PART (A) : PHYSICS

ANSWER KEY

1.	(D)	2.	(D)	3.	(C)	4.	(C)	5.	(D)
6.	(B)	7.	(A)	8.	(C)	9.	(D)	10.	(C)
11.	(C)	12.	(B)	13.	(B)	14.	(A)	15.	(C)
16.	(A)	17.	(B)	18.	(C)	19.	(D)	20.	(B)
21.	(3)	22.	(4)	23.	(5)	24.	(6)	25.	(3)
26.	(1)	27.	(3)	28.	(7)	29.	(2)	30.	(9)

SOLUTIONS

1. (D)

$$P_t = P_c\left(1+\frac{m^2}{2}\right) = 9\left(1+\frac{(0.6)^2}{2}\right) = 9\left[1+\frac{0.36}{2}\right]$$

$$= 9[1+0.18] = 9[1.18] = 10.62\,\text{kw}$$

2. (D)

Out pout of $G_1 = \overline{A}$, out put of $G_2 = \overline{B}$

Out put of $G_3 = AB$

Now $Y = \overline{AB}$ (NAND gate)

3. (C)

$$N_1 = N_0 e^{-10\lambda t}, N_2 = N_0 e^{-\lambda t} \therefore \frac{N_1}{N_2} = \frac{1}{e^{9\lambda t}}$$

$$\therefore \frac{1}{e} = \frac{1}{e^{9\lambda t}} \therefore t = \frac{1}{9\lambda}$$

4. (C)

As the number of orbit increases the velocity decreases i.e K.E decreases the potential energy becomes less negative i.e P.E increases.

5. (D)

The current in the circuit starts oscillating.

6. (B)

$$\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$$

Therefore, for a given mirror $\frac{1}{v}$ versus $\frac{1}{u}$ graph should be a straight line at $u = f$ or

$\frac{1}{u}=\frac{1}{f}, v=\infty$ or $\frac{1}{v}=0$ and viceversa

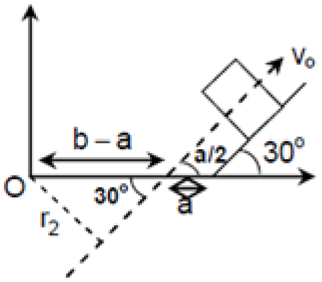
7. (A)
Fact

8. (C)

$$\rho=\frac{\pi D^2}{4L}\frac{v}{i}=\frac{(3.14)\left(2.00\times10^{-3}\right)^{-2}}{4(0.314)}\left(\frac{100.0}{10.0}\right)$$

$$\rho=1.00\times10^{-4}\Omega-m$$

9. (D)



$$r_1=(b-a)\sin 30^o$$

$$=\left(\frac{b-a}{2}\right)$$

$$\therefore L=mv_0r_2=\frac{mv_0(b-a)}{2}$$

10. (C)
Normal thrust per unit area is actually the pressure due to liquid

11. (C)

For $r\le r_1$ $V=\frac{Gm_1}{r_1}-\frac{Gm_2}{r_2}=\text{constant}$

For $r_1\le r\le r_2$ $V=-\frac{Gm_2}{r_2}-\frac{Gm_1}{r_1}$

Slope of V–x graph $dV/dr=Gm_1/r^2$

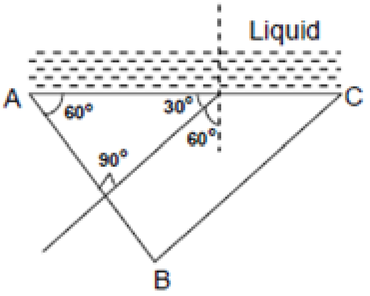
For $r\ge r_2$ $V=-\frac{G}{r}(m_1+m_2)$

Slope of V–r graph $=\frac{G}{r}(m_1+m_2)$

At the boundary of outer shell slope of V–r graph changes from

$\frac{Gm_1}{r_2^2}$ to $\frac{G(m_1+m_2)}{r_2^2}$ i.e slope increases

12. (B)



Critical angle between glass and liquid face is $\sin\theta_C = \dfrac{\mu}{3/2}$

$\therefore \sin\theta_C = \dfrac{2\mu}{3}$

Angle of incidence at face AC is 60^o

For TIR to take plane $I > \theta_C$

Or $\sin 60^o > \dfrac{2\mu}{3}$ or $\mu < \dfrac{3\sqrt{3}}{4}$.

13. (B)

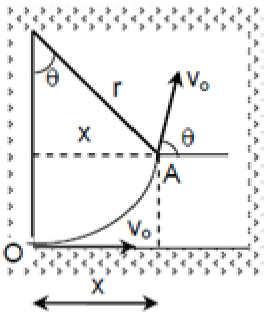
The two slabs will shift the image a distance

$$d = \left(1 - \frac{1}{\mu}\right)t = \left(1 - \frac{1}{1.5}\right)(1.5) = 1\text{cm}$$

14. (A)

No current will flow through the resistance at steady state.

15. (C)



$r = \dfrac{mv_0}{B_0 q} = \dfrac{v_0}{B_0\alpha}$

$\dfrac{x}{r} = \dfrac{\sqrt{3}}{2} = \sin\theta \therefore \theta = 60^o$

$t_{QA} = \dfrac{T}{6} = \dfrac{\pi}{3B_0\alpha}$

Therefore x–coordinate of particle at any time $t > \dfrac{\pi}{3B_0\alpha a}$ will be

$$x=\frac{\sqrt{3}}{2}\frac{v_0}{B_0\alpha}+v_0\left(t-\frac{\pi}{3B_0\alpha}\right)\cos 60^\circ \text{ So } x=\frac{\sqrt{3}}{2}\frac{v_0}{B_0\alpha}+\frac{v_0}{2}\left(t-\frac{\pi}{3B_0\alpha}\right)$$

16. (A)

$$B=2\left[\frac{\mu_0}{4\pi}.\frac{i}{d/\sqrt{2}}\right]\left(\sin 90^\circ-\sin 45^\circ\right)$$

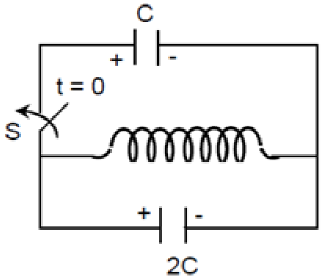
$$B=\frac{\mu_0 i}{\sqrt{2}\pi d}\left(1-\frac{1}{\sqrt{2}}\right)\otimes$$

17. (B)

Polarity of emf will be opposite in the two cases while entering and while leaving the coil only in option (b) polarity is changing

18. (C)

The equivalent circuit is as shown in fig.



$$U_{L(max)}=U_{C(max)}=\frac{1}{2}\frac{(3Q)^2}{3C}=\frac{3Q^2}{2C}$$

19. (D)

A solid with completely filled valence band is an insulator, if the energy gap between the valence band and the empty conduction band is larger than about 5 eV

20. (B)

$$C=C_V+\frac{PdV}{ndT},$$

$PV^{3/2}=k$, $PV=nRT$

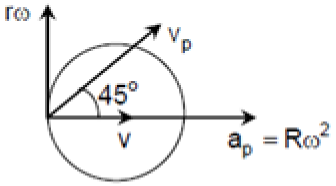
Find relation in V and T

$$\frac{dV}{dT}=-\frac{2V}{T}$$

$$\therefore\ C=C_V-2R$$

21. (3)

Angle is 45°



22. (4)

$a = g\sin\theta - (\mu_0 g\cos\theta)x \therefore \int_0^v v dv = \int_0^{x_{max}} [g\sin\theta - (\mu_0 g\cos\theta)x]dx$

After solving we get $x_{max} = \dfrac{2\tan\theta}{\mu_0}$

23. (5)

$\dfrac{d\vec{v}}{dt} = ti + 3t^2 j \quad \therefore \int_0^v d\vec{v} = \int_0^2 (ti + 3t^2 j)dt$

$\therefore \vec{v} = (2i + 8j)\,m/s$

$\therefore v = \sqrt{68}$

By work energy theorem $\therefore w = \dfrac{1}{2}mv^2 \therefore w = 34\,J$

24. (6)

Net external force on the two blocks (whether they move with same retardation or not)

$f_{ext} = (0.2)(2+1)\times 10 = 6N$

$\therefore a_{cm} = \dfrac{f_{ext}}{2+1} = 2\,m/s^2$

25. (3)

$PV = nRT \therefore pdv = nRdT \therefore dv = \left(\dfrac{v}{T}\right)dT$ ……….. (i)

$dv = \gamma v dT$ ……….. (ii)

From (i) & (ii) $\gamma = \dfrac{1}{T}$

26. (1)

$v = \sqrt{gh} \therefore \dfrac{dh}{dt} = \sqrt{gh} \therefore t = \int_0^h \dfrac{dh}{\sqrt{gh}} \text{ or } t = 2\sqrt{\dfrac{h}{g}}$

Now at the time of meeting,

Time of fall of particle = time of wave pulse on reaching up to these

$\sqrt{\dfrac{2(L-h)}{g}} = 2\sqrt{\dfrac{h}{g}} \therefore h = \dfrac{L}{3}$

27. (3)

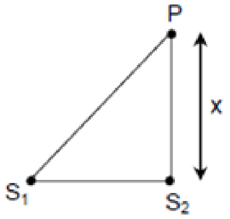
Comparing the given equation with $x = a\cos\omega t$

$\therefore \omega = \frac{\pi}{2}$ or $\frac{2\pi}{T} = \frac{\pi}{2} \therefore T = 4\sec$

The given time $t = 3\sec$ is really $\frac{3T}{4}$

$\therefore$ Distance covered will be 3 a

28. (7)



Path difference at S_2 is 2λ. Therefore for minimum intensity at P.

$S_1P - S_2P = \frac{3\lambda}{2}$ .... (i)

Or $\sqrt{4\lambda^2 + x^2} - x = \frac{3\lambda}{2}$

Solving equation, $x = \frac{7\lambda}{12}$

29. (2)

30. (9)

Series and parallel grouping of resistors.

11R/18

PART (B) : CHEMISTRY

SOLUTIONS

31.	(A)	32.	(A)	33.	(D)	34.	(B)	35.	(A)
36.	(C)	37.	(B)	38.	(C)	39.	(B)	40.	(C)
41.	(D)	42.	(D)	43.	(C)	44.	(A)	45.	(B)
46.	(B)	47.	(D)	48.	(A)	49.	(B)	50.	(C)
51.	(6)	52.	(4)	53.	(0)	54.	(4)	55.	(1)
56.	(5)	57.	(5)	58.	(4)	59.	(2)	60.	(4)

SOLUTIONS

31. (A)

No. of radial nodes = $n-\ell-1$

In 3s, $n-\ell-1=3-0-1=2$

In 2p, $n-\ell-1=2-1-1=0$

32. (A)

No. of moles of CO_2 evolved $=\dfrac{0.25\times10^{-3}}{25}=10^{-5}$

$2\,NaHCO_3+H_2C_2O_4\longrightarrow Na_2C_2O_4+2CO_2+2H_2O$

No. of moles of $NaHCO_3$ = 10^{-5}

Wt. of $NaHCO_3$ = 84 × 10^{-5} gm

% of $NaHCO_3$ $\dfrac{84\times10^{-5}}{10\times10^{-3}}\times100=8.4$

33. (D)

S of $MX=\sqrt{K_{sp}}=\sqrt{4\times10^{-8}}=2\times10^{-4}$

S of MX_2 = $\left(\dfrac{K_{sp}}{4}\right)^{1/3}=\left(\dfrac{3.2\times10^{-14}}{4}\right)^{1/3}=2\times10^{-5}$

S of M_3X = $\left(\dfrac{K_{sp}}{27}\right)^{1/4}=\left(\dfrac{2.7\times10^{-15}}{27}\right)^{1/4}=10^{-4}$

Solubility of MX > M_3X > MX_2

34. (B)

$C_2H_5OH\,(\ell) + 3O_2(g) \rightarrow 2CO_2\,(g) + 3H_2O\,(\ell)$

$\Delta H=\Delta U+\Delta ngRT$

$=-1364.47+\dfrac{(-1)8.314\times298}{1000}$

$=-1366.95\ kJ/mol$

35. (A)

For octahedral void, $\frac{r^+}{r^-} = 0.414$

$r^+ = 0.414\, r^-$

$= (0.414)(250) = 103.5 \approx 104\, pm$

36. (C)

$\Delta t_f = iK_f\ m$

$= 4 \times 1.86 \times \frac{0.1 \times 1000}{329 \times 100} = 0.0226 = 2.26 \times 10^{-2}$

$f.p. = -2.26 \times 10^{-2}\ ^oC$

37. (B)

Mass = Volume × density

= 80 × 0.005 × 10^{-1} × 10.5

= 0.42 gm

$\frac{0.42}{108} \times 1 = \frac{3 \times t}{96500}$

$t = \frac{0.42 \times 96500}{108 \times 3} sec = 125\, sec$

38. (C)

Slope = –4606 $= -\frac{E_a}{R}$

$Ea = 4606\, R$

$\log \frac{K_2}{K_1} = \frac{Ea(T_2 - T_1)}{2.303\, R\, (T_1 T_2)}$

$\log \frac{K_2}{K_1} = 1$

$K_2 = 10\, K_1 = 10^{-4}\, sec^{-1}$

39. (B)

It is negatively charged solution coagulating power $\propto$ charge.

40. (C)

When H_2O_2 acts as oxidising agent then itself is reduced to H_2O. while when it acts as reducing agent then itself is oxidised into O_2.

41. (D)

$$\begin{array}{ccccccc} & O & & & & O & \\ & \| & & & & \| & \\ O^- - & S & - & O - O - & & S & - O^- \\ & \| & & & & \| & \\ & O & & & & O & \end{array}$$

42. (D)
$B_2H_6 + 2NH_3 \rightarrow [BH_2(NH_3)_2]^+ [BH_4]^-$
Similar reaction with CH_3NH_2 & $(CH_3)_2NH$.

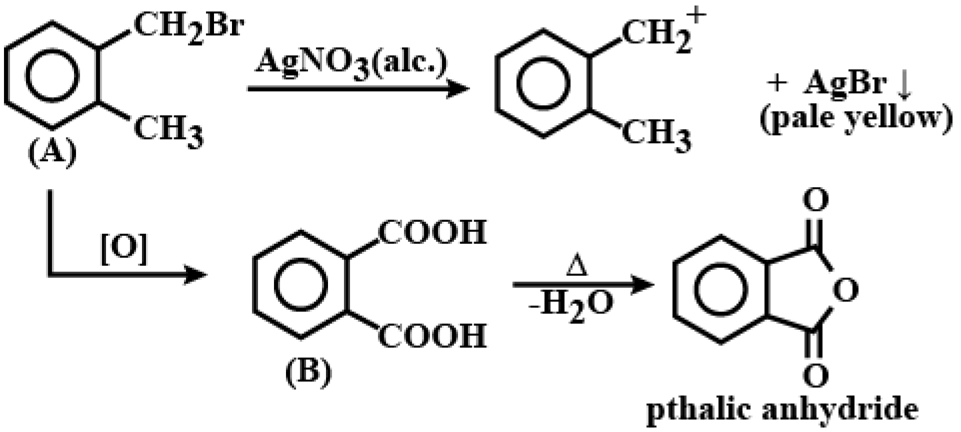
43. (C)
n factor of $MnO_4^- = 5$
n factor of $C_2O_4^{2-} = 2$
$2Mno_4^- + 5C_2O_4^{2-} + 16H^+ \rightarrow 2Mn^{2+} + 10CO_2 + 8H_2O$

44. (A)
$\underset{\text{impure}}{Ni} + 4CO \xrightarrow{\Delta} Ni(CO)_4 \xrightarrow{\Delta} \underset{\text{Pure}}{Ni} + 4CO$

45. (B)

46. (B)
$Ni(CO)_4$: Tetrahedral
$[Ni(CN)_4]^{2-}$: Square planner
$K_3[Cu(CN)_4]$: Tetrahedral
$[NiCl_4]^{2-}$: Tetrahedral

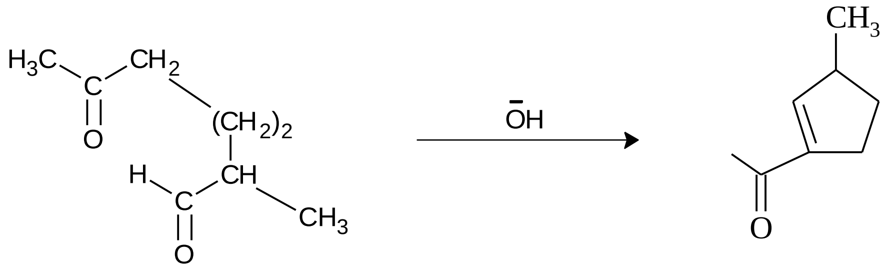
47. (D)



48. (A)
(A) on hydrolysis forms a hemiacetal which is reducing.

49. (B)
EAS will occur on the highly activated ring.

50. (C)

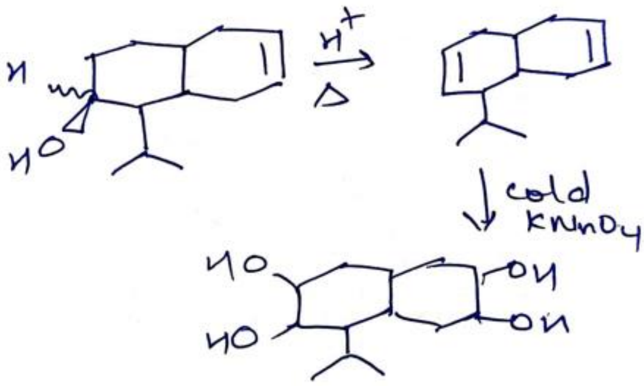


51. (6)
H_2, Li_2, C_2, N_2, Br_2, F_2 are diamagnetic.

52. (4)

53. (0)

54. (4)

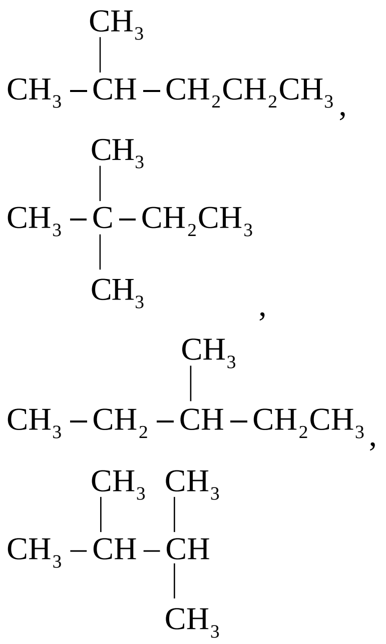


55. (1)
$x+5(0)+1+(-2)=0$
$x=+1$

56. (5)
PbS, CuS, HgS, NiS, CoS are black.

57. (5)

$CH_3-(CH_2)_4-CH_3$,



58. (4)

59. (2)

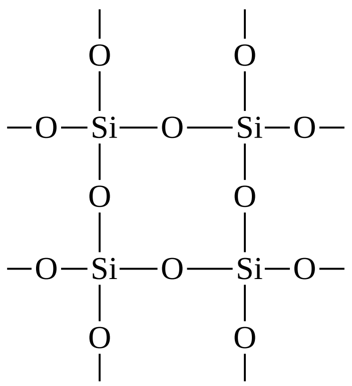
$$MX_2 \rightleftharpoons M^{2+} + 2X^-$$
$$(1-\alpha) \qquad \alpha \qquad 2\alpha$$

$i = 1-\alpha+\alpha+2\alpha$

$i = 1+2\alpha$

$i = 1+2(0.5) = 2$

60. (4)



PART (C) : MATHEMATICS

ANSWER KEY

61.	(C)	62.	(C)	63.	(D)	64.	(B)	65.	(A)
66.	(A)	67.	(C)	68.	(C)	69.	(A)	70.	(D)
71.	(C)	72.	(C)	73.	(C)	74.	(A)	75.	(C)
76.	(D)	77.	(B)	78.	(D)	79.	(A)	80.	(A)
81.	(4)	82.	(1)	83.	(7)	84.	(3)	85.	(1)
86.	(3)	87.	(2)	88.	(2)	89.	(6)	90.	(7)

SOLUTIONS

61. (C)

$x^2+bx-1=0$ & $x^2+x+b=0$ have common root α.

$\Rightarrow \quad \alpha^2+b\alpha-1=0$

$\alpha^2+\alpha+b=0$

$\Rightarrow \quad \dfrac{\alpha^2}{b^2+1}=\dfrac{\alpha}{-(b-1)}=\dfrac{1}{(1-b)} \quad \Rightarrow \quad (b+1)^2=(b^2+1)(1-b)$

$\Rightarrow \quad b^2+2b+1=b^2-b^3+1-b \quad \Rightarrow \quad b^3+3b=0$

When b = 0 then common roots is (–1) hence b = 0 rejected.

So, $b^3=-3 \Rightarrow b=\pm\sqrt{3}\,i \quad \Rightarrow \quad |b|=\sqrt{3}$

62. (C)

$$\sum_{r=1}^{15} r^2\left(\frac{^{15}C_r}{^{15}C_{r-1}}\right)=\sum_{r=1}^{15} r^2\left(\frac{15-r+1}{r}\right)=\sum_{r=1}^{15} r(16-r)=16\left(\frac{15\times16}{2}\right)-\frac{15\times16\times31}{6}=\frac{15\times16}{6}(17)=680.$$

63. (D)

$q\Rightarrow(\sim p\vee q)$

$\sim q\vee(\sim p\vee q)$

$(\sim q\vee\sim p)\vee(\sim q\vee q)$

$(\sim q\vee\sim p)\vee(T)$

T

64. (B)

$f(x)=2x^3-15x^2+36x+1$

$f'(x)=6x^2-30x+36$

$=6(x^2-5x+6)$

$=6(x-2)(x-3)$

$f(x)$ is increasing in [0, 2] and decreasing in [2, 3].

$f(x)$ is many one.

$f(0)=1$

$f(2)=29$

$f(3)=28$

Range is [1, 29].

Hence, $f(x)$ is many-one onto.

Hence, the correction is option (B).

65. (A)

Given $\sin\theta = 3\sin(\theta+2\alpha)$.

Now, $\sin(\theta+\alpha-\alpha) = 3\sin(\theta+\alpha+\alpha)$

$\Rightarrow \sin(\theta+\alpha)\cos\alpha - \cos(\theta+\alpha)\sin\alpha$

$= 3\sin(\theta+\alpha)\cos\alpha + 3\cos(\theta+\alpha)\sin\alpha$

$\Rightarrow -2\sin(\theta+\alpha)\cos\alpha = 4\cos(\theta+\alpha)\sin\alpha$

$$\Rightarrow \frac{-\sin(\theta+\alpha)}{\cos(\theta+\alpha)} = \frac{2\sin\alpha}{\cos\alpha}$$

$\Rightarrow \tan(\theta+\alpha) + 2\tan\alpha = 0$

Hence, the correct answer is option (A).

66. (A)

$f(x)$ is continuous for all non integers

For integers

$f(I^+) = I - 0 = I$

$f(I^-) = I + 1 + \sqrt{1} = I$

$\therefore f(x)$ is continuous for integers

$\therefore f(x)$ is continuous $\forall x \in R$

67. (C)

$\vec{a} : 2\hat{i} + 4\hat{j} + 5\hat{k}$; $\vec{p} = 3\hat{i} + 4\hat{j} + 5\hat{k}$

$\vec{b} : \hat{i} + 2\hat{j} + 3\hat{k}$; $\vec{q} = 2\hat{i} + 3\hat{j} + 4\hat{k}$

$$SD = \left| \frac{(\vec{b}-\vec{a})\cdot(\hat{i}-2\hat{j}+\hat{k})}{|\hat{i}-2\hat{j}+\hat{k}|} \right| = \frac{1}{\sqrt{6}}$$

68. (C)

$$\sec x = \frac{1}{1-\cos x} \Rightarrow \cos x = 1 - \cos x$$

$$\Rightarrow \cos = \frac{1}{2}$$

2 solutions in $[0, 2\pi]$

$\therefore$ 100 solutions

69. (A)

$$2xydx + x^2dy = \frac{ydx - xdy}{y^2}$$

$$\Rightarrow d\left(x^2y\right) = d\left(\frac{x}{y}\right)$$

$$\Rightarrow x^2y = \frac{x}{y} + c$$

At $x = 2; y = 1 \Rightarrow 4 = 2 + c \Rightarrow c = 2$

At $x = -1; y = -\frac{1}{y} + 2 \Rightarrow y = 1$

70. (D)

$$y = mx + \frac{2}{m}; \ y = mx \pm \sqrt{32m^2 + 8}$$

$$\therefore \ \frac{2}{m} = \pm\sqrt{32m^2 + 8}$$

$$\Rightarrow \frac{4}{m^2} = 32m^2 + 8 \Rightarrow 1 = 8m^4 + 2m^2$$

$$\Rightarrow m^2 = -\frac{1}{2}; \ m^2 = \frac{1}{4} \Rightarrow m = \frac{1}{2} \text{ or } -\frac{1}{2}$$

$\therefore$ Product $= -\frac{1}{4}$

71. (C)

$$f'(x) = 0 \text{ at } x = 0, \pm\sqrt{2}$$

$$f(x)_{\max} = \frac{4}{e^2}; \ f(x)_{\min} = 0$$

72. (C)

$$\int \frac{\log x}{x}\left(\frac{1-\log x}{x^2}\right)dx + \int \frac{dx}{x}$$

Put $\frac{\log x}{x} = t \Rightarrow \int t\,dt + \ln x + c$

$\Rightarrow \frac{1}{2}\left(\frac{\log x}{x}\right)^2 + \log x + c$

73. (C)

$\text{Var}(ax_i + b) = a^2 \text{ var}(x_i)$

Variance on doubling each observation

$= 2^2 \times 16 = 64$

Std. deviation $= \sqrt{\text{var}} = 8$

74. (A)

$\left[\vec{a}\ \vec{b}\ \vec{c}\right] = 12$

$\frac{1}{6}\left(\left[\vec{a}-\vec{b}\quad \vec{b}-\vec{c}\quad \vec{a}-\vec{c}+\vec{b}\right]\right) = \frac{1}{6}\left[\vec{a}\ \vec{b}\ \vec{c}\right] = 2$

75. (C)

$f'(x) = f(x);\ \ f(1) = 0$

$\therefore\ \frac{f'(x)}{f(x)} = 1 \Rightarrow \ln(f(x)) = x + c$

$\Rightarrow\ f(x) = k.e^x \Rightarrow k = 0$

$\therefore\ f(x) = 0$.

76. (D)

We have

$P(-\sin(\beta-\alpha), -\cos\beta)[= (x_1, y_1)$

$Q = (\cos(\beta-\alpha), \sin\beta) = (x_2, y_2)$

$R = (\cos(\beta-\alpha+\theta), \sin(\beta-\theta))$

$= (x_2\cos\theta + x_1\sin\theta), y_2\cos\theta + y_1\sin\theta)$

Let T is a point on PQ which divides PQ in $\cos\theta : \sin\theta$. Then

$T = \left(\frac{x_2\cos\theta + x_1\sin\theta}{\cos\theta + \sin\theta}, \frac{y_2\cos\theta + y_1\sin\theta}{\cos\theta + \sin\theta}\right)$

$\Rightarrow$ P, Q, R are collinear

Therefore, P, Q, R are non-collinear.

Hence, the correct answer is option (D).

77. (B)

$\sqrt{(x-2)^2 + y^2} + \sqrt{(x+2)^2 + y^2} = 4$

$\Rightarrow \sqrt{x^2 + y^2 + 4 - 4x} + \sqrt{x^2 + y^2 + 4 + 4x} = 4$

$\Rightarrow \sqrt{k - 4x} + \sqrt{k + 4x} = 4$, (where $k = x^2 + y^2 + 4$)

$\Rightarrow \sqrt{k-4x} = 4 - \sqrt{k+4x}$

Squaring both sides

$$k - 4x = 16 + k + 4x - 8\sqrt{k+4x} \Rightarrow x + 2 = \sqrt{k+4x}$$

Squaring again

$$x^2 + 4x + 4 = k + 4x \Rightarrow y^2 = 0$$

Squaring, and on simplification, it reduces to $y^2 = 0$.

Hence, the correct answer is option (B).

78. (D)

Circle touching y-axis at (0, 2) is $(x-0)^2 + (y-2)^2 + \lambda x = 0$ and it passes through (–1, 0). Therefore,

$$1 + 4 - \lambda = 0 \Rightarrow \lambda = 5$$

Therefore,

$$x^2 + y^2 + 5x - 4y + 4 = 0$$

Put $y = 0 \Rightarrow x = -1, -4$

Therefore, circle passes through (–4, 0).

Hence the correct answer is option (D).

79. (A)

The equation of the two parabolas can be written as follows:

$$y^2 = 4a(x-k)$$

and $y^2 = -4b(x+k)$

A line parallel to the common axis is $y = h$. Then

$$A = \left(\frac{h^2}{4a} + k, h\right) \text{ and } B = \left(\frac{-h^2}{4b} - k, h\right)$$

If $P \equiv (\alpha, \beta)$, then

$$\alpha = \frac{1}{2}\left(\frac{h^2}{4a} + k - k - \frac{h^2}{4b}\right)$$

Now, $\beta = h$. Therefore,

$$2\alpha = \frac{h^2}{4}\left(\frac{1}{a} - \frac{1}{b}\right)$$

The locus of P is

$$2x = \frac{y^2}{2}\left(\frac{b-a}{ab}\right)$$

Hence, the correct answer is option (A).

80. (A)

A = Area, $2s = a + b + c$

AM $\geq$ GM

$$\frac{s+(s-a)+(s-b)+(s-c)}{4} \geq \left[s(s-a)(s-b)(s-c)\right]^{1/4}$$

$$\Rightarrow \quad A \leq \frac{s^2}{4}$$

Hence, A is the correct answer.

81. (4)

$$\left|z-\frac{6}{z}\right| \geq |z|-\frac{6}{|z|} \Rightarrow 5 \geq |z|-\frac{6}{|z|}$$

$$\Rightarrow |z|^2-5|z|-6 \leq 0$$

$$\Rightarrow |z| \in [-1, 6]$$

$\therefore$ Maximum value = 6

82. (1)

$$(\vec{a}-\vec{d})\times\vec{b}=0 \Rightarrow \vec{a}=\vec{\mathbf{d}}+\lambda\vec{b}$$

Dot with $\vec{c}$

$$2+3+1=8+\lambda(1-\mathbf{1}+1)$$

$$\Rightarrow \lambda=-2$$

$$\therefore \vec{d}=\vec{a}+2\vec{b}$$

$$=4\hat{i}+\hat{j}+3\hat{k}$$

$$\therefore \vec{b}.\vec{d}=4-1+3=6$$

83. (7)

$b^2-4a \geq 0$

$b=1 \Rightarrow a \in \phi$

$b=2 \Rightarrow a \in \{1\}$ total = 7

$b=3 \Rightarrow a \in \{1, 2\}$

$b=4 \Rightarrow a \in \{1, 2, 3, 4\}$

84. (3)

$$\sin^{-1}x=2\tan^{-1}x \Rightarrow \sin^{-1}x=\sin^{-1}\frac{2x}{1+x^2}$$

$$\Rightarrow \frac{2x}{1+x^2}=x \Rightarrow x^3-x=0 \Rightarrow x(x+1)(x-1)=0 \Rightarrow x=\{-1, 1, 0\}$$

Hence, 3 solutions.

85. (1)

$$\frac{|adj\ B|}{|C|}=\frac{|adj(adj\ A)|}{|5A|}=\frac{|A|^{(3-1)^2}}{5^3|A|}=\frac{|A|^3}{125}$$

Now, $|A|=5$,

Therefore, $\frac{|adj\ B|}{|C|}=1$

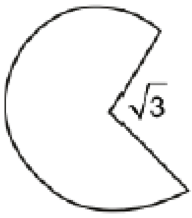
86. (3)

$$\lim_{x\to 0}\left([f(x)]+x^2\right)^{\frac{1}{\{f(x)\}}}=\lim_{x\to 0}\left(1+x^2\right)^{\frac{1}{\frac{\tan x}{x}-1}}=\lim_{x\to 0}\left(1+x^2\right)^{\frac{x}{\tan x-x}}$$

$$=e^{\lim\limits_{x\to 0}\frac{x^3}{\tan x-x}}=e^{\lim\limits_{x\to 0}\frac{3x^2}{\sec^2 x-1}}=e^{\lim\limits_{x\to 0}\frac{6x}{2\sec^2 x\tan x}}=e^3$$

Therefore, $\lambda=3$.

87. (2)



$$3=h^2+r^2$$

$$\Rightarrow\ r^2=3-h^2$$

Now, $V=\frac{1}{3}\pi r^2 h=\frac{1}{3}\pi\left(3-h^2\right)h$

Therefore,

$$\frac{dV}{dh}=\frac{1}{3}\pi\left(3-3h^2\right)$$

$$\frac{dV}{dh}=0 \text{ at } h=1$$

$$\frac{d^2V}{dh^2}<0 \text{ at } h=1$$

$$\Rightarrow\ V_{\max}=\frac{2\pi}{3}$$

Therefore, $\lambda=2$.

88. (2)

The given integral is

$$I=\int_{-\pi/2}^{\pi/2}\frac{x^2\cos x}{1+e^x}dx$$

Using the integral property, we get

$$I=\int_0^{\pi/2}\left(\frac{x^2\cos x}{1+e^x}+\frac{x^2\cos x}{1+e^{-x}}\right)dx$$

$$I=\int_0^{\pi/2}x^2\cos x\,dx$$

That is,

$$\int x^2 \cos x\,dx = x^2 \sin x - 2\int x \sin x\,dx$$
$$= x^2 \sin x - 2\left\{-x\cos x + \int \cos x\,dx\right\}$$
$$= x^2 \sin x + 2x\cos x - 2\sin x$$

Therefore,

$$I = x^2 \sin x + 2x\cos x - 2\sin x\Big|_0^{\pi/2} = \left(\frac{\pi^2}{0} + 0 - 2\right) - (0) = \frac{\pi^2}{4} - 2.$$

Hence, $a = 2$.

89. (6)

f is increasing. So, its greatest value is $f(3) = 27$.

Let the GP be $a, ar, ar^2, ...$ with $-1 < r < 1$.

$\frac{a}{1-r} = 27$ and $a - ar = 3 \Rightarrow r = \frac{4}{3}$ or $r = \frac{2}{3}$

But, $-1 < r < 1$

So, $r = \frac{2}{3}$

$\Rightarrow 9r = 6$

90. (7)

Probability of hitting the target in one fire $p = \frac{1}{5}$.

Then, the probability of hitting the target at least one in *n* fires is

$$1 - \text{(Probability of not hitting the target)} = 1 - \left(\frac{4}{5}\right)^n > \frac{3}{4}$$

$\left(\frac{4}{5}\right)^n < \frac{1}{4}$ as $\left(\frac{4}{5}\right)^6 > \frac{1}{4}$ and $\left(\frac{4}{5}\right)^7 < \frac{1}{4}$

The least value of $n = 7$.