PART (A) : PHYSICS

ANSWER KEY

1.	(B)	2.	(B)	3.	(C)	4.	(B)	5.	(B)
6.	(C)	7.	(D)	8.	(A)	9.	(A)	10.	(D)
11.	(A)	12.	(C)	13.	(C)	14.	(B)	15.	(A)
16.	(C)	17.	(A)	18.	(A)	19.	(C)	20.	(D)
21.	(5)	22.	(7)	23.	(5)	24.	(9)	25.	(3)
26.	(4)	27.	(4)	28.	(7)	29.	(2)	30.	(4)

SOLUTIONS

1. (B)

Maximum range up the inclined plane $=\dfrac{u^2}{g(1+\sin\theta)}$

Maximum range down the inclined plane $=\dfrac{u^2}{g(1-\sin\theta)}$

The maximum possible distance between the two bullets after they hit the inclined plane

$$\therefore d_{max}=\frac{u^2}{g(1+\sin\theta)}+\frac{u^2}{g(1-\sin\theta)}$$

$$d_{max}=\frac{2u^2}{g\cos^2\theta}$$

2. (B)

Use the basic concept of interference of light waves.

3. (C)

Acceleration of block $=-3\,m/s^2$

Acceleration of plank $=-7\,m/s^2$

After 3 sec, the plank will come to rest. In 3 sec, the block will move $\frac{1}{2}\times4\times3\times3=18\,m$ relative to the plank.

Now, let the block will cover remaining 18 m relative to plank in next t seconds.

$$12t-\frac{1}{2}\times3t^2=18$$

$$8t-t^2=12\Rightarrow t=2\sec$$

Total time after which the block will get off the plank, $t_0=3+2=5\sec$.

4. (B)

At θ_{min}

$$\lambda gR(1-\sin\theta)-\lambda gR(1-\cos\theta)-\mu\lambda gR(\sin\theta+\cos\theta)=0$$

$$\Rightarrow \cos\theta-\sin\theta-\frac{\sin\theta}{3}-\frac{\cos\theta}{3}=0$$

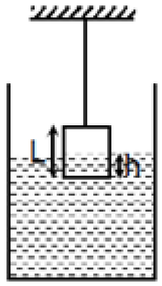
$$\therefore \theta=\tan^{-1}(1/2)$$

5. (B)

$$f \propto \sqrt{T}$$

$$T_0=n\rho_w L^3 g$$

$$T_{new}=n\rho_w L^3 g-2\rho_w L^2 hg=\rho_w L^2 g(nL-2h)$$



$$\therefore \frac{f_{new}}{f_0}=\sqrt{\frac{\rho_w L^2 g(nL-2h)}{n\rho_w L^3 g}}$$

$$f_{new}=\left(\frac{nL-2h}{nL}\right)^{1/2} f_0$$

6. (C)

$$\phi=\frac{\pi}{2}$$

$$\frac{t}{T}=\frac{\pi/2}{2\pi}$$

$$\therefore t=T/4=\frac{\pi}{2}\sqrt{\frac{m}{k}}$$

7. (D)

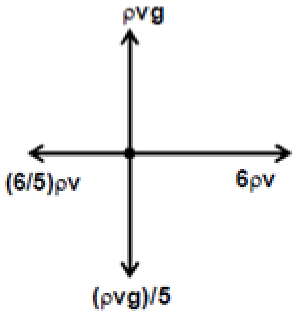
$$3L(1+\alpha_{eff}\Delta\theta)=L(1+2\alpha\Delta\theta)+2L(1+\alpha\Delta\theta)$$

$$3L\alpha_{eff}\Delta\theta=4\alpha L\Delta\theta$$

$$\therefore \alpha_{eff}=\frac{4}{3}\alpha$$

8. (A)

F.B.D.

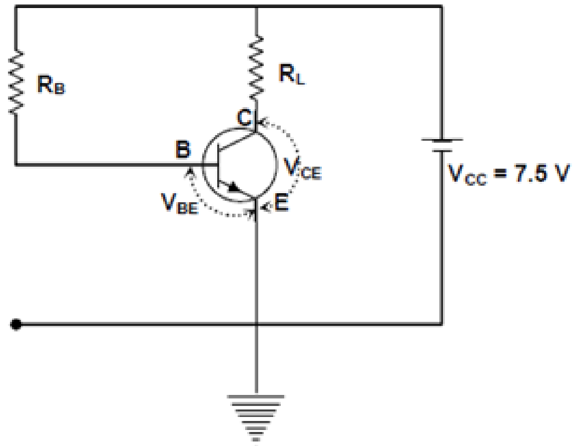


9. (A)

$I = neAv_d$

$I = kV^2$

10. (D)



$V_{CC} = I_B R_B + V_{BE}$

$I_B = \frac{V_{CC} - V_{BE}}{R_B} = \frac{7.5 - 1}{130 \times 10^3} = 50 \times 10^{-6} = 50\,\mu A$

$I_C = \beta I_B = 100 \times 5 \times 10^{-5} = 5 \times 10^{-3} A = 5mA$

Now, $I_C R_L + V_{CE} = V_{CC}$

$R_L = \frac{V_{CC} - V_{CE}}{I_C} = \frac{7.5 - 3.5}{5 \times 10^{-3}} = \frac{4000}{5} = 800\,\Omega$

11. (A)

$d(K.E.) = mv \times \Delta v$

Change in velocity Δv has fixed magnitude

Greater change in kinetic energy is achieved when impulse is parallel to the velocity and when the speed is as large as possible.

12. (C)

Then intensity of the emerging light, $I = \frac{I_0}{2}\cos^2\left(37^o\right) = \frac{I_0}{2} \times \left(\frac{4}{5}\right)^2$

$I = \frac{8I_0}{25} = 8\,W/m^2$

13. (C)

$V_0 = \frac{k\rho a^2}{\varepsilon_0} =$ potential at the centre of a cube of side ‘a’

$k\rho \times \frac{4a^2}{\varepsilon_0} = 4V_0 =$ potential at the centre of a cube of side ‘2a’

Six such pyramids make a cube

Potential at the tip of pyramid $= \frac{4V_0}{6} = \frac{2V_0}{3}$

14. (B)

$\frac{dQ_1}{T_1} + \frac{dQ_2}{T_2} = 0$

$\frac{dT_1}{T_1} = -\frac{dT_2}{T_2} \Rightarrow T_1T_2 =$ constant

When their temperatures become equal

$T_1T_2 = T^2$

$\Rightarrow$ Final temperature, $T = \sqrt{T_1T_2}$

15. (A)

$t = 1.5h$

Total charge transferred $= 4.5\,Ah$

$Q_{max} = 60\,Ah$

Percentage charged $= \frac{4.5}{60} \times 100 = 7.5\%$

16. (C)

Use Faraday’s Law

17. (A)

$nhv = \frac{100 \times 60}{100} = 60\,J$

$n = \frac{60\lambda}{hc} = \frac{60 \times 3.9 \times 10^{-7}}{6.63 \times 10^{-34} \times 3 \times 10^8} = 1.18 \times 10^{20}$

18. (A)

Using conservation of momentum

$mu = mv_1 + Mv_2$

$\Rightarrow 2v_2 + v_1 = u$ .... (i)

$v_2 - v_1 = u$ .... (ii)

From (i) and (ii)

$v_2 = 2u/3$

$\therefore$ Fraction of kinetic energy lost by neutron $= \dfrac{\frac{1}{2} \times 2m \times \left(\frac{2u}{3}\right)^2}{\frac{1}{2}mu^2} = \dfrac{8}{9}$

19. (C)

$$2 \times \frac{1}{2g}\left(\frac{p\sin\theta}{m}\right)^2 = \frac{2p\sin\theta}{mg} \times \frac{p\cos\theta}{m}$$

$$\frac{1}{2}\sin^2\theta = \sin\theta\cos\theta \Rightarrow \tan\theta = 2$$

$$\therefore \cos\theta = \frac{1}{\sqrt{5}}$$

Minimum kinetic energy $= \dfrac{(p\cos\theta)^2}{2m} = \dfrac{p^2}{2m} \times \dfrac{1}{5} = \dfrac{p^2}{10m}$

20. (D)

$$A = A_0 e^{-\frac{bt}{2m}}$$

$$A = A_0 e^{-\frac{\ell n 2}{10 \times 2} \times \frac{20}{2}} = A_0 e^{-\ell n\sqrt{2}} = \frac{A_0}{\sqrt{2}}$$

21. (5)

$H = B/\mu_0$

$ni = 2 \times 10^4$

$40 \times 100\, i = 2 \times 10^4$

$i = \dfrac{20}{4} A = 5A$

22. (7)

$m_1 g - T_1 = m_1 a$ .... (i)

$T_2 - m_2 g = m_2 a$ .... (ii)

$T_1 - T_2 \pm \dfrac{m_1 g}{2} = m_1 a$ .... (iii)

$$(m_1 - m_2)g \pm \frac{m_1 g}{2} = (2m_1 + m_2)a$$

$$a = \frac{2m_2 g \pm \frac{3gm_2}{2}}{7m_2}$$

$$a_{max} = \frac{g}{2}$$

$$a_{min} = \frac{g}{14}$$

$$\frac{a_{max}}{a_{min}} = 7$$

23. (5)

$$(\mu_S - 1)t_0 = \frac{d \times y}{D}$$

$$y = (\mu_S - 1)t_0 \frac{D}{d}$$

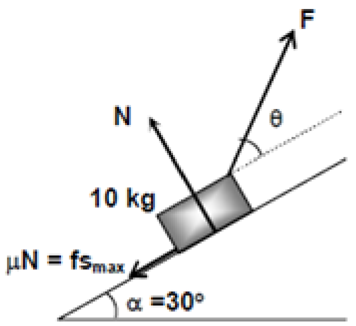
$\Rightarrow$ Velocity of central maxima,

$$v = \frac{dy}{dt} = (2t + 12) \times \frac{2.4 \times 10^{-4} \times 2}{2 \times 10^{-3}}$$

At $t = 4\,\text{sec}$

$$v = 20 \times 2.4 \times 10^{-1} = 4.80\,\text{m/s}$$

24. (9)



$$N = mg\cos\alpha - F\sin\theta$$

$$F\cos\theta = \mu N + mg\sin\alpha$$

$$F(\cos\theta + \mu\sin\theta) = mg(\sin\alpha + \mu\cos\alpha)$$

$$F = \frac{mg(\sin\alpha + \mu\cos\alpha)}{(\cos\theta + \mu\sin\theta)}$$

$$\therefore F_{min} = \frac{mg(\sin\alpha + \mu\cos\alpha)}{\sqrt{1+\mu^2}}$$ [when $\theta = \tan^{-1}(\mu) = 30^o$]

$$\therefore F_{min} = 86.60\,\text{N}$$

25. (3)

$\frac{5\lambda}{4} = \ell_1 + e$ .... (i)

$\frac{9\lambda}{4} = \ell_2 + e$ .... (ii)

From (i) and (ii)

$\lambda = \ell_2 - \ell_1 = 200 - 111 = 89\text{cm}$

From (i), end correction

$e = \frac{5\lambda}{4} - \ell_1 = \frac{5}{4} \times 89 - 111$

$\Rightarrow e = 0.25\text{cm}$

26. (4)

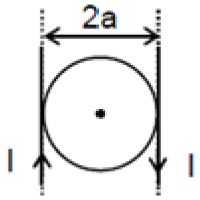
$\vec{v}_i = (v\cos\alpha)\hat{i} + (v\sin\alpha - gt)\hat{j}$

$<\vec{v}> = (v\cos\alpha)\hat{i} + \left(v\sin\alpha - \frac{gt}{2}\right)\hat{j}$

$|\vec{v}_i| = |<\vec{v}>|$

$\Rightarrow t = \frac{4v\sin\alpha}{3g}$

27. (4)



$r = a(1 - \cos\theta) \Rightarrow dr = a\sin\theta\, d\theta$

Flux through the element is

$d\phi = \frac{\mu_0 I}{2\pi a}\left[\frac{1}{(1-\cos\theta)} + \frac{1}{(1+\cos\theta)}\right] 2a^2 \sin^2\theta\, d\theta$

$\phi = \frac{\mu_0 I}{\pi}\int_0^{\pi} \frac{2a}{\sin^2\theta}\sin^2\theta\, d\theta = \frac{\mu_0 I}{\pi} 2a \int_0^{\pi} d\theta$

$\phi = \frac{\mu_0 I}{\pi}\int_0^{\pi} \frac{2a}{\sin^2\theta}\sin^2\theta\, d\theta = \frac{\mu_0 I}{\pi} 2a \int_0^{\pi} d\theta$

$\phi = 2\mu_0 I a$

$\therefore$ Mutual inductance

$M = \phi / I = 2\mu_0 a$

28. (7)

$$V = \frac{Q}{k4\pi\varepsilon_0 r}$$

$$\Rightarrow \frac{Q}{k\varepsilon_0} = 4\pi rv \qquad \text{.......... (i)}$$

$$I = \oint_s \vec{J}.\vec{ds} = \sigma\oint_s \vec{E}.\vec{ds} = \frac{\sigma Q}{k4\pi\varepsilon_0 r^2}4\pi r^2 = \frac{\sigma Q}{k\varepsilon_0} = 4\pi\sigma rv$$

29. (2)

$$\frac{q\lambda}{2\pi\varepsilon_0 r} - \frac{q\mu_0 I}{2\pi r}v = 0$$

$$v = \frac{\lambda}{\varepsilon_0\mu_0 I} = \frac{\lambda c^2}{I}$$

30. (4)

Speed of sound, $v = \sqrt{\frac{B}{\rho}}$

$$PV^2 = K$$

Bulk modulus, $B = -V\frac{dP}{dV} = \frac{2K}{V^2} = 2P$

$$v = \sqrt{\frac{2P}{\rho}}$$

PART (B) : CHEMISTRY

SOLUTIONS

31.	(B)	32.	(A)	33.	(A)	34.	(B)	35.	(C)
36.	(D)	37.	(D)	38.	(A)	39.	(B)	40.	(D)
41.	(D)	42.	(C)	43.	(A)	44.	(A)	45.	(C)
46.	(A)	47.	(A)	48.	(D)	49.	(C)	50.	(D)
51.	(1)	52.	(4)	53.	(2)	54.	(2)	53.	(4)
56.	(1)	57.	(6)	58.	(5)	59.	(44)	60.	(7)

SOLUTIONS

31. (B)

32. (A)
Buffer solution containing mixture of CH_3COOH and CH_3COONa is known as acidic buffer. When an equal volume of unchanged and acidic buffer is formed.
The acetate ion reacts with H^+ ion to form acetic acid thus, the solution contains CH_3COOH and CH_3COONa in equal amounts.

33. (A)

34. (B)

35. (C)

36. (D)

37. (D)

38. (A)

39. (B)

40. (D)

41. (D)

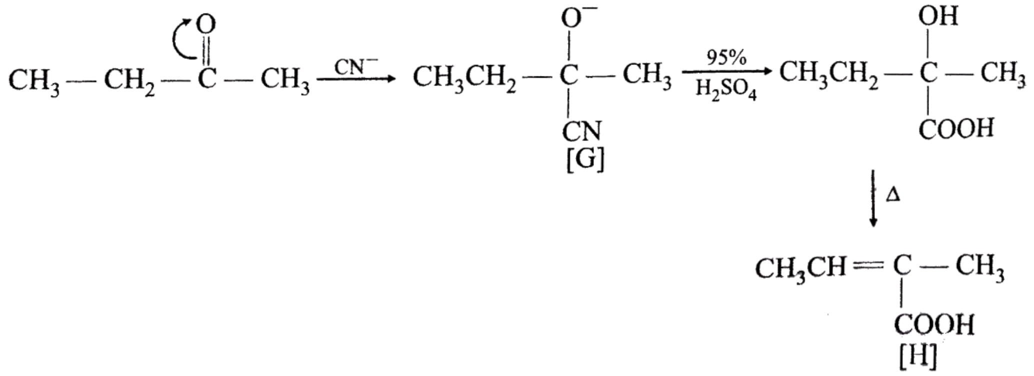
42. (C)

43. (A)

44. (A)

45. (C)

46. (A)



47. (A)

48. (D)

49. (C)

50. (D)

51. (1)
$y = 6, x = 5 \therefore y - x = 6 - 5 = 1$

52. (4)

53. (2)

54. (2)

53. (4)

56. (1)

57. (6)

58. (5)

59. 44

60. (7)

PART (C) : MATHEMATICS

ANSWER KEY

61.	(B)	62.	(A)	63.	(A)	64.	(A)	65.	(C)
66.	(A)	67.	(C)	68.	(A)	69.	(D)	70.	(C)
71.	(B)	72.	(C)	73.	(C)	74.	(B)	75.	(A)
76.	(C)	77.	(C)	78.	(B)	79.	(B)	80.	(B)
81.	(6)	82.	(1)	83.	(6)	84.	(0)	85.	(256)
86.	(3)	87.	(5)	88.	(96)	89.	(4)	90.	(4)

SOLUTIONS

61. (B)

$$\frac{x^2}{5}-\frac{y^2}{5\cos^2\alpha}=1$$

$$e_1^2=1+\frac{b^2}{a^2}=1+\frac{5\cos^2\alpha}{5}=1+\cos^2\alpha;$$

Now, $\frac{x^2}{25\cos^2\alpha}+\frac{y^2}{25}=1$ is $e_2^2=1-\frac{25\cos^2\alpha}{25}=\sin^2\alpha$; Put $e_1=\sqrt{3}\,e_2 \Rightarrow e_1^2=3e_2^2$

$$\Rightarrow\ 1+\cos^2\alpha=3\sin^2\alpha\ \Rightarrow\ 2=4\sin^2\alpha\ \Rightarrow\ \sin\alpha=\frac{1}{\sqrt{2}}$$

62. (A)
Conceptual

63. (A)

64. (A)
Use sin x expansion,

$$\sin x=x-\frac{x^3}{3!}+\frac{x^5}{5!}-........$$

65. (C)
Conceptual

66. (A)
Conceptual

67. (C)
Conceptual

68. (A)

69. (D)

Lines are $x+y+1=0; 4x+3y+4=0$ and $x+\alpha y+\beta=0$ where $\alpha^2+\beta^2=2$

$$\begin{vmatrix} 1 & 1 & 1 \\ 4 & 3 & 4 \\ 1 & \alpha & \beta \end{vmatrix}=0$$

$1(3\beta-4\alpha)-1(4\beta-4)+1(4\alpha-3)=3\beta-4\alpha-4\beta+4+4\alpha-3$

$=-\beta+1=0 \Rightarrow \beta=1$

$\alpha=\pm 1$

70. (C)

71. (B)

Conceptual

72. (C)

$f(x)=\dfrac{t+3x-x^2}{x-4}$; $f'(x)=\dfrac{(x-4)(3-2x)-(t+3x-x^2)}{(x-4)^2}$

For maximum or minimum, $f'(x)=0$

$-2x^2+11x-12-t-3x+x^2=0$

$-x^2+8x-(12+t)=0 \quad \Rightarrow d>0$

73. (C)

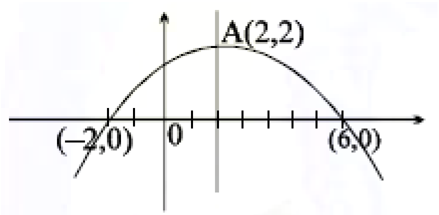
Shifting the origin at A equation is

$x^2=-8y$

Now, $(x-2)^2=-8(y-2)$

$x^2-4x+4=-8y+16$

$x^2-4x+8y-12=0$



74. (B)

$q^2-4pr=0,\ p>0,\ p>0$

$f(x)=\log\left(px^3+(p+q)x^2+(q+r)x+r\right)$

Let $g(x)=px^3+(p+q)x^2+(q+r)x+r$

$g(x)=(x+1)\left(px^2+qx+r\right)$

Discriminant of $px^2+qx+r=q^2-4pr=0$

Domain $(x+1)\left(px^2+qx+r\right)>0$

$$\Rightarrow \quad p(x+1)\left(x+\frac{q}{2p}\right)^2>0$$

$$\Rightarrow \quad x\neq -1 \text{ and } x>-1$$

$$\Rightarrow \quad x\in R-[(-\infty,-1]\cup\left\{-\frac{q}{2p}\right\}$$

75. (A)

f is not differentiable at $x=\frac{1}{2}$

g is not continuous in [0, 1] at $x = 0$

h is not continuous in [0, 1] at $x = 1$ & 0

$k(x)=(x+3)\log_2 5=(x+3)P$ where $2<p<3$

76. (C)

$x=\cos\theta,\ \theta\in[0,\pi]$

$$\cos^{-1}(\cos\theta)+\cos^{-1}\left(\cos\frac{\pi}{3}\cos\theta+\sin\frac{\pi}{3}\sin\theta\right)=\frac{\pi}{3}$$

$\theta+\cos^{-1}\cos\left(\frac{\pi}{3}-\theta\right)=\frac{\pi}{3}$ can hold only if $0\leq\frac{\pi}{3}-\theta<\pi$

$$-\frac{\pi}{3}\leq-\theta\leq\frac{2\pi}{3}$$

$$\therefore\ x\in\left[\frac{1}{2},1\right]0\quad 0\leq\theta\leq\frac{\pi}{3}$$

77. (C)

No. of integral solutions $x_1\,x_2\,x_3\,x_4=2*5*7*11$

Case-I: All $x_i>0$

No. of solutions $=(4)^4$

Case-II: All $x_i<0$

No. of solutions $=(4)^4$

Case-III: Two positive, two negative

${}^4C_2=6$ Combinations

No. of solutions $=6\times(4)^4$

Total no. of integral solutions $=8\times(4)^4$

78. (B)

Conceptual

79. (B)

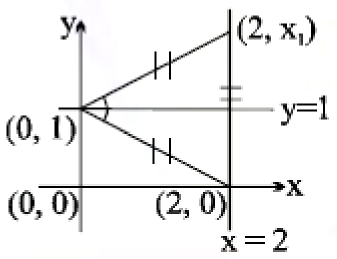
$$2^2+(x_1-1)^2=x_1^2$$

$$4+x_1^2+1-2x_1=x_1^2$$

$$5=2x_1 \text{ or } x_1=\frac{5}{2}$$

Equation of Ist altitude from $\left(2,\frac{5}{2}\right)$ to the given base

$$y-\frac{5}{2}=2(x-2)$$



$$2y-5=4(x-2)$$

At $y=1$ (since, Ist altitude meet y = 1) (i.e. orthocenter)

$$-\frac{3}{4}=x-2 \text{ or } x=\frac{5}{4}$$

80. (B)

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=\lim_{h\to 0}\frac{f(h)+|x|h+xh^2}{h}$$

$$\because\ f(0)=0 \Rightarrow f'(x)=\lim_{h\to 0}\left(\frac{f(h)-f(0)}{h}+|x|+xh\right)$$

$$f'(x)=f'(0)+|x|=|x|$$

81. (6)

$$BC=\begin{bmatrix}3 & 4\\ 2 & 3\end{bmatrix}\begin{bmatrix}3 & -4\\ -2 & 3\end{bmatrix} \Rightarrow BC=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}=I$$

$$t_r(A)+t_r\left(\frac{A}{2}\right)+t_r\left(\frac{A}{2^2}\right)+........$$

$$t_r(A)+\frac{1}{2}t_r(A)+\frac{1}{2^2}t_r(A)+........$$

$$=\frac{t_r(A)}{1-\left(\frac{1}{2}\right)}=2t_r(A)=2(2+1)=6$$

82. (1)

Hint: $A=\int_{a}^{2a}\left(\frac{x}{6}+\frac{1}{x^2}\right)dx=\left[\frac{x^2}{12}-\frac{1}{x}\right]_a^{2a}=\left(\frac{a^2}{3}-\frac{1}{2a}\right)-\left(\frac{a^2}{12}-\frac{1}{a}\right)$

$f(a)=\frac{a^2}{4}+\frac{1}{2a}$

Now, $f'(a)=\frac{a}{2}-\frac{1}{2a^2}=0 \Rightarrow a^3=1 \Rightarrow a=1$

83. (6)

$(1-2x+5x^2+10x^3)\left[C_0+C_1x+C_2x^2+....\right]=1+a_1x+a_2x^2+.....$

$a_1=n-2$ and $a_2=\frac{n(n-1)}{2}-2n+5$

Put $a_1^2=2a_2$

$(n-2)^2=n(n-1)-4n+10$

$n^2-4n+4=n^2-5n+10$

$n=6$

84. (0)

Hint : find 'c' assuming the line is the tangent to the circle. And plot the inequalities on *x*-*y* plane.

85. (256)

86. (3)

Conceptual

87. (5)

88. (96)

89. (4)

90. (4)