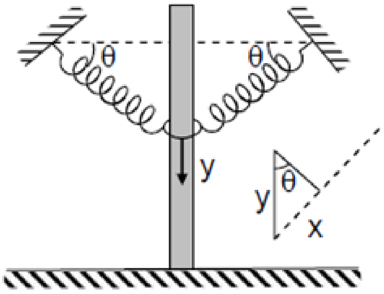
PART (A) : PHYSICS

ANSWER KEY

1.	(D)	2.	(C)	3.	(B)	4.	(A)	5.	(A)
6.	(A)	7.	(A)	8.	(A)	9.	(B)	10.	(B)
11.	(A)	12.	(B)	13.	(D)	14.	(A)	15.	(B)
16.	(D)	17.	(C)	18.	(B)	19.	(C)	20.	(B)
21.	(5)	22.	(1.5)	23.	(100)	24.	(2)	25.	(0)
26.	(50)	27.	(10)	28.	(60)	29.	(1)	30.	(35)

SOLUTIONS

1. (D)



For small displacement $y = \frac{x}{\sin\theta}$

$2kx_0 \sin\theta = mg$ .... (1)

If ring is displaced y

$mg - 2k(x_0 + y\sin\theta)\sin\theta = ma$

$a = -\frac{2k\sin^2\theta}{m} y$

2. (C)

$W_F = \Delta U + \Delta K$

Length AC

$\ell = \sqrt{(1.2)^2 + (0.9)^2}$

$300\{1.5 - 0.9\} = \frac{1}{2}K(x_2^2 - x_1^2) + \frac{1}{2}mV^2$

3. (B)

$\frac{1}{2}Ka^2 = \mu mg(a+b) + \frac{1}{2}Kb^2$

$(a-b) = 2\mu mg / K$

4. (A)

$$\tan\phi = \frac{\omega L}{R}$$

$$\cos\phi = \frac{R}{\sqrt{R^2 + X_L^2}}$$

$$i_0 = \frac{\varepsilon_0}{\sqrt{R^2 + X_L^2}}, V_L = IX_L = \frac{\varepsilon X_L}{\sqrt{R^2 + X_L^2}}$$

5. (A)

$$v = \frac{1}{2\pi\sqrt{LC}}$$

6. (A)

$$\frac{0 + V_A}{2} = V_1$$

$$\frac{V_A + V_B}{2} = V_2$$

$$\frac{V_B + 0}{2} = V_3$$

7. (A)

$$mgh = \frac{1}{2}kx^2 - mgx$$

8. (A)
About point O Ring is in pure rotation.

9. (B)

$$\Delta\theta = W + \Delta U$$

$$\Delta U - \Delta Q - W$$

10. (B)
Can be understood by graph.

11. (A)

$$dQ = mS\Delta T < 0 \Rightarrow \Delta T < 0$$

12. (B)

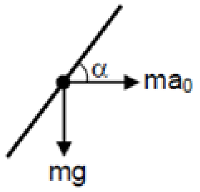
$$a = \frac{g\sin\theta}{\left(1 + \frac{I}{mr^2}\right)}, a_{ring} = \frac{g\sin\theta}{2}$$

$a_{coin} = \frac{2}{3} g \sin\theta$

13. (D)
Number of significant figures in the answer must be three.

14. (A)
$\frac{T_1}{T_2} = a = 1$
$\frac{r_1}{r_2} = \frac{\sin\theta_1}{\sin\theta_2} = b = \frac{1}{\sqrt{3}}; \frac{P_1}{P_2} = \frac{\cos\theta_1}{\cos\theta_2} = c = \sqrt{3}$

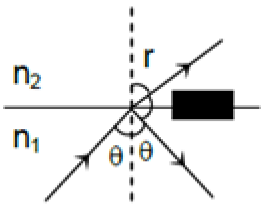
15. (B)
Escape velocity, $v = \sqrt{2gR}$
$\therefore \quad \frac{V_E}{V_M} = \sqrt{\frac{2g_E R_E}{2g_M R_M}} = \sqrt{6 \times 4} = \sqrt{24}$

16. (D)


Along the rod
$mg\sin\alpha - ma_0\cos\alpha = ma$

17. (C)
For the velocity perpendicular to incline after collision will be same for both so time for both will remains.
For the velocity along the incline after collision, velocity of the ball on smooth surface will be greater than that of rough surface.

18. (B)
$mg(h + x) - F.x = 0$

19. (C)


$n_1 \sin\theta = n_2 \sin r = n_2 \sin(60 - \theta)$

$$\sin c = \frac{n_2}{n_1} = \frac{\sin\theta}{\sin(60-\theta)} = \frac{2\sin\theta}{\sqrt{3}\cos\theta - \sin\theta}$$

20. (B)

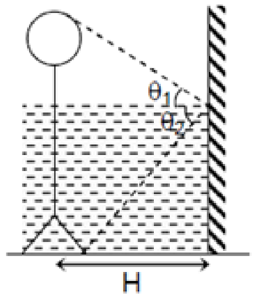
For bead to be stationary

Slope of tangent on the curve $\frac{dy}{dx} = \tan\theta \le \mu$

$\frac{2a}{y} \le \mu$.

21. (5)

$\mu_1 \cos\theta_1 = \mu_2 \cos\theta_2$



22. (1.5)

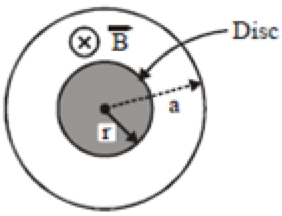
$$\frac{V_0 \times 6}{2} - \frac{\frac{V_0}{3} \times 2}{2} = 4$$

$$V_0 = \frac{3}{2}\,m/s$$

23. (100)

Kinetic energy at mean position $= 4J = \frac{1}{2} mA^2\omega^2 \Rightarrow \omega = 200\,Rad/s$.

24. (2)



$$E = \frac{x}{2}\frac{dB}{dt}$$

$$E = \frac{3Kxt^2}{2}$$

$$d\tau = \frac{3Kxt^2}{2} \times \frac{2\pi x dx}{\pi r^2} q.x$$

$$\tau = \frac{3Kt^2q}{r^2}\int_0^r x^3dx$$

$$\tau = \frac{3Kq.t^2}{4}.r^2 \qquad \text{.......... (i)}$$

Torque due to friction force

$$d\tau = \mu dmgx$$

$$\tau = 2\mu g\frac{qm}{r^2}\int_0^r x^2dx = \frac{2}{3}\mu mgr \qquad \text{......... (ii)}$$

$$\frac{3Kq.t^2r^2}{4} = \frac{2}{3}\mu mgr$$

$$t = \sqrt{\frac{8\mu mg}{9Kqr}} = 2 \text{ seconds.}$$

25. (0)

Light from any part of body can not reach the eye of man.

26. (50)

$$F_{N.ext} = 0$$

$$F.2R - F.R = I\alpha \qquad \Rightarrow \alpha = 2.5\,Rad/s^2$$

$$\omega = \alpha t$$

$$K = \frac{1}{2}I\omega^2$$

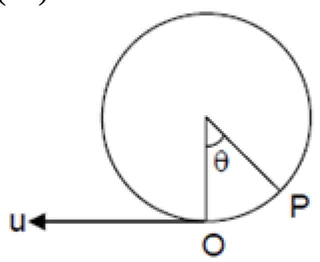
27. (10)

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

$$7mgR = \frac{1}{2}mv^2 + \frac{1}{2}\times\frac{2}{5}mv^2$$

$$mv^2 = 10mgR \Rightarrow v = \sqrt{10gR}$$

28. (60)



Tension in the string at the lowest point in vertical circle is given by

$$T_L = mg + \frac{mu^2}{\ell}$$

$2mg = mg + \frac{mu^2}{\ell}$

$u = \sqrt{g\ell}$

Using the law of conservation of energy. K.E. at $O = $ P.E. at P

$\frac{1}{2}mu^2 = mg\ell(1-\cos\theta)$

$\frac{1}{2}mg\ell = mg\ell(1-\cos\theta)$

$\cos\theta = \frac{1}{2} \Rightarrow \theta = 60^o$

29. (1)

If the tension in the string is T then for insect

$T - mg = ma$

$T = \frac{3mg}{2}$

For block $T + N = 2mg$

30. (35)

For each reaction involving both the process, mass defect

$= 3\times 2.014 - 4.001 - 1.007 = 0.026u$

Energy released per deuteron $= \frac{931.5\times 0.026}{3}$ MeV

Let n be the number of deuterons required

Total energy $= \frac{931.5\times 0.026}{3}\times n\,\text{MeV} = \frac{P\times t}{\eta} = \frac{10^{16}\times 365\times 24\times 60\times 60}{0.5}$

$n \simeq 5\times 10^{35}$.

PART (B) : CHEMISTRY

ANSWER KEY

31.	(C)	32.	(C)	33.	(A)	34.	(C)	35.	(B)
36.	(C)	37.	(A)	38.	(B)	39.	(A)	40.	(A)
41.	(B)	42.	(B)	43.	(C)	44.	(A)	45.	(C)
46.	(C)	47.	(C)	48.	(B)	49.	(D)	50.	(C)
51.	(4)	52.	(3)	53.	(1)	54.	(6)	55.	(8)
56.	(2)	57.	(6)	58.	(4)	59.	(6)	60.	(3)

SOLUTIONS

31. (C)

For non-polar MX_3, it must have triangular planar arrangement, i.e. there should be sp^2-hybridisation around M.

32. (C)

$$V_{mp} = \sqrt{\frac{2RT}{M}} \text{ i.e.} V_{mp} \propto \sqrt{\frac{T}{M}}$$

Gas	M	T(k)	$\sqrt{T/M}$
H_2	2	300	$\sqrt{300/2} = \sqrt{150}$
N_2	28	300	$\sqrt{300/28} = \sqrt{10.71}$
O_2	32	400	$\sqrt{400/32} = \sqrt{12.5}$

33. (A)

24. (C)

25. (B)

When $k_1 = k_2$

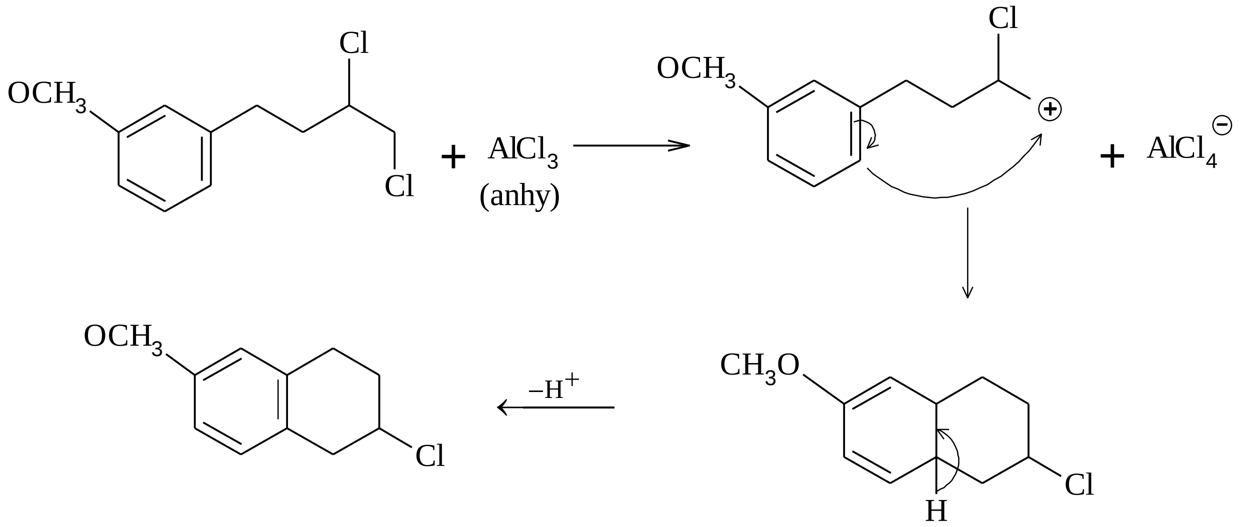
$\frac{2}{3}$rd A reacted for [A] = [C] = [D]

$$(k_1 + k_2) = \frac{1}{t} \ln \frac{[A_0]}{p[A_0]/3}$$

$$\Rightarrow t = \frac{1}{(k_1 + k_2)\ln 3}$$

$$t = \frac{1}{2k_1} \ln 3 = \frac{1}{2k_2} \ln 3$$

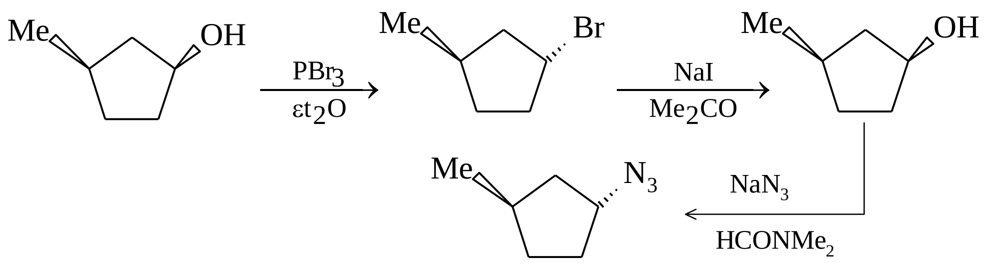
36. (C)



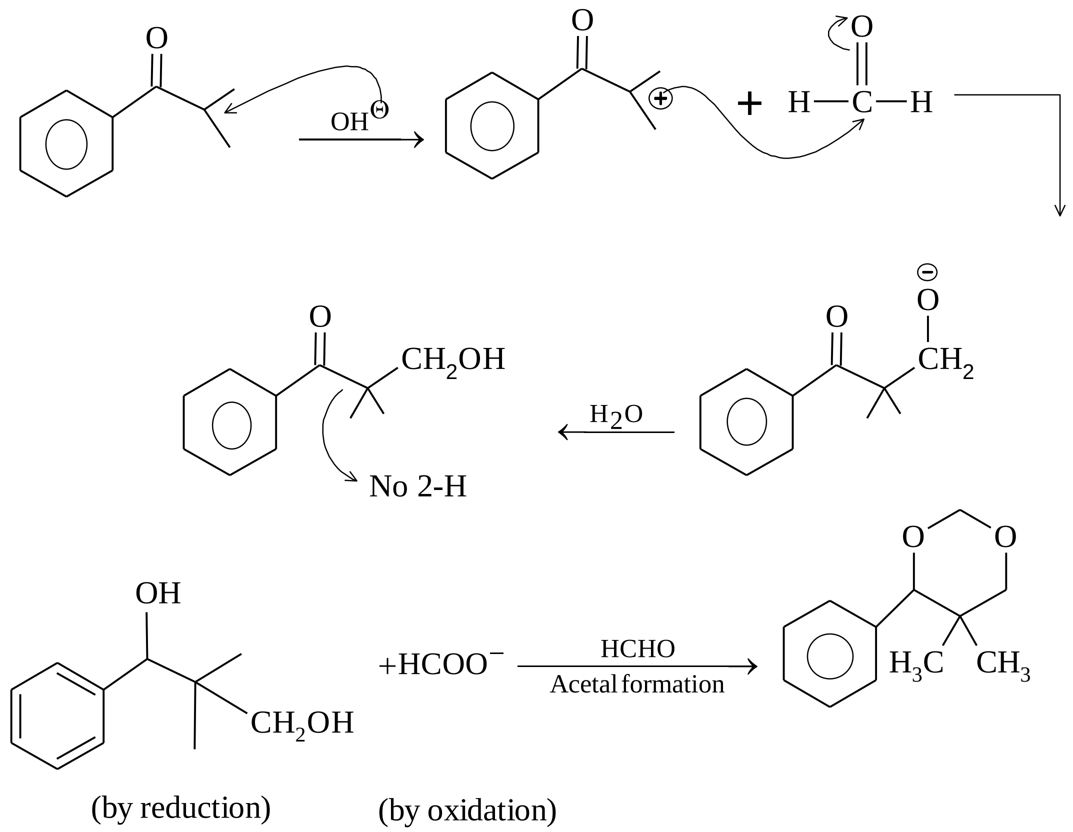
37. (A)

H_2O_2 in alkaline medium acts as a reducing agent.

38. (B)



39. (A)



40. (A)
CO_2 is an acidic oxide, H_2O is neutral, CaO is strongly basic and CuO is weakly basic

41. (B)
Magnetic moment = 2.83 BM indicates that there are 2 unpaired electrons
$\mu = \sqrt{n(n+2)}\ \text{BM} = \sqrt{8}\text{BM} = 2.82\text{BM}$

42. (B)
$\% \text{ of N} = \dfrac{\text{Mass of N}}{\text{Mass of organic compound}} \times 100\%$

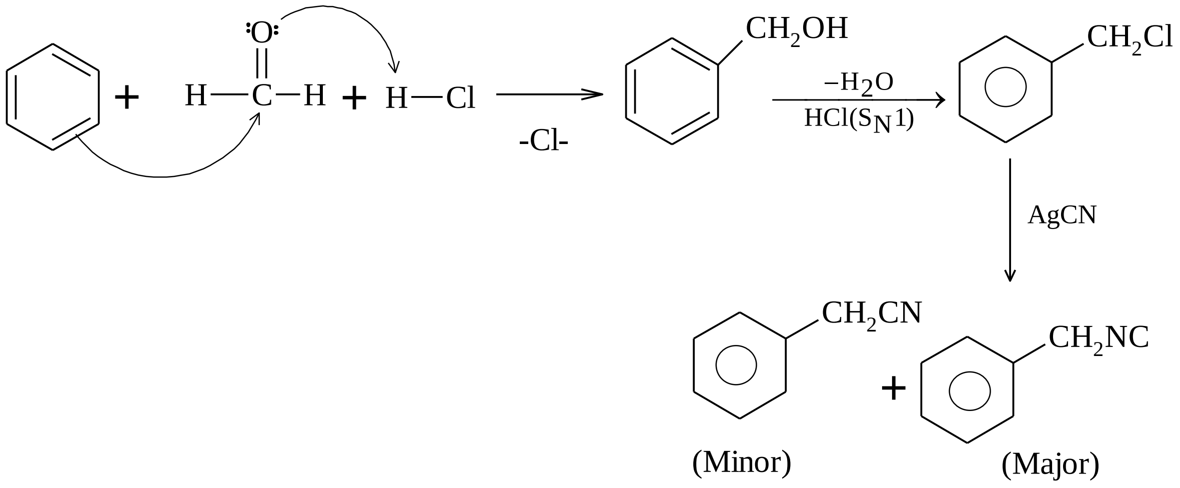
$2NH_3 + H_2SO_4 \rightarrow (NH_4)_2SO_4$

$H_2SO_4 + 2NaOH \rightarrow Na_2SO_4 + 2H_2O$

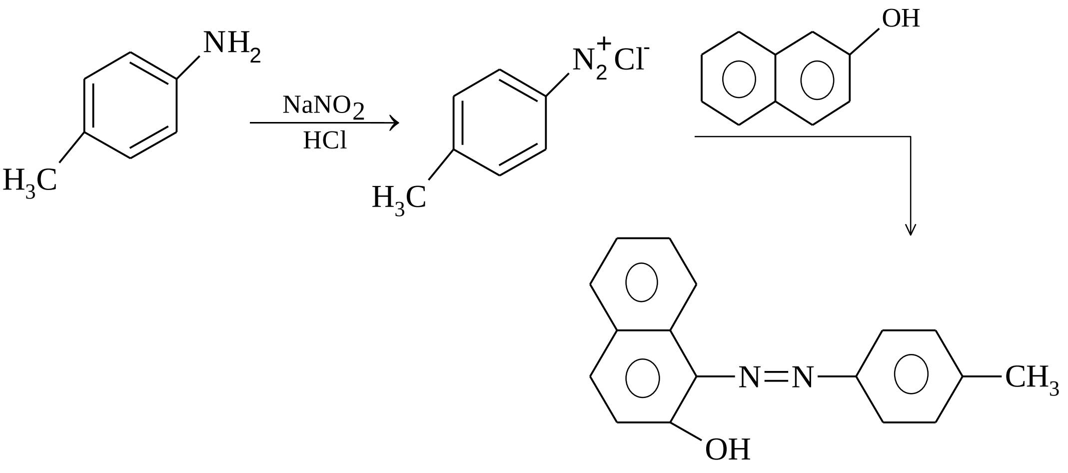
43. (C)

44. (A)
KNO_3 and other nitrates of alkali metals (except $LiNO_3$) are thermally stable

45. (C)



46. (C)



dye

47. (C)

$$\log k_p = \text{constant} - \frac{\Delta H}{2.3RT}$$

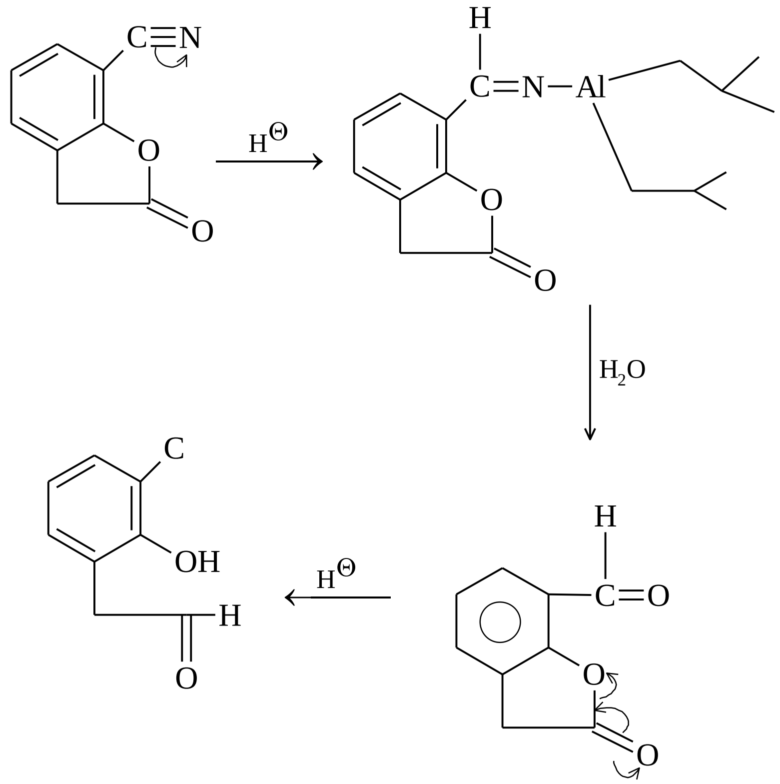
$$\log \frac{[x_0]}{[x]} = \frac{kt}{2.3}, \text{ or } \log [x] = \log[x_0] = -\frac{kt}{2.3}$$

At constant T, P vs $\frac{1}{V}$ will give a straight line

48. (B)

Acidic strength $\propto \frac{1}{\text{pKa value}}$

49. (D)



50. (C)

Black phosphorus is thermodynamically most stable allotrope of phosphorus.

51. (4)

$$m = \frac{92 \times 1000}{23 \times 1000} = 4\,\text{mol}\,\text{kg}^{-1}$$

52. (3)

In S_8, oxidation no. of S is 0

In S_2F_2. Oxidation no of S is +1

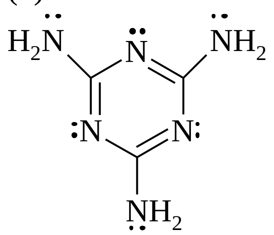
In H_2S, oxidation no of S is -2

$\Rightarrow x + y - z = 0 + 1 - (-2) = 3$

53. (1)

Only K_2CrO_4 is diamagnetic

54. (6)



55. (8)

$A^- + H^2O \rightleftharpoons AH + OH^-$

$k_h = \frac{k_w}{k_a} = 10^{-10}$

$[OH^-] = \sqrt{K_h C} = 10^{-6}$

pOH = 6 and pH =8

56. (2)

$\frac{P}{V} = 1 \Rightarrow P = V$ (i)

For 1 mole, PV = RT

$PdV + VdP = RdT$ (ii)

From (i) , PdV =VdP

Substituting in eq (ii), we get

2PdV =RdT

Or PdV = $\frac{RdT}{2}$

From FLOT, dq = C_VdT + $\frac{RdT}{2}$

$\int \frac{dq}{dT} = Cv + \frac{R}{2} = \frac{3}{2}R + \frac{R}{2} = 2R$

57. (6)

58. (4)

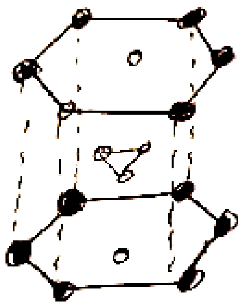
Energy consumed in converting one mole of M^+ to M^{3+}

$= -nFE^0 = 2 \times 96500 \times 0.25J = \frac{96500}{2}J$

$\Rightarrow 193 \times 10^3 = n\left(\frac{96500}{2}\right)$

$\Rightarrow n = 4$

59. (6)



Total atoms = $12 \times \frac{1}{6} + 2 \times \frac{1}{2} + 3 = 6$

60. (3)

$NaOCl + H_2 \rightarrow NaOH + HOCl$

$HO^- + HCO_3^- \rightarrow CO_3^{2-} + H_2O$

$Ca(HCO_3)_2 + Na_2CO_3 \rightarrow CaCO_3 \downarrow + 2NaHCO_3$

PART (C) : MATHEMATICS

ANSWER KEY

61.	(D)	62.	(B)	63.	(A)	64.	(D)	65.	(D)
66.	(A)	67.	(C)	68.	(C)	69.	(A)	70.	(C)
71.	(C)	72.	(D)	73.	(A)	74.	(D)	75.	(B)
76.	(D)	77.	(B)	78.	(C)	79.	(C)	80.	(B)
81.	(40)	82.	(10)	83.	(7)	84.	(98)	85.	(200)
86.	(21)	87.	(6)	88.	(85)	89.	(21)	90.	(16)

SOLUTIONS

61. (D)

$3^{2022} = 9^{1011} = (10-1)^{1011} = 10m - 1 = 10m - 5 + 4$

$= 5(2m-1) + 4$ (m is integer)

Remainder = 4

62. (B)

Here, α, β roots of equation $3x^2 + \lambda x - 1 = 0$

$\alpha + \beta = \frac{-\lambda}{3}, \ \alpha\beta = \frac{-1}{3}$

$\frac{1}{\alpha^2} + \frac{1}{\beta^2} = \frac{(\alpha+\beta)^2 - 2\alpha\beta}{\alpha^2\beta^2} = 15$

$\lambda^2 = 9$

Now, $6(\alpha^3 + \beta^3)^2 = 6\left((\alpha+\beta)\left((\alpha+\beta)^2 - 3\alpha\beta\right)\right)^2$

$= 6\left(\frac{\lambda^2}{9}\right)\left\{\frac{\lambda^2}{9} + 1\right\}^2 = 24$

63. (A)

$n = 33$, let probability of success is p and $q = 1 - p$

$3p(x=0) = p(x=1)$

$3 \cdot {}^{33}C_0 (q)^{33} = {}^{33}C_1 pq^{32}$

$p = \frac{1}{12}, q = \frac{11}{12}, \frac{q}{p} = 11$

$\frac{p(x=15)}{p(x=18)} - \frac{p(x=16)}{p(x=17)}$

$$\frac{^{33}C_{15}p^{15}q^{18}}{^{33}C_{18}p^{18}q^{15}} - \frac{^{33}C_{15}p^{16}q^{17}}{^{33}C_{18}p^{17}q^{16}} = \left(\frac{q}{p}\right)^3\left(\frac{q}{p}\right)$$

$$= (11)^3 - 11$$

$$= 1320$$

64. (D)

$$\Delta = \begin{vmatrix} 1 & 1 & \alpha \\ 3 & 1 & 1 \\ 1 & 0 & 2 \end{vmatrix} = -(\alpha + 3)$$

$$\Delta_1 = \begin{vmatrix} 2 & 1 & \alpha \\ 4 & 1 & 1 \\ 1 & 0 & 3 \end{vmatrix} = -(3 + \alpha)$$

$$\Delta_2 = \begin{vmatrix} 1 & 2 & \alpha \\ 3 & 4 & 1 \\ 1 & 1 & 2 \end{vmatrix} = -(\alpha + 3)$$

$$\Delta_3 = \begin{vmatrix} 1 & 1 & 2 \\ 3 & 1 & 4 \\ 1 & 0 & 1 \end{vmatrix} = 0$$

$\alpha \neq -3, x = 1, y = 1, z = 0,$

Now points $(\alpha, 1), (1, \alpha)$ & $(1, -1)$ are collinear

$$\begin{vmatrix} \alpha & 1 & 1 \\ 1 & \alpha & 1 \\ 1 & -1 & 1 \end{vmatrix} = 0$$

$$\Rightarrow \alpha(\alpha + 1) - 1(1 - 1) + 1(-1 - \alpha) = 0$$

$$\alpha^2 + \alpha - 1 - \alpha = 0$$

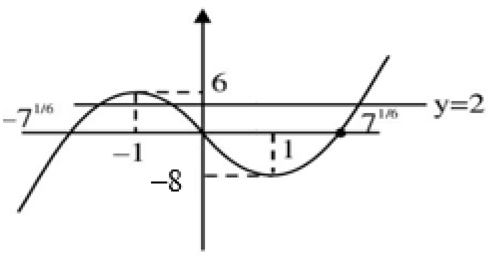
$$\alpha = \pm 1$$

65. (D)

$$x^7 - 7x - 2 = 0$$

$$f'(x) = 7(x^6 - 1) = 7(x^2 - 1)(x^4 + x^2 + 1)$$

$$f'(x) = 0 \Rightarrow x = \pm 1$$



$f(x) = 0$ has 3 real distinct solution.

66. (A)

$f(x) = x^3 + x - 5$

$\Rightarrow$ $f'(x) = 3x^2 + 1$

$\Rightarrow$ Since $f(x)$ is increasing function

$\Rightarrow$ So, $f(x)$ is invertible

$\Rightarrow$ $g(x)$ is inverse of $f(x)$

$\Rightarrow$ $g(f(x)) = x$

$\Rightarrow$ $g'(f(x))\,f'(x) = 1$

$f(x) = 63$

$\Rightarrow$ $x^3 + x - 5 = 63$

$\Rightarrow$ $x = 4$

Put $x = 4$

$g'(f(4))\,f'(4) = 1$

$g'(63) \times 49 = 1$ $\{f'(4) = 49\}$

$g'(63) = \dfrac{1}{49}$

67. (C)

$C_1 : x^2 + y^2 = 2$

$C_2 : y^2 = x$

Let tangent to parabola be $y = mx + \dfrac{1}{4m}$.

It is also a tangent of circle so distance from centre of circle (0, 0) will be $\sqrt{2}$.

$$\left|\frac{\frac{1}{4m}}{\sqrt{1+m^2}}\right| = \sqrt{2} \Rightarrow 1 = 32m^2 + 32m^4$$

By solving

$m^2 = \dfrac{3\sqrt{2}-4}{8}$, $m^2 = \dfrac{-3\sqrt{2}-4}{8}$ (rejected)

$m = \pm\sqrt{\dfrac{3\sqrt{2}-4}{8}}$

So, $8|m_1 m_2| = 3\sqrt{2} - 4$

68. (C)

$\bar{z}_1 = i\,\bar{z}_2$

$z_1 = -iz_2$

$$\arg\left(\frac{z_1}{z_2}\right)=\pi \quad \Rightarrow \arg\left(z_1\cdot z_2\right)=\pi$$

$$\arg\left(z_2\right)=\theta$$

$$-\frac{\pi}{2}+\theta+\theta=\pi$$

$$2\theta=\frac{3\pi}{2}$$

$$\arg\left(z_2\right)=\theta=\frac{3\pi}{4},\ \arg z_1=\frac{\pi}{4}$$

69. (A)

$$\lim_{x\to\frac{\pi}{2}}\tan^2 x\left[\sqrt{2\sin^2 x+3\sin x+4}-\sqrt{\sin^2 x+6\sin x+2}\right]=\lim_{x\to\frac{\pi}{2}}\frac{\tan^2 x\left(\sin x-1\right)\left(\sin x-2\right)}{\sqrt{9}\times\sqrt{9}}$$

$$=\lim_{x\to\frac{\pi}{2}}\frac{\tan^2 x\left(\sin x-1\right)\left(\sin x-2\right)}{6}$$

$$=\frac{1}{6}\lim_{x\to\frac{\pi}{2}}\tan^2 x\left(1-\sin x\right)$$

$$=\frac{1}{6}\lim_{x\to\frac{\pi}{2}}\frac{\sin^2 x\left(1-\sin x\right)}{\left(1-\sin x\right)\left(1+\sin x\right)}=\frac{1}{12}$$

70. (C)

$$P_1+\lambda P_2=0$$

$$\Rightarrow \left(x+3y-z-5\right)+\lambda\left(2x-y+z-3\right)=0$$

(2, 1, –2) lies on this plane

$\therefore$ $\lambda=1 \Rightarrow$ Plane is $3x+2y-8=0$

71. (C)

$$\frac{dy}{dx}=\frac{2\left(1+\sin t\right)\times\cos t}{1+\cos 2t}$$

$$\Rightarrow \frac{2\left(1+\sin t\right)\cos t}{2\cos^2 t}=\sqrt{3}$$

$$\Rightarrow t=\frac{\pi}{6},\ y_0=27$$

72. (D)

$$\left(\frac{x}{a}\right)^n+\left(\frac{y}{b}\right)^n=2$$

Slope of tangent at (a, b)

$$n.\left(\frac{x}{a}\right)^{n-1}.\frac{1}{a}+n\left(\frac{y}{b}\right)^{n-1}.\frac{1}{b}\frac{dy}{dx}=0$$

$$\left.\frac{dy}{dx}\right|_{(a,b)}=-\frac{b}{a}$$

$\therefore$ Equation of tangent

$$y-b=-\frac{b}{a}(x-a)$$

$$\frac{x}{a}+\frac{y}{b}=2 \ \ \forall\, n\in N$$

73. (A)

$$l+m-n=0$$

$$3l^2+m^2+cl(l+m)=0$$

$$n=l+m$$

$$3l^2+m^2+cl^2+clm=0$$

$$(3+c)l^2+clm+m^2=0$$

$$(3+c)\left(\frac{l}{m}\right)^2+c\left(\frac{l}{m}\right)+1=0$$

$\because$ lines are parallel.

$$\Rightarrow D=0$$

$$c^2-4(3+c)=0$$

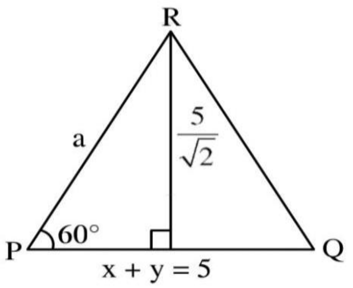
$$c^2-4x-12=0$$

$$(c-6)(c+2)=0$$

$$c=6 \text{ or } x=-2$$

+ve value of $c=6$

74. (D)

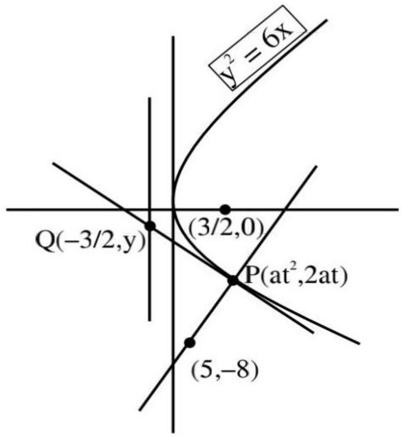


$$\sin 60^\circ=\frac{5/\sqrt{2}}{a}$$

$$a=\frac{5\sqrt{2}}{\sqrt{3}}$$

Area of $\Delta PQR=\frac{\sqrt{3}}{4}a^2=\frac{25}{2\sqrt{3}}$

75. (B)



Equation of normal : $y = -tx + 2at + at^3$ $\left(a = \frac{3}{2}\right)$

Since passing through (5, –8), we get t = –2

Co-ordinate of $Q:(6, -6)$

Equation of tangent at Q : $x + 2y + 6 = 0$

Put $x = -\frac{3}{2}$ to get $R\left(\frac{-3}{2}, \frac{-9}{4}\right)$

76. (D)

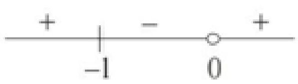
$f(x) = x - 1;\ g(x) = \frac{x^2}{x^2 - 1}$

$f(g(x)) = g(x) - 1$

$= \frac{x^2}{x^2 - 1} - 1 = \frac{x^2 - x^2 + 1}{x^2 - 1}$

$f(g(x)) = \frac{1}{x^2 - 1};\ x \neq \pm 1$, even function $\rightarrow$ Hence $f(g(x))$ is many one function

$y = \frac{1}{x^2 - 1}$



Range :- $y \in (-\infty, -1] \cup (0, \infty)$

Hence, Range $\neq$ Co-domain $\Rightarrow f(g(x))$ is into function

77. (B)

Surface area $= 76x^2 + 3\pi r^2$ = constant (K)

$V = 40x^3 + \frac{2}{3}\pi r^3$

$[76x^2 + 3\pi r^2 = K]$

$$r^2 = \frac{K - 76x^2}{3\pi}$$

$$r = \left(\frac{K - 76x^2}{3\pi}\right)^{\frac{1}{2}}$$

$$V = 40x^3 + \frac{2}{3}\pi\left(\frac{K - 76x^2}{3\pi}\right)^{\frac{3}{2}}$$

$$\frac{dV}{dx} = 120x^2 + \frac{2}{3}\pi.\frac{3}{2}\left(\frac{K - 76x^2}{3\pi}\right)^{\frac{1}{2}}.\left(\frac{-76(2x)}{3\pi}\right)$$

Put

$$\frac{dV}{dx} = 0 \Rightarrow 120x^2 + \frac{2}{3}\pi.\frac{3}{2}\left(\frac{K - 76x^2}{3\pi}\right)^{\frac{1}{2}}.\left(\frac{-76(2x)}{3\pi}\right) = 0$$

$$\Rightarrow \quad 120x^2 = \frac{152x}{3}\left(\frac{k - 76x^2}{3\pi}\right)^{\frac{1}{2}}$$

$$\Rightarrow \quad \frac{45}{19}x^2 = x\left(\frac{k - 76x^2}{3\pi}\right); x \neq 0$$

$$\Rightarrow \quad \frac{45}{19}x = \left(\frac{k - 76x^2}{3\pi}\right)^{\frac{1}{2}} \Rightarrow \left(\frac{45}{19}\right)^2 x^2 = \frac{k - 76x^2}{3\pi}$$

$$\Rightarrow \quad \left(\frac{45}{19}\right)^2 x^2 = r^2$$

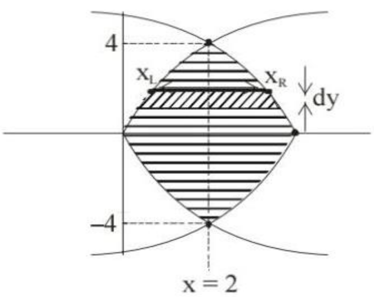
$$\Rightarrow \quad \frac{x^2}{r^2} = \left(\frac{19}{45}\right)^2$$

$$\Rightarrow \quad \frac{x}{r} = \frac{19}{45}$$

78. (C)

$$y^2 = 8x; \; y^2 = 16(3 - x)$$

$$y^2 = -16(x - 3)$$



Finding their intersection pts.

$$y^2 = 8x \;\&\; y^2 = -16(x - 3)$$

$8x = -16x + 48$
$24x = 48$
$x = 2;\ y = \pm 4$

Required Area $A = 2 \cdot \int_0^4 (x_R - x_L) dy$

$= 2 \cdot \int_0^4 \left\{ \underbrace{3 - \frac{y^2}{16}}_{(x_R} - \underbrace{\frac{y^2}{8}}_{x_L)} \right\} dy$

$= 2\left(3y - \frac{y^3}{3\times16} - \frac{y^3}{3\times8}\right)_0^4$

$= 2\left(3\times4 - \frac{4\times4\times4}{3\times16} - \frac{4\times4\times4\times2}{3\times8\times2}\right)$

$= 2\left(12 - \frac{4}{3} - \frac{8}{3}\right) = 2\times12\left(1 - \frac{1}{3}\right) = 2\times12\times\frac{2}{3} = 16$

79. (C)

$I.F. = e^{\int e^x(x^2-2)dx} = e^{\int e^x(x^2-2x+2x-2)dx}$

$= e^{e^x(x^2-2x)}$

$y \cdot e^{e^x(x^2-2x)} = \int e^{e^x(x^2-2x)} e^x (x^2 - 2x)(x^2-2)e^x\, dx$

Let $e^x(x^2 - 2x) = t$

So, $y \cdot e^{e^x(x^2-2x)} = \int e^t \cdot t\, dt$

At $x = 0, t = 0$

$= t \cdot e^t - e^t + c$

$x = 0;\ 0 \cdot 1 = 0 - 1 + c \Rightarrow c = 1$

For $x = 2;\ y \cdot 1 = 0 - 1 + 1 = 0$

$y(2) = 0$

80. (B)

$16 \sin 20^\circ \sin 40^\circ \sin 80^\circ$

$= 16 \sin 40^\circ \sin 20^\circ \sin 80^\circ$

$= 4(4\sin(60-20)\sin(20)\sin(60+20))$

$= 4 \times \sin(3\times 20^\circ)$ $\quad [\because \sin 3\theta = 4\sin(60-\theta)\times\sin\theta\times\sin(60+\theta)]$

$= 4 \times \sin 60^\circ$

$= 4 \times \frac{\sqrt{3}}{2} = 2\sqrt{3}$

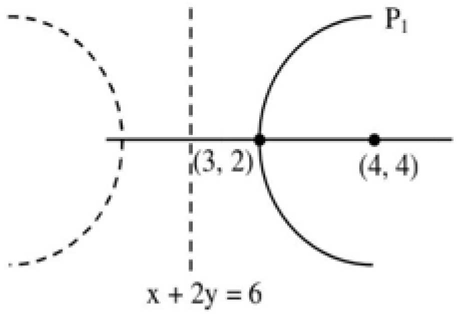
81. (40)

$x_1 + x_2 + x_3 + x_4 + x_5 = 5$

Only one possibilities 3, 3, 3, –2, –2

Number of ways is $= \frac{5!}{3!2!} \times 2 \times 2 = 40$

82. (10)



P_1 : Directorix :

$x + 2y = k$

$x + 2y - k = 0$

$\left|\frac{3+4-K}{\sqrt{5}}\right| = \sqrt{5}$

$|7-k| = 5$

$7 - K = 5$	$7 - K = -5$
$\boxed{k = 2}$	$\boxed{k = 12}$
Accepted	Rejected Passes through focus

$D_1 = x + 2y = 2$
$\ \ = x + 2y = 6$ $\Rightarrow d$ $\Rightarrow d$ $\Rightarrow$ $\boxed{c = 10}$
$D_2 = x + 2y = C$

83. (7)

$2x^2 - rx + p = 0$ → x_1, x_2

$y^2 - sy - q = 0$ → y_1, y_2

Equation of the circle with PQ as diameter is

$2(x^2 + y^2) - rx - 2sy + p - 2q = 0$

On comparing with the given equation

$r = 11, s = 7$

$p - 2q = -22$

$\therefore \quad 2r + s - 2q + p = 22 + 7 - 22 = 7$

84. (98)

$\frac{1}{3} + \frac{5}{9} + \frac{19}{27} + \frac{65}{81} + ...$

$\left(1 - \frac{2}{3}\right) + \left(1 - \frac{4}{9}\right) + \left(1 - \frac{8}{27}\right) + \left(1 - \frac{16}{81}\right) ......100 \text{ terms}$

$100 - \left[\frac{2}{3} + \left(\frac{2}{3}\right)^2 + ...\right]$

$100 - \frac{\frac{2}{3}\left(1 - \left(\frac{2}{3}\right)^{100}\right)}{1 - \frac{2}{3}}$

$100 - 2\left(1 - \left(\frac{2}{3}\right)^{100}\right)$

$S = 98 + 2\left(\frac{2}{3}\right)^{100}$

$\Rightarrow \quad [S] = 98$

85. (200)

Let the common difference is 'd'

$a_1 + a_7 + a_{16} = 40$

$\Rightarrow a_1 + a_1 + 6d + a_1 + 15d = 40$

$\Rightarrow 3a_1 + 21d = 40$

$\Rightarrow a_1 + 7d = \frac{40}{3}$

$S_{15} = \frac{15}{2}[2a_1 + 14d]$

$= 15(a_1 + 7d)$

$= 15\left(\frac{40}{3}\right)$

= 200

86. (21)

Mean deviation about mean of first n natural numbers is $\frac{n^2 - 1}{4n}$

$\therefore \quad n = 21$

87. (6)
The given function will attain a minimum value at x = 6

88. (85)

$$e^2 = 1 + \frac{b^2}{a^2} = \frac{25}{16} \Rightarrow \frac{b^2}{a^2} = \frac{9}{16} \qquad ...(1)$$

$A\left(\frac{8}{\sqrt{5}}, \frac{12}{5}\right)$ satisfies $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

Solving (1) & (2)

$$b = \frac{6}{5},\ a = \frac{8}{5}$$

Normal at A is $\frac{\sqrt{5}a^2x}{8} + \frac{5b^2y}{12} = a^2 + b^2$

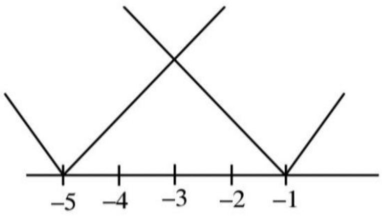
Comparing it $8\sqrt{5}x + \beta y = \lambda$

Gives $\lambda = 100, \beta = 15$

$\lambda - \beta = 85$

89. (21)

$$f(x) = \max\{|x+1|, |x+2|, |x+3|, |x+4|, |x+5|\}$$



$$\int_{-6}^{0} f(x)\,dx = \int_{-6}^{-3} |x+1|\,dx + \int_{-3}^{0} |x+5|\,dx$$

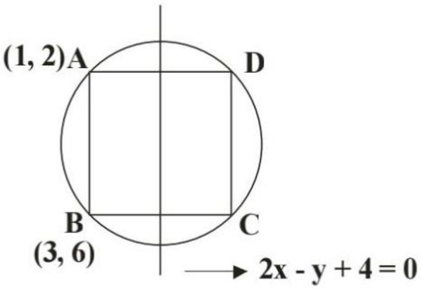
$$= -\int_{-6}^{-3} (x+1)\,dx + \int_{-3}^{0} (x+5)\,dx$$

$$= -\left[\frac{x^2}{2} + x\right]_{-6}^{-3} + \left[\frac{x^2}{2} + 5x\right]_{-3}^{0}$$

$$= -\left[\left(\frac{9}{2} - 3\right) - (18 - 6)\right] + \left[0 - \left(\frac{9}{2} - 15\right)\right]$$

$$= -\left[\frac{3}{2} - 12\right] + \frac{21}{2} = \frac{21}{2} + \frac{21}{2} = 21$$

90. (16)



Equation of line *AB*

$y = 2x$

Slope of *AB* = 2

Slope of given diameter = 2

So the diameter is parallel to *AB*

Distance between diameter and line *AB*

$= \left(\frac{4}{\sqrt{2^2 + 12}}\right) = \frac{4}{\sqrt{5}}$

Thus $BC = 2 \times \frac{4}{\sqrt{5}} = \frac{8}{\sqrt{5}}$

$AB = \sqrt{(1-3)^2 + (2-6)^2} = \sqrt{20} = 2\sqrt{5}$

Area $= AB \times BC = \frac{8}{\sqrt{5}} \times 2\sqrt{5} = 16$ Ans.