IIT – JEE: 2023 **AITS-2 (ADVANCED)** **DATE: 26/03/2023**

PAPER – I

Que.	1	2	3	4	5	6	7	8	9	10
Ans.	C	C	B	A	B	BC	ABC	ABD	ACD	BD
Que.	11	12	13	14	15	16	17	18	19	20
Ans.	AC	BC	ABC	2	4	2	6	3	C	C
Que.	21	22	23	24	25	26	27	28	29	30
Ans.	D	B	A	ABCD	ACD	BCD	AC	CD	ABCD	ABC
Que.	31	32	33	34	35	36	37	38	39	40
Ans.	AB	5	6	5	6	5	A	B	C	A
Que.	41	42	43	44	45	46	47	48	49	50
Ans.	A	ABCD	AB	B	ABC	ABCD	ACD	ABD	AB	5
Que.	51	52	53	54						
Ans.	9	5	1	0						

PAPER – II

Que.	1	2	3	4	5	6	7	8	9	10
Ans.	A	A	C	D	C	B	BC	BD	ABCD	CD
Que.	11	12	13	14	15	16	17	18	19	20
Ans.	BC	BC	AD	CD	C	C	C	C	C	A
Que.	21	22	23	24	25	26	27	28	29	30
Ans.	A	B	A	A	BCD	AC	BD	ACD	B	A
Que.	31	32	33	34	35	36	37	38	39	40
Ans.	ABD	ABC	B	A	D	A	A	A	A	A
Que.	41	42	43	44	45	46	47	48	49	50
Ans.	B	B	AC	ABCD	AB	ABC	ABCD	BCD	BD	ABD
Que.	51	52	53	54						
Ans.	B	C	D	C						

Note : **Detailed solution to this test is available Today after 05.00 pm on our website.:www.iitianspace.com**

SOLUTION

1. **(C)**

Least count $= \dfrac{0.5}{50} = 0.01\ mm$

Zero error = 5 × L.C.
= 5 × 0.01 mm
= 0.05 mm

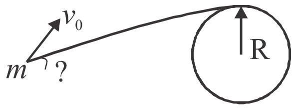
Diameter of ball = [Reading on main scale] + [Reading on circular scale × L.C.] - Zero error
= 0.5 × 2 + 25 0.01 - 0.05
= 1.20 mm

2. **(C)**

Mg = n m v (1 + e)

3. **(B)**

Let the speed of the instrument package is v when it grazes the surface of the planet.



Conserving angular momentum of the package about the centre of the planet

$mv_0 \times 5R \sin(\pi - \theta) = mvR \sin 90^o$

$\Rightarrow v = 5v_0 \sin\theta$ ....(1)

Conserving mechanical energy

$$-\frac{GMm}{5R} + \frac{1}{2}mv_0^2 = -\frac{GMm}{R} + \frac{1}{2}mv^2$$

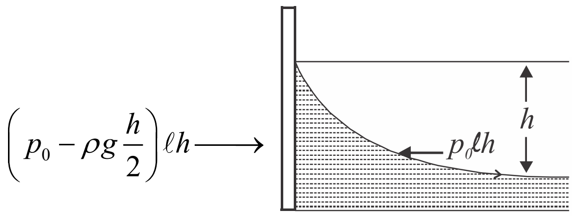
$$\Rightarrow \frac{1}{2}m(v^2 - v_0^2) = \frac{4GMm}{5R} \Rightarrow v^2 - v_0^2 = \frac{8GM}{5R} \quad ....(2)$$

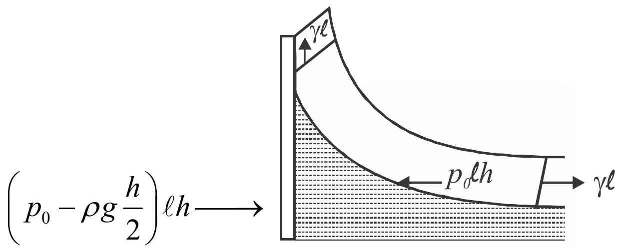
Substituting the value of V from eq. (1) in equation (2)

$$25v_0^2 \sin^2\theta - v_0^2 = \frac{8GM}{5R}$$

$$\theta = \sin^{-1}\left(\frac{1}{5}1 + \frac{8GM}{5v_0^2 R}\right)$$

4. **(A)**





Balancing forces in horizontal direction

$$\left(p_0 - \rho g\frac{h}{2}\right)\ell h + \lambda\ell - p_0\ell h \Rightarrow h = \frac{2\gamma}{\rho g}$$

5. **(B)**

After two seconds pulses will overlap each other.
According to superposition principle the string will not have any distortion and will be straight.
Hence there will be no P.E. The total energy will be only kinetic.

6. **(BC)**

There is a decrease in volume during melting of an ice slab at 273 K. Therefore, negative work is done by ice-water system on the atmosphere of positive work is done on the ice-water system by the atmosphere. Hence, option (b) is correct
Secondly heat is absorbed during melting (i.e. dQ is positive) and as we have seen, work done by ice-water system is negative (dW is negative.) Therefore, from first law of thermodynamics
dU = dQ - dW
change in internal energy of ice-water, dU will be positive or internal energy will increase.

7. **(ABC)**

We shall write the condition of the equality to zro of the potential of the sphere, and hence of any point inside it (in particular, its centre), by the moment of time t. We shall single out three time intervals :

a) $(1) t < \frac{a}{v},\ (2) \frac{a}{v} \le t < \frac{b}{v},\ \ (3) t \ge \frac{b}{v}$

Denoting the charge of the sphere q(t), we obtain the following expression for an instant t from the first time interval : $\frac{q_1}{a} + \frac{q_2}{b} + \frac{q(1)}{vt} = 0$

where q(1) = $-v\left(\frac{q_1}{a} + \frac{q_2}{b}\right)$

$I_1(t) = -v\left(\frac{q_1}{a} + \frac{q_2}{b}\right)$

For an instant t from, the second time interval, we find that the fields inside na outside the sphere are independent, and hence

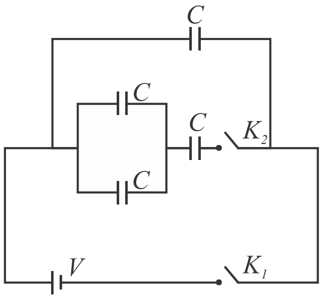
$$\frac{q(t) + q_1}{vt} = \frac{q_2}{b}, I_2(t) = -v\frac{q_2}{b}$$

Finally, as soon as the sphere absorbs the two point charges q_1 and q_2, the current will stop flowing through the "earthing" conductor, and we can write $I_3(t) = 0$. Thus,

$$I(t) = \begin{cases} -v\left(\frac{q_1}{a} + \frac{q_2}{a}\right), t < \frac{a}{v} \\ -v\frac{q_2}{b}, \qquad \frac{a}{v} < t < \frac{b}{v} \\ 0, \qquad t \ge \frac{b}{v} \end{cases}$$

8. **(ABD)**

Equivalanet circuit



When K_1 is closed, energy stored $= \frac{1}{2}CV^2$

When K_2 is also closed,

Equivalent capacitance $= \frac{5C}{3}$

When K_2 is closed, total energy stored

$$= \frac{1}{2}\frac{5C}{3}V^2 = \frac{5CV_2}{6}$$

Change in process 2 = $\frac{5CV^2}{6} - \frac{1}{2}CV^2 = \frac{1}{3}CV^2$

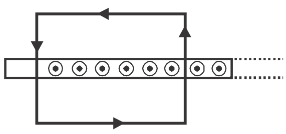
Change in process $1 = \frac{1}{2}CV^2$

9. **(ACD)**

Across all, we will get a balanced wheastone bridge and use series and parallel combination.

10. **(BD)**

Current flowing per unit length $= \sigma\frac{dx}{dt} = \sigma v$



By Ampere`s law

$$B\ell + B\ell = \mu_0\sigma v\ell \Rightarrow B = \frac{\mu_0\sigma v}{2}$$

11. **(AC)**

12. **(BC)**

Effective circuit is shown in the figure :

Applying Kirchhof loop law equations, we get :

$$L_1\frac{di_1}{di} - M\frac{di_2}{di} = \varepsilon$$

$$L_2\frac{di_2}{di} - M\frac{di_1}{di} = \varepsilon$$

Solving these we get

$$\frac{di_2}{di} = \frac{(M+L_1)\varepsilon}{L_1L_2 - M^2}$$

$$\frac{di_1}{di} = \frac{(M+L_2)\varepsilon}{L_1L_2 - M^2}$$

Solving these we get :

$$\Rightarrow \quad \frac{di}{di} = \frac{(M+L_1)\varepsilon}{L_1L_2 - M^2} = \frac{(L_1+L_2+2M)\varepsilon}{L_1L_2 - M^2}$$

$$\varepsilon = L_{eff}\frac{di}{di} = \left(\frac{(L_1L_2 - 2M^2)}{L_1+L_2+2M}\right)\frac{di}{di}$$

$$\varepsilon = L_{eff}\frac{di}{di} = \left(\frac{(L_1L_2 - 2M^2)}{L_1+L_2+2M}\right)\frac{di}{di}$$

$$\Rightarrow \quad L_{eff} = \frac{L_1L_2 - M^2)}{L_1+L_2+2M}$$

If M is negligible

$$L_{eff} = \frac{L_1L_2}{L_1+L_2}$$

13. **(ABC)**

n for liquid - n for Glass/yellow light but n for liquid < n for glass (red light) devided toward base

14. **(2)**

The light entering the rod does not emerge from the curved surface of the rod when the angle 90-r is greater than the critical angle.

i.e. $\mu \le \frac{1}{\sin C}$ where C is the critical angle.

Here C = 90 - r

$$\Rightarrow \mu \le \frac{1}{\sin(90-r)} \Rightarrow \mu \le \frac{1}{\cos\ r}$$

As a limiting case $\mu = \frac{1}{\cos r}$ .... (i)

Applying Snell`s law at A

$$\mu = \frac{\sin\alpha}{\sin r} \Rightarrow \sin r = \frac{\sin\alpha}{\mu} \quad \text{....... (ii)}$$

The smallest angle of incident on the curved surface is when $\alpha = \frac{\pi}{2}$.

This can be taken as a limiting case for angle of incidence on plane surface.

From (ii),

$$\sin r = \frac{\sin\pi/2}{\mu} \Rightarrow \mu = \frac{1}{\sin r} \quad \text{....... (iii)}$$

From (i) and (ii) sin r = cos r

$$\Rightarrow r = 45^o$$

$$\Rightarrow \mu = \frac{1}{\cos 45^o} = \frac{1}{1/\sqrt{2}}$$

$$\Rightarrow \mu = \sqrt{2} = 1.41$$

This is the least value of the refractive index of rod for light entering the rod and not leaving it from the curved surface.

15. **(4)**

$$\frac{I_{min}}{I_{max}} = \left(\frac{I - \sqrt{0.36I}}{I + \sqrt{0.36I}}\right)^2 = \left(\frac{0.4}{1.6}\right)^2 = \frac{1}{16}$$

[∵ If intensity of light falling on P directly from S is I, then the intensity of light falling at P after reflection from AB is 0.36 I]

16. **(2)**

Let e^- in hydrogen atom is excited to n^{th} level.

$$\therefore\ E_{KE(n-1)} = 8\,|E_{P.E.(n)}|$$

$$\therefore\ 13.6eV = 8\left|2\times\frac{13.6}{n^2}eV\right| \quad \Rightarrow n = 4$$

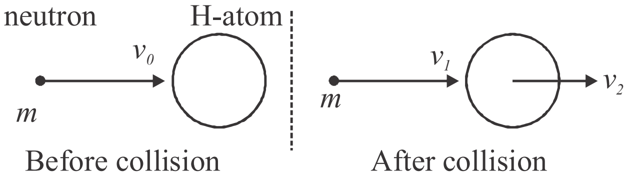
$$\therefore\ E = 13.6\left(1-\frac{1}{16}\right) = \frac{15}{16}\times 13.6eV = 12.75eV$$

Using conservation of linear momentum

$mv = mv_1 + mv_2$ ......(1)

$(v_2 - v_1) = \frac{1}{2}v$ ......(2)

$\therefore\ v_1 = \frac{v}{4}, v_2 = \frac{3v}{4}$ ......(3)



By energy conservation

$$\therefore\ \frac{1}{2}mv^2 = \frac{1}{2}mv_1^2 + \frac{1}{2}mv_2^2 + \Delta E$$

$$\Rightarrow\ \frac{1}{2}mv^2\left(1-\frac{1}{16}-\frac{9}{16}\right) = \Delta E$$

$$\Rightarrow\ \frac{1}{2}mv^2 = \frac{16}{6}\Delta E = \frac{8}{3}\times 12.75eV = 34.eV$$

K.E. of neutron = 34 eV.

17. **(6)**

According to conservation of energy

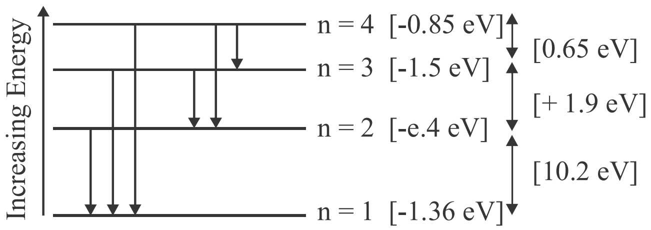
$$\frac{hc}{\lambda_1} + \frac{hc}{\lambda_2} = RhZ^2\left[\frac{1}{1^2} - \frac{1}{n^2}\right]$$

$$\frac{1}{\lambda_1}+\frac{1}{\lambda_2}=RZ^2\left[1-\frac{1}{n^2}\right]$$

$$\frac{1}{n^2}=1-\frac{\lambda_1+\lambda_2}{\lambda_1\lambda_2}\times\frac{1}{RZ^2}$$

18. **(3)**

The transition state of six different photon energies are shown.



Since after absorbing monochromatic light, some of the emitted photons have energy more and some have less than 2.7 eV, this indicated that the excited level B is n = 2. (This is because if n = 3 is the excited level then energy less than 2.7 eV is not possible).

For hydrogen like atoms we have

$$E_n=\frac{-13.6}{n^2}Z^2\ eV/atom$$

$$E_4-E_2\frac{-13.6}{16}Z^2 - \left(\frac{-13.6}{4}\right)Z^2=2.7$$

$$\Rightarrow Z^2\times13.6\left[\frac{1}{4}-\frac{1}{16}\right]=2.7$$

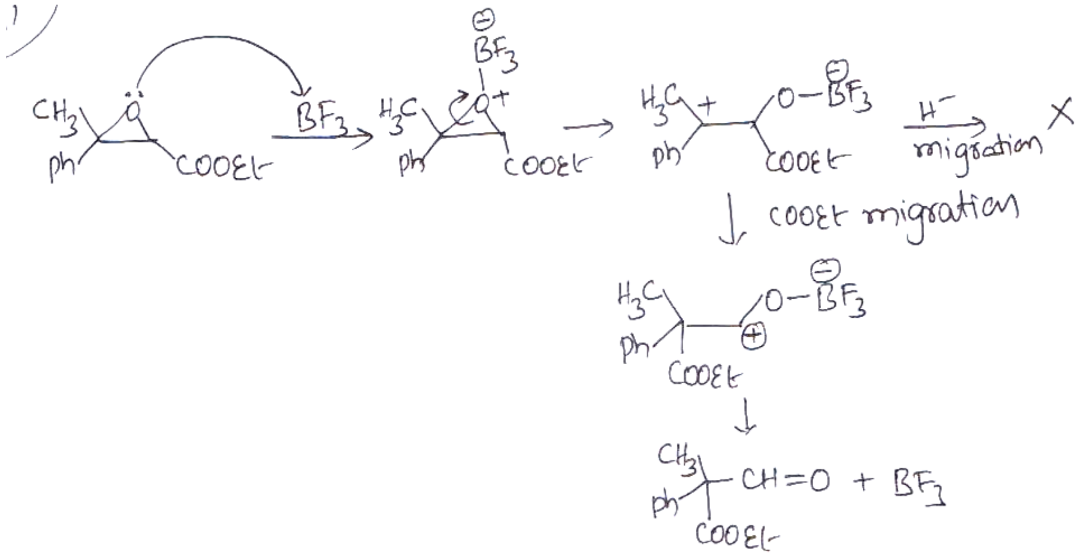
$$\Rightarrow Z^2=\frac{2.7}{13.6}\times\frac{4\times16}{12}$$

$$\Rightarrow I.E.=13.6Z^2\left(\frac{1}{1^2}-\frac{1}{\infty^2}\right)$$

$$=13.6\ \times\ \frac{2.7}{13.6}\times\frac{4\times16}{12}=14.5\text{ eV}$$

SOLUTION

19. **(C)**



20. **(C)**

$\Delta G = \Delta H - T\Delta S$

$= -nFE$

$= -2 \times F \times (1.101832)$

$= -19635\,KJ$

$\Delta S = nF\left(\frac{dE}{dT}\right)_p$

$= -9.65\ JK^{-1}$

$\Delta H = -19.6508 - T(-9.65)$

$(= -199384)$

21. **(D)**

Conceptual

22. **(B)**

23. **(A)**

$\frac{\sqrt{3}a}{4}$

24. **(ABCD)**

(A) Hemiacetal formation followed by esterification

(B) Intramolecular benzoin condensation

(C) Application of aldol condensation

(D) Acetal formation

25. **(ACD)**
(A) Free radical mechanism
(B) $ArSn^1$
(C) Free radical mechanism
(D) Free radical mechanism

26. **(BCD)**
Conceptual

27. **(AC)**
Conceptual

28. **(CD)**
Peroxy oxygen is changed to molecular oxygen. So Bond length ↓ Diamagnetic (Peroxide) changes to Paramagnetic oxygen.

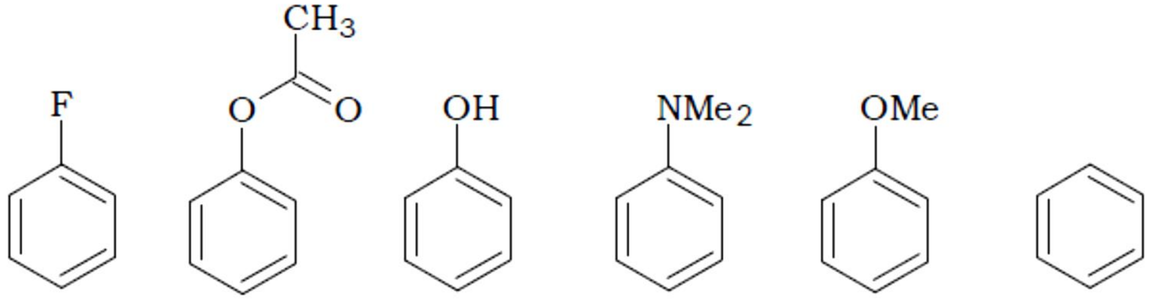
29. **(ABCD)**
Conceptual

30. **(ABC)**
Conceptual

31. **(AB)**
In B_2H_6 boron is in sp^3 hybridization. In the formation of H_3BO_3 with water and BCl_3 with chlorine there is change in hybridization of boron from sp^3 *to* sp^2. In the formation $\left[BH_2\left(NH_3\right)_2\right]^+\left[BH_4\right]^-$ and $NaBH_4$ while reacting with NH_3 at 120^0 and NaH there is no change in hybridization of boron

32. **(5)**
Unit cell is fcc with half of tetrahedral voids are occupied by C-atoms
Hence effective no. is $1+2\left(\frac{1}{8}\right)=\frac{5}{4}$

33. **(6)**



34. **(5)**

Only primary Amines can be prepared and R-X should give SN_2 so no vinyl and arylhalide can be used.

35. **(6)**

$i = 1.25$

Original mole fraction $= \frac{1}{n} = \frac{1}{1+(n-1)}$

Now $\frac{1.25}{1.25+(n-1)} = \frac{1}{5}$

$6.25 = 1.25 + (n-1)$

$n = 6$

36. **(5)**

In $FeS_2\ (-1)$, In $K_2S_2O_3\ (+2)$, In $Na_2S_2O_4\ (+3)$, In $S_2Cl_2\ (+1)$

SOLUTION

37. **(A)**

$$f'(x)=\frac{f(x)}{6f(x)-x}$$

Now, $I=\int\frac{2x\left(x-6f(x)\right)+f(x)}{\left(6f(x)-x\right)\left(x^2-f(x)\right)^2}dx$

$$\Rightarrow I=-\int\frac{2x-f('x)}{\left(x^2-f(x)\right)^2}dx=\frac{1}{x^2-f(x)}+C$$

38. **(B)**

$$X+iy=\frac{a_r+2i}{(a_r)^2+4}$$

$$X=\frac{a_r}{a_r^2+4},Y=\frac{2}{a_r^2+4}$$

$$X^2+Y^2=\frac{1}{2}Y$$

$$X^2+Y^2-\frac{1}{2}Y=0$$

So all points lie on circle $x^2+y^2-\frac{y}{2}=0$

$$r=\frac{1}{4}$$

Area of regular octagon = $8\left(\frac{1}{2}\times r^2\times\sin\left(\frac{2\pi}{8}\right)\right)=8\times\frac{1}{2}\times\frac{1}{16}\times\frac{1}{\sqrt{2}}=\frac{1}{4\sqrt{2}}$

39. **(C)**

$$A^3-A^2B=B^3-B^2A$$

$$A^2(A-B)=B^2(B-A)$$

$$A^2(A-B)+B^2(A-B)=0$$

$$\left(A^2+B^2\right)(A-B)=0$$

Let $\left|A^2+B^2\right|\neq 0$

$$\left(A^2+B^2\right)^{-1}\left(A^2+B^2\right)(A-B)=0\quad A=B$$

But A and B are different matrices. So our assumption is wrong.

40. **(A)**

$$\int_0^x\left(f(t)\right)^3dt=\frac{1}{x^2}\left(\int_0^x f(t)dt\right)^3$$

$$\therefore \left(f(x)\right)^3 = \frac{1}{x^2}.3\left(\int_0^x f(t)dt\right)^2 .f(x) - \frac{2}{x^3}.\left(\int_0^x f(t)dt\right)^3$$

$$\Rightarrow \left(\frac{xg'(x)}{g(x)}\right)^3 - 3\left(\frac{xg'(x)}{g(x)}\right) + 2 = 0$$

$$\Rightarrow \frac{xg'(x)}{g(x)} = 1\, or\, -2$$

If $\frac{xg'(x)}{g(x)} = 1 \Rightarrow f(x) = 1$

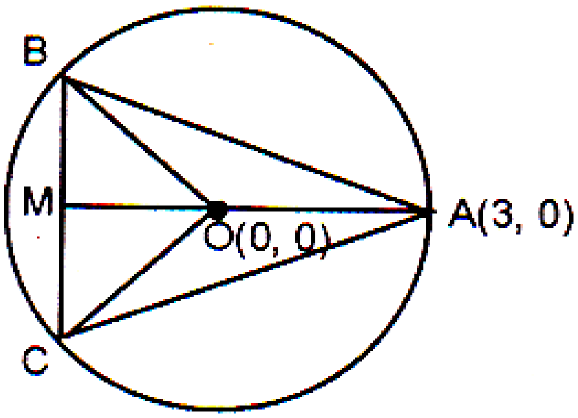
While if $\frac{xg'(x)}{g(x)} = -2 \Rightarrow f(x) = \frac{1}{x^3}$ (decreasing function)

41. **(A)**

Since $\angle B = \angle C = 75^0$

$\angle BAC = 30^0, \angle BOC = 60^0$

ΔOBC is equilateral BC=OB=3



M is the midpoint of BC

$OM = \sqrt{9 - \frac{9}{4}} = \frac{3\sqrt{3}}{2}$

Equation of BC is $x = -\frac{3\sqrt{3}}{2}$

Solving with $x^2 + y^2 = 9$, we get the points $\left(-\frac{3\sqrt{3}}{2}, \pm\frac{3}{2}\right)$

$\therefore$ Product of the ordinates of B and C $= \frac{3}{2}\left(-\frac{3}{2}\right) = -\frac{9}{4}$.

42. **(ABCD)**

$n(s) = {}^{20}C_3$

Number of A.P's with c.d's 1, 2, 3, ……., 9

are respectively 18, 16, 14, ………, 2
Total no. of A.P's = 2 + 4 + 6 + …….. + 18 = 2(1 + 2 + 3 + …….. + 9) = 90
No. of A.P's with odd c.d. = 18 + 14 + 10 + 6 + 2 = 50
Sum = even $\Rightarrow$ (3 nos. are even) or (1 even, 2 odd)
No. of fav. Case = ${}^{10}C_3 + \left({}^{10}C_1 \times {}^{10}C_2\right) = 120 + 450 = 570$

Prob. (product is odd) = $\left(\frac{{}^{10}C_3}{{}^{20}C_3}\right) = \frac{2}{19}$

43. **(AB)**

$(a_1, b_1)(a_2, b_2) = \left(2 + \sqrt{5}\left(\frac{1}{\sqrt{5}}\right), 3 + \sqrt{5}\left(\frac{2}{\sqrt{5}}\right)\right),$

$\left(2 - \sqrt{5}\left(\frac{1}{\sqrt{5}}\right), 3 - \sqrt{5}\left(\frac{2}{\sqrt{5}}\right)\right)$

$= (3,5), (1,1)$

$B \to \left|\frac{8+3-1}{2\left(\frac{1}{\sqrt{10}}\right) - \frac{3}{\sqrt{10}}}\right| = 10\sqrt{10}$

$C \to -x - 2y = -1 + 4 = 0$

$\Rightarrow x + 2y + 3 = 0$

Area = $\frac{9}{2(1)(2)} = \frac{9}{4}$

$D \to$ PQ = $\sqrt{4+4} = 2\sqrt{2}$

44. **(B)**

On each lateral face of the tetrahedron other than p_1 there are 5 points. Take any 3 out of these 5 and add p_1 to it. (e.g. $p_1 p_3 p_4 p_6$. So, there are in all $3 \times 5_{C_3}$ groups of lateral faces. Apart from these, there are 3 points on each edge containing p_1. When we add a mid point taking from the edge on the base which is not on the same plane with the edge above. we obtain another required group (e.g. $p_1 p_2 p_6 p_{10}$).There 3 groups like this.

So, total required groups = $3 \times 5_{C_3} + 3 = 33$

45. **(ABC)**

Obvious

46. **(ABCD)**

$f_1(x) = \pi/2 - \sin^2 x$

$f_2(x) = \pi/2 - \cos^2 x$

$f_3(x) = \pi/2 - \cos^2 x$

$f_4(x) = \pi/2 - \sin^2 x$

47. (ACD)

Maximum distance of a normal from origin is |b – a|. So, r ≤ |a – b| with $a^2 + b^2 = 25$. Possible value of |a – b| lies in [0, 5)

48. (ABD)

We have $\left(\frac{x}{x+1}\right)^2 + \left(\frac{x}{x-1}\right)^2 = a(a-1)$

$\Rightarrow \left(\frac{x}{x+1} + \frac{x}{x-1}\right)^2 - 2\left(\frac{x}{x+1}\right)\left(\frac{x}{x-1}\right) = a(a-1) \Rightarrow \left(\frac{2x^2}{x^2-1}\right)^2 - \frac{2x^2}{x^2-1} = a(a-1)$

$\Rightarrow z^2 - z - a(a-1) = 0,$ where $z = \frac{2x^2}{x^2-1}$ $\Rightarrow z = a \;\; or \;\; 1-a$

When, $z = a, \frac{2x^2}{x^2-1} = a \Rightarrow 2x^2 = ax^2 - a \quad \Rightarrow x = \pm\sqrt{\frac{a}{a-2}}$

When, $z = 1-a, \frac{2x^2}{x^2-1} = 1-a \Rightarrow 2x^2 = (1-a)x^2 - 1 + a \Rightarrow x = \pm\sqrt{\frac{a-1}{a+1}}$

$\therefore x = \pm\sqrt{\frac{a}{a-2}}, \pm\sqrt{\frac{a-1}{a+1}}$

If $a < -1 \Rightarrow$ *All roots are real.*

If $1 < a < 2 \Rightarrow x = \pm\sqrt{\frac{a}{2-a}}i, \pm\sqrt{\frac{a-1}{a+1}}$ $\Rightarrow$ Only two roots are real.

If $a > 2 \Rightarrow$ All roots are real.

49. (AB)

We have $\int_0^1 \frac{dt}{t^2 + 2t\cos\alpha + 1} = \int_0^1 \frac{dt}{(t+\cos\alpha)^2 + \sin^2\alpha} = \frac{1}{\sin\alpha}\tan^{-1}\left(\frac{t+\cos\alpha}{\sin\alpha}\right)_0^1$

$= \frac{1}{\sin\alpha}\left\{\tan^{-1}\left(\frac{1+\cos\alpha}{\sin\alpha}\right) - \tan^{-1}(\cot\alpha)\right\} = \frac{1}{\sin\alpha}\left\{\frac{\pi}{2} - \frac{\alpha}{2} - \frac{\pi}{2} + \alpha\right\} = \frac{\alpha}{2\sin\alpha}$

And $\int_{-3}^{3} \frac{t^2 \sin 2t}{t^2+1} dt = 0\left(It\, is\, odd\;\; function\right)$

$\therefore$ we have $\frac{\alpha x^2}{2\sin\alpha} - 2 = 0 \Rightarrow x^2 = \frac{4\sin\alpha}{\alpha}$

$x = \pm 2\sqrt{\frac{\sin\alpha}{\alpha}}$

50. (5)

Obvious

51. **(9)**

$$V=\frac{1}{3}\pi r^2\left(\sqrt{1-r^2}\right)$$

$$V^2=\frac{\pi^2}{9}\left(r^4-r^6\right)$$

$$\frac{d}{dr}\left(V^2\right)=0\Rightarrow r=\frac{\sqrt{2}}{\sqrt{3}}$$

$$\frac{\Delta_1}{\Delta_2}=\frac{\pi}{\pi-\pi\frac{\sqrt{2}}{\sqrt{3}}(1)}=\sqrt{3}\left(\sqrt{3}+\sqrt{2}\right)=3+\sqrt{6}$$

$$\therefore \quad \left.\begin{matrix}a=3\\ b=6\end{matrix}\right\} a+b=9$$

52. **(5)**

Solving first three planes P.I = (1,1,2) which also lies on 4th plane $\Rightarrow 1-2+k=0\Rightarrow k=1$

$$\left.\begin{matrix}L_1\ drs:1,0,-1\\ L_2\ drs:1,3,1\end{matrix}\right\} \text{Plane normal drs: 3,-2,3}$$

Equation of plane $3x-2y+3z=7$

$$\left.\begin{matrix}a=3\\ b=-2\\ c=3\end{matrix}\right\} \text{also k=1} \Rightarrow a+b+c+k=5$$

53. **(1)**

$$2y^2+6\ x+1\ y+5x^2+7x+6=0$$

$$\therefore y=\frac{-3\ x+1\ \pm\sqrt{\ x-1\ \ 3-x\ }}{2}$$

$$\therefore y_2-y_1=\sqrt{\ x-1\ \ 3-x\ }$$

$$\therefore \text{ Area of ellipse }=\int_1^3\sqrt{\ x-1\ \ 3-x\ }\,dx=\frac{\pi}{8}\times 4=\frac{\pi}{2}$$

$$\therefore \lambda=1$$

54. **(0)**

Put $t=\frac{1}{x}$

Then $I=\int_{\frac{1}{2}}^{2}\frac{1}{x}\operatorname{cosec}^{101}\left(x-\frac{1}{x}\right)dx$

= – I or I = 0

SOLUTION

1. **(A)**

Imagine ‘q’ to be placed at the centre of a cube of side L then base of the pyramid will become one face of that cube. Therefore, magnetic flux through the base, $\phi_{base} = -\frac{q}{6 \in_0}$

Flux through the pyramid is zero, therefore flux through all the four slanted faces will be $+\frac{q}{6 \in_0}$

Therefore, flux through a slanted face will be $\frac{1}{4}\left(+\frac{q}{6 \in_0}\right) = +\frac{q}{24 \in_0}$

2. **(A)**

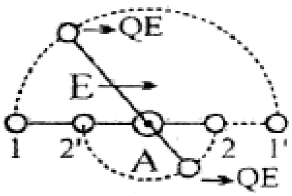
Induced emf $\int_a^b \text{Bvdx} = \int_a^b \frac{\mu_0 I}{2\pi x}\text{vdx}$

$\Rightarrow$ Induced emf $= \frac{\mu_0 Iv}{2\pi}\ell n\left(\frac{b}{a}\right) \Rightarrow$ Power dissipated $= \frac{E^2}{R}$

Also, power $= F.V \Rightarrow F = \frac{E^2}{VR}$

$\Rightarrow F = \frac{1}{VR}\left[\frac{\mu_0 IV}{2\pi}\ell n\left(\frac{b}{a}\right)\right]^2$

3. **(C)**



Position of maximum velocity is 1' and 2' for the balls as it is minimum potential energy position

Potential difference between 1 and 1' $= V_1 - V_1 = E \times \left(\frac{3L}{4} \times 2\right)$

Potential difference between 2 and 2' $= V_2 - V_2 = -E \times \left(\frac{L}{4} \times 2\right)$

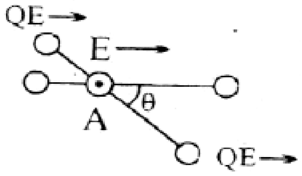
Change in electrostatic potential energy $= Q \times (V_1 - V_1) + Q \times (V_2 - V_2) = -QEL$ (loss is PE)

$= \text{Gain in KE} = \frac{1}{2}mV^2 + \frac{1}{2}m(2V)^2$

(Where velocity of the ball closer to the axis is V and farther from axis is 2V)

$\Rightarrow V = \sqrt{\frac{QEL}{5m}}$

4. **(D)**



For small angular displacement θ from equilibrium restoring torque

$= QE\sin\theta\left(\frac{3L}{4}-\frac{L}{4}\right) = I\alpha$ (since θ is very small $\sin\theta \approx \theta$)

$\Rightarrow \quad \alpha = \left(\frac{QEL}{2I}\right)(-\theta) \quad \Rightarrow \quad \omega = \sqrt{\left(\frac{QEL}{2I}\right)}$ where I (moment of inertia) $= \left(\frac{mL^2}{16}+\frac{9mL^2}{16}\right)$

Time period $(T) = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{5mL}{4QE}}$

5. **(C)**

$H = I\omega$, here $I = \frac{M(2d)^2}{12} = \frac{Md^2}{3}$

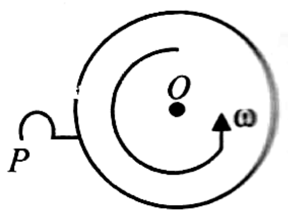
$\omega = \frac{H}{(Md^2/3)} = \frac{3H}{Md^2}$

6. **(B)**

During the impact, the impact forces pass through point P. Therefore, the torque produced by it about P is equal to zero.

Consequently, the angular momentum of the disc about P, just before and after the impact, remains the same

$\Rightarrow \quad L_2 = L_1 \quad ...(i)$



Where $L_1 =$ angular momentum of the disc about P just before the impact

$I_0\omega = \frac{1}{2}mr^2\omega$

$L_2 =$ angular momentum of the disc about P just after the impact

$I_0\omega = \left(\frac{1}{2}mr^2 + mr^2\right)\omega' = \frac{3}{2}mr^2\omega'$

Just before the impact, the disc rotates about O. But just after the impact, the disc rotates about P.

$\Rightarrow \quad \frac{1}{2}mr^2\omega = \frac{3}{2}mr^2\omega' \quad \Rightarrow \quad \omega' = \frac{1}{3}\omega$

7. **(BC)**

$\vec{B} = \frac{B}{\sqrt{3}}(\hat{i}+\hat{j}+\hat{k})$ & Area Vector

$\vec{A} = \left(\frac{\pi r^2}{2}+r^2\right)\hat{i} + r^2\hat{j} + r^2\hat{k}$

$$\therefore \phi_B = \vec{B}\cdot\vec{A} = \frac{Br^2}{\sqrt{3}}\left(\frac{\pi}{2}+3\right) \ \& \ \frac{dB}{dt} = \alpha$$

$$\vec{\tau} = \vec{\mu}\times\vec{B} \ \ \& \ \vec{\mu} = Ir^2\left[\left(\frac{\pi}{2}+1\right)\hat{i}+\hat{j}+\hat{k}\right]$$

8. **(BD)**

Let x be decrease in length l then

$$xA\rho g = Vdg \quad \Rightarrow x = \frac{Vd}{A\rho}$$

Also from volume conservation

$$A_1h - Al = A_1(h-\Delta h) - A(l-x) - V$$

Putting the value of x and solving

$$\Delta h = \frac{V}{A_1}\left(1-\frac{d}{\rho}\right)$$ i.e. level is increases

9. **(ABCD)**

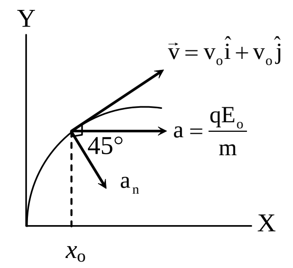
10. **(CD)**

11. **(BC)**

$$qE_o x_o = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2$$

$$v = \sqrt{2}v_o$$

$$\rho = \frac{v^2}{a_n},\ \rho = 4\sqrt{2}x_o$$



12. **(BC)**

$$T = 2\pi\sqrt{\frac{I}{\alpha}}$$

The cord is loose during one-half of the oscillation

$$\therefore T = \pi\sqrt{\frac{Ml}{2(kl+YA)}} + \pi\sqrt{\frac{M}{2k}}$$

When B_0 is switched off, there will be a torque on the charged disc due to induced electric field.

$$\int \tau dt = I(\omega_f - o)$$

$$\tau = \int \tau dt = \int_0^R \frac{r}{2}\frac{dB}{dt}\frac{Q}{\pi R^2}2\pi r dr = \frac{dB}{dt}\frac{QR^2}{4}$$

$$\therefore \int \tau dt = \frac{QR^2B_0}{4} = I\omega = \frac{1}{2}MR^2\omega \Rightarrow \omega = \frac{QB_0}{2M}$$

$\therefore$ maximum potential energy is stoned in spring when cord is loose & only spring is compressed

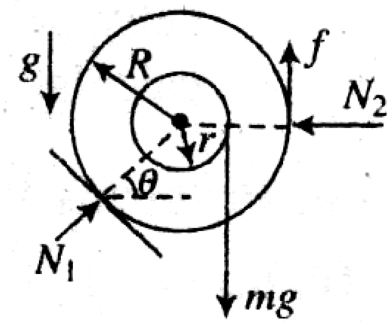
$$\therefore \frac{1}{2}I\omega^2 = \frac{1}{2}kx^2 = \frac{1}{2}\times\frac{1}{2}MR^2\left(\frac{QB_0}{2M}\right)^2 = \frac{Q^2B_0^2R^2}{16M}$$

When cord is stretched spring is also stretched.

$$\therefore \frac{1}{2}I\omega^2 = \frac{1}{2}\left(\frac{YA}{l}\right)x^2 + \frac{1}{2}kx^2$$

$$\therefore \text{ energy stored in cord } = \left(\frac{\frac{YA}{l}}{\frac{YA}{l}+k}\right)\frac{1}{2}I\omega^2 = \left(\frac{YA}{YA+kl}\right)\frac{Q^2B_0^2R^2}{16M}$$

13. **(AD)**



$mgr = fR$ ...(i)

$N_1\sin\theta + f = mg$ ...(ii)

$N_1\cos\theta = N_2$ ...(iii)

From (i), $f = \dfrac{mgr}{R}$

From (ii) and (iii),

$$N_2 = \frac{mg-f}{\tan\theta} = \frac{mg}{\tan\theta}\left[1-\frac{r}{R}\right]$$

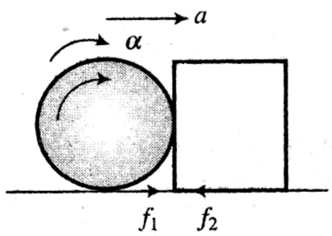
Net force at $B : F_B = \sqrt{f^2 + N_2^2}$

For minimum value of $\mu : f \le \mu N_2$

$$\Rightarrow \quad \frac{mgr}{R} \le \frac{\mu mg}{\tan\theta}\left[1-\frac{r}{R}\right] \Rightarrow \mu \ge \frac{\tan\theta}{R/r-1}$$

14. **(CD)**

$$a = \frac{f_1 - f_2}{m_1 + m_2} = \frac{0.4\times 6g - 0.5\times 3g}{6+3}$$



$$\Rightarrow \quad a = 1\,\text{m}\,\text{s}^{-2}$$

$$\alpha = \frac{\tau_{\text{applied}} - \tau_{\text{friction}}}{I} = \frac{6 - f_1R}{\frac{1}{2}6(R)^2}$$

$$= \frac{6 - 0.4\times 6g(0.2)}{\frac{1}{2}\times 6\times(0.2)^2} = 10\,\text{rad}\,\text{s}^{-2}$$

15. **(C)**

16. **(C)**

Maximum heat dissipated in

$$R_P = \left(\frac{1}{2}Li^2\right)\frac{R_P}{R_C + R_P} \cong \frac{1}{2}Li^2$$

$$= \frac{1}{2}\left(10^{-2}\right)\times\left(1.2\right)^2$$

$$= 7.2\ \text{mJ}$$

17. **(C)**

For $t = 0$ to $t_0 = RC$ seconds, the circuit is charging type.

The charging equation for this time is $q = CE\left(1 - e^{\frac{t}{RC}}\right)$

Therefore the charge capacitor at time $t_0 = RC$ is $q_0 = CE\left(1 - \frac{1}{e}\right)$

18. **(C)**

For $t = RC$ to $t = 2RC$ seconds, the circuit is of discharging type.

The charge and current equation for this time are

$q = q_0 e^{-\frac{t-t_0}{RC}}$ and $i = \frac{q_0}{RC}e^{-\frac{t-t_0}{RC}}$.

Hence charge at t = 2RC and current at t = 1.5 RC are

$= q_0 e^{-\frac{2RC-RC}{RC}} = \frac{q_0}{e} = \frac{1}{e}CE\left(1-\frac{1}{e}\right)$ and $i = \frac{q_0}{RC}e^{-\frac{1.5RC-RC}{RC}} = \frac{q_0}{\sqrt{e}RC}\left(1-\frac{1}{e}\right)$

respectively.

Since the capacitor gets more charged up from t = 2RC to $t = 3RC$ than in the interval t = 0 to t = RC, the graph representing the charge variation is as shown in figure.

SOLUTION

19. **(C)**

$$\Delta H = nF\left[T\left[\frac{\delta E}{\delta T}\right]_P - E\right]; \Delta H = 2\times 96500\left[-1.5\times 10^{-4}\times 298 - 0.03\right]$$

$$= -14417.1 \text{ Jmol}^{-1}$$

20. **(A)**

$$N_t = N_0 e^{-\lambda t_1}$$

$$Nt_2 = N_0 e^{-\lambda t_2}$$

$$N_0 - n_1 = N_0 e^{-\lambda t_1}$$

$$N_0 - n_2 = N_0 e^{-\lambda t_2}$$

$$n_1 = N_0\left[1 - e^{-\lambda t_1}\right]$$

$$n_2 = N_0\left[1 - e^{-\lambda t_2}\right]$$

$$\frac{n_2 = \left[1 - e^{-\lambda t_2}\right]}{\left[1 - e^{-\lambda t_2}\right]}$$

$$2.66 = \frac{1 - e^{-\frac{6}{5}}}{1 - e^{-\frac{2}{5}}}$$

$$2.66 = \frac{1 - x^3}{(1 - x)}$$

$$2.66 = \left(1 + x + x^2\right)$$

$$1 + x + x^2 = 2.66$$

$$x^2 + x - 1.66 = 0$$

$$x = \frac{-1 + \sqrt{1^2 + 4\times 1\times 1.66}}{2} = 0.882$$

$$e^{-2/T} = 0.882$$

$$2.303 \log 0.882 = -\frac{2}{T}$$

$$0.12555 = \frac{2}{T}$$

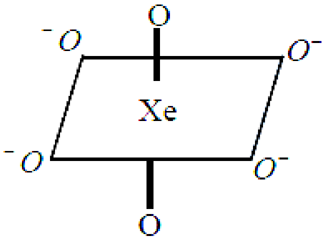
$$T = \frac{2}{0.125} = 16 \sec$$

21. **(A)**

Among 5 H_2O molecules one is in hydrogen bonding with sulphate ions, four are incoordinated bonds. Among the four coordinated water molecules again two watermolecules involve in hydrogen bonding with the 5th water molecule

22. **(B)**

XeO_4^{4-} have regular octahedron



23. **(A)**

24. **(A)**

$\Delta T_f = ik_f m$

$= 3 \times 1.855 \times \frac{24}{106} \times \frac{1000}{(159+41)}$

$= -6.31$

25. **(BCD)**

All are extension of $PV = nRT, n = \frac{W}{M}$ and $d = \frac{W}{V}$

26. **(AC)**

The complex forming ability of Cl^- ion enhances the oxidation power of NO_3^- ion

27. **(BD)**

Presence of $\alpha -$hydrogen with respect to the substituent in elimination additionreactions are must.

28. **(ACD)**

Aqueous NaOH dissolves

(A) Zn is brass not Cu

(B) Both lead and tin in solder

(C) Al in magnallium but not magnesium

(D) Sn and Zn in gun metal but not copper

29. **(B)**

$160 = x_B \times 3.3 \times 10^7$

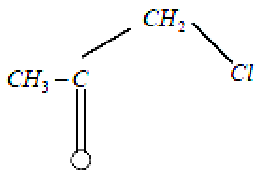
$x_B = 48.5 \times 10^{-7}$

$= \frac{48.5 \times 10^{-7} \times 1000}{18} = 2.7 \times 10^{-4}$

For dilute solutions molality = molarity

30. **(A)**

31. **(ABD)**



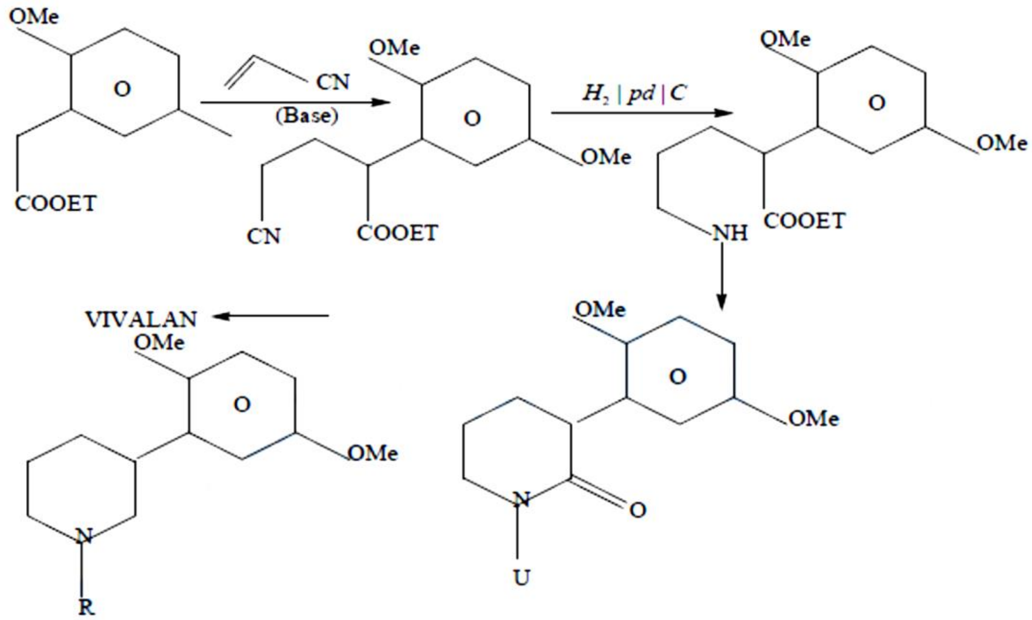
32. **(ABC)**

Conceptual

33. **(B)**

Mechanism

34. **(A)**



35. **(D)**

$$CH_3OCH_{3(g)} \to CH_{4(g)} + CO_{(g)} + H_{2(g)}$$

Initial P	P_0	-	-	-
P at time t	$P_0 - x$	P_0	P_0	P_0

P after long time

Given $3P_0 = 1.2$ $\quad \therefore p_0 = 0.4\,atm$

Also $k = \frac{0.693}{0.2} h^{-1}$

$k = \frac{2.303}{t}\log\frac{P_0}{P_0 - x}$ $\quad \therefore \frac{0.693}{0.2} = \frac{2.303}{0.6}\log\frac{0.4}{0.4 - x}$

$\therefore 0.9 = \log\frac{0.4}{0.4 - x}$ $\quad \therefore 8 = \frac{0.4}{0.4 - x}$

$\therefore x = 0.35$

$\therefore P_t = 0.4 - 0.35 + 0.35 + 0.35 + 0.35 = 1.1 \text{atm}$

36. **(A)**

$$\text{rate} = k.p_{CH_4} = \frac{0.693}{0.2} \times 0.4 = 1.386 \text{ atm h}^{-1}$$

37. **(A)**

P can be selected in 4 ways.

Number of nos when 9 is used once = $4({}^{100}C_1)9^{99}$

Number of nos when 9 is used thrice = $4({}^{100}C_3)9^{97}$

No of number formed is

$$4\left[{}^{100}C_1 9^{99} + {}^{100}C_3 9^{97} + \ldots + 9^{100} C_{97}\right]$$

$$= 4\left[\frac{1}{2}\left(10^{100} - 8^{100}\right)\right]$$

$$= 2\left(10^{100} - 8^{100}\right)$$

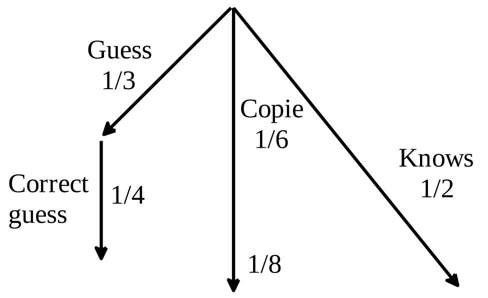
38. **(A)**

Let G, C, K & A be events where

G = guess answer C = copy answer

K = known answer A = answer correctly

P(G) = 1/3, P(C) = 1/6, P(K) = 1 – 1/3 – 1/6 = ½



Answer right $\boxed{\frac{1}{12} + \frac{1}{6}\cdot\frac{1}{8} + \frac{1}{2}} = \frac{4+1+24}{48} = \frac{29}{48}$

$$P\left(K/A\right) = \frac{24}{29}$$

39. **(A)**

$$\frac{4PS.SR}{PS+SR} = \frac{2b^2}{a}$$

$$\frac{PS+SR}{SR} = \frac{2a}{b^2}.PS$$

$$\frac{PS'+S'Q}{S'Q} = \frac{2a}{b^2}.PS'$$

$$\frac{PS}{SR} + \frac{PS'}{S'Q'} + 2 = \frac{2a}{b^2}\left(2a\right)$$

$$= \frac{4a^2}{b^2}$$

$$= \frac{4}{1-e^2}$$

$$\frac{PS}{SR} + \frac{PS'}{S'Q} = \frac{4}{1-e^2} - 2$$

$$= 2\left(\frac{2-1+e^2}{1-e^2}\right)$$

$$= 2\left(\frac{1+e^2}{1-e^2}\right)$$

$$= \left(\frac{1+3/4}{1-3/4}\right)$$

$= 14$

40. **(A)**

$$\int \frac{x^2-1}{x^3\sqrt{3x^4-2x^2+1}}.d\alpha$$

$x^2 = t$

$$= \frac{1}{2}\int \frac{t-1}{t^2\sqrt{3t^2-2t+1}}.dt$$

$t = 1/4$

$$= \left(\frac{-1}{2}\right)\int \frac{1-4}{\sqrt{4^2-24+3}}.du$$

$$= \frac{1}{4}\int\left(\frac{2x-s}{\sqrt{u^2-24+3}}\right).dx$$

$$= \frac{1}{4}\left(2\sqrt{4^4-24+3}\right)+c$$

$$= \frac{\sqrt{3/x^4-\frac{2}{x^2}+3}}{2}$$

$$= \frac{\sqrt{3x^4-2x^2+1}}{2x^2}$$

$\phi(x) = 2x^2$

$\phi'(1), \phi'(2), \phi'(3)$ are in A.P

41. **(B)**

$f(x, y) = |z| + |z-1| + |z-i| + |z-(3+4i)|$

is min when z is the point of intersection of diagonals of the quadrilateral formed by (0, 0) (1, 2) (3, 4) (0, 1)

min value = $5+\sqrt{2}$

42. **(B)**

The centre of a circle passing through M & N lies on the bisector y = 3 – x of MN. Denote the curve by C (a, 3 – a), the equation of circle $(x-a)^2(y-3+a)^2 = 2(1+a^2)$.

Chord of fixed length subtends larger angle as the radius becomes smaller. When $\angle MPN$ reaches its maximum value the circle through M, N, P will be tangent to x – axis.

$\Rightarrow 2\left(1+a^2\right)=\left(a-3\right)^2$. A = 1, or –7

Point of contact $\equiv P\left(1,0\right)$ or $P'\left(-7,0\right)$

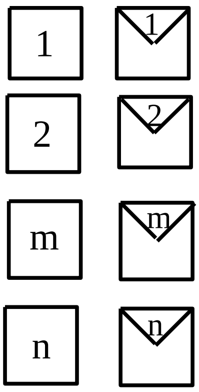
But radius of circle MNP' is larger than of circle MNP

$\therefore \angle NP'N > \angle MP'N$

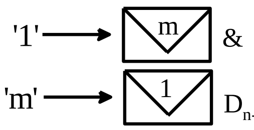
So $P\equiv\left(1,0\right)$

43. **(AC)**

If I goes in letter 'n' then m can go either 1 or not in '1'



If



D_{n-2}

otherwiseD_{n-1} ways

$D_n = \left(n-1\right)\left(D_{n-2}+D_{n-1}\right)$

$D_n - {}_nD_{n-1} = \left(n-1\right)D_{n-2} - D_{n-1}$

$D_{n-1} = \left(n-2\right)\left(D_{n-2}+D_{n-3}\right)$

$D_n - {}_nD_{n-1} = D_{n-2} - \left(n-2\right)D_{n-3}$

$= \left(n-3\right)\left(D_{n-3}+D_{n-4}\right) - \left(n-2\right)D_{n-3}$

$D_n - {}_nD_{n-1} = \left(n-3\right)D_{n-4} - D_{n-3}$

44. **(ABCD)**

(A) No of $F^n s = 4^5 = 1024$ ways

No of onto $F^n S$

$$D_5 = 50\left(1-\frac{1}{1!}+\frac{1}{2!}-\frac{1}{3!}+\frac{1}{4!}-\frac{1}{5!}\right)$$

$$D_5 = 60-20+5-1$$

= 44

So no of into $F^n S$ = 1024 – 44 = 980

(B) No of onto $F^nS \equiv \begin{pmatrix}1\\2\\3\\4\\5\\6\end{pmatrix} \begin{pmatrix}1\\2\\3\\4\end{pmatrix}$

$= 3^4 4^2 - {}^4C_1\left(3^3 2^3\right) + {}^4C_2\left(2^4 1^2\right) - {}^4C_3(0) + {}^4C_4(0)$

$= 81 \times 16 - 4(216) + 6(16)$

$= 1296 - 864 + 96$

= 528

(C) Onto F^nS = 44

(D) No of increasing $F^nS(A \to B)$

$\begin{pmatrix}1\\2\\3\\4\end{pmatrix} \quad \begin{pmatrix}1\\2\\3\\4\\5\\6\\7\end{pmatrix}$

${}^7C_4 - \left({}^6C_3 + {}^5C_2 + {}^4C_1 + 1\right) + \left({}^5C_2 + {}^4C_1 + 1 + {}^4C_1 + 1 + 1\right) - \left(1 + 1 + 1 + {}^4C_1\right) + 1$

= 35 – (20 + 10 + 5) + (10 + 11) – (7) + 1

= 15

45. **(AB)**

$$\sum_{k=0}^{7} \frac{{}^7C_k}{{}^{14}C_k}\left(\sum_{r=k}^{14} {}^rC_k \, {}^{14}C_r\right)$$

$$= \sum_{k=0}^{7} \frac{{}^7C_k}{{}^{14}C_k}\left(\sum_{r=k}^{14} \frac{7!}{k!(r-k)!} \times \frac{14!}{(14-r)!}\right)$$

$$= \sum_{k=0}^{7} \frac{{}^7C_k}{{}^{14}C_k k!}\left(\sum_{r=k}^{14} {}^{14-k}C_{r-k} \times \frac{14!}{(14+k)!}\right)$$

$$= \sum_{k=0}^{7} {}^7C_k\left({}^{14-k}C_0 + {}^{14-k}C_1 + ... + {}^{14-k}C_{14-k}\right)$$

$$= 2^{14}\left(\sum_{k=0}^{7} {}^7C_k \left(\frac{1}{2}\right)^k\right)$$

$$= 2^{14}\left(1 + 1/2\right)^7 = 2^7 3^7 = 6^7$$

$$y = \frac{\ln x}{x}$$

$$y' = \frac{x\left(\frac{1}{x}\right) - (\ln x)}{x^2} = \frac{1-\ln x}{x^2}$$

$y' \downarrow$ decrease if x > e

$6^{1/6} > 7^{1/7}$

$6^7 > 7^6$

46. **(ABC)**

Replace x by '2'

$2f(2) + 2f\left(\frac{1}{2}\right) - 2f(1) = 4$

$f(2) + f\left(\frac{1}{2}\right) = 2 + f(1)$ …(A)

Put x = 1;f(1) = –1

Put x = ½ $2f\left(\frac{1}{2}\right) + \frac{1}{2}f(2) + 2 = \frac{5}{2}$

$f\left(\frac{1}{2}\right) = 0, f(2) = 1$

$f(2) + f\left(\frac{1}{2}\right) = 1$

f(2) + f(1) = 0

$f(2) + f(1) = f\left(\frac{1}{2}\right)$

47. **(ABCD)**

3n is divisible by '3'

So sum of its digits is divisible by 3

$\Rightarrow$ n is also divisible by 3

$\Rightarrow$ 3n is divisible by 9

$\Rightarrow$ sum of divisible by 3n are divisible by 9

$\Rightarrow$ sum of divisible of n are divisible by 9

48. **(BCD)**

$f'(x) = \text{even}$, f (x) is odd, f(x) increase function

f(3) = 0

So f(0) must be negative f(3) = 0

f(g(x)) = x g(0) = 3

$f'(g(x))g'(x) = 1$ x = 0

$f'(3)g'(0) = 1$

$$g'(0) = \frac{1}{f'(3)} = 11$$

$f''(g(x))(g'(x))^2 + f'(g(x))g''(x) = 0$

Put x = 0

$f''(3)(121) + f'(3)g''(0) = 0$

$$f'(x) = \frac{1}{\sqrt{x^4 + 3x^2 + 13}}$$

$$f''(x) = (-1/2)\frac{(4x^3 + 6x)}{(x^4 + 3x^2 + 13)^{3/2}}$$

$$f^4(3) = (-1/2)\frac{(108+18)}{(11^3)}$$

$$\frac{-63\times 11^2}{11^3} + \frac{1}{11}9^x(0) = 0$$

$$g''(0) = 63$$

49. **(BD)**

Let $\frac{2}{x} = e^t$, $t = \ln 2 - \ln x$

$x = 2e^{-t}$

$dx = -2e^{-t}dt$

$$I_1 = \int_{-\infty}^{\infty} f\left(e^t + e^{-t}\right)(1n2 - t)dt$$

$$I_1 = (\ln 2)\int_{-\infty}^{\infty} f\left(e^t + e^{-t}\right)dt - I_3$$

$$I_2 = \int_{-\infty}^{\infty} f\left(e^t + e^{-t}\right)dt$$

$I_3 = 0$ odd function

$I_1 = (\ln 2)I_2$ & $I_1 = (\ln_2)I_2 + I_3$

50. **(ABD)**

$$\left[l\bar{a} + m\bar{b} + n\bar{c}, ...\right]$$

$$= \left[\bar{a}\ \bar{b}\ \bar{c}\right]\begin{vmatrix} l & m & n \\ m & n & l \\ n & l & m \end{vmatrix} = 0$$

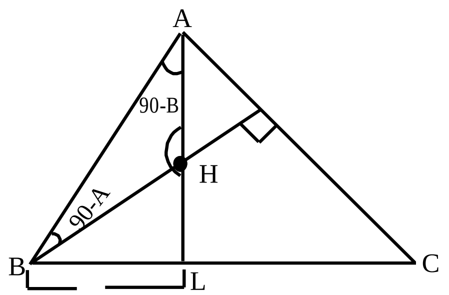
$\Rightarrow l^3 + m^3 + n^3 = 3lmn$

$\Rightarrow l + m + n = 0$

$lx^2 + mx + n = 0$

$x = 1$ is a root

51. **(B)**



$$\angle AHB = 180 - (90 - A) - (90 - B)$$
$$= A + B$$
$$R_1 = \frac{C}{2\sin(A+B)} = \frac{C}{2\sin C} = R$$
$$\therefore R_1R_2 + R_2R_3 + R_3R_1 = 3R^2$$

52. **(C)**

AH = 2R cos A, ΔAHB

BH = 2R cos B $= \frac{1}{2}(2R\cos A)(2R\cos B)\sin(A+B)$

$$= 2R^2 \cos A \cos B \sin C$$
$$\Delta BMC \equiv \Delta BHL; BL = C\cos B$$
$$\frac{HL}{BL} = \tan(90 - C)$$
$$HL = BL\cot C$$
$$\Delta BHL = \frac{1}{2}\left(c^2\cos^2 B\cot C\right)$$
$$= \frac{1}{2}\left(4R^2 \sin C\cos C\cos^2 B\right)$$
$$\frac{\Delta AHB}{\Delta BML} = \frac{2R^2 \cos A\cos B\sin C}{2R^2 \cos C \sin C \cos^2 B}$$
$$= \cos A : \cos B\cos C$$

53. **(D)**

54. **(C)**

$$\left|\frac{z_1+z_2+z_3+z_4}{d} - z_1\right| = \left|\frac{z_1+z_2+z_3+z_n}{d} - z_2\right| = \left|\frac{z_1+z_2+z_3+z_n}{d} - z_3\right|$$
$$\Rightarrow \text{circumcentre} = \frac{z_1+z_2+z_3+z_4}{d} = 0$$
$$z_1+z_2+z_3+z_4 = 0$$

53. arg ($z_1 + z_2 + z_3 + z_n$) = not defined

54. $|z_1+z_2+z_3+z_4| = 0$