PART (A) : PHYSICS

Answer Key

1.	(A)	2.	(C)	3.	(B)	4.	(D)	5.	(D)
6.	(D)	7.	(C)	8.	(B)	9.	(D)	10.	(B)
11.	(D)	12.	(A)	13.	(B)	14.	(D)	15.	(C)
16.	(B)	17.	(C)	18.	(B)	19.	(C)	20.	(D)
21.	(2)	22.	(16)	23.	(4)	24.	(120)	25.	(3)
26.	(3)	27.	(2)	28.	(2)	29.	(600)	30.	(2)

Solutions

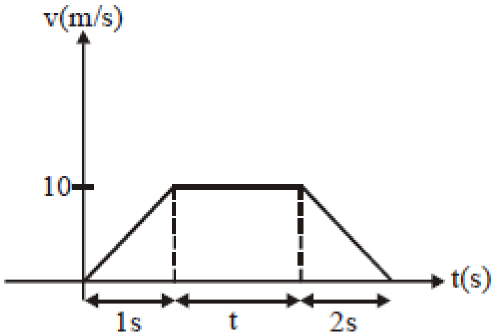
1. (A)

For $r < a, E = 0$

For $a < r < b, \vec{E} = \frac{kQ}{r}\hat{r}$

For $r > b, \vec{E} = -\frac{kQ}{r^2}\hat{r}$

2. (C)



$\frac{1}{2}(1+2)(10)+10t=135$

$t=12s$

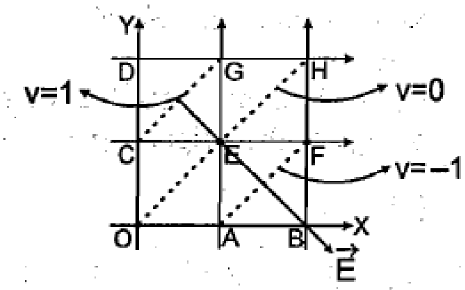
3. (B)

OEH is an equipotential surface, the uniform E.F. must be perpendicular to it pointing from higher to lower potential as shown

Hence, $\hat{E} = \left(\frac{\hat{r}-\hat{j}}{\sqrt{2}}\right)$

$E = \frac{(V_E - V_B)}{EB} = \frac{0-\sqrt{2}}{\sqrt{2}} = \sqrt{2}\ V/m$

$\vec{E} = E\cdot\hat{E} = \sqrt{2}\frac{(\hat{i}-\hat{j})}{\sqrt{2}} = (\hat{i}-\hat{j})\ V/m$

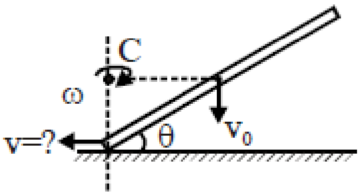


4. (D)

$\omega \frac{L}{2} \cos\theta = v_0$

$\omega \frac{L}{2} \sin\theta = v$

$\therefore \quad v = v_0 \tan\theta$



5. (D)

Let intensities are I & $I + \delta I$.

$$I_{minima} = \left[\sqrt{I+\delta I} - \sqrt{I}\right]^2 = I\left\{\left(1+\frac{\delta I}{I}\right)^{1/2} - 1\right\}^2$$

$$= \frac{(\delta I)^2}{4I} = \frac{I}{4}\left(\frac{\delta I}{I}\right)^2 = \frac{I}{4}\left(10^{-4}\right)$$

6. (D)

$y = \overline{A + B} \cdot B$

7. (C)

Using work energy theorem

8. (B)

$40 \times 10^{-3} = m v_p^2 v = 50 \times 10^{-3} \times v_p^2 \times 20$

$\frac{4}{100} = v_p^2 \Rightarrow 0.2\,m/s = v_p = 20\,cm/s$

9. (D)

I = V/Z

10. (B)

$$\overrightarrow{v_{cm}} = \frac{m\overrightarrow{v_1} + m\overrightarrow{v_2}}{2m} = \frac{1}{2}\left(\overrightarrow{v_1} + \overrightarrow{v_2}\right)$$

$$\overrightarrow{v_{1cm}} = \overrightarrow{v_1} - \overrightarrow{v_{cm}} = \frac{\overrightarrow{v_1} - \overrightarrow{v_2}}{2}$$

$$\left|\vec{v}_{1cm}\right| = \frac{\left|\overrightarrow{v_1} - \overrightarrow{v_2}\right|}{2} = \frac{v_1 - v_2}{2}$$

$$\lambda_{req} = \frac{h}{mv_{1cm}} = \frac{2h}{mv_1} = 2\lambda$$

11. (D)

1 msd = 200 × 0.005 = 1 mm

2r = 4 × 1 + 25 × 0.005 – 5 × 0.005

= 4.1

r = 2.05 mm

12. (A)

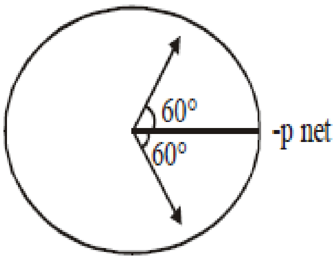
$$\frac{W}{Q}=\frac{nR\Delta T}{nC_p\Delta T}=\frac{32}{80}=\frac{2}{5}$$

$$5R=2C_p=2\left[C_v+R\right]$$

$$C_v=\frac{3R}{2}$$

13. (B)

$$\vec{p}=\sum q_i\,\overrightarrow{d_i}$$



14. (D)

Let and corrections at A and B α and β

$$\frac{15}{42+\alpha}=\frac{20}{58+\beta}\Rightarrow\frac{58+\beta}{42+\alpha}=\frac{4}{3}$$

$$\Rightarrow\ 3\beta+174=168+4\alpha$$

$$4\alpha-3\beta=6 \qquad ...(i)$$

$$\frac{20}{57+\alpha}=\frac{15}{43+\beta}\Rightarrow\frac{57+\alpha}{43+\beta}$$

$$=\frac{4}{3}\Rightarrow 171+3\alpha=172+4\beta$$

$$3\alpha-4\beta=1 \qquad ...(ii)$$

From (i) and (ii)

$$\alpha=3\text{cm},\ \beta=2\text{cm}$$

15. (C)

16. (B)

$$2\text{fm}=10\text{k Hz}$$

FM = 5 k Hz

$f_{LSB} = f_c - f_m = 95\,k\ Hz$

$f_{USB} = f_c - f_m = 105\,k\ Hz$

17. (C)

$L = 10\log\frac{I}{I_0}$

$I = I_0(10)^{\frac{L}{10}} = I_0 e^{(\ln 10)\frac{L}{10}}$

$\ln 2 = \frac{\ln 10}{10} I$

$\frac{dI}{I} = \frac{\ln 10}{10} dL$

$\int_I^{2I} \frac{dI}{I} = \frac{\ln 10}{10}\int_0^t \frac{dL}{dt} dt$

$\ln 2 = \frac{\ln 10}{10}\int_0^t (1)\,dt$

$t \simeq 3\,years$

18. (B)

Let decay constants for two modes are λ_α and λ_β, respectively.

$\frac{dN_\alpha}{dt} = -\lambda_\alpha B$

$\frac{dN_\beta}{dt} = -\lambda_\beta B$

$\frac{dN_\alpha/dt}{dN_\beta/dt} = 2 \Rightarrow \frac{\lambda_\alpha}{\lambda_\beta} = 2$ .....(i)

Also, $\frac{\ln 2}{\lambda_\alpha + \lambda_\beta} = 60$

$\frac{\ln 2}{\lambda_\alpha + \frac{\lambda_\alpha}{2}} = 60$

$\frac{\ln 2}{\lambda_\alpha} = 90$ years.

19. (C)

$\log P = m\log V$ where m is slope

$m = \frac{2.38 - 2.10}{1.1 - 1.3} = -1.4$

$\log P = -1.4\log V$

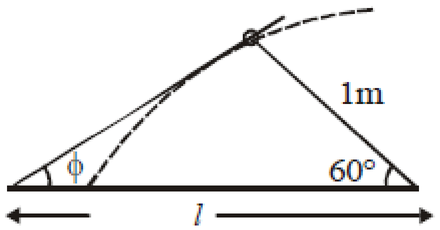
$\log PV^{1.4} = 0$

$PV^{1.4} = k$

20. (D)

21. (2)

For maximum value of ϕ, θ is 60^o. In this situation rod Q is tangent on the circle on which ring attached to P moves.



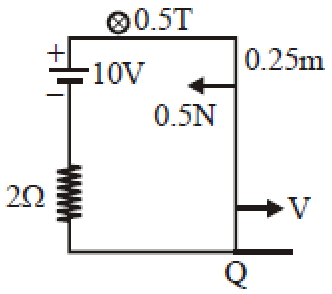
$l\cos 60 = 1$

$l = 2\text{m}$

22. (16)

Induced e.m.f = $B\ell v = 0.125\,V$

Current $I = \frac{10 - e}{R} = \frac{10 - 0.125\,V}{2}$



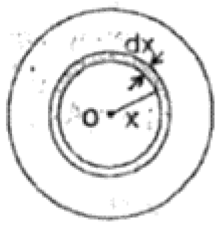
Force $BI\ell\ = 0.5\left(\frac{10 - 0.125\,V}{2}\right)0.25 = 0.5\,N$ (given)

Solving V = 16 m/s.

23. (4)

We can consider all the charge inside the sphere to be concentrated on the center of sphere

Consider an elementary shell of radius x and thickness dx.



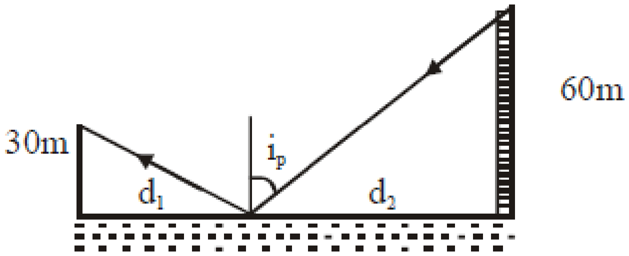
$$E = \frac{K\int dq}{r^2} = \frac{K\int_0^r 4\pi x^2\,dx\,(\alpha x)}{r^2} = \frac{k4\pi\alpha}{r^2}\int_0^r x^3\,dx = \frac{\alpha r^2}{4\varepsilon_0}$$

$\therefore\ n = 4$

24. (120)

$\tan i_p = \mu = \frac{4}{3}$

$i_p = 53^o$



$$\frac{30}{d_1} = \frac{3}{4} \Rightarrow d_1 = 40\,m$$

$$\frac{30}{d_2} = \frac{3}{4} \Rightarrow d_2 = 80\,m$$

Width = 120 m

25. (3)
Use time period of shm

26. (3)
The potential at centre of sphere in which q change in uniformly distributed throughout the volume is

$$V_C = \frac{1}{4\pi\varepsilon_0}\frac{3q}{2R}$$

By symmetry the potential at centre due to half sphere will be half of the complete sphere

$$\therefore \quad V_C = \frac{1}{4\pi\varepsilon_0}\frac{3q/2}{2R} = \frac{1}{4\pi\varepsilon_0}\frac{3Q}{2R} \quad [\because = Q]$$

27. (2)

$$F = \eta A\frac{v-0}{y} + 4\eta A\frac{V-0}{d-y}$$

$$\frac{dF}{dy} = 0$$

$$-\frac{\eta}{y^2} + \frac{4\eta}{(d-y)^2} = 0$$

$$\frac{d-y}{y} = 2$$

$$y = \frac{d}{3} \Rightarrow d_1 = \frac{d}{3},\ d_2 = \frac{2d}{3}$$

$$\frac{d_2}{d_1} = 2$$

28. (2)
Ft = 2mv (i)

$$t = \frac{T}{2} = \frac{2\pi}{2}\sqrt{\frac{m}{2k}}$$

$$t = \pi\sqrt{\frac{m}{2k}}$$

$$F\pi\sqrt{\frac{m}{2k}} = 2mv$$

$$v = \frac{F\pi}{2}\sqrt{\frac{1}{2km}}$$

29. (600)

$$W = \frac{\phi C}{cop} = \frac{60\times10^3}{60\times1.2}\,J/s = \frac{10^3}{1.2}\,J/s$$

No. of units consumed

$$= \frac{\left(\frac{10^3}{1.2}\right)(4)(30)}{1000} = 100\,\text{units}$$

Cost of energy = (6) (100) = 600 Rs.

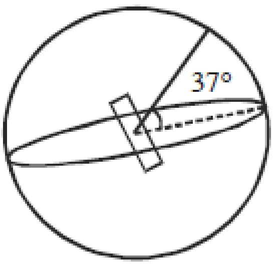
30. (2)

At magnetic equator

$$\frac{\mu_0}{4\pi}\frac{M}{r^3} = 5$$

At magnetic latitude of 37°

$$\frac{\mu_0}{4\pi}\frac{M}{r^3}\sqrt{1+3\cos^2 53^o} = \sqrt{52}\text{ units}$$

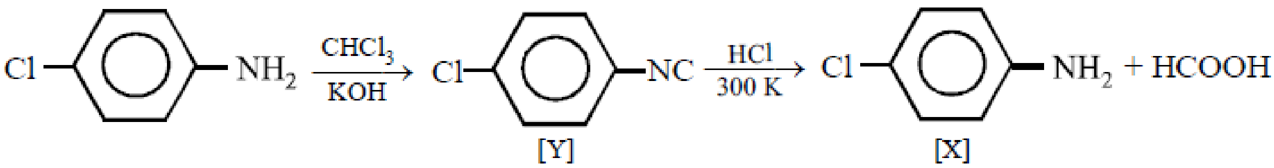


PART (B) : CHEMISTRY

SOLUTIONS

31. (A)
The order of stability of resonating structures: carrying no charge > carrying minimum charge and each atom having octet complete.

32. (C)
o-Nitrophenol is not sufficiently strong acid so as to react with $NaHCO_3$.

33. (A)


34. (C)
As Sb_2S_3 is a negative solution, so $Al_2(SO_4)_3$ will be the most effective coagulant due to higher positive charge on Al(Al^{3+}) – Hardy-Schulze rule.

35. (D)
Urotropine is used as antibiotic for urinary tract infection.

36. (D)
To convert covalent compounds into ionic compounds such as NaCN, Na_2S, NaX, etc.

37. (D)

$$Zn^{2+} + 2NH_4OH \longrightarrow \underset{\text{White ppt.}}{Zn(OH)_2} + 2NH_4^+$$

$$Zn(OH)_2 + 2NH_4OH \longrightarrow \underset{\text{Soluble}}{(NH_4)_2ZnO_2} + 2H_2O$$

$$(NH_4)_2ZnO_2 + H_2S \longrightarrow \underset{\text{White ppt.}}{ZnS} + 2NH_4OH$$

38. (C)

$$2H_2O_2 \xrightarrow{\Delta} 2H_2O + O_2$$

$$2KI + H_2O_2 \longrightarrow 2KOH + I_2$$

$$2KMnO_4 + 3H_2SO_4 + 5H_2O_2 \longrightarrow H_2SO_4 + 2MnSO_4 + 8H_2O + 5O_2$$

39. (C)

Gas equation is $PV = \frac{m}{M}RT$ ...(i)

Again $\frac{P}{2}V=\frac{m_1}{M}R.\frac{2}{3}T$ ...(ii)

Divide (i) by (ii)

$$2=\frac{m}{m_1}\times\frac{3}{2}$$

$\therefore$ $m_1=\frac{3}{4}m$. The remaining gas is $\frac{3}{4}m$.

Then gas escaped $=\frac{1}{4}m$

40. (D)

In first case the given compounds have same anion but different cations having different charge hence they will precipitate negatively charged sol i.e. 'A'.

In second case the given compounds have similar cation but different anion with different charge. Hence they will precipitate positively charged sol. i.e. 'B'.

41. (B)

Given $K_b = x\,K\,kg\,mol^{-1}$

$\Delta T_b = K_b \times m$, where m = molality

$\therefore$ $y = x \times m$

$$m=\frac{y\,K}{x\,K\,Kg\,mol^{-1}}=\frac{y}{x}mol\,Kg^{-1}$$

We know $\Delta T_f = K_f \times m$

On substituting value of m,

$$\Delta T_f=\frac{yz}{x}K$$

42. (B)

43. (B)

- Li does not form peroxide or superoxide due to its small size.
- Solubility of carbonates and biocarbonates increases on moving down the group.
- The increasing order of size of hydrated ions of alkali metals is
  $Li^+ > Na^+ > K^+ > Rb^+ > Cs^+$
- Caesium used in photoelectric cells due to its low I.E.

Hence, statements (B) is correct.

44. (B)

$\therefore$ $\Delta G^o = -nFE^o$

(i) $A^{3+}+e^- \longrightarrow A^{2+}, \Delta G_1^o = -1\,Fy_2$

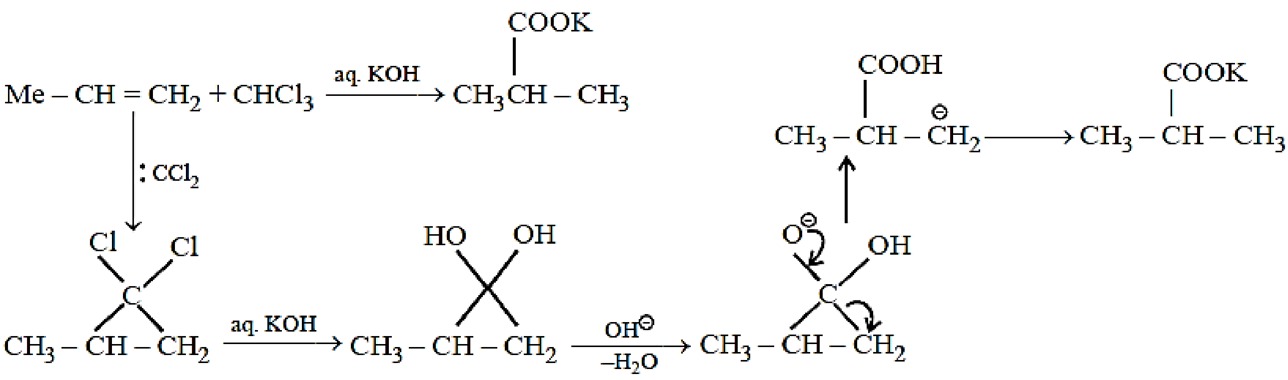
(ii) $A^{2+}+2e^- \longrightarrow A, \Delta G_2^o = -2F(-y_1) = 2Fy_1$

Add, (i) and (ii), we get

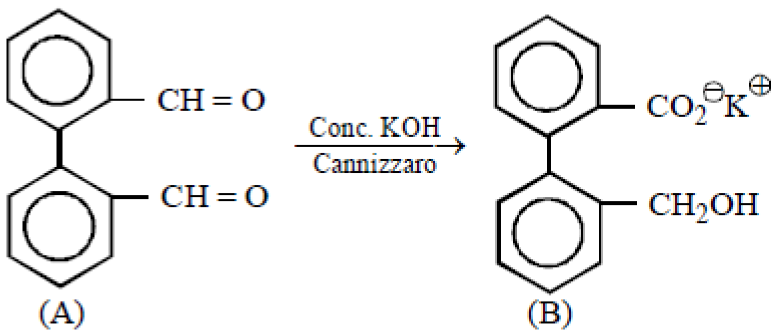
$$A^{3+} + 2e^- \longrightarrow A;\ \Delta G_3^o = \Delta G_1^o + \Delta G_2^o$$
$$-3FE^o = -Fy_2 + 2Fy_1$$
$$-3FE^o = -F(y_2 - 2y_1)$$
$$E^o = \frac{y_2 - 2y_1}{3}$$

45. (A)
The compounds of the type $M(AA)_2B_2$ exhibit both geometrical and optical isomerism.

46. (B)



47. (B)



48. (B)
In electrolysis of NaCl when Pt electrode is taken, then H_2 liberated at cathode, while with Hg cathode it forms sodium amalgam because more voltage is required to reduce H^+ at Hg than at Pt.

49. (B)

$$H_2(g) + CO_2(g) \rightleftharpoons CO(g) + H_2O(g)$$

At eq. 0.25–x 0.25–x x x

$$K_p = 0.16 = \frac{x^2}{(0.25-x)^2}$$

$$\Rightarrow 0.4 = \frac{x}{0.25-x} \Rightarrow 0.1 - 0.4x = x$$

$$x = 0.0714$$

Mole % of CO (g) $= \frac{0.0714}{0.50} \times 100 = 14.28$

50. (A)

Acid rain has pH < 5.6.

51. (210)

Osmotic pressure (π) of isotonic solutions are equal. For solution of unknown substance C_1 (concentration).

$C_1 = \frac{5.25/M}{V}$

Where M represents molar mass.

For solution of urea, C_2 (concentration)

$= \frac{1.5/60}{V}$

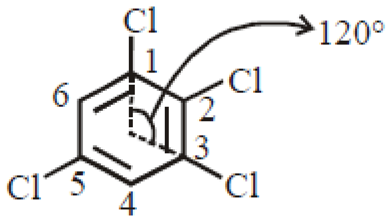
Given, $\pi_1 = \pi_2$

$\because \ \pi = CRT$

$\therefore C_1RT = C_2RT \text{ or } C_1 = C_2 \text{ or } \frac{5.25/M}{V} = \frac{1.5/60}{V}$

$\therefore \ M = 210 \text{ g/mol}$

52. (2)



Dipole moments of 2Cl and 5Cl are vectorically cancelled.

It is due 1Cl and 3Cl

$\mu^2 = \mu_1^2 + \mu_2^2 + 2\mu_1\mu_2 \cos\theta$

$= (1.5)^2 + (1.5)^2 + 2 \times 1.5 \times 1.5 \cos 120^o$

$= 2.25 + 2.25 + 4.5 \times -\frac{1}{2}$

$= 2.25 + 2.25 - 2.25$

$= 2.25 D$

$\mu^2 = 2.25$

$\therefore \ \mu = 1.5 D$

53. (2)

$A \rightarrow B$

Initial concentration	Rate of reaction
2×10^{-3} M	2.40×10^{-4} M s^{-1}

1×10^{-3} M $\qquad\qquad 0.60\times10^{-4}\ \text{M s}^{-1}$

rate of reaction $r = k[A]^x$ where x = order of reaction

Hence,

$$2.40\times10^{-4} = k\left[2\times10^{-3}\right]^x \qquad .....(i)$$

$$0.60\times10^{-4} = k\left[1\times10^{-3}\right]^x \qquad .....(ii)$$

On dividing eqn. (i) from eqn. (ii), we get

$$4 = (2)^x$$

$\therefore\quad x = 2$

i.e. order of reaction = 2

54. (60)

According to Arrhenius equation

$$\log\frac{k_2}{k_1} = \frac{E_a}{2.303R}\left(\frac{1}{T_1}-\frac{1}{T_2}\right)$$

$$\log\frac{1.3\times10^{-3}}{1.3\times10^{-4}} = \frac{E_a}{2.303\times8.314}\left[\frac{1}{373}-\frac{1}{423}\right]$$

$$1 = \frac{E_a}{2.303\times8.314}\left[\frac{1}{373}-\frac{1}{423}\right]$$

$$E_a = 60\ \text{kJ/mol}$$

55. (2)

$$\Delta v = \frac{0.001}{100}\times30,000 = 0.3\ \text{cm sec}^{-1}$$

According to uncertainty principle,

$$\Delta x.\Delta p \approx \frac{h}{4\pi};\ \Delta x.\Delta v \approx \frac{h}{4\pi m}$$

$$\Delta x\times9.1\times10^{-28}\times0.3 \approx \frac{6.625\times10^{-27}\times7}{4\times22}$$

$$\Delta x \approx 1.93\text{cm.} \approx 2$$

56. (2)

No. of $A^{2+} = \frac{1}{8}\times8 = 1$

No. of $B^{3+} = \frac{1}{2}\times4 = 2$

No. of $O^{2-} = 8\times\frac{1}{8}\times6\times\frac{1}{2} = 4$

(AB_2O_4)

$\therefore$ Value of n = 2

57. (395)

Since, work is done against constant pressure and thus, irreversible.

Given, $\Delta V = (6-2) = 4\,L;\ P = 1\,atm$

$\therefore \quad W = -1 \times 4\,L\text{-}atm = -\dfrac{1 \times 4 \times 1.987}{0.0821}\,cal$ (Since 0.0821 L-atm = 1.987 cal)

= – 96.81 cal = – 96.81 × 4.184 J ($\because$ 1 cal = 4.184 J)

= – 405.05 J

Now from 1st law of thermodynamics

q = ΔU – W

800 = ΔU + 405.05

$\therefore$ ΔU = 395 J

58. (3)

Three, these are $CH_3CH_2OCH_2CH_3$ (I), $CH_3OCH_2CH_2CH_3$ (II) and $CH_3OCH(CH_3)_2$ (III).

Here I and II, I and III are pairs of metamers.

59. (5)

$$pH = pK_a + \log\frac{[CH_3COO^-]}{[CH_3COOH]}$$

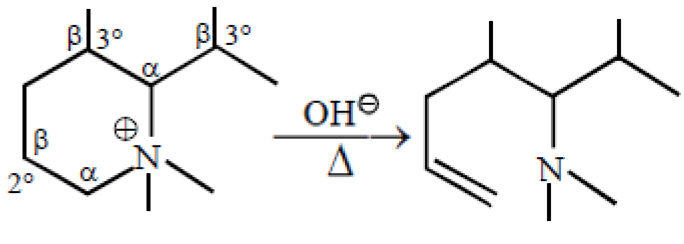
$$pK_a = -\log(1.8 \times 10^{-5}) = 4.7447$$

$$[CH_3COO^-] = 2 \times [(CH_3COO)_2Ba] = 0.2\,M$$

$$[CH_3COOH] = 0.1\,M$$

$$pH = 4.7447 + \log\frac{0.2}{0.1} = 5.046 \approx 5.0$$

60. (2)



(2α — H)

PART (C) : MATHEMATICS

Answer Key

61.	(B)	62.	(C)	63.	(D)	64.	(A)	65.	(C)
66.	(A)	67.	(C)	68.	(A)	69.	(A)	70.	(B)
71.	(A)	72.	(D)	73.	(A)	74.	(C)	75.	(D)
76.	(B)	77.	(B)	78.	(A)	79.	(A)	80.	(A)
81.	(2022)	82.	(0)	83.	(0)	84.	(1)	85.	(2)
86.	(9)	87.	(5)	88.	(11)	89.	(5)	90.	(22)

Solutions

61. (B)
Let us assume that no. of cups with handle is n and without handle is m. No. of ways of selecting one cup each is ${}^{m}C_1 . {}^{n}C_1 = 36, mn = 36$. Now apply $A.M. \geq G.M.$

$\frac{m+n}{2} \geq \sqrt{mn}$

$\frac{m+n}{2} \geq 6$

$m+n \geq 12$

So least value is '12'

62. (C)
$(x+10)(x-1) = -5$

Case (i) Either $x+10=5, x-a=-1$

$x=-5, -5-a=-1$

$a=-4$

Or $x+10=-1, x-a=5$

$x=-11, -11-a=5$

$a=-16$

Case (ii) $x+10=-5, x-a=1$

Or $x+10=1, x-a=-5$ it gives same values of ‘a’.

Sum of values of ‘a’ is $-16-4=-20$

63. (D)
$\frac{1}{x}+\frac{1}{y}+\frac{1}{z}=1; x, y, z > 0$

Let $x=\frac{1}{a}, y=\frac{1}{b}, z=\frac{1}{c} \Rightarrow a+b+c=1$

Also $(x-1)(y-1)(z-1)$

$=\left(\frac{1}{a}-1\right)\left(\frac{1}{b}-1\right)\left(\frac{1}{c}-1\right)$

$=\frac{(1-a)(1-b)(1-c)}{abc}$

$= \frac{(b+c)(a+c)(a+b)}{abc}$

Now by A.M. $\geq$ G.M.

$\frac{b+c}{2} \geq \sqrt{bc}$

$b+c \geq 2\sqrt{bc}$

$c+a \geq 2\sqrt{ac}$

$a+b \geq 2\sqrt{ab}$

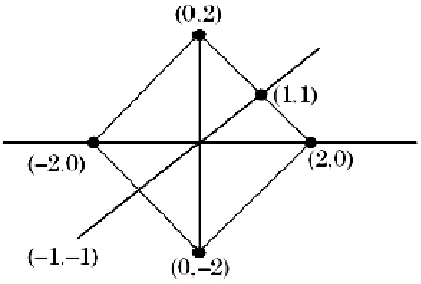
$(b+c)(c+a)(a+b) \geq 8abc$

$\frac{(b+c)(c+a)(a+b)}{abc} \geq 8$

64. (A)

$y = x + \cos x$

$\frac{dy}{dx} = 1 - \sin x = 0 \Rightarrow \sin x = 1 \Rightarrow x = 2n\pi + \frac{\pi}{2}$



Putting value of x in given function

$y = x + \cos x$

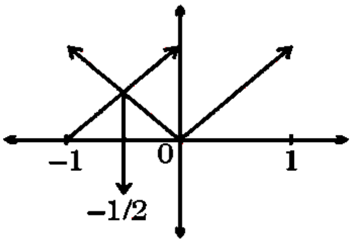
$y = 2n\pi + \frac{\pi}{2} + 0$

$y = 2n\pi + \frac{\pi}{2}$

Clearly point lie on the line $y = x$

Now x can be between -1 and 1 whereas least value of x is $\frac{\pi}{2}$, if $x > 0$ max value of x is $-\frac{\pi}{2}$ if $x < 0$. So no value of 'x'

65. (C)



Clearly, $f(x)=\begin{cases} -x, -1\le x<-\frac{1}{2} \\ x+1, \frac{-1}{2}\le x<0 \\ x, 0\le x\le 1 \end{cases}$

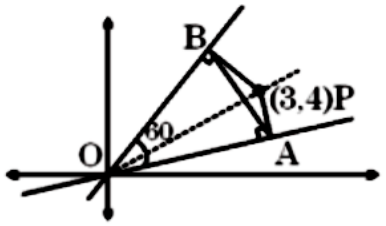
$\int_{-1}^{-1/2} -x.dx+\int_{-1/2}^{0}(x+1)dx+\int_{0}^{1}x.dx$

$-\frac{1}{2}\left[x^2\right]_{-1}^{-1/2}+\left[\frac{x^2}{2}+x\right]_{-1/2}^{0}+\left[\frac{x^2}{2}\right]_0^1$

$-\frac{1}{2}\left[\frac{1}{4}-1\right]+\left[0-\left(\frac{1}{8}-\frac{1}{2}\right)\right]+\frac{1}{2}$

$\frac{3}{8}+\left(\frac{3}{8}\right)+\frac{1}{2}=\frac{3}{4}+\frac{1}{2}=\frac{5}{4}$

66. (A)

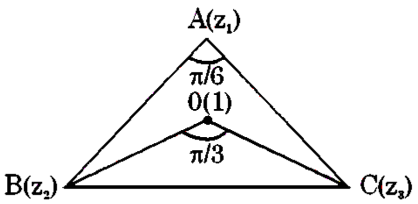


Apply sine rule in triangle OAB, $\frac{AB}{\sin 60^\circ}=OP$

$OP=\sqrt{3^2+4^2}=5$

$AB=5.\sin 60^\circ=\frac{5\sqrt{3}}{2}$

67. (C)



$z=1$ is circum center. So triangle OBC is equilateral

$z_2^2+z_3^2+1=z_2+z_3+z_2z_3$

Expression $z_2(z_2-1)-z_3(z_2+1)+(z_3+1)(z_3-1)=-2$

So value is -2

68. (A)

$1+|\sin\theta|+|\cos\theta|-1-2=0$

$\Rightarrow |\sin\theta|+|\cos\theta|=2$ which is not possible.

69. (A)

Divide throughout by x^3y, we get

$$2\frac{dx}{x}+2\frac{dy}{y}-2\frac{y^3}{x^3}dx+3.\frac{y^2}{x^2}dy=0$$

$$\Rightarrow 2\frac{dx}{x}+2\frac{dy}{y}+d\left(\frac{y^3}{x^2}\right)=0\text{, integrate}$$

$$\Rightarrow 2(\ln x+\ln y)+\frac{y^3}{x^2}=C\text{, Now put } x=y=1$$

$$\Rightarrow 2\ln(1)+1=C\Rightarrow C=1\Rightarrow 2\ln(xy)+\frac{y^3}{x^2}=1$$

$$m=3, n=2\Rightarrow m+n=5$$

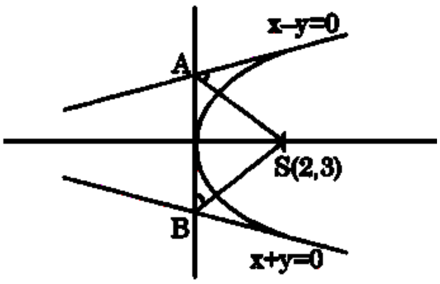
70. (B)

Minimum value of $\sqrt{(x-1)^2+(y-1)^2}$

Clearly minimum distance of circle from the point $(1,1)$

$$\sqrt{(4-1)^2+(5-1)^2}=5, 7-5=2$$

71. (A)



Equation of AS:

$(y-3)=-1(x-2)$

$y-3=-x+2$

$x+y=5$

For point ‘A’ solve

$x+y=5$

$y=x$

$2x=5$

$2x=5$

$x=\frac{5}{2}, y=\frac{5}{2}$

$A\left(\frac{5}{2},\frac{5}{2}\right)$

Equation of BS:

$(y-3)=1(x-2)$

$y-3=x-2$

$x-y=-1$

For point ‘B’ solve

$x - y = -1$
$x + y = 0$
$2x = -1$
$x = -\frac{1}{2}, y = \frac{1}{2}$

Now equation of line AB

$$y - \frac{1}{2} = \frac{\frac{5}{2} - \frac{1}{2}}{\frac{5}{2} + \frac{1}{2}}\left(x + \frac{1}{2}\right)$$

$$y - \frac{1}{2} = \frac{2}{3}\left(x + \frac{1}{2}\right)$$

$$3y - \frac{3}{2} = 2x + 1$$

$$6y - 3 = 4x + 2$$

$$4x - 6y + 5 = 0$$

Option 'A' is correct

72. (D)

$S(6,13)$ and $S'(25,8)$

$P(1,1), S'P = 25, SP = 13$

$S'P - SP = 12$

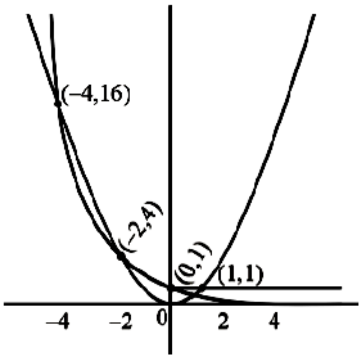
$2a = 12$

$$S'S = \sqrt{(25-6)^2 + (8-13)^2}$$

$$= \sqrt{361 + 25} = \sqrt{386} = 2ae$$

$$e = \frac{\sqrt{386}}{12}$$

73. (A)



From graph clearly '4' points of non-differentiability.

74. (C)

For $x = 1$, $0 = f(y) + f\left(\frac{1}{y}\right)$, interchanging x and y,

we have $2f(y) = f(xy) + f\left(\frac{y}{x}\right)$ adding $f(xy) = f(x) + f(y)$

Partial differentiation with respect to 'x'

$f'(xy), x = f'(x)$ ....(1)

Partial differentiation with respect to 'y'

$f'(xy), x = f'(y)$ ....(2)

$\frac{y}{x} = \frac{f'(x)}{f'(1)}$ put $y = 1$

$\frac{1}{x} = \frac{f'(x)}{f'(1)}$

$\frac{1}{\ln 6} \times \frac{1}{x} = f'(x)$

Integrate with respect to 'x'

$f(x) = \frac{1}{\ln 6}.\ln x + C$ put $x = 1$

$f(1) = 0 + C = 0$

$f(x) = \frac{\ln x}{\ln 6}$

$f(7776) = \frac{\ln(7776)}{\ln 6} = 5$

75. (D)

$\left(\left(\vec{b} \times \vec{c}\right).\vec{a}\right)\vec{c} - \left(\left(\vec{b} \times \vec{c}\right).\vec{c}\right)\vec{a} = 3\vec{c}$

$\left[\vec{a}\,\vec{b}\,\vec{c}\right]\vec{c} = 3\vec{c}$

$\left[\vec{a}\,\vec{b}\,\vec{c}\right] = 3$

We know that $\left[\vec{b} \times \vec{c} \;\; \vec{c} \times \vec{a} \;\; \vec{a} \times \vec{b}\right] = \left[\vec{a}\;\vec{b}\;\vec{c}\right]^2 = 9$

76. (B)

There will be no effect due to addition constant in variables.

77. (B)

$\begin{bmatrix} 0 & \otimes \\ -\otimes & O \end{bmatrix}$ If matrix is of order 2×2, then $\otimes$ this place can be filled in 6 ways so 6 matrices can be formed. If matrix is of order 3×3, the

$$\begin{bmatrix} 0 & \otimes & \ominus \\ -\otimes & 0 & \oplus \\ -\ominus & -\oplus & 0 \end{bmatrix}$$

$\otimes$ this place can be filled in 6 ways

$\ominus$ this place can be filled in 4ways

$\oplus$ this place can be filled in 2 ways

So total matrices $6 \times 4 \times 2 = 48$

Now total matrices $48 + 6 = 54$

78. (A)

$$\sum_{r=0}^{m} r.r! = ((r+1)-1)r! = (r+1)!-r!$$

$$\sum_{r=0}^{m} (r+1)!-r! = (m+1)!-1$$

$$\sum_{r=0}^{m} ({}^{m}C_{r})^{2} = ({}^{m}C_{0})^{2} + ({}^{m}C_{1})^{2} + ({}^{m}C_{2})^{2} + ...+ ({}^{m}C_{m})^{2} = {}^{2m}C_{m}$$

$$\sum_{r=0}^{m} (2r-1) = m^{2}-1$$

So Row 1 and Row 3 is same so value of determinant is zero.

79. (A)

Putting $(0,0,0)$ expression is –1 and putting (1, 1, 1) expression is $2+a^2+ab-1$

Both should have opposite sign so

$2+a^2+ab-1>0$

$a^2+ab+1>0$

Its discriminant is negative

$D=b^2-4<0=-2<b<2$

No. of integral values of ‘b’ is $\{-1,0,1\}$

80. (A)

We have $\sim(p\vee q)\vee(\sim p\wedge q)$

$(\sim p\wedge \sim q)\vee(\sim p\wedge q)$

$\sim p\wedge(\sim q\vee q)=\sim p$

81. (2022)

$x^4-4x^3+6x^2-4x+1=2022$

$(x-1)^4=2022$

$(x-1)^2=-\sqrt{2022}$ because only non real roots required

$x^2+1-2x+\sqrt{2022}=0$

$p=\text{product of roots }=1+\sqrt{2022}$

82. (0)

Equation of normal at $P(3\cos\theta, 2\sin\theta)$ is

$3x\sec\theta-2y\operatorname{cosec}\theta=5$

Now it is tangent to circle so

$$\frac{5}{\sqrt{9\sec^2\theta+4\operatorname{cosec}^2\theta}}=\sqrt{3}$$

But $\{9\sec^2\theta+4\operatorname{cosec}^2\theta\}\geq(3+2)^2=25$

No such θ exist

83. (0)

$\sin x.\sin\left(\frac{1}{x}\right)=1$

$\sin\left(\frac{1}{x}\right)=\operatorname{cosec} x$

Clearly $\sin\left(\frac{1}{x}\right)=\operatorname{cosec} x=1$ or

$\sin\left(\frac{1}{x}\right)=\operatorname{cosec} x=-1$ which is not possible for same value of x.

84. (1)

$x\in$ prime and $x<10\Rightarrow x=2,3,5,7$

Total no. of $A(x,y)$ pair $n(A)=4\times10=40$ and $x^2-3y^2=1\Rightarrow x^2=3y^2+1$

For above condition to be satisfied only two such pairs $(2,1)$ and $(7,4)$ are possible

$\Rightarrow P(A)=\frac{2}{40}=\frac{1}{20}=P$

85. (2)

$\lim_{k\to\infty}\int_0^{k[x]}(\{kt\})^k=\lim_{k\to\infty}\int_0^{k^2[x].1/k}(\{kt\})^k$

$=\lim_{k\to\infty}k^2[x]\int_0^{1/k}(kt)^k.dt$

$=\lim_{k\to\infty}k^2[x].k^k.\left[\frac{t^{k+1}}{k+1}\right]_0^{1/k}$

$=\lim_{k\to\infty}k^2[x].k^k.\frac{1}{k^{k+1}(k+1)}$

$=[x]\lim_{k\to\infty}\frac{k}{k+1}=[x]\text{or}\,\lambda=2$

86. (9)

$\lim_{x\to0}\left[\frac{\sin^{-1}x}{\sin x}\right]=1$ $\quad 1+2+3+........+n=45$

$\lim_{x\to0}\left[\frac{\sin^{-1}2x}{\sin x}\right]=2$ $\quad n(n+1)=90$

$\lim_{x\to0}\left[\frac{\sin^{-1}nx}{\sin x}\right]=n$ $\quad n=9$

87. (5)

$T_r=\cot^{-1}\left(4+\frac{r(r+1)}{4}\right)$

$$= \tan^{-1}\left(\frac{1}{4+\frac{r(r+1)}{4}}\right)$$

$$= \tan^{-1}\left(\frac{1}{4\left(1+\frac{r(r+1)}{16}\right)}\right)$$

$$= \tan^{-1}\left(\frac{1/4}{1+\frac{r}{4}.\frac{(r+1)}{4}}\right)$$

$$= \tan^{-1}\left(\frac{\frac{r+1}{4}-\frac{r}{4}}{1+\frac{r}{4}.\frac{r+1}{4}}\right)$$

$$T_r = \tan^{-1}\left(\frac{r+1}{4}\right)-\tan^{-1}\left(\frac{r}{4}\right)$$

$$S_n = \sum_{r=1}^{n} T_r = \tan^{-1}\left(\frac{n+1}{4}\right)-\tan^{-1}\left(\frac{1}{4}\right)$$

$$S_\infty = \tan^{-1}(\infty)-\tan^{-1}\left(\frac{1}{4}\right)$$

$$= \frac{\pi}{2}-\tan^{-1}\left(\frac{1}{4}\right) = \cot^{-1}\left(\frac{1}{4}\right) = \tan^{-1}(4)$$

So $a=4, b=1$

88. (11)

Rearrangement of 5 things

$$5!\left(\frac{1}{2!}-\frac{1}{3!}+\frac{1}{4!}-\frac{1}{5!}\right)$$

$$\frac{5!}{2!}-\frac{5!}{3!}+\frac{5!}{4!}-\frac{5!}{5!}$$

$$= 60-20+5-1=44$$

Now $f(1)$ can be 2,3,4,5 in above arrangement if $f(1)$ has to be ‘2’ then arrangements will be $\frac{44}{4}=11$.

89. (5)

From $7!$ every number will be divisible by 14

So $1!+2!+3!+4!+5!+6!=873$

Now remainder when it is divided by 14 is ‘5’.

90. (22)

Use LMVT in $[1,2]$

$$\frac{f(2)-f(1)}{2-1}=f'(C)\geq 1$$

$f(2)-3\geq 1$

$f(2)\geq 4$

Now use L.M.V.T in $[2,4]$

$$\frac{f(4)-f(2)}{4-2}=f'(C_1)\geq 1$$

$f(4)-f(2)\geq 2$

$f(2)\leq 7$

$4\leq f(2)\leq 7$

Possible integral values 4,5,6,7

Sum of values $4+5+6+7=22$