PART (A) : PHYSICS

ANSWER KEY

1.	(A)	2.	(C)	3.	(D)	4.	(A)	5.	(B)
6.	(D)	7.	(C)	8.	(A)	9.	(B)	10.	(D)
11.	(B)	12.	(C)	13.	(C)	14.	(C)	15.	(A)
16.	(A)	17.	(C)	18.	(B)	19.	(C)	20.	(B)
21.	(2)	22.	(4)	23.	(6)	24.	(5)	25.	(3)
26.	(9)	27.	(3)	28.	(2)	29.	(3)	30.	(9)

SOLUTIONS

1. (A)

Using Gauss's Law

$$E4\pi r^2 = \frac{1}{\varepsilon_0}\int_0^r \frac{\alpha}{r}4\pi r^2 dr$$

$$E4\pi r^2 = \frac{4\pi\alpha}{\varepsilon_0}\frac{r^2}{2} \Rightarrow E = \frac{\alpha}{2\varepsilon_0} = \text{constant}$$

2. (C)

Acceleration of pulley, $a_1 = 2\,m/s^2$

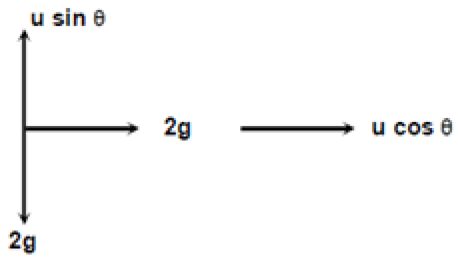
Acceleration of the block, $a_2 = a_1 + 2\alpha R$

$a_2 = a_1 + 2a_1 \quad [\because \alpha R = a_1]$

$a_2 = 3a_1$

$\Rightarrow a_2 = 3\times 2 = 6\,m/s^2$

3. (D)



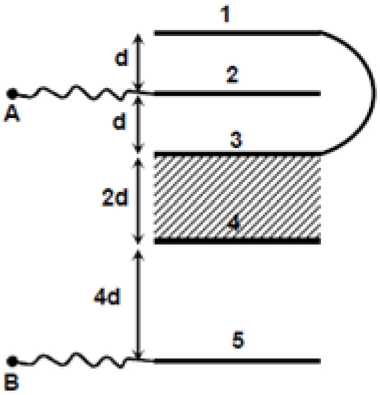
Time of flight, $T = \frac{2u\sin\theta}{2g} = \frac{u\sin\theta}{g}$

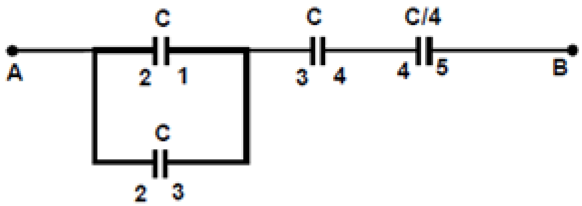
Horizontal range,

$$R = u\cos\theta\left(\frac{u\sin\theta}{g}\right) + \frac{1}{2}(2g)\left(\frac{u^2\sin^2\theta}{g^2}\right)$$

$$R=\frac{u^2\sin\theta\cos\theta}{g}+\frac{u^2\sin^2\theta}{g}=\frac{u^2}{g}\sin\theta(\cos\theta+\sin\theta)$$

4. (A)



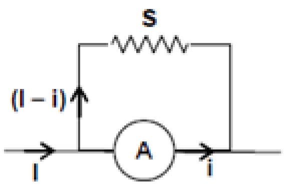


$$C=\frac{\varepsilon_0 S}{d}=1\mu F$$

$$\frac{1}{C_{AB}}=\frac{1}{2}+1+\frac{4}{1}=\frac{11}{2}$$

$$\therefore C_{AB}=\frac{2}{11}\mu F$$

5. (B)



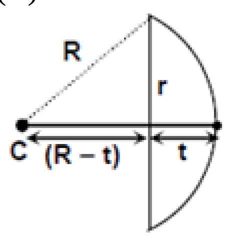
$$(I-i)S=ir$$

$$S=\frac{ir}{(I-i)}$$

6. (D)

$$E=-\frac{dV}{dr}=-\frac{ze}{4\pi\epsilon_0}\left(-\frac{1}{r^2}+\frac{2r}{2R^3}\right)=\frac{ze}{4\pi\varepsilon_0}\left(\frac{1}{r^2}-\frac{r}{R^3}\right)$$

7. (C)



Refractive index of the lens material is

$$\mu = \frac{3\times10^8}{2\times10^8} = \frac{3}{2}$$

Now, $R^2 = r^2 + (R-t)^2$

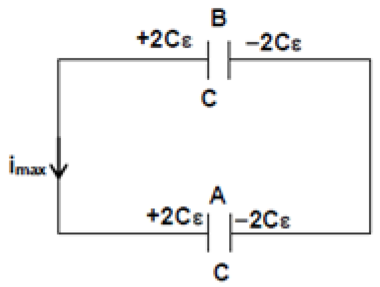
$$R^2 = r^2 + R^2 - 2Rt + t^2$$

$$R = \frac{r^2}{2t} = \frac{3\times3}{2\times0.3} = 15\,\text{cm}$$ (as t^2 is very small, it can be neglected)

$$\frac{1}{f} = \left(\frac{\mu_2}{\mu_1} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = \left(\frac{3}{2} - 1\right)\left(\frac{1}{15}\right)$$

$$f = 30\,\text{cm}$$

8. (A)



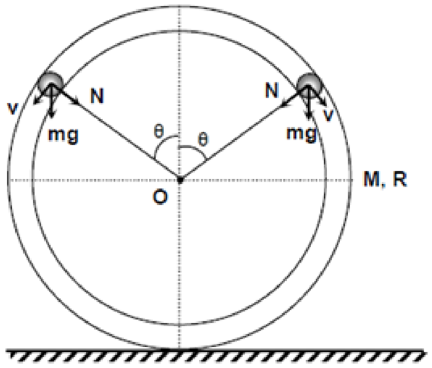
When the current through the inductor is maximum. The two capacitors will be in parallel.

Now, $$\frac{(3C\varepsilon)^2}{2C} + \frac{(C\varepsilon)^2}{2C} = 2\times\frac{(2C\varepsilon)^2}{2C} + \frac{1}{2}Li_{max}^2$$

$$\Rightarrow \frac{9C\varepsilon^2}{2} + \frac{C\varepsilon^2}{2} = 4C\varepsilon^2 + \frac{1}{2}Li_{max}^2$$

$$\Rightarrow i_{max} = \varepsilon\sqrt{\frac{2C}{L}}$$

9. (B)



Since the tube loses contact with horizontal surface when $\theta = 60^\circ$

$$2N\cos\theta = Mg$$

$$2N\cos 60^\circ = Mg$$

$$N = Mg \qquad \ldots\ldots\ldots\ldots (i)$$

Now, $mg\cos\theta + N = \frac{mv^2}{R}$

$\frac{mg}{2} + Mg = \frac{mv^2}{R}$ ………….. (ii)

Using conservation of energy

$mgR(1-\cos\theta) = \frac{1}{2}m(v^2 - u^2)$

$\frac{mgR}{2} = \frac{m}{2}(v^2 - gR)$

$gR = v^2 - gR$

$v = \sqrt{2gR}$ ………….. (iii)

From (ii) and (iii)

$\frac{mg}{2} + Mg = 2mg$

$Mg = \frac{3mg}{2}$

$M = \frac{3m}{2} = 1.5\,kg$

$M = 1.5\,kg$

10. (D)

$$\frac{W_1}{W_2} = \frac{qV_1}{qV_2} = \frac{\frac{KQ}{R}}{\frac{3}{2}\frac{KQ}{R}} = \frac{2}{3}$$

11. (B)

Energy of each photon, $E = \frac{hc}{\lambda} = \frac{1240}{200} = 6.2\,eV$

Now, $K_{max} = E - \phi = 6.2 - 3.8 = 2.4\,eV$

Also, $eVs = K_{max} = 2.4\,eV$

$V_S = 2.4\,volt$

12. (C)

Torque experienced by the loop is

$\tau = MB = \frac{\sqrt{3}}{4}\ell^2 IB$

Now, $\int \tau dt = \frac{5m\ell^2}{4}\omega$

$\frac{\sqrt{3}}{4}B\ell^2 \int I dt = \frac{5m\ell^2}{4}\omega$

$\sqrt{3}BQ = 5m\omega$

$\omega = \dfrac{\sqrt{3}BQ}{5m}$

13. (C)

$\eta = \dfrac{E_2 I_2}{E_1 I_1} \times 100 = 83.3\%$

14. (C)

Fundamental frequency of a sonometer wire, $f = \dfrac{1}{2\ell}\sqrt{\dfrac{T}{\mu}}$

When tension is increased by 44%, then

$\Rightarrow \dfrac{f_1}{f} = \sqrt{\dfrac{1.44T}{T}} = 1.2$

$\therefore f_1 = 1.2f \Rightarrow f + 6 = 1.2f$

$\Rightarrow f = 30\,Hz$

When the length is increased by 20%, then

$\dfrac{f_2}{f} = \dfrac{\ell}{\ell_2} = \dfrac{\ell}{1.2\ell}$

$f_2 = \dfrac{f}{1.2} = \dfrac{30}{1.2} = 25\,Hz$

$\therefore$ decrease in the fundamental frequency $= f - f_2 = 5\,Hz$

15. (A)

$t(\mu - 1) = 4\lambda$

$t = \dfrac{4\lambda}{(\mu - 1)} = 4.8\,\mu m$

16. (A)

To avoid disintegration

$\left(\dfrac{GM}{R^3}\right) r \geq \omega^2 r$

$\omega^2 \leq \dfrac{4\pi G\rho}{3}$

$\omega \leq \sqrt{\dfrac{4\pi G\rho}{3}}$

$\dfrac{2\pi}{T} \leq \sqrt{\dfrac{4\pi G\rho}{3}}$

$T \geq \sqrt{\dfrac{3\pi}{G\rho}}$

$$T \geq \sqrt{\frac{3\pi}{\frac{20}{3}\times10^{-11}\times\frac{9\pi}{20}\times10^{11}}}$$

$T \geq 1\text{sec}$

17. (C)

Electric field 10 V/cm

Distance moved 1 cm

Time taken $20\mu\text{sec}$

$\therefore$ velocity of minority carrier $= \frac{1\text{cm}}{20\times10^{-6}\text{ sec}} = 50,000\text{ cm/s}$

Drift velocity $v_d = \mu E$

$$\mu = \frac{V_d}{E} = \frac{50,000}{10} = 5000\text{ cm}^2/\text{V}-\text{s}$$

18. (B)

$H = i^2Rt = ms\Delta\theta$

$(0.1)^2 10^3 t = 10\times10^3\times4.2\times10$

$$t = \frac{420}{0.01} = 42000\text{ sec}$$

$t = 700\text{ minutes}$

19. (C)

Since diode is forward biased so it behaves as short circuit.

Equivalent resistance of circuit $= 15\text{k}\Omega$

Current through the cell $= \frac{30}{15} = 2\text{ mA}$

Current between A and B = 1 mA

$V_{AB} = 10\text{k}\Omega\times1\text{mA} = 10\text{V}$

20. (B)

$f_0 = 1.5\text{cm}, f_e = 5\text{cm}$

$$\frac{1}{v} - \frac{1}{-2} = \frac{1}{1.5}$$

$$\frac{1}{v} = \frac{1}{1.5} - \frac{1}{2}$$

$v = +6\text{cm}$

Hence the magnification for the adjustment for near point vision is

$$m = \frac{v}{u}\left(1+\frac{D}{f_e}\right) = \frac{-6}{2}\left(1+\frac{25}{5}\right) = -18$$

$|m| = 18$

21. (2)

Diameter $= 2.4 + 2 \times LC - (0.4 + 2 \times LC) = 2\,\text{cm}$

22. (4)

$$m\frac{d^2x}{dt^2} = -16kx$$

$$\frac{d^2x}{dt^2} = -\left(\frac{16k}{m}\right)x$$

$\therefore$ frequency of small oscillations $f = \frac{1}{2\pi}\sqrt{\frac{16k}{m}} = \frac{4}{2\pi}\sqrt{\frac{k}{m}}$

$\therefore n = 4$

23. (6)

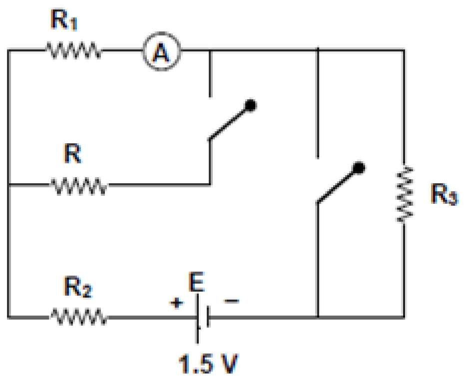
With both switches open : $I_A = \frac{E}{R_1 + R_2 + R_3}$

With both switches closed:

Current through E and R_2

$$I' = \frac{E}{R_2 + \frac{RR_1}{R + R_1}}$$

Current through the ammeter

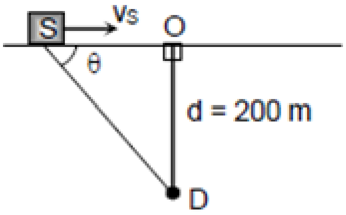


$$I'_A = \frac{RI'}{R + R_1}$$

$$I_A = I'_A$$

$$R = \frac{R_1R_2}{R_3} = 600\,\Omega$$

24. (5)



$$\cos\theta = \frac{v_s t}{vt} = \frac{v_s}{v} = \frac{66}{330} = \frac{1}{5} = 0.2$$

$$f = \left(\frac{v}{v - v_s\cos\theta}\right)f_0 = \frac{f_0}{\left(1 - \frac{1}{25}\right)} = \frac{25}{24}\times 600 = 625\,\text{Hz}$$

25. (3)

Total work done by gas per cycle, $\Delta W_{cycle} = P_0V_0$

During the process AB

$$\Delta Q_{AB} = nC_V\Delta T = n\frac{3R}{2}\Delta T = \frac{3}{2}V_0\left(2P_0 - P_0\right) = \frac{3}{2}P_0V_0$$

During the process BC

$$\Delta Q_{BC} = nC_P\Delta T = n\frac{5R}{2}\Delta T = \frac{5}{2}\times 2P_0\left(2V_0 - V_0\right) = 5P_0V_0$$

Efficiency of the cycle

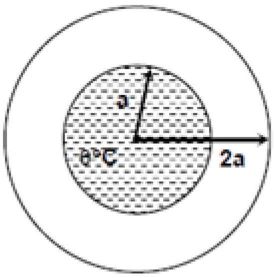
$$\eta = \frac{\Delta W_{cycle}}{\Delta Q_{supplied}}\times 100 = \frac{P_0V_0}{\left(\frac{3P_0V_0}{2} + 5P_0V_0\right)}\times 100 = \frac{2}{13}\times 100 = 15.38\%$$

26. (9)

In steady state, potential difference across $6\,\mu F$ capacitor $= 7.5V$

$\therefore$ Charge on the $6\,\mu F$ capacitor, $q = 6\times 7.5 = 45\,\mu C$

27. (3)



Thermal resistance of the spherical shell is

$$R = \int_a^{2a}\frac{dr}{K4\pi r^2} = \int_a^{2a}\frac{dr}{\frac{\alpha}{r^2}4\pi r^2} = \frac{a}{4\pi\alpha} \qquad \text{.......... (i)}$$

$$-\frac{msd\theta}{dt} = \frac{\theta - \theta_0}{R}$$

$$-mSR\int_{80}^{60}\frac{d\theta}{(\theta-\theta_0)}=\int_0^t dt$$

$$t = mSR\,\ell n\left(\frac{5}{3}\right)$$

$$t=\rho\frac{4}{3}\pi a^3 S\left(\frac{a}{4\pi\alpha}\right)\ell n\left(\frac{5}{3}\right)$$

$$t=\frac{\rho S a^4}{3\alpha}\ell n\left(\frac{5}{3}\right)$$

$$t=\frac{\rho S a^4}{3\alpha}\ell n\left(\frac{5}{3}\right)$$

28. (2)

$$A = A_0 e^{-\lambda t}$$

$$7.8 = 31.2e^{-\frac{\ell n2}{5570}t}$$

$$\Rightarrow -2\ell n2 = -\frac{\ell n2}{5570}t$$

Solving, $t = 11140$ years

29. (3)

Let v = actual velocity of fly towards right

$$v_{approach} = 4+\mu v$$

$$8 = 4+\mu v$$

$$\therefore v = 3m/s$$

30. (9)

$$T - mg = \frac{m(3v_0)^2}{\ell}$$

$$T = mg + \frac{9mv_0^2}{\ell}$$

PART (B) : CHEMISTRY

SOLUTIONS

31.	(A)	32.	(D)	33.	(D)	34.	(C)	35.	(D)
36.	(A)	37.	(D)	38.	(A)	39.	(B)	40.	(A)
41.	(C)	42.	(D)	43.	(C)	44.	(C)	45.	(C)
46.	(A)	47.	(D)	48.	(A)	49.	(C)	50.	(A)
51.	(4)	52.	(9)	53.	(5)	54.	(9)	55.	(4)
56.	(3)	57.	(7)	58.	(3)	59.	(3)	60.	(7)

SOLUTIONS

31. (A)
Among the given chlorofluorocarbons or CF_2Cl_2 are the compounds that are responsible for ozone depletion which degrades ozone into molecular oxygen. It is not a component of photochemical smog.

32. (D)

33. (D)

34. (C)

35. (D)

36. (A)

37. (D)

38. (A)

39. (B)

40. (A)

41. (C)

42. (D)

43. (C)

44. (C)

45. (C)

46. (A)

47. (D)

48. (A)

49. (C)

50. (A)

51. (4)

52. (9)

53. (5)

54. (9)

Given: $\frac{\ell \times m}{n} = 0$ (For Al)

To find : maximum no. of electrons

$\frac{\ell \times m}{n} = 0 \Rightarrow$ This means $\ell = 0$ or m = 0 electrons.

Al = $1s^2\, 2s^2\, 2p^6\, 3s^2\, 3p^1$

For n = 1, where n = shell no.

ℓ m_e $\ell =$ angular momentum quantum no.

0 0 $m_e =$ magnetic quantum no.

For n = 2

ℓ	m_e
0	0
1	$-1, 0, +1$

For n = 3

ℓ	m_e
0	0
1	$-1, 0, +1$
2	$-2, -1, 0, +1, +2$

Total no. of electrons with $\ell = 0$ or $m = 0$

$\Rightarrow 2 + 3 + 4 = 9$

Hence maximum no. of electrons = 9

55. (4)

56. (3)

57. (7)

58. (3)

59. (3)

60. (7)

PART (C) : MATHEMATICS

ANSWER KEY

61.	(C)	62.	(B)	63.	(D)	64.	(A)	65.	(B)
66.	(A)	67.	(B)	68.	(A)	69.	(B)	70.	(D)
71.	(C)	72.	(D)	73.	(A)	74.	(A)	75.	(A)
76.	(A)	77.	(A)	78.	(D)	79.	(D)	80.	(D)
81.	(2)	82.	(6)	83.	(6)	84.	(3)	85.	(0)
86.	(0)	87.	(3)	88.	(3)	89.	(25)	90.	(8.08)

SOLUTIONS

61. (C)

Equation of tangents is

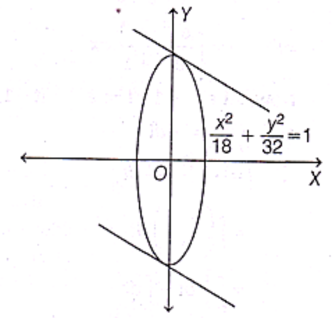
$$y = mx \pm \sqrt{a^2m^2 + b^2}$$

$$\Rightarrow \quad y = \frac{-4}{3}x \pm \sqrt{18 \times \frac{16}{9} + 32}$$

$$\Rightarrow \quad y = \frac{-4}{3}x \pm 8$$

Hence, tangents are

$$y = -\frac{4}{3}x + 8 \text{ and } y = -\frac{4}{3}x - 8$$



$$= \frac{8-(-8)}{\sqrt{1^2 + \left(\frac{4}{3}\right)^2}} = \frac{48}{5}$$

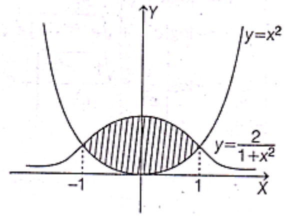
62. (B)

Given, equation of curves are

$$y = x^2 \qquad \ldots(i)$$

$$\text{and } y = \frac{2}{1+x^2} \qquad \ldots(ii)$$

For intersection point,



$$x^2 = \frac{2}{1+x^2}$$

$\Rightarrow$ $x^4 + x^2 - 2 = 0$

$\Rightarrow$ $(x^2 + 2)(x^2 - 1) = 0$

$\because$ $x^2 + 2 = 0$

(not possible in R)

$\therefore$ $x^2 - 1 = 0$

$\Rightarrow$ $x = \pm 1$

Hence, the required area

$$= 2\int_0^1 \left(\frac{2}{1+x^2} - x^2\right)dx$$

$$= 2\left[2\tan^{-1}x - \frac{x^3}{3}\right]_0^1$$

$$= 2\left(\frac{\pi}{2} - \frac{1}{3}\right) = \pi - \frac{2}{3}$$

63. (D)

Clearly,

$$f(-x) = \begin{vmatrix} -x^3 & -\sin x & -\tan x \\ \cos x & x^8 & 1 \\ e^{x^2} & \sec x & 2 \end{vmatrix} = -f(x)$$

$\Rightarrow$ f(x) is odd function.

Hence, $\int\limits_{-\pi/2}^{\pi/2} f(x)\,dx = 0$

64. (A)

From the equation, we can write

$\log_3(\sqrt{x} + |\sqrt{x} - 1|)$

$$= \frac{1}{2}\log_3[4(\sqrt{x} + |\sqrt{x} - 1|) - 3]$$

$\Rightarrow$ $\log_3(\sqrt{x} + |\sqrt{x} - 1|)^2$

$$= \log_3[4(\sqrt{x} + |\sqrt{x} - 1|) - 3]$$

$\Rightarrow$ $(\sqrt{x} + |\sqrt{x} - 1|)^2 = 4(\sqrt{x} + |\sqrt{x} - 1|) - 3$

$\Rightarrow$ $y^2 - 4y + 3 = 0$,

Where $y = \sqrt{x} + |\sqrt{x} - 1|$

$\Rightarrow$ y = 1, 3

When y = 1, then $\sqrt{x} + |\sqrt{x} - 1| = 1$

$\Rightarrow \quad |\sqrt{x}-1| = 1-\sqrt{x} = -(\sqrt{x}-1)$ this is possible only.

When $\sqrt{x}-1 \le 0 \Rightarrow \sqrt{x} \le 1$ or $x \le 1$

Hence, $0 \le x \le 1$

When y = 3, then $\sqrt{x} + |\sqrt{x}-1| = 3$

$\Rightarrow \quad |\sqrt{x}-1| = 3-\sqrt{x}$

$\Rightarrow \quad x+1-2\sqrt{x} = 9+x-6\sqrt{x}$

(on squaring)

$\Rightarrow \quad \sqrt{x} = 2 \text{ or } x = 4$

Here, x = 4 is also satisfies the equation.

Hence, the solutions are $0 \le x \le 1$, x = 4

65. (B)

$\sim(p \vee q) \vee (\sim p \wedge q) \equiv (\sim p \wedge \sim q) \vee (\sim p \wedge q)$

$\equiv (\sim p) \wedge (\sim q \wedge q)$

$\equiv (\sim p) \wedge T \equiv \sim p$

66. (A)

$\because \quad \left(x^4 + 2 + \frac{1}{x^4}\right)^{10} = \left(x^2 + \frac{1}{x^2}\right)^{20}$

General term = (r + 1)th term of $\left(x^2 + \frac{1}{x^2}\right)^{20}$ is

$$T_{r+1} = {}^{20}C_r \cdot (x^2)^{20-r} \cdot \left(\frac{1}{x^2}\right)^r$$

$$= {}^{20}C_r \cdot x^{40-4r}$$

For the coefficient of x^8,

Put 40 – 4r = 8 $\quad \Rightarrow$ r = 8

And for the coefficient of x^{-24}.

Put 40 – 4r = – 24

$\Rightarrow$ r = 16

$\therefore \quad \alpha = {}^{20}C_8$ and $\beta = {}^{20}C_{16}$

$$\Rightarrow \quad \frac{\alpha}{\beta} = \frac{20!}{8!12!} \times \frac{16!4!}{20!}$$

$$= \frac{16 \times 15 \times 14 \times 13}{8 \times 7 \times 6 \times 5} = 26$$

67. (B)

Since, f(x) is continuous at x = 0.

$\therefore \quad \lim_{x \to 0} f(x) = \lim_{x \to 0^+} f(x) = f(0)$

i.e. LHL = RHL = f(0) $\quad \ldots$(i)

Now, LHL at x = 0 i.e. $f(0^-)$

$$\lim_{x \to 0^-} f(x) = \lim_{x \to 0^-} (\cos x - \sin x)^{\operatorname{cosec} x}$$

$$= \lim_{h \to 0} (\cos h + \sin h)^{-\operatorname{cosec} h}$$

$$= \lim_{h\to 0}[1+(\cos h+\sin h-1)]$$

i.e. 1^{∞} form

$$= e^{\lim_{h\to 0}\{\cos h+\sin h-1\}\left(\frac{-1}{\sin h}\right)}$$

$$= e^{\lim_{h\to 0}\left\{-2\sin^2\frac{h}{2}+2\sin\frac{h}{2}\cos\frac{h}{2}\right\}\left(\frac{-1}{2\sin\frac{h}{2}\cdot\cos\frac{h}{2}}\right)}$$

$$= e^{\lim_{h\to 0}\left(\frac{\sin\frac{h}{2}-\cos\frac{h}{2}}{\cos\frac{h}{2}}\right)} = e^{-1} = \frac{1}{e}$$

$\Rightarrow$ LHL = $\frac{1}{e}$ ...(i)

RHL at $x=0 = \lim_{x\to 0^+} f(x)$

$$= \lim_{x\to 0^+}\frac{e^{1/x}+e^{2/x}+e^{3/x}}{ae^{2/x}+be^{3/x}}$$

$$= \lim_{x\to 0}\frac{e^{1/h}\left\{\frac{1}{e^{2/h}}+\frac{1}{e^{1/h}}+1\right\}}{e^{3/h}\left\{\frac{a}{e^{1/h}}+b\right\}}$$

$\Rightarrow$ RHL = $\frac{1}{b}$ and f(0) = a

From Eq. (i), a = e^{-1} = $\frac{1}{b}$

$\Rightarrow$ a = $\frac{1}{e}$ and b = e

68. (A)

$$S_n = \sum_{k=1}^{n}\tan^{-1}\left(\frac{2K}{1+(K^4+K^2+1)}\right)$$

$$= \sum_{k=1}^{n}\tan^{-1}\left(\frac{2K}{1+(K^2+K+1)(K^2-K+1)}\right)$$

$$= \sum_{k=1}^{n}\tan^{-1}\left(\frac{(K^2+K+1)-(K^2-K+1)}{1+(K^2+K+1)(K^2-K+1)}\right)$$

$$\Rightarrow S_n = \sum_{k=1}^{n}[\tan^{-1}(K^2+K+1)-\tan^{-1}(K^2-K+1)]$$

$$\Rightarrow S_n = [(\tan^{-1}3-\tan^{-1}1)+(\tan^{-1}7-\tan^{-1}3)+$$
$$(\tan^{-1}13-\tan^{-1}7)+....+\{\tan^{-1}(n^2+n+1)-\tan^{-1}(n^2-n+1)\}]$$

$$= \tan^{-1}(n^2+n+1)-\tan^{-1}1$$

$$= \tan^{-1}\left(\frac{n^2+n}{1+n^2+n+1}\right)$$

$$S_n = \tan^{-1}\left(\frac{n^2+n}{n^2+n+2}\right)$$

$$\lim_{n\to\infty} S_n = \lim_{n\to\infty} \tan^{-1}\left(\frac{1+\frac{1}{n}}{1+\frac{1}{n}+\frac{2}{n^2}}\right) = \frac{\pi}{4}$$

69. (B)

Given, $y(x) = f(e^x)\, e^{f(x)}$,

$f(0) = f(1) = 0,\ f'(1) = 2$

$\therefore\quad y'(x) = f'(e^x)\cdot e^x\cdot e^{f(x)} + f(e^x)\, e^{f(x)}\, f'(x)$

$y'(0) = f'(1)\cdot 1\cdot e^{f(0)} + f(1)\, e^{f(0)}\, f'(0)$

$= 2e^0 + 0\cdot e^0\cdot f'(0)$

$= 2 + 0$

$y'(0) = 2$

70. (D)

Equation of required plane is

$$\begin{vmatrix} x-1 & y-1 & z-1 \\ 1 & 2 & 3 \\ 1 & 3 & 5 \end{vmatrix} = 0$$

$\Rightarrow\quad (x - 1) - 2(y - 1) + (z - 1) = 0$

$\Rightarrow\quad x - 2y + z = 0$

71. (C)

$$\frac{52!}{(17!)^3\cdot 3!}\times 3!$$

72. (D)

The best approach to solve this problem is graphical approach.

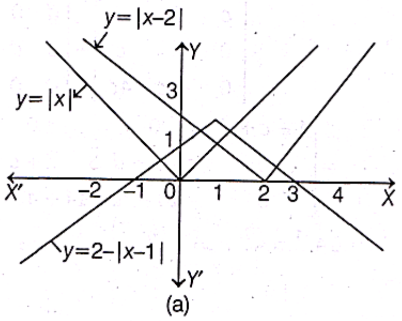
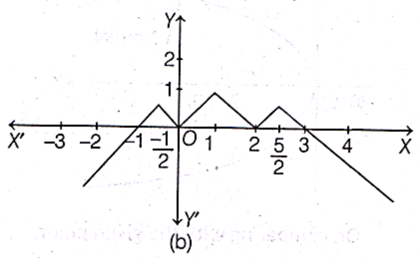


Fig. (a) graph of $y = |x|, |x-2|$ and $2-|x-1|$



It is clear from figure (b) that f(x) is continuous $\forall x \in R$ but

Non-differentiable at $x = -\frac{1}{2}, 0, 1, 2, \frac{5}{2}$.

$\therefore$ Sum of non-differentiable points

$$=-\frac{1}{2}+0+1+2+\frac{5}{2}$$
$$=\frac{-1+6+5}{2}=\frac{10}{2}=5$$

73. (A)

Method (i)

$$6A^{-1}=A^2+cA+dI$$
$\Rightarrow$ $6I = A^3 + cA^2 + dA$

$\Rightarrow$ $A^3 + cA^2 + dA - 6I = 0$ …(i)

Now, $|A-\lambda I|=\begin{vmatrix}1-\lambda & 0 & 0\\ 0 & 1-\lambda & 1\\ 0 & -2 & 4-\lambda\end{vmatrix}$

$$=(1-\lambda)(\lambda^2-5\lambda+4+2)$$
$$=-\lambda^3+6\lambda^2-11\lambda+6$$

Putting $|A-\lambda I|=0$

$\Rightarrow$ $-\lambda^3+6\lambda^2-11\lambda+6=0$

$\Rightarrow$ $\lambda^3-6\lambda^2+11\lambda-6=0$

Cayley Hamilton theorem gives

$A^3 - 6A^2 + 11A - 6I = 0$ …(ii)

From Eqs. (i) and (ii), we get c = – 6 and d = 11

Method (ii)

$|A|=\begin{vmatrix}1 & 0 & 0\\ 0 & 1 & 1\\ 0 & -2 & 4\end{vmatrix}=6\neq 0$ …(i)

i.e. A is non-singular.

adj A $=\begin{vmatrix}6 & 0 & 0\\ 0 & 4 & 2\\ 0 & -1 & 1\end{vmatrix}=\begin{vmatrix}6 & 0 & 0\\ 0 & 4 & -1\\ 0 & 2 & 1\end{vmatrix}$ …(ii)

from Eqs. (i) and (ii), we get

$$A^{-1}=\frac{\text{adj}A}{|A|}=\frac{1}{6}\begin{vmatrix}6 & 0 & 0\\ 0 & 4 & -1\\ 0 & 2 & 1\end{vmatrix}$$

$$A^2=\begin{vmatrix}1 & 0 & 0\\ 0 & 1 & 1\\ 0 & -2 & 4\end{vmatrix}\begin{vmatrix}6 & 0 & 0\\ 0 & 1 & 1\\ 0 & -2 & 4\end{vmatrix}$$

$$=\begin{vmatrix}1 & 0 & 0\\ 0 & -1 & 5\\ 0 & -10 & 14\end{vmatrix}$$

Now, $A^2 + cA + dI=\begin{vmatrix}1 & 0 & 0\\ 0 & -1 & 5\\ 0 & -10 & 14\end{vmatrix}$

$$+\begin{vmatrix} c & 0 & 0 \\ 0 & c & c \\ 0 & -2c & 4c \end{vmatrix}+\begin{vmatrix} d & 0 & 0 \\ 0 & d & 0 \\ 0 & 0 & d \end{vmatrix}$$

$$=\begin{vmatrix} 1+c+d & 0 & 0 \\ 0 & c-1+d & 5+c \\ 0 & -10-2c & 14+4c+d \end{vmatrix}$$

$6A^{-1} = A^2 + cA + dl$

$$\Rightarrow \begin{vmatrix} 6 & 0 & 0 \\ 0 & 4 & -1 \\ 0 & 2 & 1 \end{vmatrix}$$

$$=\begin{vmatrix} 1+c+d & 0 & 0 \\ 0 & c-1+d & 5+c \\ 0 & -10-2c & 14+4c+d \end{vmatrix}$$

$\Rightarrow$ $1 + c + d = 6$, $c - 1 + d = 4$

$5 + c = -1$, $-10 - 2c = 2$

$14 + 4c + d = 1$

$\Rightarrow$ $c = -6$ and $d = 11$

74. (A)

Let equation of circle passing through (p, q) be

$x^2 + y^2 + 2gx + 2fy + c = 0$ …(i)

$p^2 + q^2 + 2pg + 2qf + c = 0$ …(ii)

Since Eq. (i) cuts $x^2 + y^2 = r^2$

Orthogonally,

$(2g) \cdot 0 + (2f) \cdot 0 = c - r^2$

$\Rightarrow$ $c = r^2$

On putting in Eq. (ii),

$2pg + 2qf + (p^2 + q^2 + r^2) = 0$

Locus of centre $(-g, -f)$ is

$2px + 2qy - (p^2 + q^2 + r^2) = 0$

75. (A)

Dividing N' and D' by x^2, then the given integral can be written as

$$\int \frac{\left(1-\frac{1}{x^2}\right)dx}{\left[\left(x+\frac{1}{x}\right)^2+1\right]\tan^{-1}\left(x+\frac{1}{x}\right)}$$

$$=\int \frac{dt}{(t^2+1)\tan^{-1}t}$$

Where $t = x + \frac{1}{x}$

Let $u = \tan^{-1} t$

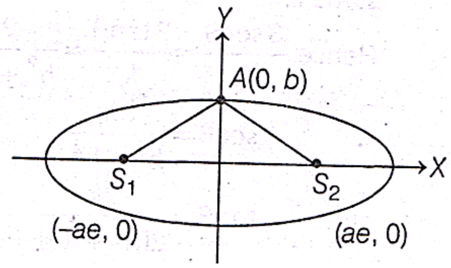
$\Rightarrow$ $du = \frac{1}{1+t^2}dt$

So, $\int \frac{dt}{(t^2+1)\tan^{-1} t} = \int \frac{du}{u} = \ln|u| + C$

$$= \ln\left|\tan^{-1}\left(\frac{x^2+1}{x}\right)\right| + C$$

On comparing, k = 1

76. (A)



$$\frac{b-0}{0-ae} \times \frac{b-0}{0+ae} = -1$$

$\Rightarrow$ $b^2 = a^2e^2$

$\Rightarrow$ $a^2(1-e^2) = a^2e^2$

$\Rightarrow$ $e = \frac{1}{\sqrt{2}}$

77. (A)

$|z|^2 \omega - z|\omega|^2 - z + \omega = 0$ ...(i)

$\omega(|z|^2+1) - z(|\omega|^2+1) = 0$

$\Rightarrow$ $\frac{\omega}{z} = \frac{|\omega|^2+1}{|z|^2+1}$

$\Rightarrow$ $\frac{\omega}{z}$ is a real number.

$\therefore$ $\frac{\omega}{z} = \frac{\overline{\omega}}{\overline{z}} \Rightarrow \overline{\omega}z = \overline{z}\omega$ ...(ii)

From Eq. (i),

$z\overline{z}\omega - z\omega\overline{\omega} - z + \omega = 0$

$(\because z\overline{z} = |z|^2)$

$\Rightarrow$ $z(\overline{z}\omega - 1) - \omega(\overline{\omega}z - 1) = 0$

[using Eq. (ii)]

$\Rightarrow$ $(z-\omega)(z\overline{\omega} - 1) = 0$

$\Rightarrow$ $z = \omega$ or $z\overline{\omega} = 1$

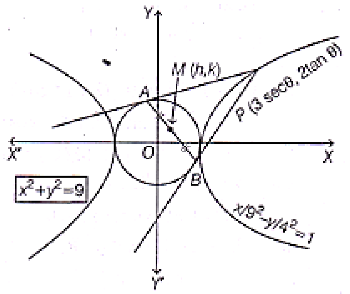
78. (D)

Equation of hyperbola and circle are

$\frac{x^2}{9} - \frac{y^2}{4} = 1$ ...(i)

And $x^2 + y^2 = 9$ ...(ii)

Any point on hyperbola (i) is taken as $P(3\sec\theta, 2\tan\theta)$



Now, equation of chord of contact of circle w.r.t. point P is T = 0

i.e. $(3\sec\theta)x+(2\tan\theta)y-9=0$ …(iii)

Also, the equation of chord AB of circle with mid-point M(h, k) is T = S_1

i.e. $hx + ky - 9 = h^2 + k^2 - 9$

$\Rightarrow hx + ky = h^2 + k^2$

$\Rightarrow hx + ky - (h^2 + k^2) = 0$ …(iv)

Since, Eqs. (iii) and (iv) represent same line.

Hence, $\frac{3\sec\theta}{h}=\frac{2\tan\theta}{k}=\frac{9}{h^2+k^2}$

$\therefore \sec\theta=\frac{3h}{h^2+k^2}$

And $\tan\theta=\frac{9k}{2(h^2+k^2)}$

As we know that,

$\sec^2\theta-\tan^2\theta=1$

$\Rightarrow \frac{9h^2}{(h^2+k^2)^2}-\frac{81h^2}{4(h^2+k^2)^2}=1$

$\Rightarrow 9(4h^2 - 9k^2) = 4(h^2 + k^2)^2$

Locus of M(h, k) is $9(4x^2-9y^2) = 4(x^2 + y^2)^2$

79. (D)

For $-1 < x < 0$; $[x] = -1$

$\therefore f(x) = \min\{x(-1), |-x-2|+2\} = -x$

$(\because -x<1, |-x-2|+2\geq 2)$

For $0\leq x<1; [x]=0$

$\therefore f(x) = \min\{0, |0-2|+2\} = 0$

For $1\leq x<2; [x]=1$

$\therefore f(x) = \min\{x, -|x-2|+2\} = x$

$(\because 1\leq x<2, |x-2|+2\geq 2)$

For $2\leq x<3, [x]=2$

$\therefore f(x) = \min\{2x, |2x-2|+2\}$

$= \min\{2x, 2x-2+2\} = 2x$

Thus, $f(x)=\begin{cases}-x & , -1<x<0\\ 0 & , 0\leq x<1\\ x & , 1\leq x<2\\ 2x & , 2\leq x<3\end{cases}$

From above we observe that f(x) is not continuous at x = 1 and x = 2

and f(x) is not differentiable at x = 0, 1 and 2.

80. (D)

Given, $\frac{dy}{dx} = (e^y - x)^{-1}$

$\Rightarrow \frac{dx}{dy} = e^y - x$

$\Rightarrow \frac{dx}{dy} + x = e^y$

So, integrating factor (IF) $= e^{\int dy} = e^y$

$\therefore$ General solution is given by

$x \times (IF) = \int e^y \times (IF) dy$

$x \cdot e^y = \frac{1}{2} e^{2y} + C$

$\Rightarrow x = \frac{1}{2} e^y + Ce^{-y}$

Also, given y(0) = 0

$\Rightarrow 0 = \frac{1}{2} e^0 + C \cdot e^{-0}$

$\Rightarrow C = -\frac{1}{2}$

Thus, $x = \frac{1}{2} e^y - \frac{1}{2} e^{-y}$

$\Rightarrow e^{2y} - 2xe^y - 1 = 0$

$\Rightarrow 2e^y = 2x \pm \sqrt{4x^2 + 4}$

Hence, $y = \text{In}(x + \sqrt{1 + x^2})$

81. (2)

Given, f(x) = a sin x + $\frac{1}{4}$ sin 4x

On differentiating w.r.t. x, we get

F'(x) = a cos x + $\frac{1}{4}$ (4 cos 4x)

F'(x) = a cos x + cos 4x

For maximum value of f(x)

F'(x) = 0

At x = $\frac{\pi}{4}$,

$F'\left(\frac{\pi}{4}\right) = 0$

$a \cos\left(\frac{\pi}{4}\right) + \cos 4\left(\frac{\pi}{4}\right) = 0$

$a = \sqrt{2} \Rightarrow a^2 = 2$

82. (6)

Method 1

As we know that latus rectum is the least focal chord.

$\therefore$ Length of latus rectum = 32 = λ (given)
$\therefore$ Divisors of 32 are 1, 2, 4, 8, 16, 32
$\therefore$ Number of divisors = 6
Method 2
As we know that, length of focal chord having one extremely $(at^2, 2at)$ is given by a $\left(t+\frac{1}{t}\right)^2$

i.e. PQ = a $\left(t+\frac{1}{t}\right)^2$

$\therefore \quad \left|t+\frac{1}{t}\right| \geq 2$ [using AM ≥ GM]

$\Rightarrow \quad \left|t+\frac{1}{t}\right|^4 \geq 4$

$\Rightarrow \quad \left(t+\frac{1}{t}\right)^2 \geq 4$

$\Rightarrow \quad a\left(t+\frac{1}{t}\right)^2 \geq 4a$ [here a = 8 > 0]

$\Rightarrow$ PQ ≥ 32
Hence, λ = 32
Therefore, number of divisors of λ = 6.

83. (6)
Let $x^2 = 4\cos^2\theta + \sin^2\theta$
Then, $(4-x^2) = 3\sin^2\theta$
And $(x^2-1) = 3\cos^2\theta$

$\therefore \quad f(x) = \sqrt{3}|\sin\theta| + \sqrt{3}|\cos\theta|$

$\Rightarrow \quad f(x)_{min} = \sqrt{3}$

And $f(x)_{max} = \sqrt{3}\left(\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}\right) = \sqrt{6}$

Hence, range of $f(x)$ is $[\sqrt{3}, \sqrt{6}]$.
$\therefore$ Maximum value of $(f(x))^2$ is 6.

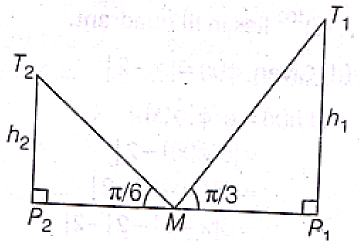
84. (3)
Let P_1T_1 and P_2T_2 be two towers, M be the mid-point of P_1P_2.
From the right angled tringles T_1MP_1 and T_2MP_2

$$\frac{P_1T_1}{MP_1} = \tan\left(\frac{\pi}{3}\right) = \sqrt{3} \quad \ldots(i)$$

And $$\frac{P_2T_2}{MP_2} = \tan\left(\frac{\pi}{6}\right) = \frac{1}{\sqrt{3}} \quad \ldots(ii)$$

From Eqs. (i) and (ii), we get

$$\frac{P_1T_1}{MP_1} \times \frac{MP_2}{P_2T_2} = \frac{1}{\sqrt{3}} = \sqrt{3}\times\sqrt{3} = 3$$



$\Rightarrow \dfrac{P_1T_1}{P_2T_2} = 3$ $(\because MP_1 = MP_2)$

$\Rightarrow \dfrac{h_1}{h_2} = 3$

85. (0)
Area bounded by X-axis, the curve
Y = f(x) and the ordinates x = 1, x = a is

$$\int_1^a f(x)dx = \sqrt{(1+a^2)} - \sqrt{2}$$

[given]
Differentiating both sides w.r.t. a, we get

$$f(a) = \frac{a}{\sqrt{1+a^2}}$$

Hence, $f(x) = \dfrac{x}{\sqrt{1+x^2}}$

So, f(0) = 0

86. (0)

Let $I = \displaystyle\int \frac{1}{\sin x + \sin 2x} dx$

$$= \int \frac{1}{\sin x(1+2\cos x)} dx$$

$$= \int \frac{1}{\sin^2 x(1+2\cos x)} \times \sin x\, dx$$

$$= \int \frac{1}{(1-\cos x)(1+\cos x)(1+2\cos x)} \times \sin x\, dx$$

Let cos x = z $\Rightarrow$ sin x dx = – dz

$\therefore \quad I = \displaystyle\int \frac{1}{(z-1)(z+1)(1+2z)} dz$

Let $\dfrac{1}{(z-1)(z+1)(1+2z)} = \dfrac{A}{z-1} + \dfrac{B}{z+1} + \dfrac{C}{1+2z}$

$\Rightarrow$ I = A(z + 1)(2z + 1) + B(z – 1) (2z + 1) + C(z^2 – 1)

On comparing the coefficient, we get

$$\left.\begin{aligned} 2A+2B+C=0 \\ 3A-B=0 \\ A-B-C=1 \end{aligned}\right\}$$

By solving, we get

$$A = \frac{1}{6}, B = \frac{1}{2}, C = -\frac{4}{3}$$

$$\therefore \quad I = \frac{1}{6}\int \frac{1}{z-1}dz + \frac{1}{2}\int \frac{1}{z+1}dz - \frac{4}{3}\int \frac{1}{2z+1}dz$$

87. (3)

88. (3)

89. (25)

90. (8.08)