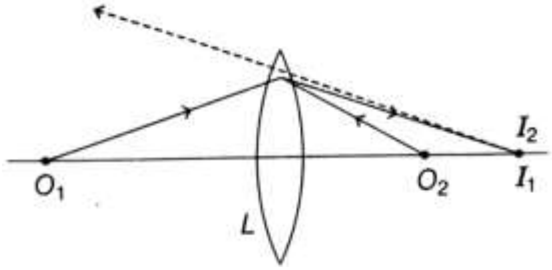
PART (A) : PHYSICS

1. (B, C, D)

Images of O_1 and O_2



May coincides, if L is placed as shown in ray diagram. If $O_1L=x$, then $LO_2 = 24 - x$.

Now for object O_1, a real image is formed at I_1

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$$

$$\Rightarrow \frac{1}{v}\frac{1}{-x}=\frac{1}{9}$$

$$\text{Or } \frac{1}{v}=\frac{1}{9}-\frac{1}{x} \qquad \text{.....(i)}$$

For object O_2 image is virtual and it also formed at I_1

$$\frac{1}{v}-\frac{1}{(24-x)}=\frac{1}{-9} \qquad \text{....(ii)}$$

As, source O_2 is on left side of lens, so focal length of lens is negative.

From eqs. (i) and (ii), we have

$$\frac{1}{9}=\frac{1}{x}=\frac{1}{9}+\frac{1}{24-x}$$

$$\Rightarrow x^2-24x+108=0$$

$$\Rightarrow x^2-24x+108=0$$

Now, substituting for x(x = 18cm) in Eq. (i) we get

$$\frac{1}{v}=\frac{1}{9}-\frac{1}{x}$$

$$\frac{1}{v}=\frac{1}{9}-\frac{1}{18}\Rightarrow\frac{1}{v}=\frac{1}{18}$$

$$\Rightarrow v=18\text{cm}$$

2. (B, C)

Using relative velocity formula, we have

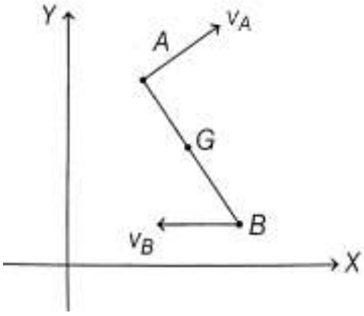
$$v_G=v_A+\omega\times r_{GA}$$

$$\text{And } v_B=v_A+\omega\times r_{AB}$$

$$v_A+\omega\hat{k}\times L(-\text{cis}\theta\hat{i}+\sin\theta\hat{j})$$

$$\Rightarrow v_B=2\hat{i}+\sqrt{3}\hat{j}-\omega L.(\cos\theta\hat{j}+\sin\theta\hat{i})$$

$$=(2\hat{i}+\sqrt{3}\hat{j})\ -\omega\left(\frac{1}{2}\right)\left(\frac{1}{2}\hat{j}+\frac{\sqrt{3}}{2}\hat{i}\right)$$



$= 3.5\hat{i} + 2.6\hat{j}$

3. (A, C)

Z_{min} = minimum level of rotational profile of liquid

Then $Z_1 = Z_{min} + \frac{r_1^2\omega^2}{2g}$

$Z_{min} + \frac{(0.1)^2(10)^2}{2\times10}$

And $Z_2 = Z_{min} + \frac{r_2^2\omega^2}{2g}$

$= Z_{min} + \frac{(02)^2\times(10)^2}{2\times10}$

Also $Z_1 + Z_2 = 2\times0.4 = 0.8m$

From above equations, we get

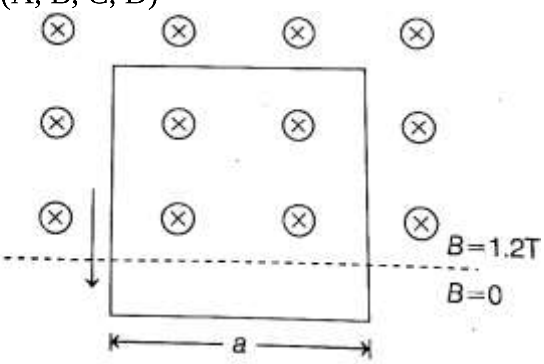
$Z_{min} = 0275m$

Also $Z_1 = 0.325sm$

And $Z_2 = 0.475m$

$\Rightarrow \frac{Z_1}{Z_2} = \frac{0.325}{0.475} = 0.684$

4. (A, B, C, D)



Emf induced in coil has a magnitude,

E = Bav

Resistance of loop is

$R = \frac{\rho(4a)}{\pi\left(\frac{d}{2}\right)^2} = \frac{16\rho a}{\pi d^2}$

Current in loop is

$$I = \frac{E}{R} = \frac{\pi B v d^2}{16\rho_c}$$

This current flows clockwise following Lenz's law.
Force on loop = Force on its upper arm

$$F = Bld = \frac{\pi B^2 v d^2 a}{16\rho}$$

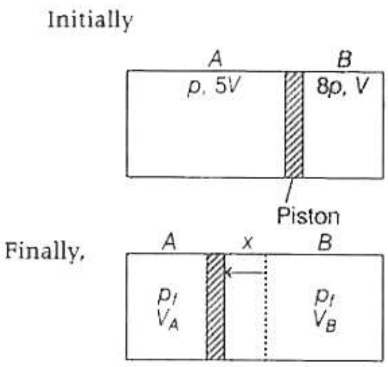
If loop reaches its terminal speed, then magnetic force is equals to weight of loop

So $\rho_m 4a\pi\left(\frac{d}{2}\right)^2 g = \frac{\pi B^2 v d^2 a}{16\rho_c}$

Here ρ_m = density of copper and ρ_c = resistivity of copper

$$\Rightarrow v = \frac{16\rho_c\rho_m g}{B^2}$$

5. (A, B)
When piston is released, it stops when pressure in both chambers A and B is same
Let final pressure is p' and volume are V_A and V_B then



For adiabatic processes in A and B part, we have $p(5V)^\gamma = p'V_A^\gamma$

And $8p(V)^\gamma = p'V_B^\gamma$

Dividing Eq. (i) by Eq. (ii), we get

$$\frac{5^\gamma}{8} = \left(\frac{V_A}{V_B}\right)^\gamma \Rightarrow \frac{V_A}{V_B} = \frac{5}{8^{\frac{1}{\gamma}}}$$

$$\Rightarrow \frac{V_A}{V_B} = \frac{5}{(2^3)^{\frac{2}{3}}} \text{ or } \frac{V_A}{V_B} = \frac{5}{4}$$

Also $V_A + V_B = 6V$

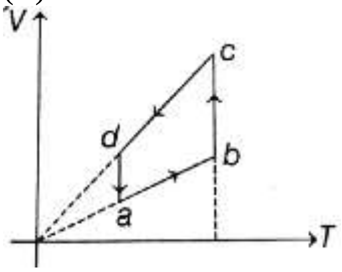
Hence $V_A = \frac{10}{3}V$

$$\text{Now, } \frac{V_{Ainitial} - V_{Afinal}}{V_{Ainitial}} = \frac{5V - \frac{10}{3}V}{5V}$$

$$= \frac{5V}{15V} = \frac{1}{3} \text{ and } \frac{V_{Bfinal} - V_{Bintial}}{V_{Binitial}}$$

$$=\frac{\frac{8}{3}V-V}{V}=\frac{5}{3}$$

6. (C)



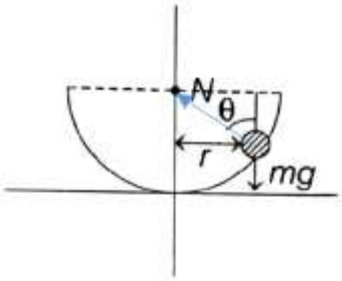
For da, temperature constant and pressure increase and volume decreases from equation pV = nRT. So V – T graph must be vertical.

For ab, temperature increases at constant pressure, so volume also increases $V=\frac{nR}{P}T$ and V – T graph straight line passing through origin, bc as ad will be vertical line, cd as ab will be a straight line passing through origin.

For bc, T constant $\rho \propto \frac{1}{V} p \downarrow V \uparrow$

For cd, p constant $v \propto T$ as $T \downarrow v \uparrow$

7. (B, C, D)



FBD of particle in bowl is

From FBD, $\tan\theta=\frac{r}{R-h}$

Also, $N\cos\theta = mg$ ……(i)

And $N\sin\theta = mr\omega^2$ …..(ii)

On dividing Eq (ii) by Eq. (i), we get

$$\tan\theta=\frac{r\omega^2}{g}$$

$$\Rightarrow \frac{r}{R-h}=\frac{r\omega^2}{g}$$

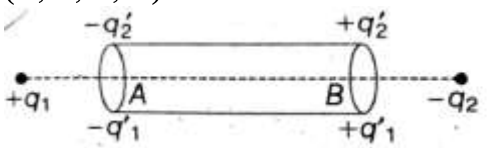
Or $\omega^2 = g/R-h$

$\Rightarrow h = R - g/\omega^2$

For non-zero h, $R > g/\omega^2$

$$\Rightarrow \omega_{min}=\sqrt{\frac{g}{R}}=\sqrt{\frac{9.8}{0.1}}=9.8 rads^{-1}$$

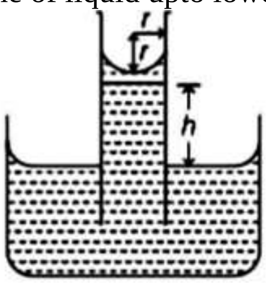
8. (A, B, C, D)



Due to induction effect, the situation is shown clearly in figure. Due to $+q_1$, net induced charges is q_1^+ at end A and q_1^+ at end B while due to $-q_2$ induced charges are $q_2^{'}$ and $q_2^{'}$ at ends A and B, respectively. Thus, the end A acquires negatively charged and B acquires positive charge. Electric force experienced by q_1 or $-q_2$ has to be computed by using principle of superposition

9. (B, C, D)

Volume of liquid upto lower level of meniscus = $\pi r^2 h$



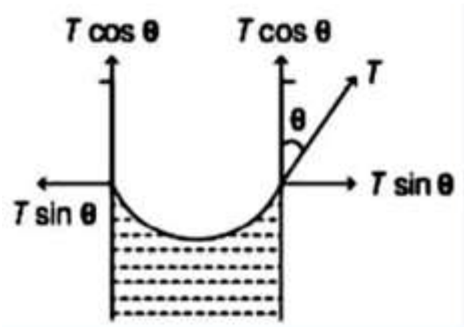
Volume of liquid above meniscus = Volume of sphere of radius *r* – Volume of hemisphere of radius *r*

$$= \pi r^3 - \frac{2}{3}\pi r^3 = \frac{1}{3}\pi r^3$$

So, total volume of liquid in capillary $= \pi r^2 h + \frac{1}{3}\pi r^3 = \pi r^2\left(h + \frac{r}{3}\right)$

For equilibrium, weight of liquid = Upward force of surface tension

$= \pi r^2\left(h + \frac{r}{3}\right) = 2\pi T\cos\theta$



$\Rightarrow h = \frac{2T\cos\theta}{r\rho g} - \frac{r}{3}$

If we neglect $\frac{r}{3}$ in comparison to *h*, then

$T = \frac{hr\rho g}{2\cos\theta}$

or $T = r\frac{\left(h + \frac{r}{3}\right)\rho g}{2\cos\theta}$

For water and glass, θ is very small $\theta = 0^0$

$$\therefore T = \frac{r\left(h+\frac{r}{3}\right)\rho g}{2}$$

10. (A, D)

Force on steels = Force on copper

$$\sigma_{steel}A_{steel} + \sigma_{Cu}A_{Cu} = 0$$

$$\sigma_{steel}.\frac{\pi}{4}d_{steel}^2 = -\sigma_{Cu}\frac{\pi}{4}.(d_2^2 - d_1^2)$$

$$\sigma_{Cu} = \frac{-\left(\frac{d}{2}\right)^2}{d^2 - \left(\frac{d}{d}\right)^2}.\sigma_{steel}$$

$$\Rightarrow \sigma_{Cu} = \frac{-\left(\frac{1}{4}\right)}{1-\left(\frac{1}{4}\right)}.\sigma_{steel}$$

$$\sigma_{Cu} = -\frac{1}{3}\sigma_{steel}$$

Also strain in copper tube

$$= \frac{\sigma_{Cu}}{Y_{Cu}} + \alpha_{Cu}\Delta T$$

11. (4.91)

Not force on system is

$$F_{net} = mg - F_m$$

$$= mg - \frac{vB^2L^2}{R}$$

$$\Rightarrow a_{net} = g - \frac{vB^2L^2}{mR}$$

When velocity is terminal velocity

$$v = \frac{v_T}{2} = \frac{mgR}{2B^2L^2}$$

Then acceleration

$$a = g - \frac{mgT}{2B^2L^2} \times \frac{B^2L^2}{mR} = \frac{g}{2}$$

$= 4.91\ ms^{-2}$

12. (2.45)

Energy density at any point in capacitor is

$$u = \frac{1}{2}\varepsilon_0 E^2 \text{ and } E = \frac{q}{2\pi\varepsilon_0 Lr}$$

When L = length of cylindrical capacitor

And r = distance of point from centre of axis.

So, $u = \frac{1}{2}\varepsilon_0 E^2 = \frac{q^2}{8\pi 2\varepsilon_0 L^2 r^2}$

Energy in cylindrical volume of radius r is

$U = \int x.dv$

Where dv = differentiable volume

$2\pi L_1 dr$

Hence, $U = \int_a^r \frac{q^2 2\pi r L_3}{8\pi^2 \varepsilon_0 L^2 r^2}.dr$

$= \frac{q^2}{4\pi\varepsilon_0 L}\log_e\left(\frac{r}{a}\right)$

For r = b, U = U_b

And $U_b = \frac{q^2}{4\pi\varepsilon^0 L_1}\log_e\left(\frac{b}{a}\right)$

For $\frac{U_r}{U_b} = \frac{1}{2}$, we have

$\log\left(\frac{r}{a}\right) = \frac{1}{2}\log\left(\frac{b}{a}\right)$

$\Rightarrow \frac{r}{a} = \sqrt{\frac{b}{a}}$

Or $r = \sqrt{ab} = \sqrt{6} = 2.45$

13. (2.50)

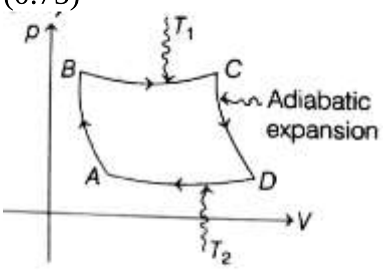
Heat rejected by water = $m_w s_w (T_i - T_f)$

Heat absorbed by ice = $m_i s_i(0 - T) + m_i s_w(T_f - 0) + m_i L$

Equating both and substituting values, we get

$T_f = 2.5^0C$

14. (0.75)



Given volume at C = v

And volume at D = 32V

For adiabatic expansion CD,

$T_C V_C^{\gamma-1} = T_S V_D^{\gamma-1}$

$\Rightarrow \frac{T_C}{T_D} = \left(\frac{V_D}{V_C}\right)^{\gamma-1} = \left(\frac{32V}{V}\right)^{\gamma-1}$

$= 32^{\frac{7}{5}-1}$

$\frac{T_C}{T_D} = (32)^{\frac{2}{5}} = 4$

Now, efficiency of Carnot's cycle

$\eta = 1 - \frac{T_D}{T_C} = 1 - \frac{1}{4}$

$= \frac{3}{4} = 0.75$

15. (1.00)

Energy of emitted photons,

$E_1 = 5eV = 8 \times 10^{-19} J$

Energy emitted by source per second = power = $32 \times 10^{-3} W$

Number of photons emitted

$n_1 = \frac{P}{E_1} = 4 \times 10^{15}$ photons/s

Photons incident per unit area at a distance of 0.8m

$n_2 = \frac{n_1}{4\pi r^2} = \frac{4 \times 10^{15}}{4\pi \times (0.8)^2} = 5 \times 10^{14}$ (per s per m^2)

Number of photons incident on sphere = $n_3 = n_2 A$

$= \pi(8 \times 10^{-3})^2 \times 5 \times 10^{14} = 10^{11} s^{-1}$

So, number of photoelectrons emitted

$n_4 = \frac{1}{10^6} \times n_3 = 1 \times 10^5 s^{-1}$

16. (39.20)

At distance y above mean position, velocity of block is $v = \omega\sqrt{A^2 - y^2}$

As block is detached, its downward acceleration is g.

So, total height attained by block above mean position is

$h = y + \frac{v^2}{2g} = y + \frac{\omega^2(A^2 - y^2)}{2g}$

For h to be maximum $\frac{dh}{dy} = 0 \Rightarrow y = \frac{g}{\omega^2} = 39.20m$

17. (8.00)

The acceleration is given by

$a = -\omega^2 x = -\frac{k}{m} x$

Or $|a_{max}| = \frac{x}{m} |x_{max}|$

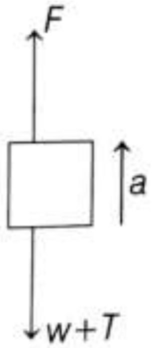
i.e. Acceleration will be maximum, when x is maximum

i.e $x_{max} = A = 0.02m$

$\therefore a = \frac{1200}{3} \times 0.02 = 8.0 ms^{-2}$

18. (3.75)

Free body diagram of the block is shown in figure



In the figure

$F = $ upthrust force

$= v\rho_{\omega}(g+a)$

$= \left(\frac{\text{mass of block}}{\text{density of block}}\right)\rho_{\omega}(g+a)$

$= \left(\frac{1}{800}\right)(1000)(10+1) = 13.75N$

$\therefore w = mg = 10N$

Equation of motion of the block is

$F - T - w = ma$

$13.75 - T - 10 = 1 \times 1$

$\therefore T = 2.75N$

When the string is cut, T = 0

$a = \frac{F-w}{m}$

$= \frac{13.75-10}{1}$

$= 3.75 \text{ m/s}^2$

PART (B) : CHEMISTRY

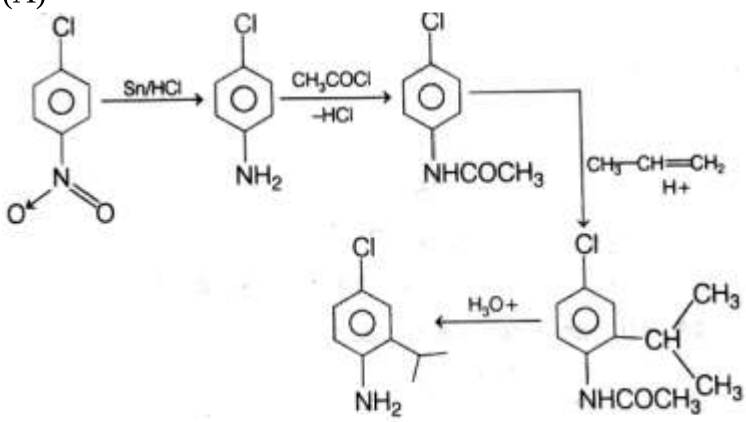
1. (B, C)
In the aqueous solution of $Pd(NH_3)_2Cl_2$, the atoms chlorine are in coordination sphere and the van't Hoff factor of the compound is unity.

2. (B)
$KClO_3$, KNO_3, sulphur and antimony contains the head of match stick. This sides of match box contains red phosphorus and sand powder
Ammonia has the highest proton affinity

$$NH_3 + H^+ \rightarrow NH_4^+$$

Molecular nitrogen is less reactive than that of oxygen because nitrogen has high dissociation energy in comparison to oxygen is shorter than oxygen because of the presence of triple bond between nitrogen atoms. So, both are true but not correct explanation.
Thomas slag or phosphatic slag is a mixture of calcium phosphate and calcium silicate $[Ca_3(PO_4)_2.CaSiO_3]$. It is used as manure.

3. (A)



4. (B, C)

$$\log k = \log A - \frac{\Delta H^0}{2.303RT}$$

$$K = Ae^{-\frac{\Delta H^0}{RT}}$$

$$\therefore \log K = 10 = OP$$

$$\therefore A = 10^{10}$$

$$\text{Slope} = \frac{\Delta H^0}{2.303R}$$

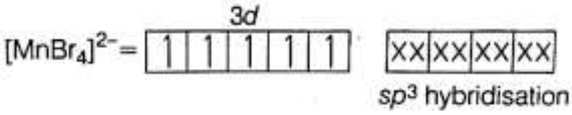
$$\Delta H^0 = \text{Slope} \times 2.303R$$

$$\therefore \Delta H^0 = 2.303 \times 8314 Jmol^{-1}K^{-1} \times 0.5 = 9.574 Jmol^{-1}$$

5. (A, B, C, D)
(a) In $[MnBr_4]^2$ Mn is present in +2 oxidation state as
$x + 4(-1) = -2$
$x = -2 + 4$

x = 2

Mn^{2+}[Z = 25]: [Ar]$3d^5$: 5 unpaired electrons



In $[MnBr_4]^{2-}$, five unpaired electrons are present, thus it has magnetic moment 5.9 BM.

(b) Magnetic moment $(\mu) = \sqrt{n(n+2)}$BM

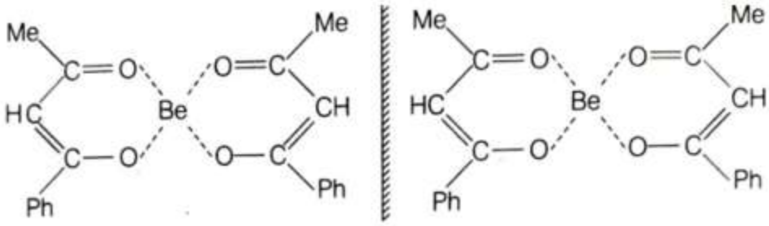
$[Cr(H_2O)_6]^{2+} = Cr^{2+} = 3d^4$ = 4 unpaired electrons

$[Fe(H_2O)_6]$2+ = Fe2+ = 3d6 = 4 unpaired electrons

So, $[Cr(H_2O)_6]^{2+}$ and $[Fe(H_2O)_6]^{2+}$ has same magnetic moment

(c) The IUPAC name of $[COCl(NH_3)_3(H_2O)_2]Cl_2$ is chlorodiaquatriammine cobalt (III) chloride.

(d) Benzoyl acetonato beryllium exhibit optical isomerism as follows:



6. (A, B)

The organic mixture contains $(NH_4)_3PO_4$ and $(NH_4)_3AsO_4$. On reaction with conc. HNO_3 and ammonium molybdate $[(NH_4)_2MoO_4]$, yellow precipitate are obtained. The reactions are as follows

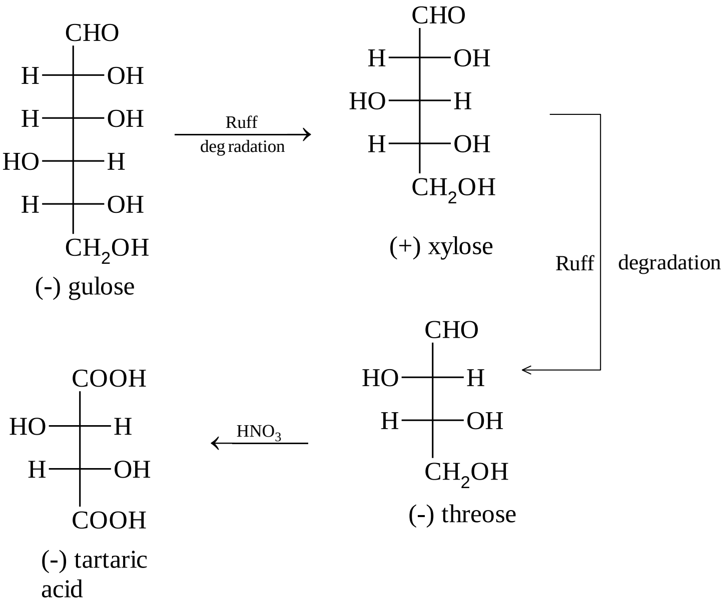
$AsO_4^{3-} + 12(NH_4)_2M_0O_4]$ yellow precipitate are obtained. The reaction are as follows

$AsO_4^{3-} + 12(NH_4)_2M_0O_4 + 21HNO_3 \rightarrow (NH_4)_3AsO_4.12MoO_3\downarrow$ yellow ppt. of ammonium arseno molybdate + $21NH_4NO_3$ + $12H_2O$

$$(NH_4)_3PO_4 + 12MoO3 + 6H2O \rightarrow \underset{\text{Ammonium phosphate molybdate (yellow ppt)}}{(NH_4)_3PO_4.12MoO_3.6H_2O\downarrow}$$

7. (C)

Following reaction takes place with (-) glucose



8. (B)

At equilibrium $\underset{4-x}{A_2(g)} + \underset{8-x}{B_2(g)} \rightleftharpoons \underset{2x}{2AB(g)}$

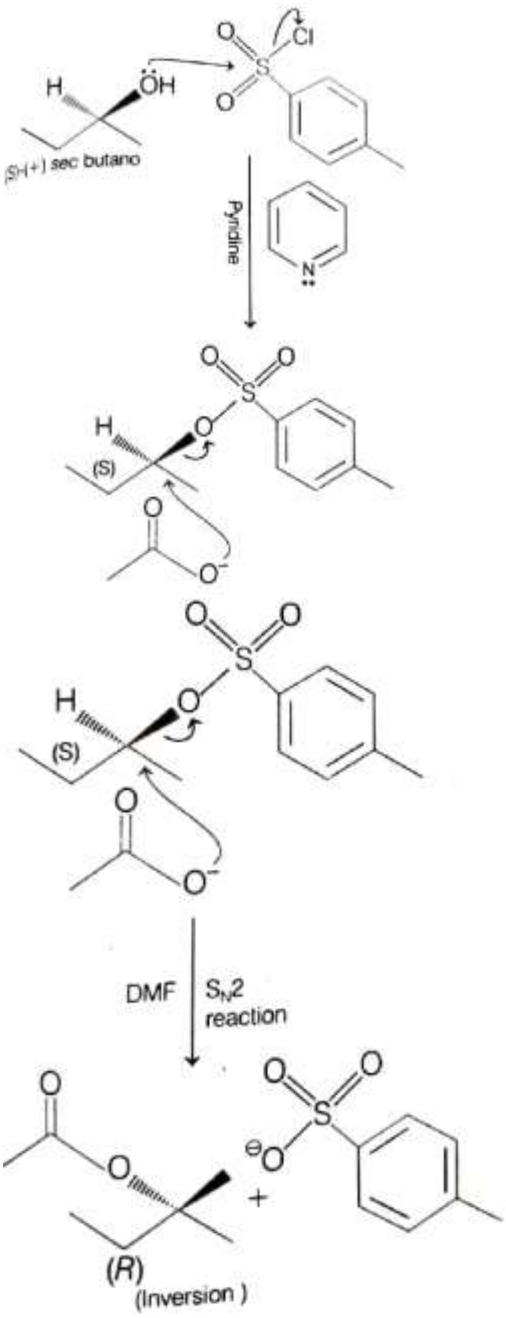
At equilibrium $K_C = \dfrac{4x^2}{(4-x)(8-x)}$

$$\Rightarrow \frac{4x^2}{(4-x)(8-x)} = 4 \Rightarrow x^2 = [32 - 12x + x^2]$$

$$x = \frac{32}{12} = \frac{8}{3} = 2.66\text{mol}$$

$$[AB] = \frac{2\times 2.66}{6} \Rightarrow [AB] = 0.886 = 0.89$$

9. (B)



10. (C)

$$A \xrightarrow[300K,2atm]{\Delta v=O} B \xrightarrow[5atm,750K]{\text{Adibatic rev,exp}} C \xrightarrow[300K]{\text{Iso,rev,comp}} \underset{\substack{300K\\2atm}}{A}$$

$C_v = 1.5R,\ C_p = 2.5R$

$\Rightarrow \gamma = \frac{5}{3}$ and $1-\gamma = \frac{-2}{3}$

$\Rightarrow \frac{\gamma}{1-\gamma} = \frac{-5}{2}$

Between B and C

$pT^{\frac{\gamma}{1-\gamma}} = \text{constant}$

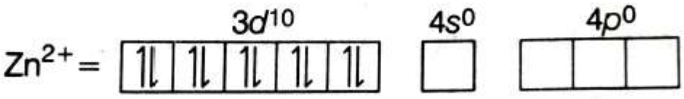
$\Rightarrow 5(750)^{\frac{-5}{2}} = pC(300)^{\frac{-5}{2}}$

$\Rightarrow pC = 5\left(\frac{750}{300}\right)^{\frac{-5}{2}} = 0.5\text{atm}$

Therefore pC < pA and hence graph(C) applies most appropriately

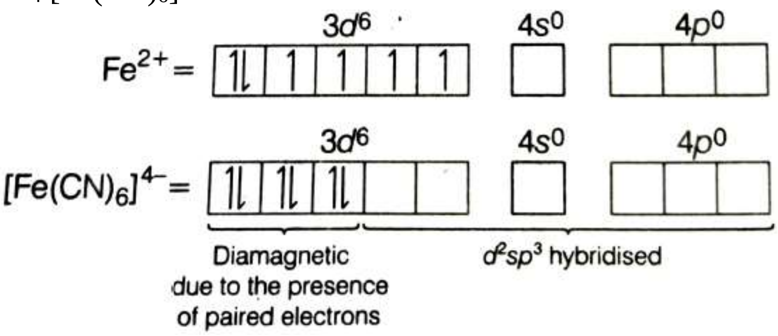
11. (4.00)

Diamagnetic molecules are $[Zn(OH)_4]^{2-}$, $K_4[Fe(CN)_6]$ $[PdBr_4]^{2-}$, $[Ni(CO)_4]$

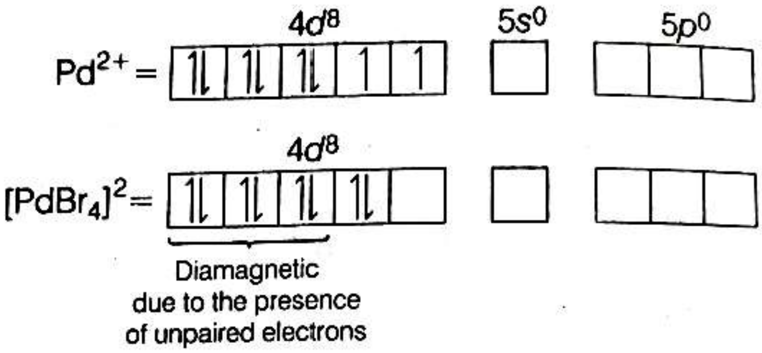


$[Zn(OH)_4]^{2-}$ is diamagnetic due to the paired electrons.

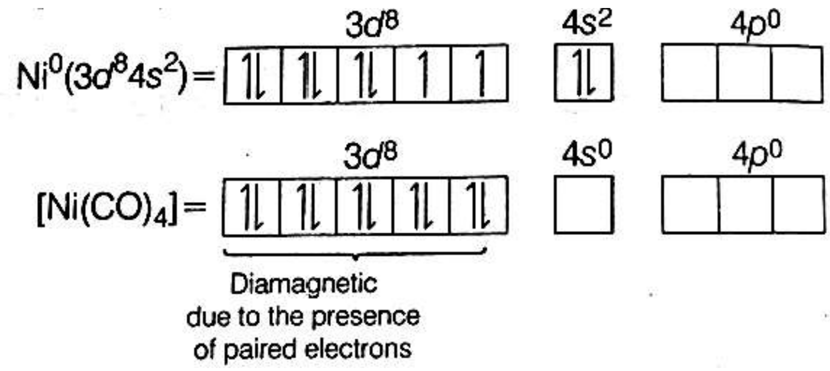
In $K_4[Fe(CN)_6]$ Fe is in +2 oxidation state



In $[PdBr_4]^{2-}$, Pd is in +2 oxidation state with electronic configuration $4d^8 5s^0$



In $[Ni(CO)_4]$Ni is in zero oxidation state



12. (0.23)

Given K_P = (1000K) = 0.48 $\Rightarrow$ Kp (700K) =0.83

As we know that

$$\ln\frac{K_P(1000K)}{K_P(700K)}=\frac{\Delta H^0}{R}\left(\frac{3}{7000}\right)=\ln\left(\frac{0.48}{0.83}\right) \quad \ldots(i)$$

$$\ln\frac{K_P(1000K)}{K_P(850K)}=\frac{\Delta H}{R}\left(\frac{15}{85000}\right)=\ln\frac{0.48}{K_P(850)} \quad \ldots.(ii)$$

$$\frac{\ln\frac{0.48}{(850K)}}{\ln\left(\frac{0.48}{0.83}\right)}=\frac{35}{85}\Rightarrow 0.48-\ln K_P(850K)=0.749$$

$$\Rightarrow K_{p(850K)}=0.23\Rightarrow 0.23=\frac{p(CO_2)}{1.21-p(CO_2)}\Rightarrow p(CO_2)=0.23$$

13. (0.72)

Cell reaction is

$$Zn+2H^+\rightleftharpoons Zn^{2+}+H_2;E^0=+0.76V$$

Also $\underset{0.1-x}{HIO_3}\rightleftharpoons \underset{x}{H^+}+\underset{x}{IO_3^-}\Rightarrow 0.2=\frac{x^2}{0.1-x}$

Or, $x^2+0.2x-0.2=0$

$$x=\frac{-0.2+\sqrt{0.04+0.08}}{2}=0.073M$$

Nernst equation is given as, E = $0.76-\frac{0.059}{2}\log\frac{[Zn^{2+}]}{[H^+]^2}$

$$=0.76-\frac{0.059}{2}\log\frac{0.1}{(0.073)^2}=0.722V$$

14. (0.08)

$p=\frac{K}{V}$ for Boyle's law under isothermal condition

Taking log on both sides, we get

$$\log p = \log \frac{1}{V} + \log K$$

Thus, log K = 0.6990 bar dm^3

Slope = tan 45^0=1

$\therefore$ K = antilog of 0.6990 = 5.0 bar dm^3

$$\frac{1}{V} = \frac{P}{K} = \frac{0.2 \text{bar}}{5 \text{bar dm}^3} = 0.04 \text{dm}^{-3}$$

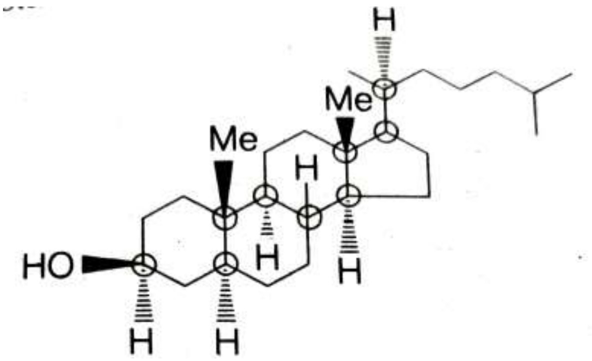
$$\therefore \text{Density} = \frac{\text{Mass}}{\text{Volume}} = 2 \times 0.04 = 0.08 \text{g dm}^{-3}$$

15. (7.00)

CH_2Cl_2, H_2O, $CHCl_3$, o-cresol, SCl_2, IBr and HCHO (in all seven) have dipole moment.

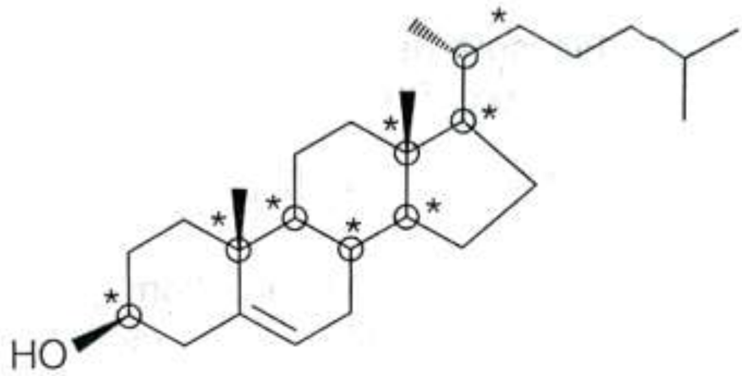
16. (9.00)

The total number of chiral centres are 9



17. (8.00)

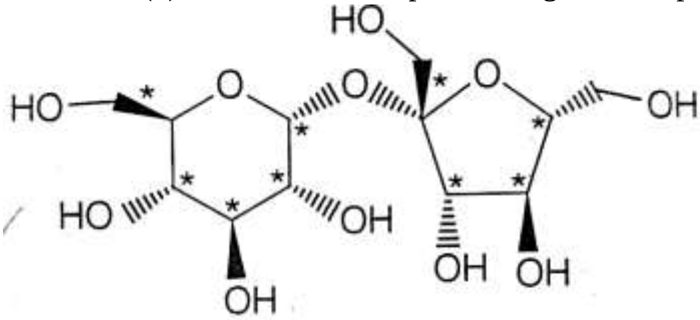
There are eight chiral centers in the given structure



* = Chiral centres

18. (512.00)

Total nine (9) stereocentres are present in given compound (sucrose)



Hence total number of stereoisomers = $2^9 = 512$.

PART (C) : MATHEMATICS

1. (B, C)

Area of quadrilateral ABCD is maximum when area of ΔACD is maximum $\Rightarrow$ Distance of D from AC is maximum i.e. $\cos\theta - \sin\theta$ is maximum

$= \sqrt{2}\cos\left(\theta + \frac{\pi}{4}\right)$ is maximum

$\Rightarrow \theta = \frac{7\pi}{4}$

And area = $\frac{6}{\sqrt{2}} \times 2\sqrt{2} = 12$ sq. unit [$\because$ ABCD is a rectangle]

2. (A, B, C)

We have $f(x) = x^2 e^{\frac{1}{1-x^2}}$

$\Rightarrow f'(x) = \left(2x + \frac{2x^3}{(1-x^2)^2}\right)e^{\frac{1}{1-x^2}}$

$\Rightarrow f'(x) = 2x\left(1 + \frac{x^2}{(1-x^2)^2}\right)e^{\frac{1}{1-x^2}}$

Clearly $f'(x) > 0$ for $x \in (0,1)$ and $f'(x) < 0$ for $x \in (-1, 0)$. So f(x) is decreasing in (-1, 0) and increasing in (0,1) $\lim\limits_{x\to\pm1} x^2 e^{\frac{1}{1-x^2}} = \infty$ and f(0) =0 $\Rightarrow f(x) = 1$ has exactly one solution in each of the intervals (-1, 0) and (0, 1)

3. (A, C, D)

We have

$$f(x,y) = \begin{vmatrix} x & x^2 & 0 \\ 1 & 2x+y & (x+y)^2 \\ 0 & 1 & 2x+3y \end{vmatrix}$$

Apply $C_2 \to C_2 - xC_1$ we get

$$f(x,y) = \begin{vmatrix} x & 0 & 0 \\ 1 & x+y & (x+y)^2 \\ 0 & 1 & 2x+3y \end{vmatrix}$$

Expand along R_1, we get

F(x,y) = x[(x + y)(2x + 3y)-(x + y)^2]

F(x, y) = x(x +y) [2x + 3y –x – y]

[(x,y) = x(x + y) (x + 2y)

Clearly x, (x+ y) and (x +2y) is a factor of f(x, y)

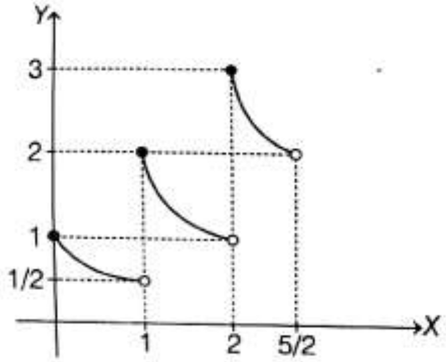
4. (C, D)

(A) $(N^TMN)^T = N^TM^TN = N^TMN$

If M s symmetric and is -N^TMN if M is skew symmetric
$\therefore$ (a) is TRUE

(B) $(MN – NM)^T$= NTMT – MNT $\rightarrow N^TM^T - M^TN^T$
= NM – MN
= - (MN – NM)
So (MN – NM) is skew symmetric
$\therefore$ (b) is TRUE
(C) $(MN)^T = N^TM^T$ = MN $\neq$ MN is M and N are symmetric. So MN is not symmetric
$\therefore$ (c) is FALSE
(D) (adj M) (adj N) = ad(NM) $\neq$ adj MN
$\therefore$ (d) is FALSE

5. (A, B, D)



we have $f : \left[0, \frac{5}{2}\right) \rightarrow \left(\frac{1}{2}, 3\right]$

$$f(x) = \frac{[x]+1}{(x)+1} \Rightarrow f(x) = \frac{[x]+1}{x-[x]+1}$$

$$f(x) = \begin{cases} \frac{1}{x+1}, & 0 \le x < 1 \\ \frac{2}{x}, & 1 \le x \le 2 \\ \frac{3}{x-1}, & 2 \le x < \frac{5}{2} \end{cases}$$

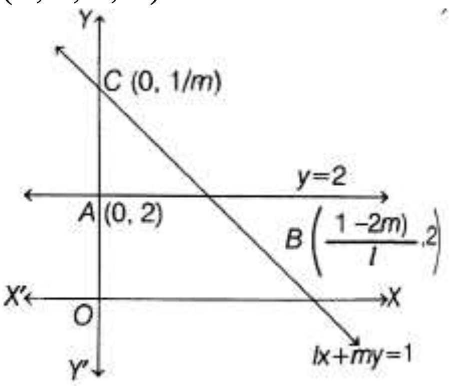
Clearly f(x) is discontinuous and bijective function

$$\lim_{x\to 1^-} f(x) = \frac{1}{2} \Rightarrow \lim_{x\to 1^+} f(x) = 2$$

$$\Rightarrow mn\left(\lim_{x\to 1^-} f(x) \lim_{x\to 1^+} f(x)\right) = \frac{1}{2} \neq f(1)$$

Point of discontinuity at 1 and 2
Max (1, 2) =2 = f(1)

6. (A, B, C, D)



The vertices of the ΔABC $A(0,2)$, $B\left(\frac{1-2M}{\ell}, 2\right)$ and $C\left(0, \frac{1}{M}\right)$

Let (h, k) be the circumcentre of ΔABC

$h = \frac{1-2m}{2\ell}, \ell = \frac{k-2}{2h(k-1)}$

Solving these equation, we get

$m = \frac{1}{2k-2}, k = \frac{1+2m}{2m}$

Since (ℓ, m) lies on $y^2 = 4ax$

$\therefore m^2 = 4a\ell$

$\Rightarrow \left(\frac{1}{2k-2}\right)^2 = 4a\left(\frac{k-2}{2h(k-1)}\right)$

$\Rightarrow h = 8a(k^2 - 3k + 2)$

Locus of (h, k) is $x = 8a(y^2 - 3y + 2)$

$\Rightarrow \left(y - \frac{3}{2}\right)^2 = \frac{1}{8a}(x + 2a)$ which represent the equation of parabola vertex is $\left(-2a, \frac{3}{2}\right)$.

$\because$ Length of smallest focal chord = length of latusrectum = $\frac{1}{8a}$

From the equation of curve C it is clear that it is symmetric about the line $y = \frac{3}{2}$.

7. A, B, D)

We have

$f(x) = \int_0^{3\pi} \cos z \cos(x - z)dx$ …….(i)

$= \int_0^{3\pi} \cos(3\pi - z)\cos(x - (3\pi - z)dz$

$= \int_0^{3\pi} (-\cos z)(\cos(3\pi - (x + z))dz$

$= \int_0^{3\pi} \cos z \cos(x + z)dz$

Adding Eqs. (i) and (ii) we get

$$2f(x)=\int_0^{3\pi}\cos z.[\cos(x-2)+\cos(x+z))dz$$

$$\int_0^{3\pi}\cos z.2.\cos x.\cos z.dz$$

$$\Rightarrow f(x)=\cos x\int_0^{3\pi}\cos^2 zdz$$

$$=\frac{\cos x}{2}\int_0^{3\pi}(1+\cos 2z)dz$$

$$=\frac{\cos x}{2}[(3\pi+0)-)]$$

$$=\frac{3\pi}{2}\cos x$$

Which is continuous and differentiable

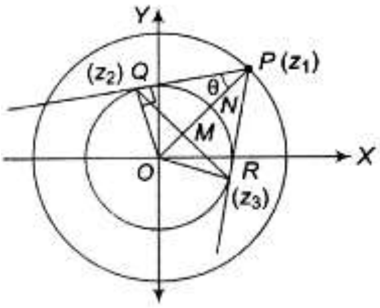
Max. f(x) = $\frac{3\pi}{2}$

And min f(x) = $=-\frac{3\pi}{2}$

Also, f(x) satisfies all the conditions of Rolle's theorem in $[0,4\pi]$, there exist $c\in(0,4\pi)$ such that $f'(c)=0$

8. (A, B, C, D)

Clearly OQ = 1 and OP = 2



$$\therefore \sin\theta=\frac{OQ}{OP}=\frac{1}{2}\Rightarrow\theta=30^0$$

$\therefore \angle QPR=60^0$

Also $PQ=PR$ (Length of tangents drawn from external point to a circle are equal)

$\therefore \Delta PRQ$ is equilateral triangle

Also, $OM\perp QR\Rightarrow\angle OMQ=90^0$

And $\angle MOQ=60^0$

$[\because \angle OQP=90^0$ and $\angle OPQ=30^0]$

$$\therefore \frac{MO}{OQ}=\cos 60^0=\frac{1}{2}$$

$MO = \frac{1}{2}$

$\Rightarrow NM = \frac{1}{2}$

And PN = 1

$\therefore N$ divides PM in the ratio 2 :1

Hence, the centroid of ΔPQR liess on |z| = 1

As PQRs is an equilateral triangle, so centroid will coincide

Now, $\left|\frac{z_1+z_2+z_3}{3}\right| = 1$ [$\because$ centroid lies on |z| = 1]

$\Rightarrow |z_1+z_2+z_3|^2 = 9$

$\Rightarrow (z_1+z_2+z_3)(\bar{z}_1+\bar{z}_2+\bar{z}_3) = 9$ ….(i)

Also $z_1\bar{z}_1 = 4$

Similarly $z_2\bar{z}_2 = 1$ and $z_3\bar{z}_3 = 1$ ….(ii)

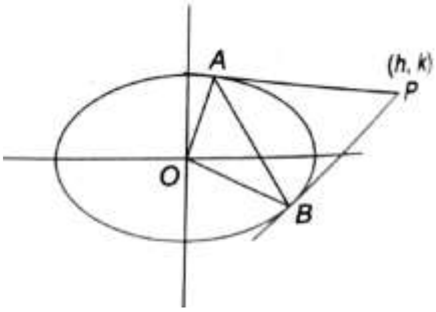
From Eqs (i) and (ii), we get

$$\left(\frac{4}{z_1}+\frac{1}{z_2}+\frac{1}{z_3}\right)\left(\frac{4}{z_1}+\frac{1}{z_2}+\frac{1}{z_3}\right) = 9$$

Note that

$$\arg\left(\frac{z_2}{z_3}\right) = \angle QOR = 120^0 = \frac{2\pi}{3}$$

9. (A, C)



Let (h, k) be the point of intersection of two tangents, then equation of AB is $\frac{xh}{4}+\frac{yk}{1} = 1$

Equation of ellipse is $\frac{x^2}{4}+y^2 = 1$

Joint equation OA and OB is (obtained by homogenizing equation of ellipse using Eq.(i))

$$\frac{x^2}{4}+\frac{y^2}{1} = \left(\frac{xh}{4}+\frac{yk}{1}\right)^2$$

$\Rightarrow x^2\left(\frac{h^2-4}{16}\right)+y^2(k^2-1)+\frac{2hk}{4}xy = 0$ ….(ii)

Given equation of OA and OB is

$x^2+4y^2+\alpha xy = 0$ ….(iii)

Since eqs. (ii) and (iii) represent same line

$\therefore \therefore \frac{h^2-4}{16}=\frac{k^2-1}{4}=\frac{hk}{2\alpha}$ ....(iv)

$\Rightarrow h^2-4=4(k^2-1)$

$\Rightarrow h^2-4k^2=0$

Locus is (x – 2y) (x+2y) = 0

10. (A, B, C)

$\lim_{x\to5^-}\frac{x^2-9x+20}{x-[x]}$

$=\lim_{x\to5^-}\frac{(x-5)(x-4)}{x-[x]}$

$=\lim_{h\to0}\frac{(5-h-5)(5-h-4)}{5-h-[5-h]}$

$=\lim_{h\to0}\frac{-h(1-h)}{5-h-4}=\lim_{h\to0}\frac{-h(1-h)}{(1-h)}$

$=\lim_{h\to0}(-h)=0$

Again, $\lim_{x\to5^+}\frac{x^2-9x+20}{x-[x]}$

$\lim_{x\to5^+}\frac{(x-5)(x-4)}{x-[x]}$

$\lim_{h\to0}\frac{(5+h-5)(5+h-4)}{5+h-[5+h]}$

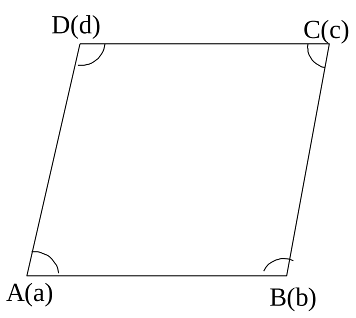
$=\lim_{h\to0}\frac{h(1+h)}{5+h-5}=\lim_{h\to0}\frac{h(1+h)}{h}$

$=\lim_{h\to0}(1+h)=1$

Hence, the limit at x = 5 does not exist.

11. (0.00)

Given A, B, C, D are the vertices of a cyclic quadrilateral ABCD and $A\neq90°$



Now, as ABCD is a cyclic quadrilateral

$\therefore A+C=180^0$

And B + D = 180^0

$\Rightarrow A=180^0-C$

$\Rightarrow$tanA = tan (180^0 – C)

$\Rightarrow \tan A + \tan C = 0$

$\Rightarrow \dfrac{\sin A}{\cos A} + \dfrac{\sin C}{\cos C} = 0$

$\Rightarrow \dfrac{|AB \times AD|}{AB.AD} + \dfrac{|CB \times CD|}{CB.CD} = 0$

$\Rightarrow \dfrac{|(b-a)\times(d-a)|}{(b-a).(d-a)} + \dfrac{|(b-c)\times(d-c)|}{(b-c).(d-c)} = 0$

$\Rightarrow \dfrac{|b\times d + a\times b + d\times a|}{(b-a).(d-a)} + \dfrac{|b\times d + d\times c + c\times b|}{(b-c).(d-c)} = 0$

12. (4.00)

Given f(x) = f(y) (f(x – y)

Replacing x by x + y, we get

$f(x+y) = f(y)f(x+y-y)$

$\Rightarrow f(x+y) = f(x)f(y)$

$\Rightarrow f(x) = e^{kx} \Rightarrow f'(x)f(y)$

$\Rightarrow f(x) = e^{kx} \Downarrow f'(x) = ke^{kx}$

But $f'(0) = \int_0^4 \{2x\}dx = 2$

$\Rightarrow f'(0)k = 2 \Rightarrow f'(x) = 2e^{2x}$

$\Rightarrow f'(-3) = 2e^{-6}$

$\Rightarrow |\alpha+\beta| = |2-6| = 4$

13. (1024.00)

We have

$(x-16)f(2x) = 16(x-1)f(x)$

Here f(x) is divisible by (x – 16) [$\because$ f(x) is polynomial function]

$\Rightarrow$f(2x) is divisible by (x – 8)

$\Rightarrow$f(x) is divisible by (x – 8)

$\Rightarrow$f(2x) is divisible by (x – 4)

$\Rightarrow$f(x) is divisible by (x – 2)

i.e. f(x) = (x – 2) (x – 4) (x – 8) (x – 16) $\phi(x)$

Putting in the given equation we get

$\phi(2x) = \phi(x) = c = 1$

$\Rightarrow f(x) = (x-2)(x-4)(x-8)(x-16)$

$\Rightarrow f(x) = 2\times4\times8\times16 = 1024$

14. (6.00)

Let L = $\lim_{x\to0} \dfrac{1-\prod_{r=2}^{n}(\cos rx)^{1/r}}{x^2}$

By using L, Hospital's rule

$$\lim_{x\to 0}\frac{\frac{d}{dx}\prod_{r=2}^{n}(\cos rx)^{1/r}}{2x}$$

$$=\lim_{x\to 0}\frac{\prod_{r=2}^{n}(\cos rx)^{\frac{1}{r}-1}\sin(rx)}{2x}$$

$$=\lim_{x\to 0}\prod_{r=2}^{n}(\cos rx)\frac{1}{r}\times\lim_{x\to 0}\frac{\sum_{r=2}^{n}\tan rx}{2x}$$

$$\Rightarrow \lim_{x\to 0}\prod_{r=2}^{n}(\cos rx)\frac{1}{r}\times\lim_{x\to 0}\frac{\sum_{r=2}^{n}\tan rx}{2x}=\frac{1}{2}\sum_{r=2}^{n}r$$

$$=\frac{1}{2}\left(\frac{n(n+1)}{2}-1\right)=\frac{1}{4}(n^2+n-2)$$

Since L = 10

$$\therefore \frac{n^2+n-2}{4}=10\Rightarrow n=6$$

15. (84.00)

Let T_{r+1} be the general term in the expansion of $(ax^{1/6}+bx^{-1/3})^9$

$$\therefore T_{r+1}={}^9C_r(ax^{1/6})^{9-r}(bx^{-1/3})^r$$

$$={}^9C_r a^{9-r}b^r x^{\frac{9-r}{6}-\frac{r}{3}}$$

$$={}^9C_r a^{9-r}b^r x^{\frac{9-3r}{6}}$$

$\because T_{r+1}$ is independent of x

$$\therefore \frac{9-3r}{6}=0\Rightarrow r=3$$

$$\therefore T_4={}^9C_3a^6b^3$$

$$=\frac{9\times 8\times 7}{3\times 2\times 1}a^6b^3=84a^6b^3$$

Now, $\frac{a2+b}{2}\geq(a^2b)^{1/2}$

$1\geq(a^2b)^{1/2}$

$\Rightarrow a^2b\leq 1\Rightarrow a^6b^3\leq 1$

$\Rightarrow 84a^6b^3\leq 84$

$\therefore$ Maximum value of T_4 is 84.

16. (11.00)

We have

$\sin\alpha+\cos\beta=\frac{1}{\sqrt{2}}$ ....(i)

$\cos\alpha+\sin\beta=\frac{\sqrt{2}}{\sqrt{3}}$ ......(ii)

Subtract Eq. (ii) from Eq (i) we get $(\sin\alpha-\sin\beta)-(\cos\alpha-\cos\beta)$

$=\frac{1}{\sqrt{2}}-\frac{\sqrt{2}}{\sqrt{3}}$

$\Rightarrow 2\cos\frac{\alpha+\beta}{2}\sin\left(\frac{\alpha-\beta}{2}\right)+2\sin\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2}=\frac{\sqrt{3}-2}{\sqrt{6}}$

$\Rightarrow 2\sin\left(\frac{\alpha-\beta}{2}\right)\left(\cos\frac{\alpha+\beta}{2}+\sin\frac{\alpha+\beta}{2}\right)=\frac{\sqrt{3}+2}{\sqrt{6}}$ ....(iv)

Eq. (iii) dividing Eq (iv) we get

$\tan\left(\frac{\alpha-\beta}{2}\right)=\frac{\sqrt{3}-2}{\sqrt{3}+2}$

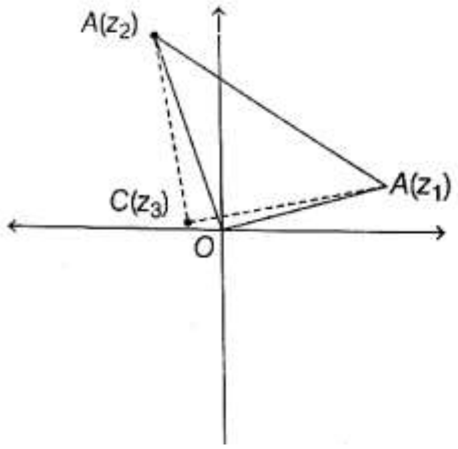
$=\frac{\left(\sqrt{3}-2\right)^2}{-1}$

$=-7+4\sqrt{3}$

$\therefore a=-7, b=4$

17. (25.00)

We have $|z_2+iz_1|=|z_1|+|z_2|$



$\Rightarrow \arg(iz_1)=\arg(z_2)$

$\Rightarrow \arg(i)+\arg(z_1)=\arg(z_2)$

$\Rightarrow \frac{\pi}{2}=\arg(z_2)-\arg(z_1)$ ....(i)

Let $z_3=\frac{z_2-iz_1}{1-i}\Rightarrow(1-i)z_3=z_2-iz_1$

$\Rightarrow z_3-z_2=i(z_3-z_1)\Rightarrow\frac{z_2-z_3}{z_1-z_3}=i$

$\Rightarrow \angle ACB = \dfrac{\pi}{2}$ and $|z_2 - z_3| = |z_1 - z_3|$

$\Rightarrow \angle ACB = \dfrac{\pi}{2}$ and $AC = BC$

$\because AB^2 = AC^2 + BC^2$

$\Rightarrow 100 = 2AC^2$ $\qquad [\because AB = 10]$

$\Rightarrow 50 = AC^2 = AC = BC = 5\sqrt{2}$

Hence area

$(\Delta ABC) = \dfrac{1}{2}, AC.BC = \dfrac{AC^2}{2} = \dfrac{1}{2}$

So = 25 sq. unit.

18. (132.00)

Let image of (x, y) on ellipse about the line

$x - y - 2 = 0$ is (α, β)

$\therefore \dfrac{\alpha - x}{1} = \dfrac{\beta - y}{-1} = \dfrac{-2(x - y - z)}{1+1}$

$\Rightarrow \dfrac{\alpha - x}{1} = \dfrac{\beta - y}{-1} = -x + y + 2$

$\Rightarrow \alpha = y + 2, \beta = x - 2$

$\therefore (x, y) = (\beta + 2, \alpha - 2)$

$\Rightarrow \dfrac{(\beta + 2 - 4)^2}{16} + \dfrac{(\alpha - 2 - 3)^2}{9} = 1$

$\Rightarrow 16\alpha^2 + 9\beta^2 - 160\alpha - 36\beta + 290 = 0$

Equation of reflection of ellipse is

$16x^2 + 9y^2 - 160x - 36y + 292 = 0$

$\therefore \lambda = 160,\ \mu = 290$

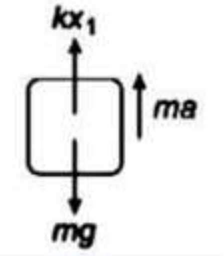
$\therefore \lambda + u = -160 + 290 = 132$

PART (A) : PHYSICS

1. (A, C)

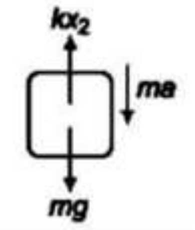
For upward moving elevator

$kx_1 - mg = ma$ …(i)



For downward moving elevator

$mg - kx_2 = ma$

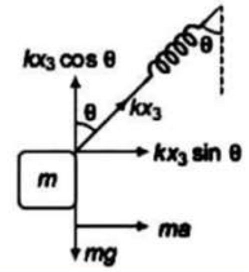


Dividing Eq. (i) by Eq. (ii) we get

$$\frac{g+a}{g-a} = \frac{x_1}{x_2} = \frac{4\sqrt{2}}{3\sqrt{2}}$$

$$\therefore a = \frac{g}{7}$$

For horizontal motion of elevator, spring will be inclined at some angle as given in figure



$kx_3 \cos\theta = mg$ …..(ii)

$kx_3 \sin\theta = ma$ ….(iv)

Squaring and adding eqs (iii) and (iv) we get

$(kx_3)^2 = m^2(g^2 + a^2)$

$$x_3 = \frac{m}{k}\sqrt{g^2+a^2} \quad \text{….(v)}$$

From Eqs. (i) and (iv) we get,

$$\frac{x_3}{x_1} = \frac{\sqrt{g^2+a^2}}{g+a}$$

$$x_3 = \left(\frac{\sqrt{g^2+a^2}}{g+a}\right)$$

Hence x_3 = 5mm

2. (B, D)

$E_{photon} = \dfrac{12400}{\lambda(\text{in } \overset{\circ}{A})} = \dfrac{12400}{4000} = 3.1 eV$

As W_0 for Zn, Fe and Ni > 3.1 eV, there will be no photoelectric emission from any surface

To emit photoelectrons from all the three metals, λ_{max} should corresponds to λ_{max} for Ni (as, it has highest W_0)

$\Rightarrow \lambda_{max}$ (to start ejection from Ni)

$= \dfrac{12400}{W_0(eV)} = \dfrac{12400}{5.9} \overset{\circ}{A} = 2101.7 \overset{\circ}{A}$

If wavelength of the radiation is less than $2000 \overset{\circ}{A}$, then photoelectrons from all the metal surface will be emitted.

3. (A)

As, the peg is moving with constant speed in the horizontal direction, then distance covered will be calculated by the multiplication of speed and the duration (t)

From figure

$OP = 2r\sin\theta = vt$

$\sin\theta = \dfrac{vt}{2r}$

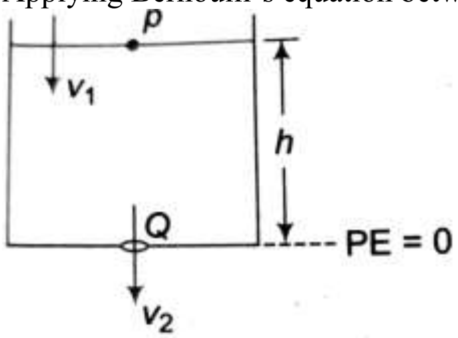
$\cos\theta \dfrac{d\theta}{dt} = \dfrac{v}{2r}$

$\dfrac{d\theta}{dt} = \dfrac{v}{2r\sqrt{1-\sin^2\theta}}$

$= \dfrac{v}{2r\sqrt{1-\dfrac{v^2t^2}{4r^2}}} = \dfrac{v}{\sqrt{4r^2 - v^2t^2}}$

4. (A, B)

Applying Bernoulli's equation between P and Q points



$p_0 + \rho gh + \dfrac{1}{2}\rho v_1^2 = p_0 + 0 + \dfrac{1}{2}\rho v_2^2$

$\rho gh = \dfrac{\rho}{2}(v_2^2 - v_1^2)$

$v_2^2 - v_1^2 = 2gh$ ......(i)

From equation of continuity

$$Av_1 = av_2 = \frac{A}{3}v_2 \Rightarrow v_2 = 3v_1$$

$$9v_1^2 - v_1^2 = 2gh \Rightarrow 8v_1^2 = 2gh$$

$$\Rightarrow v_1 = \sqrt{\frac{gh}{4}} = \frac{\sqrt{gh}}{2}$$

Acceleration of top layer

$$a = \frac{dv_1}{dt} = \frac{1}{2}\sqrt{g} \times \frac{1}{2\sqrt{h}} \times \frac{dh}{dt}$$

$$= \frac{1}{4}\sqrt{\frac{g}{h}} \times (-v_1) = \frac{-1}{4}\sqrt{\frac{g}{h}} \times \frac{\sqrt{gh}}{2}$$

$$= -\frac{g}{8} m/s^2$$

5. (A, D)

Let $\eta \propto m^a d^b v^c$

$\eta = km^a d^b v^c$

Equating dimensions, we get

$[ML^{-1}T^{-1}] = [M^a L^{b+c} T^{-c}]$

$\Rightarrow a = 1, b = -2 \text{ and } c = 1$

So $\eta = k\frac{mv}{d^2}$

Now, $d^2 = \frac{kmv}{\eta}$

At same temperature mean KE is a constant

$\Rightarrow \frac{1}{2}mv^2 = \text{constant}$

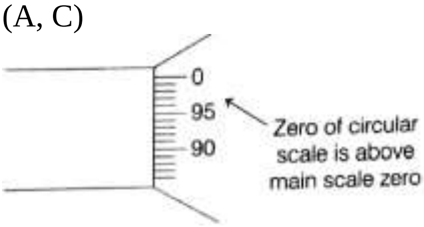
$\Rightarrow \eta d2m^{-\frac{1}{2}} = \text{a constant}$

So, $\frac{d_{CH4}}{d_{He}} = \left(\frac{\eta_{He}}{\eta_{CH4}}\right)^{1/2}\left(\frac{m_{CH4}}{m_{He}}\right)^{1/4}$

$\Rightarrow d_{CH4} == 2.1\times10^{-10}\left(\frac{2.0}{1.1}\right)^{1/2}\left(\frac{16}{4}\right)^{1/4}$

$= 4\times10^{-10} m$

6. (A, C)



Least count of instrument $= \frac{1}{100} mm = 0.01mm$

As, 95th circular scale division coincides with reference line when laws are fully closed, error is negative zero error.

$\text{Error} = -(100-95)\times\frac{1}{100}(\text{mm})$

$=-0.05\text{mm}$

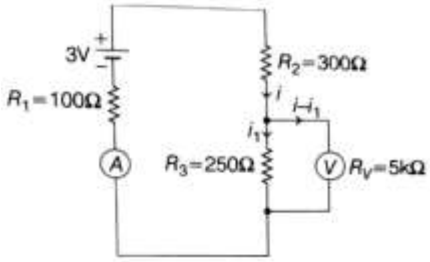
Diameter of wire

$= \text{MSR} + \text{CSR}\times\text{LC} - \text{Error}$

$= 2\text{mm} + 45\times 0.01\text{mm} - (-0.05\text{mm})$

$= 2 + 0.45 + 0.05 = 2.50\text{mm}$

7. (B, D)



By Kirchhoff's loop rule we get

$E - iR_2 - i_1R_3 - iR_1 = 0$

And $i_iR_3 - (i-i_1)R_V = 0$

From above equation we get

$i_1[(R_1+R_V)R_2 - R_VR_3$

$E = \frac{-(R_3-R_V)R_1]}{R_V}$

$\Rightarrow i_1 = 448\times10^{-5}\text{A}$

$\therefore i_1R_3 = 1.12\text{V}$

Current without voltmeter is

$i' = \frac{E}{R_1+R_2+R_3}$

Potential drop with voltmeter is

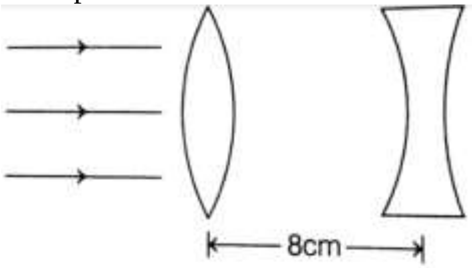
$i'R_3 = \frac{3\times250}{250+300+100} = 1.15\text{V}$

So, error in measurement

$= \frac{1.15-1.12}{1.15}\times100 = 3\%$

8. (B, C, D)

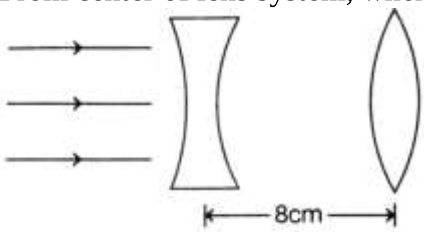
Let a parallel beam is incident on convex lens



For first lens, $f_1 = +30\text{cm}$

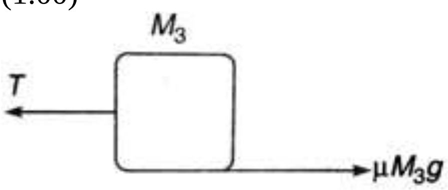
$x_1 = -\infty$

Now, $\frac{1}{v_1} = \frac{1}{f_1} + \frac{1}{u_1}$
Gives v_1 = 30 cm
For second lens,
$f_2 = -20cm$
$x_2 = 30 - 8 = +22cm$
$\therefore \frac{1}{v_2} = \frac{-1}{20} + \frac{1}{22}$
$\Rightarrow v_2 = -220cm$
So, the parallel beam appears to diverge from a point 216 cm.
From center of lens system, when parallel beam is incident over concave lens then



For first lens,
$f_1 = -20cm, u_1 = -\infty$
$\frac{1}{v_1} = \frac{1}{f_1} + \frac{1}{x_1} \Rightarrow v_1 = -20cm$
For second lens, f_2 = +30cm
So, $\frac{1}{v_2} = \frac{1}{f_2} + \frac{1}{u_2}$
Gives v_2 = -420 cm
So beam appears to diverge from a point 416 cm from the centre of lens system.
So, we do not have a simple equation true for all x (and v).
The motion of effective focal length.
$\therefore$ Does not seem to be meaningful for this system.

9. (1.00)



Consider velocity means net acceleration of the system is zero.
Or
Net pulling force on the system is zero. While calculating the pulling force, tension forces are not taken into consideration. Therefore

i) $M_1 g = M_2 g \sin 37^0 + \mu M_2 g \cos 37^0 + \mu M_3 g$ or
$M_1 = M_2 \sin 37^0 + \mu M2 \cos 37^0 + \mu M_3$
Substituting the values, we get
$M_1 = (4)\left(\frac{3}{5}\right) + (0.25)(4)\left(\frac{4}{5}\right) + (0.25)(4) = 4.2kg$

ii) Since M_3 is moving with uniform velocity

$T = \mu M_3 g$

$= (25)(4)(9.8) = 9.8N$

10. (2.82)

Between C and D block moves with a constant speed of $120 cms^{-1}$

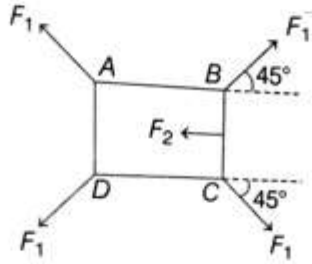
So, time period of oscillation is

$$T = t_{CD} + \frac{T_2}{2} + t_{DC} + \frac{T_1}{2}$$

$$= \frac{60}{120} + 2\pi\sqrt{\frac{m}{k_2}} + \frac{60}{120} + 2\pi\sqrt{\frac{m}{k_1}}$$

= 2.82 s

11. (3.00)



Let F_1 = electrostatic force

And F_2 = force of surface tension from diagram

For equilibrium of line BC,

$$2F_1 \cos 45^0 = F_2$$

$$\Rightarrow \frac{1}{4\pi\varepsilon_0}.\frac{q^2}{a^2}\left(2+\frac{1}{\sqrt{2}}\right) = ra$$

$$\Rightarrow a^3 = \frac{1}{4\pi\varepsilon_0}\left(2+\frac{1}{\sqrt{2}}\right)\frac{q^2}{r}$$

$$\text{or } a = \left\{\frac{1}{4\pi\varepsilon_0}\left(2+\frac{1}{\sqrt{2}}\right)\right\}^{\frac{1}{3}} \times \left(\frac{q^2}{r}\right)^{\frac{1}{3}}$$

$\therefore N = 3$

12. (2.00)

Voltage across the capacitors will increases from 0 to 10V exponentially. The voltage at time t will be given by

$$V = 10(1 - e^{-t/\tau}C)$$

Here $\tau_c = C_{net}R_{net}$

$= (1\times10^6)(4\times10^{-6}) = 4s$

$\therefore V = 10(1 - e^{-t/4})$

Substituting V = 4 volt, we have

$4 = 10(1 - e^{-t/4})$

Or $e^{-t/4} = 0.6 = \frac{3}{5}$

Taking log both sides, we have

$-\frac{t}{4} = \ln 3 - \ln 5$

Or t = 4(ln 5 – ln 3) = 2s

13. (8.00)

$\left|\frac{dN}{dt}\right| = |\text{Activity of radioactive substance}|$

$= \lambda N = \lambda N_0 e^{-\lambda t}$

Taking log both sides

$\ln\left|\frac{dN}{dt}\right| = \ln(\lambda N_0)(-\lambda t)$

Hence, $\ln\left|\frac{dN}{dt}\right|$ versus t graph is a straight line with slope -λ

From the graph, we can see

$\lambda = \frac{1}{2} = 0.5 yr^{-1}$

Now applying the equation

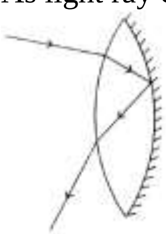
$N = N_0 e^{-\lambda t} = N_0 e^{-0.5\times4.16}$

$= N_0 e^{-2.08} = 0.125 N_0 = \frac{N_0}{8}$

i.e. nuclei decreases by a factor of B. Hence the answer is 8

14. (9.00)

As light ray crosses



Lens two times, focal length F of given system is given by

$\frac{1}{F} = \frac{2}{f_1} - \frac{1}{f_m}$

$\Rightarrow \frac{1}{F} = 2\left(\frac{2(n-1)}{R}\right) - \frac{1}{\left(-\frac{R}{2}\right)}$

$\Rightarrow F = \frac{R}{2(2n-1)} = \frac{40}{2\left(\frac{3\times2}{2}-1\right)}$

= 10 cm

$\therefore F - 1 = 9cm$

15. (A)

Current in capacitor leads voltage by $\frac{\pi}{2}$ and current amplitude in lower branch is $\frac{V}{Z}=\frac{V}{X_C}$

$$\therefore I_{capacitor}=\frac{200\sqrt{2}\sin\left(\omega t+\frac{\pi}{4}+\frac{\pi}{2}\right)}{10}$$

$$=20\sqrt{2}\sin\left(\omega t+\frac{3\pi}{4}\right)$$

16. (A)

In upper branch

$$\cos\theta_1=\frac{R_1}{Z_1}=\frac{R_1}{\sqrt{R_2+X_L^2}}$$

$$=\frac{3}{\sqrt{3^2+4^2}}=\frac{3}{5}$$

$\therefore \phi_1=53^0$ current lags behind voltage

In lower branch

$$\theta=-\frac{\pi}{2}=-90^0$$

Current leads the voltage

So, angle between current is

$=53^0-(-90^0)=143^0$

17. (A)

Conservation of energy gives

$$(E_\alpha+m_\alpha c^2)+(O+m_N c^2)=(E_P+M_P c^2)+(E_X+M_X c^2)$$

And reaction energy is

$$Q=KE_f-KE_i=(E_p+E_x)-E_\alpha$$

$$=(m_\alpha+m_N-m_P-m_X)c^2$$

$$\Rightarrow\frac{-1.26}{931.5}=(4.00388+14.00752-1.00814-M_X)$$

$$\Rightarrow m_X=17$$

So, X is ${}^{17}_{9}O$

18. (D)

Conservation of momentum gives,

$$p_\alpha=p_x+p_p\Rightarrow p_\alpha-p_p=p_x$$

$$\Rightarrow p_\alpha^2+p_p^2-2p_\alpha p_p\cos\theta=p_x^2$$

$$\Rightarrow\cos\theta=-\left\{\frac{m_x E_x-m_\alpha E_\alpha-m_p E_p}{2\sqrt{m_\alpha m_p E_\alpha E_p}}\right.$$

= 0.4429

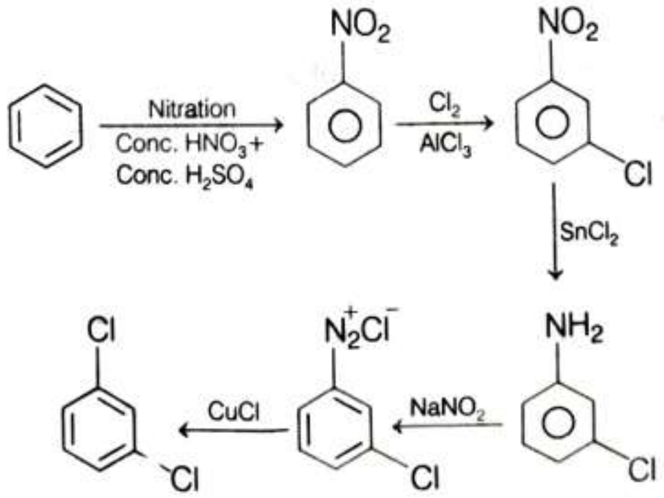
As $\cos 60^0 = \frac{1}{2}$ so above value of θ is just above 60^0

In fact $\theta = 63^0 42'$ (from tables)

PART (B) : CHEMISTRY

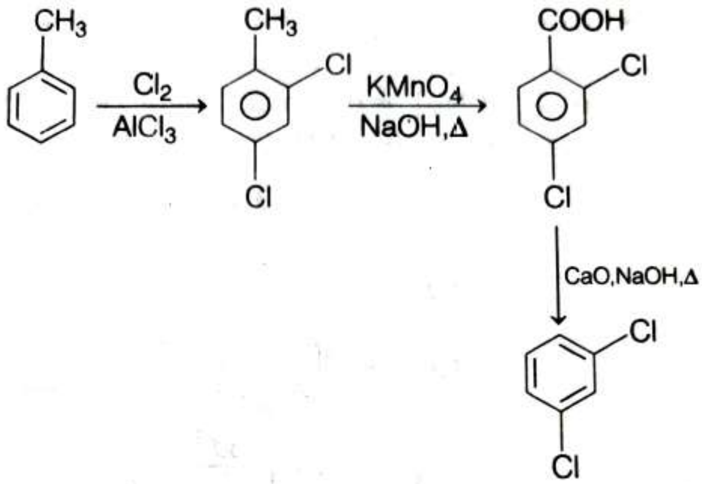
1. (A, B, D)

(a)



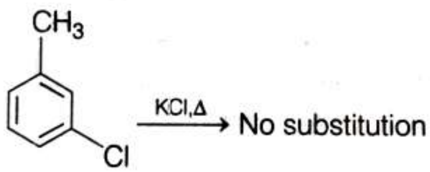
1,3-dichloro benzine

(b)

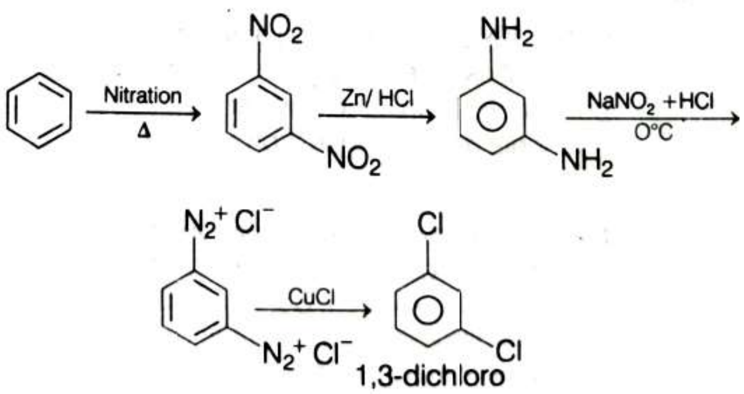


1,3-dichloro benzene

(c)



(d)



1,3-dichloro benzene

2. (C, D)

(A) SO_2 and CO_2 cannot be distinguished by using lime water as both produce the milky white of $CaCO_3$ and $CaSO_3$

$$CO_2 \text{ and } SO_2 \xrightarrow{\text{Lime water}} \text{milky solution}$$

(B) SO_2 and CO_2 on reaction with $BaCl_2$ produces the same white ppt. in the form of barium sulphate and calcium sulphate in which both are soluble in dil.HCl.

$$SO_2 + BaCl_2 \rightarrow \underset{\text{white ppt}}{BaSO_3} \downarrow$$

$$CO_2 + BaCl_2 \rightarrow \underset{\text{white ppt}}{BaSO_3} \downarrow$$

(C) SO_2 on reaction with H_2O_2 produces H_2SO_4 due to oxidation of SO_2 and H_2SO_4 on reaction with $BaCl_2$ produces $BaSO_4$

$$SO_2 + H_2O_2 \rightarrow H_2SO_4$$

$$H_2SO_4 + BaCl_2 \rightarrow \underset{\text{white ppt}}{BaSO_4} \downarrow$$

CO_2 does not reacts with H_2O_2, hence SO_2 can be separated easily.

(D) SO_2 on reaction with acidified dichromate turns orange colour of dichromate green white there is no effect of acidified $K_2Cr_2O_7$ on CO_2.

Acidified

$$K_2Cr_2O_7 \text{(orange)} \xrightarrow{SO_2} \text{Green}$$

$$\text{Acidified } K_2Cr_2O_7 \xrightarrow{CO_2} \text{No change in colour}$$

(orange)

Hence, correct choice is (c) and (d)

3. (B)

$$K_2Cr_2O_7 + 6H_2SO_4 + 4KCl \Rightarrow 6KHSO_4 + 3H_2O + \underset{\substack{(J) \\ \text{Chromyl chloride}}}{2CrO_2Cl_2}$$

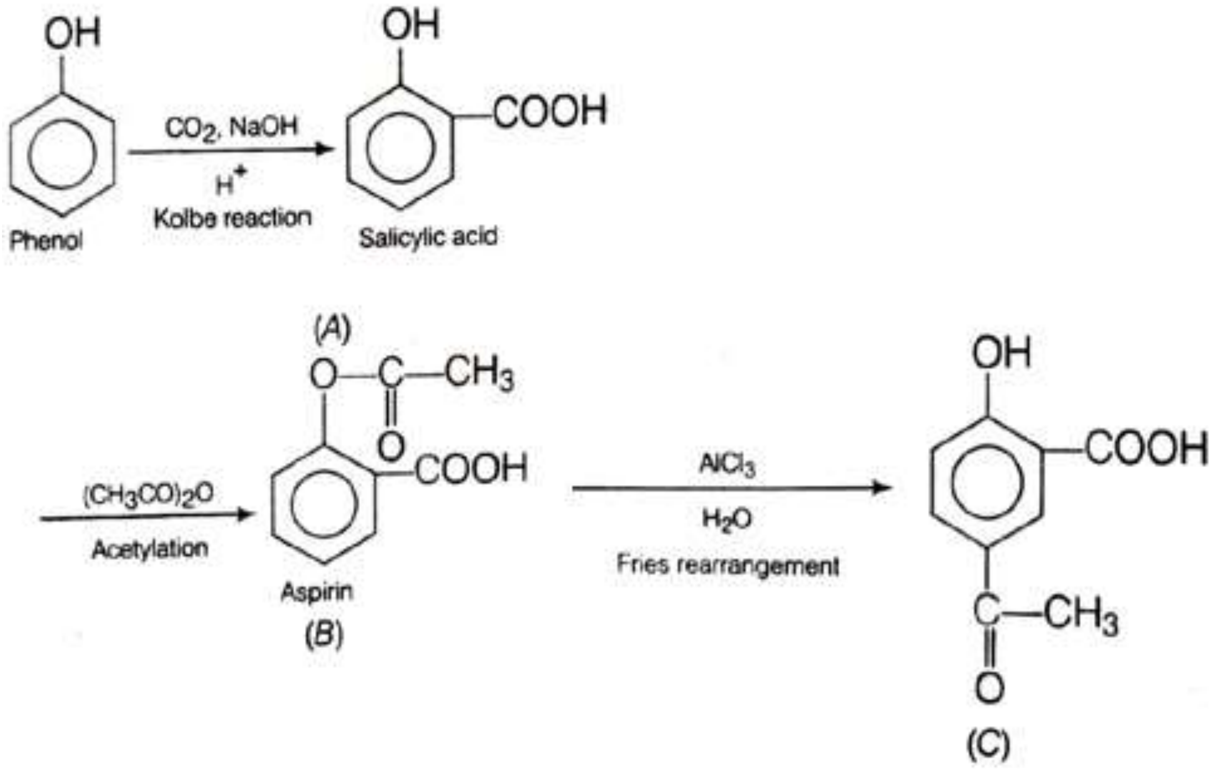
When a mixture of a metal chloride and potassium dichromate is heated with conc. H_2SO_4 orange red vapour or chromyl chloride are evolved

$$4KOH + CrO_2Cl_2 \rightarrow \underset{\substack{\text{(Yellow solution)} \\ (K)}}{K_2CrO_4} + 2KCl + 2H_2O$$

$$\underset{(K)}{2CrO_4^{2-}} + 2H^+ \rightarrow Cr_2O_7^{2-} + H_2O$$

$$Cr_2O_7^{2-} + H_2SO_4 + 3SO_2 \rightarrow SO_4^{2-} + \underset{\substack{(M) \\ \text{(Green coloured)}}}{Cr_2(SO_4)_3} + H_2O$$

4. (C, D)



5. (A, B, C)

In $Pt(NH_3)_2Cl_2$, platinum is in + 2 oxidation state with d^2 configuration. It has dsp^2 configuration with square planar geometry. Number of unpaired electron are zero. So, it is diamagnetic in nature.

6. (A, C, D)

(A) Formation of Slag When ore is treated with silica it form slag of iron silicates

$$FeO + SiO_2 \rightarrow \underset{\text{Slag}}{Fe.SiO_3}$$

(B) Reduction of Zn ZnO on reaction ZnO with coke, converts ZnO into Zn and CO. This is due to reduction of ZnO into Zn.

$$ZnO + C \rightarrow Zn + CO$$

(C) $2Al_2O_3 + 3C \rightarrow 4Al + 3CO_2$

The equation (C) represents electrolysis of alumina which is done by using electrolytic cell that contain steel cathode and graphite anode

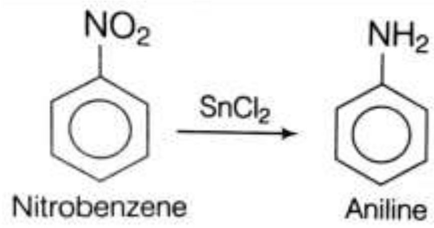
7. (A, B)

Crystal structure of diamond and corrudum is same

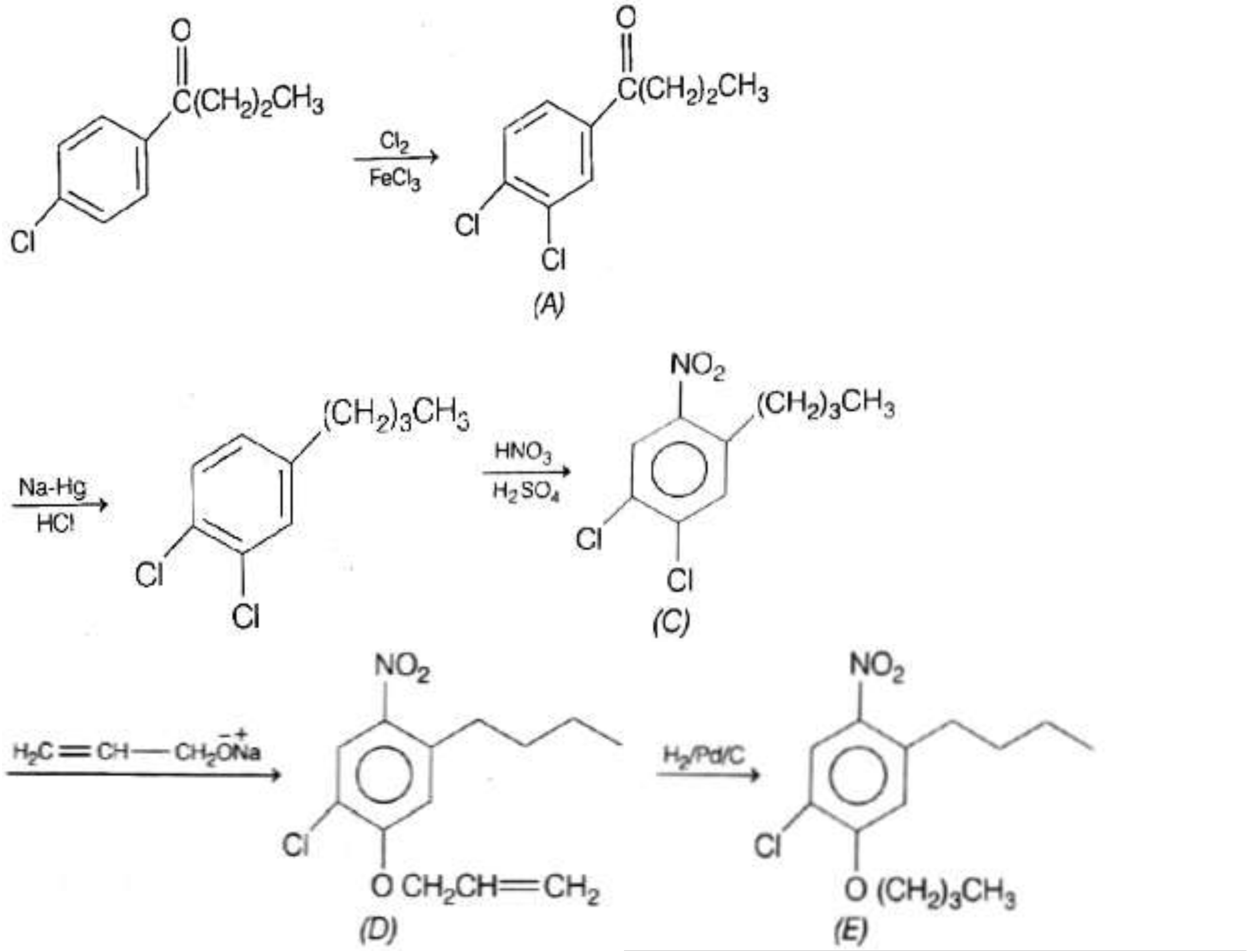
Carbone is a mixture of 5-10% CO_2 and O_2 which is used in artificial respiration of pneumonia patients

PbO is known as litharge which has yellow orange colour

$SnCl_2$ is a powerful reducing agent and is used to reduce nitrobenzene into aniline



8. (B)



9. (4.00)

$$A(g) + 2B(g) \rightleftharpoons C(g)$$

At V 3M 4M xM

At 2V $\left(\frac{3}{2}+y\right)\left(\frac{4}{2}+2y\right)\left(\frac{x}{2}-y\right)$

$$K_{eq} = \frac{x}{3\times 4^2} = \frac{x}{48} \quad \ldots..(i)$$

Given 2y + 2 = 3

$$y = \frac{1}{2} = 0.5$$

$$k_{eq} = \frac{\left(\frac{x-1}{2}\right)}{(2)(3)^2}$$

On equating Eq. (i) and (ii), we have x = 4M

10. (8.00)

$\Delta E = \frac{3}{4}\times 0.85 eV$

Photon will be in Brackett series $(\because 0.31 \le E \le 0.85)$ for Brackett

$0.85\left(1-\frac{1}{4}\right)=13.6\left(\frac{1}{4^2}-\frac{1}{n^2}\right)$

$0.85\left(1-\frac{1}{4}\right)=\frac{13.6}{16}\left(1-\left(\frac{4}{n}\right)^2\right)$

$\Rightarrow \frac{4}{n}=\frac{1}{2}\Rightarrow n=8$

Hence, n = 8

11. (4.00)

$Fe^{3+}=3d^5$ and CN^- is strong ligand paramagnetic
Co^{3+} = $3d^6$ and NH_3 is strong ligand, diamagnetic
Co^{3+} = $3d^6$ and C_2O_4 is a strong ligand, diamagnetic
Ni^{2+} = $3d^8$ and Ni octahedral complexes are always paramagnetic
Pt^{2+} = $5f^8$ and CN^- is square planar complex, diamagnetic
Zn^{2+} = $3d^{10}$ and it is diamagnetic

12. (20)

$(3.42\times10^{20})\because K_C=\frac{K_1}{K_2}=10^{10}$

$\therefore K_1=K_2\times10^{10}$

Also $K_2=10^{20}e^{-\frac{Ea}{RT}}=10^{20}e-\frac{54}{8.314\times10^{-3}\times298}$

$=10^{20}\times3.42\times10^{-10}s^{-1}=3.42\times10^{10}s^{-1}$

$\therefore K_1=K_2\times10^{10}=3.42\times10^{20}$

13. (0.01)

$p(NO)=2p(Br_2)$ (from balanced equation)

Because it is 34% dissociated

$p(NOBr)=0.66p^0(NOBr)$

$p(NO)=0.34p^0(NOBr)$ and

$P(Br_2)$ = 0.17 $p^0(NOBr)$

P(NOBr) + p(NO) + $p(Br_2)$ =025 atm

0.66 p^0 + 0.34p^0 + 0.17p^0 = 2.5 atm

1.17p^0 = 0.25 atm

$p^0(NOBr)=\frac{0.25}{1.17}=0214atm$

P(NOBr) = (0.66) (0214 atm) =0.14atm
P(NO) =(0.34)(0214 atm)=0.073 atm
P(Br_2) =0.17 (0214 atm) =0.036 min

$$\because K_P = \frac{p(NO)^2 p(Br_2)}{p(NO)}$$

$$= 0.0098 \times 10^{-3} = 0.01 \times 10^{-3}$$

14. (8.23)

At anode

$$Co(CN)_6^{4-} \rightarrow Co(CN)_6^{3-} + e^- \quad E^0_{OP} = +0.83V$$

At Cathode

$$Co^{3+} + e^- \rightarrow Co^{2+} \quad E^0_{RP} = +1.82V$$

Overall cell reaction

$$Co(CN)_6^{4-} + Co^{3+} \rightleftharpoons CO(CN)_6^{3-} + Co^{2+}$$

Nernst equation for this cell is given as

$$E_{cell} = E^0_{cell}$$

$$\frac{-0.0591}{1} \log \frac{[Co^{2+}][Co(CN)_6^{3-}]}{[Co^{3+}][Co(CN)_6^{4-}]}$$

$$\text{Or } E_{cell} = E^0_{cell} + \frac{0.0591}{1} \log \frac{[Co^{3+}][Co(CN)_6^{4-}][CN^-]^6}{[Co^{2+}][Co(CN)_6^{3-}][CN^-]^6} \quad \ldots.(i)$$

$$\text{Now, } 6CN^- + Co^{2+} \rightleftharpoons Co(CN)_6^{4-}$$

$$\therefore K_{f_2} = \frac{[Co(CN)_6^+]}{[Co^{2+}] + [CN^-]^6} = 10^{19} \quad \ldots..(ii)$$

$$\text{Also } 6CN^- + Co^{3+} \rightleftharpoons Co(CN)_6^{-3}$$

$$K_{f_2} = \frac{[Co(CN)_6^{3-}]}{[Co^{3+}][CN^-]^6} \quad \ldots..(iii)$$

On solving Eqs (i), (ii) and (iii), we have

$$E_{cell} = E^0_{cell} + \frac{0.0591}{1} \log \frac{k_{f_1}}{k_{f_2}}$$

$$\text{Or } 0 = (0.83 + 1.82) + 0.0591 \log \frac{10^{19}}{K_{f_2}}$$

$$K_{f_2} = 8.23 \times 10^{63}$$

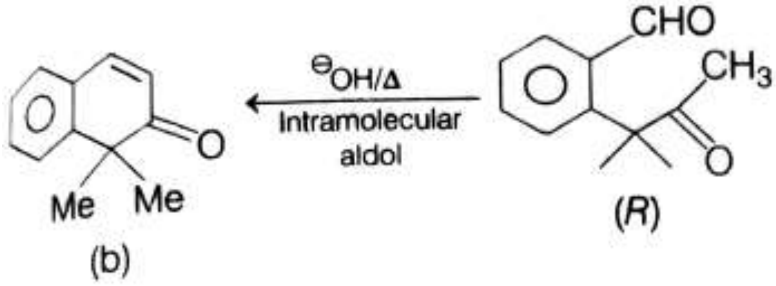
15. (D)

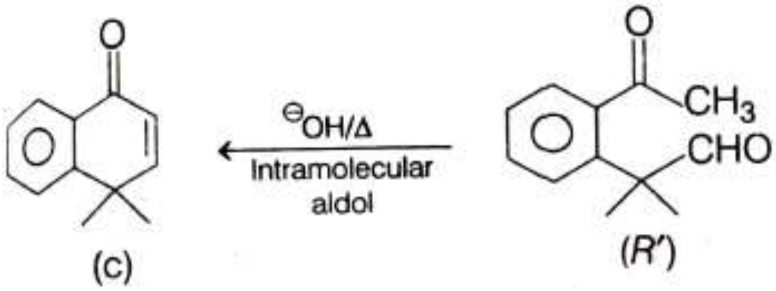
16. (A)

17. (B)

P gives iodoform test so it must be methylketone. From the given structures (P) can be either (a) or (b). Here (a) and (b) on reaction with MeMgBr followed by cyclisation would give products (Q) and

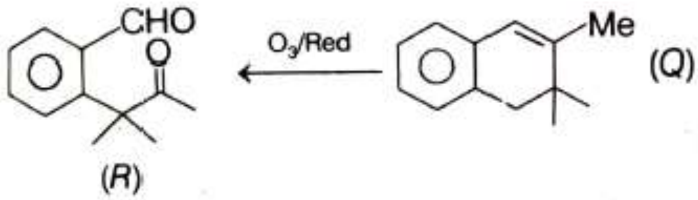
(R) respectively. It is clear from the structures of (Q) and(R) that (S) can be either (b) or (c). On proceeding reverse from (b) or (c). Products (R) and (Q) can be determined



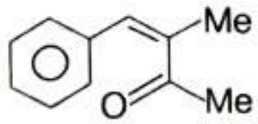


Product S is (b)

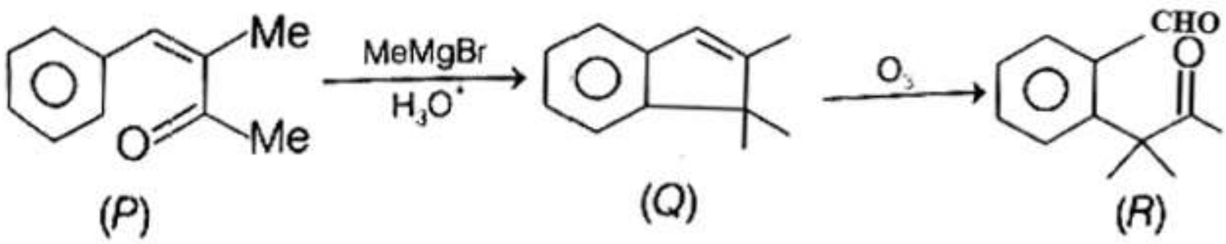
Now, proceed reverse from (R) to get back structure (Q)



The correct structure of P is



18. (A)



PART (C) : MATHEMATICS

1. (A, B)

(A) $(M - N)^2 (M + N^2) = 0$ ....(i) $[\because MN^2 = N^2M]$

$\Rightarrow |M - N^2||M + N^2| = 0$

Case I If $|M + N^2| = 0$

$|M^2 + MN^2| = 0$

Case II If $|M + N^2| \neq 0$

$\Rightarrow M + N^2$ is invertible

From Eq. (i)

$(M - N^2)(M + N^2)(M + N^2)^{-1} = 0$

$\Rightarrow M - N^2 = 0$ which is wrong

(B) $(M + N^2)(M - N^2) = 0$

Pre multiply by M

$\Rightarrow (M^2 + MN^2)(M - N^2) = 0$ ......(ii)

Let $M - N^2 = U \Rightarrow$ from eq (i) their exist some non-zero

$U(M^2 + MN^2)U = 0$

2. (A, B, C)

$\therefore f'(x) = 0$ has a root α_1 such that $0 < \alpha_1 < \alpha_0$ [using Rolle's theorem]

Now, $f'(x) = 0$ is a fourth degree equation. As imaginary roots occur in pairs f'(x) = 0 will have another real root α_2.

As f(x)=0 is an equation of the degree five, it will have atleast 3 real roots.

Thus, f'(x) will have at least two real roots and f''(x) will have atleast one real root.

3. (A, D)

We have

$z_1 = 5 + 12i, |z_2| = 4$

(a) $|z_1 + iz_2| \leq |z_1| + |iz_2| = 13 + 4 = 17$

(b) $|z_1 + (1+i)z_2| \geq ||z_1| - |1+i||z_2|| \geq |13 - 4\sqrt{2}| = 13 - 4\sqrt{2}$

$$\Rightarrow \left|z_2 + \frac{4}{z_2}\right| \leq |z_2| + \frac{4}{|z_2|} = 4 + 1 = 5$$

$$\Rightarrow \left|z_2 + \frac{4}{z_2}\right| \geq |z_2| - \frac{4}{|z_2|} = 4 - 1 = 3$$

(c) Minimum value of $\left|\frac{z_1}{z_2 + \frac{4}{z_2}}\right| = \frac{13}{5}$

(d) Maximum value of $\left|\dfrac{z_1}{z_2+\dfrac{4}{z_2}}\right| = \dfrac{13}{5}$

4. (A, B, C)

Consider the given differential equation

$$\tan^2 x\frac{dy}{dx} = \sec x(1-y) - \frac{dy}{dx}$$

$$\Rightarrow (1+\tan^2 x)\frac{dy}{dx} = \sec x(1-y)$$

$$\Rightarrow \sec^2 x\frac{dy}{dx} + y\sec x = \sec x$$

$$\Rightarrow \frac{dy}{dx} + y\cos x = \cos x$$

$$\Rightarrow \frac{dy}{dx} + y\cos x = \cos x$$

$$IF = \int e^{\sin x}\cos x dx$$

$= e^{\sin x} + C$

Now, for x = 0 and y =1 we have

1 = 1 + C $\Rightarrow$C =0

$$\Rightarrow ye^{\sin x} = e^{\sin x} \Rightarrow y = 1$$

$$\therefore f(x) = 1$$

Which is continuous periodic differential and even function

$\therefore$ Option (a, b, c) are correct answer.

5. (A, B, D)

The given question

$$\frac{x-5}{3} = \frac{y-7}{-1} = \frac{z+2}{1} \quad ....(i)$$

And $\dfrac{x+3}{-3} = \dfrac{y-3}{2} = \dfrac{z-6}{4}$ .....(ii)

Any point P on Eq. (i) is

$(3r_2 - 3, 2r_2 + 3, 4r_2 + 6)$ the direction ratio of PQ

$(3r_1 + 3r_2 + 8, -r_1 - 2r_2 + 4r_1 - 4r_2 - 87)$ ......(iii)

Suppose the line with direction ratio's 2, 7, -5 will be proportional to the Dr's given by eq. (iii)

$$\therefore \frac{3r_1 + 3r_2 + 8}{2} = \frac{-r_1 - 2r_2 + 4}{7} = \frac{r_1 - 4r_2 - 8}{-5} \quad .....(iv)$$

Solving Eq. (iv) we get $r_1 = r_2$ =-1

So the point of intersection P(2, 8, -3) and Q (0, 1, 2) and intercepted length

$$PQ = \sqrt{(2-0)^2 + (8-1)^2 + (3-2)^2}$$

$$= \sqrt{78}$$

The equation of PQ is

$$\frac{x-2}{2}=\frac{y-8}{7}=\frac{z+3}{-5}$$

6. (A, B, C, D)

(a) Graph of $y = |x^2 - 4|x| + 3|$ is shown above.
Clearly, the above equation
$|x^2 - 4|x| + 3| = 0$ has four solutions ± 1 and ± 3.

So (a) is correction (c) also f(x) = 0 and y =a intersect at 8 distinct point, if 0 < a < 1
Thus, f(x) = a has 8 real roots fpr 0 < a < 1
(d) is also the correct option as $y \geq 0$

7. (A, B, C, D)

We have, f(x) = f(1 – x)

Put $x=\frac{1}{2}+x$

$$\Rightarrow f\left(\frac{1}{2}+x\right)=f\left(\frac{1}{2}-x\right)$$

Hence $f\left(x+\frac{1}{2}\right)$ is an even function

of $f\left(x+\frac{1}{2}\right)$ sinx is an odd function

Also $f'(x)=-f'(1-x)$ for $x=\frac{1}{2}$

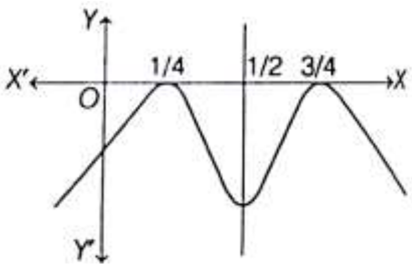
We have $f'\left(\frac{1}{2}\right)=0$

Also $\int_{1/2}^{1}(1-t)e^{\sin\pi t}=-\int_{1/2}^{0}f(y)e^{\sin\pi y}dy$ [$\because$ put 1 – t=y]

Since $f'\left(\frac{1}{4}\right)=0,\ f'\left(\frac{3}{4}\right)=0$

Also $f'\left(\frac{1}{2}\right)=0$

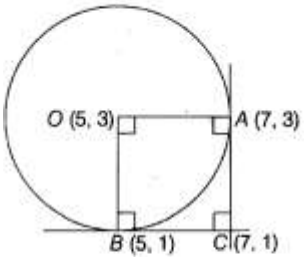
$\Rightarrow f''(x)=0$ at least twice in [0,1] by Rolle's theorem



8. (A, C, D)

We have $x^2 + y^2 - 10x - 6y + 30 = 0$
Centre O (5, 3)

Radius = $\sqrt{(5)^2+(3)^2-30} = \sqrt{4} = 2$



(a) Area of quadrilateral = $OA^2 = 2^2 = 4$

(b) Radical axis of family S = 0 is line passing through AB equation of AB = x – y = 4

(c) Smallest possible circle of family S = 0 has AB as diameter given by

$(x-5)(x-7)+(y-3)(y-1)=0$

$\Rightarrow x^2+y^2-12x-4y+38=0$

(d) The coordinate of C are (7, 1)

9. (25.00)

We have

$$f(x)=ax+\cos 2x+\sin x+\cos x$$

$$f'(x)=a-2\sin 2x+\cos x-\sin x$$

Let g(x)= -2sin2x + cosx – sinx

= 2{(cosx-sinx)^2 -1} + cosx – sinx

Put cosx – six = t

$\therefore t\in\left[-\sqrt{2},\sqrt{2}\right]$

$\therefore g(x)=-2(t^2-1)+t$

$=-2t^2+2+t$

$\Rightarrow -2-\sqrt{2}\le g(x)\le \frac{17}{8}$

Since f(x) is strictly increasing

$\therefore f'(x)\ge 0$

Hence $a\ge \frac{17}{8}$

$a\in\left[\frac{17}{8},\infty\right)$

m = 17, n = 8

$\therefore m+n=17+8=25$

10. (0.25)

We have

$$f\left(\frac{\pi}{6}+x\right)+f\left(\frac{\pi}{3}-x\right)=\frac{\pi}{2}$$

X is replaced by $\frac{\pi}{3}-x$, we get

$$f\left(\frac{\pi}{6}+\frac{\pi}{3}-x\right)+f\left(\frac{\pi}{3}-\frac{\pi}{3}+x\right)=\frac{\pi}{2}$$

$$\Rightarrow f\left(\frac{\pi}{2}-x\right)+f(x)=\frac{\pi}{2}$$

$$\Rightarrow f\left(\frac{\pi}{2}-x\right)=\frac{\pi}{2}-f(x)$$

Now, Let $I=\int_0^{\pi/2}(\cos f(x))^2 dx$

$$I=\int_0^{\pi/2}\left(\cos f\left(\frac{\pi}{2}-x\right)\right)^2 dx \qquad \left[\because \int_0^a f(x)dx=\int_0^a f(a-x)dx\right]$$

$$\Rightarrow I=\int_0^{\pi/2}\left(\cos\left(\frac{\pi}{2}-f(x)\right)\right)^2 dx$$

$$\Rightarrow I=\int_0^{\pi/2}(\sin f(x))^2 dx$$

$$\Rightarrow 2I=\int_0^{\pi/2}(\cos^2 f(x)+\sin^2 f(x))dx$$

$$\Rightarrow 2I=\int_0^{\pi/2} dx$$

$$\Rightarrow 2I=\frac{\pi}{2}\Rightarrow I=\frac{\pi}{4}$$

$$\therefore k=\frac{1}{4}=0.25$$

11. (8.5)

Line through point P (-2, 3, -4) and parallel to the given line

$\frac{x+2}{3}=\frac{2y+3}{4}=\frac{3z+4}{5}$ is

$$\Rightarrow \frac{x+2}{3}=\frac{y+3/2}{2}=\frac{z+4/3}{5/3}=\lambda$$

Any point on this line is

$$Q\left(3\lambda-2, 2\lambda-\frac{3}{2}, \frac{5}{3}\lambda-\frac{4}{3}\right)$$

Direction ratio of PQ are

$$\left[3\lambda, \frac{4\lambda-9}{2}, \frac{5\lambda+8}{3}\right]$$

Now, PQ is parallel to the given plane

$4x+12y-3z+1=0$

Hence, line is perpendicular to the normal to the plane

Thus, $4(3y)+12\left(\frac{4\lambda-9}{2}\right)-3\left(\frac{5\lambda+8}{3}\right)=0$

$$\Rightarrow \lambda=2\Rightarrow Q\left(4, \frac{5}{2}, 2\right)$$

$$\Rightarrow PQ = d = \sqrt{6^2 + \left(\frac{5}{2} - 3\right)^2 + 6^2}$$

$$\frac{17}{2} = 8.5$$

12. (75.00)

Let $\log_{10} x = a \Rightarrow x = 10^a$

$\log_{10} y = b \Rightarrow y = 10^b$

$\log_{10} z = c \Rightarrow z = 10^c$

Given xyz $= 10^{81}$

$\therefore 10^a.10^b.10^c = 10^{81}$

$\therefore 10^{a+b+c} = 10^{81}$

$\Rightarrow a + b + c = 81$

Also $(\log_{10} x)(\log_{10} yz) + (\log_{10} y)(\log_{10} z) = 468$

$\Rightarrow \log_{10} x(\log_{10} y + \log_{10} z) + (\log_{10} y)(\log_{10} z) = 468$

$\Rightarrow a(b + c) + bc = 468$

$\Rightarrow ab + bc + ac = 468$

$\Rightarrow (a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$

$\Rightarrow (81)2 = a^2 + b^2 + c^2 + 2(468)$

$\Rightarrow a^2 + b^2 + c^2 = 6561 - 936$

Now, $\sqrt{(\log_{10} x)^2 + (\log_{10} y)^2 + (\log_{10} z)^2} = \sqrt{5625} = 75$

13. (32.00)

Let (1, 1, 1), (-1, 1, 1), (1, -1, 1), (-1,-1,1) be vectors $\vec{a}, \vec{b}, \vec{c}, \vec{d}$. Rest of the vector are $-\vec{a}, \vec{b}, -\vec{c}, -\vec{d}$ and let us find the number of ways of selecting coplanar vectors.

Observe that out of any three coplanar vectors, two will be collinear (antiparallel)

Number of ways of selecting antiparallel pair = 4

Number of ways of selecting the third vector =6

Total = 24

Number of non-coplanar selection $= {}^8C_3 - 24 = 32$

Hence $\lambda = 32$

14. (2018.00)

Let y= $\lim_{n\to\infty}\left(\sum_{k=10}^{n+9} \frac{2^{\frac{11(k-9)}{n}}}{\log_2 e^{n/11}} - \sum_{k=0}^{n-1} \frac{58}{\pi\sqrt{(n-k)(n+k)}}\right)$

$$\Rightarrow y = \lim_{n\to\infty} \sum_{k=10}^{n+9} \frac{2^{\frac{11(k-9)}{n}}}{\frac{n}{11}\log_2 e} - \lim_{n\to\infty} \frac{1}{n} \sum_{k=0}^{n-1} \frac{58}{\sqrt{1 - \left(\frac{k}{n}\right)^2}}$$

$$\Rightarrow y = \lim_{n\to\infty}\frac{11}{n}\sum_{k=1}^{n}2^{\frac{11k}{n}}\log_e 2 - \lim_{n\to\infty}\frac{1}{n}\sum_{k=0}^{n-1}\frac{58}{\pi\sqrt{1-\left(\frac{k}{n}\right)^2}}$$

$$\Rightarrow y = \int_0^{11}2^x\log_e 2dx - \int_0^1\frac{58}{\pi\sqrt{1-x^2}}dx$$

$$\Rightarrow y = [2x]_0^{11} - \frac{58}{\pi}[\sin^{-1}x]_0^1$$

$$\Rightarrow y = 2^{11} - 2^0 - \frac{58}{\pi}(\sin^{-1}1 - \sin^{-1}0)$$

$$\Rightarrow y = 2048 - 1 - \frac{58}{\pi}\left(\frac{\pi}{2}\right)$$

$$\Rightarrow y = 2047 - 29 \Rightarrow y = 2018$$

15. (B)

If a family of n children contains exactly k boys, then by binomial distribution, its probability is

$${}^nC_k\left(\frac{1}{2}\right)^k\left(\frac{1}{2}\right)^{n-k}$$

Hence, by total probability law, the probability of a family of n children having exactly k boys is given by

$$\alpha p^n\ {}^nC_k\left(\frac{1}{2}\right)^k\left(\frac{1}{2}\right)^{n-k}$$

$\therefore$ Required probability is

$$= \sum_{n=k}^{\infty}\alpha p^n\ {}^nC_k\left(\frac{1}{2}\right)^k\left(\frac{1}{2}\right)^{n-k}$$

$$= \alpha\left(\frac{1}{2}\right)^k p^k\sum_{n=k}^{\infty}{}^nC_k\left(\frac{1}{2}\right)^{n-k}$$

$$= \alpha\left(\frac{1}{2}\right)^k p^k\left[1 + {}^{k+1}C_1\left(\frac{P}{2}\right) + {}^{k+2}C_2\left(\frac{P}{2}\right)^2 \ldots\ldots\right]$$

$$= \alpha\left(\frac{1}{2}\right)^k p^k\left(1-\frac{P}{2}\right)^{-(k+1)}$$

$$= 2\alpha p^k(2-p)^{-(k+1)} = 2\alpha p^k(2-p)^{-k-1}$$

16. (C)

Let A denotes the even of a family including at least one boy

Then $P(A) = 2\alpha\sum_{k=1}^{\infty}p^k(2-p)^{-(k+1)}$

$$= \frac{2\alpha}{2-p}\left(\frac{\frac{p}{2-p}}{1-\frac{p}{2-p}}\right)$$

$$= \frac{\alpha p}{(2-p)(1-p)}$$

17. (B)

Given $1, \alpha_1, \alpha_2, \alpha_3, ...\alpha_{n-1}$ are n distinct, nth roots of unity

$\therefore (z-1)(z-\alpha_1)(z-\alpha_2)......(z-\alpha_{n-1}) = z^n - 1$

Putting z = -1 we get

$(1+\alpha_1)(1+\alpha_2).....(1+\alpha_{n-1})$

$= \frac{1-(-1)^n}{2}$

Now, $|1+\alpha_k|$

$= \left|1+\cos\frac{2k\pi}{n} + i\sin\frac{2k\pi}{n}\right| = 2\left|\cos\frac{k\pi}{n}\right|$

For n = 45

$|1+\alpha_k| = 2\left|\cos\frac{k\pi}{45}\right| = 2|\cos(4k)|$

Since $\prod_{k=1}^{44}(1+\alpha_k) = \frac{1-(-1)^{45}}{2} = 1$

$\therefore 2^{44}(\cos 4^0 \cos 8^0 \cos 12^0 .... \cos 176^0) = 1$

$\Rightarrow \cos 4^0 \cos 8^0 \cos 12^0 ...... \cos 88^0 = \frac{1}{2^{22}}$

18. (A)

We have $\sum_{k=0}^{n-1} |z_1 + \omega^k z_2|^2$

$= \sum_{k=0}^{n-1}(z_1+\omega^k z_2)(\bar{z}_1 + \bar{\omega}^k \bar{z}_2)$

$= \sum_{k=0}^{n-1}(z_1\bar{z}_1 + (\omega\bar{\omega})^k z_2\bar{z}_2 + \bar{z}_1 z_2 \omega^k + z_1\bar{z}_2\bar{\omega}^k)$

$= \sum_{k=0}^{n-1}(|z_1|^2 + |z_2|^2) + (z_2 z_1)\frac{(1-\omega^n)}{1-\omega} + z_1\bar{z}_2\left(\frac{1-\bar{\omega}^n}{1-\bar{\omega}}\right)$

$= n[|z_1|^2 + |z_2|^2]$