PART (A) : PHYSICS

ANSWER KEY

1.	(A)	2.	(A)	3.	(B)	4.	(C)	5.	(C)
6.	(D)	7.	(D)	8.	(A)	9.	(B)	10.	(C)
11.	(A)	12.	(D)	13.	(A)	14.	(B)	15.	(A)
16.	(C)	17.	(D)	18.	(D)	19.	(C)	20.	(D)
21.	(3)	22.	(1)	23.	(1)	24.	(2)	25.	(2)
26.	(6)	27.	(2)	28.	(5)	29.	(2)	30.	(8)

SOLUTIONS

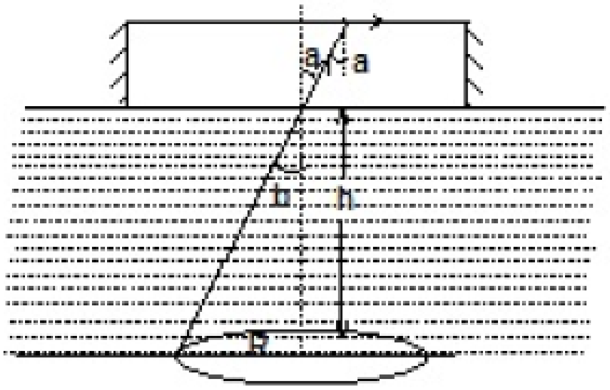
1. (A)

$\sin\alpha = \frac{1}{\mu g}$

$\mu_w \sin\beta = \mu_g \sin\alpha$

or $\mu_w \sin\beta = \mu_g \times \frac{1}{\mu_g} = 1$

$\beta = \sin^{-1}\left(\frac{1}{\mu_w}\right)$



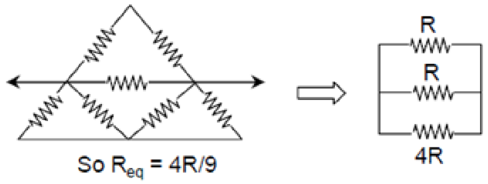
$\therefore\ A_{\text{base}} = \pi R^2 = \pi h^2 \tan^2\beta = \frac{\pi h^2}{\left(\mu_w^2 - 1\right)} \left[\because \tan\beta = \frac{1}{\sqrt{\mu_w^2 - 1}}\right]$

$= 9\pi\, m^2$

2. (A)

Fact about P–N junction.

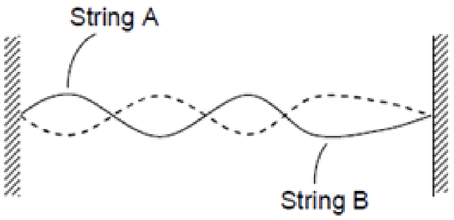
3. (B)



So $R_{eq} = 4R/9$

For $P\max, r = Reg. \therefore r = 4R/9$

4. (C)



$f_1 = f_2$

$C = fD$

$\therefore C_1 < C_2$

But $C = \sqrt{\frac{T}{\mu}}$

$\therefore \mu_1 > \mu_2$

5. (C)

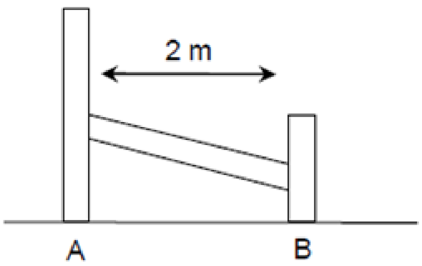
At the surface of a current carrying wire

$B = \frac{1}{2}\mu_0 Jr$

$J \propto \frac{B}{r}$

$\therefore$ as slope of wire $P >$ slope of wire Q

6. (D)



$\Sigma F_x = 0_1 \Sigma\ F_y = 0 \therefore \mu N = Mg$

$\Sigma\tau$ about $O = 0$

$\therefore \left[N\frac{\ell}{2}\sin\theta\right](2) = \mu N\frac{\ell}{2}\cos\theta$

$\therefore \tan\theta = \mu/2$

$\sec\theta = \left(\sqrt{\mu^2+4}\right)/2$

Thus $\left(\frac{\ell}{2}/1\right) = \sqrt{\frac{\mu^2+4}{2}} \therefore \ell = \sqrt{\mu^2+4}$

$\ell = \sqrt{0.25+4} \therefore \ell = \frac{\sqrt{17}}{2}$ m

7. (D)

$\frac{\mu_2}{v} = \frac{\mu_1}{v} = \frac{\mu_2-\mu_1}{R}$

$\frac{1}{v} - \frac{1.5}{-u} = \frac{1-1.5}{-R}$

For v to be +ve, $\frac{1}{2R} - \frac{3}{2u} > 0$ or $u > 3R$

8. (A)

N_B reaches maximum when activity of A and B will be same

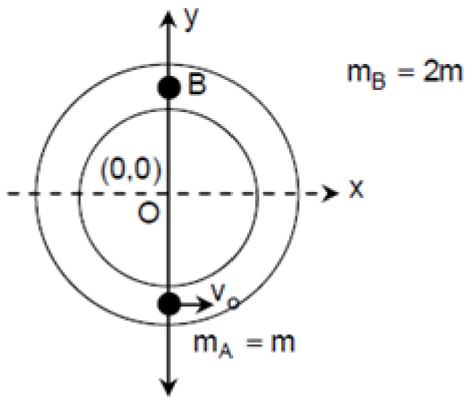
i.e $D_A N_A = D_B N_B \therefore N_B = \frac{D_A N_A}{D_B}$

9. (B)

$V_B = 0 \therefore \frac{KQ}{b} + \frac{KQ'}{b} = 0$ or $Q' = -Q$

Where Q is total charge inside A and Q^1 is charge on shell B therefore field outside B will be zero and electric field inside A will be $E = \frac{KQ}{a^3}r$ (linearly increase) between spherical shell A and B $E = KQ/r^2$.

10. (C)



$-mv_0 + 0 = mv_1 + 2mv_2$

$-v_0 = v_1 + 2v_2$ .... (i) & $e = \frac{v_2 - v_1}{-v_0} = 1$

$\therefore v_2 - v_1 = -v_0$ ……….. (ii)

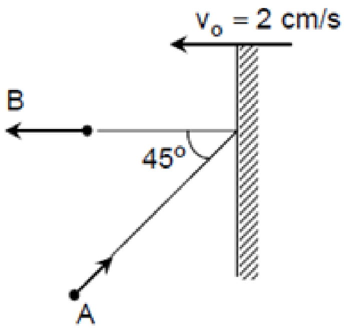
from (i) & (ii)

$v_1 = \frac{v_0}{3}, v_2 = \frac{-2v_0}{3}$

$\therefore T = \frac{2\pi R}{v_0}$

Angle rotated by $A = \frac{2\pi}{3}$, Angle rotated by $B = \frac{4\pi}{3}$ rad.

11. (A)



Velocity of image of particle B,

$V_B = 5i + 4(-i) = i$

Velocity of image of particle A,

$V_A = 10(-i) + 4(-i) - 10j$

$\therefore V_A = -14i - 10j$

Relative velocity of image

$V_{BA} = V_B - V_A$

$= i - [14(-i) - 10j]$

$= 15i + 10j$

$\therefore V_{BA} = \sqrt{325}\ \text{cm/sec}$

12. (D)

Since $P_1V_1 > P_2V_2 \Rightarrow T_A > T_B$ work done is +ve and efficiency is positive quantity

$\eta = \frac{\text{work done}}{\text{Heat given}} = 1 - \frac{Q_{released}}{Q_{absorbed}}$

Therefore there must be some heat absorbed in the process. During adiabatic process $\Delta Q = 0$, therefore during linear process heat will enter and leaves system at different times.

13. (A)

$AV = \text{const.}$

$P + \frac{1}{2}\rho v^2 = \text{constt}$

If velocity increases then pressure decreases.

14. (B)

$$\left(\frac{dy}{dx}\right)_{tangent} = 2\cos\frac{\pi x}{L}$$

And $\tan\left(90+45^o\right) = \left(\frac{dy}{dx}\right)_{normal}$

$$\Rightarrow x = \frac{L}{3}, y = \frac{\sqrt{3}L}{\pi}$$

15. (A)

Fact

16. (C)

$mvr = \frac{h}{2\pi}$ in first orbit

$$v = \frac{h}{2\pi mr}$$

$$\therefore a = \frac{v^2}{r} = \frac{h^2}{4\pi^2 m^2 r^3}$$

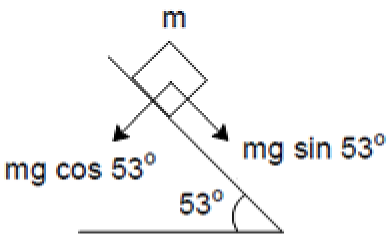
17. (D)

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f} \therefore u = \frac{fv}{f-v}$$

$$\therefore \frac{v}{u} = \frac{f-v}{f} = 1 - \frac{v}{f}$$

$|m| = \left|1 - \frac{v}{f}\right| \therefore$ slope $\frac{1}{f} = b/c$

18. (D)



$a = g\sin 53^o = \frac{4g}{5}$ time taken by block to reach the bottom

$$\therefore t = \sqrt{\frac{5}{a}} = \sqrt{\frac{25}{2\times10}} = \sqrt{\frac{5}{4}}$$

$\therefore$ Distance travelled by the inclined $= vt = 4\sqrt{\frac{5}{4}} = 2\sqrt{5}m$

19. (C)

Displacement of the winning team is zero. Hence work done by loosing team on the winning team is zero.

Ground applies a frictional force on the losing team in a direction opposite to its displacement hence work done by ground on losing team is zero.

20. (D)

Zero in fact A will be inclined away from the vertical at some angle.

21. (3)

Given that $V = V_0 \log_e (r/r_0)$

Field $E = \frac{-dV}{dr}$ or $E = -V_0\left(\frac{r_0}{r}\right)$

Now $eE = \frac{mv^2}{r}$ or $\frac{eV_0 r_0}{r} = \frac{mv_0^2}{r}$

$\therefore v = \left(\frac{ev_0 r_0}{m}\right)^{1/2}$

Now $mv = (mev_0 r_0)^{1/2} = \text{constant}$

$\therefore mvr = \frac{nh}{2\pi}$ Bohr's quantum condition

Or $r \propto n$

22. (1)

$T = 2\pi\sqrt{1/MB_H} \therefore T_1 = 2\pi\sqrt{\left[\frac{2I}{(P_m + 2P_m)B_H}\right]}$

$T_2 = 2\pi\sqrt{\left[\frac{2I}{(2P_m + P_m)B_H}\right]}$

$\therefore \frac{T_1}{T_2} = \frac{1}{\sqrt{3}}$

23. (1)

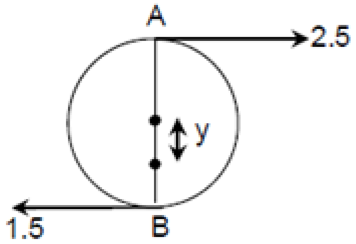
Let lengths of ABC and ADC is I_1 and I_2 respectively. Then

$\frac{I_1}{I_2} = \frac{360 - 60}{60} = 5$ Now $\frac{i_1}{i_2} = \frac{I_1}{I_2} = \therefore i_2 = 5i_1$

Now $B_1 = \frac{5}{6}\left(\frac{\mu_0 i_1}{2r}\right)$ and $B_2 = \frac{1}{6}\left(\frac{\mu_0 i_2}{2r}\right)$

$\therefore B_1 = B_2$

24. (2)



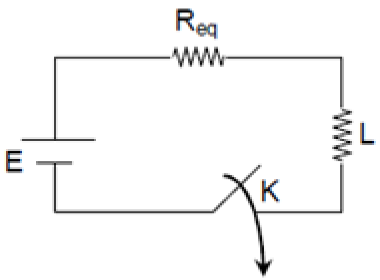
$$\frac{2.5}{2+y}=\frac{1.5}{2-y}$$

$$\therefore y = 0.5\,m$$

25. (2)

Equivalent circuit

$$Req = R + \frac{2R+2R}{4R} = 2R$$



$$\therefore \text{ time constant } t = \frac{L}{Req} = \frac{L}{2R}$$

26. (6)

By $\rho = m/v$ so percentage error can find.

1.2%

27. (2)

$$SL_2 - SL_1 = 10\log\frac{I_{max}}{I_{min}} = 10\log\left[\frac{\left(\sqrt{I_1}+\sqrt{I_2}\right)^2}{\left(\sqrt{I_1}-\sqrt{I_2}\right)^2}\right]$$

$$= 10\log\frac{\left(\frac{a_1}{a_2}+1\right)^2}{\left(\frac{a_1}{a_2}-1\right)^2} = 20\,dB$$

28. (5)

$$a = \frac{iB1}{m}$$

And $v=\left(\frac{iB1}{m}\right)t$

$\therefore v=20\,cm/s$

29. (2)

Speed $v=\sqrt{v_x^2+v_y^2}\ \therefore 2v\frac{dv}{dt}=2v_xa_x+2v_ya_y$

$\frac{dv}{dt}=\frac{v_xa_x+v_ya_y}{v}=\frac{3\times2+4\times1}{\sqrt{3^2+4^2}}=2\,m/s^2$

30. (8)

$\int E.dl=-A.\frac{dB}{dt}$

$E\left(2\pi r\right)=-17\left[\frac{d}{dt}\left\{17+0.2\sin\left(\omega t+\phi\right)\right\}\right]$

$\therefore E=\frac{-r}{2}\left(0.2\right)\omega\cos\left(\omega t+\phi\right)$

$\therefore$ magnitude of amplitude $=\frac{r}{2}\left(0.2\right)=240\,m\,N/C$

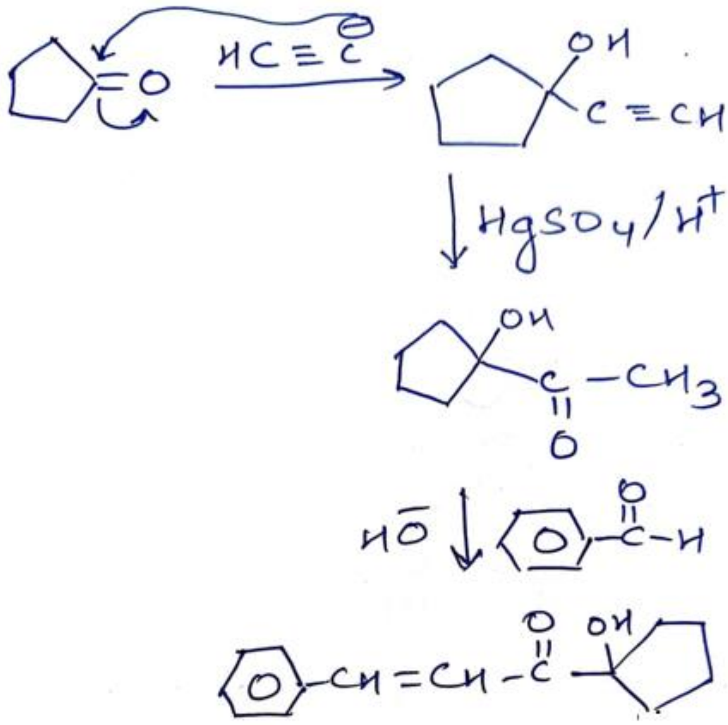
PART (B) : CHEMISTRY

ANSWER KEY

31.	(D)	32.	(C)	33.	(D)	34.	(B)	35.	(A)
36.	(C)	37.	(C)	38.	(A)	39.	(D)	40.	(C)
41.	(B)	42.	(C)	43.	(C)	44.	(D)	45.	(D)
46.	(B)	47.	(D)	48.	(A)	49.	(C)	50.	(C)
51.	(5)	52.	(6)	53.	(3)	54.	(8)	55.	(8)
56.	(6)	57.	(3)	58.	(0)	59.	(5)	60.	(1)

SOLUTIONS

31. (D)



32. (C)

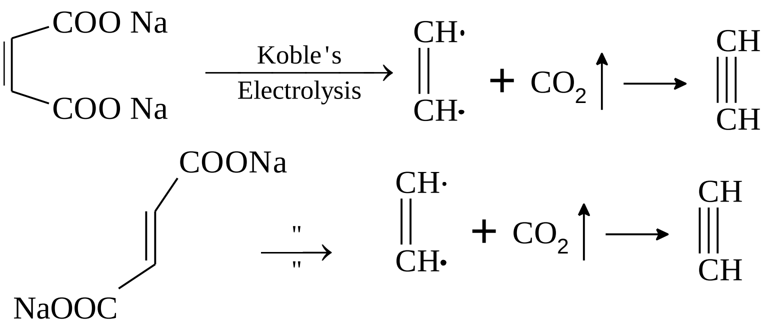
$d_A = 2d_B; 3M_A = M_B; PM = dRT$

$\frac{P_A}{P_B} \times \frac{M_A}{M_B} = \frac{d_A}{d_B} \times \frac{RT}{RT}$

$\frac{P_A}{P_B} \times \frac{1}{2} = 2$

$\frac{P_A}{P_B} = \frac{4}{1}$

33. (D)



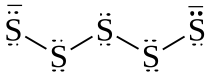
Socl Maleate

34. (B)

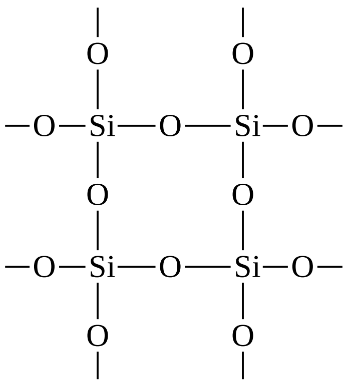
$HIO_4 < HBrO_4 < HClO_4 \Rightarrow$ acidic strength has been decided on the basic of electronegativity or charge density on central atom.

35. (A)

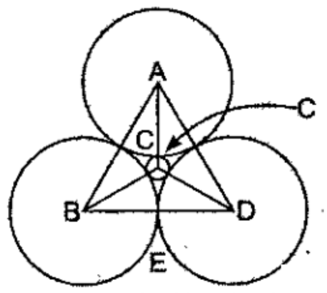
S_8: Molecular solid, in solid sulphur various S_8 molecules are bonded to one another by weak forces

S_5^{2-} :  $\Rightarrow$ Zig – Zag

SiO_2: represents to 3D silicate, that has covalent lattice as one silicon atom is bonded to four oxygen atoms, and each oxygen in turn is bonded to two silicon



36. (C)



$$\frac{BE}{BC} = \cos 30^0 = \frac{\sqrt{3}}{2}$$

Distance between the centres of two B atoms

$= 2 \times BE = \sqrt{3}\,\overset{o}{A}$

37. (C)

(a) $NaBr + H_2SO_4 \rightarrow Na_2SO_4 + HBr\uparrow \xrightarrow[\text{(Remaining part)}]{\text{Conc.}H_2SO_4} \underset{\text{reddish brown}}{Br_2\uparrow}$

(b) $NaNO_3 + H_2SO_4 \rightarrow \underset{\text{Reddish brown}}{NO_2\uparrow}$

(c) $CaF_2 + H_2SO_4 \rightarrow CaSO_4 + \underset{\text{Colourless}}{HF\uparrow}$

(d) $KI + H_2SO_4 \rightarrow K_2SO_4 + 2HI \xrightarrow[\text{(Remaining part)}]{\text{Conc.}H_2SO_4} \underset{\text{(Violet)}}{I_2\uparrow}$

38. (A)

Initial wt. of $H_2O(l)$ = 18g

$$n_{H_2O}(g) = \frac{\left(\frac{24.63}{760}\right)\times 7.6}{0.0821\times 300} \Rightarrow 0.01$$

% wt. of H_2O vaporised = $\frac{0.01\times 18}{18}\times 100$

$\Rightarrow 1\%$

39. (D)

Grignard Reagent on reaction with formaldehyde gives primary alcohol

40. (C)

$$2AgCl + Na_2CO_3 \rightarrow 2NaCl + Ag_2CO_3 \xrightarrow{\Delta} 2Ag + \frac{1}{2}O_2 + CO_2\uparrow$$

41. (B)

$[Cr(en)_2Br_2]Br$ – dibromidobis (ethylenediamine) chromium (III) bromide

42. (C)

As the size of central atom increases lone pair bond pair repulsions increases so, bond angle decreases.

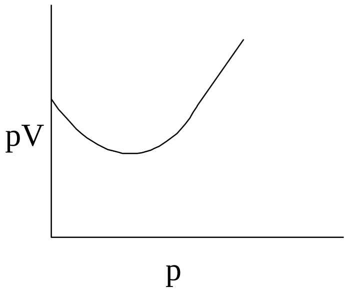
43. (C)

Lewis acids can initiate the cationic polymerization

44. (D)

Teflon is artificially made as $(\ldots.CF_2–CF_2\ldots)_n$

45. (D)



46. (B)

The 21st electron corresponds to $3d^1$. For this orbital, $n = 3, l = 2, m = 2, s = +\frac{1}{2}$

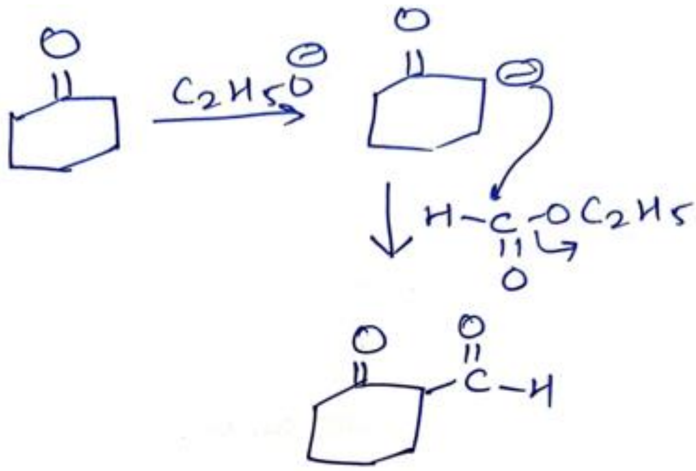
47. (D)

BF_3 has incomplete octet.

48. (A)

More stable conjugate base implies more acidic compound

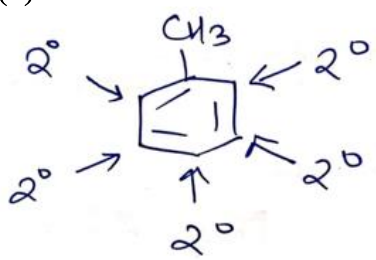
49. (C)



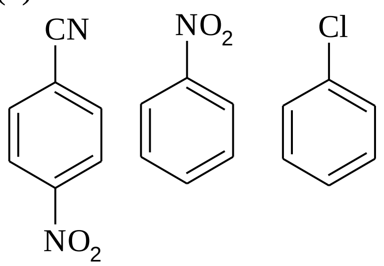
50. (C)

More stable conjugate base implies more acidic compound

51. (5)



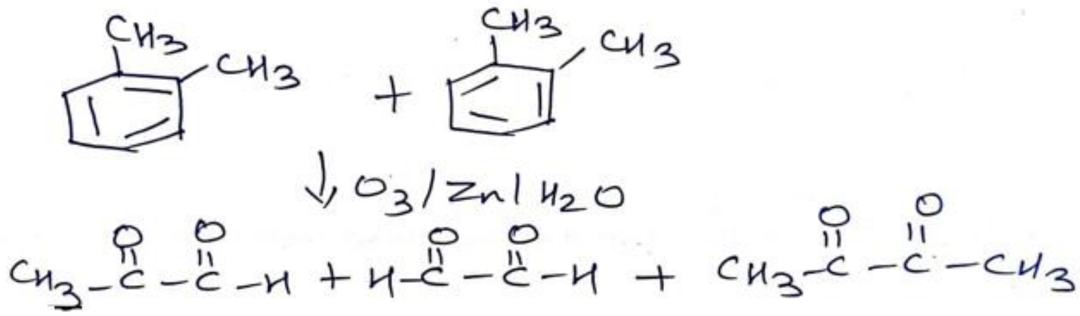
52. (6)
$H_2S_2O_8$ has one peroxide bond.

53. (3)


54. (8)
Body centred.

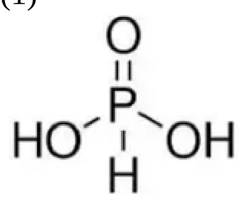
55. (8)
$\overset{-4}{C}H_4(g) + 2O_2(g) \rightarrow \overset{+4}{C}O_2(g) + 2H_2O(l)$
The oxidation number of C changes by 4 – (-4) = +8

56. (6)
C_3H_4 is formed on hydrolysis of Mg_2C_3.
$H_3C-C\equiv C-H$
$Mg_2C_3 + 4H_2O \rightarrow 2Mg(OH)_2 + CH_3C\equiv CH$

57. (3)


58. (0)

59. (5)
$HClO \xrightarrow{disp} H\overset{-1}{Cl} + H\overset{+5}{Cl}O_3$

60. (1)


PART (C) : MATHEMATICS

ANSWER KEY

61.	(A)	62.	(C)	63.	(D)	64.	(A)	65.	(B)
66.	(C)	67.	(D)	68.	(A)	69.	(A)	70.	(B)
71.	(C)	72.	(A)	73.	(A)	74.	(D)	75.	(D)
76.	(B)	77.	(A)	78.	(A)	79.	(A)	80.	(D)
81.	(5)	82.	(27)	83.	(1)	84.	(5)	85.	(36)
86.	(2)	87.	(484)	88.	(0)	89.	(8)	90.	(6)

SOLUTIONS

61. (A)

In case of function given in option (A), f(x) is continuous on [0, 1]

but not differentiable at $x = \frac{1}{2} \in (0,1)$ i.e. $-1 \neq 0$

i.e. $Lf'\left(\frac{1}{2}\right) \neq Rf'\left(\frac{1}{2}\right)$

Thus, the lagrange's mean value theorem (LMVT) is not applicable.
The function given in option (B) is continuous on [0,1] and differentiable on (0, 1) and hence the lagrange's mean value theorem (LMVT) is applicable. The function given in option (C), if $f(x) = x|x| = x \cdot x = x^2$ and in option (D), if $f(x) = |x| = x$ on [0, 1].
As both are polynomial function.
So, lagrange's mean value theorem (LMVT) is applicable.

62. (C)

Given, differential equation

$$(1+t)\frac{dy}{dt} - t \cdot y = 1$$

$$\Rightarrow \frac{dy}{dt} - \frac{t}{1+t}y = \frac{1}{1+t} \qquad \ldots(i)$$

This is linear differential equation and its integrating factor (IF)

$$= \exp\left(-\int \frac{t}{1+t}dt\right)$$

$$= e^{-t+\log_e(1+t)}$$

$$= e^{-t} \cdot (1+t)$$

$$e^{-t}(1+t)y = \int e^{-t}dt = -e^{-t} + C$$

$\Rightarrow \quad y = \frac{-1}{1+t} + \frac{Ce^t}{1+t}$ ...(ii)

Given, when t = 0, then y = –1

From Eq. (ii)

$-1 = -1 + C$

$\Rightarrow \quad C = 0$

Thus, $y = \frac{-1}{1+t}$

$\therefore \quad y(1) = \frac{-1}{2}$

63. (D)

Given, $\omega = \alpha + i\beta$, where $\beta \neq 0$ and $z \neq 1$

and $\frac{\omega - \bar{\omega}z}{1-z}$ is purely real = k (say)

$\Rightarrow \quad \omega - \bar{\omega}z = k - kz$

$\Rightarrow \quad (k - \bar{\omega})z = k - \omega$

$\Rightarrow \quad z = \frac{k-\omega}{k-\bar{\omega}}$

$\Rightarrow \quad z = \frac{k-(\alpha+i\beta)}{k-(\alpha-i\beta)} = \frac{((k-\alpha)-i\beta)}{((k-\alpha)+i\beta)}$

$\therefore \quad \beta \neq 0$

$\therefore \quad |z| = 1$ and $z \neq 1$

64. (A)

The LHS of the equation $= \begin{bmatrix} 2 & 4x+9 & 2x+5 \end{bmatrix} \begin{bmatrix} x \\ 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 2x+4x+9-2x-5 \end{bmatrix} = 4x+4$

Thus, $4x+4=0 \Rightarrow x=-1$

65. (B)

Given, $f(x) = \int_0^x \sqrt{1-f^2(t)}dt + \frac{1}{2}$

$\Rightarrow \quad f'(x) = \sqrt{1-f^2(x)}$

$\Rightarrow \quad \frac{f'(x)}{\sqrt{1-f^2(x)}} = 1 \quad \Rightarrow \quad$ sin–1 {f(x)} = x + C

For x = 0, $\sin^{-1}\left(\frac{1}{2}\right)$ = C and thus C = $\frac{\pi}{6}$

$\Rightarrow \quad f(x) = \sin\left(x + \frac{\pi}{6}\right)$

$\therefore \quad f(\pi) = \sin\left(\pi + \frac{\pi}{6}\right) = -\sin\frac{\pi}{6}$

$\Rightarrow \quad f(\pi) = \sin\left(-\frac{\pi}{6}\right)$

$\Rightarrow \quad \sin^{-1} f(\pi) = -\frac{\pi}{6}$

66. (C)

Equation of (P_1) is $y^2 - 8x = 0$ and
equation of (P_2) is $y^2 + 16x = 0$
tangent to $y^2 - 8x = 0$, passes through $(-4, 0)$

$\Rightarrow \quad m_1(-4) + \frac{2}{m_1} = 0 \Rightarrow \frac{1}{m_1^2} = 2$

Now, tangent to $y^2 + 16x = 0$
Passes through $(2, 0)$

$\Rightarrow \quad m_2 \times 2 - \frac{4}{m_2} = 0$

$\Rightarrow \quad m_2^2 = 2$

So, $\frac{1}{m_1^2} + m_2^2 = 4$

67. (D)

$$\text{Let } \Delta = \begin{vmatrix} b^2+c^2 & ab & ac \\ ab & c^2+a^2 & bc \\ ac & bc & a^2+b^2 \end{vmatrix}$$

$$\Rightarrow \quad \Delta = \frac{1}{abc}$$

$$\begin{vmatrix} a(b^2+c^2) & a^2b & a^2c \\ ab^2 & b(c^2+a^2) & b^2c \\ ac^2 & bc^2 & c(a^2+b^2) \end{vmatrix}$$

By taking a, b and c common from c_1, c_2 and c_3 respectively, we get;

$$\Delta = \frac{abc}{abc}\begin{vmatrix} b^2+c^2 & a^2 & a^2 \\ b^2 & c^2+a^2 & b^2 \\ c^2 & c^2 & a^2+b^2 \end{vmatrix}$$

Applying $R_1 \to R_1 + R_2 + R_3$, we get

$$\Delta = \begin{vmatrix} 2(b^2+c^2) & 2(c^2+a^2) & 2(a^2+b^2) \\ b^2 & c^2+a^2 & b^2 \\ c^2 & c^2 & a^2+b^2 \end{vmatrix}$$

$$= 2\begin{vmatrix} b^2+c^2 & c^2+a^2 & a^2+b^2 \\ b^2 & c^2+a^2 & b^2 \\ c^2 & c^2 & a^2+b^2 \end{vmatrix}$$

Applying $R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$, we get

$$\Delta = 2\begin{vmatrix} b^2+c^2 & c^2+a^2 & a^2+b^2 \\ -c^2 & 0 & -a^2 \\ -b^2 & -a^2 & 0 \end{vmatrix}$$

Applying $R_1 \to R_1 + R_2 + R_3$, we get

$$\Delta = 2\begin{vmatrix} 0 & c^2 & b^2 \\ -c^2 & 0 & -a^2 \\ -b^2 & -a^2 & 0 \end{vmatrix}$$

$= 2[-c^2(-a^2b^2)+b^2(c^2a^2)]$

$\Rightarrow \quad \Delta = 4a^2b^2c^2$

Comparing with the given condition, we get

$k = 4$

68. (A)

Given, $f(x) = \dfrac{x}{1+x} = 1 - \dfrac{1}{1+x}$

As, $x \geq 0$

$\Rightarrow \quad 1+x \geq 1 \Rightarrow 0 < \dfrac{1}{1+x} \leq 1$

Thus, $f(x) < 1 \forall x \in [0, \infty)$

In other words, f(x) cannot be onto.

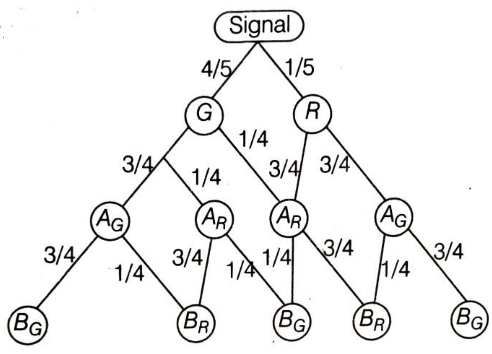
Also, $f'(x) = \dfrac{1}{(1+x)^2} > 0 \forall x \in [0, \infty)$

$\therefore$ f(x) is a strictly increasing function.

Thus, f(x) is one-one.

Hence, f(x) is one-one but not onto.

69. (A)



From the tree diagram.

$$P(B_G)=\frac{4}{5}\times\frac{3}{4}\times\frac{3}{4}+\frac{4}{5}\times\frac{1}{4}\times\frac{1}{4}+\frac{1}{5}\times\frac{3}{4}\times\frac{1}{4}+\frac{1}{5}\times\frac{1}{4}\times\frac{3}{4}=\frac{23}{40}$$

$$P(B_G/G)=\frac{3}{4}\times\frac{3}{4}+\frac{1}{4}\times\frac{1}{4}=\frac{10}{16}=\frac{5}{8}$$

$\therefore$ $P(B_G\cap G)=P(G)\cdot P(B_G/G)=\frac{4}{5}\times\frac{5}{9}=\frac{1}{2}$

$\therefore$ Required probability = $P(G/B_G)$

$$=\frac{P(B_G\cap G)}{P(B_G)}=\frac{\frac{1}{2}}{\frac{23}{40}}=\frac{20}{23}$$

70. (B)

$$\cos\frac{2\pi}{7}+\cos\frac{4\pi}{7}+\cos\frac{6\pi}{7}$$

Put $\theta=\frac{2\pi}{7}$

$$\cos\theta+\cos 2\theta+\cos 3\theta=\frac{\sin(3\theta/2)}{\sin(\theta/2)}\times\cos\left(\frac{\theta+3\theta}{2}\right)=\frac{\sin\frac{3\pi}{7}.\cos\frac{4\pi}{7}}{\sin\frac{\pi}{7}}$$

$$=\frac{-2\sin\left(\frac{3\pi}{7}\right)\cos\left(\frac{3\pi}{7}\right)}{2\sin\left(\frac{\pi}{7}\right)}=\frac{-\sin\frac{6\pi}{7}}{2\sin\left(\frac{\pi}{7}\right)}=\frac{-\sin\left(\frac{\pi}{7}\right)}{2\sin\left(\frac{\pi}{7}\right)}=-\frac{1}{2}$$

71. (C)

$$\begin{vmatrix} x & 1 & 1\\ 1 & y & 1\\ 1 & 1 & z\end{vmatrix}=x(yz-1)-1(z-1)+1(1-y)$$

$= xyz - (x + y + z) + 2$

$\therefore$ AM $\geq$ GM

$\Rightarrow$ $\frac{x+y+z}{3}\geq(xyz)^{1/3}$

$\Rightarrow$ $x+y+z\geq 3(xyz)^{1/3}$ …(i)

The value of determinant is non-negative.

$\Rightarrow$ $xyz - (x + y + z) + 2 \geq 0$

$\Rightarrow$ $xyz+2\geq x+y+z\geq 3(xyz)^{1/3}$ [from Eq. (i)]

$\Rightarrow$ $xyz-3(xyz)^{1/3}+2\geq 0$ …(ii)

Let $(xyz)^{1/3} = u \Rightarrow xyz = u^3$

From Eq. (ii),

$u^3 - 3u + 2 \geq 0$

$\Rightarrow$ $(u+2)(u-1)^2 \geq 0$

$\Rightarrow$ $u + 2 \geq 0$ $\quad [\because (u-1)^2 > 0\ \forall\ u]$

$\Rightarrow$ $u \geq -2$

$\Rightarrow$ $(xyz)^{1/3} \geq -2$

$\Rightarrow$ $xyz \geq -8$

Hence, least value of xyz is – 8.

72. (A)

Given, $(1+x^2)^4(1+x^3)^7(1+x^4)^{12}$

Power of (x) from each factor altogether is 11

$\Rightarrow$ $2x_1 + 3x_2 + 4x_3 = 11$

Total possibilities for

(x_1, x_2, x_3) are (0, 1, 2), (1, 3, 0), (2, 1, 1), (4,1, 0)

$\therefore$ Required coefficients

$$=({}^4C_0\times{}^7C_1\times{}^{12}C_2)+({}^4C_1\times{}^7C_3\times{}^{12}C_0)$$
$$+({}^4C_2\times{}^7C_1\times{}^{12}C_1)+({}^4C_4\times{}^7C_1\times{}^{12}C_0)$$

= 462 + 140 + 504 + 7 = 1113

73. (A)

Let $L = \lim_{n\to\infty}\left[\sqrt{n^2+n+1} - \sqrt{n^2+n+1}\ \right]$

$\therefore$ $n < \sqrt{n^2+n+1} < n+1$

$\therefore$ $\sqrt{n^2+n+1} = n$

So, $L = \lim_{n\to\infty}(\sqrt{n^2+n+1}-n)$

$$= \lim_{n\to\infty}\frac{(\sqrt{n^2+n+1}-n)(\sqrt{n^2+n+1}+n)}{(\sqrt{n^2+n+1}+n)}$$

$$= \lim_{n\to\infty}\frac{n^2+n+1-n^2}{n\sqrt{1+\frac{1}{n}+\frac{1}{n^2}}+n}$$

$$= \lim_{n\to\infty}\frac{n\left(\frac{1+\frac{1}{n}}{}\right)}{n\left(\sqrt{1+\frac{1}{n}+\frac{1}{n^2}}+1\right)} = \frac{1+0}{1+1} = \frac{1}{2}$$

74. (D)

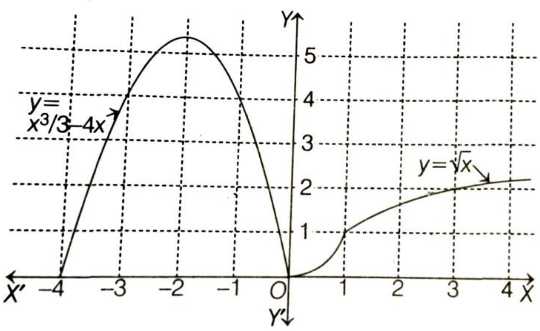
p	q	r	$p \wedge q$	$(p \wedge q) \to r$
T	T	T	T	T
T	T	F	T	F
T	F	T	F	T
T	F	F	F	T
F	T	T	F	T
F	T	F	F	T
F	F	T	F	T
F	F	F	F	T

Truth value of $(p \wedge q) \to r$ is F, if p and q both are T and r is F.

75. (D)

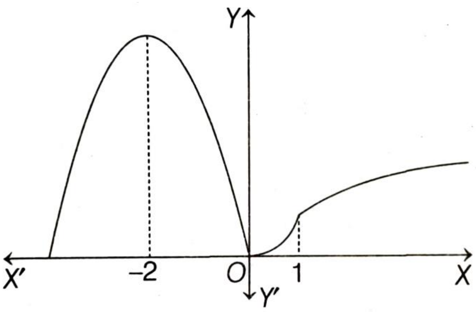
Given, $f(x) = \begin{cases} \sqrt{x} & , \quad x \geq 1 \\ x^3 & , \quad 0 \leq x \leq 1 \\ \frac{x^3}{3} - 4x & , \quad x < 0 \end{cases}$

Now, draw the graph of f(x) and then discuss.

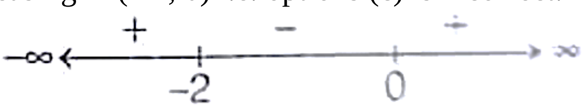


Graph of f(x)

Hence, final graph (shape) is just like below.



From graph; function is increasing in $(-\infty, -2) \cup (0, \infty)$ and decreasing in (– 2, 0) i.e. options (c) is incorrect.



That is, f'(x) changes its sign twice as x varies
from $-\infty$ to ∞, (option (d) is correct

$\therefore$ x = – 2 is local maxima point and x = 0 is local minima point.
i.e. the function attains extrema at $x_1 = -2$ and $x_2 = 0$.
So, $x_1 \cdot x_2 = 0$

$\Rightarrow$ option (b) is incorrect

Since, f(x) has corner points at x = 0 and x = 1, then f(x) is differentiable
for all $x \in R - \{0,1\}$ and it is continuous for all $x \in R$. i.e. option (a) is incorrect

76. (B)

As $\omega^n = 1$, Let n = 3m

Then,

$$(\omega + x)^{3m} = \omega^{3m} + 3m\omega^{3m-1}x + \frac{1}{2}(3m)(3m-1)\omega^{3m-2}x^2 + ...$$

$$= 1 + 3m\bar{\omega}x + \frac{1}{2}(3m)(3m-1)\omega x^2 + ...$$

On comparing, we get

3mx = 12 and $\frac{1}{2}$ (3m)(3m – 1)x^2 = 69

$$\Rightarrow \frac{3m-1}{3m} = \frac{69 \times 2}{12 \times 12} = \frac{23}{24}$$

m = 8

Thus, n = 24

And $x = \frac{1}{2}$

77. (A)

$$f(x) = \frac{x}{(1+x^n)^{1/n}}$$

$$f(f(x)) = f\left(\frac{x}{(1+x^n)^{1/n}}\right)$$

$$= \frac{\frac{x}{(1+x^n)^{1/n}}}{\left(1 + \frac{x^n}{1+x^n}\right)^{1/n}}$$

$$= \frac{x}{(1+2x^n)^{1/n}}$$

$$f(f(f(x))) = f\left(\frac{x}{(1+2x^n)^{1/n}}\right)$$

$$=\frac{\frac{x}{(1+2x^n)^{1/n}}}{\left(1+\frac{x^n}{1+2x^n}\right)^{1/n}}$$

$$=\frac{x}{(1+3x^n)^{1/n}}$$

And so on, leads to

$$\underbrace{fofofo...fof}_{n-times}(x)=\frac{x}{(1+nx^n)^{1/n}}=g(x)$$

Now, $\int g(x)\cdot x^{n-2}dx=\int\frac{x^{n-1}}{(1+nx^n)^{1/n}}dx$

Let 1 + nx^n = z $\Rightarrow$ n^2x^{n-1} dx = dz

Then, $\int g(x)x^{n-2}dx=\frac{1}{n^2}\int\frac{1}{z^{1/n}}dz$

$$=\frac{1}{n^2}\left(\frac{z^{\frac{n-1}{n}}}{\frac{n-1}{n}}\right)+C$$

$$==\frac{1}{n(n-1)}(1+nx^n)^{1-\frac{1}{n}}+C$$

78. (A)

LHL = $\lim_{x\to-2^-}f(x)=\lim_{x\to-2^-}\frac{a\cdot e^{\frac{1}{|x+2|}}-1}{2-e^{\frac{1}{|x+2|}}}$

$$=\lim_{x\to-2^-}\frac{a-e^{-1|x+2|}}{2\cdot e^{-1|x+2|}-1}=-a$$

LHL = – a $\left[\therefore\frac{1}{|x+2|}\to\infty \text{ as } x\to-2^-\right]$

RHL = $\lim_{x\to-2^+}(x)=\lim_{x\to-2^+}\sin\left(\frac{x^4-16}{x^5+32}\right)$

$$=\lim_{x\to-2^+}\sin\left[\frac{(x+2)(x-2)(x^2+2)}{(x+2)(x^4-2x^3+4x^2-8x+16)}\right]$$

$$=\sin\left[\frac{(-4)\cdot 8}{16+16+16+16+16}\right]=\sin\left(\frac{-2}{5}\right)$$

Value of function (VF)

= f(–2) = b for f(x) to be

Continuous at x = – 2; LHL = RHL = VF

i.e. b = sin $\left(\frac{-2}{5}\right)$ and – a = b = sin $\left(\frac{-2}{5}\right)$

hence, a = $\sin\left(\frac{2}{5}\right)$ and b = $\sin\left(\frac{-2}{5}\right)$

79. (A)

Let $x = \lambda b + \mu a$

Since, $x \perp b$

So, $x \cdot b = 0$

$\Rightarrow \quad (\lambda b + \mu a)\cdot b = 0$

$\Rightarrow \quad \lambda \vec{b}\cdot\vec{b} + \mu a \cdot b = 0$

$\Rightarrow \quad 5\lambda - \mu = 0 \qquad \ldots(i)$

($\therefore$ $b^2 = |b|^2 = 5$ and $a\cdot b = -1$)

Now, $x \cdot a = (\lambda b + \mu a)\cdot a$

$= \lambda a \cdot b + \mu a \cdot a = 7$

$\Rightarrow \quad 3\mu - \lambda = 7 \qquad \ldots(ii)$

From Eqs. (i) and (ii), we get

$\lambda = \frac{1}{2}$ and $\mu = \frac{5}{2}$

$\therefore \quad x = \frac{1}{2}(2\hat{i} + \hat{k}) + \frac{5}{2}(-\hat{i} + \hat{j} + \hat{k})$

$= \frac{-3}{2}\hat{i} + \frac{5}{2}\hat{j} + 3\hat{k}$

80. (D)

Taking n!, (n + 1)! And (n + 2)! Common from R_1, R_2 and R_3. respectively.

$$\Delta_n = n!(n+1)!(n+2)! \begin{vmatrix} 1 & n+1 & (n+1)(n+2) \\ 1 & n+2 & (n+2)(n+3) \\ 1 & n+3 & (n+3)(n+4) \end{vmatrix}$$

Applying $R_2 \to R_2 - R_3$ and

$R_3 \to R_3 - R_2$, we get

$$\Delta_n = n!(n+1)!(n+2)! \begin{vmatrix} 1 & n+1 & (n+1)(n+2) \\ 0 & 1 & 2(n+2) \\ 0 & 1 & 2(n+3) \end{vmatrix}$$

Expanding along C_1

$\Delta_n = 2n!(n+1)!(n+2)!$

Now, $\frac{\Delta_n}{\Delta_{n+1}} = \frac{1}{(n+1)(n+2)(n+3)}$

$\therefore \quad \lim_{n\to\infty} \frac{(-n^3+2n^2+5n)}{(n+1)(n+2)(n+3)} = -1$

81. (5)

$f(2 + x) = f(2 - x)$

$\Rightarrow \quad f'(2 + x) = -f'(2 - x)$

When $x = 0$, then $f'(2) = -f'(2) = 0$

When $x = -1$, then $f'(1) = -f'(3) = 0$

When $x = \frac{-3}{2}$, then

$f'\left(\frac{1}{2}\right) = -f'\left(\frac{7}{2}\right) = 0$

$\therefore f'\left(\frac{1}{2}\right) = f'(1) = f'(2)$

$= f'(3) = f'\left(\frac{7}{2}\right) = 0$

Using Rolle's theorem of $f'(x)$,

Minimum number of roots of $f'(x) = 0$ is 5.

82. (27)

Given, line is $r = \lambda(\hat{i} + \hat{j} + \hat{k})$

Any point on the line is $Q(\lambda, \lambda, \lambda)$

Let $P(x, y, z)$ be at a distance of 3 units from point Q

$\therefore \quad PQ^2 = 9$

$\Rightarrow \quad (x-\lambda)^2 + (y-\lambda)^2 + (2-\lambda)^2 = 9 \quad \ldots(i)$

Also, PQ is perpendicular to the given line.

$\therefore \quad QP \cdot (\hat{i} + \hat{j} + \hat{k}) = 0$

$\Rightarrow \quad x - \lambda + y - \lambda + z - \lambda = 0$

$\Rightarrow \quad \lambda = \frac{x+y+z}{3}$

Putting this value of λ in Eq. (i), we get locus of point P as

$x^2 + y^2 + z^2 - xy - yz - zx = \frac{27}{2}$

So, $\quad k = \frac{27}{2}$

$\therefore \quad 2k = \frac{27}{2} \times 2 = 27$

83. (1)

We know that

$|A| = \det(A) = a_{11}A_{11} + a_{12}A_{21} + a_{13}A_{31}$

(Because adj A is given in question)

$= (1)(-2) + 3(-1) + 2(3\alpha) = 6\alpha - 5$

And $|A|^2 = |\text{Adj}A| = -10 + 17\alpha - 6\alpha^2$

$\Rightarrow \quad (6\alpha - 5)2 = -6\alpha^2 + 17\alpha - 10$

$\Rightarrow \quad 42\alpha^2 - 77\alpha + 35 = 0 \Rightarrow \alpha = 1, \frac{5}{6}$

But $\alpha \in N \therefore \alpha = 1$

$\therefore \quad |A| = 6\alpha - 5 = 6 - 5 = 1$

84. (5)

Normal vector of the plane

$$n = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 0 & -1 \\ 1 & 2 & -1 \end{vmatrix}$$

$\Rightarrow \quad n = 2\hat{i} + 2\hat{j} + 6\hat{k} = 2(\hat{i} + \hat{j} + 3\hat{k})$

$\therefore$ Equation of plane is given by

$\Rightarrow \quad 1(x + 1) + 1(y - 2) + 3(z - 0) = 0$

$\Rightarrow \quad P : x + y + 3z = 1$

Hence, a + b + c = 1 + 1 + 3 = 5

85. (36)

Given,

$|z_1| = 1$

$\Rightarrow \quad |z_1|^2 = 1$

$\Rightarrow \quad z_1\bar{z}_1 = 1 \Rightarrow z_1 = \frac{1}{\bar{z}_1}$

$\Rightarrow \quad |z_2| = 2 \qquad \Rightarrow |z_2|^2 = 4$

$\Rightarrow \quad z_2\bar{z}_2 = 4 \Rightarrow z_2 = \frac{1}{\bar{z}_2}$

$\Rightarrow \quad |z_3| = 3 \qquad \Rightarrow |z_3|^2 = 9$

$\Rightarrow \quad z_3\bar{z}_3 = 9 \Rightarrow z_3 = \frac{1}{\bar{z}_3}$

Now, $|z_2z_3 + 8z_3z_1 + 27z_1z_2|$

$$= \left| z_1z_2z_3\left(\frac{1}{z_1} + \frac{8}{z_2} + \frac{27}{z_3}\right) \right|$$

$= |z_1||z_2||z_3||\bar{z}_1 + 2\bar{z}_2 + 3\bar{z}_3|$

$= 1 \times 2 \times 3 \times 6 = 36$

$[\because \overline{z_1 + 2z_2 + 3z_3} = \bar{z}_1 + 2\bar{z}_2 + 3\bar{z}_3]$

86. (2)

Given,

$$L = \lim_{x\to\infty} x^2 \cdot \sin\left(\log_e \sqrt{\cos\frac{\pi}{x}}\right)$$

$$\Rightarrow \quad L = \lim_{x\to\infty} \frac{x^2 \cdot \sin\left(\log_e \sqrt{\cos\frac{\pi}{x}}\right)}{\left(\log_e \sqrt{\cos\frac{\pi}{x}}\right)} \cdot \left(\log_e \sqrt{\cos\frac{\pi}{x}}\right)$$

$$= \lim_{x\to\infty} x^2 \cdot \log_e \sqrt{\cos\frac{\pi}{x}} \left(\therefore \lim_{y\to 0}\frac{\sin y}{y} = 1\right)$$

$$= \lim_{x\to\infty} \frac{x^2}{2} \cdot \log_e \left(\cos\frac{\pi}{x}\right)$$

$$= \lim_{x\to\infty} \frac{x^2}{2} \cdot \frac{\log_e\left[1 + \left(\cos\frac{\pi}{x} - 1\right)\right]}{\left(\cos\frac{\pi}{x} - 1\right)} \times \left(-2\sin^2\frac{\pi}{2x}\right)$$

$$= \lim_{x\to\infty} \frac{x^2}{2} \cdot \frac{\left(-2\cdot\sin^2\frac{\pi}{2x}\right)}{\left(\frac{\pi}{2x}\right)^2} \cdot \left(\frac{\pi}{2x}\right)^2$$

$$\Rightarrow \quad L = -\frac{\pi^2}{4}$$

Hence, $\dfrac{-8L}{\pi^2} = \left(\dfrac{-8}{\pi^2}\right)\left(\dfrac{-\pi^2}{4}\right) = 2$

87. (484)

Given, equation of curves are

$y = 4x - x^2$ ...(i)

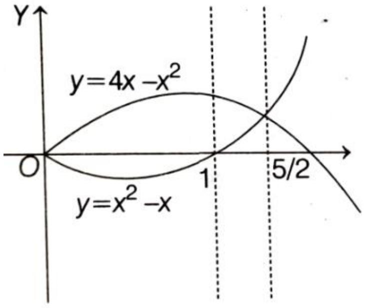
and $y = x^2 - x$ ...(ii)

on solving Eqs. (i) and (ii), we get

$4x - x^2 = x^2 - x$

$\Rightarrow$ $2x^2 = 5x$

$\Rightarrow$ $x(2x - 5) = 0$ $\quad\Rightarrow x = 0, \dfrac{5}{2}$



$$\text{Now, } a=\int_0^{5/2}(4x-x^2)dx-\int_1^{5/2}(x^2-x)dx$$

$$=\left[\frac{4}{2}\cdot(x^2)_0^{5/2}-\frac{1}{3}(x^3)_0^{5/2}\right]-\left[\frac{1}{3}(x^3)_1^{5/2}-\frac{1}{2}(x^2)_1^{5/2}\right]$$

$$=\left(\frac{4}{2}\times\frac{25}{4}-\frac{1}{3}\times\frac{125}{8}\right)-\left[\frac{1}{3}\times\left(\frac{125}{8}-1\right)-\frac{1}{2}\times\left(\frac{25}{4}-1\right)\right]$$

$$=\frac{300-125}{24}-\left(\frac{1}{3}\times\frac{117}{8}-\frac{1}{2}\times\frac{21}{4}\right)$$

$$=\frac{175}{24}-\frac{54}{24}=\frac{121}{24}$$

$$b=\left|\int_0^1(x^2-x)dx\right|=\left[\frac{1}{3}(x^3)_0^1-\frac{1}{2}(x^2)_0^1\right]$$

$$=\left[\frac{1}{3}\times(1)-\frac{1}{2}\times(1)\right]=\left|\frac{-1}{6}\right|=\frac{1}{6}$$

So, $a : b = \frac{121}{24} : \frac{1}{6} = 121 : 4$

Hence, $ab = 121\times 4 = 484$

88. (0)

As we know that, $[x + 1] < (x + 1)$

$\Rightarrow \quad \frac{x+1}{e^x} < \frac{x+1}{e^x} \qquad \ldots.(i)$

Let $f(x) = \frac{x+1}{e^x} = (x + 1)e^{-x}$

On differentiating both sides, we get

$f'(x) = -(x + 1)e^{-x} + e^{-x} = -xe^{-x}$

$\Rightarrow \quad f'(x) < 0$ for $x > 0$

i.e. $f(x)$ is decreasing function for $x > 0$

also, $\frac{x+1}{e^x} = 1$ for $x = 0$ i.e. $f(0) = 1$

for $x > 0$

$\Rightarrow \qquad f(x) < f(0)$

$(\because$ f is decreasing function)

$\Rightarrow \quad \frac{x+1}{e^x} < 1, \forall x > 0$

Hence, $\frac{x+1}{e^x} < 1$ [from Eq. (i)]

$\Rightarrow \quad \left[\frac{x+1}{e^x}\right] = 0$

$\therefore \int_0^\infty \left[\frac{x+1}{e^x}\right] dx = \int_0^\infty 0 \cdot dx = 0$

89. (8)

Given, $x = \lambda(t) = t^5 - 5t^3 - 20t + 7$

And $y = u(t) = 4t^3 - 3t^2 - 18t + 3$

Where $t \in [-2, 2]$

Now, $\frac{dx}{dt} = \lambda'(t) = 5t^4 - 15t^2 - 20$

$= 5(t^2 - 4)(t^2 + 1)$

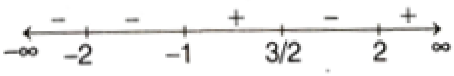
And $\frac{dy}{dt} = \mu'(t) = 12t^2 - 6t - 18$

$= 6(t+2)(2t-3)$

$\therefore \quad \frac{dy}{dx} = \frac{dt}{dx} = \frac{6(t+1)(2t-3)}{5(t^2-4)(t^2+1)}$

$= = \frac{6(t+1)(2t-3)}{5(t-2)(t+2)(t^2+1)}$

Sign scheme for $\frac{dy}{dx}$



Here, $\frac{dy}{dx} > 0$ for $t \in \left(-1, \frac{3}{2}\right)$, where y = f(x) increases.

And $\frac{dy}{dx} < 0$ for $t \in [-2, -1] \cup \left(\frac{3}{2}, 2\right]$

Where y = f(x) decreases.

Also, t = – 1 is the point of minima and

$t = \frac{3}{2}$ is the point of maxima.

Hence, $\alpha + 6\beta = (-1) + 6\left(\frac{3}{2}\right)$

$= -1 + 9 = 8$

90. (6)

Given, $y = ax^2 + bx + \frac{7}{2}$ ...(i)

On differentiating Eq. (i) w.r.t.x,

We get

$$\frac{dy}{dx} = 2ax + b$$

$$\Rightarrow \left(\frac{dy}{dx}\right)_{(1,2)} = 2a + b$$

= slope of tangent ...(A)

Since, (1, 2) lies on Eq. (i)

Hence, $2 = a + b + \frac{7}{2}$

$\Rightarrow a + b = \frac{-3}{2}$ ...(B)

For $y = x^2 + 6x + 10$ ...(ii)

$$\frac{dy}{dx} = 2x + b$$

$$\Rightarrow \left(\frac{dy}{dx}\right)_{(-2,2)} = 2$$

$\therefore$ Slope of normal $= -\frac{1}{2}$ ...(C)

From Eqs. (A) and (C),

$2a + b = \frac{-1}{2}$ ...(D)

On solving Eqs. (B) and (D) , we get

$a = 1, b = \frac{-5}{2}$

Hence, $a - 2b = 6$