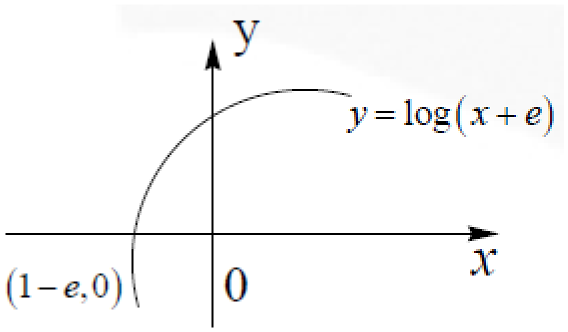
Rank of the word SACHIN $=\left(5\times\lfloor 5\right)+1=601$

83. (16)

$\bar{x}=5$, variance $=\frac{1}{n}\sum x_1^2-\left(\bar{x}\right)^2 \Rightarrow 0=\frac{1}{n}\cdot 400-25$

$\Rightarrow \quad n=\frac{400}{25}=16$

84. (1)



$A=\int_{1-e}^{0} y\,dx$

85. (7)

$$\frac{1}{\sin\frac{\pi}{n}}-\frac{1}{\sin\frac{3\pi}{n}}=\frac{1}{\sin\frac{2\pi}{n}}$$

or $$\frac{\sin\frac{3\pi}{n}-\sin\frac{\pi}{n}}{\sin\frac{\pi}{n}\sin\frac{3\pi}{n}}=\frac{1}{\sin\frac{2\pi}{n}}$$

or $$\frac{\left(2\sin\frac{\pi}{n}\cos\frac{2\pi}{n}\right)\sin\frac{2\pi}{n}}{\sin\frac{\pi}{n}\sin\frac{3\pi}{n}}=1$$

or $\sin\frac{4\pi}{n}=\sin\frac{3\pi}{n} \Rightarrow \frac{4\pi}{n}+\frac{3\pi}{n}=\pi$

$\Rightarrow$ $n=7$

86. (19)

$$\Delta=\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ (100a_1+10b_1+c_1) & (100a_2+10b_2+c_2) & (100a_3+10b_3+c_3)\end{vmatrix}$$

$[R_3 \to R_3+100R_1+10R_2]$

$$=\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ x_1 & x_2 & x_3\end{vmatrix}=\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ 19m_1 & 19m_2 & 19m_3\end{vmatrix}$$ [where each $m \in N$]

$$=19\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ m_1 & m_2 & m_3\end{vmatrix}=19n$$

Where $n=\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ m_1 & m_2 & m_3\end{vmatrix}$ is certainly an integer.

87. (88)

Let $a, A_1, A_2.....A_g, b \in \text{A.P}$

Where $a=2, b=20, n=8$

$\therefore$ Sum of the means $=\frac{n}{2}(a+b)=\frac{8}{2}(2+20)=88$

88. (28)

89. (35)

90. (12)