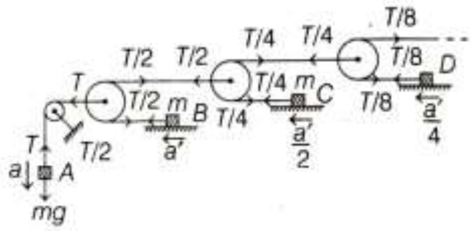
## PART (A) : PHYSICS

1. (C)

As ideal pulley is massless, net force on the pulley is zero. So, tensions can be assumed as depicted in the figure. Let acceleration of the hanging block (block A) be a and acceleration of block B and $a'$. As tension force acting on C is half that of tension acting on B, so acceleration will also be half. i.e. $\frac{a'}{2}$.

For subsequent blocks, acceleration will keep on getting halved by same argument.




As per virtual work method sum of work done by tensions on system is zero.

$\Rightarrow \Sigma T_i . x_i = 0$

Differentiating twice w.r.t. time, we get

$\Sigma T_i . a_i = 0$

$$\Rightarrow Ta\cos 180^0 + \frac{T}{2}a'\cos 0^0 + \frac{T}{4}\frac{a'}{2}\cos 0 + .... = 0$$

$$\Rightarrow -Ta + \frac{Ta'}{2}\left(1 + \frac{1}{4} + \frac{1}{16} + .....\right) = 0$$

(As for infinite G.P. summation is $\frac{A}{1-r}$ where A is first term while r is common ratio)

$$\Rightarrow a' = \frac{3}{2}a \qquad ....(i)$$

For block A, by $F_{net} = ma$, we get

$Mg - T = ma$ .....(ii)

Similarly, for B, we get

$$\frac{T}{2} = ma'$$

$$\Rightarrow \frac{T}{2} = m\left(\frac{3a}{2}\right) \qquad \text{[using Eq. (i)]}$$

$$\Rightarrow T = 3ma \qquad ....(iii)$$

From Eqs (ii) and (iii), we get mg – 3 = ma

$$\Rightarrow a = \frac{g}{4} \qquad ....(iv)$$

$$\Rightarrow a' = \frac{3a}{2} = \frac{3g}{8} \qquad \text{[Using eq (i)]}$$

Velocity of block B after time t of releasing

$$v = u + a't = 0\frac{3g}{8}t \qquad ....(v)$$

Also, tension force on block B,

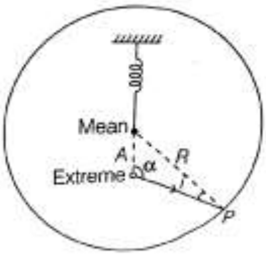
$$F = \frac{T}{2} = \frac{3ma}{2}, \frac{3mg}{8} \quad \ldots.(vi) \qquad \text{[using Eqs (iii) and (iv)]}$$

Thus, power delivered to B,

$$P = Fv = \frac{9mg^2t}{64} \qquad \text{[using Eqs. (v) and (vi)]}$$

2. (D)

By sine rule in the triangle formed in fig. 1




Angle of incidence i, at general point P, is maximum when the source is at extreme position shown in figure

$$\frac{\sin i}{A} = \frac{\sin\alpha}{R}$$

$$\Rightarrow \sin i = \frac{A}{R}\sin\alpha$$

So for given A, I maximum if point P has $\alpha = 90^0$. Also for TIR i should be equal to critical angle C.

$\therefore$ For $\alpha = 90^0$

A = R sin C

Also by Snell's law, for refraction at P,

n . sin C =1. Sin $90^0$

$$\Rightarrow \sin C = \frac{1}{n} \quad \ldots.(ii)$$

For an oscillator executing SHM, amplitude A in terms of speed v at mean position is

$$A = v\sqrt{\frac{m}{k}} \quad \ldots.(iii)$$

Putting values of A and sin C from Eqs. (ii) and (iii) in Eq. (i), we get

$$v\sqrt{\frac{m}{k}} = \frac{R}{n}$$

$$\Rightarrow v = \frac{R}{n}\sqrt{\frac{k}{m}}$$

3. (A)

From FBD of the soap film shown here in figure

$$p\pi R^2 = p_0\pi R^2 + T\sin\theta.4\pi R$$

$$\Rightarrow p = p_0 + \frac{4T\sin\theta}{R}$$

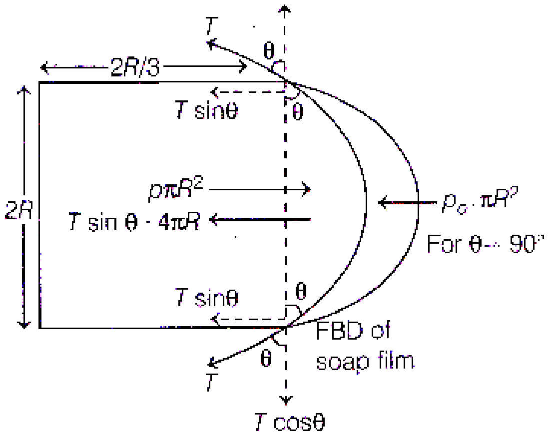


Fig. 1

p is maximum, for $\theta = 90^\circ$

$$\Rightarrow p_{max} = p_0 + \frac{4T}{R} \qquad \ldots\ldots(i)$$

Also, for $\theta = 90^\circ$, shape of the film is hemispherical as shown in figure. Final volume of the gas,

$$V_f = \pi R^2 . \frac{2R}{3} + \frac{2}{3}\pi R^3$$

$$\Rightarrow V_f = \frac{4}{3}\pi R^3 = 2V_0 \qquad \ldots..(ii)$$

(as, initial volume, $V_0 = \pi R^2 , \frac{2R}{3} = \frac{2\pi R^3}{3}$)

By conservation of number of moles , if $T_f$ is the final temperature at the instant when pressure is maximum.

$$n_{final} = n_{initial}$$

$$\Rightarrow \frac{p_{max} V_f}{T_f} = \frac{p_0 V_0}{T_0} \qquad \text{(using ideal gas equation)}$$

$$\Rightarrow T_f = \left(\frac{p_{max} V_f}{p_0 V_0}\right) T_0$$

$$= \left(1 + \frac{4T}{p_0 R}\right) 2T_0 \qquad \text{[using Eqs. (i) and (ii)]}$$

$$= \left(2 + \frac{8T}{p_0 R}\right) T_0$$

4. (C)

Let charge on each face of the cube =q

Total flux through the cube $= \frac{6q}{\varepsilon_0}$

So, flux through each face $\phi = \frac{q}{\varepsilon_0} \qquad \ldots\ldots.(i)$

Also , field due to a sheet of charge, having surface charge density $\sigma$, near its surface

$$E=\frac{\sigma}{2\varepsilon_0}$$

So, flux through each face due to charge on itself,

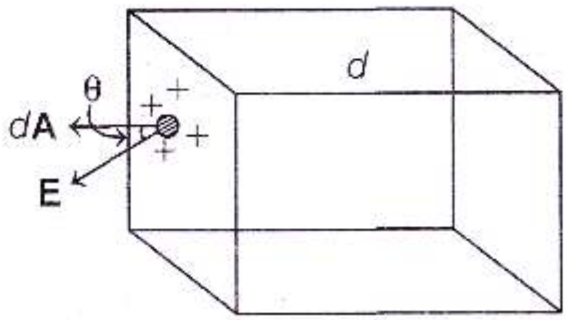
$$\phi'=\frac{\sigma}{2\varepsilon_0}A=\frac{q}{2\varepsilon_0} \qquad \text{....(ii)}$$

Thus, flux through each face due to remaining five faces.

$$\phi''=\phi-\phi'=\frac{q}{2\varepsilon_0} \qquad \text{......(iii)}$$

[using Eqs. (i) and (ii)]

Now consider a small element of surface area dA of face, on which we want to find force, as shown in figure. Let its area vector is dA and electric field at its location is E as shown.




Charge on the element, $dq=\sigma\,dA$

So, force on the element

$$|dF|=dq|E|$$

$$=\sigma dA|E|$$

$$=\sigma dAE \qquad \text{......(iv)}$$

As by symmetry, net force on the face is perpendicular to it. Therefore, net force is summation of components & forces perpendicular to the surface.

$$F=\int dF\cos\theta=\int \sigma E\,dA\cos\theta \quad \text{[using Eq. (iv)]}$$

$$\Rightarrow F=\sigma\int E.dA=\sigma\phi''$$

$$=\sigma.\frac{q}{2\varepsilon_0} \qquad \text{[Using Eq. (iii)]}$$

$$=\frac{\sigma.\sigma d^2}{2\varepsilon_0}=\frac{\sigma^2 d^2}{2\varepsilon_0} \qquad \left[\because q=\sigma d^2\right]$$

5. (B, C, D)

As, cylindrical part of the foil is long and is carrying circumferential current it can be treated as long solenoid. So field at its centre will be field inside ideal solenoid.

$$B=\mu_0\times \text{ current per unit length}$$

$$\Rightarrow B_1=\mu_0\times\frac{i}{l}$$

So, $B_2$ due to the two planar surfaces,

$$B_2 = \frac{\mu_0 i}{l} \qquad \text{....(ii)}$$

As, energy stored in magnetic field is given by

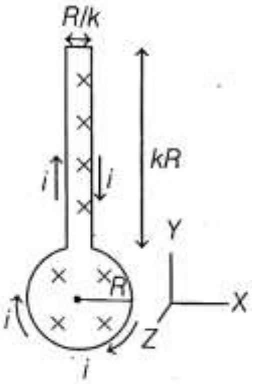
$$U = \frac{B^2}{2\mu_0} \times \text{Volume}$$

Ratio of energies in cylindrical volume $V_1$ to that in volume $V_2$ between planes.

$$\frac{U_1}{U_2} = \frac{\frac{B_1^2}{2\mu_0} \times V_1}{\frac{B_2^2}{2\mu_0} \times V_2} = \frac{V_1}{V_2} \quad [\because\ B_1 = B_2 \text{ as per Eqs (i) and (ii)}]$$

$$\Rightarrow \frac{U_1}{U_2} = \frac{\pi R^2 . i}{\frac{R}{k} . kR . I} = \pi$$

If we consider cross-sectional view of the arrangement, magnetic field is in negative z-direction as shown




Flux of the field is given by

$\phi = BAs$

$$\Rightarrow \phi = \frac{\mu_0 i}{l}\left(\pi R^2 + kR.\frac{R}{k}\right)$$

$$= \frac{\mu_0 i R^2}{I}(\pi + 1)$$

$\therefore$ Self-inductance of arrangement

$$L = \frac{\phi}{i} = \frac{\mu_0 R^2 (\pi + 1)}{l}$$

Energy stored in the arrangement can be written as

$$U = \frac{1}{2} L i^2 = \frac{1}{2}\frac{\mu_0 R^2}{I}(\pi + 1) i^2$$

6. (A, B, C)

Conveyor belt in this case is a variable mass system. Let F be force on it by rollers to keep it moving with constant speed v.

By Newton's $2^{nd}$ law

$F = \frac{dp}{dt} = \frac{d(mv)}{dt}$

$= m.\frac{dv}{dt} + v.\frac{dm}{dt} = 0 + v.\mu = \mu v$ (as v = constant) …..(i)

Power delivered by this force can be calculated by

$P = F.v = Fv\cos\theta$

$\Rightarrow P_F = \mu v.v\cos 00 = \mu v^2$ [using Eq. (i)]

By work-energy theorem

$dW_F + dW_f = dK$

Here, $dW_F, dW_f$ represent work by F, work by friction and change in kinetic energy, respectively

$\Rightarrow \frac{dW_F}{dt} + \frac{dW_f}{dt} = \frac{dK}{dt}$

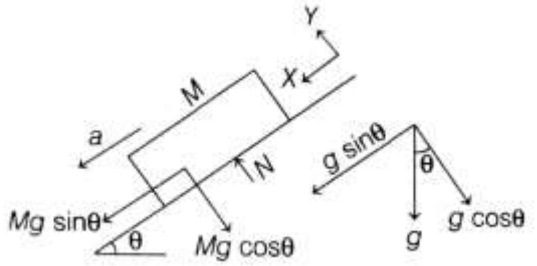
$\Rightarrow P_F = \frac{dW_f}{dt} = \frac{d\left(\frac{1}{2}mv^2\right)}{dt}$

$\Rightarrow \mu v^2 + \frac{dW_f}{dt} = \frac{v^2}{2}\frac{dm}{dt} = \frac{\mu v^2}{2}$

$\Rightarrow \frac{dW_f}{t} = -\mu\frac{v^2}{2}$

7. (A, B, C, D)

From FBD of box shown in fig 1.




Acceleration of box

$a = \frac{mg\sin\theta}{M} = g\sin\theta$

$\Rightarrow a_{box} = g\sin\theta\hat{i}$

$\therefore$ Relative acceleration due to gravity inside box

$g' = g - a$

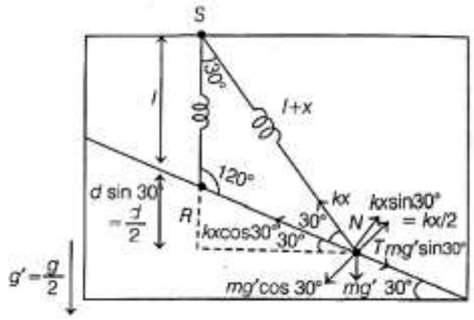
$= (g\sin\theta\hat{i} - g\cos\theta\hat{j}) - g\sin\theta\hat{i}$

$= -g\cos\theta\hat{j}$

So, relative acceleration due to gravity.

$g' = g\cos\theta = g\cos 60^0 = \frac{g}{2} = 5ms^{-2}$

If we rotate the figure of box for simplicity of analysis, we fig 2. Let natural length of spring is l (initially) and spring constant is k.



Let extension in spring in final position T be x. By sine rule in $\Delta RST$, we get

$$\frac{l+x}{\sin 120^0} = \frac{l}{\sin 30^0}$$

$$\Rightarrow l + x = \sqrt{3}l$$

$$\Rightarrow x = \left(\sqrt{3}-1\right)l = \left(\sqrt{3}-1\right)\left(\sqrt{3}+1\right) \qquad \ldots.(i)$$

$$\Rightarrow x = 2m$$

Maximum displacement of the bead is equal to

$$d = RT = I = \left(\sqrt{3}+1\right)m(\text{given}) \qquad \ldots..(ii)$$

As, bead comes to rest at T, after release, loss in its potential energy is equal to gain in spring potential energy

Therefore, $mg'\frac{d}{2} = \frac{1}{2}kx^2$

$$\Rightarrow k = \frac{mg'd}{x^2} = \frac{1\times 5\times\left(\sqrt{3}+1\right)}{4}$$

$$= \frac{5\left(\sqrt{3}+1\right)}{4} N/m \qquad \text{(using Eqs. (i) and (ii)}$$

From FBD of bead in position T as shown fig. 2. In direction perpendicular to the wire, we get

$$N + \frac{kx}{2} = \frac{\sqrt{3}mg'}{2} \Rightarrow N = \frac{\sqrt{3}mg' - kx}{2}$$

$$= \frac{\sqrt{3}\times 1\times 5 - \frac{5\left(\sqrt{3}+1\right)}{4}\times 2}{2}$$

$$= \frac{5\left(\sqrt{3}-1\right)}{4} N$$

8. (A, B, D)

$P = \text{Power} = i^2R = i^2.\frac{\rho l}{\pi r^2}$

As, i through all the wires is same.

$$\Rightarrow P \propto \frac{\rho l}{r^2}$$

$$\Rightarrow P_A : P_B : P_C = \frac{\rho_A l_A}{r_A^2} : \frac{\rho_B l_B}{r_B^2} : \frac{\rho_C l_C}{r_C^2}$$

$$= 4 : \frac{3}{2} : \frac{1}{3} = 24 : 9 : 2$$

$$\Rightarrow P_A = 24P_0, P_B = 9P_0 . P_C = 2P_0$$

$\therefore \sqrt{P_B P_C}$ = Geometric mean of $P_B$ and $P_C$

$$\sqrt{9P_0 2P_0} = \sqrt{18} P_0$$

$$\Rightarrow \frac{P_A}{\sqrt{P_B P_C}} \frac{24P_0}{\sqrt{18P_0}} = 4\sqrt{2}$$

By Ohm's law in microscopic form

J= $\sigma$E

$$\Rightarrow \frac{i}{\pi r^2} = \frac{E}{\rho} \Rightarrow E \propto \frac{\rho}{r^2}$$

$$\Rightarrow E_A : E_B : E_C = \frac{\rho_A}{r_A^2} : \frac{\pi_P}{r_B^2} : \frac{\rho_C}{r_C^2}$$

$$= \frac{2}{4} : \frac{3}{4} : \frac{1}{9} = 72 : 27 : 4$$

$$\Rightarrow E_A = 72E_0, E_B = 27E_0, E_C = 4E_0$$

$$\Rightarrow \frac{E_B^2}{E_A E_C} = \frac{27^2}{72 \times 4} = \frac{81}{32}$$

$$\Rightarrow 32E_B^2 = 81E_A E_C$$

Drift speed v is related to current I by the relations

$$v = \frac{i}{NeA} = \frac{i}{Ne\pi r^2} \Rightarrow v \propto \frac{1}{Nr^2}$$

$$\therefore v_A : v_B : V_C = 1 : \frac{1}{8} : \frac{1}{27} = 216 : 27 : 8$$

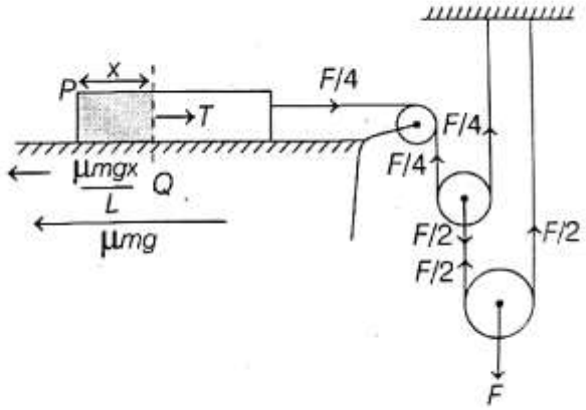
Also, potential difference

$$V = IR = \frac{l \rho l}{\pi r^2} \Rightarrow V \propto \frac{\rho l}{r^2}$$

$$\therefore v_A : v_B : v_C = 4 : \frac{3}{2} : \frac{1}{3} = 24 : 9 : 2$$

9. (5)

From FBD of pulley, tension at various points will be as shown in figure




So, acceleration of the rod,

$$a = \frac{\frac{F}{4} - \mu mg}{m} = \frac{F}{4m} - \mu g \qquad \ldots\ldots(i)$$

If we now consider portion PQ of the rod having length x, mass of PQ is $\frac{mx}{L}$. Therefore, friction on it will be $\frac{\mu mgx}{L}$. Using F = ma for PQ, we get

$$T - \frac{\mu mgx}{L} = \frac{mx}{L}a$$

$$\Rightarrow T - \frac{\mu mgx}{L} = \frac{mx}{L}\left(\frac{F}{4m} - \mu g\right) \qquad \text{[using Eq. (i)]}$$

$$\Rightarrow T = \frac{F}{4}.\frac{x}{L}$$

So, stress at point Q

$$\sigma = \frac{T}{A} = \frac{Fx}{4LA} \qquad \ldots\ldots(ii)$$

To find elastic potential energy, lets now first consider an element of a thickness dx at a distance x from left end.

As, elastic potential energy in terms of stress $\sigma$ is given by

$$U = \frac{\sigma^2}{2Y} \times \text{volume}$$

Potential energy of element

$$dU = \frac{F^2.x^2 \times Adx}{32L^2A^2Y} \qquad \text{[using Eq. (ii)]}$$

$\therefore$ Total potential energy of rod

$$U = \int dU \frac{F^2L}{96AY} = \frac{(F-)\times mg)^2 L}{(10^2-4)AY}$$

$$\Rightarrow a = 0, b = 10$$

$$\therefore \frac{b}{2+a} = 5$$

10. (3)

Refractive index of lens

$$n=\frac{\lambda_{vaccum}}{\pi_{lens}}=\frac{6000}{4000}=15$$

Focal length of lower unpolished lens ($f_1$) using lens Maker's formula is

$$\frac{1}{f_1}=(1.5-1)\left(\frac{1}{R}-\frac{1}{\infty}\right)=\frac{1}{2R}$$

Upper part can be consider as combination of lens and mirror. This combination is equivalent to a mirror, whose equivalent focal length ($f_{eq}$) can be calculate using the formula

$$\frac{1}{f_{eq}}=\frac{1}{f_{mirror}}-\frac{2}{f_{lens}}$$

As for flat plane mirror, focal length is infinite

$$\Rightarrow \frac{1}{F_{eq}}=\frac{1}{\infty}-\frac{2}{2R}=-\frac{1}{R} \qquad \text{[Using Eq. (i)]}$$

$$\Rightarrow f_{eq}=-R \qquad \text{.....(ii)}$$

Hence, the combination is equivalent to a concave mirror of focal length R. Thus, for image ($l_1$) formation by polished part, we use mirror formula

$$\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$$

$$\Rightarrow \frac{1}{v}+\frac{1}{-3R}=\frac{1}{-R} \qquad \text{[Using Eq. (ii)]}$$

$$\Rightarrow v=-\frac{3R}{2} \qquad \text{....(iii)}$$

We are given velocity of object $v_0$. As velocity $v_i$ for object moving along axis of lens is given by

$$v_i=\left(\frac{v}{u}\right)^2 v_0$$

$$\therefore \text{Velocity of image -2, } v_2=\left(\frac{6R}{-3R}\right)^2 v_0 \qquad \text{[using eq. (v)]}$$

$$\Rightarrow v_2=4v_0$$

Thus, velocity of image -1 w.r.t. to image -2 will be

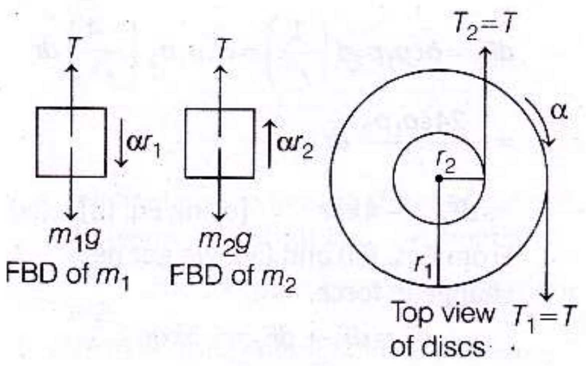
$$v_{relative}=|v_1-v_2|$$

$$=\left|\frac{-v_0}{4}-4v_0\right|=\frac{17v_0}{4}$$

$$=\frac{17}{4}\times\frac{12}{17}=3cm/s$$

11. (2)

For axis to remain stationary, net force on the composite body should be zero.




Therefore, tensions in the two strings should be equal. i.e $T_1 = T_2 = T$ (suppose).

If we assume angular acceleration of the composite disc equal to $\alpha$, accelerations of blocks of masses $m_1$ and $m_2$ are $\alpha r_1$ and $\alpha r_2$, respectively as shown in figure.

By using $\tau = I\alpha$, for composite body.

$$Tr_1 - Tr_2 = I\alpha$$

$$\Rightarrow T(0.3-0.2) = 0.25\alpha$$

$$\Rightarrow T = 2.5\alpha \qquad \ldots\ldots(i)$$

For $m_1$, using $F = ma$, we get

$$m_1g - T = m_1\alpha r_1$$

$$\Rightarrow m_1g - 2.5\alpha = m_1\alpha(0.3) \quad [\text{using Eq. (i)}]$$

$$\Rightarrow m_1g = (2.5 + 0.3m_1)\alpha \qquad \ldots\ldots(ii)$$

Similarly, for $m_2$,

$$T - m_2g = m_2\alpha r_2$$

$$\Rightarrow 2.5\alpha - 2.5g = 2.5\alpha(0.2)$$

$$\Rightarrow \alpha = \frac{5g}{4} \qquad \ldots..(iii)$$

Putting value of $\alpha$ from Eq. (iii) into Eq. (ii), we get

$$m_1g = (2.5 + 0.3m_1)\frac{5g}{4}$$

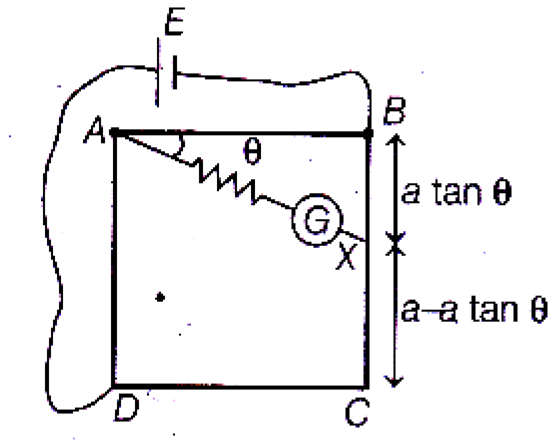
$$\Rightarrow 4m_1 = 12.5 + 1.5m_1$$

$$\Rightarrow m_1 = 5\text{kg}$$

$$\therefore \frac{m_1}{m_2} = \frac{5}{2.5} = 2$$

12. (8)

Let a be the side length of square and $\theta$ be the position where galvanometer gives zero deflection. To have zero deflection, bridge is to be balanced.




$$\Rightarrow \frac{R_{AB}}{R_{AD}} = \frac{R_{BX}}{R_{DC} + R_{CX}}$$

$[R_{DC} \text{ and } R_{CX} \text{ are in series}]$

$$\Rightarrow \frac{100}{200} = \frac{\frac{400}{a} a \tan\theta}{500 + \frac{400}{\alpha}(a - a\tan\theta)}$$

$$\Rightarrow \frac{1}{2} = \frac{400\tan\theta}{500 + 400(1 - \tan\theta)}$$

Solving this, we get $\tan\theta = 3/4$

$\Rightarrow \theta = 37^\circ$

Let t be the time taken from start, then $\theta = \omega t\,(\theta \text{ in rad})$

$$\Rightarrow \frac{\pi}{180} \times 37 = \frac{\pi}{360} \times t$$

$\Rightarrow t = 74\,s = n^2 + 10$

$\therefore n = 8$

13. (4)

Total increase in length of rods

$$= L\alpha\Delta T + \frac{L}{2}\alpha\Delta T = 3\frac{L}{2}\alpha\Delta T$$

Let the compression in spring A is $x_A$, B is $x_B$ and C is $x_C$.

$\Rightarrow k_A x_A = k_B x_B = k_C x_C$

$\Rightarrow kx_A = 2kx_B = kx_C \Rightarrow x_A = 2x_B = x_C$

And $x_A + x_B + x_C = \frac{3}{2}L\alpha\Delta T$

$$\Rightarrow x_B = \frac{3}{10}L\alpha\Delta T \qquad \ldots\ldots(i)$$

Energy stored,

$$E=\frac{1}{2}kx_A^2+\frac{1}{2}(2k)x_B^2+\frac{1}{2}kx_C^2$$

$$=\frac{1}{2}k(2x_B)^2+kx_B^2+\frac{1}{2}k(2x_B)^2$$

$$=2kx_B^2+kx_B^2+2kx_B^2=5kx_B^2 \qquad \text{.....(ii)}$$

Using Eqs. (i) and (ii), we get

$$E=5k\left(\frac{9}{100}L^2\alpha^2\Delta T^2\right)$$

$$=\frac{9}{20}k\alpha^2L^2\Delta T^2$$

$$=\frac{9}{5\beta}k\alpha^2L^2\Delta T^2$$

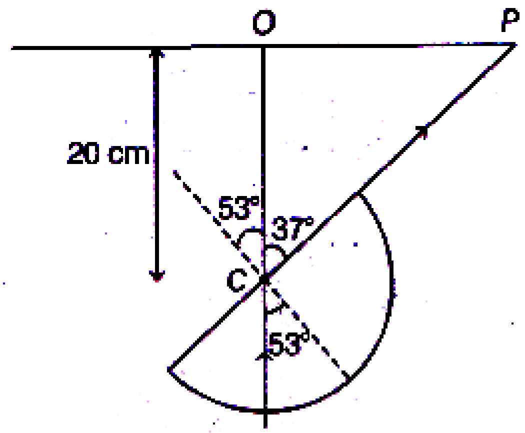
$$\Rightarrow \beta=4$$

14. (5)

Critical angle for a pair of medium is given by

$$C=\sin^{-1}\left(\frac{n_{rarer}}{n_{denser}}\right)$$

$$\Rightarrow C=\sin^{-1}\left(\frac{1}{5/4}\right)=53^\circ$$




So, when the ray is incident at $53^\circ$ as shown, it will emerge grazingly along surface making $37^\circ$ with line OC striking screen at P. Distance moved by the laser spot.

$$d=OP=20\tan 37^\circ=15\text{cm} \qquad \text{.......(i)}$$

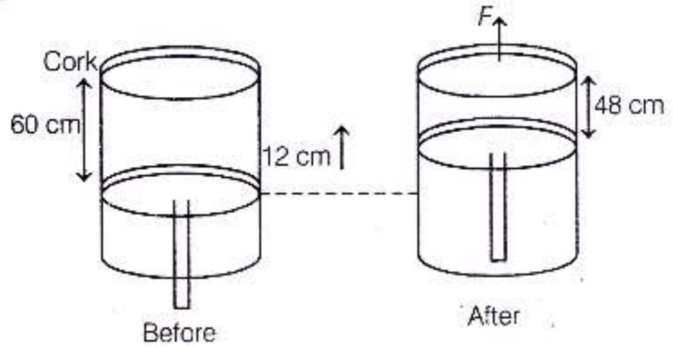
Also, time taken for rotation of the cylinder.

$$t=\frac{\theta}{\omega}=\frac{53\pi}{180\times\frac{53\pi}{540}}=3\text{s} \qquad \text{........(ii)}$$

Therefore, average speed of the laser spot $=\frac{d}{t}=5\text{cm}/\text{s}$

15. (4)

As, the tube is perfectly conducting, temperature of air will remain constant.




As, $\rho V = nRT = \text{constant}$

$\Rightarrow p_{final} V_{final} = p_{initial} V_{initial}$

$\Rightarrow \rho_{final} = p_0 \times \frac{5}{4}$

$= 1.25 \times 10^5 \text{Pa}$

$\therefore$ Force = Difference in pressure $\times$ Area

$= \left(1.25 \times 10^5 - 10^5\right) \times 1.6 \times 10^{-4} = 4\text{N}$

16. (6)

Consider the object as two portions a uniform rod an a frustum with thermal resistances $R_1$ and $R_2$ respectively, then

$R_1 = \frac{l_1}{K_1 A_1} = \frac{l}{K\pi r^2}$

And $R_2 = \frac{l_2}{K_2 A_2}$

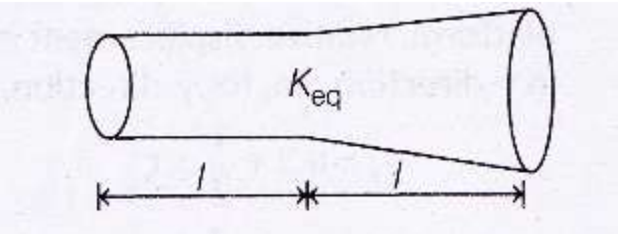
$= \frac{l}{(2K)(\pi r_1 r_2)} = \frac{l}{4K\pi r^2}$

$\therefore$ Equivalent thermal resistance,

$R_{eq} = R_1 + R_2$

$\Rightarrow R_{eq} = \frac{5l}{4K\pi r^2}$ ......(i)

Now, if we consider the same lamina with equivalent thermal conductivity $K_{eq}$, then




$R_{eq} = R_1 + R_2$

$$= \frac{l}{K_{eq}\pi r^2} + \frac{l}{K_{eq}\left(2\pi r^2\right)}$$

$$= \frac{3l}{2K_{eq}\pi r^2} \qquad \ldots.(iii)$$

By equating the terms of $R_{eq}$ from Eqs. (i) and (ii), we get

$$\frac{5l}{4K\pi r^2} = \frac{3l}{2K_{eq}\pi r^2}$$

$$K_{eq} = \frac{6K}{5}$$

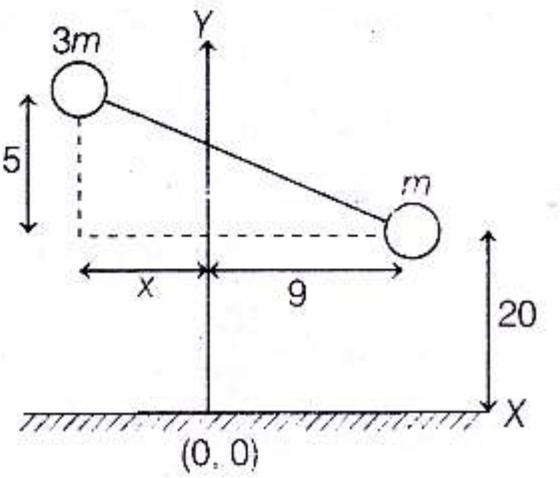
$$\Rightarrow \frac{K_{eq}}{K} = \frac{6}{5} = 1.20$$

$$\Rightarrow 5K_{eq} = 5 \times 1.2 = 6$$

17. (4)

The external force on two body systems acts along Y-axis.

The initial momentum of the two body systems is zero. Hence, the CM of two body system always moves along Y-axis




$\therefore$ CM of two body system lies along Y-axis, the x-coordinate of centre of mass of two body sytem is $X_{CM} = 0$

$$\therefore 3M \times x = m \times 9$$

$$\Rightarrow x = 3$$

$$\therefore \text{ Length of string } = \sqrt{5^2 + (9+3)^2}$$

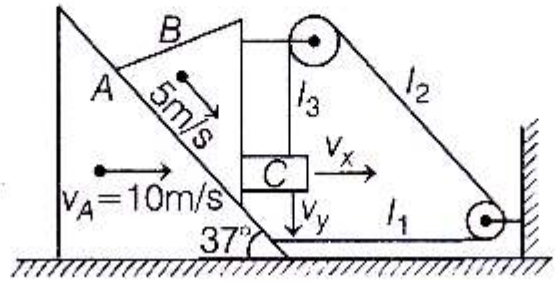
$$= 13\text{cm}$$

$$= x^2 - x + 1 (\text{Given})$$

$$\Rightarrow x = 4$$

18. (7)

Let $v_x$ and $v_y$ be the horizontal and vertical components of velocity of block C.




The component of relative velocity of B and C normal to the surface of contact is zero.

$\therefore 10+5\cos 37^{\circ}-v_x=0$ ......(i)

$v_x=14\,m/s$

From the figure, $l_1+l_2+l_3=$constant

$\therefore \frac{dl_1}{dt}+\frac{dl_2}{dt}+\frac{dl_3}{dt}=0$

$(-10)+\left(-5-10\cos 37^{\circ}\right)$

$+\left(-5\sin 37^{\circ}+v_y\right)=0$

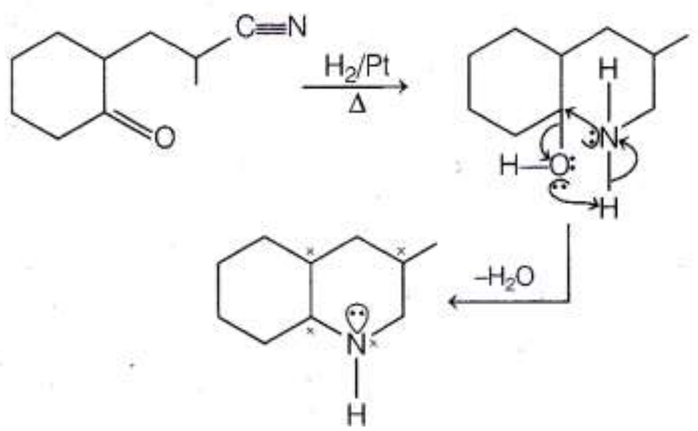
$\therefore v_y=26\,m/s$

So, ratio, $\frac{v_x}{v_y}=\frac{14}{26}=\frac{7}{13}$

$\therefore a=7$

## PART (B) : CHEMISTRY

1. (C)




The product formed has 4 chiral centres.

2. (A)

$$\ln\frac{k_2}{k_1} = \frac{E_A}{R}\left(\frac{1}{T_1} - \frac{1}{T_2}\right)$$

Greater $E_A$, greater will be increase in rate constant.

3. (A)

$Mn(CO)_6 - e^- \longrightarrow \left[Mn(CO)_6\right]^{\oplus}$ most stable because Effective atomic number
$= (25-1) + 6\times 2 = 36 =$ Atomic number of Kr

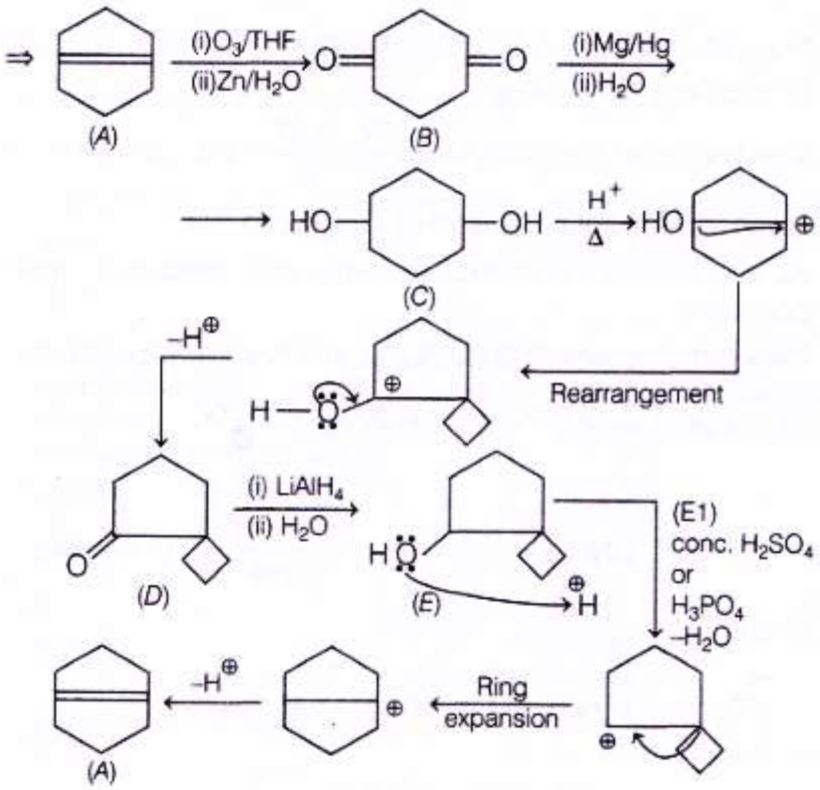
4. (A)

Molecular orbitals electronic configuration of $NO_2$ confirms the presence of unpaired electron which can easily undergo transition from ground state energy level to excited state level by absorbing light of suitable wavelength.

Cause of colour in $N_2O_3$

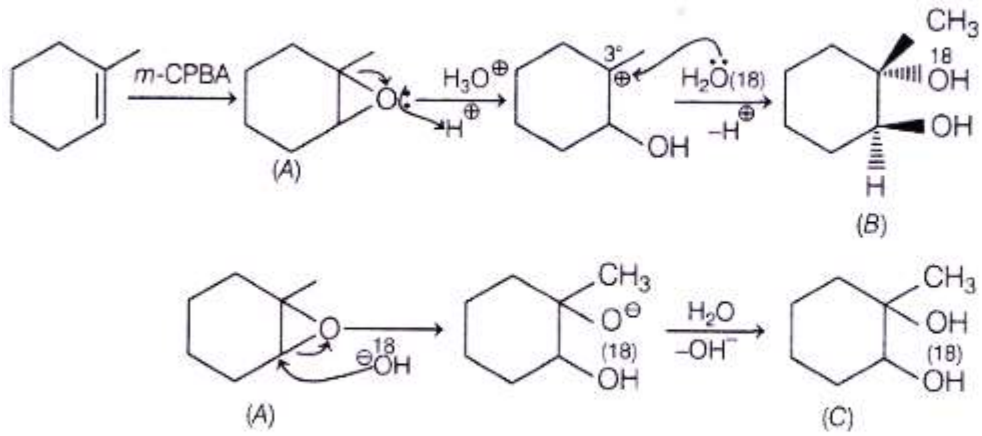
Due to low difference between occupied and unoccupied energy levels of electron among molecules of $N_2O_3$, it absorbs a part of visible light spectrum which causes colour in $N_2O_3$.

5. (B, C)

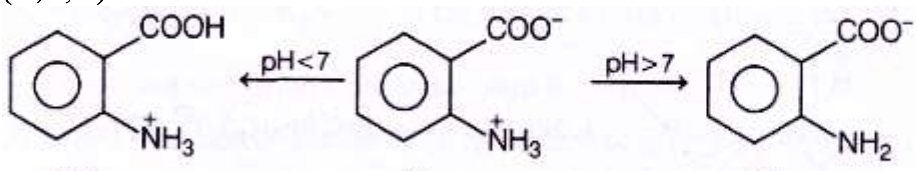



$\Rightarrow Al_2O_3$ or $ThO_2$ shows dehydration of an alcohol into alkene via E2 elimination.

6. (A, B, D)




7. (A,C,D)



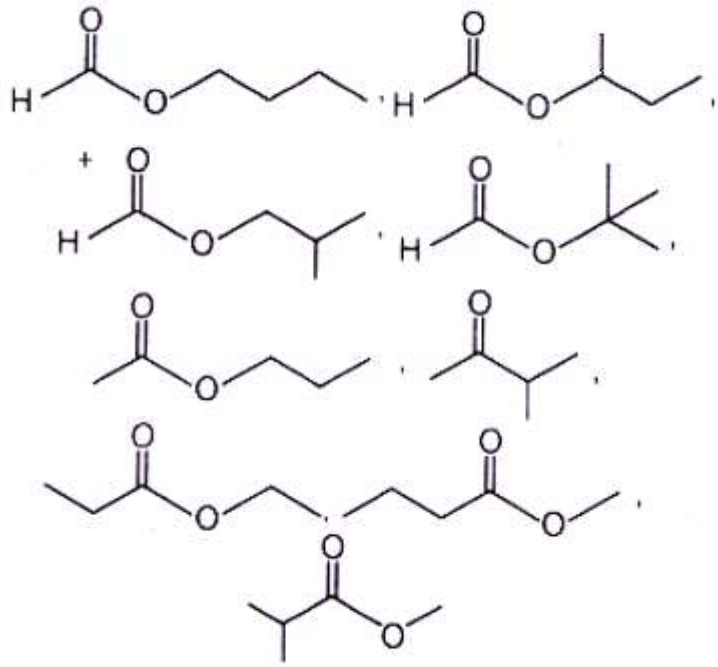

(III)
All are acidic groups

(I)
Zwitter ion
(pH = $p_I$H = 7)

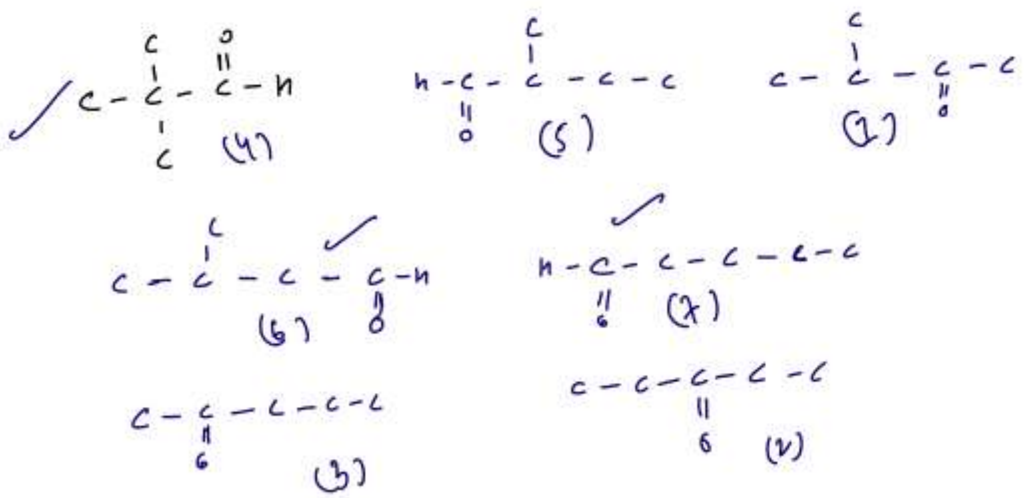
(II)
All are basic groups

8. (B, C, D)
Statements (B, C, D) are correct, whereas statement (A) is incorrect.
(A) X reaches end point earlier than Y, hence X is present in lower concentration.
(B) Greater change in pH of X at end point indicates that X is stronger acid than Y.
(C) Volume of NaOH require to neutralise Y is twice to that required for X, so $[X]=\frac{1}{100}=0.01M$ and $[Y]=\frac{2}{100}=0.02M$
(D) Weaker acid (Y) produces a stronger conjugate base (salt).

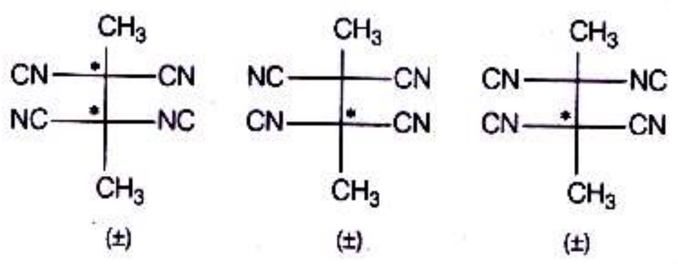
9. (9)



10. (3)
The isomeric carbonyl compounds of molecular formula, $C_5H_{10}O(86u)$ are
(4),(6) and (7) $\longrightarrow$ give racemic mixture of alcohols) $=(5)\rightarrow$ is a pure enantiomer and will give diastereomers $=(1),(2),(3)\rightarrow$ give achiral alcohol.



11. (6)



12. (5)

Only $1^{\circ}$-aliphatic amines can be prepared from Gabriel's phthalimide synthesis, where a halogen is replaced by $NH_2$ group. In chlorobenzene, Cl is attached to $sp^2$-hybridised carbob, hence aniline can not be prepared while halogen atom in 2,4- dinitrochlorobenzene is replaceable, hence compound (vi) can be prepared.
So, (i), (iv), (v), (vi) and (vii) can be prepared.

13. (2)

10 mL of 1 mM solution contains $10^{-5}$ mole

$\therefore$ Number of molecules in one face $= \frac{6\times10^{23}\times10^{-5}}{6} = 10^{18}$

Number of molecules in one edge $= \sqrt{10^{18}} = 10^{9}$

$\Rightarrow$ Area of 1 face $= \frac{0.24}{6} = 0.04\text{m}^2$,

Edge length $= \sqrt{0.04}\text{m} = 0.2\text{m}$

$10^9$ molecules are converting 0.2 m length[monoatomic layer]

$\therefore$ 1molecule is covering $2\times10^{-10}$ m length, ie. $2\,\text{Å}$.

14. (6)

Only $p\pi - p\pi : NO_3^-, CO_3^{2-}, (CN)_2$

$\Rightarrow$ only $d\pi - p\pi$:

$xeO_3, ClO_4^-, XeOF_2, H_3PO_4$

$\Rightarrow$ One $p\pi - p\pi$, rest $d\pi - p\pi : SO_3, SO_4^{2-}$

15. (2)

$\left[Ni(CN)_4\right]^{2-}, \left[Pt(Cl)_4\right]^{2-}$

Hybridisation $-dsp^2$

Shape of molecule = Square planar

$PF_5, PCl_5$

Hybridisation of molecule = Trigonal bipyramidal

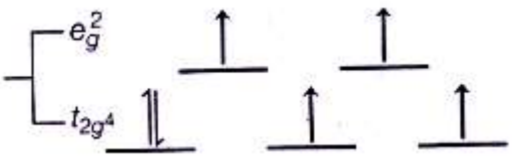
$BrF_5$ Hybrdisation is $sp^3d^2$ and shape of molecule is square pyramidal

$SF_6.[CrF_6]^{3-}, \left[Co(NH_3)_6\right]^{3+}$

Hybridisation $sp^3d^2, d^2sp^3$
Shape of molecules =Octahedral
$CH_4, NH_4^+, \left[Ni(CO)_4\right]$
Hybridisation: $sp^3$
Shape of molecule- Tetrahedral

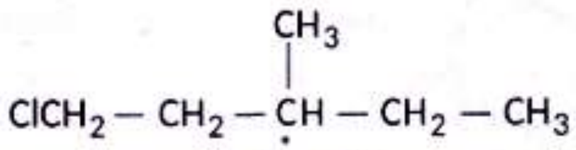
16. (9)
Number of atoms of A in fcc $= 4$ [Corners+ Face centres]
Number of atoms of B at octahedral voids
=4[ Edge centres + Body centre]
$\Rightarrow$ Number of effective atoms of A after removal
$= 4 - 2 \times \frac{1}{8} = \frac{15}{4}$ (two corners of the body diagonal are removed)
$\Rightarrow$Number of effective atoms of B after removal $= 4 - 1 = 3$ (body centre of the body diagonal is removed)
$\Rightarrow A : B = \frac{15}{4} : 3 = 5 : 4$
So, simplest formula $= A_5B_4$ and $x + y = 9$

17. (4)
$\Rightarrow H_2O$ is a weak field ligand and $\left[Fe(H_2O)_6\right]^{2+}$ is a high spin complex
$\Rightarrow Fe^{2+} : 3d^6 \Rightarrow t_{2g}^4 e_g^2$

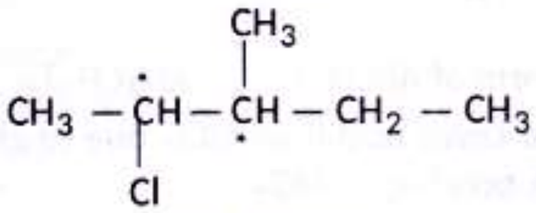



$\Rightarrow$ Number of unpaired electrons $= 4$

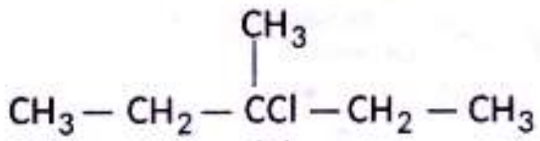
18. (8)
Total number of isomeric monochlorides $= 2 + 4 + 1 + 1 = 8$
The products (P) can be shown as,



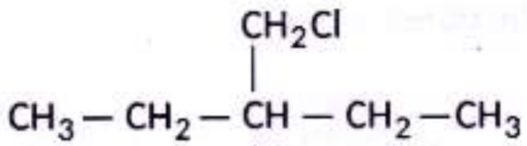
(2) $[1 \times (\pm)]$



(4) $[2 \times (\pm)]$



(1)



(1)

# PART (C) : MATHEMATICS

1. (C)

Consider the given series

$$\sum_{r=0}^{100}(-1)^r\left({}^{99}C_r+{}^{99}C_{r-1}\right)$$

$$\left[\frac{1}{2^r}+\frac{3^r}{2^{2r}}+\frac{7^r}{2^{3r}}+\text{...upto 10 terms}\right]$$

$$=\sum_{r=0}^{100}(-1)^r\,{}^{100}C_r$$

$$\left[\frac{1}{2^r}+\frac{3^f}{2^{2r}}+\frac{7^f}{2^{3r}}+\text{....upto 10 terms}\right] \qquad \left[\because {}^nC_r+{}^nC_{r-1}={}^{n+1}C_r\right]$$

$$=\sum_{r=0}^{100}(-1)^r\,{}^{100}C_r\left(\frac{1}{2}\right)^r+\sum_{r=0}^{100}(-1)^r\,{}^{100}C_r\left(\frac{3}{4}\right)^r+\sum_{r=0}^{100}(-1)^r\,{}^{100}C_r\left(\frac{7}{8}\right)^r+\text{....upto 10 terms}$$

$$=\sum_{r=0}^{100}{}^{100}C_r\left(\frac{-1}{2}\right)^r+\sum_{r=0}^{100}{}^{100}C_r\left(\frac{-3}{4}\right)^r+\sum_{r=0}^{100}{}^{100}C_r\left(\frac{-7}{8}\right)^r+\text{....upto 10 terms}$$

$$=\left(1-\frac{1}{2}\right)^{100}+\left(1-\frac{3}{4}\right)^{100}+\text{....upto 10 terms} \qquad \left[\because \sum_{r=0}^{n}{}^nC_r(-x)^r=(1-x)^n\right]$$

$$=\left(\frac{1}{2}\right)^{100}+\left(\frac{1}{4}\right)^{100}+\text{....upto 10 terms}$$

$$=\left(\frac{1}{2}\right)^{100}\left[\frac{1-\left(\left(\frac{1}{2}\right)^{100}\right)^{10}}{1-\frac{1}{2^{100}}}\right]$$

$$=\frac{1}{2^{100}}\left[\frac{1-\frac{1}{2^{1000}}}{1-\frac{1}{2^{100}}}\right]=\frac{1}{2^{1000}}\left[\frac{2^{1000}-1}{2^{100}-1}\right]$$

2. (C)

We know the prime digits are 2, 3,5, 7. If we fix 2 at the first place then rest of 2n – 1 places can be filled in $4^{2n-1}$ ways. Sum of 2 consecutive digits are (2, 3) or (2,5), thus 2 will be fixed at all alternate places i.e.

| | | | | |
|---|---|---|---|---|
| 2 | | 2 | | 2 |

For filling n places by 2, we have only 1 way and for filling rest of places by 3 or 5.

$\therefore$ Number of favourable ways = $2^n$

Required probability = $\dfrac{2^n}{4^{2n-1}}$

$$= \frac{2^n \times 4}{2^{4n}} = \frac{4}{2^{3n}}$$

3. (A)

Given functional equation is

$$2f(x-1) - f\left(\frac{1-x}{x}\right) = x......(i)$$

Replacing x by $\dfrac{1}{x}$ we get

$$2f\left(\frac{1}{x}-1\right) - f\left(\frac{1-\frac{1}{x}}{\frac{1}{x}}\right) = \frac{1}{x}$$

$$\Rightarrow 2f\left(\frac{1-x}{x}\right) - f(x-1) = \frac{1}{x} \quad .....(ii)$$

On multiplying by 2 in Eq. (i) and then adding Eqs. (i) and (ii) we get

$$2\left[2f(x-1) - f\left(\frac{1-x}{x}\right)\right] + \left[2f\left(\frac{1-x}{x}\right) - f(x-1)\right]$$

$$= 2x + \frac{1}{x} \Rightarrow 3f(x-1) = 2x + \frac{1}{x}$$

Now, replacing x by x + 1, we get [to generate f(x)]

$$3f[(x+1)-1] = 2(x+1) + \frac{1}{(x+1)}$$

$$\therefore f(x) = \frac{1}{3}\left(2(x+1) + \frac{1}{(x+1)}\right)$$

$$= \frac{2(1+x)^2+1}{3(1+x)}$$

4. (A)

$$f(x) = \int_0^x \frac{1}{f(t)}dt \Rightarrow f'(x) = \frac{1}{f(x)}$$

Now, $\dfrac{dy}{dx} = \dfrac{1}{y} \Rightarrow \int y dy = \int 1 dx$ (by separating the variables)

$$\frac{y^2}{2} = x + C \qquad ........(i)$$

Now, $f(1) = \int_0^1 \dfrac{1}{f(t)}dt = \sqrt{2}$

$$\therefore \frac{\{f(1)\}^2}{2} = 1 + C \qquad \text{[From Eq (i)]}$$

$$\Rightarrow \frac{\left(\sqrt{2}\right)^2}{2} = 1 + C \Rightarrow C = 1 - 1 = 0$$

So $\{f(x)\}^2 = 2x \Rightarrow f(x) = \sqrt{2}x$

So $f(200) = \sqrt{2.200} = \sqrt{400} = 20$

5. (A, B, C, D)

(a) We know that, the plane ax + by + cz + d = 0 contains the line $\frac{x-\alpha}{l} = \frac{y-\beta}{m} = \frac{z-\gamma}{n}$ if $a\alpha + b + c\gamma + d = 0$ and $al + bm + cn = 0$

Now, since

$(-1) - 2(3) + 7(-2) + 21 = 0$ and (-3)(1)+2(-2)+1(7) =0

The line given in (a) lies on the given plane

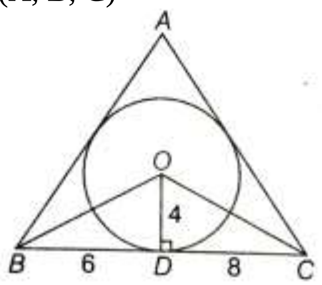
(b) Since 0 – 2(7) + 7(-1) + 21 = 0

$\therefore$ The point (0, 7, -1) lies on the plane

(c) Direction ratios of the normal to the given plane are (1, -2, 7) which are same as those of the given in (c). So the plane is perpendicular to the line.

(d) The direction ratios of the normal to the planes given in (d) are same as those of the given plane. So, the plane in (d) is parallel to the given plane.

6. (A, B, C)




And $BD = 6$, DC = 8

$$\therefore \tan\frac{B}{2} = \frac{OD}{BD} = \frac{4}{6} = \frac{2}{3}$$

$$\text{and } \tan\frac{C}{2} = \Delta\frac{OD}{DC} = \frac{4}{8} = \frac{1}{2}$$

$$\tan\left(\frac{\pi}{2} - \frac{A}{2}\right) = \tan\left(\frac{B}{2} + \frac{C}{2}\right) = \frac{\tan\frac{B}{2} + \tan\frac{C}{2}}{1 - \tan\frac{B}{2}\tan\frac{C}{2}}$$

$$\Rightarrow \cot\frac{A}{2} = \frac{\frac{2}{3} + \frac{1}{2}}{1 - \frac{2}{3} \times \frac{1}{2}} = \frac{7}{4}$$

$$\Rightarrow \tan\frac{A}{2} = \frac{4}{7}$$

$$\Rightarrow \tan\frac{B}{2}\tan\frac{C}{2} = \frac{s-a}{s}$$

$\Rightarrow \frac{s-1}{s} = \frac{1}{3} \Rightarrow 2s = 3a$

$\Rightarrow 2s = 42 \Rightarrow s = 21$

$\therefore \Delta = rs \Rightarrow \Delta = 4 \times 21 = 84 \text{sq cm}$

$\because \tan\frac{A}{2}, \tan\frac{B}{2}, \tan\frac{C}{2}$ all less than 1

$\therefore \Delta$ is acute angled

7. (B, C)

8. (A, C)

Given, plane $P_1$ contains the line $r = \hat{i} + \hat{j} + \hat{k} + \lambda(\hat{i} - \hat{j} - \hat{k})$

$\therefore$ It contains the point $\hat{i} + \hat{j} + \hat{k}$ and is normal to vector $(\hat{i} + \hat{j})$

Hence, equation of plane $P_1$ is

$(r - (i + j + k)).(i + j) = 0$ or x + y = 2

Or

Plane $P_2$ contains the line $r = \hat{i} + \hat{j} + \hat{k} + \lambda(\hat{i} - \hat{j} - \hat{k})$ and the point $\hat{j}$

$\therefore$ The equation of plane is

$$\begin{vmatrix} x-0 & y-1 & z-0 \\ 1-0 & 1-1 & 1-0 \\ 1 & -1 & -1 \end{vmatrix} = 0$$

Or x + 2y – z = 2

If $\theta$ is the acute angle between $P_1$ and $P_2$ then

$$\cos\theta = \frac{n_1.n_2}{|n_1|.|n_2|}$$

$$= \frac{(\hat{i} + \hat{j}).(\hat{i} + 2\hat{j} - \hat{k})}{\sqrt{2}\sqrt{6}}$$

$$= \frac{3}{\sqrt{2}\sqrt{6}} = \frac{\sqrt{3}}{2}$$

$$\therefore \theta = \cos^{-1}\frac{\sqrt{3}}{2} = \frac{\pi}{6}$$

As, L is contained in $P_2, \theta = 0^o$

9. (1)

$$y = \lim_{x \to \infty} \left( x + \cfrac{x}{\sqrt[3]{x} \over x + \cfrac{\sqrt[3]{x}}{x + \cfrac{\sqrt[3]{x}}{....\text{inf inity}}}} \right)$$

$$= \frac{x}{x + \frac{1}{x^{2/3}} \cdot \frac{\sqrt[3]{x}}{x + \frac{\sqrt[3]{x}}{....\text{inf inity}}}}$$

$$= \frac{x}{x + \frac{1}{x^{2/3}}.y} \Rightarrow y = \frac{x^{5/3}}{x^{5/3} + y}$$

$$\Rightarrow y^2 + (x^{5/3})y - x^{5/3} = 0$$

$$\Rightarrow y = \frac{-(x^{5/3}) \pm \sqrt{x^{10/3} + 4.x^{5/3}}}{2}$$

$$= -\frac{x^{5/3} + \sqrt{x^{10/3} + 4x^{5/3}}}{2} \quad [\because y > 0]$$

$$\Rightarrow y = \frac{4x^{5/3}}{2\left(x^{5/3} + \sqrt{x^{10/3} + 4x^{5/3}}\right)} \quad \text{[by rationalising]}$$

$$\Rightarrow y = \frac{2}{\left(1 + \sqrt{1 + 4x^{-5/3}}\right)}$$

Now, $\lim\limits_{x \to \infty} y = \lim\limits_{x \to \infty} \dfrac{2}{\left(1 + \sqrt{1 + 4x^{-5/3}}\right)}$

$$\frac{2}{1+1} = 1$$

10. (7)

Let $I = \int_0^1 (1 - x^4)^7 dx$. Then $I = \int_0^1 (1 - x^4)^7 \cdot 1\, dx$

$$= [(1 - x^4)^7 .x - \int 7(1 - x^4)^6 (-4x^3).xdx]_0^1$$

$$= [(1 - x^4)^7 .x]_0^1 + 28\int_0^1 x^4 (1 - x^4)^6 dx$$

$$= 0 - 28\int_0^1 (1 - x^4 - 1)(1 - x^4)^6 dx$$

$$= -28\int_0^1 (1 - x^4)^7 + 28\int_0^1 (1 - x^4)^6 dx$$

$$=29I=28\int_0^1 (1 - x^4)^6 dx$$

$$\Rightarrow \frac{29\int_0^1 (1-x^4)^7 dx}{4\int_0^1 (1-x^4)^6 dx}$$

11. (2)

We know, $(1+x)^{100} = {}^{100}C_0 x^{100} + {}^{100}C_1 x^{99} + .... + {}^{100}C_{100}$

And $(1+x)^{100} = {}^{100}C_0 + {}^{100}C_1 x + {}^{100}C_2 x_2 + .... + {}^{100}C_{100} x^{100}$

$\Rightarrow (1+x)^{200} = ({}^{100}C_0 x^{100} + {}^{100}C_1 x^{99} + {}^{100}C_2 x^{98} + .... + {}^{100}C_{100})$

$({}^{100}C_0 + {}^{100}C_1 x + {}^{100}C_2 x^2 + {}^{100}C_{100} x^{100})$

$\Rightarrow {}^{200}C_{102} = {}^{100}C_0 . {}^{100}C_2 + {}^{100}C_1 . {}^{100}C_3 + {}^{100}C_2 . {}^{100}C_4 + .......) = \text{coefficient of } x^{102}$

Let $a = {}^{100}C_0 . {}^{100}C_2 + {}^{100}C_2 . {}^{100}C_4 + .... + {}^{100}C_{98} . {}^{100}C_{100}$

Let $b = {}^{100}C_1 . {}^{100}C_3 + {}^{100}C_3 . {}^{100}C_5 + ... + {}^{100}C_{97} . {}^{100}C_{99}$

Then, we have

$a + b = {}^{200}C_{102} = {}^{200}C_{98}$......(i)

Clearly

$(1+x)^{100} . (1-x)^{100} = ({}^{100}C_0 x^{100} + {}^{100}C_1 x^{99} + .... + {}^{100}C_{100})$

$({}^{100}C_0 - {}^{100}C_1 x + {}^{100}C_2 x^2 - ..... + {}^{100}C_{100} x^{100})$

$\Rightarrow -{}^{100}C_{51} = {}^{100}C_0 . {}^{100}C_2 - {}^{100}C_1 . {}^{100}C_3 + {}^{100}C_2 . {}^{100}C_4 - ... + {}^{100}C_{98} . {}^{100}C_{100}$

$\Rightarrow -{}^{100}C_{49} = a - b$ .....(ii)

On adding Eqs. (i) and (ii) we get

$2a = {}^{200}C_{98} - {}^{100}C_{49}$

$a = \frac{1}{2}\left[{}^{200}C_{98} - {}^{100}C_{49}\right]$

Hence $\lambda = 2$

12. (1)

Given equation of planes are

x – cy – bz = 0 ….(i)

cx – y + az = 0 ….(ii)

and bx + ay – z = 0 …..(iii)

Now, equation of plane passing through the line of intersection of planes (i) and (ii) may be takes as

$(x - cy - bz) + \lambda(cx - y + az) = 0$

i.e. $(1+\lambda c)x + (-c-\lambda)y + (-b + a\lambda)z = 0$ ……(iv)

Clearly, the planes (ii) and (iv) are same

$\therefore \frac{1+c\lambda}{b} = \frac{(c+\lambda)}{a} = \frac{-b+a\lambda}{-1}$

By eliminating $\lambda$ we get

$a^2 + b^2 + c^2 + 2abc = 1$

13. (8)

We have $[|x|] + [|y|] = 1$

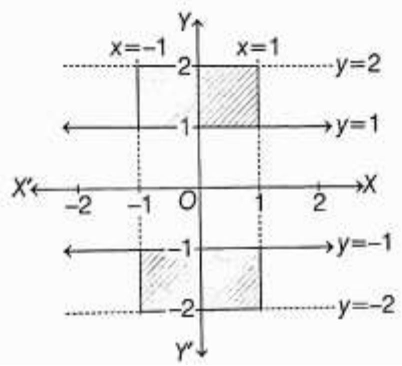
This is possible, when $[|x|]= 0$ and $[|y|] = 1$ or $[|x|]= 1$ and $[|y|] = 1$ or $[|x|]$ and $[|y|] = 0$ $\left[\because [|x|] \text{ and } [|y|] \text{ are integers}\right]$

**Case I** When $[|x|] =0$ and $[|y|] = 1$

Then, $0 \le |x| < 1$ and $1 \le |y| < 2$

$\Rightarrow |x| < 1$ and $1 \le |y| < 2$

$\Rightarrow x \in (-1,1)$ and $y \in (-2,-1) \cup [1,2)$




**Case II** when $[|x|] = 1$ and $[|y|] = 0$

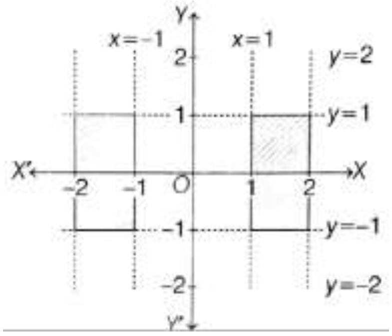
Then, $1 \le |x| < 2$ and $0 \le |y| < 1$

$\Rightarrow x \in (-2,-1) \cup [1,2)$ and $y \in (-1,1)$

Thus, we have the following graph

Hence, are of required region

$= 4(2 - 1)\{1 - (-1)\} = 8$ sq. units..




14. (7)

We have $a = x\hat{i} + y\hat{j} + z\hat{k}$

$b = \hat{i} - 2\hat{j} + 3\hat{k}$

$c = 2\hat{i} + 3\hat{j} - \hat{k}$

and $d = \hat{i} - \hat{j} + \hat{k}$

a makes equal $\theta$ with b and c

$\therefore a = \alpha(b + c) + \beta(b \times c)$

Here, $b + c = \hat{i} - 2\hat{j} + 3\hat{k} + 2\hat{i} + 3\hat{j} - \hat{k} = 3\hat{i} + \hat{j} + 2\hat{k}$ and $b \times c = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & 3 \\ 2 & 3 & -1 \end{vmatrix} = -7\hat{i} + 7\hat{j} + 7\hat{k}$

$\therefore a = \alpha(3\hat{i} + \hat{j} + 2\hat{k}) + \beta'(-\hat{i} + \hat{j} + \hat{k})$

Where $\beta' = 7\beta$

$\because$ a is perpendicular to d

$\therefore a.d = 0 \Rightarrow (\alpha(3\hat{i}+\hat{j}+2\hat{k})+\beta'(-\hat{i}+\hat{j}+\hat{k})).(\hat{i}-\hat{j}+\hat{k})=0$

$\alpha(3-1+2)+\beta'(-1-1+1)=0$

$4\alpha-\beta'=0 \Rightarrow 4\alpha=\beta'$

$\therefore a=\alpha(3\hat{i}+\hat{j}+2\hat{k})+4\alpha(-\hat{i}+\hat{j}+\hat{k})$

$a=\alpha(3\hat{i}+\hat{j}+2\hat{k}-4\hat{i}+4\hat{j}+4\hat{k})$

$=\alpha(-\hat{i}+5\hat{j}+6\hat{k})$

a.b = $|a||b|\cos\theta$

$=\alpha(-\hat{i}+5\hat{j}+6\hat{k})||(\hat{i}-2\hat{j}+3\hat{k})|\cos\theta$

$=\alpha\sqrt{(1)^2+(5)^2+(6)^2}.\sqrt{(1)^2+(2)^2+(3)^2}\cos\theta$

7= $\left(\sqrt{1+25+36}\right)\left(\sqrt{1+4+9}\right)$

$\cos^2\frac{7}{124}=124\cos^2\theta=7$

15. (0)

Let I = $\int_{1/3}^{3}\frac{1}{x}\operatorname{cos ec}\left(x-\frac{1}{x}\right)dx$

Put $x-\frac{1}{x}=t \Rightarrow \left(1+\frac{1}{x^2}\right)dx=dt$

$\left(\frac{x^2+1}{x}\right)\frac{dx}{x}=dt \Rightarrow \left(x+\frac{1}{x}\right)\frac{dx}{x}=dt$

$\Rightarrow \sqrt{\left(x-\frac{1}{x}\right)^2+2}\,\frac{dx}{x}=dt$

$\Rightarrow \frac{dx}{x}=\frac{dt}{\sqrt{t^2+2}}$

Where $x=\frac{1}{3}, t=-\frac{8}{3}$ and $x=3, t=\frac{8}{3}$

$\therefore I=\int_{-8/3}^{8/3}\frac{\operatorname{cos ec} t}{\sqrt{t^2+2}}dt$

I= 0 [$\because$ the integral is an odd function)

16. (1)

We have

f(x) + f(y)= $f\left(\frac{x+y}{1-xy}\right)$ …..(i)

Putting x = y = 0 we get f(0) = 0

Putting y = -x, we get f(x) + f(-x) = f(0) = 0

f(x) + f(-x) = f(0) = 0

$\Rightarrow f(-x)=-f(x)$

Clearly $f'(x)=\lim\limits_{n\to 0}\frac{f(x+h)-f(x)}{h}$

$\lim\limits_{h\to 0}\frac{f(x+h)+f(-x)}{h}$ [Using Eq. (ii)]

$=\lim\limits_{h\to 0}\frac{f\left(\frac{x+h-x}{1-(x+h)(-x)}\right)}{h}$ [Using Eq. (i)]

$=\lim\limits_{h\to 0}\frac{f\left(\frac{h}{1+x(x+h)}\right)}{h}$

$=\lim\limits_{h\to 0}\frac{f\left(\frac{h}{1+x(x+h)}\right)}{\frac{h}{1+x(x+h)}\times(1+x(x+h))}$

$=\lim\limits_{h\to 0}\frac{f\left(\frac{h}{1+x(x+h)}\right)}{\frac{h}{1+x(x+h)}}.\lim\limits_{n\to 0}\frac{1}{1+x(x+h)}$

$=2\times\frac{1}{1+x^2}$

$\Rightarrow f'(x)=\frac{2}{1+x^2}$

$\therefore f'(1)=\frac{2}{2}=1$

17. (5)

We have

$r=(a\times b)\sin x+(b\times c)\cos y+2(c\times a)$

$\Rightarrow r.a=[b\,c\,a]\cos y$

$\Rightarrow r.b=2[c\ a\ b]$

$\Rightarrow r.c=(a\ b\ c)\sin x$

Given $r.(a+b+c)=0$

$\therefore [a\,b\,c]\left(\cos y+2+\sin x\right)=0$

Since a, b, c are non-zero and non-coplanar

$\therefore$ sinx + osy + 2 = 0

$\Rightarrow \sin x+\cos y=-2$

It is possible when sinx = cosy = -1

$\therefore x=-\frac{\pi}{2}, y=\pi$

$\therefore$ Minimum value of

$\frac{4}{\pi^2}(x^2+y^2)=\frac{4}{\pi^2}\left(\frac{\pi^2}{4}+\pi^2\right)$

$$= \frac{4}{\pi^2} \times \frac{5\pi^2}{4} = 5$$

18. (7)

$$P_n = \frac{3!(3n-3)!(n!)^3}{((n-1)!)^3(3n)!}$$

$$= \lim_{x\to\infty} \frac{6(3n-3)!(n)^3((n-1)!)^3}{((n-1)!)^3 3n(3n-1)(3n-2)(3n-3)!}$$

$$\Rightarrow \lim_{x\to\infty} \frac{6n^3}{3n(3n-1)(3n-2)} = \frac{2}{9}$$

$$\therefore |m-n| = |2-9| = 7$$

## PART (A) : PHYSICS

1. (6.67)

Thermal current in the rod, $l = \frac{kA\Delta T}{l}$ where, k is thermal conductivity , A is cross-sectional area, $\Delta T$ is temperature difference and $l$ is length of rod.

$$\Rightarrow l = \frac{80\times10\times10\times10^{-4}\times100}{20\times10^{-2}}$$

$$= 400\text{cal}/\text{s}$$

Heat extracted from water in time t is $l\times t$ which can be equated to heat required to freeze m mass of water, i.e. mL, where L is latent heat. So, we get

$$l\times t = m\times L$$

$\therefore$ Mass of water in the vessel,

$$m = \frac{lt}{L} = \frac{400\times10}{80} = 50g$$

Now, velocity of efflux of water coming out of tank,

$$v = \sqrt{2gh}$$

$$= \sqrt{2\times10\times0.2}$$

$$= 2m/s$$

Mass flow rate of water coming out of tank,

$$\frac{dm}{dt} = \rho av = 10^3\times10^{-6}\times2$$

$$= 2\times10^{-3}kg/s = 2g/s$$

Thus, mass of water in the tank after time t will be (as 50 g was already there)

$$m' = (50+2t)g$$

It time taken to freeze is t', then we have

$$l\times t' = m'\times L$$

$$\Rightarrow 400\times t' = (50+2t')\times80$$

$$\Rightarrow t' = \frac{50}{3}s$$

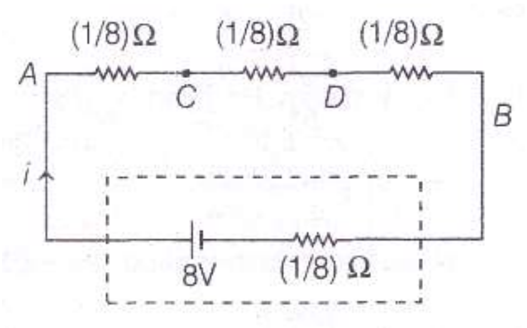
$\therefore$ Time of delay,

$$= t'-t = \frac{50}{3}-10$$

$$= 667s$$

2. (96)

By symmetry all points of octagons vertex are at same potential. So resistors connecting adjacent vertices with not have any current and so these sixteen resistors can be removed. Now, let vertices of octagon with centre at A be labelled C. So, 8 resistors between A and C will be in parallel. Similarly, if we label vertices of octagon with centre at B be D, 8 resistors between B and D will be in parallel and 8 resistors between C and D will also be in parallel. So, the equivalent circuit can be drawn as




Resistance of external circuit,

$$R = \frac{1}{8} + \frac{1}{8} + \frac{1}{8} = \frac{3}{8}\Omega$$

Effective resistance of the circuit,

$$Z = R + \frac{1}{8} = 0.5\Omega$$

Therefore, current in the circuit will be

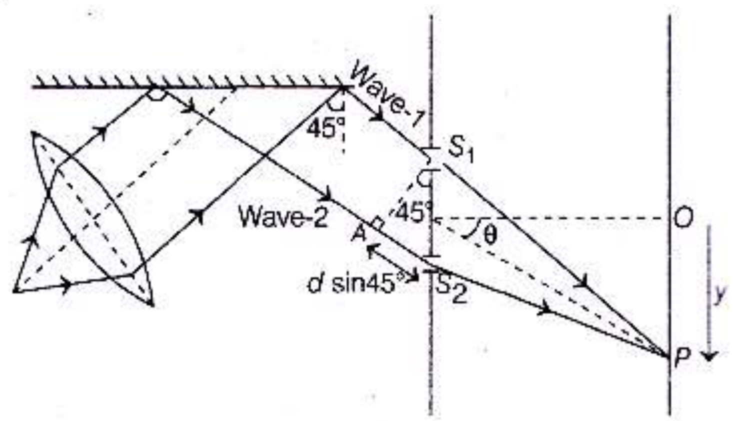
$$i = \frac{V}{Z} = \frac{8}{0.5} = 16A$$

$\therefore$ Power consumed by the circuit will be

$$P = i^2R = (16)^2 \times \frac{3}{8} = 96W$$

3. (1.50)

Consider interference of Wave-i and Wave-2 at any general point of the screen.




For journey behind the slits, extra path of Wave-2 can be determined by drawing a perpendicular from $S_1$ at A as shown in figure. Extra path is equal to

$$\Delta x_1 = AS_2 = d\sin 45^\circ \qquad \ldots\ldots(i)$$

For journey from slits to screen, at a point at angle $\theta$ with respect to line of symmetry, path difference is equal to

$$\Delta x_2 = d\sin\theta \qquad \ldots\ldots.(ii)$$

For central maxima, net path difference is zero

$$\Rightarrow \Delta x_1 = \Delta x_2$$

From Eqs. (i) and (ii), we get

$$d\sin 45^\circ = d\sin\theta$$

$$\Rightarrow \theta = 45^\circ$$

$\therefore$ Distance of central maxima from O,

$y = 1.5 \tan 45^{\circ} = 1.5 \times 1 = 1.5\,\text{m}$

4. (45)

Velocities of points Q and R of the string w.r.t. ground are as shown in Figure 1

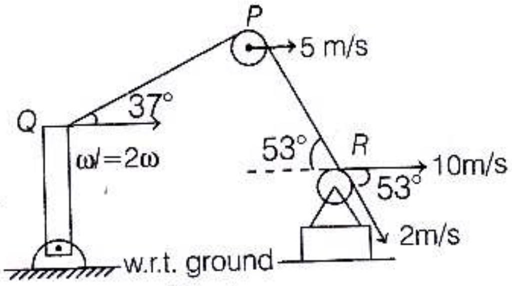



Fig. 1

With respect to pulley P, points Q and R will have an additional negative velocity of 5 m/s as shown in Fig. 2.

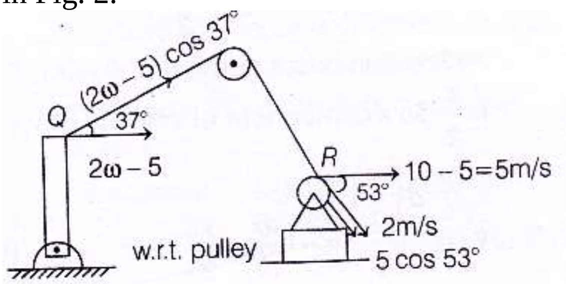



Fig. 2

Thus, velocity components of Q and R along string will be as shown in Fig 2. As the string is inextensible, these components should be equal.

$$(2\omega - 5)\cos 37^{\circ} = 5\cos 53^{\circ} + 2$$

$$\Rightarrow (2\omega - 5)\frac{4}{5} = 5 \times \frac{3}{5} + 2$$

$$\Rightarrow 2\omega - 5 = \frac{25}{4} \Rightarrow \omega = 562\,\text{rad/s}$$

5. (2.50)

FBD of B and C are as shown in the Figure. 1

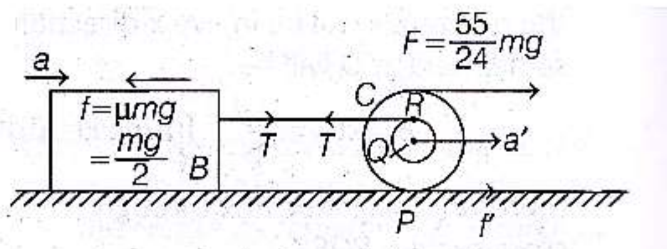



Fig.1

Assuming motion of spool as pure rotation about instantaneous axis of rotation p, accelerations of point Q and R are

$a_Q = a' = \alpha(2r)$ ……(i)

$a_R = a = \alpha(3r)$ …….(ii)

(as R is connected to B, they have same acceleration)
Dividing Eq. (i) by Eq. (ii), we get

$$a' = \frac{2}{3}a \qquad \text{.......(iii)}$$

Using , $\tau = I\alpha$ for spool about instantaneous axis P, we get

$$\frac{55}{24}mg(4r) - T(3r) = \left[mr^2 + m(2r)^2\right]\frac{a'}{2r}$$

$$\Rightarrow \frac{55}{6}mgr - 3Tr = 5mr^2 \times \left(\frac{a}{3r}\right) \qquad \text{[ using Eq. (iii)]}$$

$$\Rightarrow \frac{55}{6}mg - 3T = \frac{5ma}{3} \qquad \text{....(iv)}$$

For block B, using $F_{net} = ma$ , we get

$$T - \frac{mg}{2} = 2ma \qquad \text{....(v)}$$

Dividing Eq. (iv) by Eq. (v), we get

$$2\left(\frac{55}{6}mg - 3T\right) = \frac{5}{3}\left(T - \frac{mg}{2}\right)$$

$$\Rightarrow \frac{23T}{3} = \frac{115mg}{6}$$

$$\Rightarrow T = \frac{5mg}{2} \Rightarrow x = 2.50$$

6. (0.75)
Let us find Q, for all steps.
As, AB is adiabatic,

$$Q_{AB} = 0 \qquad \text{......(i)}$$

As, graph of BC is straight line passing through origin,

$p \propto T$

$\Rightarrow V = \text{Constant(isochoric process)}$

$\Rightarrow Q_{BC} = \Delta U + W$

$= \frac{f}{2}nR\Delta T + 0$

$$\Rightarrow Q_{BC} = \frac{f}{2}nR(T_C - T_B) \qquad \text{.....(ii)}$$

As, CD is adiabatic ,

$$Q_{CD} = 0 \qquad \text{.......(iii)}$$

As, graph of DA is straight line passing through origin.

$p \propto T$

$\Rightarrow V =$ constant (isochoric process)

$\Rightarrow Q_{DA} = \Delta U + W$

$= \frac{f}{2}nR\Delta T$

$$\Rightarrow Q_{DA} = \frac{f}{2}nR(T_A - T_D) \qquad \text{.....(iv)}$$

For the cycle ABCDA, work done by gas,

$W = Q - \Delta U = Q$

(As, $\Delta U$ is zero for cycle)

$\Rightarrow W = Q_{AB} + Q_{BC} + Q_{CD} + Q_{DA}$

$\Rightarrow W = \frac{f}{2} nR\left[\left(T_C - T_B\right) + \left(T_A - T_D\right)\right]$......(v)

[using Eqs. (i), (ii), (iii) and(iv)]

From the given diagram,

$T_C > T_B$

While $T_A < T_B$, therefore we get

$Q_{BC} > 0$ [using Eq. (ii)]

$Q_{DA} < 0$ [using Eq. (iv)]

As, heat supplied during cycle is sum of positive values of Q of steps, therefore

$Q = Q_{BC} = \frac{f}{2} nR\left(T_C - T_B\right)$ ......(vi)

Dividing Eq. (v) by Eq. (vi),we get

$\eta = \frac{W}{Q} = 1 + \frac{T_A - T_D}{T_C - T_B}$ ......(vii)

As, $T_B = T_A\left(\frac{V_A}{V_B}\right)^{2/5}$

$= T_A\left(32\right)^{2/5} = 4T_A$ .......(viii)

Similarly for CD, we get

$T_C = 4T_D$ ......(ix)

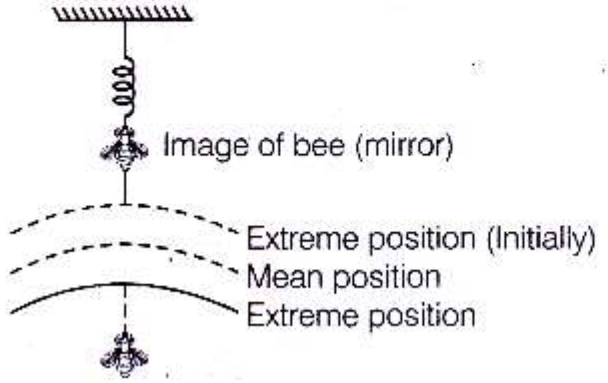
Putting values of $T_B$ and $T_C$, from Eqs. (viii) and (ix) into Eq. (vii), we get

$\eta = 1 + \frac{T_A - T_D}{4T_D - 4T_A}$

$= 1 - \frac{1}{4} = \frac{3}{4} = 0.75$

7. (54.00)

The image formation is as shown below




At mean position, $ka = mg$

$\Rightarrow$ Amplitude of oscillation of mirror

$a = \frac{mg}{k} = \frac{10^{-1} \times 10}{10^3} = 1\text{mm}$ .....(i)

Now, for bee, image position is given by

$$\frac{1}{v}+\frac{1}{u}=\frac{1}{f}\Rightarrow\frac{1}{v}+\frac{1}{-50}=\frac{1}{-100}$$

$$\Rightarrow v=1m$$

Now, to relate small displacement of object (du) and for small displacement of image (dv), lets differentiate mirror formula w.r.t.u

$$\Rightarrow\left(-\frac{1}{v^2}\right)\frac{dv}{du}-\frac{1}{u^2}=0$$

$$\Rightarrow dv=-\left(\frac{v}{u}\right)^2 du \qquad ....(ii)$$

Displacement of bee's image w.r.t. mirror when it moves from upper extreme to mean position,

$$dv=-\left(\frac{1}{-0.5}\right)(1)$$

[as, $du=A$ using Eq. (i)]

Amplitude of oscillation of bee's image,

$amm=4mm\Rightarrow a=4$ ......(iii)

Time period of oscillation of mirror and bee has to be equal, as they reach mean and extreme simultaneously,

$$\Rightarrow T=2\pi\sqrt{\frac{m}{k}}$$

$$=2\pi\sqrt{\frac{0.1}{10^3}}=\frac{\pi}{50}s$$

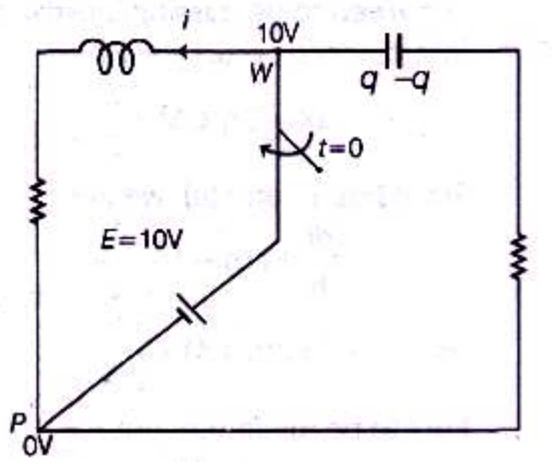
$$=\frac{\pi}{b} \qquad \text{(given)}$$

$b=50$ ......(iv)

Using Eqs. (iii) and (iv), we get

$a+b=50+4=54$

8 (0.69)




If potential of P is assumed 0 V, after the key is closed, potential of point W will be W=10 V. In branch with inductor and resistor, current i will grow as per the function

$$i=\frac{E}{R}\left(1-e^{-\frac{Rt}{L}}\right)$$

Potential difference across inductor

$$V_1 = L\frac{di}{dt} = L.\frac{E}{R}.\frac{R}{L}e^{-\frac{Rt}{L}}$$

$$\Rightarrow V_1 = Ee^{-\frac{Rt}{L}} \quad .......(i)$$

Charge on capacitor grows as per function.

$$q = CE\left(1-e^{-t/RC}\right)$$

So, potential difference across capacitor,

$$V_2 = \frac{q}{C} = E\left(1-e^{-t/RC}\right) \quad ......(ii)$$

When voltmeters read the same, $V_1 = V_2$

$$\Rightarrow Ee^{-\frac{Rt}{L}} = E\left(1-e^{-t/RC}\right)$$

$$\Rightarrow e^{-t} = 1-e^{-t}$$

$$(\because R = 1\Omega, L = 1H, C = 1F)$$

$$\Rightarrow e^{t} = 2$$

$$\Rightarrow t = \ln 2 = 0.69$$

9. (A, B)

Consider the figure shown, as distance between $C_1$ and $C_2$ is $R, \Delta C_1pC_2$ is equilateral.

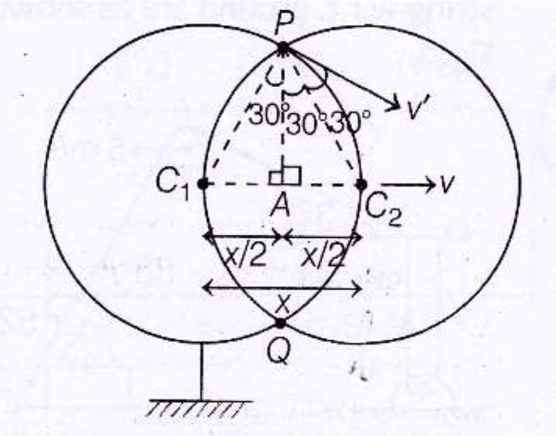



Fig.1

Let x be position of $C_2$ w.r.t. $C_1$. As $\Delta PC_1A \simeq \Delta PC_2A$, we have

$$C_1A = C_2A = \frac{x}{2}$$

$\Rightarrow$ Position (x-coordinate) of P w.r.t. $C_1$ is $\frac{x}{2}$, So x component of velocity of P,

$$v_x = \frac{d\left(\frac{x}{2}\right)}{dt} = \frac{1}{2}.\frac{dx}{dt} = \frac{v}{2} \quad ......(i)$$

Let velocity of P (which is tangential to Ring-1) is v'. Frim Fig 1, x component of velocity of p,

$$v_x = v'\sin 60^\circ \quad .......(iii)$$

From Eqs. (i) and (ii), we get

$$v'\sin 60^\circ = \frac{v}{2} \Rightarrow v' = \frac{v}{\sqrt{3}}$$

Angular velocity w.r.t. $C_1 = \frac{v'}{R} = \frac{v}{\sqrt{3}R}$

Motional emf between points P and Q (in both smaller and larger arcs) of Ring-2 will be

$$E = Bv(PQ) = Bv\left(\sqrt{3}R\right)$$

Also, resistance of small and larger arcs will be in ratio of lengths/angle subtended at centre, that will be $\frac{r}{3}$ and $\frac{2r}{3}$, respectively. As Ring-1 is at rest, no motional emf is there in the ring while resistance are as shown in the figure 2.

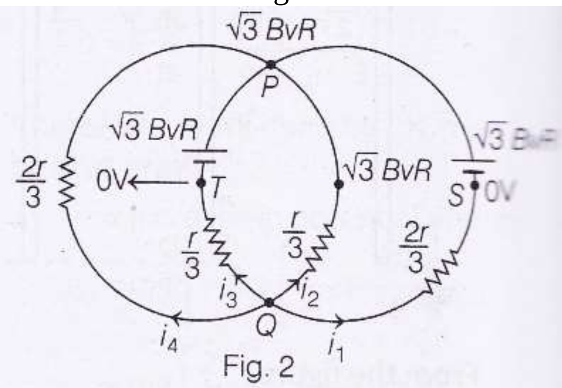


Fig. 2

Assuming potential of S equal to zero, potentials of other points are as shown. Potential of Q is assumed equal to x.

By KCL for junction at Q,

$$i_1 + i_2 + i_3 + i_4 = 0$$

$$\Rightarrow \frac{x}{\frac{2r}{3}} + \frac{x}{\frac{r}{3}} + \frac{x - \sqrt{3}BvR}{\frac{r}{3}} + \frac{x - \sqrt{3}BvR}{\frac{2r}{3}} = 0$$

$$\Rightarrow x = \frac{\sqrt{3}BvR}{2}$$

Current in various branches will therefore be as shown in Figure 3

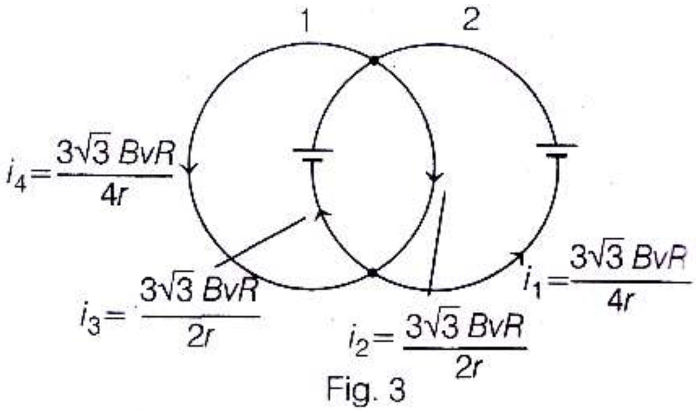


Fig. 3

Force on smaller arc of Ring-2 will be

$$F_1 = Bil = Bi_3\sqrt{3}R \text{ (leftwards)}$$

Force on larger arc or Ring-2,

$$F_2 = Bil = Bi_1\sqrt{3}R \text{ (leftwards)}$$

S, net force on Ring-2,

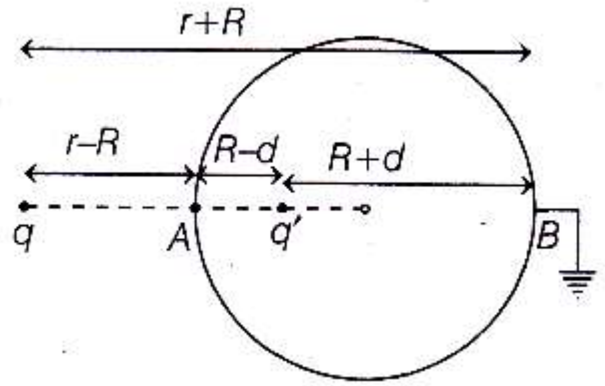
$$F = F_1 + F_2 = B(i_1 + i_3)\sqrt{3}R$$

$$= B\left(\frac{9\sqrt{3}BvR}{4r}\right)\sqrt{3}R = \frac{27B^2R^2v}{4r}$$

Similarly, net force on Ring-1 is non-zero.

10 (A, C)

As the sphere is grounded, potential at any point on its surface will be zero.




Potential at $A = 0$

$$\Rightarrow \frac{kq}{r-R} + \frac{kq'}{R-d} = 0$$

$$\Rightarrow \frac{q}{q'} = -\left(\frac{r-R}{R-d}\right) \qquad .....(i)$$

Also, potential at diametrically opposite point , (point B)=0

$$\Rightarrow \frac{kq}{r+R} + \frac{kq'}{R+d} = 0$$

$$\Rightarrow \frac{q}{q'} = -\left(\frac{r+R}{R+d}\right) \qquad ........(ii)$$

From Eqs. (i) and (ii), we get

$$\frac{r-R}{R-d} = \frac{r+R}{R+d}$$

$$\Rightarrow (r-R)(R+d) = (r+R)(R-d)$$

$$\Rightarrow rR + rd - R^2 - Rd = rR - rd + R^2 - Rd$$

$$\Rightarrow d = \frac{R^2}{r}$$

Putting this value of d in Eq. (i), we get

$$q' = -q\left(\frac{R-d}{r-R}\right)$$

$$= -q\left[\frac{R - \frac{R^2}{r}}{r-R}\right] = -\frac{qR}{r}$$

11. (A, B, C, D)

The circuit can be realised as shown below in figure.

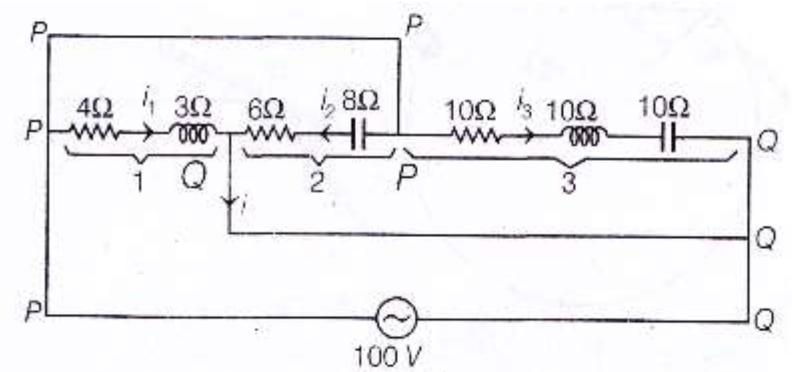



Fig. 1

To find impedance Z of each branch, rms current through them and lead of voltage in terms of phase, we can use the following formulae

$$Z = \sqrt{(X_L - X_C)^2 + R^2}$$

$$I = \frac{V}{Z}$$

$$\phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right)$$

So, branch-1,

$$Z_1 = \sqrt{(3-0)^2 + 4^2} = 5\Omega$$

$$I_1 = \frac{100}{5} = 20A$$

$$\phi_1 = \tan^{-1}\left(\frac{3}{4}\right) = 37^\circ$$

For branch-2,

$$Z_2 = \sqrt{(0-8)^2 + 6^2} = 10\Omega$$

$$I_2 = \frac{100}{10} = 10A$$

$$\phi_2 = \tan^{-1}\frac{-8}{6} = -53^\circ$$

For branch-3

$$Z_3 = \sqrt{(10-10)^2 + 10^2} = 10\Omega$$

$$I_3 = \frac{100}{10} = 10A$$

$$\phi_3 = \tan^{-1}\left(\frac{10-10}{10}\right) = 0$$

Thus, phasor diagram representing various phasors will be as shown. To find net current, resolve components of current and add as vectors.

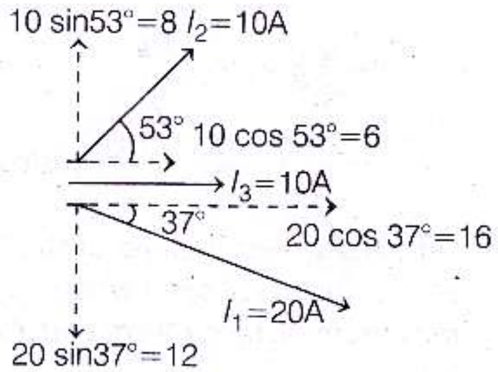



Fig. 2

Net current will therefore be

$$I=\sqrt{(10+6+16)^2+(12-8)^2}$$

$$=\sqrt{1040}\text{A}$$

Impedance of circuit,

$$Z=\frac{V}{I}=\frac{100}{\sqrt{1040}}\Omega$$

So, option (B) is correct.

As from the figure, current in branch of AC ammeter is sum of currents in branch-1 and branch-2.

$$\therefore I=\sqrt{I_1^2+I_2^2}$$

$$=\sqrt{20^2+10^2}$$

$$=10\sqrt{5}\text{A}$$

So, option (D) is correct

As DC ammeter reads average value reading, so DC ammeter reads zero. Also, readings of Voltmeter-1 and Voltmeter-2 both are equal to $V_P - V_Q$ or voltage of AC source , i.e. equal to 100V.

12. (B, C)

Force on a dipole in non-uniform field is given by $p\frac{dE}{dx}$. So, force on dipole -2, due to dipole-1,

$$F=-p_2\frac{d\left[\frac{2cp_1}{r^3}\right]}{dr}=\frac{6c\,p_1p_2}{r^4}$$

Here, $c=\frac{1}{4\pi\varepsilon_0}$

In equilibrium, F is balanced by spring force kr.

Thus, $kr=\frac{6cp_1p_2}{r^4}$

$$\Rightarrow k=\frac{6c\,p_1p_2}{r^5} \qquad \text{.......(i)}$$

So, option (B) is correct.

Electrostatic potential energy of a dipole is given by

$U=-pE\cos\theta$

So, potential energy of dipole-2 in field of dipole-1, $U=-p_2.\frac{2cp_1}{r^3}.\cos180^\circ$

$$\frac{2cp_1p_2}{r^3} \qquad .......(iii)$$

Dividing Eq (ii) by Eq. (i), we get

$$\frac{U}{k}=\frac{r^2}{3} \Rightarrow U=\frac{1}{3}kr^2$$

Also, spring potential energy,

$$U'=\frac{1}{2}kr^2$$

$\therefore$ Total potential energy of system is

$$U+U'=\frac{5}{6}kr^2$$

So, option (A) is incorrect.
If the left ball is held and right ball is slightly displaced, change in spring force.

$$dF_1=-kdr \qquad .......(iii)$$

Also, change in electrostatic force,

$$dF_2=6cp_1p_2d\left(\frac{1}{r^4}\right)=6cp_1p_2\left(\frac{-4}{r^5}\right)dr$$

$$=-\frac{24cp_1p_2}{r^5}dr$$

$$\Rightarrow dF_2=-4kdr \qquad \text{[using Eq (i)]} \; .....(iv)$$

From Eqs. (iii) and (iv), we get net change in force,

$$dF=dF_1+dF_2=-5kdr$$

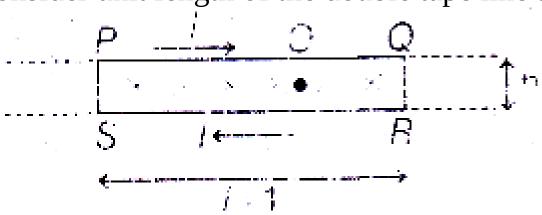
$$\Rightarrow a=\frac{dF}{m}=-\frac{5k}{m}dr$$

As acceleration in SHM is $-\omega^2x$, so on comparison, we get

$$\omega=\sqrt{\frac{5k}{m}}$$

$$\Rightarrow T=2\pi\sqrt{\frac{m}{5k}}$$

13. (A, B, C, D)
Consider unit length of the double tape line as shown in the figure.




As field due to infinite sheet of current in terms of current per unit length, is $\frac{\mu_0\lambda}{2}$, due to upper tape, then magnetic field at any general point O between plates,

$$B_0=\frac{\mu_0 l}{2b} \qquad .....(i)$$

Likewise, magnetic field due to lower tape will also be the same in magnitude. Directions of fields due to the two tapes will be in positive x-direction, so net field at O will be

$B = 2B_0 = \frac{\mu_0 l}{b}$ [using Eq. (i)]

Therefore, magnetic flux through cross-section PQRS,

$\phi = Bh = \frac{\mu_0 lh}{b}$

$\Rightarrow$ Self-inductance per unit length,

$L_0 = \frac{\phi}{l} = \frac{\mu_0 h}{b}$ .....(ii)

Also, capacitance of a parallel plate capacitor is given by $C = \frac{\varepsilon_0 A}{d}$

Where, A is plate area and d is plate separation.
Therefore, capacitance per unit length,

$C_0 = \frac{\varepsilon_0 \times b \times d}{h} = \frac{\varepsilon_0 b}{h}$ .....(iii)

As for an LC circuit, angular frequency of oscillation is given by

$\omega = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{L_0 C_0}}$

$= \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = c$ [using Eqs. (ii) and (iii)]

(As, $\frac{1}{\sqrt{\mu_0 \varepsilon_0}} = c$ as per Maxwell's theory of EM waves)

If $k_1$ is closed, the circuit will be L-C-E circuit in which charge q varies with time t as given by following equation

$q = CE(1 - \cos \omega t)$

$\therefore$ Maximum charge, $q_{max}$.

$= 2CE = 2C_0 E = \frac{2\varepsilon_0 bE}{h}$ [ using Eq. (iii)]

14. (A, D)

For $k_\alpha X -$ rays, by Moseley's law,

$(Z - 1) \propto \frac{1}{\lambda}$

Thus for impurity-1 having Z equal to Z

$\Rightarrow \left(\frac{Z_1 - 1}{Z - 1}\right) = \sqrt{\frac{\lambda_3}{\lambda_1}}$

$\Rightarrow \frac{Z_3 - 1}{Z - 1} = 2 \Rightarrow Z_1 = 2Z - 1$

Similarly, for impurity -2,

$\frac{Z_1 - 1}{Z - 1} = \sqrt{\frac{\lambda_3}{\lambda_2}} = \frac{1}{2}$

$\Rightarrow Z_2 = 0.5(Z + 1)$

15. (A, C)

For steady state,

$$\left(\frac{dQ}{dt}\right)_{in} = \left(\frac{dQ}{dt}\right)_{out}$$

$$\Rightarrow (V)(i_5) = 45(T-20)$$

$$\Rightarrow (500)(4.5) = 45(T-20)$$

$$\Rightarrow T = 70^\circ C$$

Resistance at $20^\circ C, R = \frac{V}{i} = \frac{500}{5}$

$$R_{20} = 100\Omega$$

Resistance at $70^\circ C$,

$$R = \frac{V}{i} = \frac{500}{4.5} \simeq 111\Omega$$

$$\therefore R_f = R_0(1+\alpha\Delta T)$$

$$111 = 100[1+\alpha(50)]$$

$$\Rightarrow \alpha = \frac{0.11}{50} \cong 2.2\times10^{-3} / ^\circ C$$

16. (B, C)

Capacitance of a capacitor with dielectric slabs in series is given by

$$C' = \frac{\varepsilon_0 A}{\frac{d_1}{K_1} + \frac{d_2}{K_2} + \ldots\ldots..}$$

Here, $d_1, d_2$ etc. are thicknesses of slabs while $K_1, K_2$ are respective dielectric constants. Therefore , for any of the capacitors with slab, capacitance would be

$$C' = \frac{\varepsilon_0 A}{\frac{3d}{4\times3} + \frac{d}{4}} = \frac{2\varepsilon_0 A}{d}$$

$= 2C$ (here, $C = \frac{\varepsilon_0 A}{d}$)

Thus arrangement can be visualised as shown in Figure 1

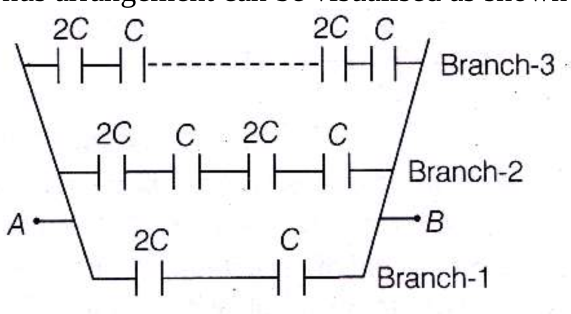



Fig.1

As capacitors in branch-1 are in series , so their equivalent capacitance

$$\frac{1}{C_{eq_1}} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{2C} + \frac{1}{C} = \frac{3}{2C}$$

$\Rightarrow C_{eq_1} = \frac{2C}{3}$ ......(i)

Similarly, for branch-2 and 3,

$C_{eq_2} = \frac{C}{3}$ ......(ii)

and $C_{eq_3} = \frac{C}{6}$ ......(iii)

As, branches -1,2,and 3 are in parallel, so final equivalent capacitance,

$C_{eq} = C_{eq_1} + C_{eq_2} + C_{eq_3}$

$= \frac{2C}{3} + \frac{C}{3} + \frac{C}{6}$

[using , Eqs. (i), (ii) and (iii)]

Or $C_{Ab} = \frac{7C}{6}$ .....(iv)

Before the slabs were inserted , the circuit was as shown in Figure 2

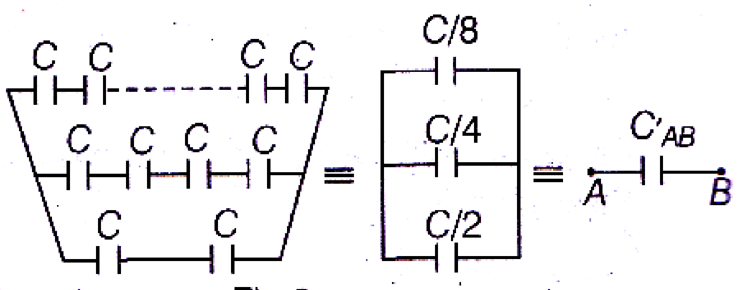



Fig.2

By using formula for parallel equivalent , equivalent capacitance between A and B,

$C'_{AB} = \frac{C}{8} + \frac{C}{4} + \frac{C}{2} = \frac{7C}{8}$ ......(v)

So, charge supplied by cell,

$Q = (C_{AB} - C'_{AB})E$

$= \left(\frac{7C}{6} - \frac{7C}{8}\right)E = \frac{7}{24}CE$

[using Eqs. (iv) and (v)]

$\therefore$ Work done by cell,

$W_a = Q.E = \frac{7E^2}{24}$ .......(vi)

Change in potential energy of the arrangement.

$\Delta U = \frac{1}{2}(C_{AB} - C'_{AB})E^2$

$= \frac{1}{2} \times \frac{7}{24}CE^2 = \frac{7}{48}CE^2$ ......(vii)

Heat dissipated is given by

$H = W - \Delta U$

$= \frac{7CE^2}{24} - \frac{7CE^2}{48} = \frac{7CE^2}{48}$

[using Eqs. (vi) and (vii)]

Consider the lowermost branch which can be seen to the a series combination of $\frac{3d}{4}$ thickness of dielectric and $\frac{5d}{4}$ thickness of air cored capacitor.

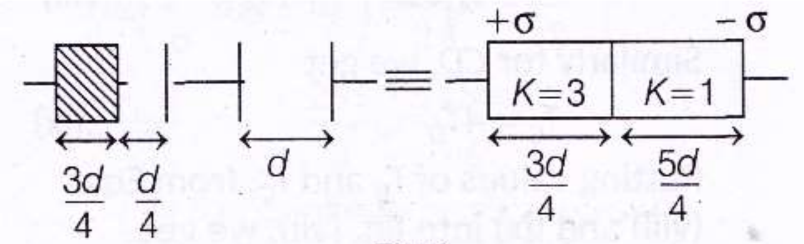


Fig.3

Energy density (energy per unit volume) is given by $\frac{\sigma^2}{2K\varepsilon_0}$

So,, if charge density on capacitor is $\sigma$, energy stored in volume V of capacitor is given by $U=\frac{\sigma^2 V}{2K\varepsilon_0}=\frac{\sigma^2 Ad}{2K\varepsilon_0}$

Thus, $U \propto \frac{d}{K}$

$$\Rightarrow \frac{U_{dielectric}}{U_{air}}=\frac{d_1}{K_1}\times\frac{K_2}{d_2}=\frac{1}{5}$$

17. (C)

As per standard Bohr's atomic model

E= Energy of electron (in eV)

$$=-\frac{13.6Z^2}{n^2}=-\frac{54.4}{n^2}$$

U= Potential energy of electron (in eV)

$$=-27.2\frac{Z^2}{n^2}=\frac{-108.8}{n^2}$$

So, for $n=1, E=-54.4eV$. As per new reference $E=0$ for n=1 which implies both E and U have been increased by 54.4 eV.

Thus, new values are

$$E'=-\frac{54.4}{n^2}+54.4eV$$

$$U'=\frac{108.0}{n^2}+54.4eV$$

I. $E'=-\frac{54.4}{2}+54.4=40.8eV$

II. Ionisation energy in ground state is difference in total energies between n=1 and $n=\infty$ which is equal to

$$IE=\frac{54.4}{1^2}-\frac{54.4}{\infty^2}=54.4eV$$

Which does not depend on reference

III. Excitation energy

$= 54.4\left(\frac{1}{1^2} - \frac{1}{2^2}\right)$

$= 40.8\text{eV}$

$= 65.28 \times 10^{-19}\text{J}$

IV. $U' = -\frac{108.8}{1^2} + 54.4 = -54.4\text{eV}$

$\Rightarrow -U' = 54.4\text{eV}$

18. (D)

Initial velocity of the particle (A) w.r.t. the platform (B) will be

$v_{AB} = v_A - v_B$

$= \left(v_2\hat{i} + 25\hat{j} + v_1\hat{k}\right) - 0$

$= v_2\hat{i} + 25\hat{j} + v_1\hat{k}$ …..(i)

Relative acceleration,

$a_{AB} = a_A - a_B$

$= -10\hat{j} - \left(2\hat{i} + 2.5\hat{j}\right)$

$= -2\hat{i} - 12.5\hat{j}$ …..(ii)

For motion of the particle till it hits the platform, relative displacement is zero in y-direction so, for y-direction.

$s_y = u_t t + \frac{1}{2}a_y t^2$

$\Rightarrow 0 = 25t + \frac{1}{2}(-12.5)t^2$

[ using Eqs. (i) and (ii)]

$\Rightarrow t = 4\text{s}$ …..(iii)

To hit the platform, relative displacement in x-direction should lie between 8m to 16m as per given location and side of the platform.

$\Rightarrow 8 \le s_x \le 16$

$\Rightarrow 8 \le u_x t + \frac{1}{2}a_x t^2 \le 16$

$\Rightarrow 8 \le v_2(4) + \frac{1}{2}(-2)(4)^2 \le 16$

[using Eqs. (i), (ii) and (iii)]

$\Rightarrow 6 \le v_2 \le 8$

Similarly, for z-direction

$16 \le s_z \le 24$

$\Rightarrow 16 \le v_1(4) \le 24$

$\Rightarrow 4 \le v_1 \le 6$

Displacement of particle w.r.t. ground at $t = 4\text{s}$

$s_y = u_y t + \frac{1}{2}a_y t^2$

$= 25(4) + \frac{1}{2}(-10)(4)^2 = 20\text{m}$

# PART (B) : CHEMISTRY

1. (0.05)

$$AgBr(s) \rightleftharpoons Ag^+(aq) + Br^-(aq); K_1 = K_{sp}$$

$$Ag^+(aq) + 2S_2O_3^{2-}(aq) \rightleftharpoons Ag(S_2O_3)_2^{3-}(aq); K_2 = K_f$$

$$AgBr(s) + 2S_2O_3^{2-}(aq) \rightleftharpoons Ag(S_2O_3)_2^{3-}(aq) + Br^-(aq); \quad K = K_{sp} \times K_f$$

| | | |
|---|---|---|
| 0.1 M | 0 | 0 |
| (0.1 – 2x)M | x | x |

$\Rightarrow K = K_{sp} \times K_f = 5.4 \times 10^{-13} \times 2.9 \times 10^{13}$
$= 15.66$

$$\Rightarrow K = \frac{x^2}{(0.1-2x)^2} = 15.66 \Rightarrow x = 0.05M$$

2. (0.90)

Given: $T_1 = 300K, p_1 = 0.8atm, V_I = V_{II} = V, n = 0.6mol$ (i.e. 0.3 mol in each bulb)

On heating flask-II at $117^\circ C$, n moles of $H_2$ gas will be diffused from flask-II to flask-I, therefore Flask-I contains $(0.3+n)$ mol and flask-II contains $(0.3-n)$ mol of $H_2$.

$$\left.\begin{array}{l} p_2 \times V = (0.3+n) \times R \times 300 \text{....in I......(i)} \\ p_2 \times V = (0.3-n) \times R \times 390 \text{......inII....(ii)} \end{array}\right]$$

$\Rightarrow n = 0.04mol$
$\Rightarrow$ Initially : $0.8 \times 2V = 0.6 \times 0.0821 \times 300 = 9.236L$
$\Rightarrow$ From Eq. (ii)

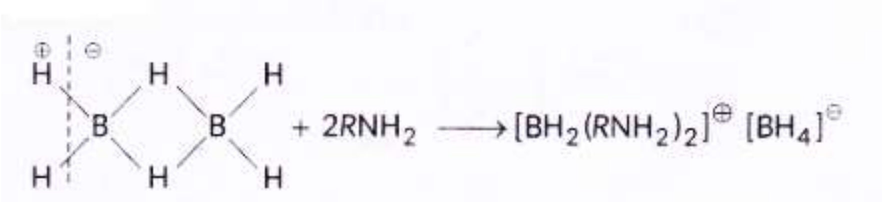
$$p_2 = \frac{(0.3-0.04) \times 0.0821 \times 390}{9.236} atm = 0.90atm$$

3. (9.60)

$$\underset{\substack{n-f=6-4 \\ =2 \\ mol=5}}{\overset{+4}{SO_2}} + \underset{\substack{n-f=7-2 \\ =5 \\ mol=2}}{\overset{+7}{MnO_4^-}} \xrightarrow{H^+} \overset{+6}{SO_4^{2-}} + Mn^{2+}$$

$\Rightarrow MnO_4^-$ required $= 240 \times 0.008$ mmol
$SO_2$ required $= 240 \times 0.008 \times 5mmol$
$= 9.60$ mmol

4. (2.00)



5. (14.00)

Equilibrium constant, $K = \frac{[B]_{eq}}{[A]_{eq}} = \frac{1.6}{0.4} = 4$

$\Rightarrow 4 = \frac{K_1}{K_2} = \frac{4\times10^{-2}}{K_2}$

$\Rightarrow K_2 = 10^{-2} s^{-1}$

For 50% completion of equilibrium concentration $(X_e)$, time taken,

$t_{eq} = \frac{1}{K_1 + K_2} \ln \frac{X_e}{50\% \text{ of } X_e}$

$= \frac{1}{4\times10^{-2} + 10^{-2}} \ln \frac{X_e}{0.5X_e}$

$= \frac{1}{5\times10^{-2}} \ln 2 = 14s$

6. (154.00)

According to given Arrhenius plot,

$\ln k = \ln A - \frac{E_a}{10^3 RT} \times 10^3$

$\ln k = \ln A + \frac{10^3}{T}\left[\frac{-E_a}{10^3 R}\right]$

Where, $\frac{-E_a}{10^3 R}$ represent slope.

$\therefore$ Slope $= \frac{-E_a}{10^3 R} = -18.5$

$\therefore\ E_a = 18.5\times10^3\times8.31 = 153.735 \simeq 154 \text{ kJ/mol}$

7. (13.90)

Since, the expansion occurs against a constant external pressure, pressure of the gas will remain constant in the given condition and it will be equal to the external pressure.

$\Rightarrow q_p = nC_p\Delta T$

Or $6236J = 3\times2.5R\Delta T$

$\Rightarrow \Delta T = \frac{6236}{7.5\times8.314} = 100K$

$\Rightarrow T = 400K$

Therefore,

In $\Delta S = nC_p \ln\frac{T_2}{T_1}$

$= 3\times 2.5R\times 2.3\log\frac{500}{400}$

$\Rightarrow \Delta S = 13.90J$

8. (74.93)

$$N_2O_4 \rightleftharpoons 2NO_2$$

| | $N_2O_4$ | $2NO_2$ |
|---|---|---|
| Initial moles | 1 | 0 |
| Moles at equil | $1-\alpha$ | $2\alpha$ |
| Moles fraction | $\frac{1-\alpha}{1+\alpha}$ | $\frac{2\alpha}{1+\alpha}$ |

$\Rightarrow K_p = \frac{4\alpha^2}{1-\alpha^2}p$

Also, $pM = \rho RT$

At 288K,

$M = \frac{\rho RT}{p} = \frac{3.62\times 0.082\times 288}{1} = 85.48$

$\Rightarrow 85.48 = \frac{92}{1+\alpha} = \alpha = 0.076$

$K_p(288K) = \frac{4(0.076)^2}{1-(0.076)^2} = 23\times 10^{-3}$

At 348 K,

$M = \frac{1.84\times 0.092\times 348}{1} = 52.5$

$\Rightarrow \frac{92}{1+\alpha} = 52.5 \text{ or } \alpha = 0.75$

$K_p(348K) = \frac{4(0.75)^2}{1-(0.75)^2} = 5.1$

Now, $\log\left[\frac{K_p(348K)}{K_p(288K)}\right] = \frac{\Delta H}{2.3\times 8.314}\left(\frac{1}{288}-\frac{1}{348}\right)$

$\log\frac{5.1}{23\times 10^{-3}} = \frac{\Delta H}{19.122}\left(\frac{60}{348\times 288}\right)$

$\Delta H = 74.93\,kJ$

9. (A, B, C, D)

$\Rightarrow E_n = -13.6\frac{Z^2}{n^2}$

$\Rightarrow -6.05 = -13.6\frac{2^2}{n^2}$

Solving, $n = 3$ and as magnetic quantum number (m) is 0, it is $3p_z$ orbital.

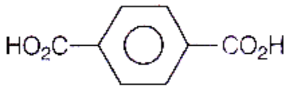
Hence, option (A) is correct

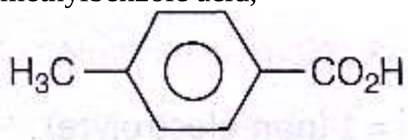
$\Rightarrow$(B) is correct as the wave function will be $\psi_{310}$.
$\Rightarrow$(C) is correct, since $\psi$ involves only $\cos\theta$.
$\Rightarrow$(D) is correct, for radial node: $6-\sigma=0$

$$\Rightarrow \sigma = 6 \Rightarrow \frac{2r}{a_0} = 6$$

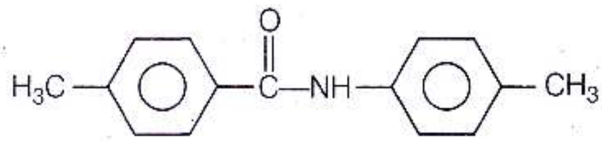
$r = 3a_0$

10. (B, D)

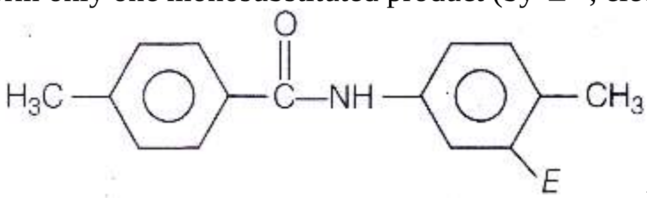
Compound (S) is terephthalic acid,  obtained from oxidaiton of (R) which is 4-methylbenzoic acid,



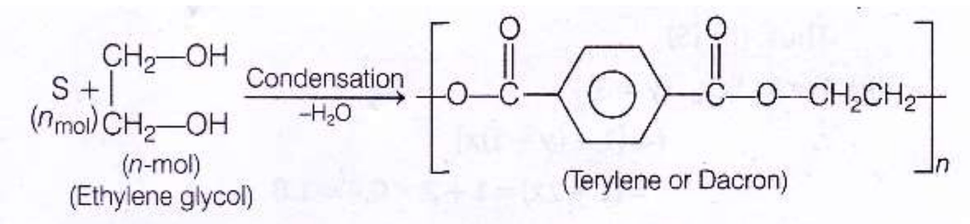
is one of the hydrolysis products of compound (P) which has structure.



, (S) is terephthalic acid and is confirmed as it can form only one monosubstituted product (by $E^+$, electrophile) as



in $ArS_E2$ reaction.




11. (A, B, D)
Option (A) is correct as the mode of reaction is intramolecular aldol where methylene carbon forms carbanion (enolate ion).
Option (B) is correct as in Hofmann degradation stereochemistry (R-configuration) of the migrating group remains same.
Option (C) is incorrect as approach of reagent is hindered due to steric effect of two methyl groups in ortho positions of aniline in carbylamine reaction.
Option (D) is correct, it is Michael's addition.

12. (A, B, C, D)
Options (A) is correct as with increase in atomic number $\left(\underset{58}{Ce} \rightarrow \underset{71}{Lu}\right)$ basicity of $Ln(OH)_3$ decreases, where $Ln =$ lanthanoids.
Option (B) is correct because of lanthanide contraction , size of $Zr^{4+}$ and $Hf^{4+}$ is nearly same.

Option (C) is correct as +3 is the most stable oxidation state of $Ln^{3+}$ ions. So , $Ce^{4+}$ easily gets reduced to $Ce^{3+}$. Hence , it acts as oxidising agent.

Option (D) is correct as ionic radii values are $La^{3+}(103)$

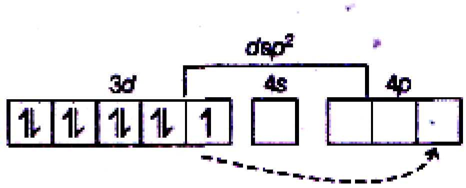
$Ce^{3+}(102), Pm^{3+}(97), Yb^{3+}(86.8)$ in picometer.

13. (A, B, D)

Statement(A), (B) and (D) are correct whereas (C) is incorrect

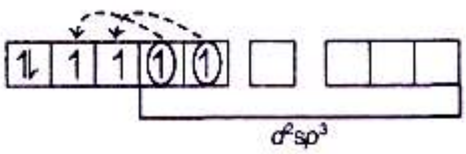
(A) $\left[Cu(NH_3)_4\right]^{2+} : 3d^9\left(Cu^{2+}\right)$

$\Rightarrow$




It is an inner orbital square planar $\left[Cu(II) : dsp^2\right]$ paramagnetic complex.

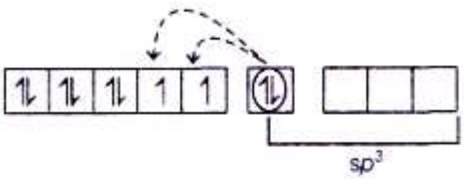
(B) $\left[Co(NH_3)_6\right]^{3+} : 3d^6\left(Co^{3+}\right)$




$\Rightarrow$

It is an inner orbital octahedral $\left[Co(III) : d^2sp^3\right]$ diamagnetic complex.

(C) $\left[Ni(CO)_4\right] : 3d^8 4s^2 \left[Ni^0\right]$




It is tetrahedral $\left[Ni(0) : sp^3\right]$ diamangetic complex. So, option – (C) is not correct.

(D) Mond process:::

$$\text{Crude Ni} \xrightleftharpoons{4CO, 350K} \underset{(\text{Volatile compound})}{\left[Ni(CO)_4\right]} \xrightarrow{470K} \underset{(\text{Pure})}{Ni} + 4CO$$

14. (A, B, C, D)

The identification test reactions of the given ions are

$$2ZnSO_4 + K_4\left[Fe(CN)_6\right] \longrightarrow \underset{\text{Bluish white ppt}}{\overset{II}{Zn_2}\left[\overset{II}{Fe}(CN)_6\right]} \downarrow + 2K_2SO_4$$

$$2CuSO_4 + K_4\left[Fe(CN)_6\right] \longrightarrow \underset{\text{Reddish–brown}}{\overset{II}{Cu_2}\left[\overset{II}{Fe}(CN)_6\right]} \downarrow + 2K_2SO_4$$

$$FeCl_3 + K_4\left[Fe(CN)_6\right] \longrightarrow \underset{\text{Prussian blue}}{\overset{II}{Fe_4}\left[\overset{II}{Fe}(CN)_6\right]_3} \downarrow + 3KCl$$

$Fe^{+2} + K_4[Fe(CN)_6] \rightarrow K_2Fe(CN)_6$ white

15. (A, B, C)
The reactions involved are as follows

(i) $\underset{(X)}{4Au + 8NaCN + 2H_2O + O_2} \longrightarrow 4Na\underset{(Y)}{[Au(CN)_2]} + 4NaOH$

(ii) $2Na\underset{(Y)}{[Au(CN)_2]} + \underset{(T)}{Zn} \longrightarrow Na_2\underset{(Z)}{[Zn(CN)_4]} + 2Au$

16. (A, B, D)
As Langmuir is based on mono-atomic layer over the surface, hence option (C) is incorrect.

$\Rightarrow$ Langmuir isotherm, $\theta = \dfrac{ap}{1+bp}$

Where, $\theta$ is the fraction of surface covered by adsorbate.
*p* is the pressure of the gas; *a* and *b* are Langmuir constants.

17. (C)

$\Delta T_b = K_b mi$

$\log \Delta T_b = \log K_b + \log(mi)$

$y = c + mi$

$(m = \text{slope}) = \tan\theta = \tan 45^\circ = 1$

$c = \text{intercept on } \log(\Delta T_b) \text{ axis} = -0.284^\circ$

Thus, when $mi = 1, \log mi = 0$

$\therefore \log K_b = \log \Delta T_b = -0.284^\circ$

(I) For 2 molal urea solution , $m = 2, i = 1$ (non-electrolyte)

$\log \Delta T_b = -0.284 + \log 2$

$= -0.284 + 0.300 = 0.016^\circ$

Thus, (I)-(R)

(II) For $NaCl(y = 2)$

$i = 1 + (y-1)x = 1 + x$

$= 1 + 1 = 2$

$\therefore \log(mi) = \log 4 = \log 2^2$

$= 2\log 2 = 0.60$

$\log \Delta T_b = -0.284 + 0.60 = 0.316$

Thus, (II)-(S)

(III) For $K_2SO_4, (y = 3)$

$\therefore\ i = [1 + (y-1)x]$

$= (1 + 2x) = 1 + 2 \times 0.4 = 1.8$

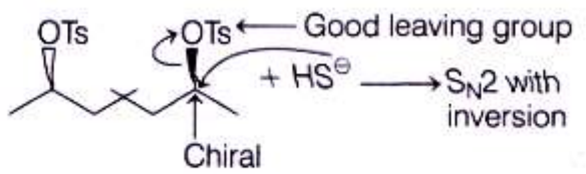
$\therefore \log(mi) = \log 3.6$

$= \log 36 - \log 10 = \log(2^2 \times 3^2) - \log 10$

$= 2\log 2 + 2\log 3 - \log 10$

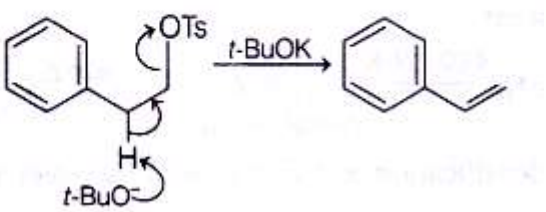
$= 2 \times 0.3 + 2 \times 0.48 - 1$
$= 0.6 + 0.96 - 1 = 0.56$
$\therefore \log \Delta T_b = -0.284 + 0.56 = 0.276$
Thus, (III)-(P)
(IV) For $K_3\left[Fe(CN)_6\right], (y = 4)$
$\therefore i = 1 + (y-1)x = 1 + 3x$
$= 1 + 3 \times 0.2 = 1.6$
$\therefore \log(mi) = \log 3.2$
$= \log 32 - \log 10$
$= 5\log 2 - \log 10$
$= 1.5 - 1.0 = 0.5$
$\therefore \log \Delta T_b = -0.284 + 0.500 = 0.216$
Thus (IV)-(Q)

18. (B)



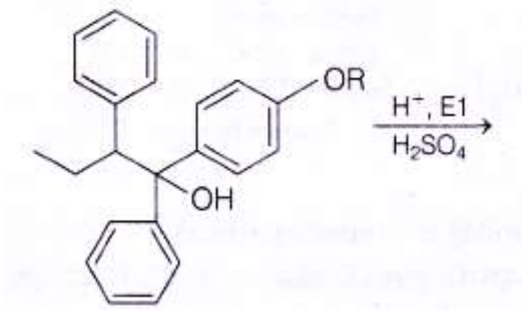

Thus, [(I)-(Q)]

(I)



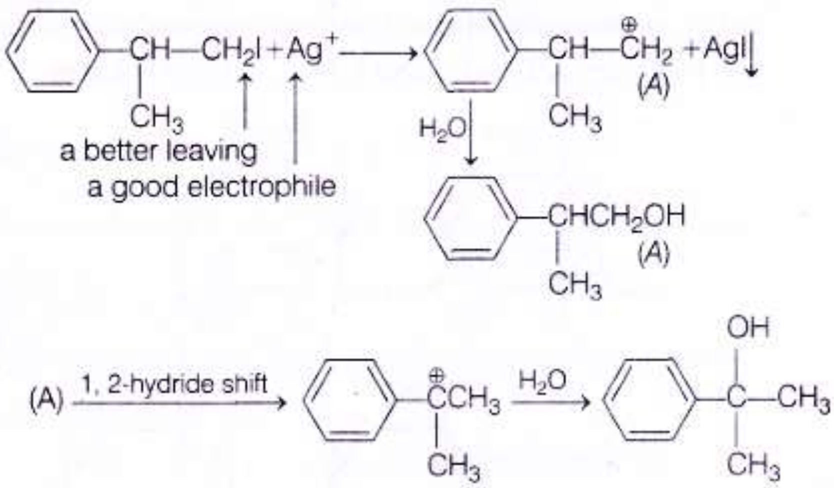

(II)

$BuO^-$ is sterically hindered base causing elimination by E2 mechanism.
Thus, $[(II)-(S)]$




(III)

$3^\circ$ alcohol changes to $\left(-OH_2^+\right)$- a good leaving group. Thus a carbocation is formed.
Thus, $[(III)-(R)]$




(IV)

Thus, $S_N1$

Thus, $\left[(IV)-(P)\right]$

## PART (C) : MATHEMATICS

1. (0.25)

We have $f(x)=\begin{cases}[x], & x\le 2\\ 0, & x>2\end{cases}$

$$I=\int_{-1}^{2}\frac{xf(x^2)}{2+f(x+1)dx}$$

$$\Rightarrow I=\int_{-1}^{0}\frac{x\times 0}{2+0}dx+\int_{0}^{1}\frac{x\times 0}{2+1}dx+\int_{1}^{\sqrt{2}}\frac{x\cdot 1}{2+0}dx$$

$$\Rightarrow I=\frac{1}{2}\left[\frac{x^2}{2}\right]_1^{\sqrt{2}}=\frac{1}{2}\left(1-\frac{1}{2}\right)=\frac{1}{4}$$

2. (21.00)

We have

|a| = |b| = 1

And |a + b| = $\sqrt{3}$

$\Rightarrow |a+b|^2=\left|\sqrt{3}\right|^2$

$\Rightarrow |a|^2+|b|^2+2a.b=3$

$2(a.b)=3-2\Rightarrow a.b=\frac{1}{2}$

Now, c = a + 2b - $3(a\times b)$

$\therefore a.c=|a|^2+2(a.b)-3a.(a\times b)$

$\Rightarrow a.c=1+1=2$

Similarly b . c = $\frac{5}{2}$

$\lambda=|(a\times b)\times c|$

$\Rightarrow \lambda=|(a.c)b-(b.c)a|$

$\Rightarrow \lambda=\left|2b-\frac{5}{2}a\right|\Rightarrow \lambda^2=\left|2b-\frac{5}{2}a\right|^2$

$\Rightarrow \lambda^2=4|b|^2+\frac{25}{4}|a|^2-10a.b$

$\Rightarrow \lambda^2=4+\frac{25}{4}-5\Rightarrow 4\lambda^2=21$

3. (33.00)

Total number of cases $={}^{100}C_1=100$

Now, consider $x+\frac{100}{x}>50$

$\Rightarrow x^2+100>50x$

$\Rightarrow x^2-50x>-100$

$\Rightarrow x^2 - 50x + 625 > 525$

$\Rightarrow (x-25)^2 > 525$

$\Rightarrow \left(x - 25 - \sqrt{525}\right)\left(x - 25 + \sqrt{525}\right) > 0$

$\Rightarrow x < 25 - \sqrt{525}$ or $x > 25 + \sqrt{525}$

Since x is a positive integer and $\sqrt{525} = 22.91$

We must have $x \le 2$ or $x \ge 48$

Thus, the favourable number of cases is 2+53 =55

Hence, the required probability is $\frac{55}{100} = \frac{11}{20}$

$\therefore m + n = 11 + 20 = 33$

4. (20.00)

If there numbers form a GP, then their exponents must be in AP.

We know, If a, b, c are in GP then $b^2 = ac$

Since, exponent of b is even, exponent of a and c must be ether both odd or both even

Now, two odd exponents or two even exponents (from 1, 2, 3……10)can be selected in ${}^5C_2$ + ${}^5C_2$ = 10 + 10=20 ways

$\therefore N = 20$

5. (14.00)

Let $g(x) = 3x^4 - 8x^3 - 6x^2 + 24x$

Then, $g'(x) = 12x^3 - 24x^2 - 12x + 24$

$= 12x(x^2 - 2x - 1) + 24$

$= 12x(x^2 - 2x + 1 - 2) + 24$

$= 12x\,(x - 1)^2 - 2) + 24$

For $x \in [1, 2), g(x)$ is decreasing

$\therefore$ min of g(t) in $1 \le t \le x$ will be g(x).

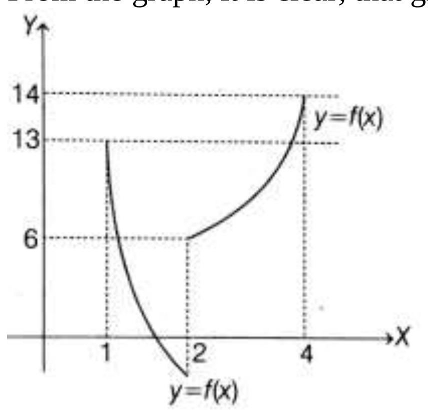
Now, let h(x) = $3x + \frac{1}{4}\sin^2 \pi x + 2$, then $h'(x) = 3 + \frac{\pi}{4}\sin(2\pi x) > 0, \forall x \in R$

$\therefore h(x)$ is increasing $\forall x \in R$

So maximum of h(x) in $2 \le 1 \le x$ will be h(x)

$$\Rightarrow f(x) = \begin{cases} 3x^4 - 8x^3 - 6x^2 + 24x, 1 \le x < 2 \\ 3x + \frac{1}{4}\sin^2 \pi x + 2, 2 \le x \le 4 \end{cases}$$

From the graph, it is clear, that greatest value of f(x) is 14



6. (149.00)

We have

$$\frac{x^2+x+1}{1-x}=a_0+a_1x+a_2x^2+....$$

$$\Rightarrow (1+x+x^2)(1-x)(1-x)^{-2}=a_0+a_1x+a_2x^2+....$$

$$\Rightarrow (1-x^3)(1-x^{-2})=a_0+a_1x+a_2x^2....$$

$$\Rightarrow (1-x^3)(1+2x+3x^2+4x^3+....)\ =\ =a_0+a_1x+a_2x^2+....$$

$$1+2x+3x^2+3x^3+3x^4+.....=a_0+a_1x+a_2x^2+...$$

On equating the coefficient of $x, x_2, x_3, x_4 ...$ respectively

We get

$a0=1, a_1=2, a_2=a_3=a_4=3$

$$\sum_{r=1}^{50} a_r = a_1+a_2+a_3....a_{50}$$

= 2 + 3 + 3 +…..49 times

= 2 + (3 × 49) = 2 + 147 = 149

7. (26.00)

Let x = y = 1, then we get

$3f(1) = 2 + (f(1))^2$

$\Rightarrow (f(1)^2-3(1)+2=0 \Rightarrow f(1)=1,2$

But it is given that f(1) $\neq 1$

$\therefore f(1)=2$

Now, put $y=\frac{1}{x}$, then we get

$$f(x)+f\left(\frac{1}{x}\right)+f(1)=2+f(x).f\left(\frac{1}{x}\right)$$

$$\Rightarrow f(x)+f\left(\frac{1}{x}\right)=f(x).f\left(\frac{1}{x}\right)$$

$$\Rightarrow f(x)=\pm x^n+1$$

$\because f(4)=17$

$\therefore \pm(4)^n+1=17 \Rightarrow n=2$

Thus, $f(x)=x^2+1$

Hence, $f(5) = 5^2 + 1 = 26$

8. (12.00)

We have $A=\begin{bmatrix}1 & -3\\ -1 & 1\end{bmatrix}$ and $A-\frac{1}{3}A^2+...\left(-\frac{1}{3}\right)^n$

$$A^{n-1}+...\infty=\frac{3}{13}\begin{bmatrix}1 & 9\\ b & 1\end{bmatrix}$$

Let $B=A-\frac{1}{3}A^2+\frac{1}{9}A^3+.....\infty.....(i)$

Premultiplied by $-\frac{A}{3}$, we get

$$-\frac{AB}{3}=-\frac{A^2}{3}+\frac{1}{9}A^3-\frac{1}{27}A^4....$$

$$\Rightarrow -\frac{AB}{3}=B-A \qquad \text{[From Eq. (i)]}$$

$$\Rightarrow A=B+\frac{AB}{3}$$

$$\Rightarrow A=B\left(I+\frac{A}{3}\right)$$

$$\Rightarrow B=3(3I+A)^{-1}A$$

$$\Rightarrow B=3\left(\begin{bmatrix}3&0\\0&3\end{bmatrix}+\begin{bmatrix}1&-3\\-1&1\end{bmatrix}\right)^{-1}\begin{bmatrix}1&-3\\-1&1\end{bmatrix}$$

$$\Rightarrow B=3\begin{bmatrix}4&-3\\-1&4\end{bmatrix}^{-1}\begin{bmatrix}1&-3\\-1&1\end{bmatrix}$$

$$\Rightarrow B=\frac{3}{13}\begin{bmatrix}4&3\\1&4\end{bmatrix}\begin{bmatrix}1&-3\\-1&1\end{bmatrix}$$

$$\Rightarrow B=\frac{3}{13}\begin{bmatrix}1&-9\\-3&1\end{bmatrix}$$

Now, $\frac{3}{13}\begin{bmatrix}1&-9\\-3&1\end{bmatrix}=\frac{3}{13}\begin{bmatrix}1&a\\b&1\end{bmatrix}$

Equating , we get a = -9, b = -3

$\therefore |a+b|=|-9-3|=12$

9. (C, D)

We have f(x) = $2^{x^4-4x^2}$

Let f(x) = y

$$\therefore y=2^{x^4-4x^2} \Rightarrow \log_2 y = x^4-4x^2$$

$$\Rightarrow x^4-4x^2+4=\log_2 y+4$$

$$\Rightarrow (x^2-2)^2=\log_2 y+4$$

$$\Rightarrow x^2=2+\sqrt{\log_2 y+4}$$

$$\Rightarrow x=\sqrt{2+\sqrt{\log_2 y+4}}$$

$$\Rightarrow f^{-1}(x)=\sqrt{2+\sqrt{\log_2 x+4}}$$

Now, $g(x)=\frac{\sin x+4}{\sin x-2}$

$$g(x)=\frac{\sin x-2+6}{\sin x-2}$$

$$g(x)=1+\frac{6}{\sin x-2}$$

$\therefore$ Range of g(x) = [–5, –2]

10. (B, C, D)

(a) If $\arg\left(\dfrac{z_1}{z_2}\right)=\dfrac{\pi}{2}$, then $z_1$ and $z_2$ subtend right angle at circumcentre origin.

$\therefore$ The chord joining $z_1$ and $z_2$ will subtend an angle $\theta$ at 'z' such that $\begin{cases}\theta=\dfrac{\pi}{4}, & \text{if } |z|=1\\ \theta<\dfrac{\pi}{4}, & \text{if } |z|>1\\ \theta>\dfrac{\pi}{4} & \text{if } |z|<1\end{cases}$

(b) $|z_1z_2+z_2z_3+z_3z_1|=|z_1||z_2||z_3|\left|\dfrac{1}{z_1}+\dfrac{1}{z_2}+\dfrac{1}{z_3}\right|$

$=|\overline{z_1+z_2+z_3}|=|z_1+z_2+z_3|$

(c) $\left(\dfrac{(z_1+z_2)(z_2+z_3)(z_3+z_1)}{z_1z_2z_3}\right)$

$=\left(\dfrac{(z_1+z_2)(z_2+z_3)(z_3+z_1)}{z_1.z_2.z_3}\right)$

$\Rightarrow\left(\dfrac{(z_1+z_2)(z_2+z_3)(z_3+z_1)}{z_1z_2z_3}\right)-\left(\dfrac{\overline{(z_1+z_2)(z_2+z_3)(z_3+z_1)}}{\overline{z_1z_2z_3}}\right)=0$

Hence

$\text{lm}\left(\dfrac{(z_1+z_2)(z_2+z_3)(z_3+z_1)}{z_1z_2z_3}\right)=0$

(d) The triangle formed by joining $z_1$, $z_3$ and $z_2$ is isosceles and right angle at $z_3$

Hence $\text{Re}\left(\dfrac{z_3-z_1}{z_3-z_2}\right)=0$

11. (A, D)

$\Delta ODE-\Delta CDA\Rightarrow\dfrac{\lambda}{a}=\dfrac{15/4}{5}$

$\Rightarrow\lambda=\dfrac{3}{4}a\Rightarrow E=\left(0,\dfrac{3}{4}a\right)$

Similarly $\Delta BFE\sim\Delta CFA$

$\Rightarrow\dfrac{BF}{CF}=\dfrac{BE}{AC}=\dfrac{a/4}{a}$

$\Rightarrow BF=\dfrac{1}{4}(a+BF)\Rightarrow BF=\dfrac{a}{3}$

$\Rightarrow F=\left(-\dfrac{a}{3},a\right)$

$$AE = \sqrt{a^2 + \left(\frac{3}{4}a\right)^2} = \frac{5}{4}a$$

$$= 5 + \frac{15}{4} = \frac{35}{4}$$

$$\frac{35}{4} = \frac{5}{4}a \Rightarrow a = 7$$

Area of square = $a^2 = (7)^2 = 49$

The coordination of $F = \left(-\frac{a}{3}, a\right) = \left(-\frac{7}{3}, 7\right)$

Hence abscissa of F is $-\frac{7}{3}$

12. (A, B, C D)

We have $z = 5 - y - x$

$\Rightarrow xy + y(5 - y - x) + (5 - y - x)x = 3$

$\Rightarrow xy + 5y - y^2 - xy + 5x - yx - x^2 = 3$

$\Rightarrow y^2 + y(x - 5) + x^2 - 5x + 3 = 0$

$\Rightarrow (x - 5)^2 - 4(x^2 - 5x + 3) \geq 0$

$\Rightarrow x^2 - 10x + 25 - 4x^2 + 20x - 12 \geq 0$

$\Rightarrow 3x^2 - 10x - 13 \leq 0$

$\Rightarrow (3x - 13)(x + 1) \leq 0$

$\Rightarrow -1 \leq x \leq \frac{13}{3}$

Similarly, $-1 \leq y \leq \frac{13}{3}$

And $-1 \leq z \leq \frac{13}{3}$

Thus, option (a), (b) and (c) are correct.

Now, required probability

$$\frac{\int_0^{13/3} dx}{\int_{-1}^{13/3} dx} = \frac{\frac{13}{3}}{\frac{13}{3} + 1} = \frac{13}{16}$$

Hence, option (d) is also correct

13. (A, B, C)

Since, angle between a and b is acute therefore

$-3x + x^2 + 2 > 0$

i.e. $x \in (-\infty, 1) \cup (2, \infty)$

Also, as the angle between a and c is obtuse, therefor

$3x^2 + 11x + x^3 - 9x^2 - 6 < 0$

i.e. $x^3 - 6x^2 + 11x - 6 < 0$

i.e., $(x - 1)(x - 2)(x - 3) < 0$

$\therefore x \in (-\infty, 1) \cup (2, 3)$
Hence option (a), (b), and (c) are correct

14. (A, C)
Let $A\left(a_1^2, 2at_1\right)$ and $B\left(a_2^2, 2at_2\right)$.
Then, we have $t_2 = -t_1 - \dfrac{2}{tl1}$.
For $AB$ to be shortest $t_1 = \pm\sqrt{2}$
$\Rightarrow \quad t_2 = \mp 2\sqrt{2}$
$\Rightarrow \quad t_1 t_2 = -4$
$\Rightarrow \quad \angle AOB$ is right angle
$\therefore$ Mid-point of $AB$ is circumcenter.
Hence, the circumcenter is $\left(5a, \sqrt{2}a\right)$ or $\left(5a, -\sqrt{2}a\right)$.

15. (A, B, D)
Given f(2 – x) = f(2 +x) …..(i)
and f(4 – x) = f(4 + x) …..(ii)
Consider
f(4 + x) = f(4 – x) = f(2 + (2 – x)))
= f(2- (2- x)) [using eq. (i)]
= f(x)
Thus, 4 is a period of f(x)
Now, consider

$$\int_0^{50} f(x)dx = \int_0^{49} f(x)dx + \int_{48}^{50} f(x)dx$$

$$= \int_0^{48} f(x)dx + \int_0^{2} f(x)dx$$

[putting x = 48 + t in second integral]

$$= 12\left(\int_0^4 f(x)dx + \int_0^2 f(4-x)dx\right) + 5 \qquad \left[\because \int_0^2 f(x)dx = 5, \text{given}\right]$$

$$= 12\left(\int_0^2 f(x)dx + \int_0^2 f(4+x)dx\right) + 5$$

$$= 24\int_0^2 f(x)dx + 5 = 125$$

Also $\displaystyle\int_0^{50} f(x)dx = \int_{-4}^{48} f(x)dx$ [putting x = 4 + t]

And

$$\int_2^{52} f(x)dx = \int_0^{52} f(x)dx - \int_0^{2} f(x)dx = 13$$

$$\int_0^4 f(x)dx - \int_0^2 f(x)dx = 13(10) - 5 = 130 - 5 = 125$$

16. (A)

Let $n_1$ and $n_2$ be the vectors normal to the planes determined by $\hat{i}, \hat{i} + \hat{j}$ and $\hat{i} - \hat{j}, \hat{i} + \hat{k}$ respectively

Then $n_1 = \hat{i} \times (\hat{i} + \hat{j})$

And $n_2 = (\hat{i} - \hat{j}) \times (\hat{i} + \hat{k})$

$\Rightarrow n_1 = \hat{k}$ and $n_2 = -\hat{j} + \hat{k} - \hat{i}$

$\therefore a \,||\, (n_1 \times n_2)$

$$\Rightarrow a = \lambda(n_1 \times n_2) = \lambda \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 0 & 1 \\ -1 & -1 & 1 \end{vmatrix}$$

$$\bar{a} = \lambda\left((-1)[-\hat{i} + \hat{j}]\right)$$

$$\vec{a} = \lambda = \left((\hat{i} - \hat{j})\right)$$

Let $\theta$ be the angle between a and $\hat{i} - 2\hat{j} + 2\hat{k}$

Then

$$\cos\theta = \frac{\lambda(\hat{i} - \hat{j}).(\hat{i} - 2\hat{j} + 2\hat{k})}{\sqrt{\lambda^2 + \lambda^2}\sqrt{1 + 4 + 4}}$$

$$= \frac{\lambda(1+2)}{\lambda\sqrt{2}.3} = \frac{1}{\sqrt{2}}$$

$$\Rightarrow \theta = \frac{\pi}{4}$$

17. (A)

Let $f(x) = a_0 + a_1x + a_2x^2 + a_3x^3 + a_4x^4 + a_5x^5 + a_6x^6$

Given $\lim\limits_{x \to 0}\left(1 + \frac{f(x)}{x^3}\right)^{\frac{1}{x}} = e^2$

$$\lim_{x \to 0} \frac{f(x)}{x^3} = 0$$

Or $a_0 = a_1 = a_2 = 0$

$$\therefore \lim_{x \to 0} e^{(a_4 + a_5x + a_6x^2)} = e^2 \Rightarrow a_4 = 2$$

$$\Rightarrow f(x) = 2x^4 + a_5x^5 + a_6x^6$$

$$\Rightarrow f'(x) = 8x^3 + 5a_5x^4 + 6a_6x^5$$

$$\Rightarrow f'(x) = x^3(8 + 5a_5x + 6a_6x^2)$$

x =1 and x = 2 are points of local maxima and local minima

$\therefore f'(1) = 0$ and $f'(2) = 0$

$\therefore 8 + 5a_5 + 6a_6 = 0$

And $4+5a_5+12a_6=0$

Solving we get $a_5=-\frac{12}{5}, a_6=\frac{2}{3}$

$\therefore f(x)=2x^4-\frac{12}{5}x^5+\frac{2}{3}x^6$

18. (D)

We have

$$f(m)=\sum_{i=0}^{m}\binom{30}{30-i}\binom{20}{m-i} = f(m)=\sum_{i=0}^{m}\binom{30}{i}\binom{20}{m-i}$$

$[\because {}^nC_r={}^nC_{n-r}]$

$={}^{20}C_m+{}^{30}C_1.{}^{20}C_{m-1}+....+{}^{30}C_m$

$={}^{50}C_m$

$[\because {}^pC_r+{}^pC_{r-1}\,{}^qC_1+{}^pC_{r-2}\,{}^qC_2+...+{}^qC_r={}^{p+q}C_r]$

P. Clearly, f(m) is maximum when m = 25

$\therefore$ Maximum value of f(m) is ${}^{50}C_{25}$

Q: Clearly $\sum_{m=0}^{50} f(m)=\sum_{m=0}^{50}{}^{50}C_m={}^{50}C_0+{}^{50}C_1+...+{}^{50}C_{50}$

$=2^{50}$

R. Clearly

$$\sum_{m=0}^{50}(f(m))^2=\sum_{m=0}^{50}({}^{50}C_m)^2=({}^{50}C_0)^2+({}^{50}C_1)^2+....+({}^{50}C_{50})^2={}^{100}C_{50}$$

$\left[\because ({}^nC_6)^2+({}^nC_1)^2+.....+({}^nC_n)^2={}^{2n}C_n\right]$

S. Consider

Consider

f(0) – 8(1) + 13f(2) – 18f(3)+…+253f(50)

$$=\sum_{m=0}^{50}(-1)^m(3+5m)f(m)$$

$$=\sum_{m=0}^{50}(-1)^m(3+5m)\,{}^{50}C_m$$

$$=3\left(\sum_{m=0}^{50}(-1)^m\,{}^{50}C_m\right)+5\left(\sum_{m=0}^{50}(-1)^m.{}^{50}C_m\right)$$

$$=3\left(\sum_{m=0}^{50}(-1)^m\,{}^{50}C_m\right)+5\left(\sum_{m=1}^{50}(-1)^m.50.{}^{49}C_{m-1}\right)$$

$$=3\left(\sum_{m=0}^{50}(-1)^m\,{}^{50}C_m\right)-250\left(\sum_{m=1}^{50}(-1)^{m-1}.{}^{49}C_{m-1}\right)$$

$= 3(1-1)^{50} - 250(1-1)^{49} = 0$