PART (A) : PHYSICS

SOLUTIONS

1. (D)

$\frac{1}{2}mv^2 = \frac{hc}{\lambda} - W_0$ or $\frac{hc}{\lambda} = \frac{1}{2}mv^2 + W_0$ and

$\frac{1}{2}mv_1^2 = \frac{hc}{(3\lambda/4)} - W_0 = \frac{4}{3}\left(\frac{1}{2}mv^2 + W_0\right) - W_0$

So, v_1 is greater than $v\left(\frac{4}{3}\right)^{1/2}$

2. (A)

Current through each bulb is same because these are connected in series. Since $\left(R = \frac{V^2}{P}\right)$, resistance of 40W bulb is more, hence greater heat is produced in the 40W bulb, it glows brightest $H = I^2Rt$

3. (A)

Process AB is isobaric and BC is isothermal expansion, CD isochoric and DA isothermic compression.

4. (C)

Electric field lines at each point of the ball must cross normally.

5. (A)

Final energy level would be n = 3

So due to de-excitation, 2 Lyman series and 1 Balmer series wavelength would emit.

6. (B)

Distance covered by lift is given by $y = t^2$

$\therefore$ Acceleration of lift upwards

$= \frac{d^2y}{dt^2} = \frac{d}{dt}(2t) = 2m/s^2 = \frac{g}{5}$

Now, $T = 2\pi\sqrt{\frac{\ell}{g}}$

$T' = 2\pi\sqrt{\frac{\ell}{g + \frac{g}{5}}} = 2\pi\sqrt{\frac{\ell}{\frac{6}{5}}} = \sqrt{\frac{5}{6}}T$

7. (B)

Angular limit of resolution of eye, $\theta = \frac{\lambda}{d}$, where, d is diameter of eye lens

Also, if y is the minimum separation between two objects at distance D from eye then $\theta = \frac{y}{D}$

$\Rightarrow \frac{y}{D} = \frac{\lambda}{d} \Rightarrow y = \frac{\lambda D}{d}$ .....(a)

Here, wavelength $\lambda = 5000\,\overset{o}{A} = 5 \times 10^{-7}\,m$, D = 50m

Diameter of eye lens = 2mm = 2×10^{-3}m

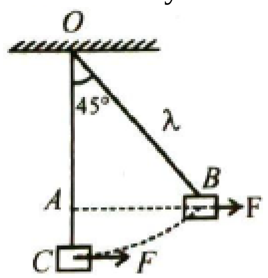
From eq. (a), minimum separation is

$$y = \frac{5 \times 10^{-7} \times 50}{2 \times 10^{-3}} = 1.25\,cm$$

8. (D)

Work done by tension + Work done by force (applied) + Work done by gravitational force = change in kinetic energy

Work done by tension is zero



$$\Rightarrow 0 + F \times AB - Mg \times AC = 0$$

$$\Rightarrow F = Mg\left(\frac{AC}{AB}\right) = Mg\left[\frac{1 - \frac{1}{\sqrt{2}}}{\frac{1}{\sqrt{2}}}\right]$$

$$\left[\because AB = \ell \sin 45^0 = \frac{\ell}{\sqrt{2}} \text{ and } AC = OC - OA = \ell - \ell\cos 45^0 = \ell\left(1 - \frac{1}{\sqrt{2}}\right) \text{where } \ell = \text{length of the string}\right]$$

$$\Rightarrow F = mg\left(\sqrt{2} - 1\right)$$

9. (A)

$$\frac{E_s}{E_p} = \frac{n_s}{n_p} \text{ or } E_s = E_p \times \left(\frac{n_s}{n_p}\right)$$

$$\therefore E_s = 120 \times \left(\frac{200}{100}\right) = 240V$$

$$\frac{I_P}{I_s} = \frac{n_s}{n_p} \text{ or } I_S = I_P\left(\frac{n_p}{n_s}\right) \therefore I_s = 10\left(\frac{100}{200}\right) = 5amp$$

10. (C)

Applying conservation of energy principle, we get

$$\frac{1}{2}mk^2v_e^2 - \frac{GMm}{R} = -\frac{GMm}{r}$$

$$\Rightarrow \frac{1}{2}mk^2\frac{2GM}{R} - \frac{GMm}{R} = -\frac{GMm}{r}$$

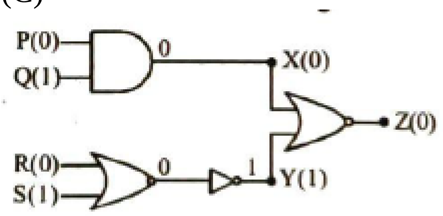
$$\Rightarrow \frac{k^2}{R} - \frac{1}{R} = -\frac{1}{r} \Rightarrow \frac{1}{r} = \frac{1}{R} - \frac{k^2}{R}$$

$$\Rightarrow \frac{1}{r} = \frac{1}{R}(1-k^2) \Rightarrow r = \frac{R}{1-k^2}$$

11. (A)

Water rises upto the top of capillary tube and stays there without overflowing.

12. (C)



13. (D)

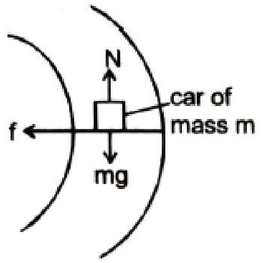
The self inductance of a long solenoid is given by

$L = \mu_r \mu_0 n^2 Al$

Self inductance of a long solenoid is independent of the current flowing through it

14. (B)

It means that car which is moving on a horizontal road and the necessary centripetal force, which is provided by friction (between car and road) is not sufficient. If μ is friction between car and road, then max speed of safely turn on horizontal road is determined from figure



$N = mg$ .....(i)

$f = \frac{mv^2}{r}$ ......(ii)

Where f is frictional force between road and car, N is the normal reaction exerted by road on the car. We know that

$f = \mu_s N = \mu_s mg$ ....(iii)

Where μ_s is static friction

So from eq (ii) and (iii) we have

$$\frac{mv^2}{r} \le \mu_s mg \Rightarrow v^2 \le \mu_s rg \text{ or } v \le \sqrt{\mu_s rg} \text{ and } v_{max} = \sqrt{\mu_s rg}$$

If the speed of car is greater than v_{max} at that road, then it will be thrown out from road, i.e., skidding.

15. (A)
Time period of simple pendulum is given by

$$T = 2\pi\sqrt{\frac{\ell}{g_{eff}}} \text{ or}, T = \frac{k}{\sqrt{g_{eff}}}$$

Now, $T_1 = \frac{k}{\sqrt{g}}$ and $T_2 = \frac{k}{\sqrt{g\left(1-\frac{d}{R}\right)}}$

So, $\frac{T_1}{T_2} = \sqrt{1-\frac{d}{R}} = \left(\frac{T_1}{T_2}\right)^2 = 1-\frac{d}{R}$

$$d = \left[1-\left(\frac{T_1}{T_2}\right)^2\right]R$$

16. (A)
Here r = 6cm = $6\times10\,m, N = 20, \omega = 40 rads^{-1}\ B = 2\times10^{-2}T, R = 8\Omega$
Maximum emf induced, $\varepsilon = NAB\omega$

$= N(\pi r^2)B\omega$

$= 20\times\pi\times(6\times10^{-2})^2\times10^{-2}\times40 = 0.18\,V$

Average value of emf induced over a full cycle

$\varepsilon_{av} = 0$

Maximum value of current in the coil

$$I = \frac{\varepsilon}{R} = \frac{0.18}{8} = 0.023A$$

Average power dissipated

$$P = \frac{\varepsilon l}{2} = \frac{0.18\times0.023}{2} = 2.07\times10^{-3}W$$

17. (C)
If ℓ is the original length of wire, then change in length of first wire, $\Delta\ell_1 = (\ell_1 - \ell)$
Change in length of second wire, $\Delta\ell_2 = (\ell_2 - \ell)$

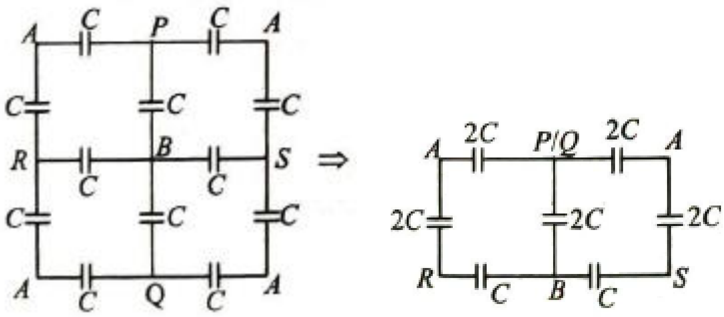
Now $Y = \frac{T_1}{A}\times\frac{\ell}{\Delta\ell_1} = \frac{T_2}{A}\times\frac{\ell}{\Delta\ell_2}$

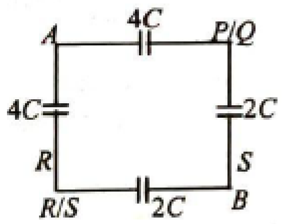
Or $\frac{T_1}{\Delta\ell_1} = \frac{T_2}{\Delta\ell_2}$ or $\frac{T_1}{\ell_1-\ell} = \frac{T_2}{\ell_2-\ell}$

Or $T_1\ell_2 - T_1\ell = T_2\ell_1 - \ell T_2$ or $\ell = \frac{T_2\ell_1 - T_1\ell_2}{T_2 - T_1}$

18. (A)
Black board paint is quite approximately equal to black bodies.

19. (B)
The effective circuit is shown in the figure





Now $C_{AB} = 2 \times \frac{4C \times 2C}{4C + 2C} = \frac{8C}{3}$

20. (C)

Angle of prism A = 60^0

By prism formula

$$\mu = \frac{\sin\left(\frac{A+\delta_m}{2}\right)}{\sin\left(\frac{A}{2}\right)} \text{ or } \sqrt{3} = \frac{\sin\left(\frac{60^0+\delta_m}{2}\right)}{\sin\left(\frac{60^0}{2}\right)}$$

$$\frac{\sqrt{3}}{2} = \sin\left(\frac{60^0+\delta_m}{2}\right); \sin 60^0 = \sin\left(\frac{60^0+\delta m}{2}\right)$$

$$60^0 = \frac{60^0+\delta_m}{2} \Rightarrow \delta_m = 60^0$$

As we know,

$$\delta_m = 2\theta - A$$

$$\theta = \frac{\delta m + A}{2} = \frac{60^0+60^0}{2} = 60^0$$

21. (8.1)

Given h = 60m, g = 10ms^{-2}

Rate of flow of water = 15 kg/s

$\therefore$ Power of the falling water

$= 15\text{kgs}^{-1} \times 10\text{ms}^{-2} \times 60\text{m} = 9000\text{watt}$

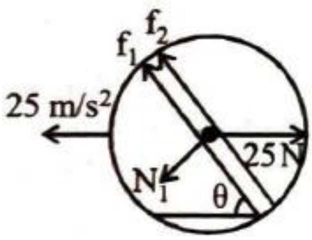
Loss in energy due to friction

$= 9000 \times \frac{10}{100} = 900\text{watt.}$

$\therefore$ Power generated by the turbine

$= (9000 - 900)\text{watt} = 8100\text{watt} = 8.1\text{kW}$

22. (10)



$N_1 = 25\sin\theta$

$\therefore f_1 = \mu N_1 = \frac{2}{5}\times 25\times\frac{3}{5} = 6N$

$f_2 = \mu N_2 = \frac{2}{5}mg = \frac{2}{5}\times 1\times 10$

Now from Newton's second law

$25\cos\theta - (f_1 + f_2) = ma$

or $25\times\frac{4}{5} - (6+4) = 1a$

$\therefore a = 10 m/s^2$.

23. (6)

M.I. of disc about tangent in a plane = $\frac{5}{4}mR^2 = I$

$\therefore mR^2 = \frac{4}{5}I$ ……(i)

M.I. of disc about tangent $\perp$ to plane $I' = \frac{3}{2}mR^2$

Substitution the value of MR^2 from equation (i) we get

$I' = \frac{3}{2}\left(\frac{4}{5}I\right) = \frac{6}{5}I$

24. (160)

Boat covers distance of 16km in a still water in 2 hours

i.e $v_B = \frac{16}{2} km/hr$

Now velocity of water $\Rightarrow v_w$ = 4km/hr

Time taken for going upstream

$t_1 = \frac{8}{v_B - v_w} = \frac{8}{8-4} = 2hr$

(As water current oppose the motion of boat)

Time taken for going down stream

$t_2 = \frac{8}{v_B + v_w} = \frac{8}{8+4} = \frac{8}{12}hr$

(As water current helps the motion of boat)

$\therefore$ Total time = $t_1 + t_2 = \left(2 + \frac{8}{12}\right)$hr or 2 hr 40 min

25. (0.01)

Current in circuit $= \frac{(3-1)V}{200\Omega} = \frac{2}{200} = 0.01A$

26. (3)

Given $t = d^{a/2}, r^{b/2}, s^{c/2}$. Substituting dimensions, we have

$(T) = (ML^{-3})^{a/2}(L)^{b/2}(MT^{-2})^{c/2}$

$= M^{(a+c)/2}.L^{(-3a/2+b/2)}T^{-c}$

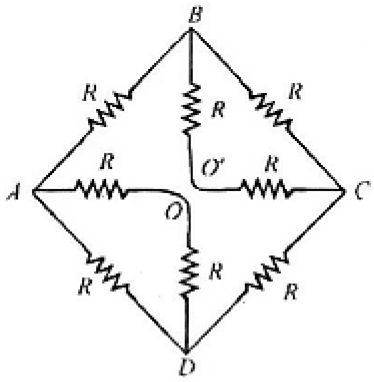
Equating powers of L, we have

$-\frac{3}{2}a + \frac{b}{2} = 0$, Given a = 1

$\therefore -\frac{3}{2} + \frac{b}{2} = 0$ or $b = 3$

27. (8)

The equivalent circuit is as shown in figure. The resistance of arm AOD (=R + R) is in parallel to the resistance R of arm AD.



Their effective resistance $R1 = \frac{2R \times R}{2R + R} = \frac{2}{3}R$

The resistance of arms AB, BC and CD is

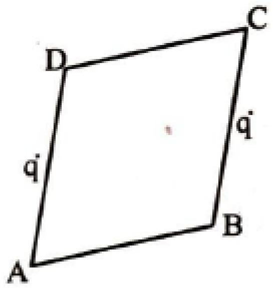
$R_2 = R + \frac{2}{3}R + R = \frac{8}{3}R$

The resistance R_1 and R_2 are in parallel. The effective resistance between A and D is

$$R_3 = \frac{R_1 \times R_2}{R_1 + R_2} = \frac{\frac{2}{3}R \times \frac{8}{3}R}{\frac{2}{3}R + \frac{8}{3}R} = \frac{8}{15}R$$

28. (0)

Both the charges are identical and placed symmetrically about ABCD. The flux crossing ABCD due to each charge is $\frac{1}{6}\left[\frac{q}{\epsilon_0}\right]$ but in opposite directions. Therefore the resultant is zero.



29. (23)

C_V for hydrogen = $\frac{5}{2}R$

C_V for helium = $\frac{3R}{2}$

C_V for water vapour - $\frac{6R}{2} = 3R$

$$\therefore (C_v)_{mix} = \frac{4\times\frac{5}{2}R + 2\times\frac{3}{2}R + 1\times 3R}{4+2+1} = \frac{16}{7}R$$

$$\therefore C_p = C_v + R$$

$$C_p = \frac{16}{7}R + R \text{ or } C_p = \frac{23}{7}R$$

30. (32)

Mg = 72N (Body weight on the surface)

$$g = \frac{GM}{R^2}$$

At a height $H = \frac{R}{2}, g' = \frac{GM}{\left(R+\frac{R}{2}\right)^2} = \frac{4GM}{9R^2}$

Body weight at height H = $\frac{R}{2}$,

$$mg' = m\times\frac{4}{9}\frac{GM}{R^2}$$

$$= m\times\frac{4}{9}\times g = \frac{4}{9}mg = \frac{4}{9}\times 72 = 32N$$

PART (B) : CHEMISTRY

SOLUTIONS

31. (D)

Since, $\sqrt{\nu} = a(Z-b)$

$\Rightarrow \sqrt{\nu} = aZ - ab$

Slope $a = \tan 45^o = 1$

Intercept ab = OX = 1

$\Rightarrow \sqrt{\nu} = 52 - 1 = 51$

$\Rightarrow \nu = 51^2$

$= 2601\,Hz$

32. (A)

At 25°C (298 K), $[H^+] = 10^{-7}$

$\Rightarrow K_w = 10^{-14}$

At 35°C (308 K), $[H^+] = 10^{-6}$

$\Rightarrow K_w = 10^{-12}$

Since, $\log\frac{K_{W_2}}{K_{W_1}} = \frac{\Delta H^o}{2.303R}\left(\frac{T_2 - T_1}{T_1T_2}\right)$

$\Rightarrow \log\frac{10^{-12}}{10^{-14}} = \frac{\Delta H^o}{2.303\times 2}\left(\frac{308-298}{308\times 298}\right)$

$\Rightarrow \Delta H^o = 84551\,cal\,mol^{-1}$

$\Rightarrow \Delta H^o = 84.55\,Kcal\,mol^{-1}$

$\downarrow$

(for $H_2O \rightarrow H^+ + OH^-$)

33. (C)

$$2SO_3(g) \rightleftharpoons 2SO_2(g) + O_2(g)$$

At equili. 400 CC 200 CC 100 CC

$P_{SO_3} = \frac{400}{700}\times 10 = \frac{40}{7}\,atm$

$P_{SO_2} = \frac{200}{700}\times 10 = \frac{20}{7}\,atm$

$P_{O_2} = \frac{100}{700}\times 10 = \frac{10}{7}\,atm$

($\because$ mol % $\propto$ vol. %)

$\Rightarrow$ (mol fraction $\propto$ vol. fraction)

$$K_P = \frac{P^2_{SO_2} \times P_{O_2}}{P_{SO_3}}$$

$$K_P = \frac{\frac{20}{7} \times \frac{20}{7} \times \frac{10}{7}}{\frac{40}{7} \times \frac{40}{7}}$$

$$\Rightarrow K_P = \frac{5}{14}$$

34. (C)

$MnO_4^- + H_2O_2 + H^+ \longrightarrow Mn^{+2} + O_2$ (H_2O_2 acting as reductant)

$PbS \xrightarrow{H_2O_2} PbSO_4$ (H_2O_2 acting as oxidant)

$SO_2 \xrightarrow{H_2O_2} H_2SO_4$ (H_2O_2 acting as oxidant)

$Na_2CO_3 + H_2O_2 \longrightarrow Na_2O_2 + H_2O + CO_2$

(Acid–Base reaction)

35. (B)

$B \equiv 801$ KJ mol^{-1}

$Al \equiv 577$ KJ mol^{-1}

$Ga \equiv 579$ KJ mol^{-1}

$In \equiv 558$ KJ mol^{-1}

$Tl \equiv 589$ KJ mol^{-1}

The irregular variation is due to poor shielding offered by d- and f-electrons which effects Z_{eff}.

36. (C)

$Pb_3O_4 + 4HNO_3 \longrightarrow 2Pb(NO_3)_2 + PbO_2 + H_2O$

37. (C)

Amphoteric oxides $\rightarrow$ Cr_2O_3, ZnO, Al_2O_3, BeO, SnO, SnO_2, PbO, PbO_2

Acidic oxides $\rightarrow$ CrO_3, B_2O_3

Basic oxide $\rightarrow$ CrO

38. (B)

$X \rightleftharpoons 2Y$, $K_C = 4$

At point (A), $Q_C = \frac{[Y]^2}{[X]} = 0$

Since, [Y] = 0.

So, (I) is incorrect.

When [X] = [Y] = 0.1 M

$Q_C = \frac{0.01}{0.1} = 0.1M$

$\Rightarrow Q_C < K_C$

Reaction is in forward direction. So, (II) is correct.

At point (D) or (E), the conc. of [X] and [Y] is constant w.r.t. time.

So, equilibrium is reached, i.e. $Q_C = K_C$

$\therefore$ (III) is correct.

39. (C)

$$CaSO_4.2H_2O \xrightarrow{120^\circ C} \underset{X(POP)}{CaSO_4.\frac{1}{2}H_2O} + \frac{3}{2}H_2O$$

$$CaSO_4.2H_2O \xrightarrow[\Delta]{200^\circ C} \underset{Y(DBP)}{CaSO_4} + 2H_2O$$

$$Y(DBP) \xrightarrow[\text{heating}]{\text{Strong}} CaO + \frac{1}{2}O_2 + \underset{(Z)}{SO_2}$$

SO_2 (Acidic gas) $\Rightarrow$ Irritating burning type smell gas.

40. (B)

Group 1 are more stable than group 2.

Thermal stability increases down the group.

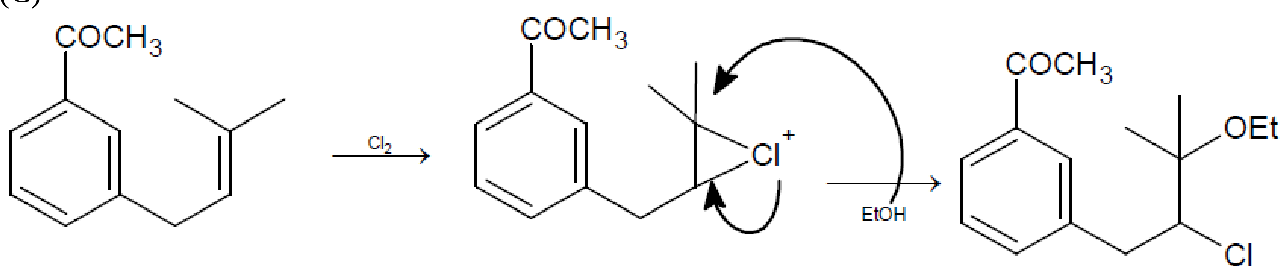
$BeCO_3 < MgCO_3 < CaCO_3 < K_2CO_3$

41. (D)

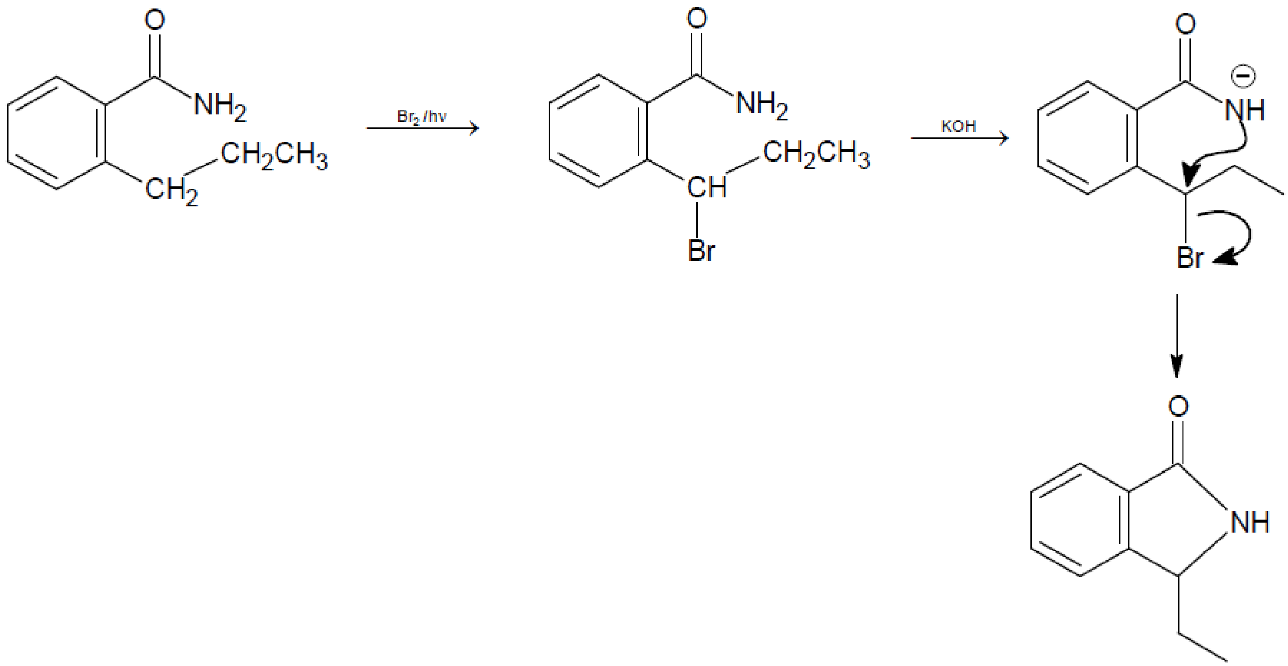
$KMnO_4$ is in conical flask and oxalic acid is added from burette.

Cannot use H_2SO_4 as it is a secondary standard.

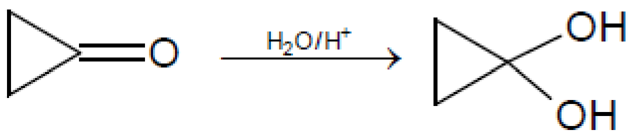
42. (C)



43. (C)

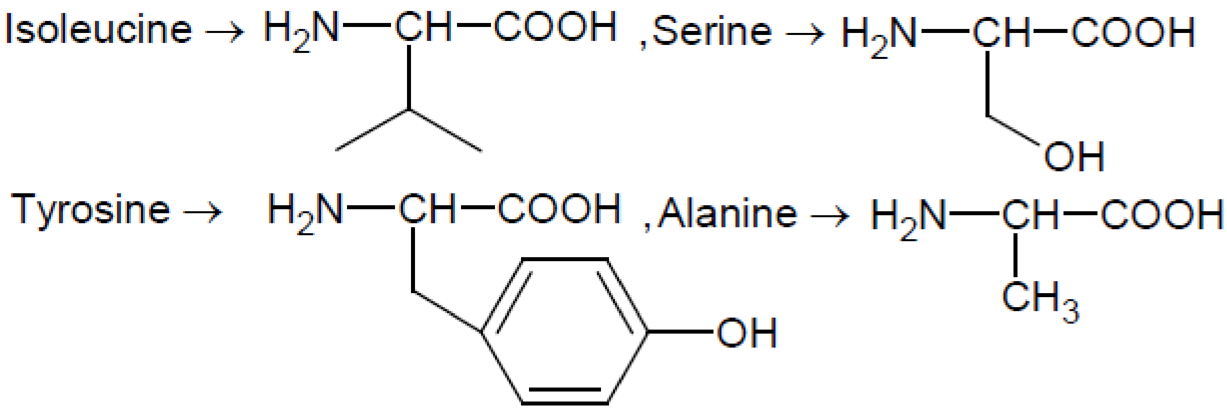


44. (A)



releases the strain and so form a stable dihydrate.

45. (D)



46. (B)

$$\frac{\text{Total molecules activated}}{\text{Total number of molecules present}} = e^{\frac{-Ea}{RT}}$$

$$\frac{\text{Molecules activated}}{N_A\,(1\text{ mole})} = e^{-\left(\frac{15.8\times10^3}{2\times298}\right)}$$

Molecules activated $= 3.067\times10^{-12}\times6.023\times10^{23}$

$$=1.847\times10^{12}$$

47. (A)

Flocculating value $\propto \dfrac{1}{\text{Flocculating power}}$

According to Hardy-Schulze rule: Flocculating power : $Sn^{+4} > Al^{+3} > Ba^{+2} > Na^{+}$

48. (D)

$$m_{KCl} = m_X$$

$$\therefore \frac{\Delta T_{f(KCl)}}{i_{KCl}} = \frac{\Delta T_{f(X)}}{i_{(X)}}$$

$$i_{(X)} = \frac{1}{2}$$

For trimerisation to 75% extent

$$i = 1 + \left(\frac{1}{6} - 1\right) \times \frac{3}{4}$$

$$i = \frac{1}{2}$$

49. (D)

All 3 belong to same group and Van-Arkel method is used for this refining.

50. (B)

$NaAlSiO_4$ is a Zeolite used for removing permanent hardness of water.

51. (600)

$$\frac{-d[RX]}{dt} = K_2[RX]\left[OH^-\right](S_N2)$$

$$\frac{-d[RX]}{dt} = K_1[RX] \qquad (S_N1)$$

$$\Rightarrow \frac{-d[RX]}{dt} = K_2[RX]\left[OH^-\right] + K_1[Rx]$$

OR

$$\underset{\downarrow\ Y}{-\frac{1}{RX}\cdot\frac{d[RX]}{dt}} = \underset{\downarrow\ \text{Slope}}{K_2}\underset{\downarrow\ X}{\left[OH^-\right]} + \underset{\downarrow\ \text{Intercept}}{K_1}$$

$$K_2 = 4\times10^3\ M^{-1}s^{-1}$$

$$K_1 = 2\times10^2\ s^{-1}$$

$$[RX] = 1M \qquad \left[OH^-\right] = .1M$$

$$\Rightarrow \frac{-d[RX]}{dt} = 4\times10^3\times0.1 + 2\times10^2$$

$\Rightarrow \dfrac{-d[RX]}{dt} = 600 \text{ mol L}^{-1} \text{ S}^{-1}$

52. (2.75)

The balanced equation is

$C_2H_5OH + 4I_2 + 6OH^- \longrightarrow CHI_3 + HCOO^- + 5H_2O + 5I^-$

$\Rightarrow x = 6, y = 4, z = 5$

$\Rightarrow \dfrac{x+z}{y} = \dfrac{6+5}{4} = \dfrac{11}{4} = 2.75$

53. (6.20)

Hydrazine is a weak base

$\therefore \text{ pOH} = \dfrac{1}{2}(pK_b - \log C)$

$2(14 - 9.7) = pK_b - \log 4 \times 10^{-3}$

$8.6 = pK_b + 3 - 2(0.30)$

$\Rightarrow pK_b = 6.20$

54. (1.25)

$$\left.\begin{aligned} N_2^+, BO &= \frac{1}{2}(9-4) = 2.5 \\ N_2^-, BO &= \frac{1}{2}(10-5) = 2.5 \\ O_2^+, BO &= \frac{1}{2}(10-5) = 2.5 \end{aligned}\right\}$$

$O_2^-, BO = \dfrac{1}{2}(10-7) = 1.5$

NO and N_2^- are isoelectronic.

So, BO (NO) = 2.5

CN and N_2^+ are isoelectronic

So, BO (CN) = 2.5

$\therefore$ Total species 'x' = 5

$\therefore \dfrac{x}{4} = 1.25$

55. (3.20)

$C_{60} \Rightarrow 5 \text{ membered ring} = 12$

6 membered rings $= \dfrac{60}{2} - 10 = 20$

T = 12 + 20 = 32

$$\therefore \frac{T}{10} = \frac{32}{10} = 3.20$$

56. (2)

$$M(s) + nH^+ \longrightarrow M^{n+}(aq) + \frac{n}{2}H_2(g)$$

$$E_{cell} = E^o_{cell} - \frac{0.059}{n}\log\frac{[M^{n+}](P_{H_2})^{n/2}}{[H^+]^n}$$

$$0.81 - 0.76 = \frac{-0.059}{n}\log(0.02)$$

$$n = 2$$

57. (9.20)

$$H_2O(g) \longrightarrow H_2O(g) \rightleftharpoons H_2O(\ell) \longrightarrow H_2O(\ell)$$

385 K 373 K 373 K 280 K

$$\Delta S = mC\ln\frac{T_2}{T_1}$$

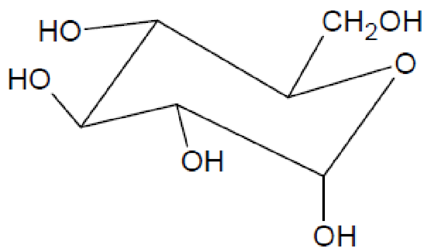
$$\Delta S_1 = 2\times2\times2.303\log\left(\frac{373}{385}\right) = -0.1267$$

$$\Delta S_2 = \frac{\Delta H_{condensation}}{T} = -\frac{2491}{373} = -6.67$$

$$\Delta S_3 = 2\times4.2\times2.303\log\left(\frac{280}{385}\right) = -2.409$$

$$\Delta S_{Total} = -9.205\text{ kJ kg}^{-1}\text{ K}^{-1}$$

58. (3)



Only 3 are equatorial position.

59. (0.22)

$[Ni(CO)_4]$ is sp^3 hybridised, hence tetrahedral.

Limiting ratio for tetrahedral is 0.225.

60. (0.67)

For node, $\psi_{2s}^{2} = 0 \quad \therefore \psi_{2s} = 0$

$$\left(2 - \frac{Zr}{a_0}\right) = 0$$

$$r = \frac{2a_0}{Z} = \frac{2a_0}{3} = 0.67\,a_0$$

PART (C) : MATHEMATICS

SOLUTIONS

61. (D)

62. (D)

$z^2+z+1=0 \Rightarrow z=\omega, \omega^2$

$\therefore \left(z+\frac{1}{z}\right)^2+\left(z^2+\frac{1}{z^2}\right)^2+\ldots\ldots+\left(z^6+\frac{1}{z^6}\right)^2$

$=\left(\omega+\omega^2\right)^2+\left(\omega+\omega^2\right)^2+\left(\omega^3+\frac{1}{\omega^3}\right)^2+\left(\omega+\omega^2\right)^2+\left(\omega+\omega^2\right)^2+\left(\omega^3+\frac{1}{\omega^3}\right)^2$

$=4\left(\omega+\omega^2\right)^2+2\left(\omega^3+\frac{1}{\omega^3}\right)^2=4(1)+2\left(2^2\right)$

$=4+8=12$

63. (B)

64. (C)

$\frac{a_2}{a_1}=\frac{a_3}{a_2}=\ldots\ldots=\frac{a_n}{a_{n-1}}=r$

Which means $a_n, a_{n+1}, a_{n+2} \in G.P. \forall n \in N$

$\Rightarrow a_{n+1}^2=a_n a_{n+2}$

$\Rightarrow 2\log a_{n+1}-\log a_n-\log a_{n+2}=0$ ....(i)

Similarly

$2\log a_{n+4}-\log a_{n+3}-\log a_{n+5}=0$ ......(ii)

And $2\log a_{n+7}-\log a_{n+6}-\log a_{n+8}=0$ ....(iii)

Using $C_1 \to C_1+C_3-2C_2$

We get $\Delta=0$

65. (B)

The roots of $bx^2+cx+a=0$ are imaginary i.e, $c^2-4ab<0 \Rightarrow c^2<4ab$

Again, the coefficient of x^2 in $3b^2x^2+6bcx+2c^2$ is $+ve$, so the minimum value of the expression

$=-\left[\frac{36b^2c^2-4\left(3b^2\right)\left(2c^2\right)}{4\left(3b^2\right)}\right]=-\frac{12b^2c^2}{12b^2}=-c^2$

As $c^2<4ab$, we have , $-c^2>-4ab$

Thus, the minimum value is $-4ab$

66. (B)

Since order of vowels is not to change, the four vowels I,I,E,E are to be taken as similar.

Hence, the required no, is $\frac{6!}{4!}-1=29$

67. (C)

Determinant of coefficients

$$=\begin{vmatrix} t & t+1 & t-1 \\ t+1 & t & t+2 \\ t-1 & t+2 & t \end{vmatrix}$$

$$=\begin{vmatrix} t & 1 & -1 \\ t+1 & -1 & 1 \\ t-1 & 3 & 1 \end{vmatrix}=\begin{vmatrix} t & 1 & -1 \\ 2t+1 & 0 & 0 \\ 2t-1 & 4 & 0 \end{vmatrix}=-4(2t+1)$$

For non-trivial solution $t=-\frac{1}{2}$

68. (C)

T_{r+1} of $\left(ax^2+\frac{1}{bx}\right)^{11}={}^{11}C_r\left(ax^2\right)^r\left(\frac{1}{bx}\right)^{11-r}$

T_{r+1} of $\left(ax-\frac{1}{bx^2}\right)^{11}={}^{11}C_r\left(ax\right)^r\left(-\frac{1}{bx^2}\right)^{11-r}$

$\therefore$ Coefficient of x^7 in $\left(ax^2+\frac{1}{bx}\right)^{11}={}^{11}C_6\frac{a^6}{b^5}$

and coefficient of x^{-7} in $\left(ax-\frac{1}{bx^2}\right)^{11}={}^{11}C_5\frac{a^5}{b^6}$

Now, ${}^{11}C_6\frac{a^6}{b^5}={}^{11}C_5\frac{a^5}{b^6}\Rightarrow ab=1.$

69. (B)

Let if possible $f'(x)=2$ for $[1,6]$

$\Rightarrow f(x)=2x+c$ (Integrating both sides w.r.t. x)

$\therefore f(1)=2+c=-2$

$c=-4$

$\therefore f(x)=2x-4$

$\therefore f(6)=2\times6-4=8 \quad \therefore f(6)\geq 8$

70. (B)

(i) Sun rises or moon sets is an "exclusive or" statement as sun rises and moon sets during day time.

(ii) Since, all integers cannot be both positive as well as negative. Therefore , statement is " exclusive or".

(iii) It is not possible for two lines to intersect and parallel at the same time. So, this statement is " exclusive or" statement.

(iv) It is fact that school is closed on holiday as well as on Sunday. So, statement "The school is closed if it is a holiday or Sunday.

Hence, the statement(iv) is an "inclusive or" statement.

71. (A)

$f'(x)=x^2+2x+2=(x+1)^2+1>0\forall x$

$\therefore f(x)$ is monotonically increasing in $[2,4]$

$\therefore$ Min $f(x)=f(2)$ and Max. $f(x)=f(4)$

$\therefore \text{Min.} f(x)=\int_0^2 (t^2+2t+2)dt=\frac{32}{3}$ and

Max. $f(x)=\int_0^4 (t^2+2t+2)dt=\frac{136}{3}$

72. (C)

Let $I=\int_{-3\pi/2}^{-\pi/2}\left[(x+\pi)^3+\cos^2(x+3\pi)\right]dx$

Put $x+\pi=t \Rightarrow dx=dt$

$\therefore I=\int_{-\pi/2}^{\pi/2}\left[t^3+\cos^2(t+2\pi)\right]dt=\int_{-\pi/2}^{\pi/2}\left[t^3+\cos^2 t\right]dt$

$=2\int_0^{\pi/2}\cos^2 t\,dt$ $\left(\because \int_{-\pi/2}^{\pi/2} t^3 dt=0\right)$

$=\int_0^{\pi/2}(1+\cos 2t)dt=\frac{\pi}{2}$

$\therefore I=\frac{\pi}{2}$

73. (B)

We are given that vectors lie in the same plane. We know, vector
$\vec{L}=a\hat{i}+a\hat{j}+c\hat{k}, \vec{M}=\hat{i}+\hat{k}, \vec{N}=c\hat{i}+c\hat{j}+b\hat{k}$, are coplanar, if

$$\left[\vec{L}\,\vec{M}\,\vec{N}\right]=0 \Rightarrow \begin{vmatrix} a & a & c \\ 1 & 0 & 1 \\ c & c & b \end{vmatrix}=0 \Rightarrow c=\sqrt{ab}$$

$\therefore c$ is geometric mean of a and b

74. (B)

Equation of the line through the points $(5,1,a)$ and $(3,b,1)$

$\frac{x-5}{3-5}=\frac{y-1}{b-1}=\frac{z-a}{1-a}=\lambda$

Now, it passes through $\left(0,\frac{17}{2},\frac{-13}{2}\right)$

$\therefore \frac{0-5}{-2}=\frac{(17/2-1)}{b-1}=\frac{(-13/2)-a}{1-a}=\lambda$

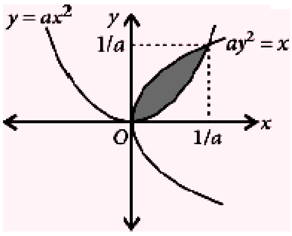
$\Rightarrow \lambda=\frac{5}{2}$

$\therefore \frac{(17/2-1)}{b-1}=\frac{5}{2}\Rightarrow b=4$

And $\dfrac{-\frac{13}{2}-a}{1-a}=\dfrac{5}{2}\Rightarrow a=6$

75. (B)
Since we are given rate per rupees, harmonic mean will give the correct answer.
$$\text{H.M.}=\frac{4}{\frac{1}{1}+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}}=\frac{4\times 12}{25}=1.92\,\text{kg per rupee}$$

76. (A)
The curves meet at $(0,0)$ and $\left(\frac{1}{a},\frac{1}{a}\right)$



$\therefore$ Required area $=\displaystyle\int_0^{1/a}\left(\sqrt{\frac{x}{a}}-ax^2\right)dx=1$

$\Rightarrow \dfrac{2}{3a^2}-\dfrac{1}{3a^2}=1\Rightarrow \dfrac{1}{3a^2}=1\Rightarrow a=\dfrac{1}{\sqrt{3}}$

77. (A)
$\tan^{-1}\theta+\cot^{-1}\theta=\dfrac{\pi}{2}=x$

$\therefore \sin x=\sin\dfrac{\pi}{2}=1$

78. (D)
Let $I=\displaystyle\int\frac{(x\cos x+1)}{x\sqrt{2xe^{\sin x}+1}}dx$
Put $t^2=2xe^{\sin x}+1\Rightarrow t^2-1=2xe^{\sin x}$
Taking log on both sides, we get
$\log\left(t^2-1\right)=\log 2+\log x+\sin x$

$\Rightarrow \dfrac{2t}{t^2-1}dt=\dfrac{x\cos x+1}{x}dx$

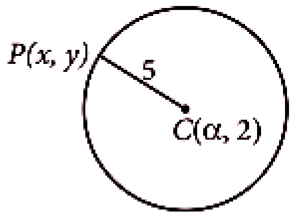
$\therefore I=\displaystyle\int\frac{2dt}{t^2-1}=\ln\left(\frac{t-1}{t+1}\right)+c$

$=\ln\left(\dfrac{\sqrt{2xe^{\sin x}+1}-1}{\sqrt{2xe^{\sin x}+1}+1}\right)+c$

79. (D)

The points are $A\left(0,\frac{8}{3}\right), B(1,3)$ and $C(82,30)$ Slope of $AB=\frac{1}{3}=$ Slope of BC = Slope of AC. So, the given points lie on same line.

80. (B)



Given, centre lies on the line $y=2$

$\therefore C(\alpha,2)$ and radius of circle $=5$ units

$\therefore$ Equation of circle be

$(x-\alpha)^2+(y-2)^2=25$ …….(i)

Differentiating (i) w.r.t. to x, we get

$(x-\alpha)+(y-2)y'=0$

$\Rightarrow (x-\alpha)^2=(y-2)^2(y')^2$ ……(ii)

Putting (ii) in (i), we have

$(y-2)^2(y')^2+(y-2)^2=25$

$\Rightarrow (y-2)^2(y')^2=25-(y-2)^2$

81. (1)

We have, $f(x)=2x^3+bx^2+cx$

Now, $f(1)=f(-1)$ and $f'\left(\frac{1}{2}\right)=0$ [$\because$ $f(x)$ satisfies Rolle's theorem]

So, $f(1)=2+b+c$

$f(-1)=-2+b-c$

$\therefore f(1)=f(-1)\Rightarrow c=-2$ ……(i)

Also, $f'(x)=6x^2+2bx+c$

$\Rightarrow f'\left(\frac{1}{2}\right)=\frac{3}{2}+b+c=0\Rightarrow\frac{3}{2}+b-2=0$

$\Rightarrow b=\frac{1}{2}$ …….(ii)

Hence, $2b+c=\left(2\times\frac{1}{2}\right)+(-2)=-1$ (using (i) and (ii))

82. (3)

Let $h=Ar+B$

$\Rightarrow\frac{dh}{dt}=\frac{Adr}{dt}=\frac{3dr}{dt}\Rightarrow A=3$

Given, $6 = A + B \Rightarrow B = 3$[from (i)]
Hence, $h = 3r + 3$
Volume (V) $= \pi r^2 h = \pi r^2 (3r + 3)$

$$\frac{dV}{dt} = 3\pi\left[3r^2 + 2r\right]\frac{dr}{dt}$$

$$\Rightarrow n = 3\pi(27 + 6)\frac{1}{3\pi} = 33$$

$$\therefore \frac{n}{11} = 3$$

83. (3)
Consider the two curves $y^2 = 4ax$ and $x^2 = 4ay$. Each is mirror image of the other in the line $x - y = 0$
$\therefore$ The slope of the required line is -1.
$\therefore$ The normal $y = mx - 2am - am^3$ becomes

$$x + y = 3a = \frac{3}{4}\left[\because a = \frac{1}{4}\right]$$

The distance of the origin from it is $\frac{3}{4\sqrt{2}}$

84. (1)
Let (x_1, y_1) is the required solution. Since both the equations are symmetric w.r.t. y-axis. So $(-x_1 y_1)$ is also a solution.
But unique solution $\Rightarrow x_1 = -x_1 \Rightarrow x_1 = 0$
So $y_1 = \pm 1$. If $y_1 = 1 \Rightarrow a = 0$
If $y_1 = -1 \Rightarrow a = 2$
For $a = 0, 2^{|x|} + |x| = y + x^2 \Rightarrow (0,1)$ only one solution.
For $a = 2, 2^{|x|} + |x| = y + x^2 + 2$
$\Rightarrow (0,-1), (2,0), (1,0), (-1,0)$
Hence $a = 0$ is acceptable.

85. (2)
We have, $\frac{8^x + 27^x}{12^x + 18^x} = \frac{7}{6}$

$$\Rightarrow \frac{(8)^x}{(12)^x} \times \frac{\left(1 + (27/8)^x\right)}{\left(1 + (18/12)^x\right)} = \frac{7}{6}$$

$$\Rightarrow \left(\frac{2}{3}\right)^x \left(\frac{1 + (3/2)^{3x}}{1 + (3/2)^x}\right) = \frac{7}{6} \qquad \ldots\ldots(i)$$

Let $\left(\frac{3}{2}\right)^x = t$
So, (i) becomes

$\frac{1+t^3}{t(1+t)}=\frac{7}{6}$ $\quad [\text{where } t+1\neq 0]$

$\Rightarrow \frac{(1+t)(t^2+1-t)}{t(1+t)}=\frac{7}{6}$

$\Rightarrow \frac{t^2+1-t}{t}=\frac{7}{6}\Rightarrow t=\frac{2}{3} \text{ or } \frac{3}{2}$

$\left(\frac{3}{2}\right)^x=\frac{2}{3} \text{ or } \left(\frac{3}{2}\right)^x=\frac{3}{2}\Rightarrow x=-1 \text{ or } 1$

86. (1)

Normal to $x^2=4y$ is $x=\frac{y}{m}-\frac{2}{m}-\frac{1}{m^3}$. It passes through $(1,2)$

$\Rightarrow 1=2/m-2/m-1/m^3\Rightarrow m^3=-1$ or $m=-1$

87. (2)

$\lim_{x\to 5\pi/4}\left[\sqrt{2}\sin(x+\pi/4)\right]$

$\text{R.H.L.}=\lim_{h\to 0}\left[\sqrt{2}\sin\left(\frac{5\pi}{4}+\frac{\pi}{4}+h\right)\right]$

$=\lim_{h\to 0}\left[-\sqrt{2}\cos h\right]=-2$

$\text{L.H.L.}=\lim_{h\to 0}\left[\sqrt{2}\sin\left(\frac{5\pi}{4}+\frac{\pi}{4}-h\right)\right]$

$=\lim_{h\to 0}\left[-\sqrt{2}\cos h\right]=-2$

88. (8)

$I=\int_{-\pi/2}^{\pi/2}\frac{\sin^2 x}{1+e^x}dx\Rightarrow I=\int_{-\pi/2}^{\pi/2}\frac{\sin^2(-x)}{1+e^{-x}}dx$

$\Rightarrow I+I=\int_{-\pi/2}^{\pi/2}\sin^2 x\left(\frac{e^x+1}{1+e^x}\right)dx\Rightarrow I=\frac{1}{2}\int_{-\pi/2}^{\pi/2}\sin^2 x dx$

Or $I=\int_0^{\pi/2}\sin^2 x\, dx$ ….. (i) or $I=\int_0^{\pi/2}\cos^2 x\, dx$ …….(ii)

Adding (i) and (ii)

$\Rightarrow 2I=\int_0^{\pi/2}(\sin^2 x+\cos^2 x)dx=\frac{\pi}{2}\Rightarrow I=\frac{\pi}{4}$

89. (1)

We have, $y=\frac{x+c}{1+x^2}\Rightarrow y+x^2y=x+c$

Now differentiating both sides w.r.t.x, we get

$\frac{dy}{dx}+x^2\frac{dy}{dx}+y(2x)=1$

$\because \frac{dy}{dx} = 0$ (y is stationary) $\therefore xy = \frac{1}{2}$

90. (1)

Here, $f(x) = x^3 + 3x^2 + 4x + b\sin x + c\cos x$

$\Rightarrow f'(x) = 3x^2 + 6x + 4 + b\cos x - c\sin x$

Now for $f(x)$ to be one-one only possibility is

$f'(x) \geq 0, \forall x \in R$

i.e, $3x^2 + 6x + 4 + b\cos x - c\sin x \geq 0, \forall x \in R$

i.e., $3x^2 + 6x + 4 \geq c\sin x - b\cos x, \forall x \in R$

i.e., $\sqrt{b^2 + c^2} \leq 3(x^2 + 2x + 1) + 1, \forall x \in R$

$\Rightarrow \sqrt{b^2 + c^2} \leq 3(x+1)^2 + 1, \forall x \in R$

$\Rightarrow \sqrt{b^2 + c^2} \leq 1, \forall x \in R \Rightarrow b^2 + c^2 \leq 1, \forall x \in R$