PART (A) : PHYSICS

SOLUTIONS

1. (D)
PV = constant. As V is decreased, P increases.

2. (B)
Sound cannot be polarised.

3. (B)
Using activity and half life concept

4. (A)
Slope of graph $= \frac{1}{R}$

5. (D)
In head on elastic collision between identical bodies velocities get interchanged.

6. (C)
Use equation of continuity $A_1V_1 = A_2V_2$
The liquid has to flow faster through the narrower section because if unit volume passes a one point in the pipe every second, that same volume must flow through any other point each second otherwise there would be an accumulation of an incompressible fluid in the pipe.

7. (C)
Taking the collision between A and C as elastic, it can be concluded that 'C' comes to rest immediately after collision and A starts moving with v_0 (the spring force, if any, during collision can be treated as non-impulsive one).
At the instant of maximum compression or elongation, relative velocity of A with respect to B would be zero and hence A and B would be moving with a common velocity. From conservation of momentum
$mv_0 = 2mv \Rightarrow v = v_0/2$
Hence, from conservation of energy

$$(P.E)_{max} = \frac{1}{2}mv_0^2 - \frac{1}{2}(2m)\left(\frac{v_0}{2}\right)^2$$

$$= \frac{1}{2}mv_0^2 - \frac{1}{4}mv_0^2 = \frac{1}{4}mv_0^2$$

8. (C)

$$KE = \left(KE_{hollow\,sphere}\right) + \left(KE_{liquid}\right) = \left(KE_{h.s.}\right)_{trans} + \left(KE\right)_{rot} + \left(KE_{liquid}\right)_{trans}$$

$$= \frac{1}{2}\left(mv^2\right) + \frac{1}{2}\left(\frac{2}{3}mr^2\right)\left(\frac{v}{r}\right)^2 + \frac{1}{2}mv^2 = \frac{4}{3}mv^2$$

Due to the absence of friction , the liquid does not rotate. It only translates that mean it does not possesses rotation K.E.

9. (A)

Conserving angular momentum of rod A about centre of mass

$$mv\frac{a}{4} = I\omega = \frac{Ma^2}{12}\omega$$

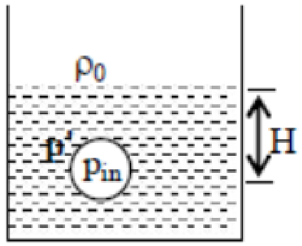
$$\Rightarrow \omega = \frac{3mv}{Ma}$$

10. (B)

$$\left(\frac{2\pi x_1}{\lambda} - \omega t + \frac{\pi}{6}\right) - \left(\frac{2\pi x_2}{\lambda} - \omega t + \frac{\pi}{8}\right) = 2\pi n$$

$$\Rightarrow x_1 - x_2 = \left(n - \frac{1}{48}\right)\lambda$$

11. (D)



$$P_{in} - p' = \frac{2S}{r}$$

$$\Rightarrow p_{in} = \frac{2S}{r} + p' = \frac{2S}{r} + p_0 + \rho g H$$

12. (B)

$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$, $k =$ spring constant of the uncut spring.

After cutting spring constant becomes $2k \Rightarrow f' = \frac{1}{2\pi}\sqrt{\frac{2k}{m}} = \sqrt{2}f$

13. (A)

$$f' = \left(\frac{v}{v - v_s}\right)f,\ f'' = \left(\frac{v}{v + v_s}\right)f,\ v = 330\ m/s$$

Beat frequency of $f' - f'' = 3 \Rightarrow \left(\frac{330}{330 - v_s} - \frac{330}{330 + v_s}\right)340 = 3 \Rightarrow v_s \approx 1.45\ m/s$

14. (D)

$N_1 / N_0 = e^{-\lambda_1 t}$ ....(1)

$N_2 / N_2 = e^{-\lambda_2 t}$ ....(2)

$\Rightarrow N_1 / N_0 = e^{-(\lambda_1 - \lambda_2)t}$

Putting $N_1 / N_2 = 1/e$ we obtain $(\lambda_1 - \lambda_2)t = 1$

$\Rightarrow t = 1/(\lambda_1 - \lambda_2) = 1/(10\lambda - \lambda) = 1/9\lambda$

15. (A)

$$\bar{F}_A = \bar{F}_{AB} + \bar{F}_{AC} = 2\left[\frac{GM^2}{a^2}\right]\cos 30^\circ = \left[\frac{GM^2}{a^2}\times\sqrt{3}\right]$$

$$r\cos 30^o = \frac{a}{2} \Rightarrow r = \frac{a}{\sqrt{3}}$$

Now, $\frac{Mv^2}{\frac{a}{\sqrt{3}}} = \frac{GM^2}{a^2}\cdot\sqrt{3}$

$$v = \sqrt{\frac{MG}{a}}$$

16. (A)

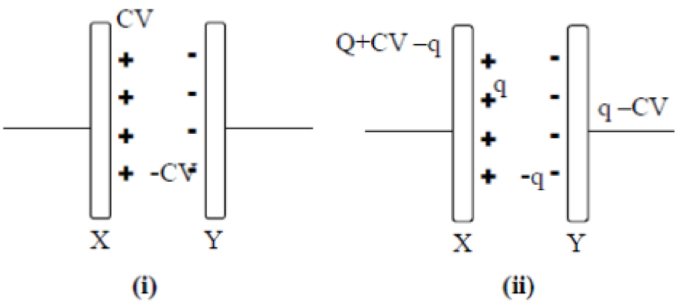
Induced emf is rate of change of flux.

$$e = \frac{-d\phi}{dt} = -\frac{d(B.A)}{dt} = -A\frac{dB}{dt}$$

$$= -\frac{\pi a^2}{2}\alpha$$

17. (C)

In the adjacent figure, let X and Y be the positive and negative plates. After charging from the cell, the inner faces of X and Y have charges $\pm CV$, as shown in (i). The outer surfaces have no charge.



When charge Q is given to X, let the inner faces of X and Y have charges $\pm q$. Then, by the principle of charge conservation, the outer faces have charges $(Q + CV - q)$ for X and $(q - CV)$ for Y, as shown in (ii). Now, the outer faces must have equal charges.

$\therefore Q + CV - q = q - CV$

Or $2q = 2CV + Q$

Or $q = CV + \frac{Q}{2}$

Potential difference $= \frac{q}{C} = V + \frac{Q}{2C}$

18. (D)

$\bar{o}R / R = 2\bar{o}u / u + 2\cot 2\theta\bar{o}\theta$

$\ln R = 2\ln u + \ln\sin 2\theta - \ln g$

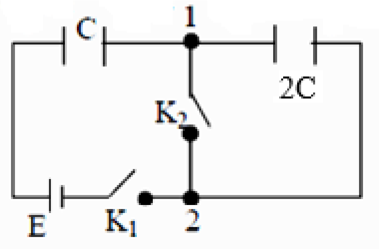
$$\Rightarrow \frac{\Delta R}{R} = \frac{2\Delta u}{u} + \frac{1}{\sin 2\theta}\cdot\cos 2\theta\cdot 2\Delta\theta$$

$$\frac{\Delta\theta}{\theta}\times 100 = 2 \Rightarrow \Delta\theta = \frac{2\theta}{100}$$

$$= 2\times 1 + \frac{1}{\sqrt{3}}\times 2\times \frac{\pi}{6}(-2) = 0.85\%$$

19. (C)

Initially charge configuration (before K_2 is closed) is



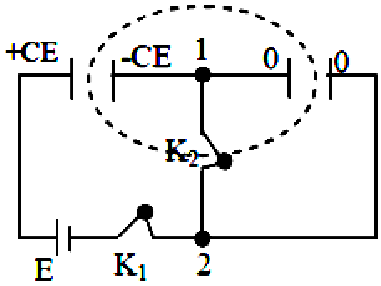
Final charge configuration (after K_2 is closed) is

The net change of charge in the encircled region can happen only because of charge flowing in section $1\to 2$.

Initial charge $= 0$

Final charge $= -CE$

$\Rightarrow$ Charge flown $= -CE$ from 2 to 1



20. (B)

Here, one might be tempted to think that since the refractive index of benzene is not given. But actually we do not need this data

$$\mu_1 \sin i_1 = \mu_2 \sin i_2 = \mu_3 \sin i_3$$

$$\therefore \sin i_3 = \frac{\mu_1 \sin i_1}{\mu_3} = \frac{3\sqrt{3}}{8}$$

21. (39)

$$\frac{1}{2}mv^2 = \frac{1}{2}\times 5\times 10^{-26}\times (500)^2 = 39$$

22. (110)

Viscous force in downward direction

$$F_v = 0.1\times 1m^2\times \frac{10^{-1}}{10^{-3}} = 10N$$

$$T = mg\sin 30^o + F_v$$

$$T = \tau + 100 = 110N$$

23. (790)

As shown in figure, if R is the unknown resistance, current in the circuit

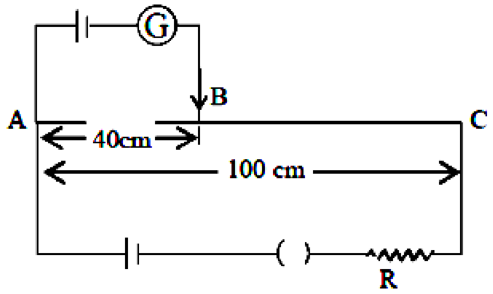
$$I=\frac{V}{(r+R)}=\frac{2}{(10+R)}$$

Now, as 100 cm wire resistance 10Ω, the resistance of 40 cm wire will be

$40\times(10/100)=4\text{ohm}$

Potential drop across 40 cm wire will be $V=I\times 4$

But $V=10\,\text{mV}(\text{given})$



$$\Rightarrow 10\times10^{-3}=\frac{2}{(10+R)}\times 4\Rightarrow R=790\Omega$$

24. (29)

$$\lambda_\alpha=\frac{4}{3R(Z-1)^2}$$

Cut-off wavelengths: $\lambda_1=\frac{hc}{eV_1};\lambda_2=\frac{hc}{eV_2}$

Using given condition: $(\lambda_2-\lambda_\alpha)=3(\lambda_1-\lambda_\alpha)$

Or $3\lambda_1-\lambda_2=2\lambda_\alpha$ or $\frac{hc}{e}\left[\frac{3}{V_1}-\frac{1}{V_2}\right]=\frac{8}{3R(Z-1)^2}$

Taking $V_1=10\times10^3\,V$ and $V_2=20\times10^3\,V$, we get

$Z=29$

25. (4)

$$T=2\pi\sqrt{\frac{L}{g}}\Rightarrow g=\frac{4\pi^2L}{T^2}$$

$$\Rightarrow\frac{\Delta g}{g}=\frac{\Delta L}{L}+2\frac{\Delta T}{T}$$

$$\Rightarrow\left(\%\frac{\Delta g}{g}\right)=\left(\%\frac{\Delta L}{L}\right)+2\left(\%\frac{\Delta T}{T}\right)$$

$$\Rightarrow\frac{\Delta g}{g}(\text{in}\%)=2\%+2\times1\%=4\%$$

26. (0)

There is no change in λ_{K_α} as it is characteristic X-ray.

27. (3)

$$E_{min} = 2\left[\frac{9.1\times10^{-31}\times\left(3\times10^{8}\right)^{2}}{1.6\times10^{-13}}\text{MeV} + 0.95\,\text{MeV}\right]$$

$= 2.924$

28. (1)

Energy of incident photon

$$= \frac{hc}{\lambda} = \frac{12400}{\lambda}\text{eV} = \frac{12400}{4000}\text{eV} = 3.1\text{eV}$$

$k_{max} = h\nu - \phi$

$= 3.1\text{eV} - 2.0\text{eV} = 1.1\text{eV}$

Hence, stopping potential 1.1V

29. (5)

$$\Delta W = P_0V_0 + \frac{1}{2}\pi 2P_0V_0\times\frac{1}{2} = P_0V_0\left(1+\frac{\pi}{2}\right)$$

$$\Delta U = \frac{3}{2}\left(P_0 2V_0 - P_0V_0\right) = \frac{3}{2}P_0V_0$$

$$\Delta Q = P_0V_0\left(1+\frac{\pi}{2}+\frac{3}{2}\right) = P_0V_0\left(\frac{5+\pi}{2}\right)$$

30. (8)

According to given problem,

$$I = \frac{V}{Z} = \frac{V}{\left[R^2 + \left(1/C^2\omega^2\right)\right]^{\frac{1}{2}}} \qquad \text{.......(1)}$$

And $$\frac{I}{2} = \frac{V}{\left[R^2 + \left(9/C^2\omega^2\right)\right]^{1/2}} \qquad \text{........(2)}$$

Substituting the value of I from equation (1) in (2)

$$4\left(R^2 + \frac{1}{C^2\omega^2}\right) = R^2 + \frac{9}{C^2\omega^2}, \text{i.e.,} \frac{1}{C^2\omega^2} = \frac{3}{5}R^2$$

So than $$\frac{X}{R} = \frac{(1/C\omega)}{R} = \frac{\left[(3/5)R^2\right]^{1/2}}{R} = \sqrt{\frac{3}{5}}$$

PART (B) : CHEMISTRY

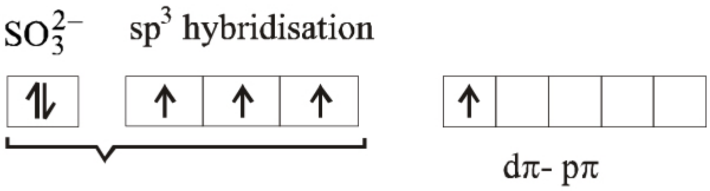
SOLUTIONS

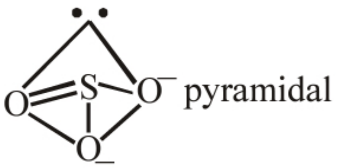
31. (C)

32. (A)
CrO_2 is amphoteric in nature

33. (B)
$XeF_4 + H_2O \rightarrow 2HF + XeOF_2$

34. (B)
$p\pi - d\pi$ bonding is present in SO_3^{2-}, N, B, C have no vacant d atomic orbitals.





pyramidal

35. (D)
$Al_2O_3.2H_2O \rightarrow Al_2O_3 + 2H_2O$ is calcination.

36. (C)
In physical adsorption the forces between adsorbate and adsorbent are weak van der waal's forces.

37. (A)
$$t = \frac{2.303}{k}\log\frac{a}{a-x} = \frac{2.303}{6}\log\frac{0.5}{0.05} = 0.384\,\text{min}.$$

38. (C)
$$\Delta T_f = 2 \times K_f \times m = 2 \times 1.86 \times 0.1 = 0.372$$
$$\therefore\ T_f = -0.372^{o}C$$

39. (C)
AgBr exhibit Frenkel defect.

40. (C)

$$CaSO_4.\frac{1}{2}H_2O \xrightarrow{\text{setting}} CaSO_4.2H_2O \xrightarrow{\text{hardening}} CaSO_4.2H_2O$$

Plaster of paris — Gypsum (orthorhombic) — Gypsum (monoclinic)

41. (A)

$HA + BOH \rightleftharpoons BA + H_2O$ (BA is Salt)

$\therefore \quad K_H = \dfrac{K_w}{K_a \times K_b}$.

The inverse of K_H is $K_c = \dfrac{K_a \times K_b}{K_w}$

$= \dfrac{2\times10^{-5}\times5\times10^{-6}}{1\times10^{-14}} = 1.0\times10^4$

42. (A)

Since the ratio of He and O_3 atoms is 1:1.

The ratio moles will be $1:\dfrac{1}{3}$.

Total moles $= 1+\dfrac{1}{3} = \dfrac{4}{3}$.

Total pressure P.

Hence, $p_{O_3} = \dfrac{\frac{1}{3}}{\frac{4}{3}}.P = 0.25P$

43. (A)

$r_n = a_0 \times n^2$

$r_4 = a_0 \times (4)^2 = 16a_0$

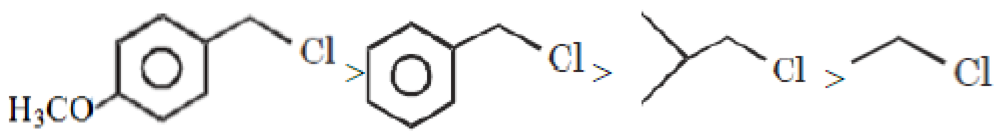
$mvr = \dfrac{nh}{2\pi}$; $mv = \dfrac{4h}{2\pi\times16a_0}$;

$\lambda = \dfrac{h}{mv} = \dfrac{h}{h/8\pi a_0} = 8\pi a_0$

44. (B)

45. (B)

Correct order



46. (A)

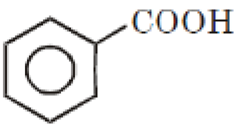
A: Saccharic acid $\rightarrow$ No. of stereoisomers = 10

B: Gluconic acid $\rightarrow$No. of enantiomeric pair = 8

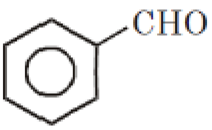
C : n-hexane $\rightarrow$No. of structure isomers = 5

D: 5 moles of acid is obtained

47. (A)

B $\rightarrow$ 

C $\rightarrow$ CHI_3

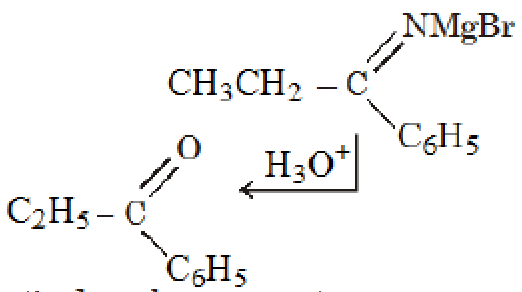
D $\rightarrow$ 

48. (C)

Gabriel synthesis is not used for preparation of aromatic amines

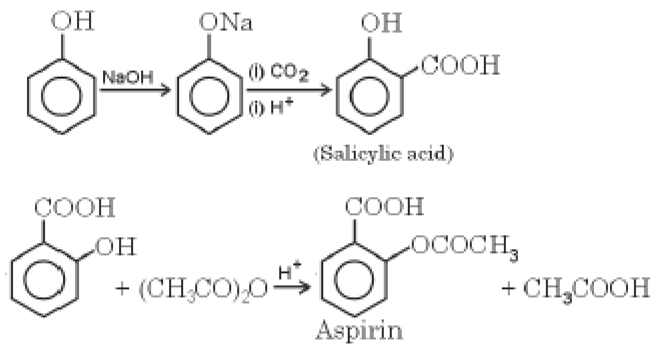
49. (A)

$CH_3–CH_2–C\equiv N + C_6H_5MgBr \xrightarrow{\text{ether}}$



(1-phenylpropanone)

50. (B)



51. (2)
$x = 2, y = 1, z = 1$
$x + y - z = 2$

52. (2123)
$W = -\Delta G = nFE^{o}_{cell} = 2 \times 96500 \times 1.1 J = 212.3 \, kJ$.

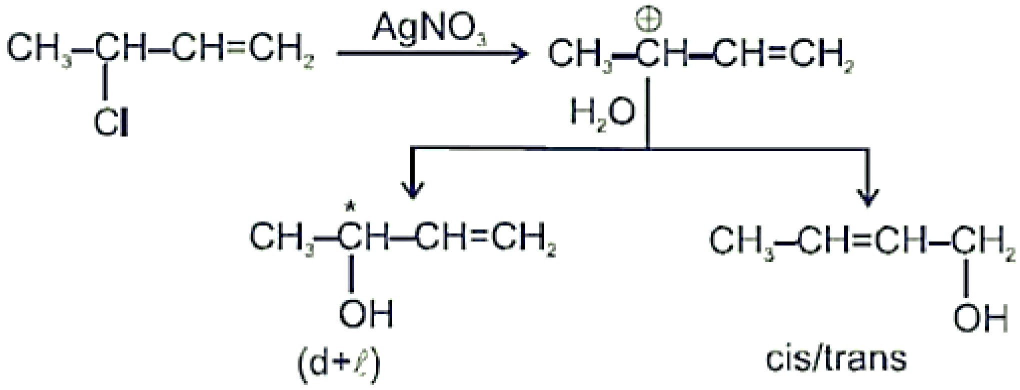
53. (3)

54. (3)
O.N. of iodine in I_3^- is –1/3

55. (100)
$\frac{1}{2}H_2 + \frac{1}{2}X_2 \longrightarrow HX$
Let the bond enthalpy of X–X bond be x.
$\Delta H_f(HX) = -50 = \frac{1}{2}\Delta H_{H-H} + \frac{1}{2}\Delta H_{X-X} - \Delta H_{H-X}$
$= \frac{1}{2}2x + \frac{1}{2}x - 2x = \frac{-x}{2}$
$\therefore \quad x = 50 \times 2 = 100 \, kJ \, mol^{-1}$

56. (6)

57. (4)
The reactions are given below:
From this, it is clear that the total number of products formed is 4 (*d* + *l* = 2 and *cis* + *trans* = 2).

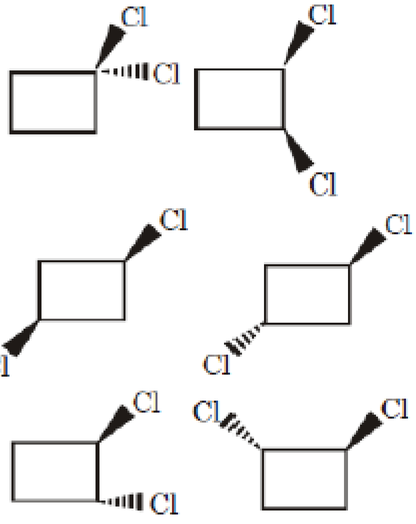


58. (5)
X = 3
Y = 1

59. (2)

x = 4

y = 2



60. (12)

$$\begin{array}{c} CH_3 \\ | \\ OH-Si-OH \\ | \\ CH_3 \end{array}$$

PART (C) : MATHEMATICS

SOLUTIONS

61. (B)

End point of L.R. $\left(\pm ae, \pm \frac{b^2}{a}\right)$

$\Rightarrow$ tangents are $\pm \frac{e}{a} x \pm \frac{1}{a} y = 1$

$\Rightarrow$ Area $= \frac{2a^2}{e} - a^2 e^2$

$\Rightarrow e^3 = 2$

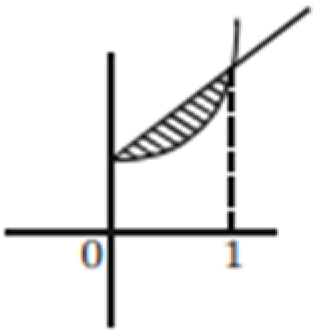
62. (C)

$x = \sqrt{y-1}$

$y = x + 1$

$\int_0^1 \left(x + 1 - x^2 - 1\right) dx$

$= \frac{1}{6}$



63. (B)

$\frac{dy}{dx} + x \sin^2 y = \sin y \cos y$

$\operatorname{cosec}^2 y \frac{dy}{dx} + x = \cot y$

Let $-\cot y = v$

$\frac{dv}{dx} + v = x$

$\therefore -\cot y . e^x = \int x e^x dx$

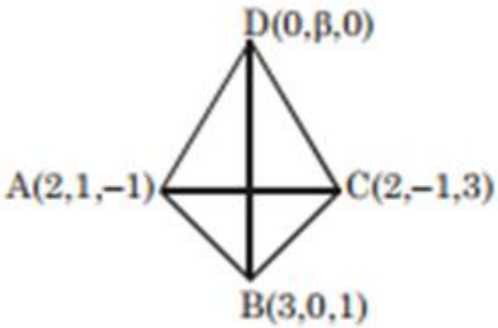
$\Rightarrow \cot y = (x - 1) + Ce^{-x}$

64. (C)

$$\begin{vmatrix} -x & x & 2 \\ 2 & x & -x \\ x & -2 & -x \end{vmatrix} = 0 \Rightarrow x = 2, -2$$

$$\Rightarrow n = 2 \Rightarrow \Delta(n) = 0$$

65. (C)



$$\left| \frac{1}{6} \begin{vmatrix} 2 & 1-\beta & -1 \\ 3 & -\beta & 1 \\ 2 & -1-\beta & 3 \end{vmatrix} \right| = 5$$

$$\Rightarrow \beta = 8, -7$$

Sum =1

66. (A)

$$S = (1-\omega)(1-\omega^2) + ... + (2017-\omega)(2017-\omega^2)$$

$$S = \sum_{n=1}^{2017} (n-\omega)(n-\omega^2) = \sum_{n=1}^{2017} (n^2+n+1)$$

$$= \frac{2017.2018.4035}{6} + \frac{2017.2018}{2} + 2017$$

$$\frac{S.\pi}{2017} = \left(\frac{2018.4035}{6} + \underbrace{1009+1}_{\text{even}} \right)$$

$$= (\text{odd} + \text{even})\pi = \text{odd} \times \pi$$

$$= \cos\left(\frac{S\pi}{2017}\right) = \cos(\text{oddx}\pi) = -1$$

67. (D)

Reflexive, symmetric but not transitive.

68. (D)

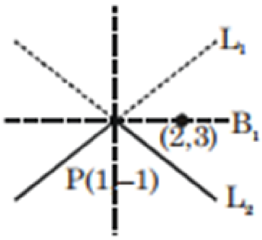
$$^{2017}C_0 + ^{2017}C_1 + ... ^{2017}C_{1008} = 2^{2016} = \lambda^2$$

$$\lambda = 2^{1008} \Rightarrow 8.32^{201} = 8(33-1)^{201} = -8 = 25$$

69. (A)

Fixed point of family is $(1,-1)$

$\Rightarrow$ other bisector is



$$y+1=-\frac{1}{4}(x-1)$$

$$x+4y+3=0$$

70. (B)

$$\lim_{x^2\to a}\frac{b-\cos(x^2-a)}{(x^2-a)\sin(x^2-a)}$$

Let $x^2-a=t$

$$\lim_{t\to 0}\frac{b-\cos t}{t\sin(x(t+a)-a)}$$

$$\Rightarrow \frac{b-1}{0}\Rightarrow b=1$$

$$\lim_{t\to 0}\frac{1-\cos t}{t\sin(ct+a(c-1))}=\lim_{t\to 0}\frac{2\sin^2\frac{t}{2}}{t\sin(ct+a(c-1))}$$

$$\lim_{t\to 0}\frac{\sin\frac{t}{2}}{(ct+a(c-1))}=\frac{0}{\sin a(c-1)}\Rightarrow c=1$$

$$\lim_{t\to 0}\frac{\sin\frac{t}{2}}{\sin t}=\frac{1}{2}\Rightarrow L=\frac{1}{2}$$

71. (A)

$$\frac{1}{2}+\frac{1}{2\sin\frac{x}{2}}2\sin\frac{x}{2}(\cos x+\cos 2x+\cos 3x+\cos 4x)=0$$

$$=\frac{1}{2}+\frac{1}{2\sin\frac{x}{2}}\left(\sin\frac{9x}{2}-\sin\frac{x}{2}\right)=0$$

$$=\frac{\sin\left(\frac{9x}{2}\right)}{\sin\left(\frac{x}{2}\right)}=0\Rightarrow x=\frac{2n\pi}{9}, n\neq 9m\, m\in I$$

72. (A)

$$\tan^{-1}(x+2)+\tan^{-1}(x-2)=\tan^{-1}\left(\frac{1}{2}\right)$$

$$\tan^{-1}\left(\frac{x+2+x-2}{1-(x+2)(x-2)}\right)=\tan^{-1}\left(\frac{1}{2}\right)$$

$$x=1,-5(\text{reject})$$

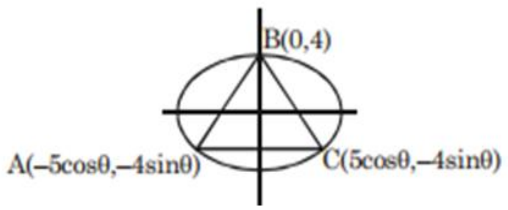
73. (C)

34 terms so mean of 17^{th} and 18^{th} term is median

$$x_{10+n}=148+(n-1)(-2)=x_{17}=136, x_{18}=134$$

Hence, median $=135$

74. (C)



$$\text{Area}=\frac{1}{2}10\cos\theta(4+4\sin\theta)$$

$$\frac{dA}{d\theta}=0\Rightarrow\theta=\frac{\pi}{6}$$

$$A_{max}=15\sqrt{3}$$

75. (D)

Let point be $\left(8\lambda+\frac{1}{3},3\lambda,-6\lambda\right)$ which also satisfies both the plane $P_1=0=P_2$

Put in $P_2: 16\lambda+\frac{2}{3}+3\lambda-24\lambda+1=0$

$$\Rightarrow -5\lambda+\frac{5}{3}=0\Rightarrow\lambda=\frac{1}{3}$$

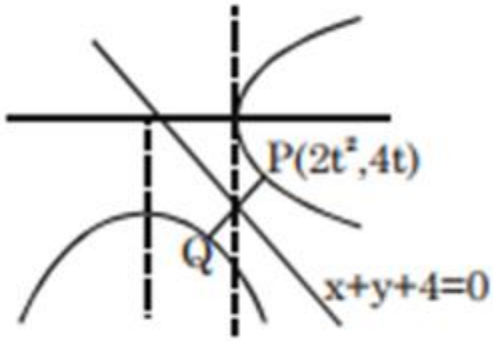
$(\alpha,\beta,\gamma)=(3,1,-2)$ it also satisfy P_1

$$\alpha+\beta+\gamma=2$$

76. (B)

p	~p	q	$p \to q$	$q \vee \sim p$	$(p \to q) \leftrightarrow (q \vee \sim p)$
T	F	T	T	T	T
T	F	F	F	F	T
F	T	T	T	T	T
F	T	F	T	T	T

77. (A)



For minimum distance $\left.\frac{dy}{dx}\right|_P = -1$

$\Rightarrow t = -1$

$\Rightarrow$ min distance $= PQ = 2\sqrt{2}$

78. (D)

Total cases $\Rightarrow {}^{15}C_2.2! = 15.14$

$2x = 3y \Rightarrow (3,2),(6,4),(9,6),(12,8),(15,10)$

Favorable cases =5

Probability $= \frac{5}{15.14} = \frac{1}{42}$

79. (C)

$$\cot x = \frac{1}{2}\left(\cot\frac{x}{2} - \tan\frac{x}{2}\right)$$

$$\cot x = \frac{1}{2}\left\{\frac{1}{2}\left(\cot\frac{x}{4} - \tan\frac{x}{4}\right) - \tan\frac{x}{2}\right\}$$

$$= \frac{1}{4}\cot\frac{x}{4} - \frac{1}{4}\tan\frac{x}{4} - \frac{1}{2}\tan\frac{x}{2}$$

$$= \frac{1}{8}\left(\cot\frac{x}{8} - \tan\frac{x}{8}\right) - \frac{1}{4}\tan\frac{x}{4} - \frac{1}{2}\tan\frac{x}{2}$$

$$\Rightarrow \cot x = \frac{2}{2^n}\cot\left(\frac{x}{2^n}\right) - \frac{1}{2^{n-1}}\tan\left(\frac{x}{2^{n-1}}\right) - \frac{1}{8}\tan\left(\frac{x}{8}\right) - \frac{1}{4}\tan\left(\frac{x}{4}\right) - \frac{1}{2}\tan\left(\frac{x}{2}\right)$$

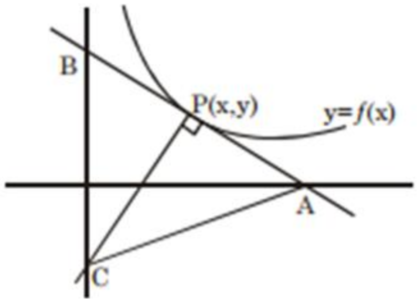
$\Rightarrow \frac{1}{2}\tan\frac{x}{2}+\frac{1}{4}\tan\frac{x}{2}+\frac{1}{8}\tan\frac{x}{8}.......... = \frac{1}{2^n}\cot\left(\frac{x}{2^n}\right)-\cot x$

$\Rightarrow \frac{1}{2}\tan\frac{x}{2}+\frac{1}{4}\tan\frac{x}{4}+.......\infty$ terms

$= \lim_{n\to\infty}\frac{1}{2^n}\cot\left(\frac{x}{2^n}\right)-\cot x = \frac{1}{x}-\cot x$

Put $x=\frac{\pi}{2}$

80. (A)



$\because AC = BC$

$\therefore$ P is mid point of AB

$\Rightarrow A(2x,0)\ \&\ B(0,2y)$

$\Rightarrow \frac{dy}{dx} = -\frac{y}{x}$

$\Rightarrow xy = c$

$\Rightarrow xy = 6$

81. (4)

$a = 11, b = 15, x = K$ and ΔABC is obtuse angled

$\Rightarrow \cos B < 0$ or $\cos C < 0$

$\Rightarrow K^2 < 104$ or $K^2 < 346$ (using cosine rule)

$\therefore K = 5, 6, ..., 10$ or $K = 19, 20, ..., 25$ and $p = 6 + 7 = 13$ $[\therefore b - a < c < a + b]$

Sum of the digits of $p = 4$

82. (0)

Conceptual

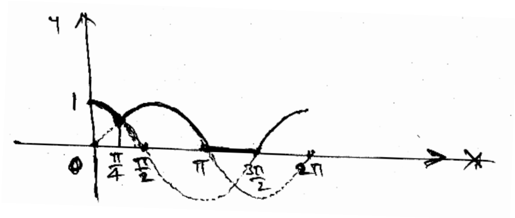
83. (3)

$$f(x)=\begin{cases}\cos x & \text{for } 0<x<\frac{\pi}{4}\\ \sin x & \text{for } \frac{\pi}{4}\le x<\pi\\ 0 & \text{for } \pi\le x<\frac{3\pi}{2}\\ \cos x & \text{for } \frac{3\pi}{2}\le x<2\pi\end{cases}$$

Also $f(x)$ has period 2π (or consider below graph)

Curve $y=f(x)$ has corner points at $x=\frac{\pi}{4},\pi,\frac{3\pi}{2}$ in $(0,2\pi)$

$\Rightarrow$ Number of points in $(0,2n\pi)$ where $f(x)$ is not differentiable $=3n$



84. (2)
Conceptual

85. (2)
Let $P=(7,-1,2),Q=(\alpha,\beta,\gamma)$

$M=\text{Mid-point } \overline{PQ}=\left(\frac{\alpha+7}{2},\frac{\beta-1}{2},\frac{\gamma+2}{2}\right)$ lies on given line.

$\Rightarrow\frac{\alpha+7}{2}=9+\lambda,\frac{\beta-1}{2}=5+3\lambda,\frac{\gamma+2}{2}=5+5\lambda$

$\therefore\alpha=11+2\lambda,\beta=11+6\lambda,\gamma=8+10\lambda$

$\overline{PQ}$ is $\perp$ to $\bar{i}+3\bar{j}+5\bar{k}$

$\Rightarrow(1)(\alpha-7)+(3)(\beta+1)+(5)(\gamma-2)=0$

$\Rightarrow\alpha+3\beta+5\gamma=14\Rightarrow\gamma=-1$

$(\alpha,\beta,\gamma)=(9,5,-2)$

86. (3)

$$=\left(\left(1-\frac{x^2}{2!}+\frac{x^4}{4!}-...\right)-1\right)\left[\left(1-\frac{x^2}{2!}+\frac{x^4}{4!}-...\right)-\left(1+x+\frac{x^2}{2!}+\frac{x^3}{3!}...\right)\right]\left(\frac{1}{x^n}\right)$$

$$=x^2\left(-\frac{1}{2}+\frac{x^2}{4!}...\right)\left(-x-x^2-\frac{x^3}{3!}...\right)\left(\frac{1}{x^n}\right)$$

$$= x^3\left(-\frac{1}{2}+\frac{x^2}{4!}...\right)\left(-x-x^2-\frac{x^3}{3!}...\right)\left(\frac{1}{x^n}\right)$$

$$\Rightarrow \frac{(\cos x-1)(\cos x-e^x)}{x^3}=\left(\frac{1}{2}+\text{terms containing } x \text{ and higher powers of } x\right)$$

87. (15)

The given expression is equal to

$1+\tan^2(\tan^{-1}2)+1+\cot^2(\cot^{-1}3)$

$=1+4+1+9=15$

88. (5)

The given equation can be written as $1-2\sin^2 x+a\sin x=2a-7$

$\Rightarrow\ 2\sin^2 x-a\sin x+2a-8=0$

$$\Rightarrow\ \sin x=\frac{a\pm\sqrt{a^2-8(2a-8)}}{4}=\frac{a\pm(a-8)}{4}$$

$\Rightarrow\ \sin x=\dfrac{a-4}{2}$ which is possible if $-1\le\dfrac{a-4}{2}\le 1$ or $2\le a\le 6$.

So the required values of a are 2, 3, 4, 5, 6 and hence the required number is 5.

89. (13)

Any point on the given line is $(3r+2, 4r-1, 12r+2)$ which lies on the given plane is $3r+2-(4r-1)+12r+2=5 \Rightarrow r=0$

So the point of intersection of the line and the plane is (2, –1, 2) and the required distance is

$\sqrt{(2+1)^2+(-1+5)^2+(2+10)^2}=13$

90. (7)

Squaring and adding the given equations of the lines we get $x^2+y^2=a^2+b^2$ as the locus of the point of intersection of these lines.

Since (3, 4) lies on the locus, we get

$9+16=a^2+b^2$ i.e. $a^2+b^2=25$ ......(i)

Also, (a, b) lies on $3x-4y=0$

So, $3a-4b=0$ $\Rightarrow b=\left(\dfrac{3}{4}\right)a$ .....(ii)

From (i), $a^2+\left(\dfrac{9}{16}\right)a^2=25\Rightarrow a^2=16$

So, that $|a+b|^2=\left(\dfrac{7}{4}\right)^2 a^2=49$

$|a+b|=7$