PACE-IIT & MEDICAL

MUMBAI / DELHI-NCR / PUNE / NASHIK / AKOLA / GOA / JALGOAN / BOKARO / AMRAVATI / DHULE

IIT – JEE: 2023 **AITS – 14 (ADVANCED)** **DATE: 28/05/23**

ANSWER KEY

PAPER – I

Que.	1	2	3	4	5	6	7	8	9	10
Ans.	A	D	B	C	D	B	B	ABD	AD	B
Que.	11	12	13	14	15	16	17	18	19	20
Ans.	D	ACD	ABC	AB	AB	6	0	4	5	2
Que.	21	22	23	24	25	26	27	28	29	30
Ans.	D	C	A	C	B	A	C	ABCD	AB	AB
Que.	31	32	33	34	35	36	37	38	39	40
Ans.	ACD	ABCD	ABCD	AD	AB	8	6	6	9	4
Que.	41	42	43	44	45	46	47	48	49	50
Ans.	D	D	D	A	B	B	C	AB	ABCD	ABD
Que.	51	52	53	54	55	56	57	58	59	60
Ans.	ABD	ACD	ABC	AB	ABC	7	0	2	5	3

PAPER – II

Que.	1	2	3	4	5	6	7	8	9	10
Ans.	AC	ABC	ABC	C	B	ABD	AD	AC	BC	ABD
Que.	11	12	13	14	15	16	17	18	19	20
Ans.	1	2	2	4	1	3	8	2	1	8
Que.	21	22	23	24	25	26	27	28	29	30
Ans.	C	B	B	ABC	AC	ABCD	ABD	ABCD	AC	AB
Que.	31	32	33	34	35	36	37	38	39	40
Ans.	5	8	5	6	5	3	5	0.04	4	6
Que.	41	42	43	44	45	46	47	48	49	50
Ans.	ABC	BD	ABD	AC	B	D	A	C	ABC	ABC
Que.	51	52	53	54	55	56	57	58	59	60
Ans.	5	8	5	2	8	2	1	2	2	6

Note : Detailed solution to this test is available on Tuesday after 02.00 pm on our website.:www.iitianspace.com

PART (A) : PHYSICS

SOLUTIONS

1. **(A)**

2. **(D)**

$$P - P_0 = \frac{4T}{R}$$

$$P = 2\times10^5\, N/m^2$$

3. **(B)**

$$P = \frac{nhc}{\lambda t}$$

$$i = \left(\frac{n}{t}\right)ex\% = \frac{p\lambda_e}{hc}x\% = \frac{1.55\times10^{-3}\times4\times10^{-7}}{6.63\times10^{-34}\times3\times10^{8}}$$

4. **(C)**

$$\frac{1}{\lambda_{k\alpha}} = R\left(\frac{1}{1^2}-\frac{1}{2^2}\right)(z-b)^2$$

$$\frac{1}{\lambda_{k\beta}} = R\left(\frac{1}{1^2}-\frac{1}{3^2}\right)(z-b)^2$$

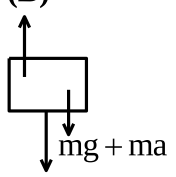
$$\frac{\lambda_{k_\beta}}{\lambda_{k_\alpha}} = \frac{27}{32}$$

5. **(D)**

$$Av = A_1v_1 + A_2v_2$$

$$A\times10 = \frac{A}{2}\times6+\frac{A}{3}\times v_2$$

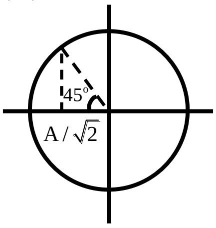
6. **(B)**



$$\therefore T = m(g+a)\left[\frac{1}{y}-1\right]$$

Y = 0.8

7. **(B)**



$$a = -\omega x$$
$$= -\omega^2 \frac{A}{\sqrt{2}}$$
$$= -\frac{K}{m}\frac{A}{\sqrt{2}}$$

8. **(ABD)**

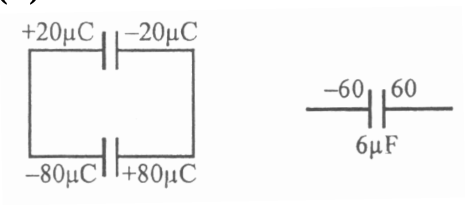
$$r = \frac{n^2 h^2 \in_0}{\pi m e^2} \propto h^2$$
$$v = \frac{e^2}{2n \in_0} \propto \frac{1}{n} \quad \Rightarrow \quad E \propto \frac{1}{r}$$

9. **(AD)**

(i) $u = V_{CM} - r\omega$

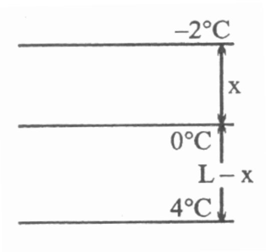
(ii) $u = V_{CM} + r\omega$ and $\omega = \frac{V_{CM}}{R}$ in both cases.

10. **(B)**



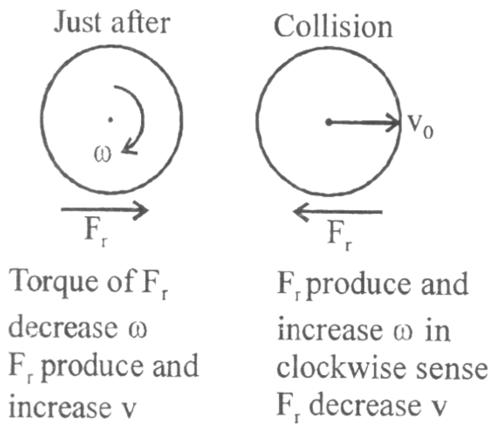
$$\Delta H = U_i - U_f$$
$$= \frac{1}{2}\times 2\times 10^2 + \frac{1}{2}\times 4^2 \times 20^2 - \frac{1}{2}\times\frac{60^2}{6} = 600\,\mu J$$

11. **(D)**



$$\frac{k_w A(4-0)}{L-x} = \frac{k_i A(0-(-2))}{x}$$
$$\frac{4^2}{L-x} = \frac{6^3}{x}$$
$$2x = 3L = -3x$$
$$x = \frac{3L}{5}$$

12. **(ACD)**



13. **(ABC)**

From charge distribution $Q_1 = Q_4$ net electric field between plates is $E \times d$

Potential Difference $= \frac{Q_2 - Q_3}{2A\varepsilon_0} d = \frac{Q_2 - Q_3}{2C}$

14. **(AB)**

(A) $165\left[\frac{330+10}{330} = 170\right]$

(B) $165\left[\frac{330}{330-10}\right] = 170.16 > 170$

(C) $165\left[\frac{330-10}{330}\right] = 160$

(D) $165\left[\frac{330}{330+10}\right] = 160.15$

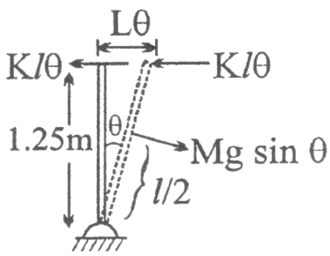
15. **(AB)**

16. **(6)**

$$I = \frac{ml^2}{3}$$

$$\omega = \sqrt{\frac{C}{I}} = 6\text{rad/sec}$$

$$\tau_{res} = (2kl\theta)l - Mg\sin\theta\left(\frac{l}{2}\right) = \left(\frac{3Mgl}{2}\right)\theta \propto \theta$$



17. **(0)**

Charge on capacitor will increases by $160\mu C$

As $C' = 16\mu F \times 2 = 32\mu F$

Heat generate $= \omega_{batt} - \Delta U = \omega_{batt} - (U_f - U_i)$

$\omega_{batt} = 160 \times 10 = 1600\mu J$

$$U_i = \frac{1}{2}(16)(100) = 800\text{mJ}$$
$$U_f = \frac{1}{2}(32)(100) = 1600\text{mJ}$$
$$H = 800\text{mJ}$$

18. **(4)**

$$m_1 g\left(\frac{l}{2}\right) - m_2 g\left(\frac{l}{2}\right) = I\alpha$$
$$\alpha = 12\text{rad}/\sec^2$$
$$(m_1 + m_2)g - N = (m_1 + m_2)a_{T_{CM}}$$

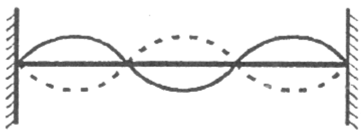
Where $a_{T_{CM}} = \left(\frac{l}{2} - \frac{l}{5}\right) \propto$ Putting values N = 32 N

19. **(5)**

(1) $i_0 = 0$ (2) $i_0 = \frac{V}{R}$ (3) $i_0 = \frac{V}{2R}$

20. **(2)**

$$f_0 = \frac{3}{2L}\sqrt{\frac{T}{\mu}} = \frac{3}{2\times 1}\sqrt{\frac{T}{m/L}} = \frac{3}{2}\sqrt{\frac{YA\alpha\Delta T \times L}{m}}$$
$$= \frac{3}{2}\sqrt{\frac{Y\times 10^{-6}\times 1.21\times 10^{-5}\times 20\times 1}{0.1}}$$



$$f = \frac{c+v}{c} f_0$$
$$40 = \frac{330+70}{330} f_0$$
$$f_0 = 11\text{Hz} = \frac{3}{2}\sqrt{y\times 1.21\times 10^{-9}\times 2}$$
$$4\times 121 = y\times 121\times 10^{-11}\times 2$$
$$Y = 2\times 10^{11}\ \text{N}/\text{m}^2$$

PART (B) : CHEMISTRY

SOLUTIONS

21. **(D)**

22. **(C)**

23. **(A)**
ΔH_1^o and ΔH_2^o are same because of same kind of bond break and form. Now from relation
$\Delta G^0 = \Delta H^0 - T\Delta S^0$
$\Delta S_2^o - \Delta S_1^o = 178.06\ J/k - mole$

24. **(C)**

25. **(B)**

26. **(A)**

27. **(C)**

28. **(ABCD)**
Draw canonical structure of (I) and (II) and compare bond length on the basis of major contributing structure.

29. **(AB)**
Formation of phenoxide from phenol is endothermic $\left(\Delta H = 89 \pm 1\ k\,cal/mol\right)$ and hence phenol is more stable. Phenoxide has better resonance stabilization.

30. **(AB)**
according to C.I.P rules, if the relative priorities of these substituent's need to be established,
R takes priority over (S). It is optically inactive due to plane of symmetry.

31. **(ACD)**
When medium is strongly acidic, 's' atoms will get protonated isoelectric point is $\frac{pKa_1 + pKa_2}{2}$.

32. **(ABCD)**

33. **(ABCD)**

34. **(AD)**
$\Delta H = Cp\Delta T$ and $Cp > Cv$
$\Delta U = Cv\Delta T$

35. **(AB)**
Liquid He is having more than one phase

for a given change $\Delta H_{rev} = \Delta H_{irr}$.
Black phosphorus is most stable but white phosphorus is thermodynamic reference state.

36. **(8)**
Formula will be Cu_4Ag_3Au

37. **(6)**
In metal carbonyl effective atomic number is equal to the atomic number of next inert gas. Thus E.A.N of Cr should be 36.
E.A.N. = 24 – 0 + no. of coordinate bond $\times 2$ = 36
x = 6

38. **(6)**
Solubility $= 2\times10^{-4}$

39. **(9)**
$\Delta Ty = k_f \times i \times 1 = i \times 1.86 \times 1$

$i = 2.5$

$H_2SO_4 \rightarrow H^+ + HSO_4^-$

$1 \quad\quad 1 \quad\quad (1-\propto)$

$HSO_4^- \rightleftharpoons H^+ + SO_4^{-2}$

$1-\propto \quad\quad \propto \quad\quad \propto$

$i = 1 + 1 - \propto + \propto + \propto = 2.5$

$2 + \propto = 2.5$

$\propto = 0.5$

$$ka_2 = \frac{[H^+](SO_4^{-2})}{[HSO_4^-]} = \frac{1.5\times0.5}{(1-0.5)} = 1.5$$

$= 1.5 \times 6 = 9$

40. **(4)**

PART (C) : MATHEMATICS

SOLUTIONS

41. **(D)**

$$w_k = \frac{z_k^2}{r_k^2 z_k} = \frac{1}{z_k}$$

$$\therefore\ w_1 + w_2 + w_3 = \frac{1}{z_1} + \frac{1}{z_2} + \frac{1}{z_3} = \left(\overline{\frac{1}{z_1} + \frac{1}{z_2} + \frac{1}{z_3}}\right) = 0$$

$\therefore$ 0 is centroid

42. **(D)**

$$p\left(x^5\right) = \left(x^{20} - 1\right) + \left(x^{15} - 1\right) + \left(x^{10} - 1\right) + \left(x^5 - 1\right) + 5$$

$$= \left(x^5 - 1\right)\left(x\right) + 5$$

$\therefore R = 5$

43. **(D)**

$$(a - 2b)^2 + (a - c)^2 = 0$$

$\Rightarrow a = 2b \quad a = c$

$$\therefore \cos B = \frac{c^2 + a^2 - b^2}{2ac} = \frac{7}{8}$$

44. **(A)**

$$(a + b) \times (a \times b) = a \times (a \times b) + b \times (a \times b)$$

$$= \left(\vec{a} \cdot \vec{b}\right)\vec{a} - |\vec{a}|^2 \vec{b} + |\vec{b}|^2 \vec{a} - \left(\vec{a} \cdot \vec{b}\right)\vec{b}$$

$$= \vec{a} \cdot \vec{b}\left(\vec{a} - \vec{b}\right) - \left(\vec{a} - \vec{b}\right)$$

$$= \left(\vec{a} - \vec{b}\right)\left(\vec{a} \cdot \vec{b} - 1\right)$$

45. **(B)**

Total ways $= 7^4$

No. of ways to select two teachers $= {}^7C_2$

The ways in which they can check 4 papers $= 2^4$

$\therefore$ The ways in which 4 papers can be checked by exactly 2 teachers $= 2^4 - 2 = 14$

$$\therefore\ p = \left({}^7C_2 \times 14\right)/7^4 = 6/49$$

46. **(B)**

$$I = \int \frac{x^8\left(x^2 + 20\right)}{\left(x^5 \sin x + 5x^4 \cos x\right)^2} dx = \int \frac{\left(x^5 + 20x^3\right)\cos x}{\left(x^5 \sin x + 5x^4 \cos x\right)^2} \cdot \left(\frac{x^5}{\cos x}\right) dx$$

$$\frac{d}{dx}\left(x^5 \sin x + 5x^4 \cos x\right) = \left(x^5 + 20x^3\right)\cos x$$

$\therefore$ Integrating by parts

$$I=\frac{-x}{\cos x\left(x\sin x+5\cos x\right)}+\tan x+c$$

47. **(C)**

$$2x^{100}+3x^{99}+2x^{99}+3x^{98}+2x^{98}+....3x+2x+3=0$$

$$\Rightarrow(2x+3)\left(x^{99}+x^{98}+....1\right)=0$$

$$\Rightarrow x=-\frac{3}{2}$$

48. **(AB)**

49. **(ABCD)**

P (successful) = P[(A $P\left[(A\cap B\cap C')\cup(A\cap B'\cap C)\cup(A\cup B\cap C)\right]$

$$=pq\left(1-\frac{1}{2}\right)+P(1-q)\left(\frac{1}{2}\right)+pq\cdot\frac{1}{2}$$

$$=\frac{1}{2}p(1+q)$$

$$\therefore\ \frac{1}{2}=\frac{1}{2}P(1+q)\Rightarrow p(1+q)=1$$

50. **(ABD)**

$$I_{n+2}=\frac{1}{n+3}\left[x^{n+3}\tan^{-1}x\right]_0^1-\frac{1}{n+3}\int_0^1\frac{x^{n+3}}{1+x^2}dx$$

$$\Rightarrow(n+3)I_{n+2}=\frac{\pi}{4}-\int_0^1\frac{x^{n+3}}{1+x^2}dx$$

$$\therefore(n+1)I_n=\frac{\pi}{4}-\int_0^1\frac{x^{n+1}}{1+x^2}dx$$

$$\therefore(n+3)I_{n+2}+(n+1)I_n=\frac{\pi}{2}-\int_0^1x^{n+2}dx=\frac{\pi}{2}-\frac{1}{n+2}$$

51. **(ABD)**

$$|z|^2=\frac{\alpha^2+\beta^2}{(1-\gamma)^2}=\frac{\gamma(1-\gamma)}{(1-\gamma)^2}=\frac{\gamma}{1-\gamma}$$

$$1+|z|^2=\frac{1}{1-\gamma}\Rightarrow\gamma=\frac{|z|^2}{1+|z|^2}$$

$$\frac{2\alpha}{1-\gamma}=z+\bar{z}\Rightarrow\alpha=\frac{z+\bar{z}}{2\left(1+|z|^2\right)}$$

$$\beta=\frac{(\bar{z}-z)i}{2\left(1+|z|^2\right)}$$

52. **(ACD)**

$$\left(x\frac{dy}{dx}+3y\right)\left(x\frac{dy}{dx}-2y\right)=0$$

$$x\frac{dy}{dx}+3y=0\Rightarrow yx^3=C$$

$$x\frac{dy}{dx}-2y=0\Rightarrow y=Cx^2\Rightarrow\frac{1}{2}\log y=C+\log x$$

53. **(ABC)**

Taking a,b,c common from $R_1, R_2 \& R_3$ respectively and then multiplying a, b, c in C_1, C_2, C_3 respectively, then $C_1 \to C_1+C_2+C_3$

$$\begin{vmatrix} 1 & b^2(1-\cos\phi) & c^2(1-\cos\phi) \\ 1 & b^2+(c^2+a^2)\cos\phi & c^2(1-\cos\phi) \\ 1 & b^2(1-\cos\phi) & c^2+(a^2+b^2)\cos\phi \end{vmatrix}$$

$R_2 \to R_2 - R_1$ & $R_3 \to R_3 - R_2$, then $=\cos^2\phi$

54. **(AB)**

$$f(x)=\begin{cases} 2\tan^{-1}g(x) & , \ |g(x)|\le 1 \\ 2\cot^{-1}g(x) & , \ |g(x)|>1 \end{cases}$$

$$f'(x)=\begin{cases} \dfrac{2}{1+(g(x))^2}g^1(x) & , \ |g(x)|\le 1 \\ \dfrac{-2}{1+(g(x))^2}g^1(x) & , \ |g(x)|>1 \end{cases}$$

55. **(ABC)**

$$x=r\sin\theta\cos\phi,\ y=r\sin\theta\sin\phi,\ \ z=r\cos\theta,\ r=\sqrt{14}$$

$$\therefore\ \sin\theta\cos\phi=\frac{1}{\sqrt{14}},\ \ \sin\theta\sin\phi=\frac{2}{\sqrt{14}},\ \cos\theta=\frac{3}{\sqrt{14}}$$

$$\tan\theta=\frac{\sqrt{5}}{3},\ \ \tan\phi=2$$

56. **(7)**

$$A^2=B^{-1}AB$$

$$A^4=\left(B^{-1}\right)^2AB^2\ \Rightarrow\ A^8=\left(B^{-1}\right)^3AB^3$$

$$\therefore\ A^7=I$$

57. **(0)**

$$p(x)-1=(x-3)p_1(x)$$

Put $x = 1$

$$\Rightarrow\ 1=-2\ P_1(x)$$

Which is not possible

58. **(2)**

$$\lim_{x\to 0}\left(\frac{\sin x}{x}\right)^{\frac{\sin x}{x-\sin x}} = \lim_{x\to 0}\left[\left(1+\left(\frac{\sin x}{x}-1\right)\right)^{\frac{1}{x/\sin x^{-1}}}\right]^{\frac{\sin x}{x}}$$

$$e^{-1}$$

$$\& \ \lim_{x\to 1}\frac{1}{x^{1-x}} = e^{-1}$$

59. **(5)**

60. **(3)**

$$I_n = (-1)^{n+1} - n\,I_{n-1}$$

$$\therefore\ I_1 = 1 - I_0 = 1-(e-1) = 2-e$$

$$I_2 = z_e - 5 \quad \& \quad I_3 = 16-6e$$

PART (A) : PHYSICS

SOLUTIONS

1. Conceptual

2. Conceptual

3. Conceptual

4. Conceptual

5. In s_1 the man stretch the spring with an average force $\frac{kl}{2}$ and the distance $L - l$.

$\therefore w = \frac{1}{2} kl(L - l)$

6. Conceptual

7. Consider an elemental area ΔS. Let the tangential component of induced electric field be E.
Total torque experienced by the ring is

$$\tau = \frac{Q}{2\pi} \oint E_t \Delta S$$

$$\oint E \Delta S = \frac{-\Delta\phi}{\Delta t} = -\pi r^2 \frac{\Delta B}{\Delta t}$$

$$\Delta\omega = \alpha \Delta t = \frac{\tau}{I} \Delta t = \frac{-Q}{2m} \Delta B$$

Since magnetic field strength increases from zero to B, the final angular velocity of the ring will be

$$\omega = \frac{-QB}{2m}$$

8. From I^{st} law of thermodynamics Q=dU+w
Q=w for isothermal process. $\therefore$ slope=1
For isobaric process

$$Q = {}^nC_p dt \ = \frac{nR\gamma}{\gamma - 1}.$$

$$w = nRdt \Rightarrow Q = \frac{w\gamma}{\gamma - 1}$$

$$\Rightarrow w = Q \frac{\gamma - 1}{\gamma}$$

$$= Q\left(1 - \frac{1}{\gamma}\right)$$

for He $\quad w = \frac{2}{5} Q$

for CO_2 $\ w = \frac{2}{7} Q$

Slope for He is more than CO_2

9. Conceptual

10. conceptual

11. Applying law of conservation of momentum
(38+19)25=19x27+38 V
V=24 m/s.
Velocity with respect to center of mass
=25-24=1 m/s.

12. The equivalent resistance of original grid bout any two neighboring points will be $\frac{R}{2}$. Let the resistance of the present grid is R_1 . we can consider the original grid as a circuit of two resistors R_1 and R connected in parallel, the latter being the removed resistor $\therefore \frac{R}{2} = \frac{RR_1}{R+R_1} \Rightarrow R_1 = R$.

13. $F_C = \frac{\Delta l}{l} A_C Y_C$ and $F_I = \frac{\Delta l}{l} A_I Y_I$
it follows that $\frac{F_C}{F_I} = 2$

14. At the maximum level H of water rate of filling of vessel =rate of outflow.
$\therefore \frac{dV}{dt} = \alpha = a\left(\sqrt{2gh}\right)$

15.

$\alpha -$ emission	β-emission	Deuteron
$\lambda_1 = 4\lambda_3$	$\lambda_2 = 2\lambda_3$	λ_3

Activity $= \frac{dN}{dt} = -\lambda N$
These activities are same at t=0
$\Rightarrow N_1 : N_2 : N_3 = 1:2:4$ where N_1, N_2, N_3 are the number of initial nuclei of α, β and deuteron respectively.

$$P_{\alpha 1600} = \frac{\frac{\lambda_1 N_1}{16}}{\frac{\lambda_1 N_1}{16} + \frac{\lambda_2 N_2}{4} + \frac{\lambda_3 N_3}{2}} = \frac{1}{1+4+8} = \frac{1}{13}$$

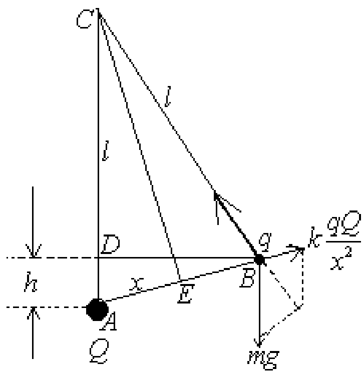
16.

For equilibrium $\frac{mg}{F} = \frac{l}{x}$

$F = \frac{kqQ}{x^2}$; ABD and CAE are similar.

$\frac{x}{2} : l = h : x$ $\quad \therefore$ final electrostatic potential energy is $\frac{kqQ}{x}$
=2mgh

Work done is sum of increase in electrostatic and gravitational potential energy is 3mgh



17. Conceptual

18. $\frac{1.5}{v_1} \frac{1}{-6} = \frac{1.5-1}{R}$

$\frac{1.5}{v_1} = \frac{1}{2R} - \frac{1}{6} = \frac{3-R}{6R}$

$v_1 = \frac{9R}{3-R}$

$u_2 \frac{9R}{3-R} - R = \frac{6R+R^2}{3-R}$

$\frac{R^2}{1.5} = \frac{32}{3}$

$R^2 + 22R - 48 = 0$

$R = -\frac{-22 \pm \sqrt{484+192}}{2} = 2cm$

19. Current through capacitor is zero at steady state

20. Standard expression

PART (C) : CHEMISTRY

SOLUTIONS

21. **(C)**

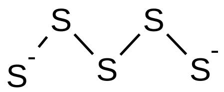
$$E_{cell} = E^o_{Zn^{2+}/Zn} + E^o_{Cl^-/Cl_2(g)}$$
$$= 0.76 + 1.36 = 2.12\ V$$

22. **(B)**

Due to +R effect of N attached to ring

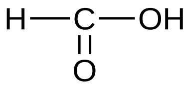
23. **(B)**

$$Na_2S_2O_3 \cdot 5H_2O \xrightarrow{\Delta} Na_2S_2O_3 + 5H_2O$$
$$4Na_2S_2O_3 \xrightarrow{\Delta} 3Na_2SO_4 + Na_2S_5$$

S_5^{2-} has the structure 

24. **(ABC)**

25. **(AC)**



Given, Fehling and Schiff's reagent.

26. **(ABCD)**

(A) Because of solution formation number of sulphur particle will be less in water

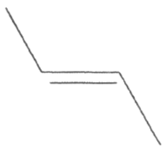
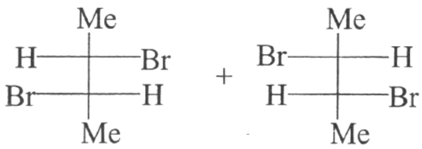
(B) CMC α 1/ size of hydrophobic portion.

(C) It is applicable for limited range of pressure.

(D) CO is polar so will be adsorbed more and replace O_2

27. **(ABD)**

Major product of the reactions will be

(A)  (B)  (D) 

28. **(ABCD)**

Carbonates of $Mg^{2+}, Hg^{2+}, Pb^{2+}$ and Ag^+ are water insoluble whereas their bicarbonates are water soluble.

29. **(AC)**

30. **(AB)**

(A) Zeolites are shapes selective catalyst. Exp ZSm-5

(B) Micelle formation occur with decrease in entropy

31. **(5)**

32. **(8)**

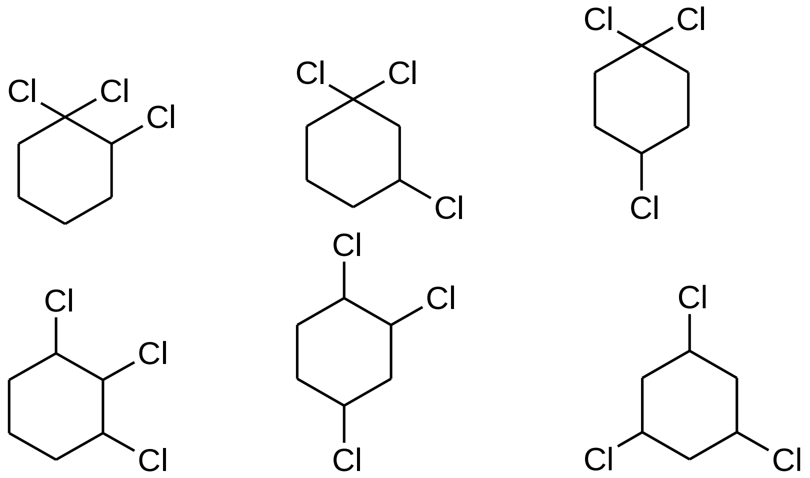
All cations will precipitate whose hydroxides are insoluble. All hydroxides are insoluble except IA, $NH_4^+, Ca^{+2}, Ba^{+2}, Sr^{+2}$.

So $Zn^{+2}, Al^{+3}, Fe^{+3}, Mn^{+2}, Mg^{+2}, Pb^{+2}, Bi^{+3}, Ni^{+2}$ will precipitate.

33. **(5)**

$NH_4NO_3, NH_4NO_2, NH_4IO_3, CH_3COONH_4, NH_4ClO_4$

34. **(6)**



35. **(5)**

36. **(3)**

$$a = 3.375 \simeq 3$$

37. **(5)**

$$\psi(r,\theta,\phi) = \sqrt{\frac{3}{4\pi}} \times \frac{1}{4\sqrt{6}}\left(\frac{1}{a_0}\right)^{3/2} e^{-\sigma/2} \cdot \left(\sigma^2 - 5\sigma + 6\right)\sin\theta\cos\phi$$

For Radial nodes

$$\psi^2(r) = 0$$

$$\left(\sigma^2 - 5\sigma + 6\right) = 0 \qquad \Rightarrow \sigma_1 = 3,\ \sigma_2 = 2$$

$$\frac{r_1}{2a_0} = 3; \qquad \frac{r_2}{2a_0} = 2$$

$$r_1 = 6a_0 \qquad r_2 = 4a_0$$

$$r_1 = 3\overset{o}{A} \qquad r_2 = 2\overset{o}{A}$$

$$r_1 + r_2 = 5\overset{o}{A}$$

38. **(0.04)**

39. **(4)**

$T_{Boyle's} = \frac{a}{bR} = \frac{10^4}{41.05}$

$Z = 1 + \frac{1}{V_m}\left(b - \frac{a}{RT}\right) + \frac{b^2}{V_m^2} + ...$ ...(i)

Virial Equation :

$Z = 1 + \frac{B}{V_m} + \frac{C}{V_m^2} + ...$ ...(ii)

On comparing coefficients

$b^2 - 4\times10^{-2} \Rightarrow b = 0.2\,\text{Lmole}^{-1}$

$\frac{a}{0.0821\times0.2} = \frac{10^4}{41.05}$

$a = 4$

40. **(6)**

$A(g) \longrightarrow B(g) + 2C(g)$

P_0	–	–
$P_0 - x$	x	2x
–	P_0	$2P_0$

If N is pressure of inert gas

$P_0 + N = 200$

$3p_0 + N = 500$

$\Rightarrow$ $P_0 = 150; N = 50$

At $t = 70\,\text{sec}$.

$Pt = 350 = P_0 + 2x + N$

$\Rightarrow$ $x = 75$

$\Rightarrow$ $t = t_{1/2} = 70\,\text{sec}$.

$k(hr_{-1}) = \frac{0.693}{t_{1/2}\,\text{sec}} \times 3600$

PART (C) : MATHEMATICS

SOLUTIONS

41. Equation of the circle will be of the form $\left(x-\frac{1}{2}\right)^2+(y-1)^2+\lambda(2x-2y+1)=0$

Where $2x-2y+1=0$ is tangent to parabola at $\left(\frac{1}{2},1\right)$

42. Interpret geometrically

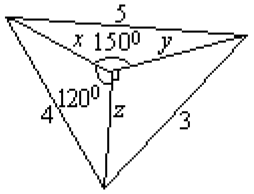
43. $f(x)=\frac{1}{2}(2-\sin^2 2x-\sin 2x)=\frac{9}{8}-\frac{1}{2}\left(\sin 2x+\frac{1}{2}\right)^2$

44. $\frac{3}{4}\le P(A\cup B)\le 1$

$\Rightarrow \frac{3}{4}\le P(A)+P(B)-P(A\cap B)\le 1$

$\therefore P(A)+P(B)\ge \frac{3}{4}+P(A\cap B)\ge \frac{3}{4}+\frac{1}{8}$ and $P(A)+P(B)\le 1+P(AB)\le 1+\frac{3}{8}$

45. Given $x^2+y^2-2xy\cos\frac{5\pi}{6}=25; x^2+z^2-2xz\cos\frac{2\pi}{3}=16$ and $y^2+z^2=9$.



46. Integrand = $\frac{1}{2}(\sin x+\cos x-1)$

47. given equation is $(x^2+x+1)(x^2-\lambda x+1)=0$

48. $|Z|^2\le 4\Rightarrow 2a(\cos\theta-2\sin\theta)\le 3-5a^2$

$|\cos\theta-2\sin\theta|\le\sqrt{5}\ \forall\theta\in R$

49. given $\frac{1}{2}\left(aP_1+bP_2+cP_3\right)=K\Rightarrow y=\frac{a}{P_1}+\frac{b}{P_2}+\frac{c}{P_3}$ is minimum.

when $y=\frac{1}{2K}\left(aP_1+bP_2+cP_3\right)\left(\frac{a}{P_1}+\frac{b}{P_2}+\frac{c}{P_3}\right)$ is minimum.

but , $y=\frac{1}{2K}\left(a^2+b^2+c^2+ab\left(\frac{P_1}{P_2}+\frac{P_2}{P_1}\right)+bc\left(\frac{P_2}{P_3}+\frac{P_3}{P_2}\right)+ab\left(\frac{P_1}{P_3}+\frac{P_3}{P_1}\right)\right)$

$$\geq \frac{1}{2K}\left(a^2+b^2+c^2+2ab+2bc+2ac\right)$$

$$\Rightarrow y \geq \frac{(a+b+c)^2}{2K} \quad when \quad \frac{P_1}{P_2}=\frac{P_2}{P_1}=\frac{P_2}{P_3}=\frac{P_3}{P_2}=\frac{P_1}{P_3}=\frac{P_3}{P_1}=\mathbf{1}$$

$\Rightarrow i.e, when\, P_1 = P_2 = P_3$

$\therefore P$ *is incentre of* ΔABC

50. If the sides are a, a, b then the triangle forms only when 2a > b .So for any $a \in N$, b can change from 1 to 2a –1 ,where $a \leq 1004 \Rightarrow$ no. Of triangles =1+2+3+---+(2(1004)-1)= $(1004)^2$

And if $1005 \leq a \leq 2008$, b can take any value from 1 to 2008

But a has 1004 possibilities hence

No . Of triangles = 1004×2008

= $2(1004)^2$

$\therefore$ Total no. Of isosceles triangles $=3(1004)^2$

51. $$f(x)=\begin{cases}1+x \text{ if } -1\leq x\leq 0\\ 1-x \text{ if } 0\leq x\leq 1\\ 0 \text{ if } x>1 \text{ or } x<-1\end{cases}$$

$$g(x)=f(x-1)+f(x+1)=\begin{cases}0 & \text{if } x<-2\\ 1-|x+1| & \text{if } -2\leq x\leq 0\\ 1-|x-1| & \text{if } 0\leq x\leq 2\\ 0 & \text{if } x>2\end{cases}$$

Points where f is not differentiable are – 2, -1 , 0, 1 and 2.

52. $f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x+0)}{h} \Rightarrow \lim_{h\to 0}\frac{f(x)+f(h)+h\sqrt{f(x)}-f(x)-f(0)-0\sqrt{f(x)}}{h}$

$$\Rightarrow \lim_{h\to 0}\left(\frac{f(h)-f(0)}{h-0}\right)+\sqrt{f(x)}$$

$f'(x) = \sqrt{f(x)}$

$$\int \frac{f'(x)}{\sqrt{f(x)}}dx=\int dx$$

$2\sqrt{f(x)} = x + c$

$f(x) = \dfrac{x^2}{4}$

when $\alpha = 0$ area is minimum

required minimum area = $2\int_0^9 2\sqrt{y}\,dy$

$$\Rightarrow 4\left(\frac{y^{3/2}}{3/2}\right)_0^9 = 72 \text{ sq. unit.}$$

53. Let Y_i be the subset of X such that $y_i = 7m + i$, $m \in I$
$Y_0 = \{7, 14, ... 98\}$, $n(Y_0) = 14$
$Y_1 = \{1, 8, 15 ...99\}$, $n(Y_1) = 15$
$Y_2 = \{2, 9, 16 ... , 100\}$, $n(Y_2) = 15$
$Y_3 = \{3, 10, 16 .... 94\}$, $nY_3) = 14$
$Y_4 = \{4, 11, 18 ... 95\}$, $n(Y_4) = 14$
$Y_5 = \{5, 12, ... 96\}$, $nY_5) = 14$
$Y_6 = \{6, 13, ... 97\}$, $n(Y_6) = 14$
The largest Y will consist of (!) an element of Y_0 (ii) Y_1 (iii) Y_2 (iv) Y_3 or Y_4
$\Rightarrow$ The maximum possible number of elements in Y = 1 + 15 + 15 + 14 = 45.

54. $a_{ij} = 0\ \forall\ i \neq j$ and $a_{ij} = (n-1)^2 + i\ \forall\ i = j$
Sum of all the element of $A_n = \sum_{i=1}^{2n-1}[(n-1)^2 + i]$
$= (2n-1)(n-1)^2 + (2n-1)n = 2n^3 - 3n^2 + 3n - 1 = n^3 + (n-1)^3$
So, $T_n = (-1)^n [n^3 + (n-1)^3] = (-1)^n n^3 - (-1)^{n-1}(n-1)^3 = V_n - V_{n-1}$
$\Rightarrow \sum_{n=1}^{102} T_n = \sum_{n=1}^{102} (V_n - V_{n-1}) = V_{102} - V_0 = (102)^3$
$\left[\frac{\sum_{n=1}^{102} T_n}{520200}\right] = 2$.

55. $II_1 = AI_1 - AI$
$= (r_1 - r)\cos cA/2$
$= a\tan A/2 \cos cA/2$
$= \frac{a}{\cos A/2} = 8$

56. $\sin(2\sin x) = \sin\left(\frac{\pi}{2} - 2\cos x\right)$
$\sin x + \cos x = \frac{\pi}{4}$
s.o.b.s
$1 + \sin 2x = \frac{\pi^2}{16}$
$\sin 2x = \frac{\pi^2 - 16}{16}$
$\therefore \tan x + \cot x = \frac{2}{\sin 2x} = \frac{2 \times 16}{\pi^2 - 16} = \frac{32}{\pi^2 - 16}$
$\therefore a = 32, b = 16, c = 2$
$\frac{a+b+c}{25} = 2$

57. $[x]=\dfrac{5\pm\sqrt{25+4\sin x-24}}{2.1}$

$=\dfrac{5\pm\sqrt{1+4\sin x}}{2}$

$-1\le \sin x\le 1$

$-4\le 4\sin x\le 4$

$-3\le 1+4\sin\le 5$

$0\le 1+4\sin x\le 5$

$\Rightarrow [x]$ is an integer $\Leftrightarrow \sin x=0$

$\Rightarrow [x]=3$

$\Rightarrow x=\pi$

58. $f(x)+f(1-x)=1$

59. Let x be the $(2009)^{th}$ root of unity $\neq 1$, then

$x^{2009}-1=(x-1)(x-w).....(x-w^{2008})$

Taking log on both sides, we get

$\ln(x^{2009}-1)=\ln(x-1)+\ln(x-w)+\ln(x-w^2).....+\ln(x-w^{2008})$

$\therefore$ On differentiate both the side w.r.t. x, we get

$\dfrac{(2009)x^{2008}}{x^{2009}-1}=\dfrac{1}{x-1}+\sum_{r=1}^{2008}\dfrac{1}{x-w^r}$.......(2)

Putting x = 2 in equation (2) , we get

$\Rightarrow 1+\sum_{r=1}^{2008}\dfrac{1}{2-w^r}=\dfrac{2009(2^{2008})}{2^{2009}-1}$

Multiplying both sides of above equation by $(2^{2009}-1)$, we get

$\therefore (2^{2009}-1)\sum_{r=1}^{2008}\dfrac{1}{2-w^r}=2009.2^{2008}-2^{2009}+1$

$=2^{2008}(2009-2)+1=2^{2008}.2007+1=\left[(a)(2^b)+c\right]$

$\therefore a=2007, b=2008, c=1$

Hence a + b + c = 4016

60. $|8z_2z_3+27z_3z_1+64z_1z_2|=|z_1||z_2||z_3|\left|\dfrac{8}{z_1}+\dfrac{27}{z_2}+\dfrac{64}{z_3}\right|=(2)(3)(4)\left|\dfrac{8\bar{z}_1}{|z_1|^2}+\dfrac{27\bar{z}_2}{|z_2|^2}+\dfrac{64\bar{z}_3}{|z_3|^2}\right|$

$=24|2\bar{z}_1+3\bar{z}_2+4\bar{z}_3|=24|\overline{2z_1+3z_2+4z_3}|=24|2z_1+3z_2+4z_3|$

= (24) (4) = 96