PACE-IIT & MEDICAL

MUMBAI / DELHI-NCR / PUNE / NASHIK / AKOLA / GOA / JALGOAN / BOKARO / AMRAVATI / DHULE

IIT – JEE: 2023 **AITS – 12 (ADVANCED)** **DATE: 21/05/23**

ANSWER KEY

PAPER – I

Que.	1	2	3	4	5	6	7	8	9	10
Ans.	D	ABCD	ABCD	ACBD	BD	AC	ACD	ABC	BC	AC
Que.	11	12	13	14	15	16	17	18	19	20
Ans.	2	2	3	5	8	4	2	3	(A) – Q R (B) – P R (C) – P R S (D) – Q R S	(A) – Q R (B) – R S T (C) – Q (D) – P R S T
Que.	21	22	23	24	25	26	27	28	29	30
Ans.	CD	AC	ABD	ABC	AD	A	BC	C	B	B
Que.	31	32	33	34	35	36	37	38	39	40
Ans.	6	7	9	4	4	6	2	4	(A) – R T (B) – P S (C) – Q (D) – P Q R	(A) – P R S (B) – Q R (C) – P T (D) – P S T
Que.	41	42	43	44	45	46	47	48	49	50
Ans.	BCD	AC	CD	ACD	ABCD	ABC	ABC	ABCD	BD	ACD
Que.	51	52	53	54	55	56	57	58	59	60
Ans.	2	3	7	7	5	4	8	8	(A) – Q (B) – R (C) – S (D) – Q	(A) – R (B) – T (C) – Q (D) – P Q T

PAPER – II

Que.	1	2	3	4	5	6	7	8	9	10
Ans.	BCD	BC	BCD	BC	BC	ABC	BC	AC	B	C
Que.	11	12	13	14	15	16	17	18	19	20
Ans.	B	B	2	9	3	2	3	7	2	6
Que.	21	22	23	24	25	26	27	28	29	30
Ans.	ABC	BCD	ABC	BD	BC	C	A	ACD	D	B
Que.	31	32	33	34	35	36	37	38	39	40
Ans.	B	A	5	9	6	4	8	6	6	4
Que.	41	42	43	44	45	46	47	48	49	50
Ans.	ABC	BC	BCD	ABD	BCD	BCD	AD	AB	CD	ABD
Que.	51	52	53	54	55	56	57	58	59	60
Ans.	BCD	CD	7	1	4	9	4	1	7	2

SOLUTION

1. **(D)**

$$I=\frac{2\times\frac{M}{2}\left(\frac{1}{2}\right)^2}{3}=\frac{ML^2}{12}$$

$$\frac{1}{2}Iw^2=Mg\frac{L}{8}$$

$$w=\sqrt{\frac{3g}{L}}$$

$$N=Mg=M\frac{3g}{L}.\frac{L}{8}$$

2. **(ABCD)**

$$P_AV_A=RT_0$$

$$P_B=2P_A$$

$$V_B=V_C=2V_A$$

$$T_B=\frac{P_BV_B}{R}=4T_0$$

$$\Delta W=\left(\frac{V_C-V_A}{2}\right)(P_B-P_C)=0.5RT_0$$

$$\Delta W_{AB}=\frac{1}{2}(P_A+P_B)(V_B-V_A)=\frac{3}{2}P_AV_A=1.5RT_0$$

$$\Delta U_{AB}=\frac{3}{2}R(T_B-T_A)=4.5RT_0$$

$$\Delta Q_{AB}=\Delta W_{AB}+\Delta U_{AB}=8RT_0$$

$$C=\left(\frac{\Delta Q_{AB}}{T_B-T_A}\right)=2R$$

$$\eta=\frac{\Delta W_{NET}}{\Delta Q_{AB}}=\frac{1}{12}=8.33$$

3. **(ABCD)**

$$w=\int\vec{f}_k.\vec{ds}>0$$

if f_k and ds both are parallel to each other friction force acts up the incline to produce angular acceleration.

4. **(ACBD)**

Orbital speed $(v_0)=\sqrt{\frac{GM}{R}}$

escapes speed $(v_e)=\sqrt{\frac{2GM}{R}}$

5. **(BD)**

$$\Delta\phi = \frac{2\pi}{\lambda}(x_1 - x_2) + \left(\frac{\pi}{4} - \frac{\pi}{6}\right)$$

When $x_1 - x_2 = 23\lambda/24$

then $\Delta\phi = 2\pi$ (constructive interference)

When $x_1 - x_2 = 11\lambda/24$

Then $\Delta\phi = \pi$ (destructive interference)

6. **(AC)**

$I = \frac{MR^2}{2}$ & M is constant $\Rightarrow I \ \alpha \ R^2$

$\therefore \frac{\Delta I}{I} = 2\left(\frac{\Delta R}{R}\right) = 2\alpha(\Delta T)$ on heating

Also no external torque present

$\therefore L - I\omega = \text{constant}$

$\therefore \omega \ \alpha \ \frac{1}{I}$

$\therefore \frac{\Delta\omega}{\omega} = -\frac{\Delta I}{I} = -2\left(\frac{\Delta R}{R}\right) = -2 \ \alpha \ (\Delta T)$

7. **(ACD)**

$i = \frac{24}{3+9+6} = \frac{4}{3}A$

$V_1 = \frac{4}{3} \times 9 = 12$

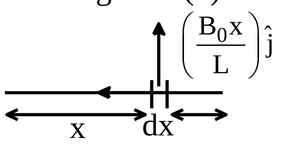
$V_2 = \frac{4}{3} \times 6 = 8V$

At $t = \infty$

$V_2 = V_1 = 24$

8. **(ABC)**

Wire segment (1)



$\int(d\vec{f}) = \int_0^L i \ dx\left(\frac{B_0 x}{2}\right) = \frac{iB_0 L}{2}$

$d\vec{f} = \frac{-iB_0 L}{2}\hat{k}$

Similary for $\vec{f}_2 = -\frac{-iB_0 L}{2}\hat{k}$

$\vec{f}_3 = \frac{iB_0 L}{2}\hat{k}$

$$\vec{f}_4 = \frac{iB_0L}{2}\hat{k}$$

$$\vec{f}_n = 0$$

If coil is constrained to rotate about

$$y - \text{axis}\left|\vec{\tau}\right| = 1\left(\frac{B_0L}{2}\right)L$$

$$= \frac{iAB_0}{2}$$

$\Rightarrow$ Similarly for torque about torque has same magnitude

9. **(BC)**

10. **(AC)**

At resonance $X_L = X_C$ and $Z = Z_{min} = R$

$X_L = \omega L$ and $\frac{1}{\omega C} = X_C$

If 'f ' is decreased then 'ω' will decrease and hence X_C will increase therefore at $f < f_r$,circuit behaves as capacitive.

V_L and V_C always difference in phase by 180° at any frequency.

11. **(2)**

$$\varepsilon = \vec{B}\left(\vec{V}_{cm} \times \vec{L}\right)$$

$$= \left(6\hat{k}\right)\left(\left(\frac{3}{2}\hat{i} - \frac{4}{2}\hat{j}\right) \times \left(4\hat{i} + 3\hat{j}\right)\right) = 21$$

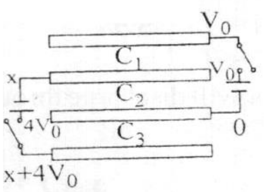
12. **(2)**

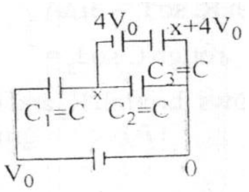
$$P = 700 \times 10^3 \times 1.6 \times 10^{-19} \times \frac{dN}{dt} = 10 \times 10^{-3}$$

$$\frac{dN}{dt} = \frac{10^{-2}}{10^{-14}} \times \frac{1}{7 \times 16} = \frac{10^{12}}{11.2} = \lambda\ N_0$$

$$\lambda = \frac{l\,n\,2}{14 \times 86400} \Rightarrow N_0 = \frac{14 \times 86400 \times 10^{12}}{11.2\,l\,n\,2} = 154 \times 10^{15} = 1.54 \times 10^{17} = 2 \times 10^{17}$$

13. **(3)**





$$C\left(x - V_0\right) + C\left(x - 0\right) + C\left(x + 4V_0\right) = 0$$

$$3Cx = -3CV_0$$

$$Q = \frac{3V_0\varepsilon_0 A}{L} \qquad \Rightarrow \qquad x = 3$$

14. **(5)**

Temperature is constant $\Rightarrow$ $\Delta E = 0, dW = nRT\frac{dv}{v} = nRT\frac{Adx}{AL/2}$

$Q = \Delta E + W$

$\Rightarrow \frac{dW}{dt} = \frac{nRT}{L/2}\frac{dx}{dt}$

$\frac{dQ}{dt} = \frac{dW}{dt} \Rightarrow k\frac{1}{900}\frac{\Delta T}{L} = \frac{2nRT}{L}\left(\frac{dx}{dt}\right)$

$\Rightarrow \frac{dx}{dt} = \frac{k \times 27}{900.nRT} = \frac{415.5 \times 27 \times 2}{900 \times 0.5 \times 8.31 \times 300} = \frac{1}{200} m/s = 5mm/s$

15. **(8)**

Radius of curvature of the curve $r = \frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\frac{d^2y}{dx}}$

$y = 2x^2, \frac{dy}{dx} = 4x, \frac{d^2y}{dx^2} = 4$

$r = \frac{1+16x^2}{4} \Rightarrow at\ x = 0, r = \frac{1}{4}$

$r = \frac{mv}{qB} \Rightarrow B = \frac{mV}{qr} = 8$

16. **(4)**

$q = CV = C\ \ Blv \quad \frac{dq}{dt} = Blc\frac{dv}{dt} \quad - 1 \quad mg - B\left(\frac{dq}{dt}\right)l = m\frac{dv}{dt} \quad mg = \ m + B^2l^2C\ \ \frac{dv}{dt}$

$a = \frac{dv}{dt} = \frac{mg}{mB^2l^2C} = \frac{mg}{m+4m} = \frac{9}{5} = 2ms^{-2} \quad s = \frac{1}{2}at^2 = 4m$

17. **(2)**

$\lambda = \frac{hc}{ev}\ ;\ \lambda_c = \frac{hc}{\propto z^2} \quad 4\left[\frac{hc}{\propto z^2} - \frac{hc}{ev}\right] = \frac{hc}{\propto\left(\frac{z}{k}\right)^2} - \frac{hc}{e\frac{v}{4}} \quad \Rightarrow k = 2$

18. **(3)**

$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$

$\frac{\delta R}{R^2} = \left|\frac{\delta R_1}{R_1^2}\right| + \left|\frac{\delta R_2}{R_2^2}\right|$

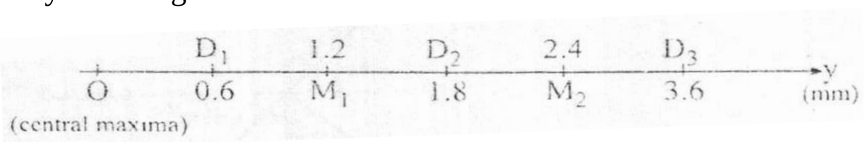
$$\frac{\delta R}{R}\times 100 = 100R\left[\frac{\delta R_1}{R_1^2}+\frac{\delta R_2}{R_2^2}\right]$$

$$=100\times\frac{5\times 10}{15}\left[\frac{0.2}{5\times 5}+\frac{0.1}{10\times 10}\right] \quad =\frac{10}{3}\ \ 4\times 0.2+0.1 \quad =\frac{9}{3} \ =3\%$$

19. **(A) - Q, R; (B) - P,R; (C) - P,R,S; (D) - Q,R,S**

Fringe width $W=\frac{\lambda D}{d}=\frac{6\times 10^{-7}\times 1}{0.5\times 10^{-3}}=12\times 10^{-4}=1.2$ mm

On y line fringes are as shown below



Ist, IInd maxima $M_1\ \&\ M_2$

Ist, IInd & IIIrd minima respectively D_1, D_2, D_3

Phase difference $\Delta\phi=\frac{yd}{D}\times\frac{2\pi}{\lambda}$

$I=I_1+I_2+2\sqrt{I_1}\sqrt{I_2}\cos\Delta\phi=I_0\left(5+4\cos\Delta\phi\right)$

In Case (A) $\quad \Delta\phi=3\pi+\pi/3 \quad \Rightarrow \quad \cos\Delta\phi=-1/2$

In Case (B) $\quad \Delta\phi=4\pi+\pi/3 \quad \Rightarrow \quad \cos\Delta\phi=1/2$

In Case (C) $\quad \Delta\phi=4\pi+\pi/3 \quad \Rightarrow \quad \cos\Delta\phi=1/2$

In Case (D) $\quad \Delta\phi=4\pi+2\pi/3 \quad \Rightarrow \quad \cos\Delta\phi=-1/2$

20. **(A) - Q, R; (B) - R, S, T; (C) - Q; (D) - P, R, S, T**

(A) If torque zero about centre of circular motion then angular momentum will be constant

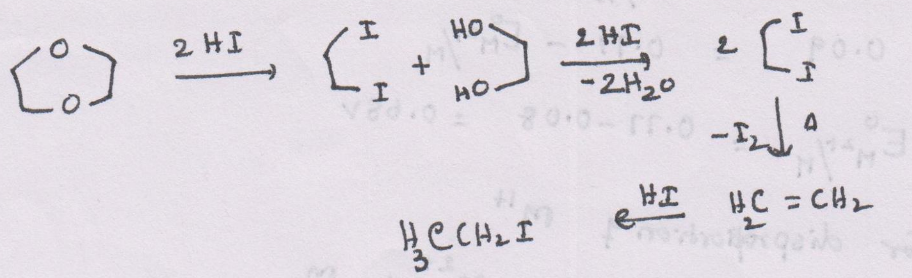
(B) $\therefore I\omega=$constant I decreases then ω increases

(C) In pure Rolling work done by frictional force zero

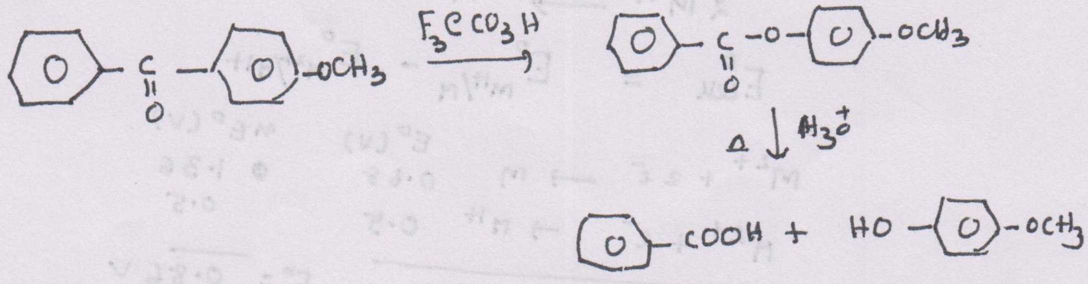
(D) An external force increases the speed of centre of mass

Paper I Open Test Solutions

(26)



(27)



(28)

$$AgCl_{(s)} \rightleftharpoons Ag^+ + Cl^- \qquad K_{sp} = 2\times10^{-10}$$

$$Ag^+ + 2NH_3 \rightleftharpoons [Ag(NH_3)_2]^+ \qquad K_f = 1.55\times10^{7}$$

$$AgCl_{(s)} + 2NH_3 \rightleftharpoons [Ag(NH_3)_2]^+ + Cl^- \qquad K_{eq} = \frac{[Ag(NH_3)_2]^+[Cl^-]}{[NH_3]^2}$$

$$= K_{sp}\times K_f = 3.1\times10^{-3}$$

$$[NH_3]^2 = \frac{0.05\times0.05}{3.1\times10^{-3}} = \frac{25\times10^{-4}}{3.1\times10^{-3}} = 8.1\times10^{-1} = 0.81$$

$$[NH_3] = 0.9$$

no. of moles of NH_3 at equilibrium = 0.9

no. moles of NH_3 required for complex formation = $2\times0.05 = 0.1$

Total moles of NH_3 required = $0.9 + 0.1 = 1.0$

(29)

	$2Fe^{3+}$	$+\ M$	$\rightleftharpoons\ 2Fe^{2+}$	$+\ M^{2+}$
Initial no. of m.moles	10		0	0
m.moles at equilibrium	0.1		9.9	4.95
Conc. at eq	$\frac{0.1}{50}$		$\frac{9.9}{50} \approx \frac{1}{5}$	$\frac{4.95}{50} \approx \frac{1}{10}$

$$K_{eq} = \frac{1\times1\times1\times50\times50}{5\times5\times10\times0.1\times0.1} = 1000$$

$$E_{cell} = \frac{0.06\times\log10^3}{2} = 0.09$$

(P.T.O)

$$E_{cell} = E^{0}_{Fe^{3+}/Fe^{2+}} - E^{0}_{M^{+2}/M}$$

$$0.09 = 0.77 - E^{0}_{M^{2+}/M}$$

$$E^{0}_{M^{2+}/M} = 0.77 - 0.08 = 0.68V$$

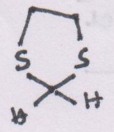
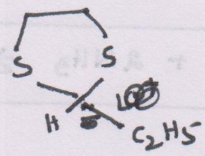
for disproportion of M^{1+}

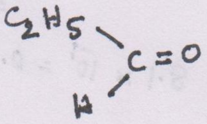
$$2M^{1+} \longrightarrow M^{2+} + M$$

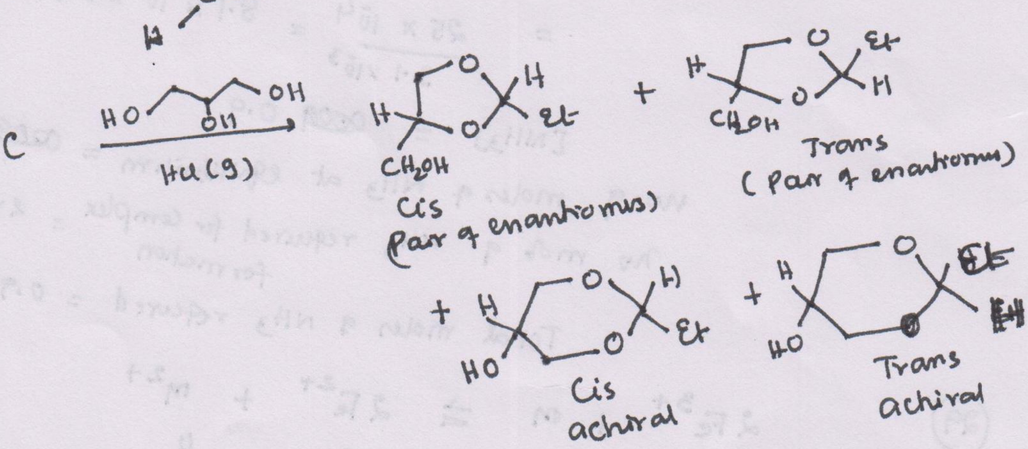
$$E^{0}_{cell} = E^{0}_{M^{1+}/M} - E^{0}_{M^{2+}/M^{1+}}$$

	E^0 (V)	nE^0 (V)
$M^{2+} + 2e^- \rightarrow M$	0.68	1.36
$M^{2+} + e^- \rightarrow M^{1+}$	0.5	0.5
$M^{1+} + e^- \rightarrow M$		$E^0 = 0.86V$

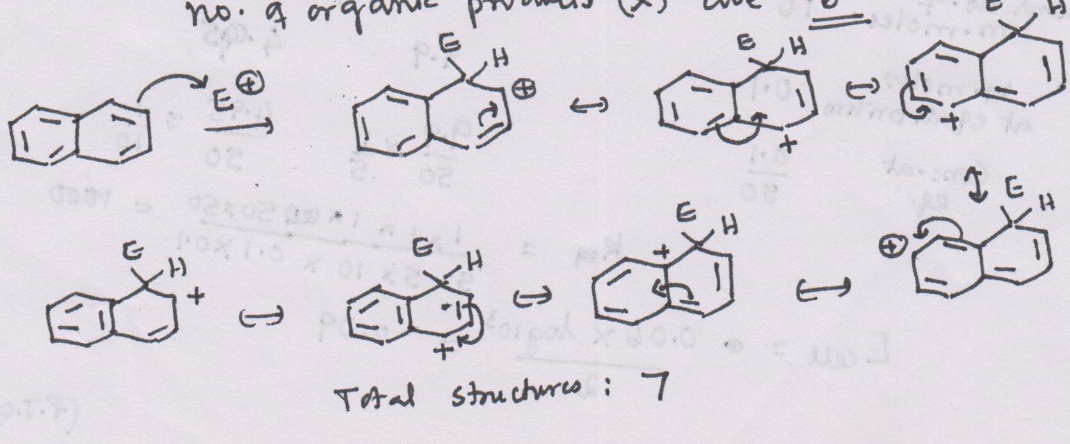
$$E^{0}_{cell} = 0.86 - 0.5 = 0.36V$$

31. A is  B is 

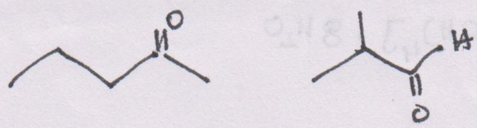
C 

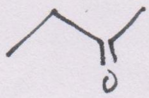


no. of organic products (X) are 6

32. 

Total structures : 7

33  aldelydes

 Ketone

$H_3C\,CH_2\,CH = CH-OH$ (Cis & Trans) , $H_3CCH_2-\overset{OH}{\overset{|}{C}} = CH_2$, $H_3C-\overset{OH}{\overset{|}{C}} = CH-CH_3$ Cis & Trans

$H_3C - \overset{CH_3}{\overset{|}{C}} = CH-OH$ enols

Total = 9

34. $C_2H_4 + 3O_2 \longrightarrow 2CO_2 + 2H_2O$

$x \qquad\qquad 2x$

$C_3H_8 + 5O_2 \longrightarrow 3CO_2 + 4H_2O$

$10-x \qquad\qquad 3(10-x)$

$2x + 3(10-x) = 24$

$x = 6$ ml

$10-x = 4$ ml

Ans: 4

35. $2Cr(OH)_3 + IO_3^- + 4\bar{O}H \longrightarrow 2CrO_4^{2-} + I^- + 5H_2O$

$\frac{1}{4} \times \frac{-d[\bar{O}H]}{dt} = \frac{-d[IO_3^-]}{dt}$

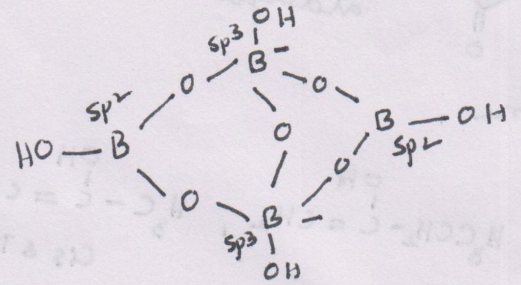
$-\frac{d[\bar{O}H]}{dt} = 4 \times \frac{-d[IO_3^-]}{dt}$

36. $PCl_5 \rightleftharpoons PCl_3 + Cl_2$

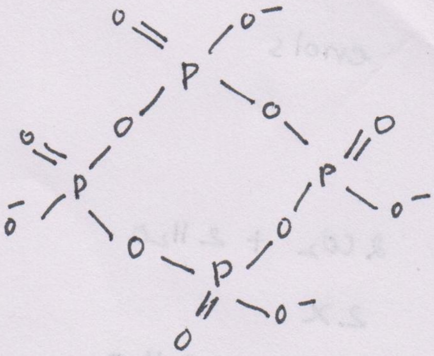
	PCl_5	PCl_3	Cl_2
Initial	6	0	0
moles at equilibrm	3	3	3
Conc. at eq	3/0.5	3/0.5	3/0.5
	6	6	6

$K_c = \frac{6 \times 6}{6} = 6$

37. Borax is $Na_2[B_4O_5(OH)_4] \cdot 8H_2O$



38.



39. Naturally occuring Uranium fuel is U^{235}.

It is enriched by fractional diffusion of a mixture of $U^{235}F_6$ & $U^{238}F_6$

Pu^{239} & U^{233} are synthetic fuels obtained by the action of fast neutron on U^{238} & Th^{232} resp.

$$U^{238} + \underset{\text{fast}}{{}_0n^1} \longrightarrow U^{239} \xrightarrow{-\beta} {}^{239}_{93}Np \xrightarrow{-\beta} {}^{239}_{94}Pu$$

(Np: first trans-uranic element; Pu: fuel)

$$Th^{233} + \underset{\text{fast}}{{}_0n^1} \longrightarrow Th^{233} \xrightarrow{-\beta} {}^{233}_{91}Pa \xrightarrow{-\beta} {}^{233}_{92}U$$

(U: fuel)

SOLUTION

41. **(BCD)**

Verify that $f'(0^+) = f'(0^-) = 0$

$\Rightarrow$ f is differentiable at $x = 0$. $\Rightarrow$ **(B) is correct.**

Let $f'(x) = g(x) = \begin{cases} 2x\tan^{-1}\frac{1}{x} - \frac{x^2}{1+x^2} & \text{if} \quad x \neq 0 \\ 0 & \text{if} \quad x = 0 \end{cases}$

Now, $f'(x)$ os obviously continuous at $x = 0$.

Also, $g'(0^+) = \lim\limits_{h\to 0} \dfrac{2h\tan^{-1}\left(\frac{1}{h}\right) - \frac{h^2}{1+h^2}}{h} = \pi$

$g'(0^-) = \lim\limits_{h\to 0} \dfrac{+2h\tan^{-1}\left(\frac{1}{h}\right) - \frac{h^2}{1+h^2}}{-h} = -\pi$

Hence $g'(0)$ i.e., $f''(0)$ does not axist. Ans.]

42. **(AC)**

Here $\Delta = \begin{vmatrix} 1 & 2 & 1 \\ 2 & 1 & 1 \\ 4 & 5 & 3 \end{vmatrix} = 0$

$\Rightarrow$ system of equations has either infinitely many solutions or no solution.

Now, $\Delta_1 = \begin{vmatrix} 1 & 2 & 1 \\ \alpha & 1 & 1 \\ \alpha^2 & 5 & 3 \end{vmatrix} = \alpha^2 - \alpha - 2 = (\alpha - 2)(\alpha + 1)$

$\Delta_2 = \begin{vmatrix} 1 & 1 & 1 \\ 2 & \alpha & 1 \\ 4 & \alpha^2 & 3 \end{vmatrix} = \alpha^2 - \alpha - 2(\alpha - 2)(\alpha + 1)$

and $\Delta_3 = \begin{vmatrix} 1 & 2 & 1 \\ 2 & 1 & \alpha \\ 4 & 5 & \alpha^2 \end{vmatrix} = -3(\alpha^2 - \alpha - 2) = -3(\alpha - 2)(+1)$

$\therefore$ The system has infinitely many solution when $\alpha = -1$ or 2 and no solution when $\alpha \in R - \{-1, 2\}$

$\Rightarrow$ (A) and (C) are correct. Ans.]

Aliter :

Given, $x + 2y + z = 1$ .... (1)

$2x + y + z = \alpha$ .... (2)

$4x + 5y + 3z = \alpha^2$ .... (3)

(1) and (2) $\Rightarrow$ $x - y = \alpha - 1 \Rightarrow x = y - 1 + \alpha$ .... (4)

$\therefore$ Putting above value of x in equation (1), we get

$\therefore$ $z = 1 - x - 2y = 1 - y + 1 - \alpha - 2y$

$z = 2 - 3y - \alpha$ .... (5)

Now, putting x and z from equation (4) and equation (5) in equation (3), we get

$$4(y - 1 + \alpha) + 5y + 3(2 - 3y - \alpha) = \alpha^2$$

$$4y - 4 + 4\alpha + 5y + 6 - 9y - 3\alpha = \alpha^2$$

$$\alpha^2 - \alpha - 2 = 0$$

$$(\alpha - 2)(\alpha + 1) = 0$$

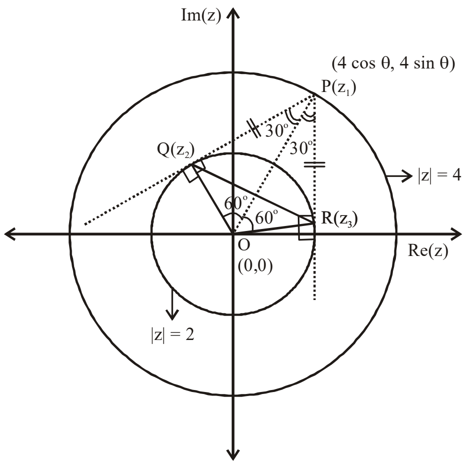
$\therefore$ If $\alpha = 2$ or $-1 \Rightarrow$ infinite solution

and $\alpha \in R - \{-1, 2\} \Rightarrow$ No solution.

$\Rightarrow$ (A) and (C) are correct. Ans.]

43. **(CD)**

$\therefore$ From above figure, $\cos(\angle POR) = \dfrac{OR}{OP}\quad \dfrac{2}{4} = \dfrac{1}{2}$



$\Rightarrow \angle POR = \dfrac{\pi}{3} = \angle POQ \quad \Rightarrow \quad \angle OPR = \angle OPQ = 30°$

$\Rightarrow \angle QPR = 60°$ .... (1)

Also, in ΔPQR, $PQ = PR$ .... (2)

$\therefore$ From (1) and (2), we get

ΔPQR is equilateral $\Rightarrow |z| = 2 \Rightarrow$ **(A) is incorrect.**

Also, PQOR are concylic and $\angle OQP$ and $\angle ORP = 90°$

So, circumcentre of ΔPQR passes through O(0, 0) and OP is diameter of it.

So, circumcentre of ΔPQR = mid point of OP

$$= \left(\frac{0 + 4\cos\theta}{2}, \frac{0 + 4\sin\theta}{2}\right) = (2\cos\theta, 2\sin\theta)$$

= centroid of ΔPQR [As, ΔPQR is equilateral.]

$\therefore$ The locus of cetroid of ΔPQR is $|z| = 2 \Rightarrow$ **(B) is incorrect.**

Also, circumradius of $\Delta PQR = \frac{OP}{2} = \frac{4}{2} = 2 \Rightarrow$ **(C) is correct.**

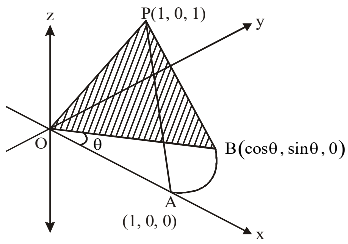
As, $r = \frac{R}{2} = \frac{2}{2} = 1$ (As, ΔPQR is equilateral.)

$\Rightarrow$ radius of circle inscribed in ΔPQR is $1. \Rightarrow$ **(D) is correct.**

44. **(ACD)**

Volume of tetrahedron $OAPB = \frac{1}{6}\left|\left[\overrightarrow{OA}\ \overrightarrow{OB}\ \overrightarrow{OP}\right]\right|$

$$= \frac{1}{6}\left|\begin{vmatrix} 1 & 0 & 0 \\ \cos\theta & \sin\theta & 0 \\ 1 & 0 & 1 \end{vmatrix}\right| = \left|\frac{\sin\theta}{6}\right|$$



Also, $\vec{n} = \overrightarrow{OP} \times \overrightarrow{OB} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 0 & 1 \\ \cos\theta & \sin\theta & 0 \end{vmatrix} = -\sin\theta\hat{i} - \hat{j}(-\cos\theta) + \hat{k}(\sin\theta)$

As, $\vec{r}\cdot\vec{n} = 0 \Rightarrow -x\sin\theta + y\cos\theta + z\sin\theta = 0$

$$d = \frac{|-\sin\theta|}{\sqrt{(\sin^2\theta + \cos^2\theta + \sin^2\theta)}} = \frac{|\sin\theta|}{\sqrt{(1+\sin^2\theta)}}$$. Ans.]

45. **(ABCD)**

Given $f(x) = \sin x\int_0^{\pi} f(t)dt + \cos x\int_0^{\frac{\pi}{2}} f'(t)dt + 2$ or $f(x) = a\sin x + b\cos x + 2$

where $a = \int_0^{\pi}(a\sin t + b\cos t + 2)dt = 2a + 2\pi \quad \Rightarrow a = -2\pi$

and $\quad b = \int_0^{\frac{\pi}{2}}(a\cos t - b\sin t)dt = a - b \quad$ or $\quad b = \frac{a}{2} = -\pi$

$\Rightarrow \qquad f(x) = -2\pi\sin x - \pi\cos x + 2.$

Now verify alternatives.]

46. **(ABC)**

$\alpha_1.\gamma_2 = \beta_1.\beta_2 = \gamma_1\alpha_2 = 36$

47. **(ABC)**

Let $A=\alpha-\beta$, $B=\alpha$, $C=\alpha+\beta$

Now, $A+B+C=\frac{3\pi}{4}\Rightarrow 3\alpha=\frac{3\pi}{4}\Rightarrow\alpha=\frac{\pi}{4}$.

Also, $\sin\alpha\sin(\alpha-\beta)\sin(\alpha+\beta)=\frac{12}{25\sqrt{2}}\ \Rightarrow\sin(\alpha-\beta)\sin(\alpha+\beta)=\frac{12}{25}$

$\Rightarrow 2\sin(\alpha-\beta)\sin(\alpha+\beta)=\frac{24}{25}\ \Rightarrow\ \cos 2\beta-\cos 2\alpha=\frac{24}{25}\quad\left(\text{As}, \alpha=\frac{\pi}{4}\right)$

So, $\cos 2\beta=\frac{24}{25}=\cos(C-A)$

As, $\cos 2\beta=\frac{24}{25}\Rightarrow\tan^2\beta=\frac{1-\cos 2\beta}{1+\cos 2\beta}=\frac{1-\frac{24}{25}}{1+\frac{24}{25}}\Rightarrow\tan\beta=\pm\frac{1}{7}$.

Now, $\tan A=\tan(\alpha-\beta)=\frac{\tan\alpha-\tan\beta}{1+\tan\alpha\cdot\tan\beta}=\frac{1\pm\frac{1}{7}}{1\mp\frac{1}{7}}=\frac{4}{3}\text{ or }\frac{3}{4}.\quad\left(\text{As}, \alpha=\frac{\pi}{4}\right)$

Also, $\cos A\cos C=\cos(\alpha-\beta)\cos(\alpha+\beta)=\cos^2\alpha-\sin^2\beta$

$$=\cos^2\frac{\pi}{4}-\left(\frac{1-\cos 2\beta}{2}\right)=\frac{1}{2}-\frac{1}{2}+\frac{1}{2}\left(\frac{24}{25}\right)=\frac{12}{25}.$$

Also, $\tan^2\left(\frac{C-A}{2}\right)=\frac{1-\cos(C-A)}{1+\cos(C-A)}=\frac{1-\frac{24}{25}}{1+\frac{24}{25}}=\frac{1}{49}\Rightarrow\cot^2\left(\frac{C-A}{2}\right)=49.$

Also, $(A+C)=\frac{3\pi}{4}-\frac{\pi}{2}=\frac{\pi}{2}\Rightarrow\sin(A+C)+\text{coses}(A+C)=1+1=2.$

Now verify alternatives. Ans.]

48. **(ABCD)**

We have

$(B^TAB)^T=B^TA^T(B^T)^T=B^TA^TB=B^TAB$, iff A is symmetric.

$\therefore B^TAB$ is symmetric iff A is symmetric.

Also, $(B^TAB)^T=B^TA^TB=-B^TAB$, iff A is skew-symmetric matrix.

49. **(BD)**

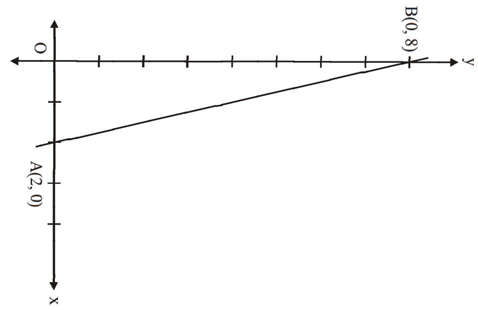
Let the line be

$$\frac{x}{a}+\frac{y}{b}=1,\ a,\ b>0$$

where $\frac{1}{a}+\frac{4}{b}=1\quad\Rightarrow\quad\frac{4}{b}=1-\frac{1}{a}\quad\Rightarrow\quad\frac{4}{a}=\frac{a-1}{a}$

$$b = \frac{4a}{a-1}$$

Now,



$$2A = ab = a\left(\frac{4a}{a-1}\right) = \frac{4\left(a^2-1+1\right)}{a-1} = 4\left[\left(a+1\right)+\frac{1}{a-1}\right]$$

$$2\frac{dA}{da} = 4\left[1-\frac{1}{\left(a-1\right)^2}\right] = 0$$

$a-1=1$ or -1

$a = 2;$ $b = 8, m = -4$

$$A\Big]_{min} = \frac{16}{2} = 8$$

Equation is $\left(y-4\right) = -4\left(x-1\right)$ $\Rightarrow$ $y-4=-4x+4$ $\Rightarrow$ $4x+y=8$ Ans.

Also, $r = \dfrac{\Delta}{s} = \dfrac{\frac{1}{2}\times 2\times 8}{\frac{2+8+\sqrt{68}}{2}} = \dfrac{8}{5+\sqrt{17}} = 5-\sqrt{17}$

Now verify alternatives.]

50. **(ACD)**

$S = \{HTH, THH, TTH, HHH, HTT, THT, TTT, HHT\}$

$A = \{HTH, HHH, HTT, HHT\}$

$B = \{HTH, TTH, HTT, TTT\}$

$C = \{HTH, THH, TTH, HHH\}.$

$D = \{TTH, HTT, THT\}$

$E = \{HHH, TTT\}$

$A \cap B \cap C = \{HTH\}$

As, $P\left(C\right) = \frac{1}{2}$, $P\left(D\right) = \frac{3}{8}$, $P\left(E\right) = \frac{1}{4}$

$\Rightarrow$ $2P\left(D\right) = \frac{3}{4} = P\left(C\right) + P\left(E\right)$ $\Rightarrow \left(A\right)$ **is correct**

Also, $A \cup B \cup C = \{HTH, THH, TTH, HHH, HTT, TTT, HHT\}$

$\Rightarrow$ $A \cup B \cup C \neq S$, as THT is not incuded in $A \cup B \cup C$.

$\therefore$ A, B, C are not exhaustive $\Rightarrow$ $\left(B\right)$ **is correct**

Also, $P(A\cap B\cap C)=\frac{1}{8}P(A)P(B)P(C)$

$\Rightarrow$ A, B, C are independent events. $\Rightarrow$ (C) **is correct**

Note that, $P(A)=\frac{1}{2}=P(B)=P(C)$

$\Rightarrow$ A, B, C are equally likely. $\Rightarrow(D)$ **is correct** Ans]

51. **(2)**

Equation of S is $(x-1)^2+(y-1)^2+\lambda(x+y-2)=0$

$x^2+y^2+x(\lambda-2)+2(1-\lambda)=0$ .....(1)

$x^2+y^2+2x+2y-2=0$ ......(2)

Given (1) nad (2) are orhtogonal, so

$$\frac{2(\lambda-2)}{2}(1)+2\left(\frac{\lambda-2}{2}\right)(1)=2(1-\lambda)-2\Rightarrow\lambda-2+\lambda-2=-2\lambda$$

$$4\lambda=4\Rightarrow\lambda=1$$

Hence equation of S is $x^2+y^2-x-y=0$.

Now, length of tangent from (2, 2) is $\sqrt{4+4-2-2}=2.$ Ans]

52. **(3)**

$$A_n=\left[\frac{2i+j}{3^{2n}}\right]=\frac{1}{3^{2n}}[2i+j]\backslash$$

$$3^n A_n=\frac{1}{3^n}[2i+j]$$

$$Tr.(3^n A_n)=\frac{1}{3^n}(3+6+9)=\frac{18}{3^n}\ \forall n\in N$$

$$\therefore Tr.(3A_1+3^2A_2+......+3^nA_n)=Tr\cdot(3A_1)+Tr\cdot(3^2A_2)+.....+Tr\cdot(3^nA_n)$$

$$=6+2+\frac{2}{3}+......+\frac{18}{3^n}$$

$$\therefore\quad l=\lim_{n\to\infty}tr(3A_1+3^2A_2+.....+3^nA_n)$$

$$=\lim_{n\to\infty}\left(6+2+\frac{2}{3}+.....+\frac{18}{3^n}\right)=\frac{6}{1-\frac{1}{3}}=9$$

$|||^{ly}$ $\quad Tr\cdot(2^nB_n)=\frac{1}{2^n}(2+4+6)=\frac{12}{2^n}$

$$Tr\cdot(2B_1+2^2B_2+.....+2^nB_n)=tr(2B_1)+tr(2^2B_2)+......+tr(2^nB_n)$$

$$=\frac{12}{2}+\frac{12}{2^2}+.....+\frac{12}{2^n}$$

$\therefore \quad m = \lim_{n\to\infty} tr\left(2B_1 + 2^2B^2 + \dots + 2^nB_n\right) = \lim_{n\to\infty}\left(\frac{12}{2}+\frac{12}{2^2}+\dots+\frac{12}{2^n}\right) = \frac{6}{1-\frac{1}{2}} = 12$

$\therefore \quad l + m = 9 + 12 = 21$ Ans]

53. **(7)**

Given, $\quad f(x) = x\int_0^x 3^t\left(3^t-4\right)dt - \int_0^x 3^t\cdot t\left(3^t-4\right)dt$

For maxima / minima, we have

$f'(x) = 0$

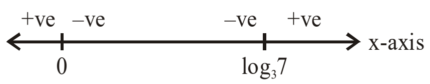
$\Rightarrow \quad \int_0^x 3^t\left(3^t-4\right)dt + x\cdot 3^x\left(3^x-4\right) - 3^x\cdot x\left(3^x-4\right)$

$\therefore \quad f'(x) = \int_0^x 3^t\left(3^t-4\right)dt$

$f'(x) = \frac{1}{2\,ln 3}\left(3^{2x} - 8\cdot 3^x + 7\right)$

$f'(x) = \frac{1}{2\,ln 3}\left(3^x-1\right)\left(3^x-7\right)$

$f'(x) = 0 \quad \Rightarrow \quad x = 0, \log_3 7$



sign scheme of f '(x)

$\therefore \quad x = \log_3 7$ is the point of minima.

Hence, $3^a = 3^{\log_3 7} = 7.$ Ans.]

54. **(7)**

Let $\quad z = r\,e^{i\theta}$; $|z| = r$; $\arg z = \theta$

Now, $\left|2z + \frac{1}{z}\right|^2 = 1 \qquad$ (squaring the given relation)

$\Rightarrow \quad \left(2z+\frac{1}{z}\right)\left(2\bar{z}+\frac{1}{\bar{z}}\right) = 1 \quad \Rightarrow \quad 4z\bar{z} + 2\left(\frac{z}{\bar{z}}+\frac{\bar{z}}{z}\right) + \frac{1}{z\bar{z}} = 1$

$4r^2 + 2\left(e^{i2\theta} + e^{-i2\theta}\right) + \frac{1}{r^2} = 1 \quad \Rightarrow \quad 4r^2 + \frac{1}{r^2} + 4\cos 2\theta = 1$

$\left(2r - \frac{1}{r}\right)^2 + 4 + 4\left(1 - 2\sin^2\theta\right) = 1 \quad \Rightarrow \quad \left(2r-\frac{1}{r}\right)^2 3 + 4 = 8\sin^2\theta$

$\therefore 8\sin^2\theta = 7 + \left(2r - \frac{1}{r}\right)^2$

Hence, $8\sin^2\theta\Big|_{min} = 7$ when $2r = \frac{1}{r}$ i.e. $r^2 = \frac{1}{2}$. Ans.]

55. **(5)**

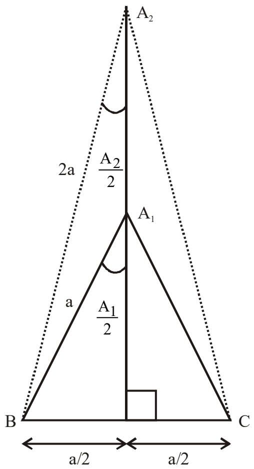
$$\sin\frac{A_1}{2}=\frac{1}{2} \qquad (A_1 = 60°)$$

$$\sin\frac{A_2}{2}=\frac{a}{2\times 2a}=\frac{1}{4}$$

$$\sin\frac{A_3}{2}=\frac{a}{2\times 4a}=\frac{1}{8}$$

$$\vdots \quad \vdots \qquad \vdots$$

$$\sin\frac{A_n}{2}=\frac{a}{2\times 2^{n-1}a}=\frac{1}{2^n}$$



$$\sum_{n=1}^{\infty}(1-\cos A_n)=2\sum_{n=1}^{\infty}\left(\sin^2\frac{A_n}{2}\right)=\lim_{n\to\infty}\left(\frac{1}{2^2}+\frac{1}{4^2}+\frac{1}{8^2}+\ldots\ldots+\frac{1}{2^{2n}}\right)=\frac{2\times\frac{1}{2^2}}{1-\frac{1}{4}}=\frac{2}{3}.$$ Ans.]

56. **(4)**

$$I=\int_1^2\left(\cot^{-1}\sqrt{x-1}\right)^2 dx \text{ ;Put, } \quad x-1=t^1 \Rightarrow dx = 2t\,dt$$

$$=2\int_0^1 \underset{I}{t}\underbrace{\left(\cot^{-1}t\right)^2}_{\substack{II\\(I.B.P)}}dt=2\left[\left(\left(\cot^{-1}t\right)^2\frac{t^2}{2}\right)_0^1+\int_0^1\left(\frac{2\cot^{-1}t}{1+t^2}\right)\frac{t^2}{2}dt\right]$$

$$=2\left[\left(\frac{\pi^2}{32}-0\right)+\int_0^1\left(\left(1+t^2\right)-1\right)\frac{\cot^{-1}t}{1+t^2}dt\right]=2\left[\frac{\pi^2}{32}+\int_0^1 1\cdot\underset{(II)}{\cot^{-1}}\underbrace{\underset{(I)}{t}}_{(I.B.P)}\,dt-\int_0^1\frac{\cot^{-1}t}{1+t^2}dt\right]$$

$$=2\left[\frac{\pi^2}{32}+\left(t\cot^{-1}t\right)_0^1+\int_0^1\frac{t}{1+t^2}dt+\left[\frac{1}{2}\left(\cot^{-1}t\right)^2\right]_0^1\right]=2\left[\frac{\pi^2}{32}+\frac{\pi}{4}+\frac{1}{2}\ln 2+\frac{1}{2}\left(\frac{\pi^2}{16}-\frac{\pi^2}{4}\right)\right]$$

$$=2\left[-\frac{\pi^2}{16}+\frac{\pi}{4}+\frac{1}{2}\ln 2\right]=\frac{-\pi^2}{8}+\frac{\pi}{2}+\ln 2.$$

Hence, $\dfrac{\pi^2+8I-8\ln 2}{\pi}=4.$ Ans.]

57. **(8)**

$x^2+y^2+y-1+k(x+y-1)=0$ .... (1)

Hencve C passes through the intersection of $x^2+y^2+y-1=0$ and $y=1-x$

Now proceed.]

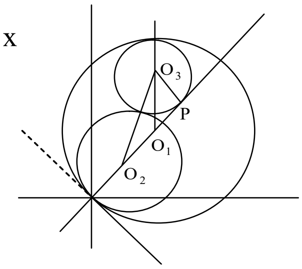
58. **(8)**

Let O_1 and O_2 be the centres of the circles of radius R = 5 and r = 3 respectively and O_3 be the centre of the third circle. Let x be the radius of the third circle and P be the point of tangency of the circle and the diameter O_1O_2.

$$O_2O_3^2 = O_3P^2 + (O_2O_1 + \sqrt{O_1O_3^2 - O_3P^2})^2$$

since $O_2O_3 = r + x$ $\quad O_3P = x$, $O_2O_1 = R - r$, $O_1O_3 = R - x$

$$\therefore \quad (r+x)^2 = x^2 + (R - r + \sqrt{(R-x)^2 - x^2})^2$$



solving we get $\quad x = \dfrac{4Rr(R-r)}{(R+r)^2} \quad = \dfrac{4 \times 5 \times 3 \times 2}{8^2} = \dfrac{15}{8}$

$$\frac{2p-1}{p} = \frac{15}{8} \qquad \Rightarrow p = 8$$

59. **(A)$\rightarrow$(Q); (B)$\rightarrow$(R); (C)$\rightarrow$(S); (D)$\rightarrow$(Q)**

(A) Let first term of G.P. be a and common ratio is r.

Now, $\dfrac{a(1-r^{201})}{1-r} = 625$ .... (1)

Also, $\displaystyle\sum_{i=0}^{201} \frac{1}{a_i} = \frac{1}{a_1} + \frac{1}{a_2} + \ldots\ldots + \frac{1}{a_{201}}$

$$= \frac{1}{a} + \frac{1}{ar} + \frac{1}{ar^2} + \ldots\ldots + \frac{1}{ar^{200}} = \frac{\frac{1}{a}\left(\left(\frac{1}{r}\right)^{201} - 1\right)}{\left(\frac{1}{r} - 1\right)} = \frac{1}{a}\left(\frac{1-r^{201}}{1-r}\right) \cdot \frac{1}{r^{200}}$$

$$= \frac{1}{a} \times \frac{625}{a} \times \frac{1}{r^{200}} \text{ (Using (1))} = \frac{625}{(ar^{100})^2} = \frac{625}{(a_{101})^2} = \frac{625}{625} = 1. \text{ Ans.]}$$

(B) Consider $\displaystyle\lim_{x \to \infty}\left(\sqrt{x^2+ax} - \sqrt{x^2+bx}\right) \quad (\infty - \infty)$

Rationalisation, $\displaystyle\lim_{x \to \infty} \frac{(a-b)x}{\sqrt{x^2+ax} - \sqrt{x^2+bx}} = \lim_{x \to \infty} \frac{(a-b)x}{x\left[\sqrt{1+\frac{a}{x}} + \sqrt{1+\frac{b}{x}}\right]} = \frac{a-b}{2} = 2$

$\Rightarrow (a-b) = 4$ .... (1)

Now, $\displaystyle\lim_{x \to a} \frac{\sqrt{x-b} - \sqrt{a-b}}{x^2 - a^2} = \frac{1}{4a\sqrt{a-b}} = \frac{1}{64} \Rightarrow \frac{1}{4a\sqrt{4}} = \frac{1}{64} \Rightarrow \frac{1}{8a} = \frac{1}{64}$

$\Rightarrow \quad a = 8$ and $b = 4 \quad \Rightarrow \quad a/b = 2$. Ans.]

(C) We have

$$x^2 - 2mx + m^2 - 1 = 0$$

$$\Rightarrow \quad (x-m)^2 - 1 = 0 \Rightarrow (x-m+1)(x-m-1) = 0$$

$$\Rightarrow \quad x = m-1, m+1$$

Now, $-2 < m-1$ and $m+1 < 4$

$\Rightarrow \quad -1 < m < 3.$ Ans.]

(D) Clearly, $f(x)=\begin{cases} \tan(-x); x \in \left(\frac{-\pi}{2}, 0\right] \\ |x| = x; x \in \left(0, \frac{\pi}{2}\right) \end{cases}$ Graph Pending

$\therefore$ Number of points of non-defferentiability of $f(x)$ is 1 i.e. $x=0$]

60. **(A)$\rightarrow$(R); (B)$\rightarrow$(T); (C)$\rightarrow$(Q); (D)$\rightarrow$(P, Q, T)**

(A) Given curves are $x^3+kxy^2=-2$ and $3x^2y-y^3=2$

Now, $y' = \frac{-(3x^2+ky^2)}{2kxy}$ and $y' = \frac{-2xy}{x^2-y^2}$

As $m_1 \times m_2 = -1 \Rightarrow \frac{(3x^2+ky^2)}{2kxy} \times \frac{2xy}{x^2-y^2} = -1 \quad \Rightarrow 3x^2+ky^2+kx^2-ky^2=0 \Rightarrow (3+k)x^2=0$

Hence, $k=-3$

So, absolute value of k equals 3.]

(B) $2\int_0^1 (2+2x)^5\,dx = 64\int_0^1 (1+x)^5\,dx = 64\cdot\left.\frac{(1+x)^6}{6}\right]_0^1 = \frac{64\cdot 63}{6} = 672$]

(C) Given, $A + A^T = I$

So, $\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix} + \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 2\cos\alpha & 0 \\ 0 & 2\cos\alpha \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\therefore \quad 2\cos\alpha = 1 \Rightarrow \cos\alpha = \frac{1}{2}$

$\therefore \quad \alpha = \frac{\pi}{3}$ or $\frac{5\pi}{3}$

So, number of values of $\alpha \in (0, \pi)$ are two.]

(D) Let $I = \int_{-6}^{4} \frac{x^2+14x+49}{x^2+2x+37}dx$ ......(1)

Also, $I = \int_{-6}^{4} \frac{x^2+10x+25}{x^2+2x+37}dx$ ....(2) (By using king property)

$\therefore$ Adding (1) and (2), we get

$2I = \int_{-6}^{4} \frac{2(x^2+2x+37)}{(x^2+2x+37)}dx \Rightarrow I = 10.$ Ans.]

SOLUTION

1. **(BCD)**

$< x >= u \ if \ sin \ wt = 0$

2. **(BC)**

Volume decreases during melting so work done on atomosphere is negative Volume increases during evaporation so work done on atmosphere is positive

3. **(BCD)**

$Q_1 = Q_4 \ \& \ Q_2 + Q_3 = 0 \quad V = \left|\frac{Q_2}{C}\right| = \left|\frac{Q_3}{C}\right| = \frac{1}{2C} \quad Q_1 + Q_2 \ - \ Q_3 + Q_4$

4. **(BC)**

Y hears a direct pulse, a reflected pulse from surface of water and a reflected pulse from bottom of the ocean

5. **(BC)**

$X_C = \frac{1}{\omega C} = \frac{20}{\pi}$

$X_L = \omega L = \pi$

$X_C > X_L$

As ω increases X_C decreases, X_L Increases

6. **(ABC)**

$\frac{1}{2}\text{mv}^2 = \text{mg}l(1-\cos\theta)$

1D elastic colic between masses.

7. **(BC)**

Blue filter and green filter allows blue and green light only across it. So fringe pattern will be seen in both case. Fringe width for blue will be less than that at green

8. **(AC)**

$w^2 A = 5\pi^2 \qquad i \quad v^2 = w^2 \ A^2 - x^2 \qquad ii \qquad 3\pi^{\ 2} = w^2 \ A^2 - 4^2 \quad A = 5cm, T = 2sec$

9. **(B)**

$dN = C\sqrt{E} \ \ E_{max} - E^{\ 2} dE$ Average energy of electrons $\frac{\int_0^{E_{max}} EdN}{\int_0^{E_{max}} dN} = \frac{E_{max}}{3}$

Heat evolved $= \frac{E_{max}}{3} \times 1milicurie = 2.03 \times 10^6 meV / sec.$

10. **(C)**

Most probable kE at emitted electrons $= \frac{d}{dE}\left(\frac{dN}{dE}\right) = 0 = \frac{E_{max}}{5} = 33keV$.

11. **(B)** 12. **(B)**

$$\frac{q}{C} + \frac{Ldi}{dt} = \varepsilon$$

$$\frac{i}{C} + \frac{Ld^2i}{dt^2} = 0$$

$$\frac{d^2i}{dt^2} = \frac{-i}{LC} = -\omega^2 i$$

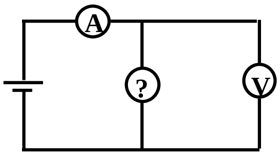
$$i = i_0 \sin(\omega t + \phi) \qquad \therefore \phi = 0$$

$$\frac{di}{dt} = i_0\omega\cos(\omega t + \phi) \therefore \frac{q}{C} = i_0\omega$$

$$i = \frac{\varepsilon}{\omega L}\sin\omega t$$

$$q = \frac{-\varepsilon}{\omega^2 L}\cos\omega t + C$$

$$q = \frac{\varepsilon}{\omega^2 L}(I - \cos\omega t)$$



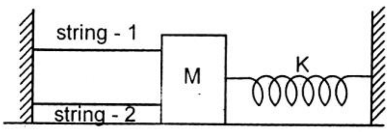
13. **(2)**

$$Y = \frac{\text{Stress}}{\text{Strain}} = \frac{T/A}{\ell/L} \Rightarrow Y = \frac{TL}{A\ell} = \frac{4TL}{\pi d^2\ell} \quad \frac{\Delta Y}{Y} = \frac{\Delta L}{L} + \frac{\Delta\ell}{\ell} + 2\frac{\Delta d}{d}$$

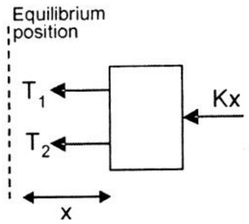
Since $\frac{4T}{\pi}$ is a constant, so it does not contribute anything to the net error.

$$\Rightarrow \frac{\Delta Y}{Y} = \frac{0.01}{110} + \frac{0.001}{0.125} + 2\left(\frac{0.00005}{0.01}\right) \Rightarrow \frac{\Delta Y}{Y} \times 100\% = 0.009 + 0.8 + 1 = 1.809\%$$

14. **(9)**



In this question the most important point to keep in mind is that there is no tension developed in string if string gets sag. In present situation if we displace the block by x towards right then both the strings get taut & spring gets compressed as a result all three components (two strings & a spring) contribute in restoring force, but when the block comes towards left of its equilibrium position then string got stretched and only the spring force is helping in bringing the block to its equilibrium position. It means equation of motion for the displacement of block on two sides of its equilibrium position is different.



For displacement towards right by x – $\frac{T_1/A}{x/\ell} = Y \Rightarrow T_1 = \frac{YAx}{\ell}$ $\frac{T_2/\frac{A}{2}}{x/\ell} = 2Y \Rightarrow T_2 = \frac{YAx}{\ell}$

Restoring force, $F = T_1 + T_2 + Kx = \left(\frac{2yA}{\ell} + K\right)x \;\Rightarrow a = \frac{1}{m}\left(\frac{2YA}{\ell} + K\right)x$ $\omega_1 = \sqrt{\frac{1}{m}\left(\frac{2YA}{\ell} + K\right)}$

$\Rightarrow T_1 = 2\pi\sqrt{\frac{M\ell}{2AY + K\ell}} = \pi\sqrt{\frac{M}{K}}$ For displacement towards left by x- the situation is similar to a spring pendulum with time period, $T_2 = 2\pi\sqrt{\frac{M}{K}}$ Required time period of oscillation is,

$$T = \frac{T_1}{2} + \frac{T_2}{2} = \pi\left[\sqrt{\frac{M\ell}{2AY + K\ell}} + \sqrt{\frac{M}{K}}\right] = \frac{3\pi}{2}\sqrt{\frac{M}{K}}$$

15. **(3)**

$$F_1 = \frac{\mu_0}{4\pi}\frac{2I_1I_2}{b}\ln(1 + b/a)$$

16. **(2)**

First send current through larger loop and calculate flux through smaller one to calculate M.

We get $\left\{\frac{\mu_o i}{2b}\right\}\pi a^2 = Mi \Rightarrow M = \frac{\mu_o \pi a^2}{2b}$ .Now for the current going through smaller loop.

Emf in larger loop = $M\frac{di}{dt} = \frac{\mu_o \pi a^2}{2b}(4t) = \frac{2\mu_o \pi a^2 t}{b}$

$$\varepsilon = \frac{q}{c} + iR \Rightarrow \frac{d\varepsilon}{dt} = \frac{i}{c} + \frac{di}{dt}R \quad \frac{2\mu_o \pi a^2}{b} = \frac{i}{c} + \frac{di}{dt}R$$

$\Rightarrow$ long time current = i when $\frac{di}{dt} = 0 = \frac{2\mu_o \pi a^2 c}{b}$

17. **(3)**

$$f_D = \left[\frac{330 + V_w}{330 + V_w - V_s \sin 30^o}\right]f$$

% increase = $\left(\frac{f_o - f}{f}\right) \times 100$

18. **(7)**

The resistance shorting the capacitor plates may be considered in parallel with the capacitor and at time $t = 0$ the capacitor behaves as a short circuit

19. **(2)**

20. **(6)**

$(2P_0)(2V_0)^y = P_C V_C^y$ ....(i)

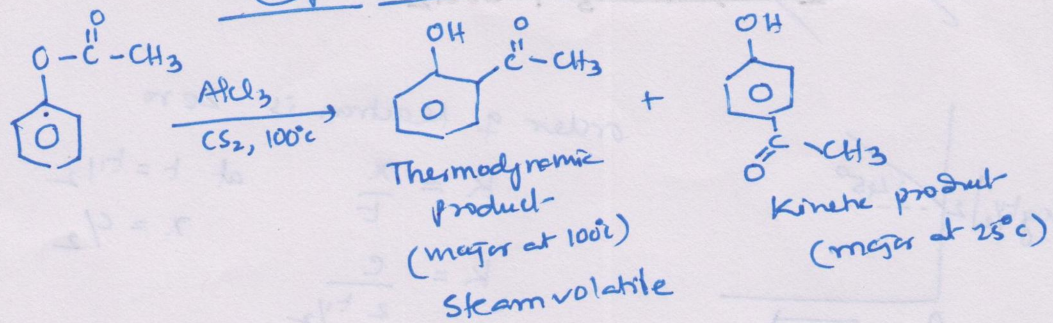
$P_0 V_0 = P_C V_C$ ....(ii)

For diatomic gas r = 7/5

$\therefore\ V_C = 2^6 V_0$

Paper II (1)

~~Key~~ Solutions

21. 

23. $P_4 + 3NaOH + 3H_2O \xrightarrow{\Delta} PH_3 + 3NaH_2PO_2$

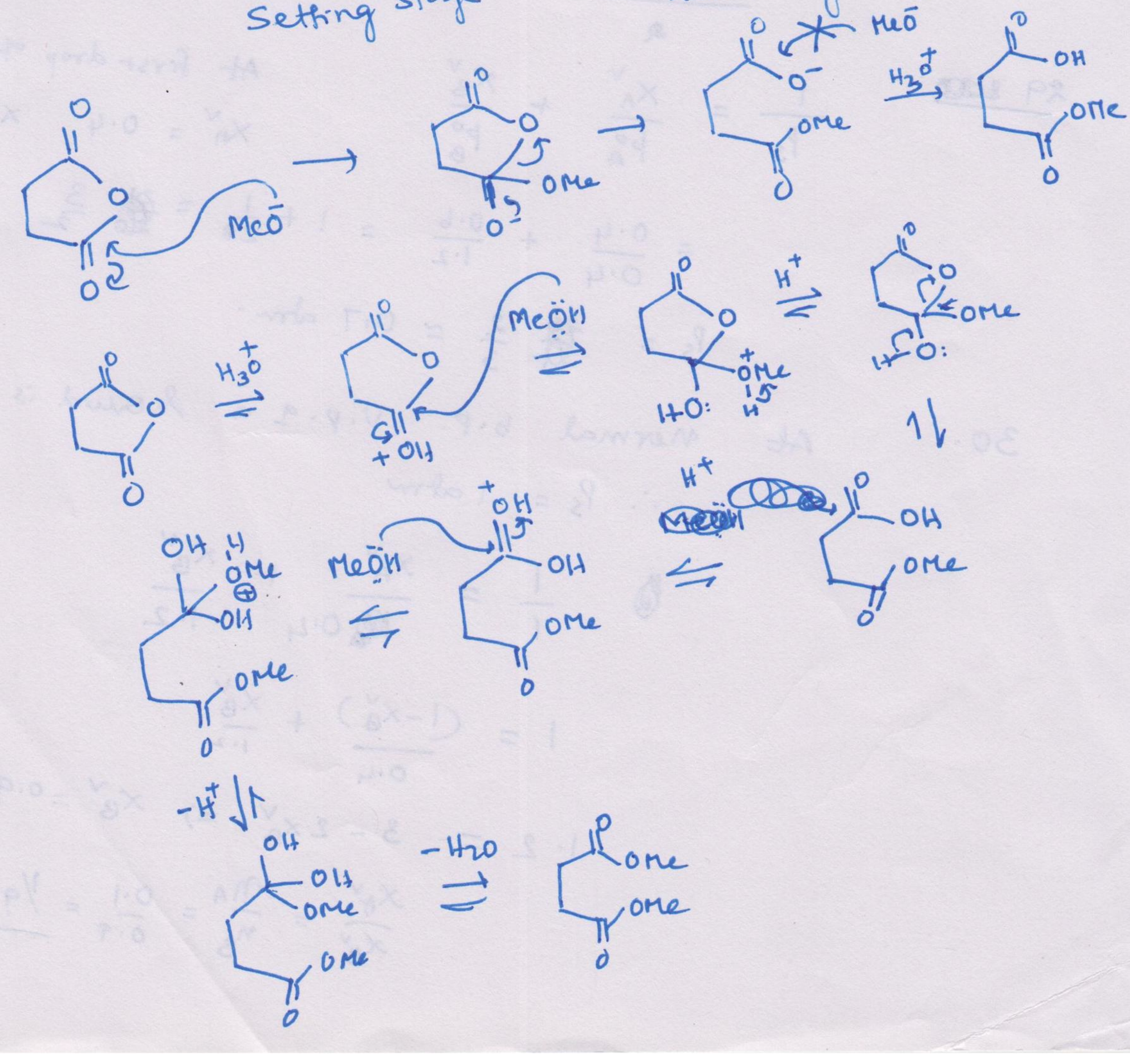
$2H_3PO_2 \xrightarrow{\Delta} H_3PO_4 + PH_3$
phosphinic acid

$4H_3PO_3 \xrightarrow{\Delta} 3H_3PO_4 + PH_3$
phosphonic acid

24. $CaSO_4 \cdot \frac{1}{2}H_2O \xrightarrow{3/2\,H_2O} CaSO_4 \cdot 2H_2O \longrightarrow CaSO_4 \cdot 2H_2O$

Plaster of Paris → orthorhombic gypsum → monoclinic gypsum

Setting stage (Plaster of Paris → orthorhombic gypsum); hardening stage (orthorhombic gypsum → monoclinic gypsum)

25. 

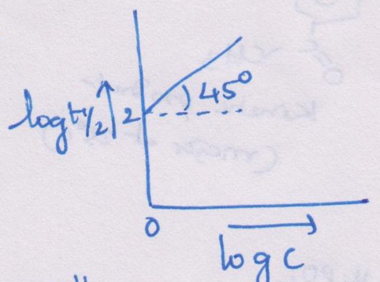
~~PAPER-I~~ / OPEN ~~TEST~~

~~Solutions~~

(2)

~~26. $PCl_5 + SO_2 \rightarrow POCl_3 + SOCl_2$~~

27.



order of reaction is zero

$k = \frac{x}{t}$ at $t = t_{1/2}$

$x = c/2$

$k = \frac{c}{2 t_{1/2}}$

$t_{1/2} = \frac{c}{2k}$

$\log t_{1/2} = \log c + \log \frac{1}{2k}$

$\log \frac{1}{2k} = \log 2$

$\frac{1}{2k} = 10^2$

$k = \frac{1}{200} = 5 \times 10^{-3}\ M s^{-1}$

$K_{eq} = \frac{2 \times 10^{-11}}{\frac{3}{2} \times \frac{3}{2} \times 10^{-10}}$

$= \frac{8}{9} \times 10^{-1}$

$= \frac{0.8}{9}$

$9 \times 9 = 81$

Comprehension

29. $\frac{1}{P_S} = \frac{X_A^V}{P_A^o} + \frac{X_B^V}{P_B^o}$

At first drop of liquid form

$X_A^V = 0.4 \quad X_B^V = 0.6$

$= \frac{0.4}{0.4} + \frac{0.6}{1.2} = 1 + \frac{1}{2} = \frac{3}{2}$

$P_S = \frac{2}{3} \simeq 0.7$ atm.

30. At normal b.p. V.p. of a liquid is 1 atm

$\therefore P_S = 1$ atm

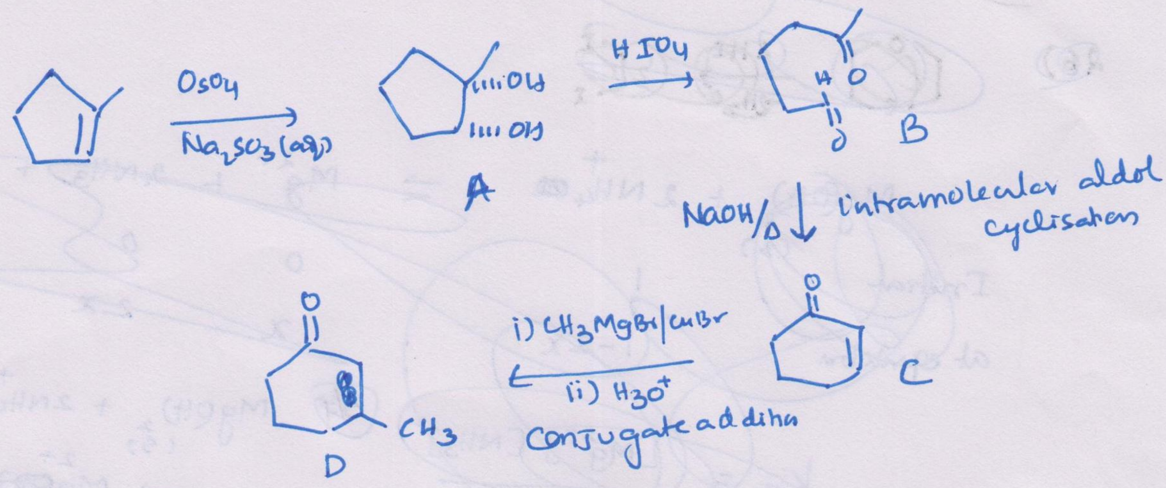
$\therefore \frac{1}{1} = \frac{X_A^V}{0.4} + \frac{X_B^V}{1.2}$

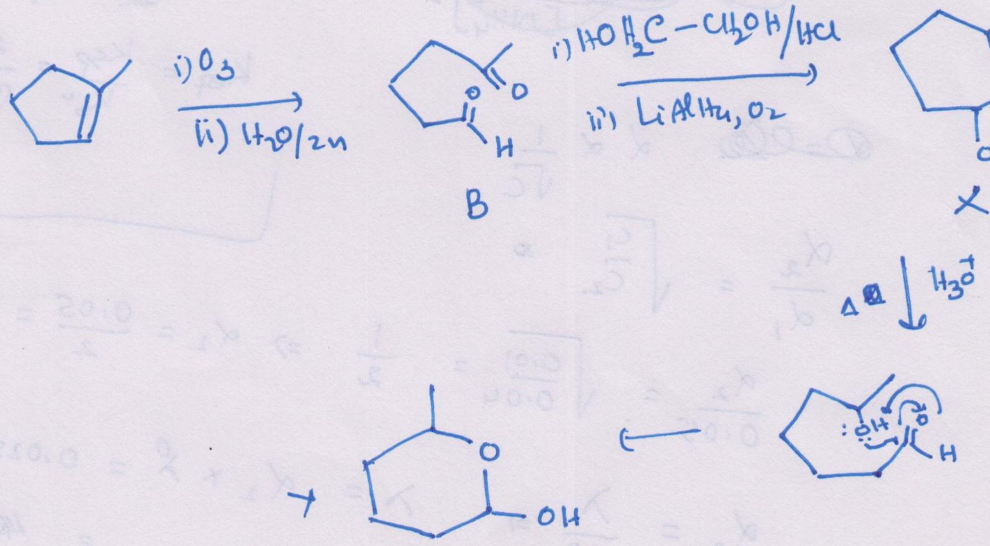
$1 = \frac{(1 - X_B^V)}{0.4} + \frac{X_B^V}{1.2}$

$1.2 = 3 - 2X_B^V \Rightarrow X_B^V = 0.9$ & $X_A^V = 0.1$

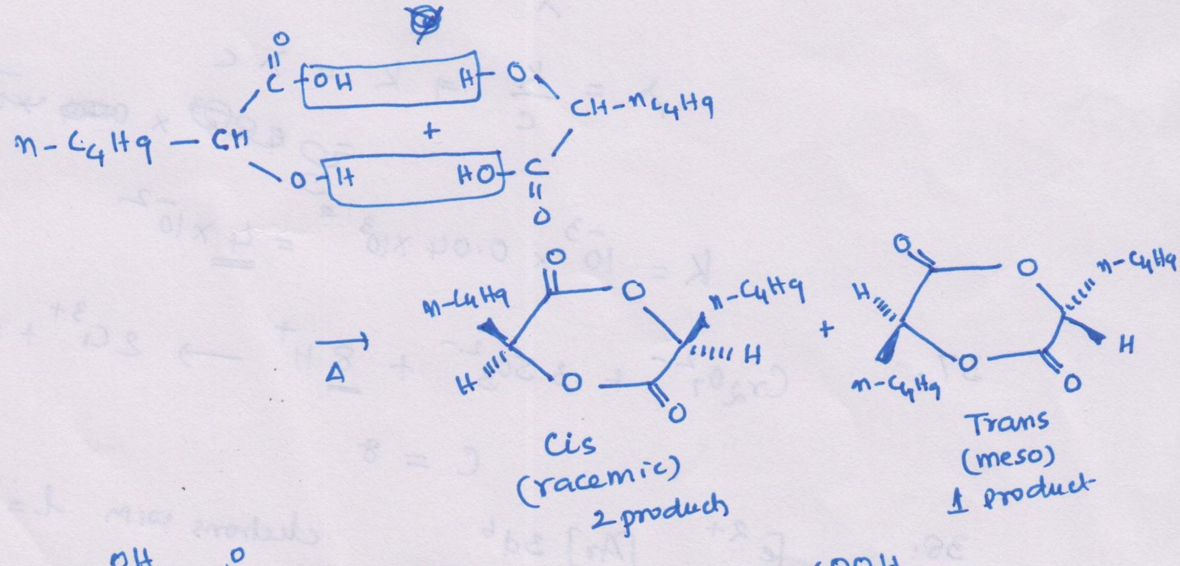
$\frac{X_A^V}{X_B^V} = \frac{n_A}{n_B} = \frac{0.1}{0.9} = 1/9$

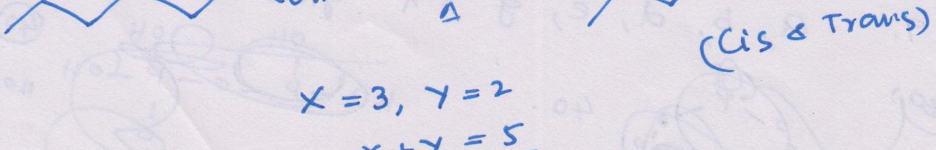
31





33.





$X = 3,\ Y = 2$

$X + Y = \underline{5}$

(4)

JEE-ADVANCE OPEN TEST

Paper-I

34. $Mg(OH)_{2(s)} + 2NH_4^+ \rightleftharpoons Mg^{2+} + 2NH_3$

$$K_{eq} = \frac{K_{sp}}{K_b^2} = \frac{2\times10^{-11}}{(1.5\times10^{-5})^2} = \frac{0.8}{9} \approx 0.09 = 9\times10^{-2}$$

36. $\alpha \propto \frac{1}{\sqrt{C}}$

$$\frac{\alpha_2}{\alpha_1} = \sqrt{\frac{C_1}{C_2}}$$

$$\frac{\alpha_2}{0.05} = \sqrt{\frac{0.01}{0.04}} = \frac{1}{2} \Rightarrow \alpha_2 = \frac{0.05}{2} = 0.025$$

$$\alpha_2 = \frac{\lambda}{\lambda^0} \Rightarrow \lambda = \alpha_2 \times \lambda^0 = 0.025 \times 4\times10^{-2} = 10^{-3}$$

$$\lambda = \frac{K}{C} \Rightarrow K = \lambda C$$

$$K = 10^{-3} \times 0.04\times10^{3} = 4\times10^{-2}$$

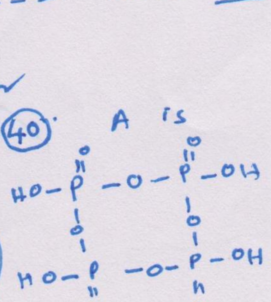
37. $Cr_2O_7^{2-} + 3SO_3^{2-} + 8H^+ \longrightarrow 2Cr^{3+} + 3SO_4^{2-} + 4H_2O$

$C = 8$

38. Fe^{2+} $[Ar]3d^6$ electrons with $l = 2$ are 6

39. a, b, d, e, g & h gives Iodoform

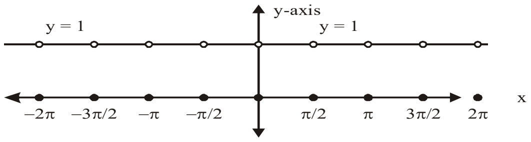
40. A is



SOLUTION

41. **(ABC)**

$$f(x)=\text{sgn}\{\sin x\}=\begin{cases}1, & -1<\sin x<1, \quad \sin x\neq 0\\ 0, & \sin x=0, \quad \pm 1\end{cases}$$



Graph of f(x) = sgn {sin x}

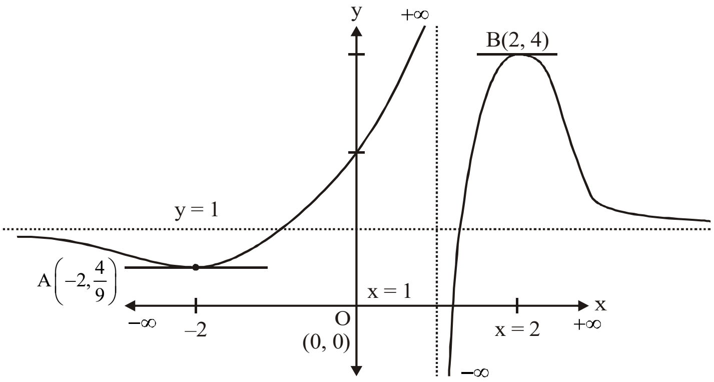
$\Rightarrow$ $f(x)$ is a periodic function.

As, $f(-x)=f(x)\Rightarrow f(x)$ is an even function.

$f(x)\in\{0,1\}\Rightarrow f(x)$ is an into function.

Note that $\lim\limits_{x\to\frac{\pi}{2}} f(x)=1$ and $\int_{-\pi}^{\pi} f(x)dx=2\pi$. Ans.]

42. **(BC)**



Graph of $f(x)=\dfrac{(x^3-4)}{(x-1)3}$

Now, verify alternatives. Ans.]

43. **(BCD)**

As, $\vec{r}_3=(\vec{r}_1\times\vec{r}_2)\times\vec{r}_2=(\vec{r}_1\cdot\vec{r}_2)\vec{r}_2-(\vec{r}_2\times\vec{r}_2)\vec{r}_1=6(2\hat{i}+\hat{j}+\hat{k})=6\hat{i}-12\hat{k}$

$\therefore$ Row 3 vector $\pm\dfrac{6(\hat{i}-2\hat{k})}{6\sqrt{5}}\cdot\sqrt{5}=\hat{i}-2\hat{k}$ or $-\hat{i}+2\hat{k}$

$$A=\begin{bmatrix}1&1&3\\2&1&1\\1&0&-2\end{bmatrix}\text{or}\begin{bmatrix}1&1&3\\2&1&1\\-1&0&2\end{bmatrix}\Rightarrow \qquad \text{Tr.}(A)=0,4.$$

Also $\left[\vec{r}_2\ \vec{r}_3\ \vec{r}_2\times\vec{r}_3\right]=\left|\vec{r}_2\right|\left|\vec{r}_3\right|\left|\vec{r}_2\times\vec{r}_3\right|=\sqrt{6}\cdot\sqrt{5}\cdot\sqrt{6}\cdot\sqrt{5}=30.$

Since $\vec{r}_1, \vec{r}_2$ and $\vec{r}_3$ are coplanar $\Rightarrow$ they are linearly dependent

$\therefore \quad \left[\vec{r}_1\times\vec{r}_2\ \vec{r}_2\times\vec{r}_3\ \vec{r}_3\times\vec{r}_1\right]-\left[\vec{r}_1\ \vec{r}_2\ \vec{r}_3\right]^2=0.$

44. **(ABD)**

(A) $P(A/B)=P(B/A) \Rightarrow P(A)=P(B)$

Now, $P(A\cup B)=P(A)+P(B)-P(A\cap B)\Rightarrow P(A\cap B)=P(A)+P(B)-P(A\cup B)$

$\Rightarrow \quad P(A\cap B)=2P(A)-1>0\ ;\ P(A)>\frac{1}{2} \quad \Rightarrow$ true

(B) $P(B)=\frac{3}{4}\ ;\ P(A/B)=\frac{1}{2}$

$\frac{P(A\cap B)}{P(B)}=\frac{1}{2} \Rightarrow P(A\cap B)=\frac{1}{2}\cdot\frac{3}{4}=\frac{3}{8}$

Now, $P(A)+P(B)-P(A\cap B)=P(A\cup B)\le 1$

$P(A)+\frac{3}{4}-\frac{3}{8}\le 1$

$P(A)\le\frac{5}{8}$

$\therefore \quad P(a)\big]_{max.}=\frac{5}{8} \Rightarrow$ true

(C) $P\left(A^C\cup B^C\right)^C=\left[(a_3+a_4)\cap(a_4+a_4)\right]^C=(a_4)^C=a_1+a_2+a_3=P(A+B)$

$P\left(A^C\cap B^C\right)^C=(a_1+a_3+a_4)^C=a_2=P(AB)$

Hence $P\left(A^C\cap B^C\right)+P\left(A^C\cup B^C\right)=P(A)+P(B)\big]=\frac{1}{2}+\frac{1}{3}=\frac{3+2}{6}=\frac{5}{6}.$

(D) Given, A is subset of B.

Now, $P(B/A)=\frac{P(B\cap A)}{P(A)}=\frac{P(A)}{P(A)}=1.$

45. **(BCD)**

Equating the coefficient of x^{20}, we get

$2^{20}-a^{20}=1\Rightarrow a=\sqrt[20]{2^{20}-1}\Rightarrow(C)$

Put $x=\frac{1}{2}$, we get

$0-\left(\frac{a}{2}+b\right)^{20}=\left(\frac{1}{4}+\frac{p}{2}+q\right)^{10}$

$\therefore \quad \left(\frac{a}{2}+b\right)^{20}+\left(\frac{1}{4}-\frac{p}{2}+q\right)^{10}=0$

$\frac{a}{2}=-b$ and $\frac{1}{4}+\frac{p}{2}+q=0$

$a+2b=0 \Rightarrow (B)$

Put, $x=0$ we get

$1-b^{20}=q^{10}$

$\Rightarrow \quad 1-\left(\frac{-\sqrt[20]{2^{20}-1}}{2}\right)^{20}=q^{10}$

$1-\frac{\left(2^{20}-1\right)}{2^{20}}=q^{10}$

$\frac{1}{2^{20}}=q^{10} \Rightarrow q=\frac{1}{4}$

Using, $\frac{1}{4}+\frac{p}{2}+q=0$

$\frac{1}{4}+\frac{p}{2}+\frac{1}{4}=0 \Rightarrow p=-1$

Hence B, C, D. Ans]

46. **(BCD)**

$f(x)=\left(\frac{\pi}{2}-\cos^{-1}(\cos x)\right)\left(\frac{\pi}{2}\sin^{-1}(\sin x)\right)$

$$=\begin{cases}\left(\frac{\pi}{2}-x\right)^2, & 0\le x\le\frac{\pi}{2}\\ \left(\frac{\pi}{2}-x\right)\left(\frac{\pi}{2}-(x-\pi)\right)=-\left(\frac{\pi}{2}-x\right)^2, & \frac{\pi}{2}<x\le\pi\\ \left(\frac{\pi}{2}-(2\pi-x)\right)\cdot\left(\frac{\pi}{2}-(\pi-x)\right)=\left(x-\frac{3\pi}{2}\right)\left(x-\frac{\pi}{2}\right)=(x-\pi)^2-\frac{\pi^2}{4}, & \pi<x\le\frac{3\pi}{2}\\ \left(\frac{\pi}{2}-(2\pi-x)\right)\cdot\left(\frac{\pi}{2}-(x-2\pi)\right)^2=\left(x-\frac{3\pi}{2}\right)\left(\frac{5\pi}{2}-x\right)=\frac{\pi^2}{4}-(x-2\pi)^2, & 3\pi<x\le 2\pi\end{cases}$$

$f(x)=\left(\frac{\pi}{2}-\cos^{-1}(\cos x)\right)\left(\frac{\pi}{2}-\sin^{-1}(\sin x)\right)$

$f(x)=\left(\frac{\pi}{2}-x\right)^2 \ \forall x\in\left[0,\frac{\pi}{2}\right],$

$f(x)=\left(\frac{\pi}{2}-x\right)\left(\frac{\pi}{2}-(\pi-x)\right)$

$f(x) = -\left(\frac{\pi}{2} - x\right); \forall x \in \left(\frac{\pi}{2}, \pi\right]$

$f(x) = (\pi - x)^2 - \frac{\pi}{4}, \pi < x \le \frac{3\pi}{2},$

$f(x) = \frac{\pi^2}{4} - (2\pi - x)^2 \ \forall \frac{3\pi}{2} < x \le 2\pi$

At $x = \pi$, $f(x)$ is not differentiable

At $x = \pi$, it is local as well as global minimum.

Range : $\left[\frac{-\pi^2}{4}, \frac{\pi^2}{4}\right],$

$$\int_0^{\frac{\pi}{2}} f(x)\,dx = \int_0^{\frac{\pi}{2}} \left(\frac{\pi}{2} - x\right)^2 dx$$

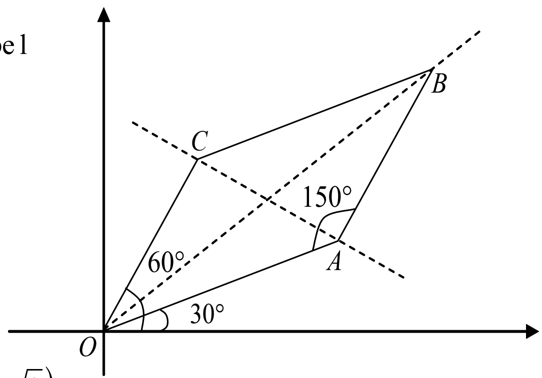
Using king

$$I = \int_0^{\frac{\pi}{2}} x^2 dx = \left.\frac{x^3}{3}\right]_0^{\frac{\pi}{2}} = \frac{\pi^3}{24}.$$ Ans]

47. **(AD)**

ΔABC is similar to ΔPQR ,which is isosceles right triangle.

48. **(AB)**



Let the length of this sides of the rhombus be l

$\Rightarrow Area = l^2 \sin 30° = 2 \Rightarrow l = 2$ unit

$OB^2 = OA^2 + AB^2 - 2OA.AB \cos 150°$

$= 4 + 4 - 2(4).\left(-\frac{\sqrt{3}}{2}\right) = 4\left(2 + \sqrt{3}\right)$

$OB = 2\sqrt{2 + \sqrt{3}}$

$B \equiv \left(\sqrt{4 + 2\sqrt{3}}, \sqrt{4 + 2\sqrt{3}}\right) \equiv \left(1 + \sqrt{3},\ 1 + \sqrt{3}\right)$

Hence coordinates of B can be $\left(1 + \sqrt{3},\ 1 + \sqrt{3}\right)$ or $\left(-1\ - \sqrt{3},\ -1 - \sqrt{3}\right)$

49. **(CD)** 50. **(ABD)**

Let degree of $P(x) = n$

$\Rightarrow$ degree of $P'(x) = n - 1$

So, L.H.S. has a degree n
and R.H.S. has a degree 2 $\Rightarrow n = 2$

Let $P(x) = ax^2 + bx + c.$

Now, $ax^2 + bx + c - (2ax + b) = x^2 + 2x + 1$

$\Rightarrow \quad a = 1; \quad b = 4 \quad$ and $\quad c = 5$

Hence, $\quad P(x) = x^2 + 4x + 5.$

Given, $$F(x) = \begin{cases} \left(\dfrac{P(x)}{10}\right)^{\frac{1}{\sin(3\sin(4x-4))}}, & x \neq 1 \\ e^{k^2-9k+40}, & x = 1 \end{cases}$$

As, $\lim\limits_{x\to 1} F(x) = e^L \left(1^\infty \text{ from}\right)$, where $L = \lim\limits_{x\to 1} \dfrac{1}{\sin(3\sin(4x-4))}\left(\dfrac{x^2+4x+5}{10} - 1\right)$

$$= \lim_{x\to 1} \frac{1}{12(x-1)} \frac{(x+5)(x-1)}{10} = \frac{6}{120} = \frac{1}{20}.$$

49. Hence, continuity at $x = 1 \Rightarrow = \lim\limits_{x\to 1} F(x) = F(1)$. So, $\quad e^{\frac{1}{20}} = e^{\frac{1}{k^2-9k+40}}$

Hence, $k^2 - 9k + 40 = 20 \Rightarrow k^2 - 9k + 20 = 0 \Rightarrow k = 4, 5.$

50. $$\int_0^1 \frac{1}{P(x)} dx = \int_0^1 \frac{1}{(x+2)^2 + 1} dx = \tan^{-1}(x+2)\Big]_0^1 = \tan^{-1}3 - \tan^{-1}2 = \tan^{-1}\frac{1}{7} = \cot^{-1}7$$

Now, use $\tan^{-1}1 + \tan^{-1}2 + \tan^{-1}3 = \pi$, to get other options.

51. **(BCD)** 52. **(CD)**

Any tangent to given parabola is

$x \cdot \sec\theta - y \cdot \tan\theta = 1 \qquad$ ...... (1)

Let $\quad$ P(h, k) be the point of intersection of tangents to the parabola $y^2 = 4x$,

so equation of chord of contact of (h, k) if $ky = 2(x+h) \qquad$ ..... (2)

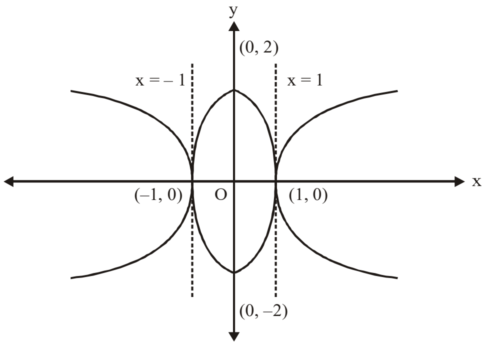
As, $\quad$ (1) and (2) are identical, so

$$\frac{\sec\theta}{2} = \frac{\tan\theta}{k} = \frac{-1}{2h} \text{ (on comparing)}$$

$\Rightarrow \quad \sec\theta = \dfrac{-1}{h}$ and $\tan\theta = \dfrac{k}{2h}$

As, $\quad \sec^2\theta - \tan^2\theta = 1$

$\Rightarrow \dfrac{1}{h^2} - \dfrac{k^2}{4h^2} = 1 \quad \Rightarrow \quad 4 - k^2 = 4h^2$



$\therefore$ Locus of P(h, k) is

$E : 4x^2 + y^2 = 4$, where is an ellipse or $\frac{x^2}{1} + \frac{y^2}{4} = 1$ ..... (3)

51. As, $e^2 = 1 - \frac{1}{4} = \frac{3}{4} \Rightarrow e = \frac{\sqrt{3}}{2} = \sin 60° \Rightarrow$ **(A) is correct**

The foci of ellipse are (0, ± be) 3i.e., (0, $\pm\sqrt{3}$) $\Rightarrow$ **(B) is correct**

Also, $l(\text{L.R}) = \frac{2a^2}{b} = \frac{2\times(1)^2}{2} = 1 \Rightarrow$ **(C) is correct.**

Note, that distance between vertices of ellipse E
= distance between (0, – 2) and (0, 2) = 4 $\Rightarrow$ **(D) is correct**

52. The circle described on vertices of ellipse E as diameter is $x^2 + y^2 = 4$ ..... (4)

As, circle in equation (4) intersects orhtogonally the circle $x^2 + y^2 - 4 - 2y + k^2 = 0$,

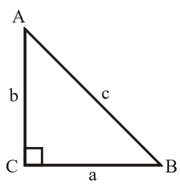
so using condition of orhtogonality, we get $0 = k^2 - 4 \Rightarrow k = \pm 2 \Rightarrow$ **(C) and (D) are correct.**

53. **(7)**

Using $\frac{c}{\sin C} = 2R \Rightarrow c = 6$ (As, R = 3 given)

$\therefore a^2 + b^2 = 36$ ....(1)

Also, $r = (s - c)\tan\frac{C}{2}$



$\Rightarrow 1 = s - c \Rightarrow 1 = \left(\frac{a+b+c}{2} - c\right) \Rightarrow 2 = a + b - c \Rightarrow a + b = 8$ .... (2)

Noe, area of triangle ABC $= \frac{1}{2}ab = \frac{1}{2}\left[\frac{(a+b)^2 - (a^2+b^2)}{2}\right] = \frac{1}{4}(64-36) = \frac{28}{4} = 7.$ Ans.]

Aliter : We know that $r_3 - r = 4R\sin\frac{C}{2}\cdot\cos\frac{A}{2}\cdot\cos\frac{B}{2} - 4R\sin\frac{C}{2}\cdot\sin\frac{A}{2}\cdot\sin\frac{B}{2}$

$\Rightarrow \quad r_3 - r = 4R\sin^2\frac{C}{2} \Rightarrow r_3 - 1 = (4\times 3)\times \sin^2 45° \Rightarrow r_3 = 7$ .... (1)

[As, r = 1, R = 3 and C= 90 90°]

Also, $\frac{c}{\sin C} = 2R$ (Using sine law) $\Rightarrow c = 6$ .... (2)

and $r_3 = s\cdot\tan\frac{C}{2} \Rightarrow 7 = s\cdot\tan 45° \Rightarrow s = 7$ .... (3)

$\therefore$ Using $r_3 = \frac{\Delta}{s-c} \Rightarrow \Delta = (s-c)r_3 = (7-6)\cdot 7 = 7.$ [Using (1), (2) and (3)]. Ans.]

54. (1)

For σ, $T_n = \frac{n}{1\cdot 3\cdot 5\ldots\ldots(2n+1)} = \frac{1}{2}\left(\frac{2n+1-1}{1\cdot 3\cdot 5\ldots\ldots(2n+1)}\right)$

$$T_n = \frac{1}{2}\left(\frac{1}{1\cdot 3\cdot 5\ldots\ldots(2n-1)} - \frac{1}{1\cdot 3\cdot 5\ldots\ldots(2n+1)}\right)$$

$$\therefore \quad T_1 = \frac{1}{2}\left(1-\frac{1}{3}\right)$$

$$T_2 = \frac{1}{2}\left(\frac{1}{3}-\frac{1}{1\cdot 3\cdot 5}\right)$$

$$\vdots \quad \vdots \quad \vdots$$

$$T_n = \frac{1}{2}\left(\frac{1}{1\cdot 3\cdot 5\ldots\ldots(2n-1)} - \frac{1}{1\cdot 3\cdot 5\ldots\ldots(2n+1)}\right)$$

$$\sigma = \lim_{n\to\infty}(T_1 + T_2 + \ldots\ldots + T_n) = \frac{1}{2}$$

For ρ, $t_n = \frac{1\cdot 3\cdot 5\ldots\ldots(2n-1)}{2\cdot 4\cdot 6\ldots\ldots(2n+2)} = \frac{1\cdot 3\cdot 5\ldots\ldots(2n-1)((2n+2)-(2n+1))}{2\cdot 4\cdot 6\ldots\ldots(2n+2)}$

$$t_n = \frac{1\cdot 3\cdot 5\ldots\ldots(2n-1)}{2\cdot 4\cdot 6\ldots\ldots 2n} - \frac{1\cdot 3\cdot 5\ldots\ldots(2n+1)}{2\cdot 4\cdot 6\ldots\ldots(2n+2)}$$

$$t_1 = \frac{1}{2} - \frac{1\cdot 3}{2\cdot 4}$$

$$t_2 = \frac{1\cdot 3}{2\cdot 4} - \frac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6}$$

$$\vdots \quad \vdots \quad \vdots$$

$$t_n = \frac{1\cdot 3\cdot 5\ldots\ldots(2n-1)}{2\cdot 4\cdot 6\ldots\ldots 2n} - \frac{1\cdot 3\cdot 5\ldots\ldots(2n+1)}{2\cdot 4\cdot 6\ldots\ldots(2n+2)}$$

$$\rho = \lim_{n\to\infty}\left(t_1 + t_2 + \ldots\ldots + t_n\right) = \frac{1}{2}$$

$\therefore \quad \sigma = k\rho \quad \Rightarrow \quad \frac{1}{2} = \frac{k}{2} \qquad \Rightarrow k = 1$. Ans.]

55. **(4)**

$y = \frac{(x-a)^2}{4}$ and $y = e^x$

$\frac{dy}{dx} = \frac{x-a}{2}$ and $\frac{dy}{dx} = e^x$

$\therefore \quad \frac{x-a}{2} = e^x \Rightarrow (x-a) = 2e^x$ ...... (1)

Also $\frac{(x-a)^2}{4} = e^x$ .... (2)

$\therefore \quad \frac{4e^{2x}}{4} = e^x$ [Using (1)]

$\Rightarrow \quad e^x\left(e^x - 1\right) = 0$. Hence $e^x = 1 \quad \Rightarrow x = 0$

if $x = 0$ then $y = 1$

$\therefore \quad 1 = \frac{a^2}{4} \Rightarrow a^2 = 4 \quad \Rightarrow \quad a = -2$ or $a = 2$ (rejected)

As, for $a = 2$ curves do not intersect.
Hence, sum of the aquares of all possible values of a is 4. Ans.]

56. **(9)**

$F(1) \int x e^{-x} dx = -e^{-x}x - e^{-x} = -e^{-x}(x+1)$

$F(2) \int e^{-x} x^2 dx = -e^{-x}\left(x^2 + 2x + 2\right)$

$\therefore \quad \frac{F(1)}{F(2)} = \frac{(x+1)}{(x+1)^2 + 1}$

$\Rightarrow \quad \int \frac{F(1)}{F(2)} dx = \int \frac{(x+1)}{(x+1)^2 + 1} dx$

$(x+1)^2 + 1 = t \Rightarrow \qquad (x+1)dx = \frac{1}{2}dt$

$= \frac{1}{2}\int \frac{dt}{t} = \frac{1}{2} ln(t) = ln\sqrt{t}$

$\therefore \quad \left(e^{\int \frac{F(1)}{F(2)}dx}\right)^2 = \left(\sqrt{t}\right)^2 = t = (x+1)^2 + 1$

Hence $\int_2^5\left((x+1)^2+1\right)dx=\left.\frac{(x+1)^3}{3}+x\right]_2^5=(72+5)-(9-2)=77-11=66.$

57. **(4)**

Clearly, $\frac{y+3}{2y+5}=3-(1-\cos x)^2$, where $x\in R$

But $0\le(1-\cos x)^2\le 4 \Rightarrow -1\le\frac{y+3}{2y+5}\le 3 \quad y\in\left(-\infty,\frac{-8}{3}\right]\cup\left[\frac{-12}{5},\infty\right)$

$\therefore \quad a=\frac{-8}{3}$ and $b=\frac{-12}{5}$

Hence, $(3a-5b)=3\left(\frac{-8}{3}\right)-5\left(\frac{-12}{5}\right)=-8+12=4.$ Ans.]

58. **(1)**

As, $\frac{\det.(\text{adj}.B)}{\det.(C)}=\frac{\det.(\text{adj}.(\text{adj}.B))}{\det.(5A)}=\frac{(\text{adj}.(A))^{(3-1)^2}}{5^3\det.(A)}=\frac{|A|^3}{125}.$

As, $\det.(A)=5$

So, $\frac{\det.(\text{adj}.B)}{\det.(C)}=\frac{(5)^3}{125}=1$. Ans.]

59. **(7)**

Given $S_k=\underbrace{\frac{1}{k}+\frac{1}{k^2}+\frac{1}{k^3}+\ldots\ldots\infty}_{\text{Infinite G.P.}}$, where $k\in N$ and $k\ge 2$

For $k>1$,

$$S_k=\frac{\frac{1}{k}}{1-\frac{1}{k}}=\frac{1}{k-1}$$

$$E=\frac{S_k\cdot S_{k+2}}{(S_{k+1})^2}$$

Hence, $E=\left(\frac{1}{k-1}\cdot\frac{1}{k+1}\right)\div\frac{1}{k^2}=\frac{k^2}{k^2-1}$

So, $E=1+\frac{1}{k^2-1}\Rightarrow E_{max}=1+\frac{1}{4-1}=\frac{4}{3}=\frac{p}{q}$ (Given)

Hence, $p=4$ and $q=3 \Rightarrow (p+q)=7$ Ans.]

60. **(2)**

Given, $f(x) = 2x^3 + 3(1-3a)x^2 + 6(a^2 - a)x + b$

$\therefore \quad f'(x) = 6\left[x^2 + (1-3a)x + a(a-1)\right]$ .... (1)

As, $f(x)$ has positive point of local maximum, so the equation $f'(x) = 0$ must have both roots positive and distinct roots.

$\therefore \; D > 0 \;\Rightarrow\; (1-3a)^2 > 4a(a-1) \;\Rightarrow\; 1 - 6a + 9a^2 - 4a$

$\Rightarrow \; 5a^2 - 2a + 1 > 0$, true $\forall\, a \in R$. .... (2)

Also, sum of roots $> 0 \;\Rightarrow\; -(1-3a) > 0 \;\Rightarrow\; 3a > 1 \;\Rightarrow\; a > \frac{1}{3}$ .... (3)

and, Product of roots $> 0 \;\Rightarrow\; a(a-1) > 0 \;\Rightarrow\; a < 0$ or $a > 1$ .... (4)

Hence, $(2) \cap (3) \cap (4)$

$\Rightarrow \quad a \in (1, \infty)$

So, the smallest integral value of 'a' equals 2. Ans.]