PART (A) : PHYSICS

SOLUTIONS

1. (AC)

$\tau = NiAB = I\alpha$

$$N \cdot \frac{dq}{dt} AB = I \cdot \alpha$$

$$NAB\int_0^Q dq = I\int_0^\omega \alpha dt$$

$NAB\,Q = I\omega$

$$\omega = \frac{NABQ}{I}$$

Kinetic energy $= \frac{1}{2}I\omega^2 = \frac{1}{2}CQ_0^2$

2. (A, D)

Point A and C are on the same line passing

through origin $\Rightarrow \frac{P_A}{V_A} = \frac{P_C}{V_C}$ ...(1)

Also $T_A = 200\text{ K} = \frac{P_A V_A}{nR}$ and Also $T_C = 1800$

$T_C = \frac{P_C V_C}{nR} \Rightarrow \frac{P_A V_A}{P_C V_C} = \frac{1}{9}$ ...(2)

From (1) and (2)

$$\frac{V_A}{V_C} = \frac{1}{3}$$

3. **(C)**

Consider the frame moving with the final velocity of the plank (it is also the frame of mass center). In this frame initial kinetic energy of the block is completely consumed into the work done against the friction force and spring force.

Vcm = m v_0/ M+m.

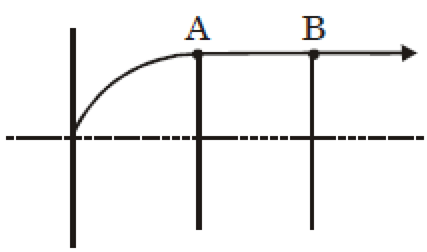
Velocity of particle in center of mass frame

$$= \frac{Mv_0}{M+m}$$

$\frac{1}{2}m\left[\frac{mv_0}{M+m}\right]^2$ = energy stored in spring + work done against friction.

$$v_0 = \mu g\sqrt{\frac{3m(m+M)}{kM}} = 0.4 \text{ m/s}$$

4. **(A, C)**



Velocity is perpendicular to magnetic field hence, parable with perform circular motion.

$$r = \frac{mv}{qB} = \frac{1\times 20}{2\times 5} = 2\text{m}$$

$v_A = 20$ ms

F = qE

$$a = \frac{qE}{m} = 2\times 50 = 100$$

$$v_B^2 = v_A^2 + 2a\times 2$$

$$v_B^2 = (20)^2 + 400 = 800$$

$$v_B = 20\sqrt{2}\hat{i}$$

5. **(B, C)**

6. **(A)**

We can choose an arbitrary center of rotation to solve this problem. In order to simplify our solution, let us choose the center of rotation so that the torque due to static friction is zero. That is accomplished by picking the point of contact with the ground as the center of rotation, because the lever arm of static friction is then 0.

The magnitude of the torque due to force T around the contact point with the ground is (τ_1 = T.2a). We want it to match the torque caused by the hole. To more experienced problem solvers it will be obvious that

$$\tau_2 = W.\frac{2}{5}a,$$

where W is the weight of the removed material. Let us derive this for those less experienced. Let us denote the point of constant by C, the center of the cross section of the cylinder by O, and the center of the hole by M. By definition, the magnitude of torque τ_2 is

$$\tau_2 = W.\ |CM|.\sin\theta,$$

where $\theta = \angle(CO, CM)$. Since [CM]. $\sin\theta = \frac{2a}{5}$, we see that indeed

$$\tau_2 = W.\frac{2}{5}a,$$

Therefore,

$$\tau_1 = \tau_2 \Rightarrow 2aT = \frac{2aW}{5}$$

$$\Rightarrow T = \frac{W}{5} = 3\text{Nm}$$

so the closest answer is A.

7. **(A, B, C)**

Conservation of angular momentum about centre.

$$\left[\frac{mR^2}{2}+\frac{mR^2}{4}\right]\times\omega_0=\left[\frac{mR^2}{2}+mR^2\right]\times\omega$$

$\omega=\frac{\omega_0}{2}$ = Angular velocity of disc

Conservation of energy

$$\frac{1}{2}\times\left[\frac{mR^2}{2}+\frac{mR^2}{4}\right]\times\omega_0^2$$

$$=\frac{1}{2}\times kx^2+\frac{1}{2}\times\frac{mR^2}{2}\times\left(\frac{\omega_0}{2}\right)^2+\frac{1}{2}\times m\left(\left(\frac{\omega R}{2}\right)^2+V_R^2\right)$$

$$V_R^2=\frac{\omega_0^2}{4}\times R^2$$

$V_R=\frac{\omega_0 R}{2}$ Radial velocity

$$E=\frac{1}{2}kx^2$$

$\frac{dE}{dt}$: Rate of change of energy $= kx.\frac{dx}{dt}=kx.V_R$

$$=\frac{m\omega_0^2}{2}\times\frac{R}{2}\times\frac{\omega_0 R}{2}=\frac{m\omega_0^3R^2}{8}$$

8. **(A, B, D)**

Initial geometrical centre

$$=\left(\frac{4.4+16.1}{2}\right)cm=10.25\text{ cm}$$

Final geometrical centre

$$=\left(\frac{20+36.5}{2}\right)=28.25\text{ cm}$$

Displacement w.r.t. scale = 18 cm

$(0.5+0.25)x=0.25\times 18$

$$x=\frac{0.25\times 18}{0.75}=6cm$$

Initial length = 16.1 – 4.1 = 11.7 cm

Final length = 36.5 – 20 = 16.5 cm

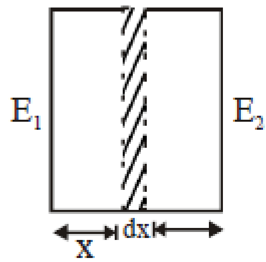
change = (16.5 – 11.5) cm = 4.8 cm

$$\Delta l_i=0.2\,cm$$

$$\Delta l_f = 0.2 \text{ cm}$$

Net error = 0.4 cm

9. **(2.00)**



$$E(x+dx) - E(x) = \frac{\rho dx}{\epsilon_0}$$

$$\epsilon_0 \, dE = \rho dx$$

$$dF = Edq = E\rho A dx$$

$$dF = E(\epsilon_0 \, dE)\, dA$$

$$\frac{F}{A} = \epsilon_0 \int_{E_1}^{E_2} EdE$$

$$\frac{F}{A} = \frac{\epsilon_0}{2}(E_2+E_1)(E_2-E_1)$$

$$E_2 - E_1 = \frac{\sigma}{\epsilon_0}$$

$$\frac{F}{A} = \frac{\sigma}{2}(E_2+E_1)$$

10. **(7.00)**

$$dR = \frac{dx}{60(10+x)}; \quad \lambda T_B = b; \; T_B = 10^3 \text{ K}$$

$$R = \frac{1}{60}\left[\ln\left(\frac{3}{2}\right)\right]$$

$$\frac{(T_A - T_B)}{R} = eA\sigma\left[T_B^4 - (300)^4\right]$$

$$T_A \approx 1400 \text{K}$$

11. **(9.00)**

$$\frac{\rho_1}{\rho_2} = \frac{1}{2}$$

$(A - \ell^2)\, h' = \ell^2 h$

$$\frac{\ell}{2} = h + h'; \qquad h = \left(\frac{\ell}{3}\right)$$

$F = (h + h')\rho_2\, \ell^2 g$

$F = 1.5\, \ell^2 \rho_2 g h$

$$W = \int_0^{\ell/3} F\, dh = 6.75\, J$$

12. **(4.00)**

$$dE = Bv\,dy = \frac{\mu_0 i_0}{2\pi y} v\,dy$$

$$E = \int dE = \int_a^{a+b} \frac{\mu_0 i_0 v}{2\pi y} dy = \frac{\mu_0 i_0 v}{2\pi} \ln\left(\frac{a+b}{a}\right)$$

Total resistance $R_0 = R + 2\lambda x = (R + 2\lambda v t)$

$$i = \frac{E}{R_0} = \frac{\mu_0 i_0 v}{2\pi[R + 2vt\lambda]} \ln\left(\frac{a+b}{a}\right)$$

$$F = \int dF = Bi\,dy = \frac{\mu_0^2 i_0^2 v}{4\pi^2 (R + 2vt\lambda)} \ln\left(\frac{a+b}{a}\right) \int_a^{a+b} \frac{dy}{y}$$

$$= \frac{\mu_0^2 i_0^2 v}{4\pi^2 (R + 2vt\lambda)} \left[\ln\left(\frac{a+b}{a}\right)\right]^2$$

13. **(4.00)**

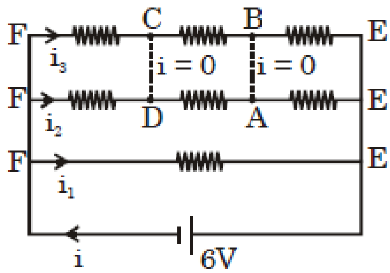
by symetring, potential at A and potential at B will be equal.

Similary $\qquad V_C = V_D$

Hence, $\qquad i_{AB} = 0$

$\qquad i_{CD} = 0$

circuit will be as



$$\frac{1}{R_{eq}} = \frac{1}{15} + \frac{1}{15} + \frac{1}{5}$$

$$R_{eq} = 3\Omega$$

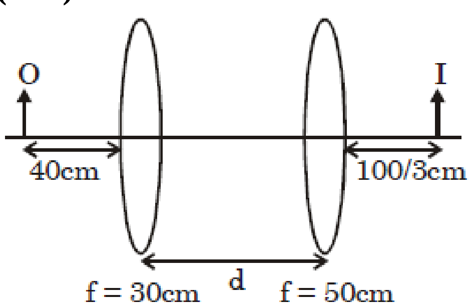
$$i = \frac{6}{3} = 2A$$

$$i_{AD} = i_2 = \frac{6}{15} = \frac{2}{5} \text{ ampere}$$

$$\alpha = \frac{2}{5}$$

$$10\alpha = 4$$

14. (2.00)



for first lens

$u_1 = -40 cm, f_1 = 30 cm$

$v_1 = ?$

$\frac{1}{v_1} - \frac{1}{-40} = \frac{1}{30}$ [Lens formula]

$\frac{1}{v_1} = \frac{1}{30} - \frac{1}{40}$

$v_1 = 120$ cm

for second lens

$u_2 = (u_1 - d)$

$u_2 = (120 - d)$

$f_2 = 50$ cm

$v_2 = \frac{100}{3}$ cm

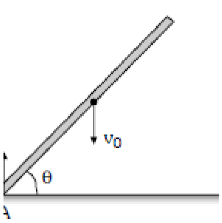
$\frac{1}{100/3} - \frac{1}{(120-d)} = \frac{1}{50}$

$d = 20$ cm

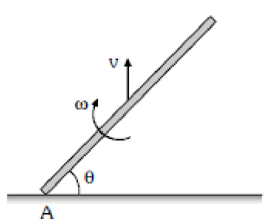
15. (5.00)

16. (6.00)

Let, v = linear velocity of rod after impact (upwards), ω = angular velocity of rod



At the time of impact



After impact

and J = linear impulse at A during impact

Then, $J = \Delta P = P_f - P_i$

$J = mv - (-mv_0)$

$\therefore J = m(v + v_0)$

Angular impulse = ΔL

$\therefore \quad J\left(\frac{l}{2}\cos\theta\right) = I\omega = \frac{ml^2}{12}\omega$

Collision is elastic (e = 1)

$\therefore$ Relative speed of approach = Relative speed of separation at point of impact

$$v_0 = v + \frac{l}{2}\omega\cos\theta$$

solving above equation, we get $\omega = \frac{6v_0\cos\theta}{l(1+3\cos^2\theta)}$

17. (20.00)

$$P_0 - \rho g(4\text{mm}) + \rho g(3\text{mm}) + \frac{2s}{R} = P_0$$

$$\Rightarrow \frac{2s}{R} = \rho g(1\text{mm})$$

$$\Rightarrow R = 200 \text{ mm}$$

18. (1.57)

$$Q_{max} = \sigma_{max} 4\pi R^2 = (\varepsilon_0 E_0) 4\pi R^2 = 4\pi\varepsilon_0 E_0 R^2$$

$$F = \frac{Q^2}{32\pi\varepsilon_0 R^2} \Rightarrow F = \frac{\pi\varepsilon_0 E_0^2 R^2}{2}$$

PART (B) : CHEMISTRY

SOLUTIONS

1. **(A, B, C)**

(A) For solution Z_2 at P_1 pressure

$X_A = 0.25$ $X_B = 0.75$

$Y_A = 0.5$ $Y_B = 0.5$

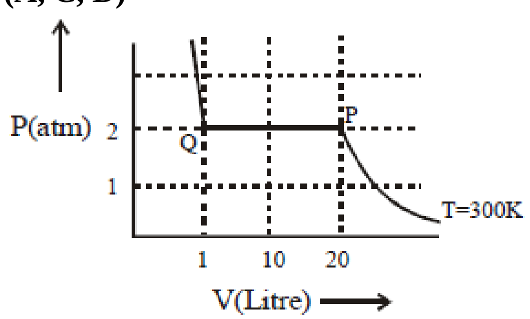
(B) For solution Z_2 at P_3 pressure $\rightarrow$ solution will not vapourise so

$X_A = 0.4$; $X_B = 0.6$

(C) For sulution Z_1 at P_2 pressure $\rightarrow$ solution will not vapourise so

$X_A = 0.2$; $X_B = 0.8$

2. **(A, C, D)**



(A) A real gas condense if external pressure is just greater than vapour pressure (B,C,D)

At point 'P' whole real gas is in gaseous phase so density of gas $= \frac{1000}{20 \times 1000} = 0.05 \text{gm/ml}$

At point 'Q' whole substance is in liquid phase so density of liquid $= \frac{1000}{1 \times 1000} = 1 \text{gm/ml}$

Note : during condensation at 300K density of gas and liquid remains constant, only amount varies.

3. **(D)**

In terms of edge length 'a'

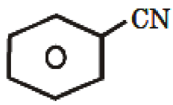
$$X = \frac{\sqrt{2}}{2}a$$

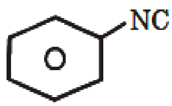
$$Y = \frac{a}{2}$$

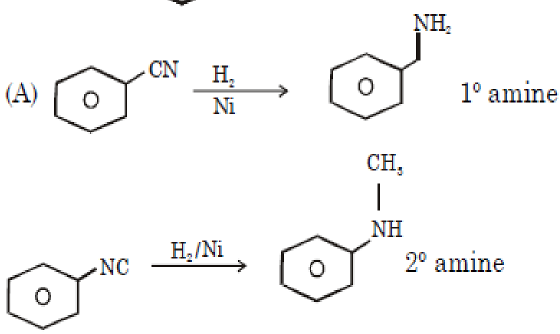
$$Z = \frac{\sqrt{3}a}{4}$$

4. **(C, D)**

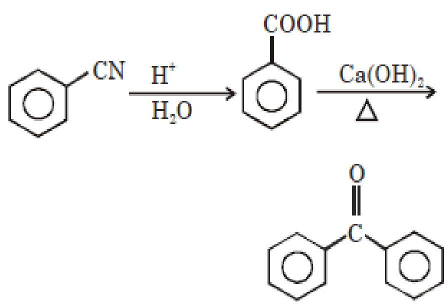
5. (B, C, D)

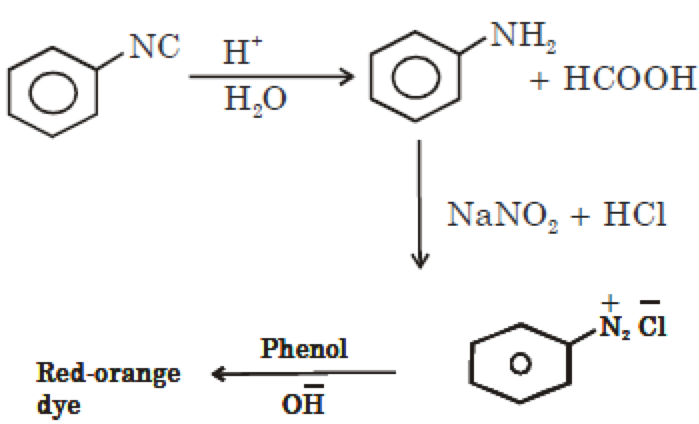
P = 

Q = 

(A) 

Functional isomer not metamer

(B) 

(C) 

(D) $CHCl_3 + \overline{O}H \;+\; \longrightarrow \; :CCl_2$

6. (A, B, C, D)

(A) Correct statement.

As on decrease in pressure reaction move in direction where no. of gaseous molecules increase

(B) Correct statement

At the start of reaction $Q_P < K_P$ so dissociation of X_2 take place spontaneously.

(C) Paragraph-1

	$x_2(g) \rightleftharpoons$	$2x(g)$
Initial mode	1	0
$t = t_{eq.}$	$(1-\alpha)$	2α

Given $2\alpha = \beta_{equilibrium}$

So $\alpha = \dfrac{\beta_{equilibrium}}{2}$

Total mole at equilibrium = $(1 + \alpha) = 1 + \left(\dfrac{\beta_{eq.}}{2}\right)$

$$P_{x_2} = \left[\frac{1-\frac{\beta_{eq.}}{2}}{1+\frac{\beta_{eq.}}{2}}P_{total}\right] = \left[\frac{2-\beta_{eq.}}{2+\beta_{eq.}}P_{total}\right] = \left[\frac{2-\beta_{eq.}}{2+\beta_{eq.}}P_{total}\right]$$

$$P_{x(g)} = \left[\frac{\beta_{eq.}}{1+\frac{\beta_{eq.}}{2}}P_{total}\right] = \left[\frac{2\beta_{eq.}}{2+\beta_{eq.}}\right]P_{total}$$

$$\text{So } K_P = \frac{(P_x)^2}{(P_{x_2})} = \frac{\left[\frac{2\beta_{eq.}}{2+\beta_{eq.}} \times P_{total}\right]^2}{\left[\frac{2-\beta_{eq.}}{2+\beta_{eq.}} \times P_{total}\right]}$$

$$K_P = \frac{4\beta_{eq.}^2}{4-\beta_{eq.}^2} \times P_{total} = \left(\frac{8\beta_{eq.}^2}{4-\beta_{eq.}^2}\right)$$

(D) Correct statement

As $\Delta G° > 0$ & $\Delta G° = - RT\ln K_P$

$\Delta G° > 0$, So K_P should be less than 1.

So $K_P < 1$

$K_P = K_C(RT)^{\Delta ng}$ $(RT > 1)$

$$K_C = \frac{K_P}{RT}$$

$K_C < K_P$ So $K_C < 1$

7. **(A, B, C)**

8. **(A, C, D)**

9. **(6.00)**

(c), (e), (d), (f), (g) and (h).

10. **(12.00)**

$$\underset{20mL}{C_xH_y} + \left(x + \frac{y}{4}\right)O_2 \rightarrow \underset{(80mL)}{xCO_2} + \frac{y}{2}H_2O$$

$CO_2 \Rightarrow 80$ mL

$20x = 80$

$x = 4$

Now

Let initial volume of $O_2 = v_1$

volume contraction is given as 60 mL

$$20 + V_1 - \left\{80 + \left[V_1 - \left(x + \frac{y}{4}\right)20\right]\right\} = 60$$

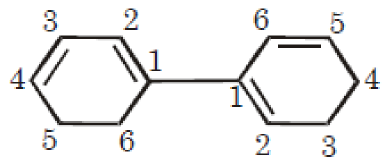
$-60 + 80 + 5y = 60$

$20 + 5y = 60$

$5y = 40$

$y = 8$

11. **(10.00)**



1 – (cyclo hex – 1, 5– dienyl) cyclohexa –1, 3– diene

Sum = 1 + 5 + 1 + 3 = 10

12. **(100.00)**

$$\frac{10^{-3} \times 100}{1000} \times 10^{6} \Rightarrow 100\text{ppm.}$$

13. **(3.00)**

14. **(6.00)**

Acidic group → Alkyne, Ketone, aldehyde
→ Alcohol, Amine, Acid will
react with grignard reagent.

15. **(4.00)**

16. **(6.00)**

17. **(4.00)**

VGA, VAG, AGV, AVG.

18. **(4.00)**

PART (C) : MATHEMATICS

SOLUTIONS

1. (A, B, C, D)

Let $I = \int_{-a}^{\alpha} (e^x + \cos x \, \ell n(x + \sqrt{1 + x^2}))dx$

$= \int_{-\alpha}^{\alpha} e^x \, dx + \int_{-\alpha}^{\alpha} \cos x \, \ell n(x + \sqrt{1 + x^2}))dx$

since cos x is an even function and $\ell n(x + \sqrt{1 + x^2})$

is an odd function, so $\cos x \, \ell n(x + \sqrt{1 + x^2})$ is an odd

function.

Hence $\int_{-\alpha}^{\alpha} e^x \, dx = e^{\alpha} - e^{-\alpha}$

Now since $I > \frac{3}{2}$

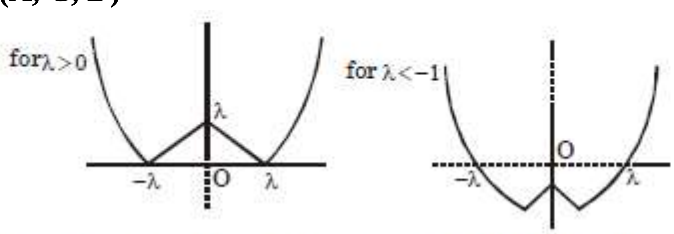
$\Rightarrow \quad e^{\alpha} - e^{-\alpha} > \frac{3}{2} \Rightarrow 2e^{2\alpha} - 3e^{\alpha} - 2 > 0$

$e^{\alpha} < -\frac{1}{2}$ or $e^{\alpha} > 2$

But $e^{\alpha} > 0 \ \forall \ \alpha \in R$

$\Rightarrow \quad e^{\alpha} > 2 \Rightarrow \alpha \in (\log_e 2, \infty)$

2. (A, C, D)



for $-1 \le \lambda \le 0$ $\quad f(x) = x^2 - \lambda^2$ which is always differentiable

3. (C, D)

$$\tan^2\frac{\pi}{12}=\tan\left(\frac{\pi}{12}-x\right)\tan\left(\frac{\pi}{12}+x\right)$$

$$\tan^2\frac{\pi}{12}=\frac{\tan\frac{\pi}{12}-\tan x}{\tan\frac{\pi}{12}-\tan x}.\frac{\tan\frac{\pi}{12}+\tan x}{1-\tan\frac{\pi}{12}\tan x}$$

$$=\frac{\tan^2\frac{\pi}{12}-\tan^2 x}{1-\tan^2\frac{\pi}{12}\tan^2 x}$$

$$\tan^2\frac{\pi}{12}-\tan^4 x\frac{\pi}{12}\tan^2 x=\tan^2\frac{\pi}{12}-\tan^2 x$$

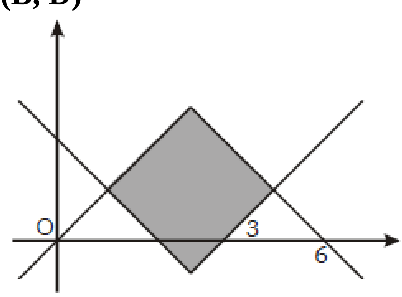
$\Rightarrow$ $\tan^2 x\left(\tan^4\frac{\pi}{12}-1\right)=0;\ \tan x=0$

$\Rightarrow$ $x = k\pi$

sum of solutions is $\pi(1 + 2 + 3)$

$= 6\pi \Rightarrow k = 6$

4. **(B, D)**



$$\text{Area} = \frac{3}{\sqrt{2}}\times\frac{3}{\sqrt{2}}=\frac{9}{2}$$

5. **(A, C)**

....04 12
....20 24
....40 32

$n(A) = 6\times3 + 4\times3 = 30$

1,2,3,0 ; 4,2,3,0

$n(B) = 18+18 = 36$

Using 1,2,3,0

.....12,....20,.......32

$1 + 2 + 1 = 4$

Using 4,2,3,0

...04,....20,...24,....32,...40

$2+2+1+1+2=8$

$n(A\cap B)=8$

6. **(A, B, C, D)**

$$f(x) = \sin 2\alpha(\sin(\alpha-\beta)) - \sin 2\beta(\sin(\alpha-2))$$

$f(x)=k$

7. **(A, B, C, D)**

(A) For many one in this case function must be continuous.

(B) $P(A/B) = P(A)$

$\Rightarrow$ A & B are independent events.

(C) n must be odd natural number.

(D) whole curve may lies below x-axis.

8. **(A, B, D)**

Let $a_n = 2dn + P$ and $b_n + dn + Q$ are n^{th} terms of A.P., then

$$A_n = dn(n+1) + Pn = dn^2 + (d+P)n$$

$$B_n = \frac{dn(n+1)}{2} + Qn = \frac{d}{2}n^2 + \left(\frac{d}{2} + Q\right)n$$

(A) $\lim\limits_{n\to\infty} \frac{A_n - B_n}{[n]^2 + [n^2]} = \frac{\frac{d}{2}n^2 + \left(\frac{d}{2} + P - Q\right)n}{2n^2} = \frac{d}{4}$

(B) $A_n + B_n = 0 \Rightarrow \frac{3}{2}n^2 + \left(\frac{3}{2} + P + Q\right)n = 0$

$$\Rightarrow n = -\left(\frac{\frac{3}{2} + P + Q}{\frac{3}{2}}\right) \ge 1 \Rightarrow P + Q \le -3$$

(C) $10d + P = 5d + Q \Rightarrow P - Q = -5d$

$f(n) = A_n - B_n$

$$= \frac{d}{2}n^2 + \left(\frac{d}{2} + P - Q\right)n = \frac{d}{2}n(n-9)$$

$$f(x) = x \Rightarrow \frac{d}{2}\,\frac{9}{2}\left(\frac{-9}{2}\right) = \frac{9}{2} \Rightarrow d = -\frac{4}{9}$$

(D) Sum of coeff. of expansion $(2dn + P) = 243$

Sum of coeff. of expansion $(dn + Q)^8 = 256$

9. **(4.00)**

$$I(n) = \int_0^{\pi} \ell n(1 - 2n\cos(\pi - x) + n^2)dx$$

$$I(n) = \int_0^{\pi} \ell n(1 + 2n\cos x + n^2)dx \Rightarrow I(-n) = I(n)$$

Also $I(n) + I(-n) = \int_0^{\pi} \ell n(1 - 2n^2\cos 2x + n^4)dx$

Put $2x = t$

$\Rightarrow \quad I(n) + I(-n) = I(n^2) \quad \Rightarrow \quad 2I(n) = I(n^2)$

10. (3)

Take dot product with $\vec{q}$ both the sides

$$\vec{p}.\vec{q} = \vec{r}.\vec{p}$$

$$\vec{p} - \vec{q} \times \vec{p} = \vec{r}$$

Squaring $p^2 + p^2\sin^2\theta = 1$

$$p^2 = \frac{1}{1+\sin^2\theta}$$

$$\vec{p} = \vec{q} \times \vec{p} + \vec{r}$$

Squaring $p^2 = p^2\sin^2\theta + 2[\vec{q}\ \ \vec{p}\ \ \vec{r}] + 1$

$$2[\vec{p}\ \ \vec{q}\ \ \vec{r}] = 1 - \frac{(1-\sin^2\theta)}{(1+\sin^2\theta)} = \frac{2\sin^2\theta}{(1+\sin^2\theta)}$$

$$2[\vec{p}\ \ \vec{q}\ \ \vec{r}] = 2\left(1 - \frac{1}{1+\sin^2\theta}\right)$$

$$[\vec{p}\ \ \vec{q}\ \ \vec{r}] = 1 - \frac{1}{(1+\sin^2\theta)}$$

When $\sin^2\theta = 0 \Rightarrow [\vec{p}\ \vec{q}\ \vec{r}] = 0$

When $\sin^2\theta = 1 \ \Rightarrow\ [\vec{p}\,\vec{q}\,\vec{r}] = \frac{1}{2}$

11. (8.00)

$$\int_0^{\frac{\pi}{4}}\left[3\tan^2 x\right]dx$$

$$\int_0^{\frac{\pi}{6}} 0\,dx + \int_{\frac{\pi}{6}}^{\tan^{-1}\sqrt{\frac{2}{3}}} 1\,dx + \int_{\tan^{-1}\sqrt{\frac{2}{3}}}^{\frac{\pi}{4}} 2\,dx$$

$$0+\left(\tan^{-1}\sqrt{\frac{2}{3}}-\frac{\pi}{6}\right)+2\left(\frac{\pi}{4}-\tan^{-1}\sqrt{\frac{2}{3}}\right)$$

$$=\frac{\pi}{3}-\tan^{-1}\left(\sqrt{\frac{2}{3}}\right)$$

12. **(2.50)**

$$y\frac{dx}{dy}+2x=y^2\ln y$$

$$\frac{dx}{dy}+\left(\frac{2}{y}\right)x=y\ln y$$

$$\text{I.F.}=e^{\int\frac{2}{y}}=y^2$$

$$\Rightarrow y^2 x = \int(y\ln y)y^2 dy$$

$$xy^2=\frac{y^4(\ln y)}{4}-\frac{y^4}{16}+c$$

curve passes through $\left(\frac{-1}{16},1\right) \Rightarrow c = 0$

$$\Rightarrow x=\frac{y^2\ln y}{4}-\frac{y^2}{16}$$

$\Rightarrow 16x + y^2 = 4y^2 \ln y$

a=1, b=4

13. **(0.60)**

$m_{PA} = \frac{3-2}{3-4} = -1$

$\Rightarrow$ Equation of tangent at P

$y - 3 = x - 3 \Rightarrow y = x$ ...(1)

$m_{QA} = \frac{-1-2}{1-4} = 1$

$\Rightarrow$ equation of tangent at Q

$y + 1 = -(x - 1) \Rightarrow x + y = 0$ ...(2)

Tangents at P and Q are mutually perpendicular

$\Rightarrow$ PQ is a focal chord, equation of PQ is $y = 2x - 3$

Slope of axis of the parabola $= \frac{2-0}{4-0} = \frac{1}{2}$

$\Rightarrow$ Equation of directrix $2x + y = 0$

$S(\alpha, \beta)$ lies on focal chord

$\Rightarrow \beta = 2\alpha - 3$

Image of $S(\alpha, \beta)$ with respect to $y = x$ line lies on $2x + y = 0$

$\Rightarrow 2\beta + \alpha = 0$

$\Rightarrow \alpha = \frac{6}{5},\ \beta = \frac{-3}{5}$

$\Rightarrow \alpha + \beta = \frac{3}{5} = 0.60$

14. **(0.50)**

$6x + 3y + 2z - 6 = 0$

$P(x_1, y_1, z_1)$

$A(0, y_1, z_1), B(x_1, 0, z_1), C(x_1, y_1, 0)$

$$\overline{N} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ x_1 & -y_1 & 0 \\ 0 & y_1 & -z_1 \end{vmatrix}$$

$= (y_1 z_1)\hat{i} + (x_1 z_1)\hat{j} + (x_1 y_1)\hat{k}$

$\Rightarrow$ Equation of plane containing points A, B & C

$\frac{x}{x_1} + \frac{y}{y_1} + \frac{z}{z_1} = 2$ ...(1)

Let $P(\alpha, \beta, \gamma)$ be the foot of perpendicular from (0,0,0) to the plane (1)

$$\Rightarrow \frac{\alpha}{1/x_1} = \frac{\beta}{1/y_2} = \frac{\gamma}{1/z_1} = \lambda$$

$$x_1 = \frac{\lambda}{\alpha},\ y_1 = \frac{\lambda}{\beta},\ z_1 = \frac{\lambda}{\gamma}$$

(x_1, y_1, z_1) lies in $6x + 3y + 2z - 6 = 0$

$$\Rightarrow \frac{6\lambda}{\alpha} + \frac{3\lambda}{\beta} + \frac{2\lambda}{\gamma} = 6 \quad ...(2)$$

(α, β, γ) lies in (1)

$$\Rightarrow \frac{\alpha^2}{\lambda} + \frac{\beta^2}{\lambda} + \frac{\gamma^2}{\lambda} = 2 \quad ...(3)$$

from (2) & (3)

$$(\alpha^2 + \beta^2 + \gamma^2)\left(\frac{6}{\alpha} + \frac{3}{\beta} + \frac{2}{\gamma}\right) = 12$$

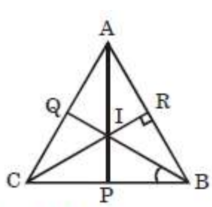
$$\Rightarrow \text{locus is } (x^2 + y^2 + z^2)\left(\frac{1}{x} + \frac{1}{2y} + \frac{1}{3z}\right) = 2$$

15. **(6.00)**

$\text{sgn}(x) = 0$ and $\text{sgn}(x) \neq \pm 1$

$\Rightarrow$ $n = 0$, $m \neq -2, 4$

16. **(2.00)**



M(1, 2, 6) and I(1, 2, 0)
I is orthocentre of ΔABC

$$\text{Volume (V)} = \frac{1}{3}(\text{height})(\text{area of } \Delta IBC)$$

$$= \frac{1}{3}.6.\frac{1}{2}.6.3\tan\frac{A}{2}$$

$$V = 18\tan\frac{A}{2}$$

$$\lim_{A \to \frac{\pi}{2}^+} \frac{e^{18\tan\frac{A}{2}} - e^k}{\sqrt{1 - \sin A}} = \frac{e^k}{L} \Rightarrow k = 18$$

$$= 18\sqrt{2}e^{18}$$

17. (0.00)

(1, 2) and (3, 6) are focii of ellipse

$\Rightarrow \quad 2ae = 2\sqrt{5} \Rightarrow 2a = \frac{2\sqrt{5}}{e} = 2 + \sqrt{5} + \sqrt{45}$

$\therefore \quad \sqrt{(\sin\theta - 1)^2 + (\cos\theta - 2)^2}$

$+\sqrt{(\sin\theta - 3)^2 + (\cos\theta - 6)^2} = (1 + \sqrt{45}) + (1 + \sqrt{52})$

$(1+\sqrt{5})$ and $(1+\sqrt{45})$ are maximum distance of (1, 2) and (3, 6) from circle $x^2 + y^2 = 1$ or $x = \sin\theta$, $y = \cos\theta$

$\Rightarrow$ (1, 2), (3, 6) and $(\sin\theta, \cos\theta)$ are collinear

$(3\cos\theta - 6\sin\theta) = 0$

18. (3.00)

Case-I : I II III IV

W W B W $\rightarrow \frac{3}{10} \times \frac{2}{10} \times \frac{9}{10} \times \frac{1}{10}$

Case-II : I II III IV

W B W W $\rightarrow \frac{3}{10} \times \frac{8}{10} \times \frac{2}{10} \times \frac{1}{10}$

Case-III : I II III IV

B W W W $\rightarrow \frac{7}{10} \times \frac{3}{10} \times \frac{2}{10} \times \frac{1}{10}$

Required probability $p = \frac{144}{10^4}$

PART (A) : PHYSICS

SOLUTIONS

1. **(B)**

$h\nu = 17\text{ eV}$

$P = \frac{h}{\lambda} = \sqrt{4me} \Rightarrow (K.E.)_{max} = \frac{P^2}{2m} = (2e)J = 2\text{ eV}$

Now for metal,

$h\nu = \phi + (KE)_{max}$

$\Rightarrow 17\text{ eV} = \phi + 2\text{ eV} \Rightarrow \phi = 15\text{ eV}$

2. **(B)**

$\frac{1}{V} - \frac{1.5}{\infty} = \frac{1-1.5}{-5}$

$V = 10$ cm

3. **(A, D)**

Apply bernaulli equation at E and top most point.

$P_0 + P + \rho gh + 0 = P_0 + \frac{1}{2}\rho V_E^2$

$\sqrt{\frac{2(P+\rho gh)}{\rho}} = V_E$

Equation of continuity at point e and E.

$A \times V_C = \frac{A}{2} \times \sqrt{\frac{2(P+\rho gh)}{\rho}} = \sqrt{\frac{P+\rho gh}{2\rho}}$

discharge rate $(\theta) = A \times V_C = A\left(\frac{P+\rho gh}{2\rho}\right)^{1/2}$

4. **(B, C, D)**

If $\mu_2 > \tan(\theta)$

then wedge will not move for any value of μ_1.

5. **(A, C, D)**

6. **(A, C, D)**

$$\frac{n\lambda}{2} = L \Rightarrow \lambda = \frac{2L}{n}$$

$$\Rightarrow f = \frac{v}{\lambda} = \frac{n}{2L}\sqrt{\frac{T}{\mu}}$$

(A) for same n, T & μ, if L is doubled, f is halved.

(C) for same f, L & μ, $n\sqrt{T} = \text{constant}$

$$(n+1)\sqrt{T'} = n\sqrt{T} \Rightarrow T' = \frac{n^2}{(n+1)^2}.T$$

(D) $f = \frac{n}{2L}\sqrt{\frac{T}{\mu}}$

$$3f = \frac{(2n)}{2\left(\frac{L}{2}\right)}\sqrt{\frac{T'}{\mu}}$$

$$T' = \frac{9T}{16}$$

7. **(9.00)**

Since water resistance $(\vec{F} = -k\vec{v}_{pr})$ is proportional to the velocity of plank relative to the water, impulse of water resistance equals to product of the proportionality constant k and displacement of the plank relative to the water. And this impulse stops the block-plank system at the other bank, therefore equals to the initial momentum of the block.

$$mu_{min} = k\int v\,dt$$

$$\Rightarrow mu_{min} = k(b-l)$$

$$u_{min} = \frac{k(b-\ell)}{m} = 9.0 m/s$$

8. **(4.50)**

$W_b = \frac{9}{2}C\varepsilon^2$, $\Delta U = \frac{9}{4}C\varepsilon^2$, So $\frac{9}{4}C\varepsilon^2 = \frac{1}{2}LI^2$,

$I_{max} = 3\varepsilon\sqrt{\frac{C}{2L}}$

9. **(0.25)**

$z_{RL} = \sqrt{R^2 + \omega^2 L^2}$

$z_{RLC} = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$

$\cos\varnothing_{RL} = \frac{R}{\sqrt{R^2 + \omega^2 L^2}}$

$\cos\varnothing_{RLC} = \frac{R}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$

According to question

$(\omega L)^2 = \left(\omega L - \frac{1}{C\omega}\right)^2 \Rightarrow C = \frac{1}{2\omega^2 L}$

10. **(0.83)**

$\frac{\lambda_0 g L^3}{3YA}$

11. **(4.00)**

$\frac{hc}{\lambda} = 13.6(3)^2\left(\frac{1}{3^2} - \frac{1}{4^2}\right)$

$\frac{hc}{\lambda} = 13.6 \times \frac{7}{16} = KE_{max} + \phi$

$V_{max} = \frac{qBR}{m}$

$KE_{max} = \frac{1}{2}mv_{max}^2$

$\phi = 4eV$

12. **(6.32 to 6.33)**

$$mvR = \left(mR^2 + \frac{MR^2}{2}\right)\omega$$

$$\omega = \frac{mvR}{mR^2 + \frac{MR^2}{2}} = \frac{V}{2R}$$

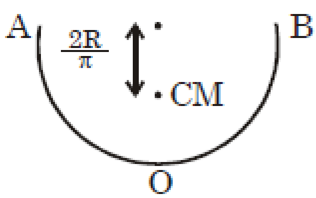
$$-mgR = 0 - \frac{1}{2}\left(mR^2 + \frac{MR^2}{2}\right)\omega^2$$

$$V = \sqrt{40}$$

$$V = 6.324 \text{ m/s}$$

13. **(3.00)**

Semicircular arc AOB



$$K.E. = \frac{1}{2}Mv_{cm}^2 + \frac{1}{2}I_{cm}\omega^2$$

$$M = m/2$$

$$v_{cm} = v_0 - \omega\left(\frac{2R}{\pi}\right) = v_0 - \frac{2}{\pi}v_0 = v_0\left(1 - \frac{2}{\pi}\right)$$

$$I_{cm} = \left(\frac{m}{2}\right)R^2 - \frac{m}{2}\left(\frac{2R}{\pi}\right)^2 = \frac{mR^2}{2}\left(1 - \frac{4}{\pi^2}\right)$$

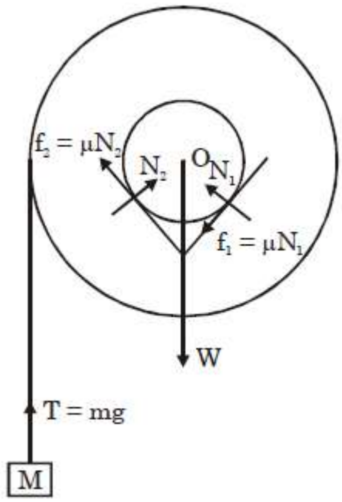
$$\omega = \frac{v_0}{R}$$

$$KE = mv_0^2\left[\frac{1}{1} - \frac{2}{\pi}\right]$$

$$\alpha = 1, \beta = 2$$

$$\alpha + \beta = 3$$

14. **(2.00)**



Torqure about 'O'

$Mg \times b = \mu(N_1 + N_2)a$ ...(i)

Force analysis

$N_2\cos\theta = \mu(N_1 + N_2)\sin\theta + N_1\cos\theta$ .....(ii)

$Mg + w + \mu N_1 \cos\theta = \mu N_2 \cos\theta + (N_1 + N_2)\sin\theta$ ......(iii)

on solving (i), (ii), (iii)

$$M = 2\left[\frac{w.a}{g(5b\sin\theta - 2a)}\right]$$

15. **(A)**

16. **(B)**

17. **(B)**

18. **(B)**

Solution for Q. No. 17 & 18

$$\lambda = \frac{V}{f} = \frac{330}{440} = \frac{3}{4}$$

In second overtone $\ell = \frac{5\lambda}{4} = \frac{15}{16}$ m

Let upper end be at x = 0

$$\therefore \Delta P = \Delta P_0 \sin\frac{2\pi x}{\lambda}$$

At $x = \frac{L}{2}$, $\Delta P = \frac{\Delta P_0}{\sqrt{2}}$

PART (B) : CHEMISTRY

SOLUTIONS

1. **(B, D)**

2. **(A, B, C)**

3. **(A, B, C, D)**

4. **(A, C, D)**

   It is acidic catalysed ester hydrolysis.

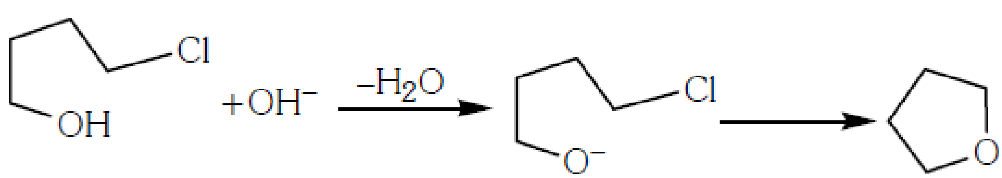
5. **(A, B, C, D)**

   Lanthanides have +3 oxidation state more stable.

6. **(A, B, C, D)**

   All reaction undergo S_N2 mechanism.

   $$CH_3CH_2O^- + CH_3CH_2Cl \xrightarrow{S_N2} CH_3CH_2OCH_2CH_3 \cdot$$

   $$(CH_3)_3C - CH_2 - CH_2 - Br + NH_3 \rightarrow (CH_3)_3C - CH_2 - CH_2NH_2$$

   $$CH_3CH_2OH + HBr \rightarrow CH_3CH_2Br + H_2O$$

7. **(2.00)**

8. **(2.00)**

9. **(6.00)**

10. **(7.00)**

11. **(20.00)**

12. **(200.00)**

$$\Delta T_f = i\ K_f m = K_f \cdot \frac{w_B \times 1000}{\overline{M} \times w_A}$$

$$0.093 = 1.86 \times \frac{1}{\overline{M}} \times \frac{1000}{100}$$

$$M = \frac{18.6}{0.093} = 200$$

13. **(3.00)**

(i) $2Ag + PCl_5 \xrightarrow{\Delta} 2AgCl + PCl_3$

(ii) $PH_3 \xrightarrow{CaOCl_2} PCl_3 + HCl + Ca(OH)_2$

(iii) $NH_3 \xrightarrow[\text{of } Cl_2]{\text{excess}} NCl_3 + HCl$

(iv) $H_3PO_2 \xrightarrow{\Delta} PH_3 + H_3PO_4$

(v) $NH_4NO_2 \xrightarrow{\Delta} N_2 + H_2O$

(vi) $P_4 \xrightarrow{SO_2Cl_2} PCl_5 + SO_2$

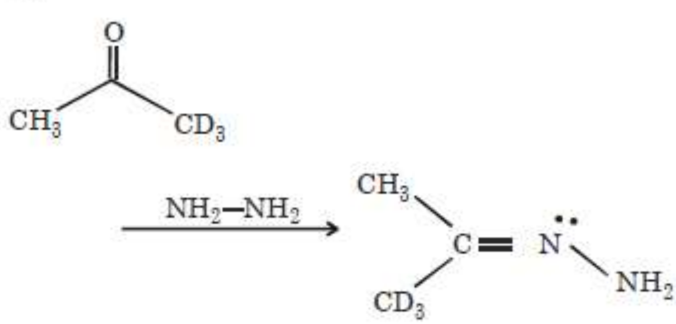
14. **(5.00)**

15. **(D)**

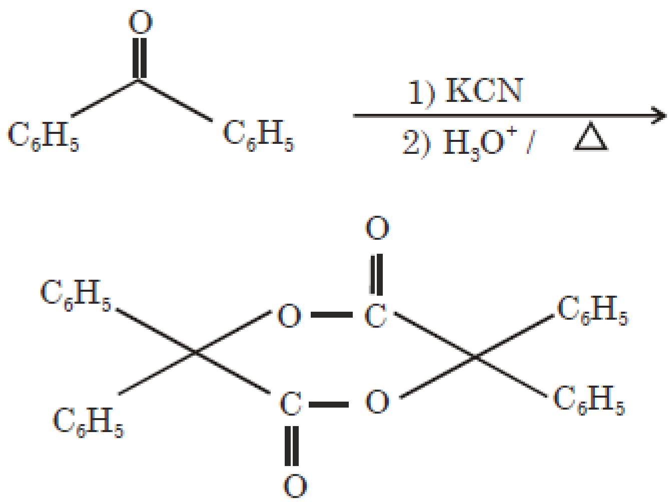
16. **(C)**

Solution for Q. No. 15 & 16

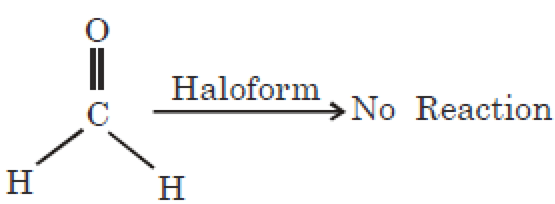
(1)



POS present

(2) 

COS present

(3) 

17. **(C)**

18. **(B)**

PART (C) : MATHEMATICS

SOLUTIONS

1. **(A, C, D)**

$z_1, z_2 ...... z_9$ are roots of equation $z^{10} = \alpha^{10}$ other than $z = \alpha$

$\Rightarrow$ all are vertices of a regular decagon

for $\alpha = 1$ roots are tenth roots of unity other than 1

$$\Rightarrow \quad z_1^r + z_2^r + ... + z_9^r = \begin{cases} -1 \; ; \; r \neq 10\lambda \\ 9 \quad ; \; r = 10\lambda \end{cases} \forall \lambda \in N$$

$$\Rightarrow \quad \sum_{r=0}^{37} \sum_{i=1}^{9} z_i^r = 36 - 34 = 2$$

$\text{Arg}\left(\frac{z_i}{z_j}\right) \neq \pm\frac{\pi}{5} \Rightarrow z_i$ & z_j can't be adjecnt vertices.

$\therefore$ (B) ${}^7C_2 = 35$

(C) ${}^6C_4 = 15$

(D) $|z_i - z_j|^2 = 1 + 1 - 2\cos\frac{4\pi}{5} = \frac{5+\sqrt{5}}{2}$

2. **(B, C, D)**

$L_1 : \vec{r} = (fg + gh + hf) + \lambda(\hat{i} + 2\hat{j} + 3\hat{k})$

$L_2 : \vec{r} = (\hat{i} + \hat{j} + \hat{k}) + \mu(4\hat{i} + 5\hat{j} + 6\hat{k})$

Shortest distance between L_1 & L_2 is

$d = fg - 2gh + hf$

(A) d may be zero for some 'x'

(B) $d = 0 = fg - 2gh + hf$

$\Rightarrow f(g' + h') + g(f' - 2h') + h(f' - 2g') = 0$

$f' = g' = h' = 0$

(C) $fg + hf = 2gh$

$$\Rightarrow \quad \frac{f}{h} + \frac{f}{g} = 2 \Rightarrow \frac{f'h - fh'}{h^2} + \frac{f'g - fg'}{g^2} = 0$$

(D) $fg + hf = 2gh$

$$\frac{1}{h} + \frac{1}{g} = \frac{2}{f} \Rightarrow \frac{h'}{h^2} + \frac{g'}{g^2} = \frac{2f'}{f^2}$$

3. **(A, B, D)**

Let $a_n = 2dn + P$ and $b_n + dn + Q$ are n^{th} terms of A.P., then

$$A_n = dn(n+1) + Pn = dn^2 + (d+P)n$$

$$B_n = \frac{dn(n+1)}{2} + Qn = \frac{d}{2}n^2 + \left(\frac{d}{2}+Q\right)n$$

(A) $\lim\limits_{n\to\infty} \frac{A_n - B_n}{[n]^2 + [n^2]} = \frac{\frac{d}{2}n^2 + \left(\frac{d}{2}+P-Q\right)n}{2n^2} = \frac{d}{4}$

(B) $A_n + B_n = 0 \Rightarrow \frac{3}{2}n^2 + \left(\frac{3}{2}+P+Q\right)n = 0$

$$\Rightarrow n = -\left(\frac{\frac{3}{2}+P+Q}{\frac{3}{2}}\right) \geq 1 \Rightarrow P+Q \leq -3$$

(C) $10d + P = 5d + Q \Rightarrow P - Q = -5d$

$f(n) = A_n - B_n$

$= \frac{d}{2}n^2 + \left(\frac{d}{2}+P-Q\right)n = \frac{d}{2}n(n-9)$

$f(x) = x \Rightarrow \frac{d}{2}\frac{9}{2}\left(\frac{-9}{2}\right) = \frac{9}{2} \Rightarrow d = -\frac{4}{9}$

(D) Sum of coeff. of expansion $(2dn + P) = 243$

Sum of coeff. of expansion $(dn + Q)^8 = 256$

4. **(B, D)**

The given equation can be written as

$(x+y)(x^2 - xy + y^2) + 3xy(x+y) - (x^2 - y^2) = 0$

$\Rightarrow x + y = 0$ or $(x+y)^2 = (x-y)$ {Parabola}

$$\Rightarrow \frac{\left(\frac{x+y}{\sqrt{2}}\right)^2}{\left|\frac{x-y}{\sqrt{2}}\right|} = \frac{1}{\sqrt{2}} \Rightarrow LR = \frac{1}{\sqrt{2}}$$ and $x+y=0$ is axis of conic.

5. **(A, B, C, D)**

$P(1) = 1, P(2) = 0$ and $P(n) = \frac{1}{3}(1 - P(n-1))$ ...(1)

$\Rightarrow P(n+1) = \frac{1}{3}(1 - P(n))$ ...(2)

(1) - (2) $\Rightarrow P(n+1) - P(n) = \frac{1}{3}(P(n-1) - P(n))$

Let $Q(n) = P(n+1) - P(n)$ ...(3)

$\Rightarrow Q(n) = -\frac{1}{3}Q(n-1)$

$\Rightarrow Q(n)$ is GP with 1st term $Q(1) = -1$

and common ratio $-\frac{1}{3}$.

$\Rightarrow \sum_{i=1}^{n} Q(i) = P(n+1) - P(1)$

{summation on (3)}

$\Rightarrow P(n) = \sum_{i=1}^{n-1} Q(i) + P(1)$

$$\Rightarrow P(n) = -1\left\{\frac{\left(-\frac{1}{3}\right)^{n-1} - 1}{-\frac{4}{3}}\right\} + 1$$

6. **(A, B, C, D)**

(A), (B), (C), (D) Given expression

$$E = -\frac{2}{(n+1)}\sum_{r=0}^{n} {}^{n+1}C_{r+1}\left(-\sqrt{k}\right)^{r+1}$$

$$= \frac{2}{n+1}\left(-\left(1-\sqrt{k}\right)^{n+1} + 1\right).$$

Evaluate (B) and (C) by putting $\sqrt{x} = y$ and $t^2 = y$ respectively, and then integrating

In (D)

$$t_r = (-1)^r .2\frac{{}^nC_r\left(\sqrt{k}\right)^{r+1}}{r+1} = \frac{2}{n+1}\ {}^{n+1}C_{r+1}(-1)^r\left(\sqrt{k}\right)^{r+1}$$

= general term of E.

7. **(0.00)**

(1, 2) and (3, 6) are focii of ellipse

$$\Rightarrow \quad 2ae = 2\sqrt{5} \Rightarrow 2a = \frac{2\sqrt{5}}{e} = 2 + \sqrt{5} + \sqrt{45}$$

$$\therefore \quad \sqrt{(\sin\theta - 1)^2 + (\cos\theta - 2)^2}$$

$$+\sqrt{(\sin\theta - 3)^2 + (\cos\theta - 6)^2} = (1 + \sqrt{45}) + (1 + \sqrt{52})$$

$(1+\sqrt{5})$ and $(1+\sqrt{45})$ are maximum distance of (1, 2) and (3, 6) from circle

$x^2 + y^2 = 1$ or $x = \sin\theta$, $y = \cos\theta$

$\Rightarrow$ (1, 2), (3, 6) and $(\sin\theta, \cos\theta)$ are collinear

$(3\cos\theta - 6\sin\theta) = 0$

8. **(6.00)**

Let $z = re^{i\theta}$

$$\Rightarrow \left|z + \frac{1}{z}\right| = 1 \Rightarrow \left(z + \frac{1}{z}\right)\left(\bar{z} + \frac{1}{\bar{z}}\right) = 1$$

$$\Rightarrow r^2 + \frac{1}{r^2} + \frac{z}{\bar{z}} + \frac{\bar{z}}{z} = 1$$

$$\Rightarrow r^2 + \frac{1}{r^2} + 2\cos 2\theta = 1 \quad \Rightarrow r^2 + \frac{1}{r^2} = 1 - 2\cos 2\theta$$

Now, $2 \le 1 - 2\cos 2\theta \le 3 \Rightarrow -3 \le 2\cos 2\theta - 1 \le -2$

$$\Rightarrow -1 \le \cos 2\theta \le -\frac{1}{2}$$

$$2n\pi + \frac{2\pi}{3} \le 2\pi \le \frac{4\pi}{3} + 2n\pi$$

$$\Rightarrow n\pi + \frac{\pi}{3} \le \theta \le n\pi + \frac{2\pi}{3} \Rightarrow \theta \in \left[\frac{\pi}{3}, \frac{2\pi}{3}\right]$$

$0 > 0$

9. **(8.00)**

$$\ell = 3 + \tan^2\frac{\pi}{7} + \tan^2\frac{2\pi}{7} + \tan^2\frac{3\pi}{7}$$

$\because \quad \tan 7\theta = 0$ (where $\theta = \frac{\pi}{7}$)

$\therefore \quad {}^7C_1 \tan\theta - {}^7C_3 \tan^3\theta + {}^7C_5 \tan^5\theta - {}^7C_7 \tan^7\theta = 0$

$\Rightarrow \quad x^6 - {}^7C_5 x^4 + {}^7C_3 x^2 - {}^7C_1 = 0$

has roots $\tan\frac{\pi}{7}, \tan\frac{2\pi}{7}, \ldots\ldots, \tan\frac{6\pi}{7}$

$\Rightarrow \quad t^3 - {}^7C_5 t^2 + {}^7C_3 t - {}^7C_1 = 0$

has roots $\tan^2\frac{\pi}{7}$, $\tan^2\frac{2\pi}{7}$ & $\tan^2\frac{3\pi}{7}$

$\Rightarrow \tan^2\frac{\pi}{7} + \tan^2\frac{2\pi}{7} + \tan^2\frac{3\pi}{7} = {}^7C_5 = 21$

10. (0.00)

$y = x - [x] = \{x\}$

then

$(K-2)\, y^2 + 2y + k^2 = 0$, where $0 < y < 1$

x is not an integer

for $3 < x < 4 \Rightarrow [x] = 3$

y has exactly one solution is (0, 1)

$f(0).\, f(1) < 0 \Rightarrow K \in (-1, 0)$

11. (4.00)

$z = (1 + ai)(1 + bi)(1 + ci)$

$|z|^2 = (1 + a^2)(1 + b^2)(1 + c^2)$

$= 25 \Rightarrow |z|\ 5$

We have $-|z| \le \text{Re}(z) \le |z|$

$\Rightarrow -5 \le 1 - ab - bc - ca \le 5$

$\Rightarrow -4 \le ab + bc + ca \le 6$

$\lambda = -4$, at $a = 2$, $b = -2$, $c = 0$

12. **(4.20)**

$a_1 = 1,\ a_2 = 0,\ a_3 = 1,$

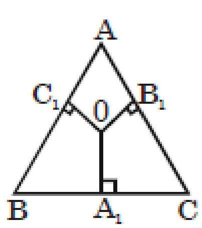
$a_n = a_{n-2} + a_{n-3},\ n > 3$

$a_4 = 1,\ a_5 = 1,\ a_6 = 2,\ a_7 = 2$

$a_8 = 3,\ a_9 = 4,\ a_{10} = 5,\ a_{11} = 7$

$a_{12} = 9,\ a_{13} = 12,\ a_{14} = 16$

13. **(0.00)**



$\frac{\overline{OA_1}}{OA_1}BC$ is image of $\overline{BC}$

rotated by 90°.

$\Rightarrow$ Required vector sum is image of vector

sum $\overline{BC} + \overline{CA} + \overline{AB}$ rotated by 90°.

$\Rightarrow$ required vector sum is $\bar{O}$.

14. **(1.00)**

$$f(1,2,3)=\lim_{x\to 0}\left(\frac{1^{x^3}+2^{x^3}+3^{x^3}}{1^{x^2}+2^{x^2}+3^{x^2}}\right)^{\frac{1}{3}}=1$$

$$\Rightarrow 1=\frac{1}{\sqrt[3]{k}}\Rightarrow k=1$$

15. **(B)**

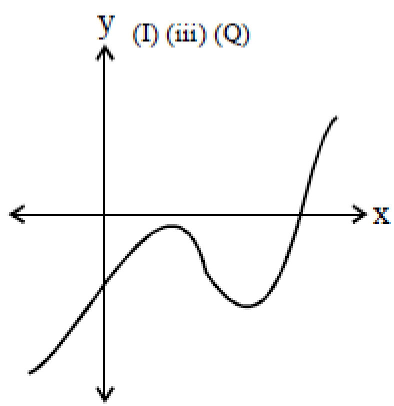
16. **(D)**

Solution for Q. No. 15 & 16

(I). $f(x)=x^3-6x^2+9x-12$

$f(x)=3x^2-12x+9=3(x-1)(x-3)$

(I) (iii) (Q)



2 points of local Exterma

1 solution of f(x) = 0

(II) $f(x)=6x-3\sin x-4\cos x-2020$

$f(x)=6-3\cos x+4\sin x>0$

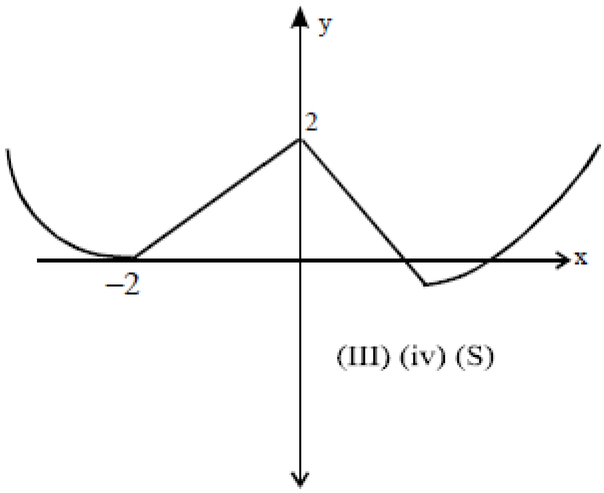
Zero points of Extrema

one solution of f(x)=0

(II)(i)(Q)

(III) $f(x) = (2-|x|, x^2-x-6)$

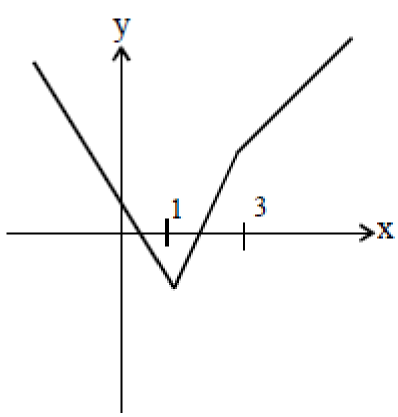
3 points of local Extrema



3 solution of $f(x) = 0$

(IV) $f(x) = |2x-|x-3||-1$

$$f(x) = \begin{cases} x+2 & ; \quad x \geq 3 \\ 3x-4 & ; \quad 1 \leq x < 3 \\ 2-3x & ; \quad x < 1 \end{cases}$$



(IV)(ii)(R)
1 Point of local Exterma
2 Solution of $f(x) = 0$

17. **(D)**

18. **(D)**

Solution for Q. 17 & 18

(I) $\vec{r} = \lambda\vec{a} + \mu\vec{b} + \gamma\left(\vec{a}\times\vec{b}\right)$

$\vec{r}.\vec{a} = 0 \Rightarrow \lambda + \frac{\mu}{2} = 0$

$\vec{r}.\vec{a} = 2 \Rightarrow \frac{\lambda}{2} + \mu = 2$

$\left(\vec{r}\times\vec{a}\right).\vec{b} = 1 \Rightarrow \frac{3\gamma}{4} = 1$

Solving $\lambda = \frac{-4}{3}, \mu = \frac{8}{3}, \gamma = \frac{4}{3}$

(I)(iii)(R)

(II) $\vec{r}.\vec{a} = 0 \Rightarrow \lambda + \frac{\mu}{2} = 0$

$\vec{r}.\vec{b} = 1 \Rightarrow \frac{\lambda}{2} + \mu = 1$

$\gamma = 0; \lambda = \frac{-2}{3}\mu = \frac{4}{3}$

(II)(ii)(P)

(III) $\vec{r}\times\vec{a} = 3\vec{a}\times\vec{b}$

$\left(\vec{r} + 3\vec{b}\right)\times\vec{a} = 0$

$\vec{r} + 3\vec{b} = \lambda\vec{a}$

$\vec{r} = \lambda\vec{a} - 3\vec{b}$

$\vec{r}.\vec{a} = 1 \Rightarrow \lambda\frac{-3}{2} = 1 \Rightarrow \lambda = \frac{5}{2}$

$\vec{r} = \frac{5}{2}\vec{a} - 3\vec{b}$

$\lambda = \frac{5}{2}, \mu = -3, \lambda = 0$

(III)(iv)(P)

(IV) $\vec{a}\times\left(\vec{b}\times\vec{r}\right)=\vec{a}\times\vec{b}$

$$\left(\vec{a}.\vec{r}\right)\vec{b}-\left(\vec{a}.\vec{b}\right)\vec{r}=\vec{a}\times\vec{b}$$

$$2\vec{b}-\frac{1}{2}\vec{r}=\vec{a}\times\vec{b}$$

$$\vec{r}=4\vec{b}-2\left(\vec{a}\times\vec{b}\right)$$

$\lambda=0,\ \mu=4,\ r=2$

(IV)(iii)(S)