PACE-IIT & MEDICAL

MUMBAI / DELHI-NCR / PUNE / NASHIK / AKOLA / GOA / JALGOAN / BOKARO / AMRAVATI / DHULE

IIT – JEE: 2023 **AITS - 3** **DATE: 31/03/23**

ANSWER KEY

PAPER – I

Que.	1	2	3	4	5	6	7	8	9	10
Ans.	BCD	BC	BC	AC	AD	CD	ACD	AD	BCD	BCD
Que.	11	12	13	14	15	16	17	18	19	20
Ans.	2	2	3	4	7	4	4	6	1	5
Que.	21	22	23	24	25	26	27	28	29	30
Ans.	AD	BC	C	AB	BCD	ABD	BD	B	B	AC
Que.	31	32	33	34	35	36	37	38	39	40
Ans.	8	2	8	5	2	4	3	0	4	3
Que.	41	42	43	44	45	46	47	48	49	50
Ans.	ABCD	BC	ABC	AC	AD	BC	ABD	ACD	AC	BCD
Que.	51	52	53	54	55	56	57	58	59	60
Ans.	8	5	2	5	8	1	8	9	7	2

PAPER – II

Que.	1	2	3	4	5	6	7	8	9	10
Ans.	CD	AB	ABC	AC	BC	AC	ABC	AB	ABC	D
Que.	11	12	13	14	15	16	17	18	19	20
Ans.	C	C	A	B	B	D	B	A	A	C
Que.	21	22	23	24	25	26	27	28	29	30
Ans.	C	B	ABC	ACD	BD	A	BC	AC	C	ABCD
Que.	31	32	33	34	35	36	37	38	39	40
Ans.	A	B	C	A	C	D	D	A	B	A
Que.	41	42	43	44	45	46	47	48	49	50
Ans.	ABD	AC	BC	AC	AD	BC	AD	ABCD	BD	AB
Que.	51	52	53	54	**55**	56	57	58	59	60
Ans.	B	ABC	C	C	**C**	A	A	B	B	D

PHYSICS PAPER – I : SOLUTION

1. ground state n = 1
first excited state n = 2
KE = $\frac{1}{4\pi\varepsilon_0}\frac{e^2}{2r}$ (z = 1) $\therefore$ KE = $\frac{14.4\times10^{-10}}{2r}$ eV
Now r = 0.53 n^2 A° (z = 1)
$(KE)_1 = \frac{14.4\times10^{-10}}{2\times0.53\times10^{-10}}$ eV = 13.58 eV
$\therefore$ $(KE)_2 = \frac{14.4\times10^{-10}}{2\times0.53\times10^{-10}\times4}$ ev = 3.39 ev
$\therefore$ KE decreases by = 10.2 ev
Now PE = $\frac{-1}{4\pi\varepsilon_0}\frac{e^2}{r} = \frac{-14.4\times10^{-10}}{r}$ ev
$\therefore$ $(PE)_1 = \frac{-14.4\times10^{-10}}{0.53\times10^{-10}}$ ev = –27.1 ev
$(PE)_2 = \frac{-14.4\times10^{-10}}{0.53\times10^{-10}\times4}$ = –6.79ev
$\therefore$ PE increases by = 20.4 ev
Now Angular momentum ;
$$L = mvr = \frac{nh}{2\pi}$$

2. The diagram is balanced wheat stone bridge

3. The blocks will be performing SHM in CM frame. At the given instant the blocks are at mean position of the SHM in CM frame, hence velocity at this instant in CM frame is equal to ωA. Also the time period will be $2\pi\sqrt{\frac{\mu}{k}}$ where μ is reduced mass. Time taken by blocks to reach maximum compression is 3T/4.

4. The particle is doing SHM because of the frictional force only. The contact force on the block is F = $\sqrt{(mg)^2+f^2} = m\sqrt{g^2+a^2}$ **=** $m\sqrt{g^2+(\omega^2x)^2}$
This is not proportional to x but proportional to m. The contact force on the platform is F only and this is independent of M.

5. **(AD)**

6. $\lambda = \frac{c}{f\mu} \Rightarrow \lambda = \frac{k}{\mu}$ Where k is same for the three media.
For A, $\lambda_A = \frac{k}{1.5}$. For B, $\lambda_B = \frac{k}{\mu}$. For c, $\lambda_C = \frac{k}{1.4}$
No. of moves in $A = \frac{t}{\lambda_A} = \frac{1.5t}{k}$

No. of moves in $B = \frac{t}{3\lambda_B} = \frac{\mu t}{3k}$

No. of moves in $C = \frac{2t}{3\lambda_A} = \frac{1.4t \times 2}{3 \times k}$

$1.5 = \frac{\mu}{3} + \frac{2.8}{3}$

$\therefore\ \mu = 1.7$

7. **(ACD)**

8. Let $A \rightarrow$ cross sectional area of the base of the cylinder
Given $T_1 = 273 + 27 = 300\ K$
$T_2 = 273 + 177 = 450K$
At constant pressure
From Charles Law

$\frac{V_1}{T_1} = \frac{V_2}{T_2}$

$\Rightarrow \frac{Ah_1}{T_1} = \frac{Ah_2}{T_2}$

$\Rightarrow h_2 = \frac{\mathrm{T_2}}{\mathrm{T_1}} \times h_1 \frac{450}{300} \times 20$

= 30 cm
$\because$ The second process is adiabatic for which

$T_2 V_2^{(y-1)} = T_3 V_3^{(y-1)}$

Where $V_2 = Ah_2$ and $V_3 = Ah_1$
For diatomic gas $v = 1.4$

$\therefore T_3 = T_2 \times \left(\frac{V_2}{V_3}\right)^{p-1}$

$= 450 \times \left(\frac{30}{20}\right)^{1.4-1}$

$= 450 \times (1.5)^{0.4}$

$= 450 \times 1.18 = 531K$

$= 258^o C$

9. $\frac{1}{2}\mathrm{mv}^2 = \mathrm{qV}$

$\Rightarrow \quad \mathrm{v} = \sqrt{\frac{2\mathrm{qV}}{\mathrm{m}}}$

$\therefore \quad \mathrm{r} = \frac{\mathrm{mv}}{\mathrm{qB}} = \frac{1}{\mathrm{B}}\sqrt{\frac{2\mathrm{mV}}{\mathrm{q}}}$.

10. Maximum current through the inductor is $I_{max} = \frac{Q_0}{\sqrt{LC}}$

When only switch k_1 is closed then $I_{max} = \frac{10\times10^{-3}}{5\times2\times10^{-1}} = 10mA$

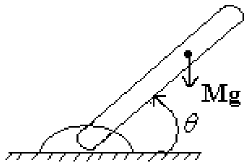
When only switch k_2 is closed then $I_{max} = \frac{10\times10^{-3}}{10\times2\times10^{-1}} = 5mA$

When both the switches are closed then maximum current through L_2 is

$$I_2 = Q_0\sqrt{\frac{L_1}{L_2\left(L_1+L_2\right)C}} = \sqrt{5}mA$$

11. **(2)**

12. $Mg\frac{L}{2}\left(1-Sin\theta\right) = \frac{1}{2}\frac{ML^2}{3}W^2$ ……….. (I)



$Mg\frac{L}{2}Cos\theta = \frac{ML^2}{3}\alpha$ …………(II)

If acceleration of center of mass is vertical only then

$W^2\frac{L}{2}Cos\theta = \frac{L}{2}\,\alpha\,Sin\theta$ …………(III)

From (I) (II) and (III) we get

$Sin\theta = \frac{2}{3}$ $\therefore\ 3Sin\theta = 2$

13. $-\frac{GMm}{2R} - \frac{G4Mm}{4R} = -\frac{GMm}{5R} - \frac{G4Mm}{R} + \frac{1}{2}mV^2 \Rightarrow V = 3\sqrt{\frac{3GM}{5R}}$

14. $mg\frac{l}{2} = \frac{mg\sigma}{2p}\frac{l}{4} + \frac{1}{2}mu^2 \Rightarrow u\sqrt{gl\left(1-\frac{\sigma}{4\rho}\right)}$

15. $\frac{dw}{d\phi} = \frac{d\phi - dU}{d\phi} = 1 - \frac{dU}{d\phi} = 1 - \frac{nc_v dT}{nc_p dT} = 1 - \frac{1}{\gamma} = 1 - \frac{5}{7} = \frac{2}{7}$

16. I = 2 sin t^2

Induced emf $e = -L\frac{di}{dt}$

Work done during small time interval dt

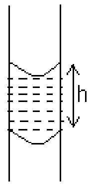
dW = ei dt.

$\therefore$ Total work done in raising the current against induced emf in given time interval 0 to t

$$W = \int_0^t ei\,dt = +\int_0^t Li\frac{di}{dt}dt \Rightarrow \int_0^2 Li\,di = L\left[\frac{i^2}{2}\right]_0^2$$

$$= \frac{1}{2}L\left(2\right)^2 = 2L = 2\times2 = 4\,\text{joule}$$

17.



$$\left(P_o + \frac{2T}{r}\right) - \left(P_o - \frac{2T}{r}\right) = pgh$$

18. Velocity of the centre = v_0 and the angular velocity about he centre = $\frac{v_0}{2r}$. Thus, $v_0 > \omega_0 r$.

The sphere slips forward and thus the friction by the plane on the sphere will act backward. As the friction is kinetic, its value is $\mu N = \mu Mg$. hence

$v(t) = v_0 - \frac{f}{M}t.$ ….(i)

this friction will also have a torque $\tau = fr$ about the centre. this torque is clockwise and in the direction of ω_0. Hence the angular acceleration about the centre will be

$$\alpha = f\frac{r}{\frac{2}{5}Mr^2} = \frac{5f}{2Mr}$$

and the clockwise angular velocity at time t will be

$$\omega(t) = \omega_0 + \frac{5f}{2Mr}t = \frac{v_0}{2r} + \frac{5f}{2Mr}t$$

pure rolling starts when v(t) = $r\omega(t)$

i.e. $v(t) = \frac{v_0}{2} + \frac{5f}{2M}t$ …(ii)

eliminating t from (i) and (ii),

$$\frac{5}{2}v(t) + v(t) = \frac{5}{2}v_0 + \frac{v_0}{2}$$

or $v(t) = \frac{2}{7} \times 2v_0 = \frac{6}{7}v_0$

Thus, the sphere rolls with translational velocity $\frac{6v_0}{7}$ in the forward direction.

19. $\frac{1}{f} = (n-1)\left(\frac{2}{R}\right) \Rightarrow R = 2f(n-1)$

Consider a point A on the second surface.
It's image is formed at V by 1st surface then

$$\frac{1}{V} - \frac{n}{-t} = \frac{1-n}{-R} \Rightarrow \frac{1}{V} = \frac{n-1}{R} - \frac{n}{t} = \frac{1}{2f} - \frac{n}{t}$$

$$\therefore V = \frac{2ft}{t - 2fn}$$

$\therefore$ thickness appears to be $= \frac{2ft}{2fn - t} \simeq \frac{t}{n}(as\ t << f)$

20. As the current lags behind the potential difference, the circuit contains resistance and inductance.

Power, P = $v_{rms} \times i_{rms} \times \cos\phi$

Here, $i_{rms} = \frac{V_{rms}}{Z}$, where $Z = \sqrt{[(R^2 + (\omega L)^2)]}$

$\therefore \quad P = \frac{V_{rms}^2 \times \cos\phi}{Z}$ or $Z = \frac{v_{rms}^2 \times \cos\phi}{P}$

So, $Z = \frac{(220)^2 \times 0.8}{550} = 70.4$ ohm

Now, power factor cos $\theta = \frac{R}{Z}$ or R = Z cos ϕ

R = 70.4 × 0.8 = 56.32 ohm

Further, $Z^2 = R^2 + (\omega L)^2$ or $(\omega L) = \sqrt{(Z^2 - R^2)}$

Or $\omega L = \sqrt{(70.4)^2 - (56.32)^2}$ = 42.2 ohm

When the capacitor is connected in the circuit,

$$Z = \sqrt{\left[R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2\right]}$$

and $$\cos\phi = \frac{R}{\sqrt{\left[R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2\right]}}$$

When $\cos\phi = 1,\ \omega L = \frac{1}{\omega C}$

$$\therefore \quad C = \frac{1}{\omega(\omega L)} = \frac{1}{2\pi f\,(\omega L)}$$

$$= \frac{1}{(2 \times 3.14 \times 50) \times (42.2)}$$

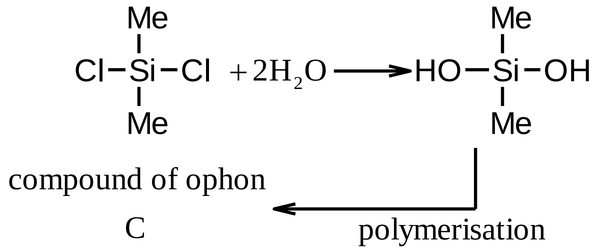
= 75 × 10^{-6} f = 75 μF.

CHEMISTRY PAPER – I SOLUTION

21. (AD)
Fact

22. (BC)
Highly electropositive metars with their salt undergoes electrolytic reduction

23. (C)



28. (B)

Anode : $Ag(s) \rightarrow Ag + (aq) + e^-$

Cathode: $Ag^+ + e^- \rightarrow Ag$

$Ag^+{}_{Ca(Be)} \rightarrow Ag^+ \left(Ag_2CO_3\right)$

$$Ecell = E^0 cell - \frac{0.59}{n} \log \frac{canode}{cethode}$$

$$\theta = 0 + \frac{0.591}{1} \log \frac{(KSP)\, PgBe \left| Br^- \right.}{\sqrt{(KSP)\, arz\, CO_3 \left| CO_3{}^{2-} \right.}}$$

$$\frac{KSP(AgBs)}{Br^-} = \sqrt{\frac{KSP(Ag_2CO_3)}{CO_3{}^2}}$$

$$\frac{4 \times 10^{-13}}{\sqrt{8 \times 10^{-12}}} = \frac{Br^-}{\sqrt{CO_3{}^{2-}}}$$

$$\frac{Br^-}{\sqrt{CO_3{}^{2-}}} = \sqrt{2} \times 10^{-7}$$

29. (b)
Bx observing fact ABCD and bx this PQRS can be considered as a face of a unit cell which is simple cubic

$$P.E = \frac{\frac{4}{3}\pi\left(\frac{9}{2}\right)3}{9^3}$$

$$= \frac{11}{21}$$

30. (A or AC)
Fact

31. (8)

$$\Delta H^0{}_R = 2 \times \Delta \overset{0}{H}_{NH_3} - 3 \times \Delta H^0_{H_2} - \Delta^{H^0}_{N_2}$$

$= -20\text{KCal}$

$\Delta S^0 R = 2\times S^0_{NH_3} - 3S^0_{H_2} - S^0_{N_2}$

$= -40\,\text{CalK} = -0.4\,\text{Cal}\,K$

$\Delta G^0 = \Delta H^0 - T\Delta S^0$

$= -8\text{KCal}$

32. (2)

$2NO + O_2 \rightarrow 2NO_2$

$2 \quad .5 \quad 0$

$2-2\times.5 \;\; 0 \quad 2\times.5$

$nF = 1+1 \quad ni = 2\times.5$

$\Delta h = 2.5 - 2 = .5$

Change in press $= \dfrac{\Delta nRT}{k}$

$= .5\times\dfrac{1}{12}\times\dfrac{300}{6.25}$

33. (8)

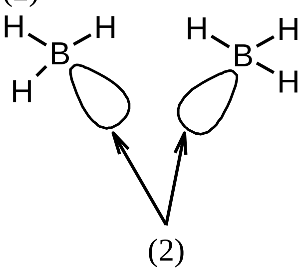
$\Delta E = \dfrac{3}{4}\times.85ex$

$.85\left(1-\dfrac{1}{4}\right) = 13.6\left(\dfrac{1}{42}-\dfrac{1}{n^2}\right)$

$\dfrac{4}{n} = \dfrac{1}{2}$

$n = 8$

35. (2)



36. (4)

37. (3)

Kr

$S - e^- = 8$

$P - e^- = 18$

$P - S = 10$

Find $= 10$

$6-3=3$

MATHEMATICS PAPER – I : SOLUTION

41. **(ABCD)**

$\because\ \vec{a}.\vec{b} = 0$ and $\vec{c}.\vec{a} = \vec{c}.\vec{b} = 1.1.\cos\theta$

$\therefore\ \vec{a}\times\vec{b} = 1.1.\sin 90°\ \hat{n}\ \therefore\ (\vec{a}\times\vec{b}) = 1$

$[|\vec{c}|^2 = x^2 + y^2 + z^2,\ \vec{c}.\vec{a} = x = \cos\theta,\ \vec{c}.\vec{b} = y = \cos\theta$. Hence a, b, c, d.]

$\vec{a}.(\vec{a}\times\vec{b}) = [\vec{a}\ \vec{a}\ \vec{b}] = 0$

$\vec{b}.(\vec{a}\times b) = [\vec{b}\,\vec{a}\,\vec{b}] = 0$

and $\vec{c}.(\vec{a}\times\vec{b}) = [\vec{a}\,\vec{b}\,\vec{c}]$

Now $\vec{c} = x\vec{a} + y\vec{b} + z(\vec{a}\times\vec{b})$ c

Multiplying both sides scalarly by $\vec{a}$ and $\vec{b}$

$\cos\theta = x.1 + 0 + 0 = x$, $\cos\theta = y.1 = y$

$\vec{c}.(\vec{a}\times\vec{b}) = x.0 + y.0 + z(a \times (B)^2 = z.1 = z$

$[\,\vec{a}\,\vec{b}\,\vec{c}\,] = z$

$$\text{Now } [\,\vec{a}\,\vec{b}\,\vec{c}\,]^2 = \begin{vmatrix} \vec{a}.\vec{a} & \vec{a}.\vec{b} & \vec{a}.\vec{c} \\ \vec{b}.\vec{a} & \vec{b}.\vec{b} & \vec{b}.\vec{c} \\ \vec{c}.\vec{a} & \vec{c}.\vec{b} & \vec{c}.\vec{c} \end{vmatrix}$$

$$\text{or } z^2 = \begin{vmatrix} 1 & 0 & \cos\theta \\ 0 & 1 & \cos\theta \\ \cos\theta & \cos\theta & 1 \end{vmatrix} = 1 - 2\cos^2\theta$$

$\therefore\ z^2 = 1 - 2x^2 = 1 - 2y^2$

Also $x^2 = y^2$

42. **(BC)**

$$\int_{-1}^{10} \operatorname{sgn}\{x\}\,dx$$

$$= 11.\int_0^1 \operatorname{sgn}\{x\}\,dx = 11.\int_0^1 \operatorname{sgn} x\ dx = 11\int_0^1 1.\ dx = 11$$

$$\int_{1/e}^{e^2} \left|\frac{\ln x}{x}\right| dx = \int_{\frac{1}{e}}^{e^2} \frac{|\ln x|}{x}\ dx$$

$$= -\int_{1/e}^{1} \frac{\ln x}{x}\,dx + \int_1^{e^2} \frac{\ln x}{x}\ dx$$

$$= \frac{1}{2}\Big[(\ln x)^2\Big]_1^{\frac{1}{e}} + \frac{1}{2}\Big[(\ln x)^2\Big]_1^{e^2} = \frac{1}{2}\times 1 + \frac{1}{2}\times 4 = \frac{5}{2}$$

$$\sum_{r=1}^{n}\int_0^1 f(r-1+x)\ dx$$

$$= \int_0^1 f(x)\,dx + \int_0^1 f(1+x)\,dx + \int_0^1 f(2+x)\,dx + \ldots\ldots\ldots\ldots + \int_0^1 f(n-1+x)\,dx$$

$$= \int_0^1 f(x)\,dx + \int_1^2 f(x)\,dx + \int_2^3 f(x)\,dx + \ldots\ldots\ldots\ldots + \int_{n-1}^{n} f(x)\,dx$$

$= \int_0^n f(x)\,dx$

$I = \int_0^1 \sin\left(2\tan^{-1}\sqrt{\frac{1-x}{1+x}}\right)dx$

put $x = \cos\theta \Rightarrow dx = -\sin\theta d\theta$

$I = -\int_{\pi/2}^{0} \sin\left(2.\frac{\theta}{2}\right)\sin\theta d\theta = \int_0^{\pi/2}\sin^2\theta d\theta = \frac{\pi}{4}$

43. **(ABC)**

Solving for $\frac{dy}{dx}$,

we get $\frac{dy}{dx} = \frac{2y\cot x \pm \sqrt{4y^2\cot^2 x + 4y^2}}{2}$

= y (-cot x + cosec x) dx

$\therefore$ dy/y = (-cot x + cosec x) dx

$\therefore$ log y = -log sin x + log tan $\frac{x}{2}$ + log C

$\therefore$ y = $\frac{C}{1-\cos x}$ 1 – cos x = $\frac{C}{y}$

$\therefore$ x = 2 sin^{-1} $\sqrt{\frac{C}{2y}}$

Also x can be takes 2cos^{-1} $\sqrt{\frac{C}{2y}}$ $\left[\because y = \frac{C}{1+\cos x} = \frac{C}{2\cos^2\frac{x}{2}}\right]$

$\therefore$(A), (B), (C), (D) all are correct.

44. **(AC)**

System of equation has no solution. $|A| = 0 \Rightarrow \left[\vec{n_1}\ \ \vec{n_2}\ \ \vec{n_3}\right] = 0 \Rightarrow$ normals of plane are coplanar hence they are not intersecting at any point and forming a triangular prism.

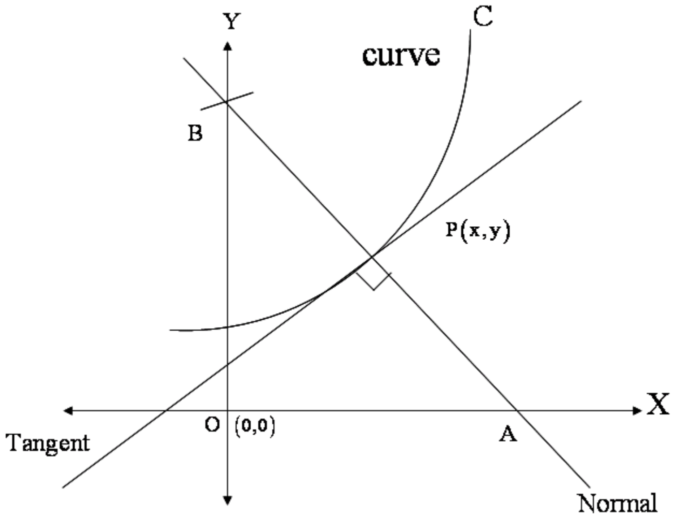
$(x, y, z) = (r, 2r, 3r)$ does not satisfy by any plane for any value of 'r' hence $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$ is parallel to each plane. (x, y, z) = (r, 3r, 4r) satisfy by plane (1) & plane (2) for some value of 'r' but not satisfy by plane '3' for any value of r. hence line $\frac{x}{1} = \frac{y}{3} = \frac{z}{4}$ does not interest plane '3'.

45. **(AD)**

The equation of normal of $P(x, y)$ is

$$(Y-y)=\frac{-1}{\frac{dy}{dx}}(X-x)$$

$$\therefore A\left(x+y\frac{dy}{dx},0\right) \text{ and } B\left(0, y+\frac{x}{\frac{dy}{dx}}\right)$$



Now

$$\frac{1\left(x+y\frac{dy}{dx}\right)+2(0)}{1+2}=x \Rightarrow x+y\frac{dy}{dx}=3x$$

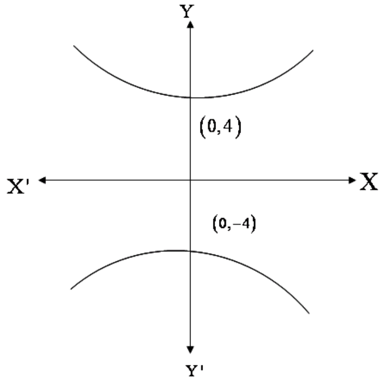
$$y\frac{dy}{dx}=2x \;\ldots..(1)$$

$$\Rightarrow \int y dy=\int 2x\, dx \Rightarrow \frac{y^2}{2}=x^2+C$$

Also (0,4) satisfy it, so C = 8

$$\therefore y^2=2x^2+16 \text{ (equation of curve)}$$

Which represent a hyperbola



Also $\left.\frac{dy}{dx}\right]_{(4,4\sqrt{3})}=\frac{2(4)}{4\sqrt{3}}=\frac{2}{\sqrt{3}}$

46. **(BC)**

Let $P_i(x_i, y_i)$ i = 1, 2, 3, …. n

$y = x^3$ (1) $\Rightarrow$ $\frac{dy}{dx}=3x^2$

Equation of tangent at $P_1(x_1, y_1)$

$y-x_1^3=3x_1^2(x-x_1)$ (2)

Solving (1) and (2), $x=-2x_1$

$\therefore$ $x_2 = -2x_1$ and $y_2 = -8x_1^3$

$P_2 \equiv (-2x_1, -8x_1^3)$

and like wise $P_3 \equiv (-2(-2x_1), +64x_1^3)$

Abscissa of $P_1, P_2, \ldots, P_n$ are given by

$x_1, -2x_1, 4x_1, -8x_1, \ldots..$ which is G. P. with common ratio –2.

$$\text{Area of } (\Delta P_1P_2P_3)=\frac{1}{2}\begin{vmatrix} x_1 & x_1^3 & 1 \\ -2x_1 & -8x_1^3 & 1 \\ 4x_1 & 64x_1^3 & 1 \end{vmatrix}=\frac{x_1^4}{2}\begin{vmatrix} 1 & 1 & 1 \\ -2 & -8 & 1 \\ 4 & 64 & 1 \end{vmatrix}$$

Area of $\left(\Delta P_2P_3P_4\right)=\frac{1}{2}\begin{vmatrix}-2x_1 & -8x_1^3 & 1\\ 4x_1 & 64x_1^3 & 1\\ -8x_1 & -512x_1^3 & 1\end{vmatrix}=8x_1^4\begin{vmatrix}1 & 1 & 1\\ -2 & -8 & 1\\ 4 & 64 & 1\end{vmatrix}$

So area of $\left(\Delta P_1P_2P_3\right)$ = $\frac{1}{16}$ area of $\left(\Delta P_2P_3P_4\right)$

47. **(ABD)**

(A) $x-iy=i\left(x^2-y^2+i2xy\right)$

$x=-2xy$ & $x^2-y^2=-y$

$x=0\,or\;y=\frac{-1}{2}$ $\quad x=0$

$y=0\,or\;1$

$y=\frac{-1}{2}$

$x=\pm\frac{\sqrt{3}}{2}$

(B) $\left(\frac{z+1}{z}\right)^7+1=0$

$\frac{z+1}{z}=\left(cis\left(2k+1\right)\pi\right)\frac{1}{7}$

$1+\frac{1}{z}=cis\frac{\left(2k+1\right)\pi}{7}$

$-\frac{1}{z}=1-cis\theta\;\; where\;\theta=\frac{\left(2k+1\right)\pi}{7}$

$-z=\frac{1}{1-cis\theta}$

$z=\frac{-1}{1-\cos\theta-i\sin\theta}$

$=\frac{-1}{2\sin\frac{\theta}{2}\left(\sin\frac{\theta}{2}-i\cos\frac{\theta}{2}\right)}$

$=\frac{-\left(\sin\frac{\theta}{2}+i\cos\frac{\theta}{2}\right)}{2\sin\frac{\theta}{2}}$

$\text{Re}\left(z\right)=\frac{-1}{2}$

$\sum_{k=0}^{6}\text{Re}\left(zk\right)=\frac{-7}{2}$

(C) $\overline{z_1}=z_1.cis\left(\frac{-2\pi}{n}\right)$

$\left(x-iy\right)=\left(x+iy\right)\left(\cos\frac{2\pi}{n}-i\sin\frac{2\pi}{n}\right)$

$$n\cos\frac{2\pi}{n}+y\sin\frac{3\pi}{n}=n \ \& \ -n\sin\frac{2\pi}{n}+y\cos\frac{2\pi}{n}=-y$$

$$n\left(1-\cos\frac{2\pi}{n}\right)=y\sin\frac{2\pi}{n}$$

$$n\sin\frac{2\pi}{n}=y\left(1+\cos\frac{2\pi}{n}\right)$$

$$\frac{y}{n}=\sqrt{2}-1 \text{ (even)}$$

$$n.2\sin^2\frac{\pi}{n}=n\left(\sqrt{2}-1\right)2\sin\frac{\pi}{n}\cos\frac{\pi}{n}$$

$$\sin\frac{\pi}{n}=\left(\sqrt{2}-1\right),\cos\frac{\pi}{n}$$

$$\tan\frac{\pi}{n}=\left(\sqrt{2}-1\right)$$

$$=\tan\frac{\pi}{8}$$

$$\therefore n=8$$

(D) $\dfrac{3+2i\sin\theta}{1-2i\sin\theta}$ is purely real

$$\Rightarrow \text{Im}\left(\frac{3+2i\sin\theta}{1-2i\sin\theta}\right)=0$$

$$1-2\sin\theta+3\left(2\sin\theta\right)=0$$

$$\sin\theta=0;\theta=n\pi$$

$$\theta\in\left(-\pi,\pi\right);\quad \therefore\theta=0$$

48. (ACD)

$$\text{I}=\int_0^{\text{I}} f(3^{\text{x}})\text{dx}$$

$$\text{I}=\int_0^{\text{I}} f(3^{1-\text{x}})\text{dx}$$

$$2\text{I}=\int_0^{1} f(3)\text{dx}$$

Also $f(\text{a.b})=f(\text{a})+f(\text{b})$

$\Rightarrow f(\text{a}^2)=2f(\text{a})$

Using this C & D also correct.

49. **(AC)**

Equation of the circle will be of the form $\left(x-\dfrac{1}{2}\right)^2+(y-1)^2+\lambda(2x-2y+1)=0$

Where $2x-2y+1=0$ is tangent to parabola at $\left(\dfrac{1}{2},1\right)$

50. **(BCD)**

$f(1)=2;\ \underset{x\to 1+}{lt} f(x)=3$

$\therefore\ at\ x=1;\ f(x)$ is discontinuous

$f(2)=4;\ \underset{x\to 2-}{lt} f(x)=3$ $\therefore\ at\ x=2;\ f(x)$ is discontinuous

51. **(8)**

$If\ -1<x<1;\ f(x)=x+4$

$1<x<3;\ f(x)=3x+c$

f is continuous at $x=1 \Rightarrow c=2$

$\therefore\ f(2)=8$

52. **(5)**

$$\frac{{}^5C_5.{}^nC_r+{}^5C_4.{}^nC_{r+1}+{}^5C_3.{}^nC_{r+2}+{}^5C_2.{}^nC_{r+3}+{}^5C_1.{}^nC_{r+4}+{}^5C_0.{}^nC_{r+5}}{{}^4C_4.{}^nC_r+{}^4C_3.{}^nC_{r+1}+{}^4C_2.{}^nC_{r+2}+{}^4C_1.{}^nC_{r+3}+{}^4C_0.{}^nC_{r+4}}=\frac{n+5}{r+5}$$

Then k = 5

53. **(2)**

$$P(E)=\frac{2.{}^4C_2}{10.9}$$

54. **(5)**

$4x^2+y^2=1$

Let any point on it is

$\left(\frac{\cos\theta}{2},\sin\theta\right)$

$f(x,y)=12x^2-3y^2+16xy$

$=12.\frac{\cos^2\theta}{4}-3\sin^2\theta+16\frac{\cos\theta}{2}.\sin\theta$

$=3\cos 2\theta+4\sin 2\theta$

Maximum is 5

55. **(8)**

$\frac{1}{x^2}+\frac{1}{\left(f(x)\right)^2}=1\ \forall x>1$

$\therefore\ f(x)=\frac{x}{\sqrt{x^2-1}}$

$f\left(f(x)\right)=x;\ f\left(f\left(f(x)\right)\right)=\frac{x}{\sqrt{x^2-1}}$

$\therefore \int_3^5 x\,dx=\frac{x^2}{2}\int_3^5 dx=8$

56. **(1)**

$2x-y+2z=2$

$2x-4y+2z=-8$

$3y = 10;\ y = \frac{10}{3}$

$2x - y + 2z = 2$

$2x + 2y + 2\lambda z = 8$

$3y + (2\lambda - 2)z = 6$

To have no solution

$2\lambda - 2 = 0;\ \lambda = 1$

57. **(8)**

58. **(9)**

Number of ways of wrong found = $= 4!\sum_{r=2}^{4}\frac{(-1)^2}{r!}$ q

59. **(7)**

$(5!)^2, (6!)^2, (7!)^2$ And so on will contain 0 in units place

$\therefore 1 + 4 + 36 + 576 \Rightarrow$ Units place is 7

60. **(2)**

$\sin A\ \sin B \sin C = \frac{3+\sqrt{3}}{8} : \cos A\ \cos B\ \cos C = \frac{\sqrt{3}-1}{8}$

$\tan A\ \tan B \tan C = \frac{3+\sqrt{3}}{\sqrt{3}-1} = \tan A + \tan B + \tan C\ \ in\ a\ \Delta ABC$

$\tan A \tan B + \tan B \tan C + \tan C \tan A = 1 + \sec A\ \ \sec B\ \ \sec C$

$\therefore \tan A,\ \tan B, \tan C$ Are the roots of

$x^3 - x^2\left(\frac{3+\sqrt{3}}{\sqrt{3}-1}\right) + x\left(\frac{7+\sqrt{3}}{\sqrt{3}-1}\right) - \frac{3+\sqrt{3}}{\sqrt{3}-1} = 0$

$(x-1)\left(x^2\left(\sqrt{3}-1\right) - 4x + \left(3+\sqrt{3}\right)\right) = 0$

$\therefore roots\ \ are\ 1, \sqrt{3},\ \frac{\sqrt{3}+1}{\sqrt{3}-1}$

$Angles\ are\ 45^0, 60^0, 75^0$

$Least\ \ Angle\ is\ \ 45^0$

$x^3 + 3x^2 - 7x + 3 = \sqrt{3}\left(x^3 - x^2 + x - 1\right)$

$\therefore k = \sqrt{3}$

$\frac{R}{r} = \frac{R}{4R\ \sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}} = \frac{2\sqrt{2}}{\sqrt{3}-\sqrt{2}+1}$

PHYSICS PAPER – II : SOLUTION

1. Displacement of image of particle A is in phase with the displacement of particle A hence the displacement equation of particle A' s image is

$Y'_A = 0.2 \sin(\pi t)$ .....(1)

Displacement of image of particle B is out of phase with the displacement of particle B therefore the displacement of equation of particle B's image is

$Y_A' = 0.2\sin(\pi t + \pi)$ ...(2)

Relative vertical displacement of A.w.r.t. to B is

$Y_{AB}' = Y_A' - Y_B'$

$= 0.2\sin(\pi t) - 0.2\sin(\pi t + \pi)$

$= 0.4\sin(\pi t)$ ...(3)

2. $i_{rms} = 5A$

Reading of voltmeter $= i_{rms}\sqrt{R^2 + X_c^2}$

From this $X_C = 20\Omega \Rightarrow \omega = 10^3 rad/s$

$\therefore X_L = \omega L = 20\Omega$

Therefore the circuit is in resonance

$E_{rms} = i_{rms}R = 50V$ also power factor = 1

Av, power $= E_{rms}i_{rms}\cos\phi = 250W$

3. A, B, C

4. (A, C)

5. B,C

At time t_1, velocity of the particle is negative i.e. going towards $-X_m$. From the graph, at time t_1, its speed is decreasing. Therefore particle lies in between $-X_m$ and 0. At time t_2, velocity is positive and its magnitude is less than maximum i.e. it has yet not crossed O

It lies in between $-X_m$ and 0

Phase of particle at time t_1 *is* $(180 + \theta_1)$

Phase of particle at time t_2 *is* $(270 + \theta_2)$

Phase difference is $90 + (\theta_2 - \theta_1)$

$\theta_2 - \theta_1$ can be negative making $\Delta\phi < 90^o$ but can not be more than 90^o

6. Part 'A' has both the convex surfaces of same radius of curvature. Its focal length remains the same and so it is power 'P' $P = \frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = \frac{(\mu-1)2}{R_1}$

Part 'B' has one plane surface $P' = \frac{1}{f'} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{\infty}\right) = \frac{(\mu - 1)}{R_1}$

$P' = \frac{P}{2}$

7. For a field to be conservative

$$\frac{\partial E_x}{\partial y} = \frac{\partial E_y}{\partial x}$$
$$\frac{\partial E_y}{\partial z} = \frac{\partial E_z}{\partial y}$$
$$\frac{\partial E_x}{\partial z} = \frac{\partial E_z}{\partial x}$$

Options A,B and C are satisfying these conditions.

8. For a virtual object, a plane mirror produces a real image. A convex mirror can produce a real image

9. Using the conservation of charge principle, distribute the charge on all the surfaces of the plates a, b, c, d, e, and f. Then use the concept, electric field is zero inside the plates.

10. D

11. Let I be the intensity of light emitted by each slit. Resultant intensity can be expressed as,

$I_R = I_1 + I_2 + 2\sqrt{I_2 I_2}\cos\phi = I + I + 2I\cos\phi = 2I(1 + \cos\phi)$

$I_R = 4I\cos^2\phi/2$

Intensity of a bright fringe, say, the central maximum formed at *O* is I_0. At $O, \phi = 0°$.

$\therefore \quad I_0 = 4I$

At *P*, $\Delta = 11250\,Å$ = 2.25λ (as $\lambda = 5000\,Å$) $\Rightarrow \phi = \frac{2\pi}{\lambda} \times (2.25\lambda)$

So $\quad \phi = 4.5\pi$ or effectively $0.5\pi = \pi/2$

$I_R = I_0\cos^2\frac{\pi}{4} = \frac{I_0}{2}$

12. Point *P* is at distance *y* = 1 mm from *O*. Therefore, path difference at $P, \Delta = \frac{yd}{D} = \frac{10^{-3}d}{D}$

but $\quad \Delta = 11250\,Å = 2.25\lambda$

$\therefore \quad \frac{10^{-3}d}{D} = 2.25\lambda$

or $\quad \frac{\lambda D}{d} = \frac{10^{-3}}{2.25}\text{m} = 0.44 \times 10^{-3}\text{m}$

$\therefore \quad \frac{\lambda D}{d} = 0.44$ mm

Since $\quad \beta = \frac{\lambda D}{d}$

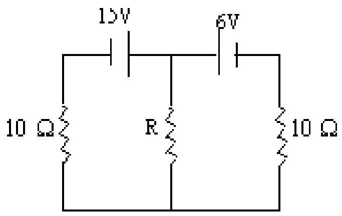
$\therefore \quad$ fringe width $\beta = 0.44\,\text{mm}$

13. $I \propto 1/r^2$

14. Power absorbed by the sphere $P = \dfrac{3.2\times10^{-3}}{4\pi(0.8)^2}\pi(8\times10^{-3})^2$

No. of photons incident on it per sec $n\dfrac{hc}{\lambda} = P$

15–16



17. F = Bil

i = $\dfrac{F}{Bl} = \dfrac{3.2\times10^{-5}}{2\times10^{-2}\times8\times10^{-2}} = 0.02A$

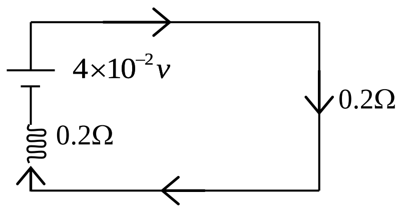
$Blv = 0.02\times2 = 0.04$

$v = \dfrac{0.04}{Bl} = \dfrac{0.04}{0.02\times8\times10^{-2}} = \dfrac{200}{8} = 25m/s$

Emf induced = Blv = $4\times10^{-2}volt$

$V_a - V_b = 0.2\times0.02 - 4\times10^{-2}$

$= -3.6\times10^{-2}volt$



18. $V_b - V_c = 0.8\times0.02 = 1.6\times10^{-2}V$

19. A

20. C

CHEMISTRY PAPER – II SOLUTION

21. (C)

$N_2(g)+3H_2(g) \rightleftharpoons 2NH_3(g)$

11 mole 12 mole 0

$11-x$ $12-3x$ $2x$

At equation: $12-3x=6$

$x=6\text{mole}$

$NH_2=6\,\text{mole}$

$NNH_2=11-2=9\,\text{mole}$

$NNH_3=4\text{mole}$

Applying Ideal gas equation

$PV=nRT$

$P(20-3.58)=(nN_2+NH_2)RT$

$P=22.5\,\text{atm}$

24. (ACD)

ΔH is state function so its's not dependent on path.

27. (AC)

$KNO_3 \rightarrow KNO_2+O_2\uparrow$

$NaNO_3 \rightarrow NaNO_2+O_2\uparrow$

$Cl(NO_3)_2 \rightarrow CuO+NO_2\uparrow$

$Pb(NO_3)_2 \rightarrow PbO+NO_2\uparrow$

29. (C)

For spontaneous mixing

$\Delta G_{mix}<0,\ TDS_{mix}>\Delta H_{mix}$

30. (ABCD) 31. (A) 32. (B) 33. (C) 34. (A)

35. (C) 36. (D) 37. (D) 38. (A) 39. (B)

40. (A)

MATHEMATICS PAPER – II : SOLUTION

41. **(ABD)**

Graph is symmetrical about (4, 0)

$\Rightarrow f(4+x) = -f(4-x)$

$\Rightarrow f(x) = -f(8-x)$ ....(i)

now let f(x) = 2010

then f(8-x) = -2010

$\Rightarrow f^{-1}(2010) + f^{-1}(-2010) = 8$

and $\int_{-2010}^{4} f(x)dx = -\int_{4}^{2018} f(x)dx$

also D = $(f'(10))^2 + 4f'(10) > 0$ as

$f'(4+x) = f'(4-x)$

$f'(x)$ is symmetric about x = 4

42. **(AC)**

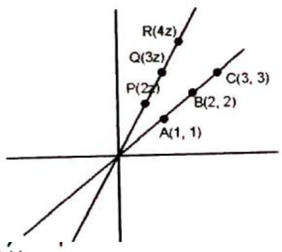
$$f(x) = \begin{cases} \frac{\pi}{2}, & -\ell n2 < x < 0 \\ \frac{\pi}{2}, & 0 < x < \ell n2 \\ \pi, & x = 0 \end{cases}$$

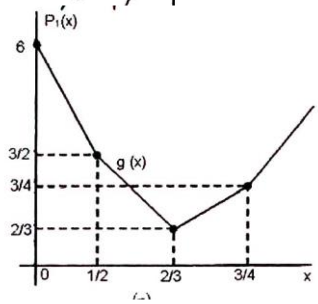
43. **(BC)**

Clearly z, 2z, 3z and 4z will lie on the same line, also P(z) will be minimum when AP + BQ + CR is minimum. Now for any fixed |z| = r, P (z) will be minimum when line PR coincides with line AB. So for minimum value of P(z), PQ and AB should coincides so for minimum value of P(z), we can minimize $P_1(x) = |2x - 1| + |3x - 2| + |4x - 3|$.

Clearly $P_1(x)]_{min} = \frac{2}{3}$ (from the graph)

$\Rightarrow$ $P(z)]_{min} = \frac{2}{3}$ and corresponding $z = \frac{2}{3} + \frac{2}{3}i$. (As AB : x = y).





44. **(AC)**

$I(k) = \int_{-k}^{k} [x]\{x^2\}dx = \int_{-k}^{k} [-x]\{x^2\} dx$

$= \int_{-k}^{k} (-1-[x])\{x^2\}dx = -\int_{-k}^{k} \{x^2\}dx - I(k)$ (x is not an integer)

$\Rightarrow$ $I(k) = \int_{0}^{k} ([x^2] - x^2)dx$

$= \int_{0}^{1} 0\,dx + \int_{1}^{\sqrt{2}} 1\,dx + \int_{\sqrt{2}}^{\sqrt{3}} 2\,dx + \int_{\sqrt{3}}^{\sqrt{4}} 3\,dx + ... + \int_{\sqrt{k^2-1}}^{k} (k^2 - 1)dx - \frac{k^3}{3}$

$\Rightarrow \quad I(k)+\frac{k^3}{3}=0+1\left(\sqrt{2}-1\right)+2\left(\sqrt{3}-\sqrt{2}\right)+3\left(\sqrt{4}-\sqrt{3}\right)+...+\left(k^2-1\right)\left(k-\sqrt{k^2-1}\right)$

$\Rightarrow \quad I(k)+\frac{k^3}{3}=-1-\sqrt{2}-\sqrt{3}-\sqrt{4}-....-\sqrt{k^2-1}+k\left(k^2-1\right)$

$\Rightarrow \quad I(k)=(k-1)k(k+1)-\frac{k^3}{3}-\sum_{r=1}^{k^2-1}\sqrt{r}$.

45. **(AD)**

$\frac{a^2+b^2-c^2}{2ab}=\frac{1}{2}$

$\cos C = \frac{1}{2}\Rightarrow C=60^0$

now $B\ge 60^0$ and $A\le 60^0$ $\quad B\ge A$

$\Rightarrow B\ge C\Rightarrow b\ge c$

$B\ge A\Rightarrow b\ge a$

$\Rightarrow (b-c)(a-b)\le 0$

46. **(BC)**

$(a-1)x^2-(a^2+2)x+(a^2+2a)=0$ has roots $a, \frac{a+2}{a-1}$

$(b-1)x^2-(b^2+2)x+(b^2+2a)=0$ has roots $b, \frac{b+2}{b-1}$

$a\ne b$

$\Rightarrow \frac{a+2}{a-1}=b\Rightarrow (a-1)(b-1)=3$

$\Rightarrow a=4, b=2$ or a = 2, b = 4

47. **(AD)**

Bisector of two line are

$(k_1u-k_2v)=\pm(k_1u+k_2v)$

$\Rightarrow u=0, v=0$

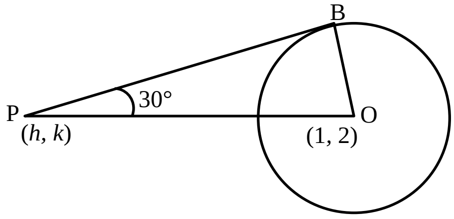
48. **(D)**

$\left(\sin^2 a\right)^3+\left(\cos^2 b\right)^3+(-1)^3=3(-1)\left(\sin^2 a\right)\left(\cos^2 b\right)$

$\Rightarrow \sin^2 a=\cos^2 b=-1\Rightarrow$ Not possible, or $\sin^2 a+\cos^2 b=1$

or $\sin^2 a+\cos^2 b=1\Rightarrow a=b$

49. **(BD)**



$\frac{1}{\sqrt{3}}=\frac{3}{\sqrt{h^2+k^2-2h-4k-4}}$

$\Rightarrow x^2+y^2-2x-4y-31=0$

So locus of P is a circle of radius 6

Locus of Q is $(x-1)^2+(y-2)^2=\left(6\sqrt{2}\right)^2$

50. **(AB)**

$b-a=c-b=d-c=\lambda$ (say)

$$f(x)=\begin{vmatrix}x+a & x+b & x+a-c\\ x+b & x+c & x-1\\ x+c & x+d & x-b+d\end{vmatrix}=\begin{vmatrix}x+a & x+b & x+a-c\\ \lambda & \lambda & -1+2\lambda\\ \lambda & \lambda & 1+2\lambda\end{vmatrix}=\begin{vmatrix}x+a & \lambda & x+a-c\\ \lambda & 0 & -1+2\lambda\\ \lambda & 0 & 1+2\lambda\end{vmatrix}$$

$\downarrow$ $R_2 \to R_2 - R_1$, $R_3 \to R_3 - R_2$; $\downarrow$ $C_2 \to C_2 - C_1$

$f(x)=-\lambda(\lambda+2\lambda^2+\lambda-2\lambda^2)=-2\lambda^2$

$\int_0^2 -2\lambda^2 dx=-4 \Rightarrow \quad 2\lambda^2[x]_0^2=4 \Rightarrow 2\lambda^2\times 2=4 \Rightarrow \lambda^2=1 \Rightarrow \lambda=\pm 1$

51. **(B)**

$\frac{d}{dx}\left(ye^{\phi(x)}\right)=H(x)e^{\int \phi'(x)}>0\ \forall x\in R$

$\Rightarrow ye^{\phi(x)}$ is Increasing function

$\Rightarrow ye^{\phi(x)}=\lambda$ will have exactly one solution.

52. **(ABC)**

Given $\frac{d}{dx}(xy)$ = x (x – 1) (x – 2) …………...(x – 100)

Number of zeroes of $\left(\frac{d(xy)}{dx}\right)$ are 101

Then minimum number of zeroes of $\frac{d}{dx}\left(\frac{d}{dx}(xy)\right)$ are 100 .

53. **(C)**

As $f'(1)=-3$ $\therefore$ This belongs to case A.

$\therefore f(1)-\frac{1}{2}=I \Rightarrow f(1)=I+\frac{1}{2}$.

54. **(C)**

$f(x)=\left(1-e^x\right)\frac{\sin x}{x} \qquad x>0$

$\left(e^x-1\right)\frac{\sin x}{x} \qquad x<0$

$\Rightarrow f(x)<0$ for $x>0$

$f(x)<0$ for $x<0$

$\Rightarrow \lim_{x\to 0}\left[f(x)\right]=-1$

55. **(C)**

Any normal to the parabola is $y = mx - 2m - m^3$ and any normal to the ellipse is

$y=mx-\frac{(a^2-b^2)m}{\sqrt{a^2+b^2m^2}} \Rightarrow 2m+m^3=\frac{(a^2-b^2)m}{\sqrt{a^2+b^2m^2}} \Rightarrow m=0 \text{ or } 2+m^2=\frac{(a^2-63)}{\sqrt{a^2+63m^2}}$ …(1)

for one common normal the equation (1) does not have any solutions.

$\Rightarrow 2 > \frac{a^2 - 63}{|a|} \Rightarrow a^2 - 2|a| - 63 < 0 \Rightarrow (|a| + 7)(|a| - 9) < 0 \Rightarrow a = 8.$

56. **(A)**

Since the sum of the slopes of the normals is zero.
They are always concurrent.

57. **(A)** 58. **(B)**

Case I: Let the line L cuts AO and AB at distances X and Y from A.
$\Rightarrow$area of the triangle with sides x and y is

$\frac{3xy}{10} = 12 \Rightarrow xy = 40$

also, x + y = 12 (using perimeter bisection)
This is not possible.

Case II: If the line L cuts OB and BA at distances y and x from B then we have xy = 30 and x + y = 12

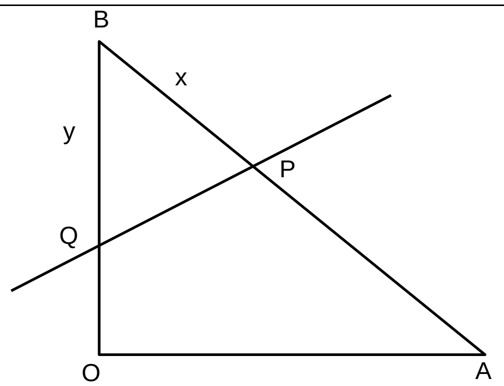
$\Rightarrow x = 6 + \sqrt{6}$ and $y = 6 - \sqrt{6}$.

Case III: If the line L cuts the sides OA and OB at distance x and y from O then
x + y = 12 and xy = 24
$x, y = 6 \pm 2\sqrt{3}$ (not possible)
so there is a unique line possible.

Let point P be (α, β)
Using parametric equation of AB

$\beta = 6 - \frac{3}{5}(6 + \sqrt{6})$ and $\alpha = \frac{4}{5}(6 + \sqrt{6})$

$\Rightarrow$slope of PQ is $\frac{\beta - \sqrt{6}}{\alpha - 0} = \frac{10 - 5\sqrt{6}}{10}$.



59. **(B)**

f(x) = odd degree polynomial + bounded function $\cot^{-1}x \in (0, \pi)$
also, $f'(x) > 0$.

60. **(D)**

$f(x) = x^4 + 1 + \frac{1}{x^2 + x + 1}$ = even degree polynomial + bounded function $\frac{1}{x^2 + x + 1} \in \left(0, \frac{4}{3}\right]$

$f'(x) = \frac{4x^3(x^2 + x + 1)^2 - 2x - 1}{(x^2 + x + 1)^2}$

$\Rightarrow f'(x) = 0$ has atleast one root which is repeated odd number of times or it has one root which is not repeated since numerator of $f'(x)$ is a polynomial of degree 7.
$\Rightarrow$ f(x) = 0 has a point of extrema.