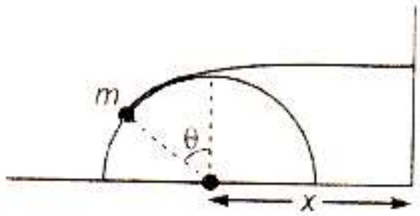
PART (A) : PHYSICS

1. (D)



From constraint equation

$x + R\theta = \text{constant}$

$\frac{dx}{dt} + R\frac{d\theta}{dt} = 0$

$\Rightarrow \frac{d^2x}{dt^2} + R\frac{d^2\theta}{dt^2} = 0$

Where $\frac{dx}{dt} = v$, velocity of hemi-sphere and $\frac{d^2x}{dt^2} = a$, acceleration of hemi-sphere

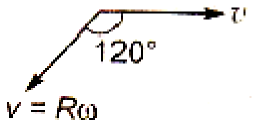
$\Rightarrow R\omega = -v \Rightarrow \omega = \frac{v}{R}$

[Taking direction into consideration]

Similarly $\alpha = \frac{a}{R}$ [ω and α are angular velocity and accelerations of particle with respect to hemi-sphere is, $a_r = R\alpha$ and velocity is, $v_1 = R\omega$. Velocity of particle with respect to ground

$v_{PG} = v_{PH} + v_{HG}$

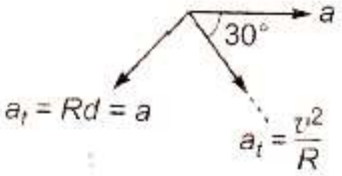
$\Rightarrow v_{PG} = \sqrt{v^2 + v^2 + 2v^2\cos 120} = v$



Acceleration of particle with respect to ground

$a_{PG} = a_{PH} + a_{HG}$

$\Rightarrow a = \sqrt{\left(\frac{v^2}{R} + \frac{\sqrt{3}a}{2}\right)^2 + \left(\frac{a}{2}\right)^2}$



2. (A)

$A \propto x$ (given). Velocity $v \propto \frac{1}{x}$

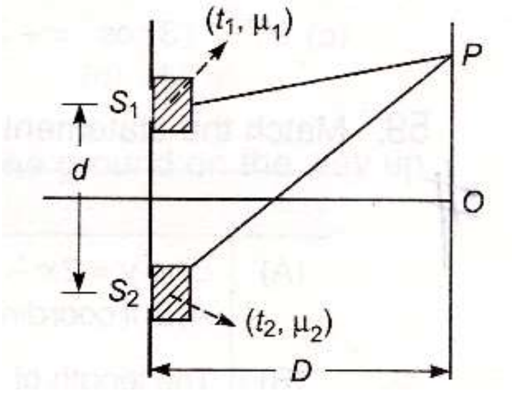
$p+\frac{1}{2}pv^2=\text{constant}$

3. (B)
Apply conservation of energy.
Kinetic energy of the rod is converted into heat dissipated in resistance R.

4. (A)
$$\frac{\mu_1}{F}=\frac{2(\mu_f-\mu_1)}{+R_1}+\frac{2(\mu_3-\mu_2)}{+R_2}-\frac{2\mu_3}{-R_3}$$

5. (D)
The focal length of the lens remains unchanged by changing its aperture.
The intensity of the image is proportional to the uncovered area.
$$\frac{I_f}{I_i}=\frac{A_f}{A_i}=\frac{\frac{\pi}{4}\left[d^2-\frac{d^2}{4}\right]}{\frac{\pi}{4}d^2}=\frac{3}{4}$$
$$I_f=\frac{3}{4}I_i=\frac{3}{4}I$$

6. (A)
Path difference,
$$\Delta x=\{(S_2P-t_2)+\mu_2t_2\}-\{(S_1P-t_1)+\mu_1t_1\}$$
$$=S_2P-S_1P+(\mu_2-1)t_2-(\mu_1-1)t_1$$
For nth order maxima



$$n\lambda=\frac{D}{d}\{(\mu_2-1)t_2-(\mu_1-1)t_1\}+\frac{dy}{D}$$
$\Rightarrow$ For zero order maxima
$$y_0=\frac{D}{d}\{(u_2-1)t_2-(\mu_1-1)t_1\}$$
When both sheets have same average thickness $\frac{t_1+t_2}{2}$ and refractive index μ_1 and μ_2
$$y_1=\frac{D}{d}\left\{(\mu_2-\mu_1)\frac{t_1+t_2}{2}\right\}$$

$\Rightarrow \dfrac{5\times10^{-3}\times1\times10^{-3}}{1}=(1.6-1.4)\dfrac{t_1+t_2}{2}$

$\therefore t_1+t_2=5\times10^{-5}$ …..(i)

7. (B,C)

ϕ crossing through Gaussian surface does not depend on location of charge while E depends. If q crosses the boundary, then $q_{enclosed}$ charges and hence flux and E.

8. (B,C,D)

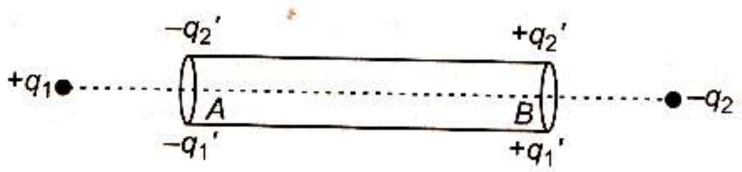
$\tan\phi=\dfrac{X_C-X_L}{R}=0$

$\Rightarrow \phi=0$ as $X_C=X_L$

$\Rightarrow Z=R$

$\Rightarrow V_C=IX_C$ and $I=\varepsilon_0/Z=\varepsilon_0/R$ When $R(\downarrow), I(\uparrow)$ and hence $V_C(\uparrow)$

9. (A,B,C,D)



Due to induction effect the situation is shown clearly in figure. Due to $+q_1$, let induced charges is $-q_1$ at end A and $+q_1$ at end B while due to $-q_2$ induced charges are $-q_2$ and q_2 at ends A and B respectively. Thus the end A acquires negatively charged and B acquires positive charge. Electric force experienced by q_1 or $-q_2$ has to be computed by using principle of superposition.

For $+q_1 \rightarrow$ Due to $-q_2$ towards right

Due to rod towards right

Hence, total force experienced by $+q_1$ in present situation is greater than as compared to the case without rod.

Same is the situation for $-q_2$

10. (B,D)

Current in wire $=\dfrac{3}{30}=0.1A$

Maximum voltage can read $=2V$

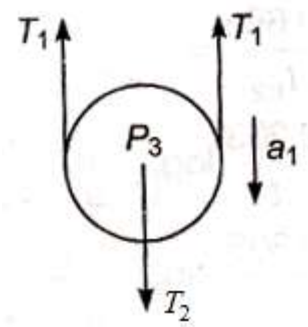
$\lambda=\dfrac{V}{I=0.2\,mV/mm}=\dfrac{2}{10\times10^3}=0.2mV/mm$

Least account of scale =1mm

Accuracy =0.2mV

11. (A,C)

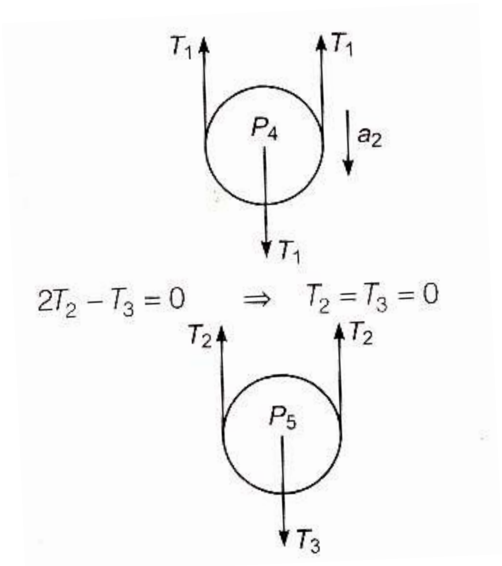
First of all draw FBD of P_3 let tension in 3 strings are T_1, T_2 and T_3 respectively.



$2T_1 - T_1 = 0 \times a \Rightarrow T_1 = 0$

Now draw FBD of P_4 and P_5

$2T_1 - T_2 = 0 \Rightarrow T_2 = 0$



Similarly, for acceleration draw the FBD of P_6 and P_7 and get the value of acceleration.

12. (A,D)

Magnitude of induced electric field due to change in magnetic flux is given by

$$\oint E.dl = \frac{d\phi}{dt} = S.\frac{dB}{dt}$$

Or $E.l = \pi R^2 (2B_0 t)$ $\left(\text{Since } \frac{dB}{dt} = 2B_0 t\right)$

Here, E= induced electric field due to change in magnetic flux.

Or $E(2\pi R) = 2\pi R^2 B_0 t$

Or $E = B_0 R t$

Hence, $F = QE = B_0 Q R t$

This force is tangential to ring. Ring starts rotting when torque of this force is greater than the

$\tau_{F\max} = (\mu mg)$ or when

Torque due to maximum friction

$\tau_F \geq \tau_{f\max}$

This is the limiting case

$\tau_F = \tau_{i\max}$ or $F.R = (\mu mg)R$

Or $F = \mu mg$ or $B_0QRt = \mu mg$

It is given that ring starts rotating after $2s$. So, putting $t = 2$, we get

$\mu = \dfrac{2B_0RQ}{mg}$

After two seconds

$\tau_F > \tau_{i\,max}$

Therefore, net torque is

$\tau = \tau_F - \tau_{f\,max} = B_0QR^2t - \mu mgR$

Substituting $\mu = \dfrac{2B_0RQ}{mg}$, we get

Or $\tau = B_0QR^2(t-2)$

Or $I\left(\dfrac{d\omega}{dt}\right) = B_0QR^2(t-2)$

Or $mR^2\left(\dfrac{d\omega}{dt}\right) = B_0QR^2(t-2)$

Or $\int_0^{\omega} d\omega = \dfrac{B_0Q}{m}\int_2^4 (t-2)dt$

Or $\omega = \dfrac{2B_0Q}{m}$

Now, magnetic field is switched off i.e. only retarding torque is present due to friction. So, angular retardation will we

$\alpha = \dfrac{\tau_{f\,max}}{I} = \dfrac{\mu mgR}{mR^2} = \dfrac{\mu g}{R}$

Therefore, applying $\omega^2 = \omega_0^2 - 2\alpha\theta$

Or $0 = \left(\dfrac{2B_0Q}{m}\right)^2 - 2\left(\dfrac{\mu g}{R}\right)\theta$

Or $\theta = \dfrac{2B_0^2Q^2R}{\mu m^2 g}$

Substituting $\mu = \dfrac{2B_0RQ}{mg}$

We get, $\theta = \dfrac{B_0Q}{m}$

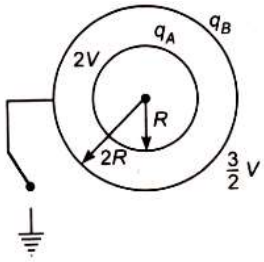
13. (A,D)

$2V = k.\left(\dfrac{q_A}{R} + \dfrac{q_B}{2R}\right)$

$2V = k\left(\dfrac{2q_A + q_B}{2R}\right)$

$\dfrac{3}{2}V = k.\left(\dfrac{q_A}{2R} + \dfrac{q_B}{2R}\right)$

$\dfrac{3}{2}V = k.\left(\dfrac{q_A + q_B}{2R}\right)$ ……(ii)

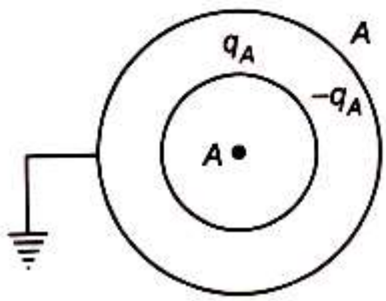


$\Rightarrow \frac{4}{3} = \frac{2q_A + q_B}{q_A + q_B}$

$\Rightarrow q_A = \frac{1}{2} q_B$

i.e., $\frac{q_A}{q_B} = \frac{1}{2}$

$\frac{q_A'}{q_B'} = -1$



Potential of $A = k.\left(\frac{q_A}{R} - \frac{q_A}{2R}\right)$

$= k\frac{q_A}{2R} = \frac{1}{2}V \qquad \left(V = \frac{k}{R}q_A\right)$

$\therefore$ Potential difference between A and B is $\frac{1}{2}V$ (after earthin)

14. (A,B,C,D)

15. (C)

It is given that electric potential energy stored in the capacitor is 7.5×10^{-4} J. In the charged state, since the capacitance is C_f and the battery has remained connected.

Therefore, $\frac{1}{2} C_f V^2 = 7.5 \times 10^{-4}$

$\frac{1}{2} C_f (25)^2 = 7.5 \times 10^{-4}$

$C_f = 2.4 \times 10^{-6}$ F

Therefore, $C_f = 2.4 \mu F$

Or $C_i = \frac{C_f}{3} = 0.8 \mu F$

16. (A)

When the switch S is closed, the capacitor becomes charged and the plates A and B, then carry +Q and $-Q$ charges. Consequently , the plates develop a force of attraction given by

$$F=\frac{Q^2}{2\varepsilon_0 A}=\frac{1}{2}\frac{Q^2}{C_i d}\left(\text{because}\,\varepsilon_0 A=C_i d\right)$$

Where, $C_i=0.8\mu F$ and $d=6mm$

$F=0.375N$

(Each plate attracts the others by this force)

For equilibrium of plate $A:(2k)x_1=0.375$

For equilibrium of plate $B:kx_2=0.375$

Or $x_2=2x_1$ ......(i)

Due to the extension of the springs , the separation between the plates will reduce by

$x_1+x_2=4mm$ ......(ii)

(Because the plates separation reduces 6 mm to 2mm)

On solving these two Eqs. (i) and (ii), we get

$x_2=2.67mm$

17. (6.00)

Here, $n=660\,hz, v=330\,ms^{-1}$

$$\lambda=\frac{v}{f}=\frac{330}{660}=0.5m$$

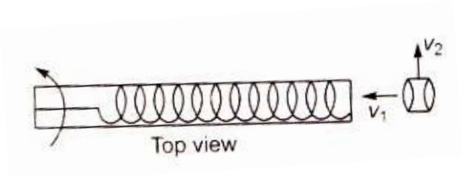
Resonance lengths are $\frac{\lambda}{4},\frac{3\lambda}{4},\frac{5\lambda}{4},\frac{7\lambda}{4},\frac{9\lambda}{4},\frac{11\lambda}{4},\frac{13\lambda}{4}$ and so on.

$$\frac{13\lambda}{4}=\frac{13\times0.5}{4}=1.6m$$

Which is greater than 1.5 m therefore total number of resonances heard is 6.

18. (6.00)

For entering without jerk $v_2=l\omega_0=2rad/s$. Using work energy theorem on sleeve after



Entering in the frame of rod

$W_{spring+W_{centrifugal}}=\Delta K$

$$-\frac{1}{2}kl^2-\frac{1}{2}m\omega^2 l^2=0-\frac{1}{2}mv_1^2$$

$\Rightarrow v_1^2=8$

Now, $K=\frac{1}{2}m\left(v_1^2+v_2^2\right)=6$

19. (5.00)

$\tan 37^\circ = \frac{l}{h}, \frac{3}{4} = \frac{l}{h}, l = \frac{3h}{4}$

$x =$ extension in spring

$= \sqrt{h^2 + i^2} - h$

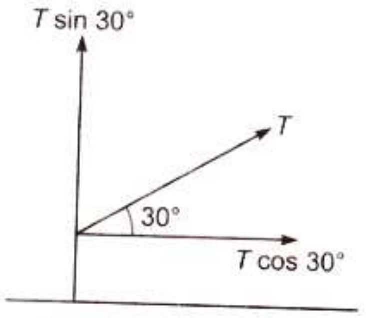
$= \sqrt{\frac{gh^2 + 16h^2}{16}} - h = \frac{h}{4}$

$\frac{1}{2}kx^2 = \frac{1}{2}mv^2$

$\Rightarrow \frac{K}{m}\left(\frac{h}{4}\right)^2 = v^2$

$\Rightarrow v = \frac{h}{4}\sqrt{\frac{k}{m}} = 5m/s$

20. (3.00)



peg will come out if $T\sin 30^\circ = 100N$

$T = 200N(\max)$

For monkey,

$T - mg = ma$

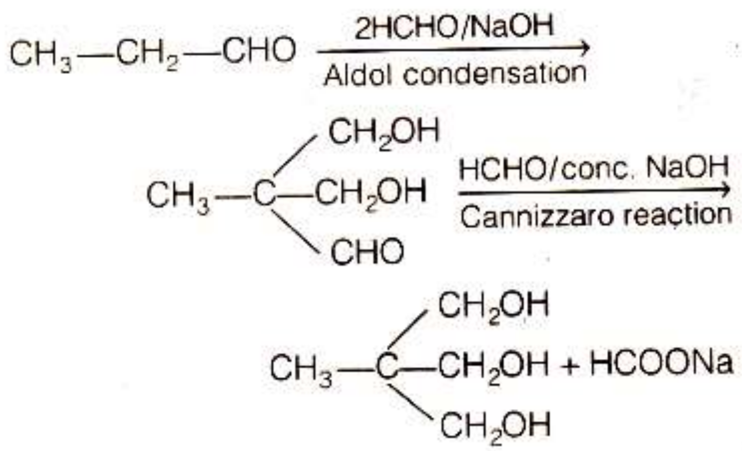
$200 - 15 \times 10 = 15a$

$a = \frac{50}{15} = \frac{10}{3} m/s^2$

PART (B) : CHEMISTRY

21. (B)
In the central atom has lesser tendency or no tendency or capability to donate a pair of electron to $<-C$ the stability of $C-O$ bond is more the pair of electron on the central atom and its donating tendency decreases the $C=O$ bond stability.

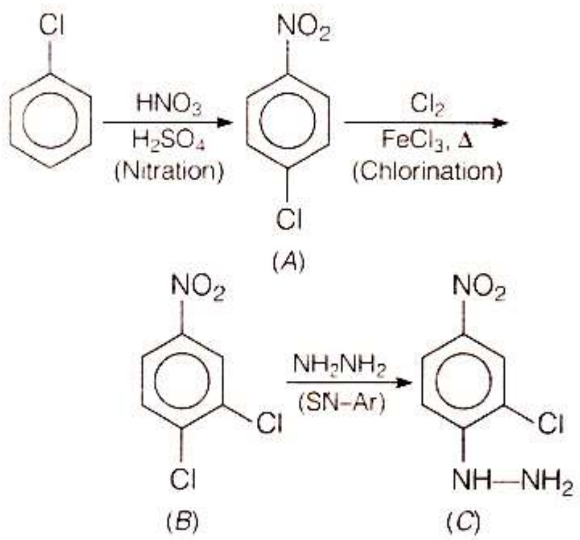
22. (A)



(C) $CH_3-HC(OH)-CH_2-CHO$ $HCOO^-Na^+; CH_3OH$

(D) $CH_3-CH=CH-CHO$ $CH_3COO^-NA^+; HCOO^-Na^+$

23. (A)



24. (C)
Compound given in option A,B and D do not have any hemiacetal group so can't be oxidised by Tollen's reagent and will not given Tollen's test.

25. (B)

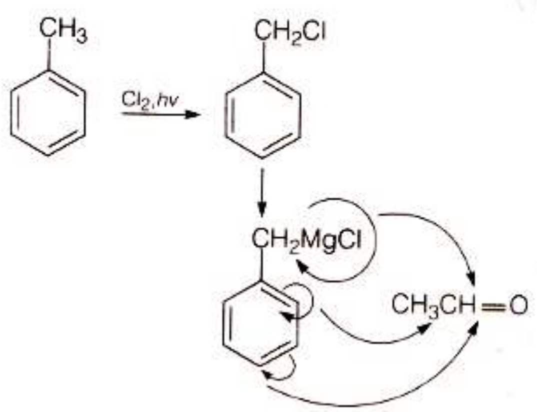
26. (D)
When all particles along body diagonal are removed 2X atoms from corner are removed one Y particle removed and 2Z particle removed.

X particle $= \frac{1}{8} \times 6 + \frac{1}{2} \times 6 = \frac{15}{4}$

(Y particle=3, Z particle =6)

$\therefore X_{15}Y_3Z_6 = X_5Y_5Z_8$

27. (A,B,C)



28. (A,B,C)
N_2H_4 is a basic substance called hydrazine. Rest all are pseudohalogenic acids.

29. (A,C)

$$\log \frac{K_{p_2}}{K_{p_1}} = \frac{\Delta H}{2.303R}\left[\frac{1}{T_1} - \frac{1}{T_2}\right]$$

30. (A,C)
Adipic acid on heating gives cyclopentanone. Malonic acid on heating gives acetic acid.

31. (C)

$$C_2O_4^{2-} \longrightarrow 2CO_2$$

+3 +4

Equivalent weight $= \frac{M}{4}$

$$Ca(HC_2O_4)_2 \rightleftharpoons 2HC_2O_4^-$$

$$HC_2O_4^- \rightleftharpoons H^+ + C_2O_4^{2-}$$

Equivalent weight $=\frac{M}{2}$

$C_2O_4^{2-}$ can be estimated by MnO_4^- / H^+, (C) is also true

32. (B,C)

Correct name for $Na_2\left[Ni\left(EDTA\right)\right]$-Sodium ethylenediaminetetratonickelate (II)

33. (ABD)

34. (A,B)

$NH_4NO_3 + NaOH \longrightarrow NH_3\uparrow + NaNO_3 + H_2O$

$NH_4NO_2 + NaOH \longrightarrow NH_3\uparrow + NaNO_2 + H_2O$

NH_3 is non-inflammable

$NaNO_3 + 4Zn + 7NaOH \longrightarrow 4Na_2ZnO_2 + NH_3 + 2H_2O$

$NaNO_2 + 3Zn + 5NaOH \longrightarrow 3Na_2ZnO_2 + NH_3 + H_2O$

35. (C)

No. of Ca atom $=8\times\frac{1}{8}=1$

No. of O atom $=6\times\frac{1}{2}=3$

No. of Tia atom $=1\times1=1$

Total no. of atoms =5

36. (B)

$CaTiO_3$

$2+x-6=0$

$x=4$

37. (3.00)

$AgCl, Zn\left(OH\right)_2, Cu\left(OH\right)_2$ will dissolve in excess of NH_4OH

38. (3.00)

39. (2.00)

$2Cl^- \longrightarrow Cl_2 + 2e^-$

$Cl_2 + 2NaOH \longrightarrow NaClO + NaCl + H_2O$

$10^6\times1\times\frac{7.45}{100}$

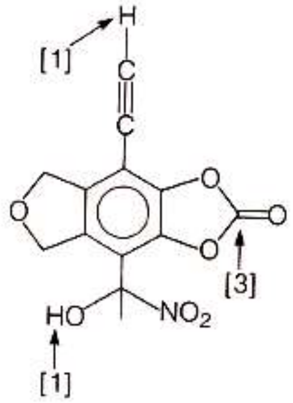
Moles of Cl_2 required $=10^3$

$2\times10^3\times96500=9.65\times t$

Equation of Cl_2 required $=2\times10^3$

$2\times10^7=t$

40. (5.00)



PART (C) : MATHEMATICS

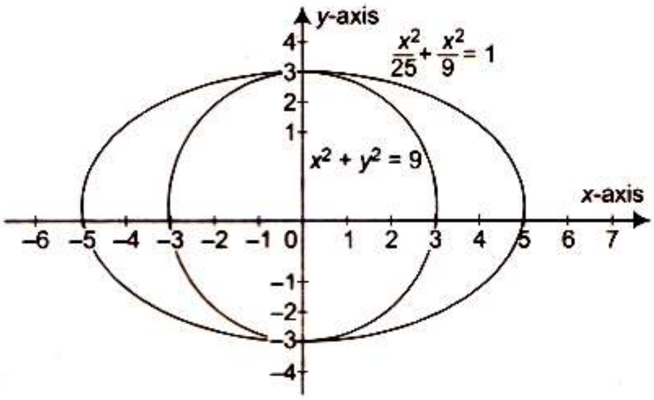
41. (B)

Concept involved

If radius of the circle is greater than difference of length of semi major axis and semi-minor axis of the given ellipse, then we cannot have any normal to the ellipse which can be tangent to the circle.

Let $P(a\cos\theta, b\sin\theta)$ be any point on the ellipse

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$



Equation of normal at $(a\cos\theta, b\sin\theta)$ is

$$\frac{ax}{a\cos\theta}-\frac{by}{\sin\theta}=a^2-b^2 \qquad \ldots\ldots\text{(i)}$$

If this line is also tangent to a circle $x^2+y^2=r^2$,then $r=$ Length of perpendicular from centre $(0,0)$ of the circle to the normal Eq.(i)

$$r=\frac{a^2-b^2}{\sqrt{\frac{a^2}{\cos^2\theta}+\frac{b^2}{\sin^2\theta}}}$$

Now, $\frac{a^2}{\cos^2\theta}+\frac{b^2}{\sin^2\theta}=a^2\left(1+\tan^2\theta\right)+b^2\left(1+\cot^2\theta\right)$

$=a^2+b^2+a^2\tan^2\theta+b^2\cot^2\theta$

On applying AM $\geq$ GM, we get

$a^2+b^2+a^2\tan^2\theta+b^2\cot^2\theta \geq a^2+b^2=2\times a\times b$

$$\Rightarrow \frac{a^2}{\cos^2\theta}+\frac{b^2}{\sin^2\theta}\geq(a+b)^2$$

$$\Rightarrow \frac{1}{\sqrt{\frac{a^2}{\cos^2\theta}+\frac{b^2}{\sin^2\theta}}}\leq\frac{1}{\sqrt{(a+b)^2}}$$

$$\Rightarrow \frac{a^2-b^2}{\sqrt{\frac{a^2}{\cos^2\theta}+\frac{b^2}{\sin^2\theta}}}\leq\frac{a^2-b^2}{|a+b|}$$

$\Rightarrow r\leq a-b$

Obviously, if radius of the circle (r) is greater than (a — b), then we cannot draw any normal to the given ellipse which can also be tangent to the given circle.
Here, radius of the circle $=3>(5-3)$. Hence , we cannot draw any a normal to the ellipse which can also be tangent to the given circle

42. (A)
Given functional equation is

$$2f(x-1)-f\left(\frac{1-x}{x}\right)=x \quad \text{......(i)}$$

Replacing x by $\frac{1}{x}$, we get

$$2f\left(\frac{1}{x}-1\right)-f\left(\frac{1-\frac{1}{x}}{\frac{1}{x}}\right)=\frac{1}{x}$$

$$\Rightarrow 2f\left(\frac{1-x}{x}\right)-f(x-1)=\frac{1}{x} \quad \text{.......(ii)}$$

On multiplying by 2 in Eq. (i) and the adding Eqs. (i) and (ii), we get

$$2\left[2f(x-1)-f\left(\frac{1-x}{x}\right)\right]+\left[2f\left(\frac{1-x}{x}\right)-f(x-1)\right]$$

$$=2x+\frac{1}{x}$$

$$\Rightarrow 3f(x-1)=2x+\frac{1}{x}$$

Now, replacing x by $x+1$, we get [to generate $f(x)$]

$$3f\left[(x+1)-1\right]=2(x+1)+\frac{1}{(x+1)}$$

$$\therefore f(x)=\frac{1}{3}\left(2(x+1)+\frac{1}{(x+1)}\right)$$

$$=\frac{2(1+x)^2+1}{3(1+x)}$$

43. (B)

Term independent of x in $\left[(1+x)\left(\frac{1-x}{x}\right)\right]^{2n}$

=Term independent of x in $(1+x)^{2n}\left(\frac{1-x}{x}\right)^{2n}$

=Term independent of x in $\frac{(1+x)^{2n}(1-x)^{2n}}{x^{2n}}$

=Coefficient of x^0 in $\frac{(1-x^2)^{2n}}{x^{2n}}$

$=$Coefficient of x^{2n} in $\left(1-x^2\right)^{2n}$

$=$Coefficient of x^{2n} in $\sum_{r=0}^{2n} {}^{2n}C_r\left(-x^2\right)^r$

$=(-1)^n\ {}^{2n}C_n$

44. (B)

Let $S=\sum_{k=1}^{n}\frac{6^k}{\left(3^k-2^k\right)\left(3^{k+1}-2^{k+1}\right)}$

Kth term of the above summation is given by

$$T_k=\frac{6^k}{\left(3^k-2^k\right)\left(3^{k+1}-2^{k+1}\right)}$$

$$=\frac{3^k\times2^k}{\left(3^k-2^k\right)\left(3^{k+1}-2^{k+1}\right)}$$

$$=\frac{3^k\times2^k(3-2)}{\left(3^k-2^k\right)\left(3^{k+1}-2^{k-1}\right)}$$

$$=\frac{3^k\times2^k(3-2)}{\left(3^k-2^k\right)\left(3^{k+1}-2^{k+1}\right)}$$

$$=\frac{3^k\left(3^{k+1}-2^{k+1}\right)-3^{k+1}\left(3^k-2^k\right)}{\left(3^k-2^k\right)\left(3^{k+1}-2^{k+1}\right)}$$

$$\left(\because 3^k2^k(3-2)=3^{k+1}2^k-3^k2^{k+1}=3^k\left(3^{k+1}-2^{k+1}\right)-3^{k+1}\left(3^k-2^k\right)\right)$$

$$\Rightarrow T_k=\frac{3^k}{3^k-2^k}-\frac{3^{k+1}}{3^{k+1}-2^{k+1}}$$

$$\therefore S=\sum_{k=1}^{n}\frac{6^k}{\left(3^k-2^k\right)\left(3^{k+1}-2^{k+1}\right)}$$

$$\therefore \lim_{n\to\infty}\left[\sum_{k=1}^{n}\frac{6^k}{\left(3^k-2^k\right)\left(3^{k+1}-2^{k+1}\right)}\right]$$

$$=\lim_{n\to\infty}\left(\frac{3}{3-2}-\frac{3^{n+1}}{3^{n+1}-2^{n+1}}\right)$$

$$=3-\lim_{n\to\infty}\left(\frac{3^{n+1}}{3^{n+1}-2^{n+1}}\right)$$

$$=3-\lim_{n\to\infty}\left[\frac{1}{1-\left(\frac{2}{3}\right)^{n+1}}\right]=3-\frac{1}{1-0}2$$

Alternatively

The general term is given by

$$T_k = \frac{6^k}{\left(3^k - 2^k\right)\left(3^{k+1} - 2^{k+1}\right)}$$

On dividing numerator and denominator 2^{2k+1}, we get

$$T_k = \frac{\left(\frac{1}{2}\right)\left(\frac{6}{4}\right)^k}{\left[\left(\frac{3}{2}\right)^k - 1\right]\left[\left(\frac{3}{2}\right)^{k+1} - 1\right]}$$

$$= \frac{\left(\frac{3}{2}\right)^k\left(\frac{3}{2} - 1\right)}{\left[\left(\frac{3}{2}\right)^k - 1\right]\left[\left(\frac{3}{2}\right)^{k+1} - 1\right]}$$

$$= \frac{\left(\frac{3}{2}\right)^{k+1} - \left(\frac{3}{2}\right)^k}{\left[\left(\frac{3}{2}\right)^k - 1\right]\left[\left(\frac{3}{2}\right)^{k+1} - 1\right]}$$

$$= \frac{\left[\left(\frac{3}{2}\right)^{k+1} - 1\right] - \left[\left(\frac{3}{2}\right)^k - 1\right]}{\left[\left(\frac{3}{2}\right)^k - 1\right]\left[\left(\frac{3}{2}\right)^{k+1} - 1\right]}$$

$$= \frac{1}{\left(\frac{3}{2}\right)^k - 1} - \frac{1}{\left(\frac{3}{2}\right)^{k+1} - 1}$$

$$\therefore \lim_{n\to\infty}\left[\sum_{k=1}^{n} \frac{6^k}{\left(3^k - 2^k\right)\left(3^{k+1} - 2^{k+1}\right)}\right]$$

$$= \lim_{n\to\infty}\sum_{k=1}^{n}\left[\frac{1}{\left(\frac{3}{2}\right)^1 - 1} - \frac{1}{\left(\frac{3}{2}\right)^{n+1} - 1}\right]$$

$$= 2 - \lim_{n\to\infty}\left[\frac{1}{\left(\frac{3}{2}\right)^{n+1} - 1}\right] = 2 - 0 = 2$$

45. (D)

Probability exactly one of A and B occurs

$P(E_1) = P(A \cap \overline{B}) \cup P(\overline{A} \cap B)$

$= P(A) + P(B) - 2P(A \cap B)$

Probability that exactly one of B and C occurs,

$P(E_2) = (B \cap \overline{C}) \cup (\overline{B} \cap C)$

$= P(B) + P(C) - 2P(B \cap C)$

Probability that exactly one of C and A occurs,

$P(E_3) = (C \cap \overline{A}) \cup (\overline{C} \cap A)$

$= P(A) + P(C) - 2P(A \cap C)$

Probability that all of A,B and C occurs,

$P(E_4) = P(A \cap B \cap C)$

Now, probability that atleast of A,B and C occurs

$P(E_5) = P(A \cup B \cup C)$

$= P(A) + P(B) + P(C) - P(A \cap B)$

$-P(C \cap B) - P(A \cap C) + P(A \cap B \cap C)$

$= \dfrac{P(E_1) + (E_2) + P(E_3)}{2} + P(E_4)$

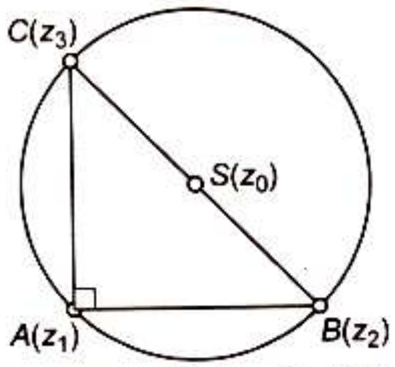
$= \dfrac{1}{2} + \dfrac{1}{9} = \dfrac{11}{18}$

46. (B)

Since, the ΔABC is a right angled isosceles triangle, with right angle at z_1 and $AB = AC$, hence it follows that

$AB = AC = \dfrac{BC}{\sqrt{2}}$

$\Rightarrow |z_1 - z_2| = |z_1 - z_3| = \dfrac{|z_2 - z_3|}{\sqrt{2}}$



Circulamcircle of ΔABC

Also, $\angle BAC = 90^{\circ}$

$\Rightarrow \arg\left(\dfrac{z_1 - z_3}{z_1 - z_2}\right) = \dfrac{\pi}{2}$

$\Rightarrow \dfrac{z_1 - z_3}{z_1 - z_2} = \left|\dfrac{z_1 - z_3}{z_1 - z_2}\right| e^{\left(\frac{i\pi}{2}\right)}$

$\Rightarrow \dfrac{z_1 - z_3}{z_1 - z_2} = i$

$\Rightarrow \left(\dfrac{z_1 - z_3}{z_1 - z_2}\right) = i^2 = -1$

$\Rightarrow (z_1 - z_3)^2 + (z_1 - z_2)^2 = 0$

$\Rightarrow 2z_1^2 + z_2^2 + z_3^2 = 2z_1(z_2 + z_3)$

Since, the ΔABC is a right angled at z_1, hence its circumcentre (z_0) is the mid-point of its hypotenuse.

$\Rightarrow z_0 = \dfrac{z_2 + z_3}{2}$

$\Rightarrow 2z_1^2 + (z_2 + z_3)^2 = 2\sum z_1 z_2$

$\Rightarrow 2\sum z_1 z_2 = 2z_1^2 + 4z_0^2$

$\Rightarrow \sum z_1 z_2 = z_1^2 + 2z_0^2$

47. (A,B,D)

Concepts Involved Binomial theorem and Mathematical induction

(A) Since, n is an even integer. Let us assume that $n = 2m$, where $m \in N$

$$\left(\sqrt{2}-1\right)^{2m} = {}^{2m}C_0\left(\sqrt{2}\right)^{2m} - {}^{2m}C_1\left(\sqrt{2}\right)^{2m-1} + {}^{2m}C_2\left(\sqrt{2}\right)^{2m-2} - .... + (-1)^{2m}$$

$$= \left({}^{2m}C_0\left(\sqrt{2}\right)^{2m} + {}^{2m}C_2\left(\sqrt{2}\right)^{2m-2} + {}^{2m}C_4\left(\sqrt{2}\right)^{2m-4} .....\right) - \left({}^{2m}C_1\left(\sqrt{2}\right)^{2m-1} + {}^{2m}C_3\left(\sqrt{2}\right)^{2m-3} + {}^{2m}C_5\left(\sqrt{2}\right)^{2m-5} + ...\right)$$

$$= \left({}^{2m}C_0(2)^{m} + {}^{2m}C_2(2)^{m-1} + {}^{2m}C_4(2)^{m-2} + ...\right) - \sqrt{2}\left({}^{2m}C_1(2)^{m-1} + {}^{2m}C_3(2)^{m-2} + {}^{2m}C_5(2)^{m-3} + ...\right)$$

$= A - B\sqrt{2}$ where A and $B \in I^+$

(B) Since, n is an odd integer. Let us assume that $n = 2m+1$, where $m \in N$

$$\left(\sqrt{2}-1\right)^{2m+1} = {}^{2m+1}C_0\left(\sqrt{2}\right)^{2m+1} = {}^{2m+1}C_1\left(\sqrt{2}\right)^{2m} + {}^{2m+1}C_2\left(\sqrt{2}\right)^{2m-1} - ..... + (-1)^{2m+1}$$

$$= \left[{}^{2m+1}C_0\left(\sqrt{2}\right)^{2m+1} + {}^{2m+1}C_2\left(\sqrt{2}\right)^{2m-1} + {}^{2m+1}C_4\left(\sqrt{2}\right)^{2m-3} + ...\right]$$

$-\left[{}^{2m+1}C_1\left(\sqrt{2}\right)^{2m}+{}^{2m+1}C_3\left(\sqrt{2}\right)^{2m-2}+{}^{2m+1}C_5\left(\sqrt{2}\right)^{2m-4}+.......\right]$

$=\sqrt{2}\left[{}^{2m+1}C_0\left(2\right)^{m}+{}^{2m+1}C_2\left(2\right)^{m-1}+{}^{2m+1}C_4\left(2\right)^{m-2}+.......\right]-$

$\left[{}^{2m+1}C_1\left(2\right)^{m}+{}^{2m+1}C_3\left(2\right)^{m-1}+{}^{2m+1}C_5\left(2\right)^{m-2}+......\right]$

$=A\sqrt{2}-B$, where A and B $\in I^+$

(C) This is false

(D) For $n=1,\left(\sqrt{2}-1\right)^n=\sqrt{2}-1=\sqrt{2}-\sqrt{1}$

For $n=2$

$\left(\sqrt{2}-1\right)^n=\left(\sqrt{2}-1\right)^2=3-2\sqrt{2}=\sqrt{9}-\sqrt{8}$

For $n=3,\left(\sqrt{2}-1\right)^n=\left(\sqrt{2}-1\right)^3=5\sqrt{2}-7$

$=\sqrt{50}-\sqrt{49}$

For $n=m,\text{let}\left(\sqrt{2}-1\right)^m=\sqrt{N}-\sqrt{(N-1)}$ is true

For $n=m+1$

$\left(\sqrt{2}-1\right)^n=\left(\sqrt{2}-1\right)^{m+1}=\left(\sqrt{2}-1\right)\left(\sqrt{2}-1\right)^n$

$=\left(\sqrt{2}-1\right)\left[\sqrt{N}-\sqrt{(N-1)}\right]$

$=\sqrt{2N}-\sqrt{N}-\sqrt{2(N-1)}+\sqrt{N-1}$

$=\left(\sqrt{2N}+\sqrt{N-1}\right)=\left[\sqrt{N}+\sqrt{2(N-1)}\right]$

$=\sqrt{3N-1+2\sqrt{2N(N-1)}}$

$-\sqrt{3N-2+2\sqrt{2N(N-1)}}$

$=\sqrt{M}-\sqrt{M-1}$

But what is the gurantee that

$M=3N-1+2\sqrt{2N(N-1)}$ is a positive integer?

Now, to prove that M is also an integer, we have to prove that $\sqrt{2N(N-1)}$ is also a positive integer.

$M=3N-1+2\sqrt{2N(N-1)}$

Also, $\sqrt{N}-\sqrt{(N-1)}=\left(\sqrt{2}-1\right)^m$

$=A+B\sqrt{2}$

Where $A,Bm\in I^+$

On squaring both sides, we get

$2N-1+\sqrt{2}\sqrt{N(N-1)}=A^2+2B^2+2\sqrt{2}AB$

On comparing rational and irrational parts, we get

$\sqrt{N(N-1)}=\sqrt{2}AB$

$\Rightarrow\sqrt{2N(N-1)}=2AB\left(\in I^+\right)$

$\therefore \left(\sqrt{2}-1\right)^n = \sqrt{N}-\sqrt{(N-1)}$ is true for $n = m+1$

$\Rightarrow \left(\sqrt{2}-1\right)^n = \sqrt{N}-\sqrt{(N-1)}$ is true for all natural numbers by mathematical induction.

48. (A,B)

$$\sum_{r=1}^{n}(-1)^{r-1}\left(\frac{r^2+3r+1}{r^3+2r^2+r}\right)C_r$$

$$=\sum_{r=1}^{n}(-1)^{r-1}\left(\frac{C_r}{r}\right)+\sum_{r=1}^{n}(-1)^{r-1}\left(\frac{C_r}{(r+1)^2}\right) \text{........(i)}$$

Now, $\frac{1-(1-x)^n}{x} = C_1 - C_2x + C_3x^2 - .... + (-1)^{n-1}C_nx^{n-1}$

On integrating both the sides from $x = 0$ to $x = 1$, we get

$$\int_0^1 \frac{1-(1-x)^n}{1-(1-x)}dx = \int_0^1\left[C_1 - C_2x + C_3x^2 - ..... + (-1)^{n-1}C_nx^{n-1}\right]dx$$

$$\Rightarrow \int_0^1 \frac{1-(x)^n}{1-(x)}dx = \left[C_1 - C_2\frac{x^2}{2} + C_3\frac{x^3}{3} - .... + (-1)^{n-1}C_n\frac{x^n}{n}\right]_0^1$$

$$\Rightarrow \int_0^1\left(1+x+x^2+.....+x^{n-1}\right)dx = \left[C_1 - \frac{C_2}{2}+\frac{C_3}{3} - ...... + (-1)^{n-1}\frac{C_n}{n}\right]$$

$$\Rightarrow 1+\frac{1}{2}+\frac{1}{3}+......+\frac{1}{n} = \sum_{r=1}^{n}(-1)^{r-1}\frac{C_r}{r}$$

$$\Rightarrow \sum_{r=1}^{n}(-1)^{r-1}\frac{C_r}{r} = 1+\frac{1}{2}+\frac{1}{3}+......+\frac{1}{n} \qquad \text{......(ii)}$$

Now , by considering,

$$\frac{1-(1-x)^{n+1}}{x(n+1)} = C_0 - \frac{C_1}{2}x + \frac{C_2}{3}x^2 + ... + (-1)^n\frac{C_nx^n}{n+1}$$

$$= \frac{1}{n+1}\int_0^1 \frac{1-(1-x)^{n+1}}{1-(1-n)}dx$$

$$= C_0 - \frac{C_1}{2^2} + \frac{C_3}{3^3} - ...... + (1)^n\frac{C_n}{(n+1)^2}$$

$$\Rightarrow \frac{1}{(n+1)}\left[1+\frac{1}{2}+\frac{1}{3}+.....+\frac{1}{n+1}\right] = 1 - \frac{C_1}{2^2}+\frac{C_2}{3^2} - ...... + (-1)^n\frac{C_n}{(n+1)^2}$$

$$\Rightarrow -\frac{1}{(n+1)}\left(1+\frac{1}{2}+\frac{1}{3}+......+\frac{1}{n}\right) = \frac{C_1}{2}-\frac{C_2}{3^2}+\frac{C_3}{4^2} - ...... + (-1)^{n-1}\frac{C_n}{(n+1)^2}$$

$$\Rightarrow \frac{-1}{(n+1)}\left(1+\frac{1}{2}+\frac{1}{3}+.....+\frac{1}{n}\right) = \sum_{r=1}^{n}\frac{(-1)^{r-1}C_r}{(r+1)^2}$$

$$\Rightarrow \sum_{r=1}^{n} \frac{(-1)^{r-1} C_r}{(r+1)^2} = \frac{-1}{(n+1)}\left(1+\frac{1}{2}+\frac{1}{3}+.....+\frac{1}{n}\right) \quad ....(iii)$$

From Eqs. (i), (ii) and (iii), we get

$$\sum_{n=1}^{r}(-1)^{r-1}\left(\frac{r^2+3r+1}{r^3+2r^2+r}\right)C_r = \left(\frac{n}{n+1}\right)\left(1+\frac{1}{2}+\frac{1}{3}+......+\frac{1}{n}\right)$$

On substituting various values of n, we get the option (A) is correct and option(B) is correct.

49. (A)

Concept Involved

If a matrix A is orthogonal, then it satisfies $AA^T = A^TA = I$

Given that, $Q = PAP^T$

On multiplying both sides by P^T from LHS, we get

$$P^T(Q) = P^T(PAP^T)$$

$$\Rightarrow P^TQ = P^TPAP^T$$

$$\Rightarrow P^TQ = AP^T \qquad (\because P^TP = I)$$

Now, multiplying both sides by P from RHS, we get

$$\Rightarrow P^TQP = AP^TP$$

$$\Rightarrow P^TQP = A$$

Now, $A = P^TQP$

$$\Rightarrow A^2 = (P^TQP)(P^TQP)$$

$$= P^TQPP^TQP$$

$$= P^TQ(PP^T)QP = P^TQ^2P$$

Similarly, we can say that

$$A^3 = P^TQ^3P, A^4 = P^TQ^4P......A^{2005}$$

$$= P^TQ^{2005}P$$

$$\Rightarrow P^TQ^{2005}P = A^{2005} = \begin{bmatrix} 1 & 2005 \\ 0 & 1 \end{bmatrix}$$

Explanation

$$A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow A^2 = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow A^3 = AA^2 = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix}$$

$\dots \qquad \dots \qquad \dots$

$$\Rightarrow A^{2005} = \begin{bmatrix} 1 & 2005 \\ 0 & 1 \end{bmatrix}$$

Alternatively

Given that, $Q = PAP^T$

On multiplying both sides by P^T from LHS, we get

$$P^T(Q) = P^T\left(PAP^T\right)$$

$$\Rightarrow P^TA = P^TPAP^T$$

$$\Rightarrow P^TQ = AP^T \qquad \left(\because P^TP = I\right)$$

Hence, required matrix

$$P^TQ^{2005}P = \left(P^TQ\right)Q^{2004}P$$

$$= \left(AP^T\right)Q^{2004}P\left(\because P^TQ = AP^T\right)$$

$$= AP^TQ^{2004}P$$

Proceeding in similar way, we can write

$$P^TQ^{2005}P = AP^TQ^{2004}P$$

$$= A^2P^TQ^{2003}P$$

$$= A^3P^TQ^{2002}P$$

$$= ....$$

$$= A^{2004}P^TQP$$

$$= A^{2004}\left(AP^T\right)P$$

$$= A^{2005}$$

$$= \begin{bmatrix} 1 & 2005 \\ 0 & 1 \end{bmatrix}$$

50. (C)

$$\left(1+2x+2x^2\right)^n = a_0 + a_1x + a_2x^2 + a_3x^3 + ...+ a_{2n}x^{2n} \qquad ....(i)$$

Replacing $x \to -x$, we get

$$\left(1-2x+2x^2\right)^n = a_0 - a_1x + a_2x^2 - a_3x^3 + ...+ a_{2n}x^{2n} \qquad ....(ii)$$

On multiplying Eqs. (i) and (ii), we get

$$\left(1+2x+2x^2\right)^n\left(1-2x+2x^2\right)^n$$

$$= \left(a_0 + a_1x + a_2x^2 + a_3x^3 + ...+ a_{2n}x^{2n}\right)$$

$$\left(a_0 - a_1x + a_2x^2 - a_3x^3 + ...+ a_{2n}x^{2n}\right)$$

Required sum is $S = a_0a_{2n} - a_1a_{2n-1} + a_2a_{2n-2} - ......+ a_{2n}a_0$

=Coefficient of x^{2n} in

$$\left[\left(1+2x+2x^2\right)\left(1-2x+2x^2\right)\right]^n$$

=Coefficient of x^{2n} in $\left[\left(1+2x^2\right)^2 - \left(2x^2\right)\right]^n$

=Coefficient of x^{2n} in $\left(1+4x^4\right)^n$

$$= {}^nC_{n/2} \times \left(2^n\right)$$

51. (D)

Given function is $h(x)=\sin^{-1}x-\cos^{-1}\sqrt{1-x^2}$,

Now, $\cos^{-1}\sqrt{1-x^2}=\begin{cases}\sin^{-1}x, \text{if } x>0\\ -\sin^{-1}x, \text{if } x<0\end{cases}$

$\therefore h(x)=\begin{cases}2\sin^{-1}x, x<0\\ 0, \quad x\geq 0\end{cases}$

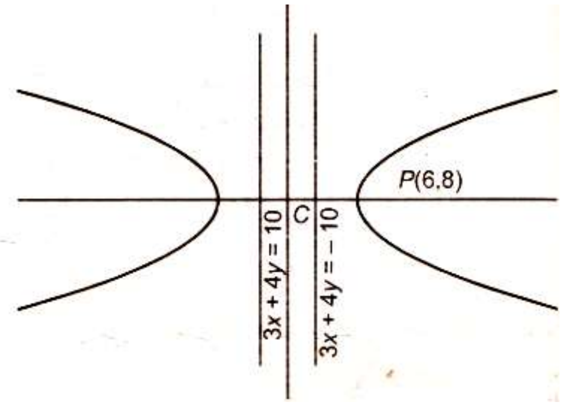
$\therefore h(x)$ is a non-decreasing function.

52. (B,C)

Let us assume that the given directrices are $L_1:3x+4y+10=0$ and $L_2:3x+4y-10=0$

$d(P,L_1)=$ distance between P and L_1

$=\left|\frac{18+32-10}{5}\right|=8$



$d(P,L_2)=$ distance between P and L_2

$=\left|\frac{18+32+10}{5}\right|=12$

Hence, P is closer to the directrix

$3x+4y-10=0$

$\Rightarrow ae-\frac{a}{e}=8 \text{ and } ae+\frac{a}{e}=12$

On solving , we get $ae=10$ and $\frac{a}{e}=2$

$\Rightarrow CP=ae=10$

On dividing the equations

$ae=10 \text{ and } \frac{a}{e}=2$, we get

$\frac{ae}{a/e}=\frac{10}{2}\Rightarrow e^2=5\Rightarrow e=\sqrt{5}$

53. (C)

Given function are as $f:A\rightarrow B, f(x)=\sin^{-1}\left(\frac{[x]}{\{x\}}\right)$ and $g:C\rightarrow D, g(x)=\cos^{-1}\left(\frac{[x]}{\{x\}}\right)$

finding the domains of $f(x)$ and $g(x)$, we proceed in the following manner. Domain of $y=\sin^{-1}x$ and $y=\cos^{-1}x$ is $[-1,1]$

$\therefore -1\le\frac{[x]}{\{x\}}\le 1$

$\Rightarrow \left|\frac{[x]}{\{x\}}\right|\le 1$

$\Rightarrow |[x]|\le\{x\}$

$\Rightarrow x\in(0,1)$

$\Rightarrow A, C=(0,1)$

$\Rightarrow A\cup C=(0,1)$

54. (D)

Given lines are $\frac{x-1}{1}=\frac{y+3}{-k}=\frac{z-1}{k}$

And $\frac{2x}{1}=\frac{y-1}{1}=\frac{z-2}{-1}$

Rearranging the given equations, we get

$\frac{x-1}{1}=\frac{y+3}{-k}=\frac{z-1}{k}$

And $\frac{x-0}{1}=\frac{y-1}{2}=\frac{z-2}{-2}$

If the above given two lines are coplanar, then the vectors $(i-4j-k),(i-kj+kk)$ and $(i+2j-2k)$ must be coplanar

$\therefore \begin{vmatrix}1 & -4 & -1\\ 1 & -k & k\\ 1 & 2 & -2\end{vmatrix}=0$

Applying the operations $R_1\to R_1-R_3$ and $R_2\to R_2-R_3$, we get

$\begin{vmatrix}0 & -6 & 1\\ 0 & -k-2 & k+2\\ 1 & 2 & -2\end{vmatrix}=0$

Now, expanding the above determinant along C_1, we get

$0-0+1(-6k+12+k+2)=0$

$\therefore k=-2$

55. (B)

$|adj(adjA)|=|adjA|^{n-1}$

$=\left[|A|^{(n-1)}\right]^{n-1}=|A|^{(n-1)^2}$

$\therefore |adj(adjC)|=|C|^{(3-1)^2}$

$= |C|^4 = (-1)^4 = 1$

56. (B)

$|adj(CB)| = |CB|^2$

$$= \left| \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 5 & 6 & 8 \end{bmatrix} \begin{bmatrix} 3 & 2 & 5 \\ 2 & 3 & 8 \\ 7 & 2 & 9 \end{bmatrix} \right|$$

$$= \begin{vmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 5 & 6 & 8 \end{vmatrix} \begin{vmatrix} 3 & 2 & 5 \\ 2 & 3 & 8 \\ 7 & 2 & 9 \end{vmatrix}$$

$= \left[(-1) \times 24\right]^2 = 24^2$

57. (1)

The system of homogenous equations $\begin{cases} a_1x + b_1y + c_1z = 0 \\ a_2x + b_2y + c_2z = 0 \\ a_3x + b_3y + c_3z = 0 \end{cases}$ will have a non- trivial solution, if determinant of coefficients is equal to zero, i.e.,

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 0$$

If the given system of homogenous equations has a non-trivial solution, then determinant of coefficients is equal to zero, i.e.,

$$\Delta = \begin{vmatrix} a & a^2 & 1 \\ b & b^2 & 1 \\ c & c^2 & 1 \end{vmatrix} = 0$$

Applying $R_1 \to R_1 - R_2$ and $R_2 \to R_2 - R_3$, we get

$$\begin{vmatrix} a-b & a^2-b^2 & 0 \\ b-c & b^2-c^2 & 0 \\ c & c^2 & 1 \end{vmatrix} = 0$$

Taking $(a-b)$ and $(b-c)$ common from R_1 and R_2 respectively, we get

$$(a-b)(b-c)\begin{vmatrix} 1 & a+b & 0 \\ 1 & b+c & 0 \\ c & c^2 & 1 \end{vmatrix} = 0$$

Now, expanding along C_3, we get

$(a-b)(b-c)\left[(b+c)-(a+b)\right] = 0$

$(a-b)(b-c)(c-a) = 0$

$\Rightarrow a = b \text{ or } b = c \text{ or } c = a$

Since, a,b and c are in GP Hence,
$a=b$ and $b=c \Rightarrow r=1$
But $c=a$
$\Rightarrow r=\pm 1$
$\therefore r=\pm 1$
$\Rightarrow |r|=1$
$\Rightarrow k=1$
$\therefore \frac{k+2}{4-k}=\frac{1+2}{4-1}=\frac{3}{3}=1$

58. (0)

$|z+(a-ib)|=\sqrt{a^2+b^2} \Rightarrow z$ lies on a circle with centre $(-a+ib)$ (say C_1) and radius equal to $\sqrt{a^2+b^2}$ (say r_1) units

$|z|=a^2+b^2+a \Rightarrow z$ lies on a circle (say C_2) with center at $(-0-0i)$ (say C_2) and radius equal to $a^2+b^2+1(\text{say } r_2)$ units.

Now, the big question is whether the two circles above intersect or not.
Distance between their centres

$C_1C_2=|(-a+ib)-(-0-0i)|=\sqrt{a^2+b^2}$

Now, $a^2+b^2+1-2\sqrt{a^2+b^2}=\left(\sqrt{a^2+b^2}-1\right)^2$

$\Rightarrow a^2+b^2+1-2\sqrt{a^2+b^2}>0$

$\Rightarrow \left(a^2+b^2+1\right)-\sqrt{a^2+b^2}>\sqrt{a^2+b^2}$

$\Rightarrow r_2-r_1>C_1C_2 \Rightarrow C_1C_2<r_2-r_1$

$\therefore$ Circle with centre C_1 lies completely inside the circle with centre C_2.
The given circles don't intersect.
The number of points of intersection are zero
Alternatively
$|z|$ can be written as $|z|=|z+(a-ib)-(a-ib)|$
Now, applying triangle's inequality, we get

$|z+(a-ib)-(a-ib)|\le|z+a-ib|+|a-ib|$

$\Rightarrow |z|\le\sqrt{a^2+b^2}+\sqrt{a^2+b^2}$

$\Rightarrow a^2+b^2+1\le 2\sqrt{a^2+b^2}$

$\Rightarrow \left(\sqrt{a^2+b^2}-1\right)^2\le 0$

($\because$ square of a real number cannot be negative.)
$\sqrt{a^2+b^2}=1$ is the only possibility but it is given that $a^2+b^2\neq 1$
$\therefore$ No real values of a and b are possible.

59. (1)

$\text{LHS}=\frac{2}{1!3!}+\frac{2}{3!11!}+\frac{2}{5!9!}+\frac{1}{7!7!}$

$= \frac{1}{14!}\left(2.\frac{14!}{1!3!} + 2.\frac{14!}{3!11!} + 2.\frac{14!}{5!9!} + \frac{14!}{7!7!}\right)$

$= \frac{1}{14!}\left(2\,{}^{14}C_1 + 2\,{}^{14}C_3 + 2\,{}^{14}C_5 + {}^{14}C_7\right)$

$= \frac{1}{14!}\left({}^{14}C_1 + {}^{14}C_3 + {}^{14}C_5 + {}^{14}C_7 + {}^{14}C_9 + {}^{14}C_{11} + {}^{14}C_{13}\right)$

$= \frac{2^{14-1}}{14!} = \frac{2^{13}}{14!} \Rightarrow \frac{2^m}{n!} = \frac{2^{13}}{14!}$

$\therefore m = 13$ and $n = 14$

$\Rightarrow n - m = 14 - 13 = 1$

60. (4)

Number of ways to draw 1 st pair $= \left({}^nC_1 \times {}^nC_1\right)$

Number of ways to draw 2 nd pair $= \left({}^{n-1}C_1 \times {}^{n-1}C_1\right)$

Number of ways to draw 2 rd pair

$= \left({}^{n-2}C_1 \times {}^{n-2}C_1\right)$

……..

Number of ways to draw last i.e., nth pair $= \left({}^1C_1 \times {}^1C_1\right)$

Combining all the above results, we get

Number of ways to draw n pairs

$= \left({}^nC_1 \times {}^nC_1\right)\left({}^{n-1}C_1 \times {}^{n-1}C_1\right)....\left({}^2C_1 \times {}^2C_1\right)\left({}^1C_1 \times {}^1C_1\right)$

$\Rightarrow \left({}^nC_1 \times {}^nC_1\right)\left({}^{n-1}C_1 \times {}^{n-1}C_1\right).....\left({}^2C_1 \times {}^2C_1\right)\left({}^1C_1 \times {}^1C_1\right) = 576$

$\Rightarrow n^2 (n-1)^2 ......2^2 \times 1^2 = (24)^2$

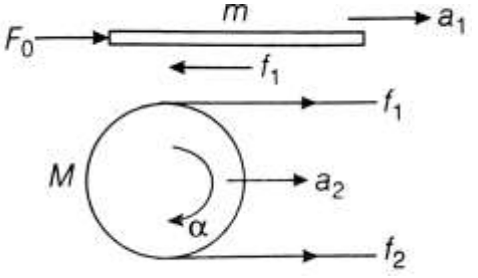
$\Rightarrow (n!)^2 = (24)^2$

$\Rightarrow n! = 24 = 4!$

$\therefore n = 4$

PART (A) : PHYSICS

1. (B, C)
For the no slipping, there will not be any relative motion
So, $v = \omega R$
Also $\tau_{net} = l\alpha$
Where τ_{net} is net torque
l is moment of inertia and α is angular acceleration.
The forces on different surfaces are shown below



Here in above diagram, all the force are shown
Where
f_1 = friction between plank and cylinder
f_2 = friction between cylinder and ground.
a_1 = acceleration of plank and
a_2 = acceleration of Cm of cylinder
As, there is no slipping anywhere

$a_1 = 2a_2$ ….(i)

$a_1 = \frac{F_0 - t_1}{m}$ ….(ii)

$a_2 = \frac{f_1 + f_2}{M}$ …..(iii)

$\therefore \tau_{net} = l\alpha \Rightarrow \alpha = \frac{\tau_{net}}{l}$

$\Rightarrow \alpha = \frac{(f_1 - f_2)R}{l} = \frac{(f_1 - f_2)R}{\frac{1}{2}MR^2}$

$= \frac{2(f_1 - f_2)}{MR}$ …….(iv)

$a_2 = R\alpha = \frac{2(t_1 - t_2)}{M}$ ……..(v)

Solving Eqs. (i) and (v) we get

$a_1 = \frac{BF_0}{3M + 8m}$

$\Rightarrow a2 = \frac{4F_0}{3M + 8m}$

$f1 = \frac{3MF_0}{3M + 8m} \Rightarrow f_2 = \frac{MF_0}{3M + 8m}$

2. (A, B)

This question can be solved, If right hand side chamber is assumed open, so that its pressure remains constant even, if the positon shifts towards right

$pV = nRT$

$\Rightarrow P \propto \frac{T}{V}$

Temperature is made three times and volume is doubled

$p_2 = \frac{3}{2}p_1$

Further

$x = \frac{\Delta V}{A} = \frac{V_2 - V_1}{A} = \frac{2V_1 - V_1}{A} = \frac{V_1}{A}$

$p_2 = \frac{3p_1}{2} = p_1 + \frac{kx}{A}$

$\Rightarrow kx = \frac{p_1 A}{3}$

Energy of spring

$\frac{1}{2}kx^2 = \frac{p_1 V_1}{4}$

Also, $\Delta U = nC_V \Delta T$

$= \frac{p_1 V_1}{RT_1} \times \frac{3}{2} R \times (3T_1 - T_1)$

$= 3p_1 v_1$

3. (A)

As, shown in the figure e

For $0 \le x \le a$

$\phi = Bx^2$

$\therefore$ Induced emf, $\varepsilon = -\frac{d\phi}{dt} = -2Bx.\frac{dx}{dt}$

$\varepsilon = -\frac{d\phi}{dt} = -2Bx.\frac{dx}{dt}$

Or $\varepsilon = -2Bx.v$

At $x = 0, \varepsilon = 0$

At x = a, $\varepsilon = -2Bav$

At $x = 2a$, $ge = 0$ as $\phi = \text{constant}$

(i.e. when the loop starts coming out from the field after sometime, then

$\phi = B(2a - x_1^2)$

$\therefore \varepsilon = -2B(2a - x)(-v)$

Or $\varepsilon = 2B(2a - x).v$

At $x = 0, \varepsilon = 4BaV$

Hence (a) is the correct option

4. (B, C)

Motion of m_2 starts, when $kx = \mu . m_2 g$, when x = extension in the spring. $x = \mu m_2 g / k$

The minimum force will be such that m_1 has no kinetic energy. Applying, work-energy principle on body of m_1.

$$\int_0^x (F_{min} - \mu m_1 g - kx)dx = 0$$

$$\Rightarrow F_{min} x - \mu m_1 gx - \frac{1}{2}kx^2 = 0$$

$$\Rightarrow F_{min} = \left[\mu m_1 g + \frac{1}{2}kx\right]$$

$$= \left[\mu m_1 g + \frac{\mu m^2 g}{2}\right]$$

$$\Rightarrow F_{min} = \mu m_1 g + \frac{\mu m_2 g}{2}$$

5. (A, C)

In the given figure, Process AB is an isobaric process

During the process $V \propto T$

But $pV = nRT \Rightarrow pV \propto T$

Thus, during this process, pressure P remains constant.

In process BC, temperature decreases, while volume remains constant.

Process CA is an isothermal process. Hence. On T_V diagram process AB will be a straight line parallel to the T-axis, during which temperature increase. Process BC will be a straight line passing through origin. During which temperature and process CA will be a straight line parallel to the V-axis. Hence, option (a) is correct.

On p-V diagram, process AB will be a straight line parallel to the V-axis. Process BC will be a straight line parallel to the p-axis and CA will be a rectangular hyperbola.

Hence (C) is the correct option.

6. (B, C)

Use relations, $Q = Q_0(1 - e^{-t/RC})$ and $i = i_0 e^{-t/RC}$

7. (3.00)

$$T \propto r^{3/2} \Rightarrow r2 = \left(\frac{T_2}{T_1}\right)^{2/3} .r_1$$

$$= \left(\frac{8}{1}\right)^{2/3} \times 10^4 = 4 \times 10^4 \text{km}$$

Now, $v_1 = \frac{2\pi \times r_1}{T_1} = 2\pi \times 10^4 \text{km/h}$

and $v_2 = \frac{2\pi r_2}{T_2} = \pi \times 10^4 \text{km/h}$

So ω_r = angular speed of S_2 relative to S_1

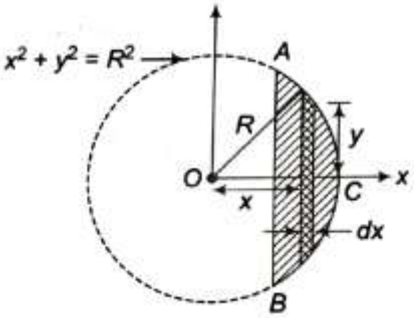
$$=\left|\frac{v_2-v_1}{r_2-r_1}\right|=3.0\times10^{-4}\,rad/s$$

8. (5.00)

For the moment of inertia of the disc of mass dm at distance x about the given axis, we will use

$$dl=\frac{1}{2}dmy^2, dm=\rho dV$$

Where y will be radius



Density of spherical segment is

$$\rho=\frac{M}{V}=\frac{M}{\int_{\frac{R}{2}}^{R}\pi y^2dx}=\frac{M}{\int_{R/2}^{R}\pi(R^2-x^2)dx}$$

$$\Rightarrow\rho=\frac{M}{\frac{5\pi R^3}{24}}=\frac{24M}{5\pi R^3}$$

Consider a small disc of radius y and thickness dx as shown in figure.

Mass of small disc, dm $=\rho.\pi y^2dx$. Moment of inertia of disc about O, X –axis

$$dl=\frac{1}{2}dm.y^2dl$$

$$dl=\frac{1}{2}\rho\pi y^2dx\times y^2=\frac{\rho\pi}{2}y^4dx$$

Total moment of inertia of segment of sphere is

$$I=\int dl=\frac{\rho\pi}{2}\int_{\frac{R}{2}}^{R}y^4dx$$

$$I=\frac{\rho\pi}{2}\int_{\frac{R}{2}}^{R}(R^2-x^2)^2dx$$

$$=\frac{\rho\pi}{2}\int_{\frac{R}{2}}^{R}(R^4-2R^2x^2+x^4)dx$$

$$i=\frac{24M}{5\pi R}\times\frac{\pi}{2}\times\left[R^4x-\frac{2R^2x^3}{3}+\frac{x^5}{5}\right]_{R/2}^{R}$$

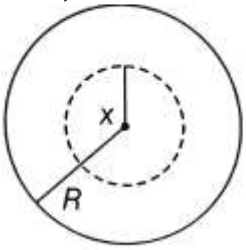
$$= \frac{24M}{5\pi R^3} \times \frac{\pi}{2} \times \frac{53R^5}{480} l = \frac{53}{100} MR^2$$

$$I = \frac{53}{200} MR^2 = \frac{53mR^2}{40x}$$

$$\Rightarrow x = 5$$

9. (5.00)

Here, we have to use the concept of flux as well as current . We will also use the relation



Sphere

dV = -E . dr

Electric field is along the direction of maximum change in potential .The equation used for electric flux is

$$\phi = \oint E.dS$$

Where dS is the surface area.

Consider the diagram shown below

We have considered a surface of thickness dx at distance x. Surface area of the considered surface = $4\pi x^2$

Now we know that

$$E = \frac{dV}{dx} \Rightarrow dV = Edx$$

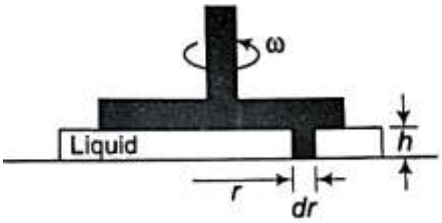
Resistance R = $\frac{\rho dx}{4\pi x^2}$

Current I = $\frac{dV}{R} = \frac{Edx}{\frac{\rho dx}{4\pi x^2}} = \frac{E.4\pi x^2}{\rho}$

$$\Rightarrow i = \frac{\phi}{\rho} = \frac{10}{2} = 5A$$

10. (4.00)

Consider an element of disc at a radius r and having a width dr. Linear velocity at this radisu = ωr.



Shear stress $\tau = \mu \frac{du}{dy}$

Assuming the gap h to be small so that the velocity distribution may be assumed linear

$\tau = \mu \times \frac{v}{h} = \mu \frac{\omega r}{h}$

Viscous force, $dF = \tau \times \text{Area}$

$= \tau \times 2\pi / dr$

Torque dT on the element

$dT = dF \times r = \tau 2\pi r^2 dr$

Or $d \frac{\mu\omega r}{h} \times 2\pi r^2 dr = \frac{2\pi\mu r^3 dr}{h}$

Total torque

$$T = \int_0^{d/2} \frac{2\pi\mu\omega r^3 dr}{h} = \frac{\mu\pi d^4\omega}{4h}$$

Thus x = 4

11. (6.00)

$\Rightarrow F + fs = m \times a$ ....(i)

$\Rightarrow (F - f_s)f = I\alpha$ ....(ii)

For pure rolling a = $R\alpha$ ....(iii)

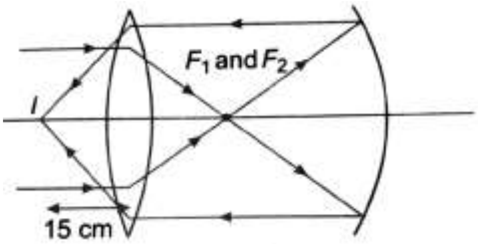
By solving above three equations, we get

$f = \frac{3}{7}F$

Put F = 14N

F = $\frac{3}{7} \times 14 = 6N$

12. (4.00)

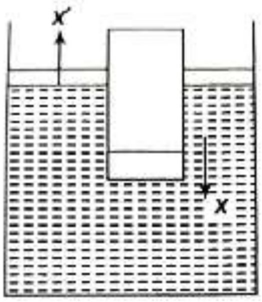


The condition is possible only when the foci of lens and mirror coincides. Then, only the final image will be at 15 cm left to lens.

So x = 15 + 21 = 36 cm

Hence $\frac{x}{9} = \frac{36}{9} = 4$ cm

13. (6.00)



Cylinder can perform SHM only till it is partially submersed. When cylinder goes down by x inside the liquid level comes up by x'(say)

$(4a - a)x' = xa$

$\Rightarrow x' = \frac{x}{3}$

So, the centre of the cylinder goes down by (w.r.t. the liquid surface)

$(x + x') = \frac{4}{3}x \leq \frac{l}{10}$

$\Rightarrow x \leq \frac{3l}{40} = 6cm$

14. (6.00)

From the given conditions, we get

$E_n - E_2 = (10.2 + 17)eV$

$= 27.2eV$ ……(i)

$= E_n - E_3 = (4.25 + 5.95)eV$

$=10.2$ eV ……(ii)

Subtracting Eq (ii) from Eq. (i) we get

$E_3 - E_2 = 17.0eV$

$Z^2(13.6)\left[\frac{1}{4} - \frac{1}{9}\right]$

$Z^2(13.6)\left(\frac{5}{36}\right) = 17.0$

$Z^2 = 9$ or $Z = 3$

From Eq. (i) we get

$Z^2(13.6)\left[\frac{1}{4} - \frac{1}{n^2}\right] = 27.2$

$3^2(13.6)\left[\frac{1}{4} - \frac{1}{n^2}\right] = 27.2$

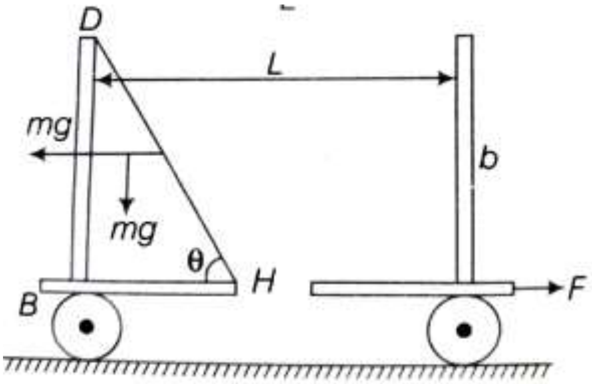
$\frac{1}{4} - \frac{1}{n^2} = 0.222$

$\Rightarrow \frac{1}{n^2} = 0.0278 \Rightarrow n^2 = 36 \Rightarrow n = 6$

15. (3)

As the cart is drawn by a force F the water in the vessel takes up a slant position rising upward at the back wall of the vessel. To prevent water flowing out of the hole H, the acceleration of the vessel should have such a value that it occupies a face area DBH and a width of vessel given by $\frac{A}{L}$

Area of $\Delta DBH = \frac{1}{2}bc$



Volume of liquid retained = $\frac{1}{2}bc \times \frac{A}{L}$

Mass of cart and water = $M + \frac{bcA\rho}{2L}$

$\tan\theta = \frac{ma}{mg}$

$a = g\tan\theta = g \times \frac{b}{c}$

Required force = $\left(M + \frac{bcA\rho}{2L}\right)\frac{gb}{c}$

$= [1 + 0.5] \times 50$

$= 1.5 \times 50 = 75N$

$\therefore \frac{F}{25} = 3$

16. (3.00)

As $\phi = \frac{d\delta}{d\lambda}$

This can be written as

$\phi = \frac{d\delta}{d\mu} \times \frac{d\mu}{d\lambda}, \delta = (\mu - 1)A_0$

$\frac{d\delta}{d\mu} = A_0$

$\mu = A + \frac{B}{\lambda^2} \Rightarrow \frac{d\mu}{d\lambda} = -\frac{2B}{\lambda^3}$

So, $\phi = \frac{2BA_0}{\lambda^3} \Rightarrow \phi \propto \frac{1}{\lambda^3}$

So, N = 3

17. (C)

In ray 1, there is no phase shift ue to reflection at a soft boundary

In ray 2, there is a phase shift of half circle due to reflection from a hard boundary.

Now as the thickness t of the air wedge at each point is proportional to the distance from the line contact, we have

$\frac{t}{x} = \frac{h}{l}$

And for destructive interference. $2t = n\lambda_0$ where n = 0, 1,2 ……

Combining both equations, we get

$$\frac{2hx}{l} = n\lambda_0$$

$$\Rightarrow x = n.\frac{l\lambda_0}{2h}$$

$$= n.\frac{0.1 \times 500 \times 10^{-9}}{2 \times 0.02 \times 10^{-3}}$$

= n (125mm)

So, successive dark fringes are spaced 1.25 mm apart

18. (A)

If we substitute x = 0, which is the location of contact line of slides, we get n =0

Hence at line of contact a dark fringe appears.

19. (D)

20. (A)

Here, the loop has given a push and left, it means that its speed will keep on decreasing due to magnetic force $[l(I \times B)]$. But observed carefully that magnetic retarding force will also decrease with time.

If we just pushed the lop and let it, then due to magnetic foece $[-lI \times B]$, the speed of the loop will start decreasing

Here $F = \frac{-vB^2l^2}{R}$ [where R = resistance of the loop]

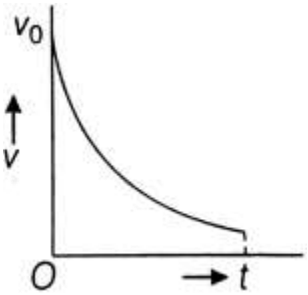
$$\frac{dv}{dt} \propto -v$$

$$\frac{dv}{dt} = -kv$$ [where k = constant]

$$\int \frac{dv}{v} = \int -kdt$$

$$\Rightarrow \log v = -kt$$

$\Rightarrow$ So, graph is

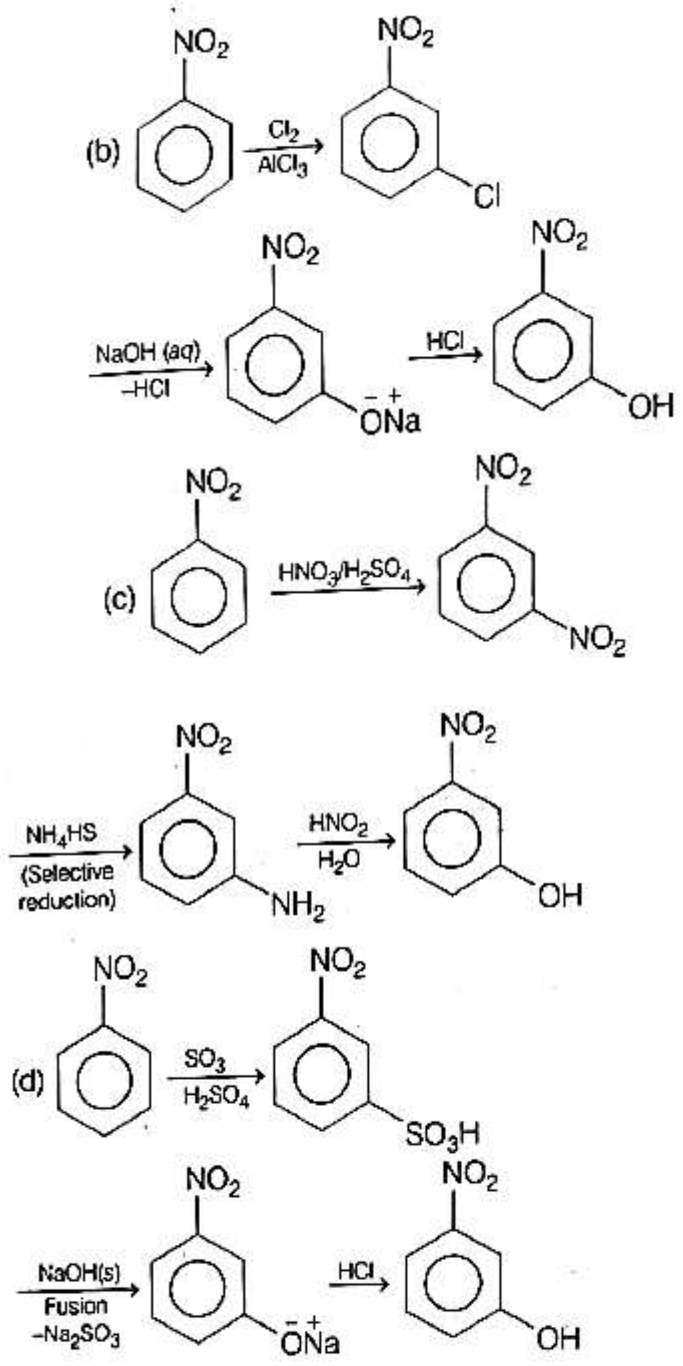


PART (B) : CHEMISTRY

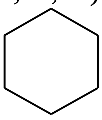
21. (A, B, C)
Li^+ ion being the smallest in size and has the highest charge/size ratio amongst the alkali metal ions, get much more hydrated (i.e. holds more water molecules in tis hydration sphere) than Na^+ ion and the latter gets more hydrated than K^+ ion and so on.

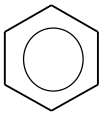
22. (A)
Reactant is $\beta - D$ glucopyranosyl, so the product formed will also be D-glucitol. In this reaction $NaBH_4$ reduces – CHO group into $-CH_2OH$

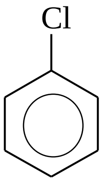
23. (BCD)

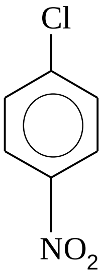


24. (A, B, C, D)

(A)  → Gives free radical substitution

(B)  → Gives S_E reaction

(C)  → Gives S_E reaction

(D)  → Gives substitution reaction

25. (A, B, C)

use $K_P = K_C RT^{\Delta n}$ to solve this problem

$Se_6(g) \rightleftharpoons 3Se_2(g)$

Initially	1	0
At equilibrium	[1 – x]	3x

Total pressure at equilibrium = 1 – x + 3x = 1 + 2x

Pressure of $Se_2\left(P_{se2}\right) = \frac{3x}{1+2x} \times p$

Pressure of $Se_6(p_{se_6}) = \frac{1-x}{1+2x} \times p$

$$Kp = \frac{Pse_2^3}{P_{Se_6}} = \frac{\left[\frac{3x}{1+2x} \times p\right]}{p_{se_6}}$$

=0.1687

Now, $K_p = K_C.RT^{\Delta n}$

$$K_C = \frac{K_P}{RT^{\Delta n}}$$

$$= \frac{0.1687}{(0.0821 \times 973)^2}$$

$$= 0.2645 \times 10^{-4}$$

26. (A, C, D)

Charge transfer spectrum The charge transfer spectrum is obtained due to transfer of charge from one position to another position as in case of $KMnO_4$ where charge on oxygen atoms transfer from O to metal. Metal accept electron density in its vacant d-orbital. In Agl, I^{Θ} transfers its electron to vacant d-orbital of Ag^+

27. (6)

SO_3, XeO_3, H_3PO_4, $ClO_4^{\ominus}, SO_4^{2-}$, $XeOF_2$ have $d\pi - p\pi$ bonding. This type of bonding is important in the compounds containing third (or higher) period elements (Si, P, S, Cl, etc). These vacant d-orbital from (d – p)π bonding when Si, P, S etc, are bonded with N, O, F which have lone pair electrons in their $p-$orbitals.

28. (11.36)

At 15 min, let α be the degree of dissociation of N_2O_5.

$$N_2O_5 \rightarrow 2NO_2 + \frac{1}{2}O_2$$

Initially	1	0	0
At 15 min	$1-\alpha$	2α	$\frac{\alpha}{2}$

Now,

$$\frac{\text{rate of effusion of } NO_2}{\text{rate of effusion of } O_2} = \frac{2\alpha}{\frac{\alpha}{2}}\sqrt{\frac{32}{46}}$$

$$= \left(\frac{\text{mole fraction of } NO_2}{\text{mole fraction } O_2}\right)_{\text{outside}}$$

$$\Rightarrow 4\sqrt{\frac{32}{46}} = \frac{0.66}{x_{O_2}}$$

$$x_{O_2} = \frac{0.66\sqrt{46}}{4\times\sqrt{32}} = 02$$

$\Rightarrow$Mole fraction of N2O5 in mixture collected outside after 15 min =0.14

$$\therefore \frac{r_{N_2O_5}}{f_{NO_2}} = \frac{1-\alpha}{2\alpha}\sqrt{\frac{46}{108}} = \frac{0.14}{0.66}$$

$$\Rightarrow \frac{1-\alpha}{2\alpha} = 0.33 \Rightarrow \alpha = 0.60$$

Now, applying first order kinetics

$$k \times 15 = \ln\frac{1}{1-0.6}$$

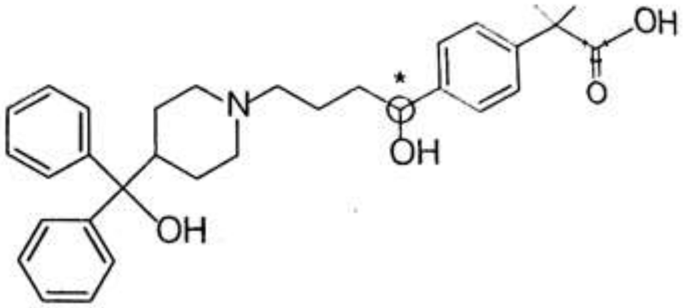
$$\Rightarrow k = \frac{1}{15}\ln\frac{5}{2}$$

$$\Rightarrow k = 0.061$$

$$\therefore t^{1/2} = \frac{\ln 2}{0.061} = 11.36\,\text{min}$$

29. (3)

Total number of chiral centres present in the given molecules are 2. (two)



Hence, the total number of stereoisomers = $2^2 = 4$

30. (-1.27)
From the combustion of benzoic acid, the heat capacity of calorimeter and its content C can be determined as"

$$\frac{3251}{122} \times 0.825 \times 10^3 J = 1.94 \times C$$

$\therefore C = 11332.07 JK^{-1}$

Heat produced in combustion of 0.727 g D-ribose = 0.910×11332.07

$= 10312.18 J$

$\Rightarrow$ Internal energy of combustion reaction is

$C_5H_{10}O_5(s) + 5O_2(g) \rightarrow 5CO_2(g) + 5H_2O(l)$

$\because \Delta n_g = 0,\ \Delta H = \Delta E$

$\Rightarrow -2.13 \times 10^6 = -5(394 + 286) \times 1000$

$-\Delta H_f^0(D - ribose)$

$\Rightarrow \Delta H_f^0(D - ribose) = -1.27 \times 10^6 J$

31. (0.05)
m mol of HCl dropped

$= \frac{1}{6} + 1.5 = 0.25$

$\Rightarrow$ mmol of Al reacted = $\frac{1}{3} \times$ mmol of HCl

Mass of Al reacted = $\frac{0.25}{3} \times 10^{-3} \times 27$

Volume of Al removed = $= \frac{0.25 \times 10^{-3}}{2.7 gcm^{-3}} g = 83.3 \times 10^{-4} cm^3$

Also $V = \pi r^2 \times$ thickness

$\Rightarrow r^2 = \frac{8.33 \times 10^{-4} cm^3}{3.14 \times 0.1 cm}$

32. (4)

Complex	EAN (Z – O. N +2 x C.N)
$K_3[Fe(CN_6]$ $[Ru(CO)_5]$	26 - + 12 = 35 44 – 0 + 10 = 54 (noble gas)
$[Cr(NH_3)_6]^{3+}$	24 – 3 + 12 = 35

$[CO(NH_3)_6]^{3+}$	27 – 3 + 12 = 36 (noble gas)
$[Ni(NH_3)_6]^{2+}$	28 – 2 + 12 = 38
$[Fe(CO)_5]$	26-0+10=36 (noble gas)
$[W(CO)_6]$	74-0+12=86 (noble gas)

33. (4)
The given compound X is twistane. It has four equivalent chiral centres. In the bridged ring compound, certain diastereomers cannot from due to steric reason. So, exists only in two enantiomeric form.

34. (0.79)

$$2Hg + 2Fe^{3+} \rightarrow Hg_2^{2+} + +2Fe^{2+}$$

At equili. Excess $\frac{10^{-3}\times 5}{100}$ $\quad\quad$ $\frac{10^{-3}\times 95}{100}$ $\frac{10^{-3}\times 95}{100}$

At equilibrium E_{cell} =0

$$\therefore 0 = E^0_{cell} - \frac{0.0591}{2}\log\frac{[Hg_2^{2+}][Fe^{2+}]^2}{[Fe^{3+}]^2}$$

$$\Rightarrow [E^0_{Hg/Hg^{2+}} + E^0_{Fe^{3+}/Fe^{2+}}]$$

$$-\frac{0.0591}{2}\log\frac{\left(\frac{10^{-3}\times 95}{2\times 100}\right)\left(\frac{10^{-3}\times 95}{100}\right)}{\left(\frac{10^{-3}\times 5}{100}\right)}$$

$$\Rightarrow E^0_{Hg/Hg_2^{2+}} = -0.77 + \frac{0.591}{2}\log\frac{(95)^3\times 10^{-6}}{25\times 2}$$

$$= (-0.77 + 0.226) = -0.7926V$$

$$\therefore E^0_{Hg_2^{2+}/Hg} = +0.793V$$

35. (4)
(i) (A) gives mononromoalkane $(B) \rightarrow (A)$ is alkene

(ii) Since $2gBr_2$ reacts completely with

$= 0.70g$ of (A)

$\therefore 160gBr_2$ reacts completely with

$$= \frac{0.70\times 160}{2}$$

$= 56g$ of (A)

$\therefore$ Molecular weight $= 56$

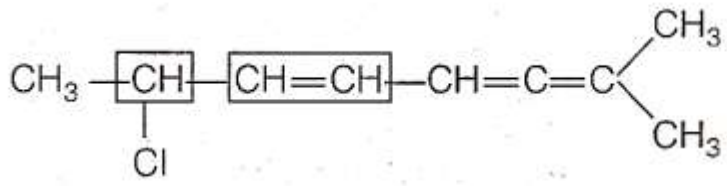
$C_nH_{2n} = 56$

(Since, compound is alkene)

$12n + 2n = 56$

$n = 4$

36. (4)

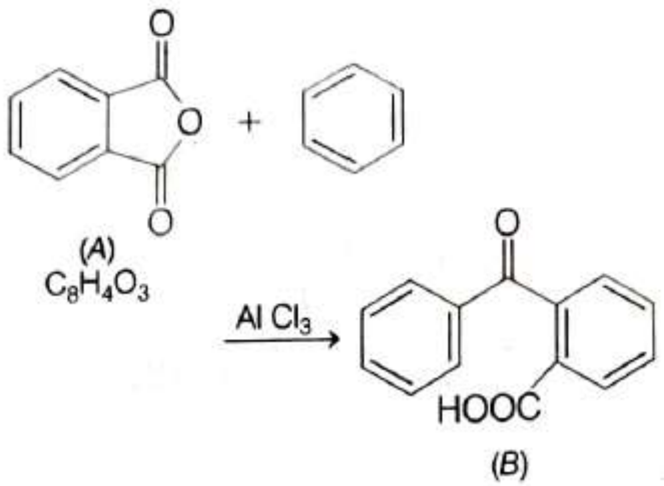


$(*) n = 2$

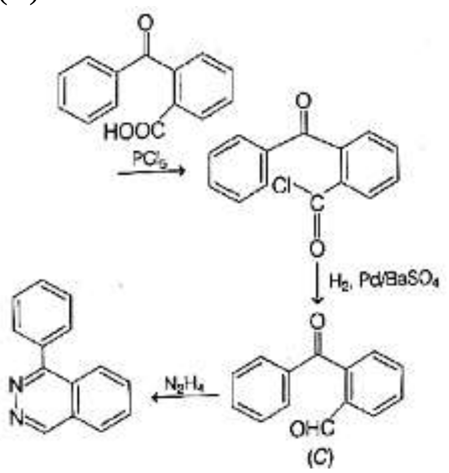
$(*)$ No symmetry

So, stereoisomer $= 2^n = 2^2 = 4$

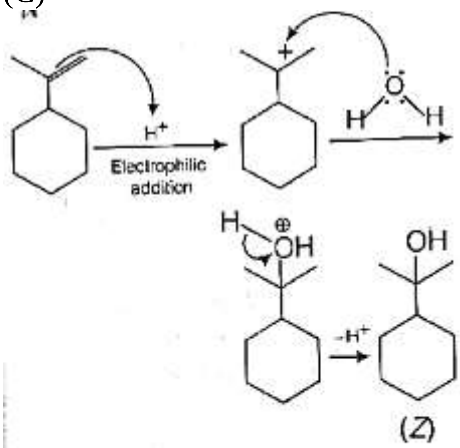
37. (C)



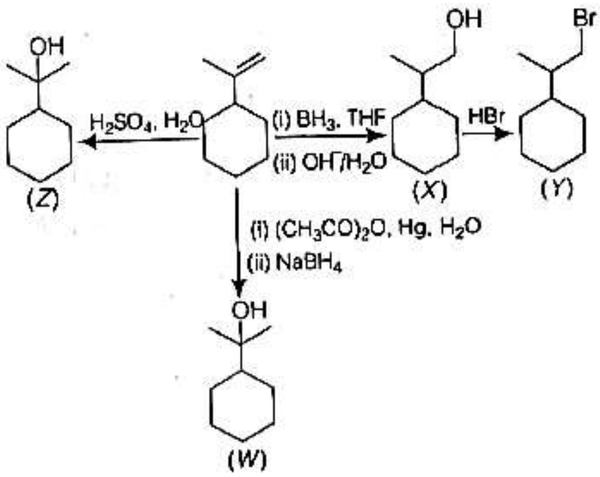
38. (C)



39. (C)


H+
Electrophilic
addition
(Z)

40. (C)


OH
Br
H2SO4, H2O
(i) BH3, THF
(ii) OH-/H2O
HBr
(Z)
(X)
(Y)
(i) (CH3CO)2O, Hg, H2O
(ii) NaBH4
(W)

PART (C) : MATHEMATICS

41. (A, B, C , D)

we have

$c_1 \equiv x^2 + y^2 + 2x + 4y - 20 = 0$

$c_2 \equiv x^2 + y^2 + 6x - 8y + 10 = 0$

Centre of $c_1 \equiv (-1, -2)$

And radius $r_1 = \sqrt{1+4+20} = 5$

Centre of $c_2 \equiv (-3, 4)$

And radius $r_2 = \sqrt{9+16-10} = \sqrt{15}$

Distance between the centre

$d = \sqrt{(-3,1)^2 + (4+2)^2}$

$= \sqrt{4+36} = \sqrt{40}$

$\therefore r_1 + r_2 > d > r_1 - r_2$

Hence, the circle intersect at two distinct point.

There are two common tangents

Also

$2g_1g_2 + 2f_1f_2 = 2(1)(3) + 2(2)(-4)$

= - 10

And $c_1 + c_2 = -20 + 10 = -10$

Thus, the two circles are orthogonal

Length of common tangent

$= \sqrt{d^2 - (r_1 - 2)^2}$

$= \sqrt{40 - (5 - \sqrt{15})^2} = 5\left(\frac{12}{5}\right)^{1/4}$

Length of common chord

$= \frac{2r_1r_2}{\sqrt{r_1^2 + r_2^2}} = 5\sqrt{\frac{3}{2}}$

42. (A, B, C)

We have

$f(x+y) = f(x) + f(y) + 3xy(x+y)$ ….(i)

On differentiating w.r.t. x y as constant

$f'(x+y) = f'(x) + 6xy + 3y^2$

Put x = 0

$f'(y) = f'(0) + 3y^2$

$f'(y) = 3y^2 - 4 \quad [\because f'(0) = 4]$

On integrating, we get

$f(x) = x^3 - 4x + c$

Now Put x = y = 0 in Eq. (i) we get

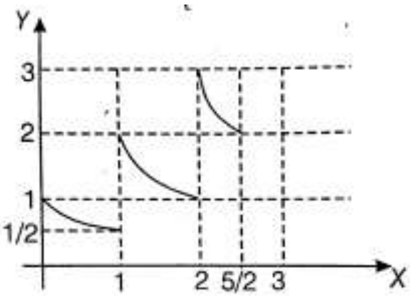
$f(0) = 0$

$f(0) = 0 + c \Rightarrow c = 0$

$\therefore f(x) = x^3 - 4x$
$f(x) = x(x+2)(x-2)$
$f(x) = 0$
$\Rightarrow x(x+2)(x-2) = 0$
$x = 0, -2, 2$
Hence three roots f(x) is passing through (2, 0) and (-1, 3)
$\sqrt{f(x)} = \sqrt{x^3 - 4x}$ is defined
$x^3 - 4x \ge 0$ or $x \in [-2, 0] \cup [2, \infty)$
$\therefore$ the domain of $\sqrt{f(x)}$ is $[-2, 0] \cup [2, \infty)$

43. (A,B, D)

$$f(x) = \begin{cases} \frac{1}{x+1}, & 0 \le x < 1 \\ \frac{2}{x}, & 1 \le x < 2 \\ \frac{3}{x-1} & 2 \le x < 5/2 \end{cases}$$



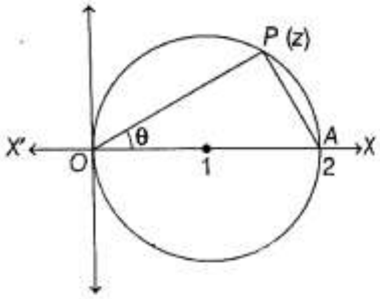
From the graph, f(x) is discontinuous and bijective function. It is also not differentaibale
Hence options a and b are correct

$\lim_{x\to 1^-} f(x) = \frac{1}{2}, \lim_{x\to 1^+} f(x) = 2$
f(1) = 2
$\therefore \min\left(\lim_{x\to 1^-} f(x) \ \lim_{x\to 1^+} f(x) \ne f(1)\right)$
Maximum points of discontinuity = 2

44. (A, B, D)

Since arg $\left(\frac{z-1-i}{z}\right)$ is the angle subtended by the chord joining the points O and 1 + i at the circumcentre of the circle |z -1 | = 1

So art $\arg\left(\frac{z-1-i}{z}\right) = -\frac{\pi}{4}$



$\arg\left(\frac{z-2}{z}\right)=\pm\frac{\pi}{2}$

$\therefore \frac{z-2}{z}$ is purely imaginary

We have $\angle OPA=\frac{\pi}{2}$

$\therefore \arg\left(\frac{z-2}{z-0}\right)=\frac{\pi}{2}\Rightarrow\frac{z-2}{z}=\frac{AP}{OP}i$

Now in ΔOAP

$\tan\theta=\frac{AP}{OP}$

$\therefore \frac{z-2}{z}=i\tan\theta$

45. (B)

We have $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ ……..(i)

And $(x-1)^2+y^2=1$ ……(ii)

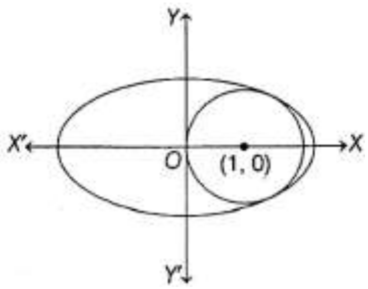
Solving both the equation, we have

$\frac{x^2}{a^2}+\frac{1-(x-1)^2}{b^2}=1$

$\Rightarrow(b^2-a^2)x^2+2a^2x-a^2b^2=0$

For least area, the circle must touch ellipse

Therefore, D= 0



$4a^4+4a^2b^2(b^2-a^2)=0$

$\Rightarrow a^2+b^2(b^2-a^2)=0$

$\Rightarrow a^2+b^2(-a^2e^2)=0$

$\Rightarrow b=\frac{1}{e}$

Also $a^2 = \frac{b^2}{1-e^2} - \frac{1}{e^2(1-e^2)}$

$\Rightarrow a = \frac{1}{e\sqrt{1-e^2}}$

Let S be area of the ellipse.Then,

$S = \pi ab = \frac{\pi}{e^2\sqrt{1-e^2}}$

$= \frac{\pi}{\sqrt{e^4 - e^6}}$

The area is minimum if f(e) = $e^4 - e^6$ is maximum when

$f'(e) = 4e^3 - 6e^5 = 0$

$\Rightarrow e = \sqrt{2/3}$

So, S is the least when = e= $\sqrt{2/3}$

Therefore the ellipse s

$2x^2 + 6y^2 = 9$

The equation of auxiliary equation is

$x^2 + y^2 = 9/2$

Length of latusrectum of ellipse

$= \frac{2b^2}{a} = \frac{2\times 3/2}{3/2} = \sqrt{2}$

Foci of ellipse $(\pm\sqrt{3}, 0)$

46. (BD)

At $x = 1 \quad y = 1$

$\therefore \ -1 = 1 + p + q$

$p + q = -2$

Vertex $\left(-\frac{p}{2}, \frac{-\left(p^2 - 4q\right)}{4}\right)$

$\left|\frac{4q - p^2}{4}\right| = \left|\frac{p^2 + 4q + 8}{4}\right|_{min}$

At $p = -2 \qquad q = 0$

Distance = 1

47. (1010)

Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$\therefore A^{-1} = \frac{1}{ad - bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$ ....(i)

$A^{-2} = A^{-1} \times A^{-1} = \frac{1}{(ad - bc)^2}$

$$\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

$$\Rightarrow A^{-2} = \frac{1}{(ad-bc)^2}\begin{bmatrix} d^2+bc & -bd-ba \\ -cd-ac & a^2+bc \end{bmatrix} \qquad .....(ii)$$

$$\Rightarrow tr(A^{-1}) = \frac{a+d}{ad-bc}, tr(A^{-2})$$

$$= \frac{a^2+2bc+d^2}{(ad-bc)^2}$$

Now $(tr(A^{-1}))^2 - tr(A^{-2})$

$$= \frac{(a+d)^2}{(ad-bc)^2} - \frac{a^2+2bc+d^2}{(ad-bc)^2}$$

$$= \frac{a^2+2ad+d^2-a^2-2bc+d^2}{(ad-bc)^2}$$

$$= \frac{2(ad-bc)}{(ad-bc)^2} = \frac{2}{ad-bc}$$

$$\frac{2}{|A|} \quad [\because |A| = ad-bc]$$

$$= \frac{2}{2018} \qquad [\because |A| = 2018]$$

$$= \frac{1}{1009}$$

$\therefore m+n = 1+1009 = 1010$

48. (0)

We have f(x) = $f(x) = \int_{-1}^{x}\sqrt{4-t^2}dt$ and $g(x) = \int_{x}^{1}\sqrt{4+t^2}dt$

Let h(x) = f(x) . g(x)

$\therefore h'(x) = (f(x).g(x))' = f(x)g'(x) + f'(x)g.(x)$

$$\Rightarrow h'(x) = -\int_{-1}^{x}\sqrt{4+t^2}dt\sqrt{4+x^2} + \sqrt{4+x^2}\int_{x}^{1}\sqrt{4+t^2}dt$$

$$\Rightarrow h'(x) = \sqrt{4+x^2}\left[-\int_{-1}^{x}\sqrt{4+t^2}dt + \int_{x}^{1}\sqrt{4+t^2}dt\right]$$

$$\Rightarrow h'(0) = \sqrt{4+0}\left[\int_{0}^{1}\sqrt{4+t^2}dt - \int_{-1}^{0}\sqrt{4+t^2}dt\right]$$

$$\Rightarrow h'(0) = 2\left[\int_{0}^{1}\sqrt{4+t^2}dt - \int_{-1}^{0}\sqrt{4+t^2}dt\right]$$

$$\Rightarrow h'(0) = 2\left[\int_{0}^{1}4+t^2dt + \sqrt{4+y^2}dy\right]$$

Where t = $-y \Rightarrow h'(0) = 0$

49. (4)

We have $|\vec{a}| = |\vec{b}| = |\vec{c}| = \frac{1}{2}$

$\vec{a}.\vec{b} = \vec{b}.\vec{c} = \vec{c}.\vec{a} = \frac{1}{2}$

Also $(\vec{a}\times\vec{b}) + (\vec{b}\times\vec{c}) = p\vec{a} + q\vec{b} + r\vec{c}$

$\Rightarrow \vec{a}.(\vec{b}\times\vec{c}) = p + q(\vec{a}.\vec{b}) + r(\vec{a}.\vec{c})$

$\Rightarrow [\vec{a}\,\vec{b}\,\vec{c}] = p + \frac{q}{2} + \frac{2r}{2}$ ......(ii)

Similarly taking dot product with vector $\vec{b}$ we get

$[\vec{a}\ \vec{b}\ \vec{c}] = \frac{p}{2} + \frac{q}{2} + r$ ......(iii)

Solving equs. (i) (ii) and (iii) we get

P = r = -q

$\therefore \frac{p^2 + 2q^2 + r^2}{q^2} = \frac{q^2 + 2q^2 + q^2}{q^2} = 4$

50. (409)

Let N = $\left(81^{\frac{1}{\log_5 9}} + (3)^{\frac{3}{\log_{\sqrt{6}} 3}}\right)\left(\left(\sqrt{7}\right)^{\frac{2}{\log_{25} 7}} - (125)^{\log_{25} 6}\right)$

$\Rightarrow N = ((3^4)^{\log_9 5} + 3^{\log_3}\left(\sqrt{6}\right)^3\left(7^{\log_7 25} - 5^{3\log_5 2^6}\right)$

$\Rightarrow N = \left(3^{\log_3 25} + 3^{\log_3 6\sqrt{6}}\right)\left(25 - 6\sqrt{6}\right)$

$\Rightarrow N = \left(25 + 6\sqrt{6}\right)\left(25 - 6\sqrt{6}\right)$

$\Rightarrow N = 625 - 216 = 409$

51. (0)

If $0 < x < \frac{\pi}{2}$, $\int_{-2}^{x} |\cos x|\,dx = \int_{-2}^{-\pi/2} |\cos x|\,dx + \int_{-\pi/2}^{x} |\cos x|\,dx$

$= \int_{-2}^{-\pi/2} -\cos x dx + \int_{-\pi/2}^{x} \cos x dx = \int_{-2}^{x} |\cos x|\,dx = 0$

$\Rightarrow |-\sin x|_{-2}^{-\pi/2} + |-\sin x|_{-\frac{\pi}{2}}^{x} = 0$

$\Rightarrow 1 - \sin 2 + \sin x + 1 = 0$

$\Rightarrow 2 - \sin 2 + \sin x = 0$

$\Rightarrow \sin x = \sin 2 - < -1$ not possible

$\therefore$ No solution exist in $\left(0, \frac{\pi}{2}\right)$

So number of solution =0

52. (30)

$p = z\bar{z} + (z-3)(\bar{z}-3) + (z-6i)(\bar{z}+6i)$

$= 3z\bar{z} - 3(z+\bar{z}) + 9 + 6(z-\bar{z})i + 36$

$= 3(x^2+y^2) - 3(2x) + 9 + 6(2iy)i + 36 \left\{\begin{array}{l} \text{let } z = iy \\ \text{then } \bar{z} = x - iy \\ z + \bar{z} = 2x \\ z - \bar{z} = 2iy \end{array}\right\}$

$=3(x^2 + y^2) - 6x + 9 - 12y + 36$

$= 3[x^2 + y^2 - 2x - 4y + 15]$

$= 3[(x-1)^2 (y-2)^2 + 10]$

For minimum value of p, x = 1, y = 2.

Minimum value of p = 3(10) = 30

53. (23)

$a_1, a_2, a_3, \ldots . a_n$ is an odd number not divisible by a prime greater than 5. So, a_i(I = 1,2,3,…..n) can be written as $a_i = 3^a 5^b$ where a, b are non-negative integer.

Thus, for all $n \in N$

$$\frac{1}{a_1} + \frac{1}{a_2} + \frac{1}{a_3} + \ldots . + \frac{1}{a_n} < \left(1 + \frac{1}{3} + \frac{1}{3^2}\right)\left(1 + \frac{1}{5} + \frac{1}{5^2} \ldots .\right)$$

$$\Rightarrow \frac{1}{a_1} + \frac{1}{a_2} + \frac{1}{a_3} + \ldots . + \frac{1}{a_n} < \left(\frac{1}{1-\frac{1}{3}}\right)\left(\frac{1}{1-\frac{1}{5}}\right)$$

$$= \frac{15}{8}$$

Hence $\frac{1}{a_1} + \frac{1}{a_2} + \frac{1}{a_3} \ldots . + \frac{1}{a_n}$ is less than $\frac{15}{8}$

$\Rightarrow m + n = 23$

54. (6)

Equation of plane containing the lines

$\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-2}{3} = \frac{y-3}{4} = \frac{z-4}{5}$ is $\begin{vmatrix} x-1 & y-2 & z-3 \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{vmatrix} = 0$

$\Rightarrow (x-1)(15-16) - (y-2)(10-12) + z(z-3)(8-9) = 0$

$\Rightarrow -(x-1) + 2(y-2) - (z-3) = 0$

$\Rightarrow x - 2y + z = 0$ or $\frac{|d|}{\sqrt{6}} = \sqrt{6}$

$\Rightarrow |d| = 6$

55. (96)

We have $I = \lim_{n\to\infty} \left(n^{-\frac{3}{2}}\right) \sum_{r=1}^{6n} \sqrt{r}$

$$= \lim_{n\to\infty} \frac{\sqrt{1} + \sqrt{2} + \sqrt{3} + \ldots . + \sqrt{6n}}{n\sqrt{n}}$$

$$\lim_{n\to\infty}\frac{1}{n}\sum_{r=1}^{6n}\sqrt{\frac{r}{n}}$$

$$=\int_0^6\sqrt{x}\,dx=\left[\frac{2}{3}x^{\frac{3}{2}}\right]_0^6=\frac{2}{3}6\sqrt{6}$$

$$=\sqrt{96}\Rightarrow\lambda=96$$

56. (0)

$$x^2f(x)-2f\left(\frac{1}{x}\right)=g(x) \qquad .....(i)$$

Replacing x by $\frac{1}{x}$, we have

$$\frac{1}{x^2}f\left(\frac{1}{x}\right)-2f(x)=g\left(\frac{1}{x}\right)$$

Multiplying by $2x^2$

$$2f\left(\frac{1}{x}\right)-4x^2f(x)=2x^2g\left(\frac{1}{x}\right) \qquad ...(ii)$$

Adding Eqs. (i) and (ii), we get

$$-3x^2f(x)=g(x)+2x^2g\left(\frac{1}{x}\right)$$

$$\Rightarrow f(x)=-\left[\frac{g(x)+2x^2g\left(\frac{1}{x}\right)}{3x^2}\right]$$

Now, $f(-x)=-\left[\frac{g(-x)+2x^2g(-1/x)}{3x^2}\right]$

$$=\left[\frac{g(x)+2x^2g(1/x)}{3x^2}\right]$$

$\therefore f(x)=-f(-x)$

f(x) is an odd function

But f(x) is given to be an even function $\therefore f(x)=0\,\forall x\Rightarrow f(5)=0$

57. (B)

For any complex number

$z=x+ly, x, y\in R$ is the arg $z=\tan^{-1}\left(\frac{y}{x}\right)$ always give the principal value

We know that arg $(\bar{z})=-\arg(z)$

$\arg(-z)-\arg(z)=\arg(-z)+\arg(-\bar{z})=-\pi \text{ as } \arg(z)>0$

58. (B)

$3\lambda_1=\pi$ and $2\lambda_2=-\pi$

$\therefore\ \lambda_1=\frac{\pi}{3}\ \lambda_2=-\frac{\pi}{2}$

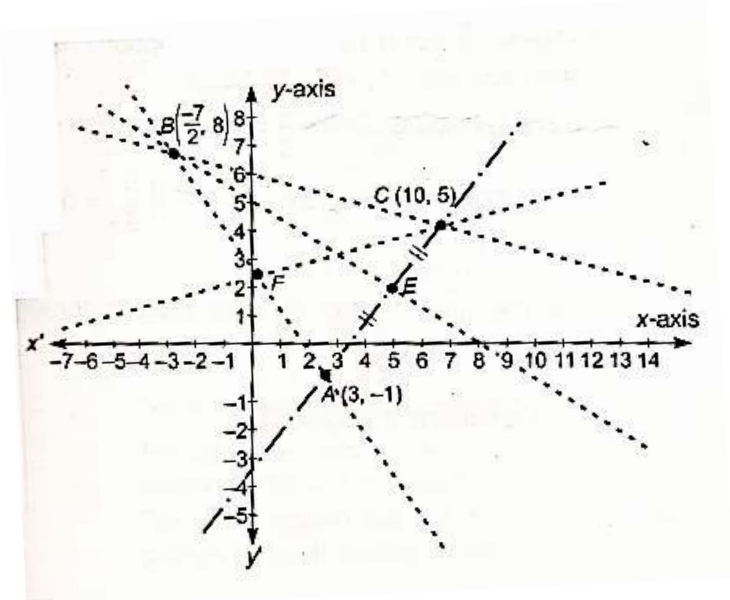
59. (D)

Slope of $BC = \frac{5-8}{\left(10+\frac{7}{2}\right)} = \frac{-2}{9}$ $\left[\because B = \left(\frac{-7}{2}, 8\right) \text{ and } C(10,5)\right]$

Hence, the acute angle made by BC with positive x-axis is

$\tan^{-1}\left(-\frac{2}{9}\right) = \pi - \tan^{-1}\left(\frac{2}{9}\right)$

60. (C)



$$y+1 = \frac{8+1}{\left(\frac{-7}{2}-3\right)}(x-3)$$

$$\Rightarrow y+1 = \frac{-18}{13}(x-3)$$

$$\therefore 18x+13y = 41$$