PART (A) :PHYSICS

SOLUTION

1. **(C)**

Diameter $D = M.S.R. + (C.S.R) \times L.C.$

$$D = 2.5 + 20 \times \frac{0.5}{50}$$

$$D = 2.70\,mm$$

The uncertainty in the measurement of diameter

$\Delta D = 0.01mm$. (least error is taken to be least count)

We know that

$$\rho = \frac{Mass}{Volume} = \frac{M}{V} = \frac{M}{\frac{4}{3}\pi\left(\frac{D}{2}\right)^3}$$

$$\therefore \quad \frac{\Delta\rho}{\rho}\times 100 = \frac{\Delta M}{M}\times 100 + 3\frac{\Delta D}{D}\times 100$$

$$= 2 + 3\times\frac{0.01}{2.70}\times 100 = 3.1\%$$

2. **(D)**

Dimensionally $\varepsilon_0 L = \text{Capacitance (c)}$

$$\therefore \quad \varepsilon_0 L\frac{\Delta V}{\Delta t} = \frac{C\Delta V}{\Delta t} = \frac{q}{\Delta t} = I$$

3. **(D)**

$$P = nh\mu$$

On increasing frequency, no. of photons decreases

$\Rightarrow$ Stopping potential increases but photo current decreases.

$\Rightarrow$ Correct choice is D.

4. **(B)**

5. **(A)**

$$\frac{kQ^2}{3} + \frac{1}{2}mv_0^2 = \frac{kQ^2}{1} + 0$$

$$\frac{1}{2}mv^2 = 9\times 10^9 \times 16\times 10^{-12} \times \frac{2}{3}$$

$$v^2 = 1920$$

$$v = \sqrt{19200} = 80\sqrt{3}\,cm/s$$

6. **(B)**

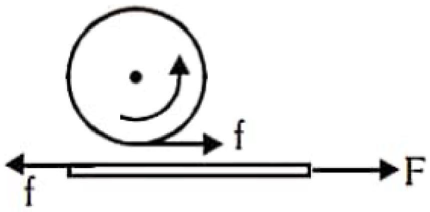
$$R_1 = \frac{200^2}{40} = 1000\Omega$$

$$R_2 = \frac{200^2}{100} = 400\Omega$$

$$i = \frac{200}{900} = \frac{2}{9}$$

$$P = \frac{4}{81}\times 900 = \frac{400}{9}W = 44.4W$$

7. **(B)**



$\Rightarrow$ The ball moves to the right and rotates anti clockwise.

8. **(B)**

Deviation = Zero

So, $\delta = \delta_1 + \delta_2 = 0$

$$\Rightarrow (\mu_1 - 1)A_1 + (\mu_2 - 1)A_2 = 0$$

$$\Rightarrow A_2(1.75-1) = -1(1.5-1)4.5^\circ$$

$$\Rightarrow A_2 = -\frac{0.5}{0.75}\times 4.5^\circ$$

Or $A_2 = 3^\circ$

Negative sign shows that the second prism is inverted with respect to the first.

9. **(A)**

Magnification

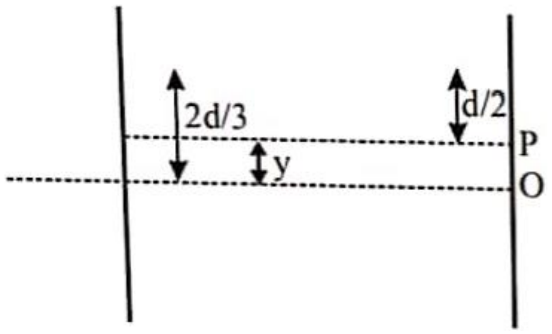
$$= \frac{f_0}{f_e} = \frac{\text{Angle substended by final image on the eye}}{\text{Angle subtended by the object on eye (or objective)}}$$

$$\Rightarrow \frac{0.3}{3cm} = \frac{\beta}{0.5^\circ} \Rightarrow \frac{30cm}{3cm} = \frac{\beta}{0.5^\circ}$$

10. **(C)**

The nearest white spot will be at P, the central maxima

$$\therefore \quad y = \frac{2d}{3} - \frac{d}{2} = \frac{d}{6}$$



11. (C)

$$f_2 = \frac{f_0 v}{v + v_0}$$

The wave which reaches wall f_1 is reflected.

$$f_1 = \frac{f_0 v}{v - v_0}$$

The reflected frequency is f_1 as the wall is at rest.

Beats $= f_1 - f_2 = \frac{f_0 v}{v - v_0} - \frac{f_0 v}{v + v_0} = \frac{2 f_0 v v_0}{v^2 - v_0^2}$

12. (C)

$$I = \frac{1}{3} M (2L)^2 = \frac{4}{3} ML^2$$

Force applied by the spring is $F = -kx$

$\Rightarrow F = -k(2L\theta)$

(θ is the angular displacement from the equilibrium position). Further

$$\tau = \left|\vec{\ell} \times \vec{F}\right| = 4L^2 k \sin\theta = -4L^2 k\theta$$

Also, $\tau = I\alpha = I\ddot{\theta} = -4L^2 k\theta$

$$\Rightarrow \ddot{\theta} + \frac{3k}{M}\theta = 0$$

$$\Rightarrow \omega_0 = \sqrt{\frac{3k}{M}}$$

13. (C)

The given graph does not obey Hooke's law. And there is no well defined plastic region. So the graph represents elastomers.

14. (A)

Fluid resistance is given by $R = \frac{8\eta L}{\pi r^4}$

When two capillary tubes of same size are joined in parallel, then equivalent fluid resistance is

$$R_S = R_1 + R_2 = \frac{8\eta L}{\pi R^4} + \frac{8\eta \times 2L}{\pi(2R)^4} = \left(\frac{8\eta L}{\pi R^4}\right) \times \frac{9}{8}$$

$$\text{Rate of flow } = \frac{P}{R_S} = \frac{\pi P R^4}{8\eta L} \times \frac{8}{9} = \frac{8}{9} X \left[\text{as } X = \frac{\pi P R^4}{8\eta L} \right]$$

15. **(B)**

Electro magnet should be amenable to magnetization and demagnetization

∴ retentivity and coercivity should be low.

16. **(A)**

$$W = \int P dV = \int \frac{nRT}{(V-b)} dV$$

$$= nRT \ln(V-b)\Big|_{v}^{2v}$$

$$W = nRT \ln\left(\frac{2V-b}{V-b}\right)$$

17. **(A)**

$$mv = mv' + \frac{h}{\lambda}$$

$$v' = v - \frac{h}{M\lambda} = v - \frac{hf}{Mc}$$

By energy cons.

$$En + \frac{1}{2} mv^2 = E_m + \frac{1}{2} mv'^2 + hf$$

$$hf = +\frac{1}{2} mv^2 + (E_n - E_m) - \frac{1}{2} Mv'^2$$

$$= \frac{1}{2} mv^2 - \frac{1}{2} m\left(v - \frac{h+}{mc}\right)^2 + hf_0$$

$$f_0 = f\left(1 - \frac{v}{c}\right)$$

18. **(B)**

At $t = 0$, no current will flow through L and R_1

$$\therefore \text{ Current through battery } = \frac{V}{R_2}$$

At $t = \infty$,

$$\text{Effective resistance, } R_{eff} = \frac{R_1 R_2}{R_1 + R_2}$$

$$\therefore \text{ Current through battery } = \frac{V}{R_{eff}}$$

$$= \frac{V(R_1 + R_2)}{R_1 R_2}$$

19. (B)

$I = I_0 \cos^2 \theta$

20. (B)

$dq = \frac{-d\phi}{R}$ $\qquad [\because \phi_f = -\phi_i]$

$\therefore \qquad dq = \frac{2\phi_i}{R} = \frac{2\mu_0 Ia}{R.2\pi} \ln\left(\frac{2a}{a}\right) = \frac{\mu_0 Ia.\ln 2}{R\pi}$

$$\left[\phi = \int_a^{2a} \frac{\mu_0 I}{2\pi x} a\, dx = \frac{\mu_0 Ia}{2\pi} \ln \frac{2a}{a}\right]$$

21. (8.00)

$U = \frac{kz^2e^2}{r}$

$r = r_0 A^{1/3} = 6\times10^{-15} m$

$U = \frac{9\times10^9 \times 49^2 \times 1.6\times10^{-19}}{2\times6\times10^{-15}} \times 1.6\times10^{-19}$

$= 2881.2\times10^5 eV$

$= 2.8812\times10^8 eV$

22. (24.00)

$Q = CV$

$Q_P = Q\left(1-\frac{1}{k}\right) = kC_0 V\left(1-\frac{1}{k}\right)$

$= (k-1)CV = 4\times120\times50$

$= 24000 PC = 24 nC$

23. (168)

$\frac{dQ}{dt} = \frac{mL}{t} = 400\times\frac{100\times10^{-4}}{20\times10^{-2}}(90-0)$

$\Rightarrow \qquad \frac{900\times80\times4.2}{t} = 1800$

$t = 168 sec$

24. (414.72)

$I\omega = I_0\omega_0$

$$\left(80\times 2^2+\frac{1}{2}\times 200\times 2^2\right)\times 1.2=\left(\frac{200}{2}\times 2^2\right)\omega$$

$$\omega=\frac{10.8}{5}$$

$$\text{work}=\frac{1}{2}I\omega^2-\frac{1}{2}I_0\omega_0^2$$

25. **(5.00)**

Assume $a=4z$

$$y=\frac{16M\times 2z-M\left(\frac{3z}{2}+\frac{5z}{2}+\frac{7z}{2}\right)}{13M}$$

26. **(3)**

Consider a ring of thickness dx

Torque on this ring = $QE\times x$

$$E=2\pi x=\pi x^2\times\frac{dB}{dt}$$

$$E=\frac{x}{2}\times 2Kxt-Kx^2t$$

Charge on ring $=\frac{Q}{\pi R^2}\times 2\pi x dx$

Torque on ring $=\frac{2Q}{R^2}X\times Kx^2t\times xdx=\frac{2KQ}{R^2}x^4t\,dx$

Total torque $=\int_0^R\frac{2KQ}{R^2}x^4t\,dx=\left[\frac{2KQtx^5}{R^2\times 5}\right]_0^R$

$$=\frac{2KQR^3t}{5}=3\,N-m$$

27. **(5.00)**

$$T=\frac{2u\sin 30^o}{g\cos 30^o}=2$$

$$u=10\sqrt{3}\,m/s$$

$$\ell=\left(10\sqrt{3}\cos 30^o\right)2-\frac{1}{2}\left(10\sin 30^o\right)4$$

$$\ell=30-10\Rightarrow\ell=20m\Rightarrow\ell/4=5$$

28. **(8.00)**

$$I_{AB} = \frac{5}{100} = \frac{1}{20}A \Rightarrow V_1 = \frac{1}{20}(40) = 2V$$

$$I_{CD} = \frac{E}{80} \Rightarrow V_2 = \frac{E}{80} \times 20 = \frac{E}{4}$$

Now $V_1 = V_2$

$$\frac{E}{4} = 2$$

$E = 8$ volt

29. **(6.00)**

$$\left(\frac{n_1}{n_2}\right)^3 = \frac{1}{64}$$

$$\frac{n_1}{n_2} = \frac{1}{4}$$

$n_2 = 4$

No. of resulting spectrum are $= 4_{C_2} = 6$

30. **(6.00)**

I_1 is the image of object O formed by the lens.

$$\frac{1}{v_1} - \frac{1}{u_1} = \frac{1}{f}; u_1 = -15, f_1 = 10$$

Solving we get, $v_1 = 30\text{cm}$.

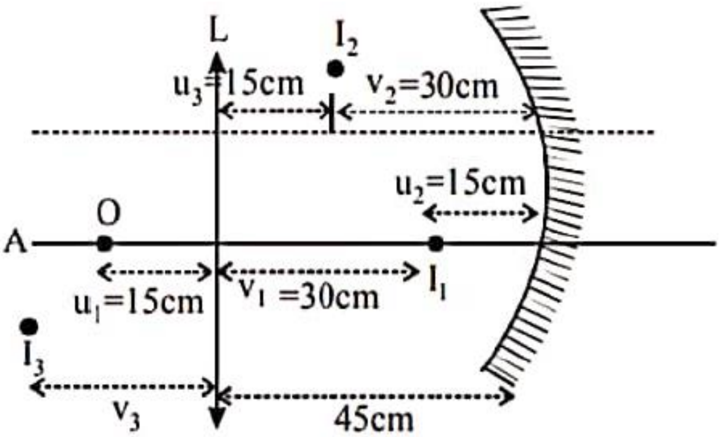
I_1 acts as source for mirror

$\therefore \quad u_2 = -(45 - v_1) = -15\text{cm}$.

I_2 is the image formed by the mirror

$$\therefore \quad \frac{1}{v_2} = \frac{1}{f_m} - \frac{1}{u_2} = -\frac{1}{10} + \frac{1}{15}$$

$\therefore \quad v_2 = -30\text{cm}$



The height of I_2 above principal axis of lens is

$$= \frac{v_2}{u_2} \times 1 + 1 = 3\text{cm}$$

I_2 acts as a source for lens, $u_3 = -(45 - v_2) = -15\text{cm}$.

Hence, the lens forms an image I_3 at a distance

$v_3 = 30\text{cm}$ to the left of lens.

The height of I_3 below the principal axis of lens

$$= v_3 \times 3 = 6\,\text{cm}.$$

$$\sqrt{30^2 + 6^2} = 6\sqrt{26}\ \text{cm}.$$

PART (B) : CHEMISTRY

SOLUTION

31. **(D)**

(A) There is a gradual decrease in the radii of the lanthanoids with increasing atomic number - a case of lanthanoids contraction, thus true.

(B) Ionisation potential for the formation of Lu^{3+} is comparatively low, hence +3state is favourable, thus true.

(C) Duo to lanthanide contraction,–Zr and Hf;Nb and Ta, Mo and W have the same size and thus similar properties and thus separation is not easy, thus true.

(D) Formation of +4 state requires very high energy, thus incorrect.

Thus, option (D) is wrong.

32. **(D)**

van der Waals, constant a is due to force of attraction and *b* due to finite size of molecules. Thus, greater the value a and smaller the value of *b*, larger the liquefaction.

Thus, $a(Cl_2) > a(C_2H_6)$ and $b(Cl_2) < b(C_2H_6)$

$a : Cl_2 = 6.579, C_2H_6 = 5.562$

$b : Cl_2 = 0.05622, C_2H_6 = 0.0638$

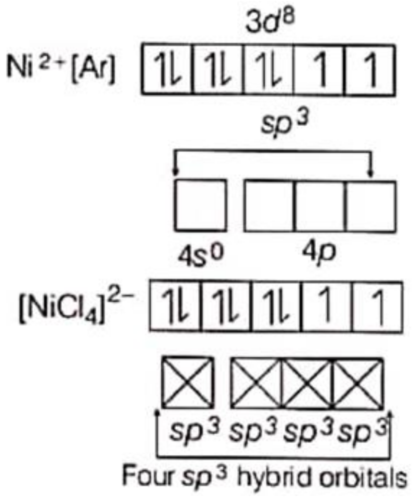
Thus, option (D) is correct.

33. **(C)**

$[NiCl_4]^{2-}$; oxidation number of Ni, $x - 4 = -2$

$\therefore \quad x = +2$

$Ni_{(28)} = [Ar]3d^8 4s^2$



sp^3 - hybride orbitals, tetrahedral

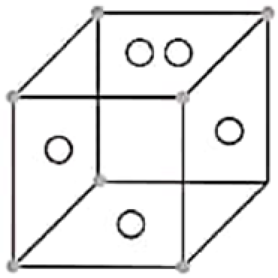
Cl^- is a weak ligand and thus unpaired electrons are not paired. Lone pairs from $4Cl^-$ are accommodated in four sp^3 hybrid orbitals

N = unpaired electron = 2, paramagnetic
Magnetic moment (spin only)

$$= \sqrt{N(N+2)}BM = \sqrt{8} = 2.828\ BM$$

Thus, (C) option is correct

34. **(D)**
Calculate the number of atoms (or ratio) of elements A and B as a chemical formula represents the number of atoms of different elements ina compound. Derive the formula



Number of atoms (A) per unit cell $= 8 \times \frac{1}{8} = 1$

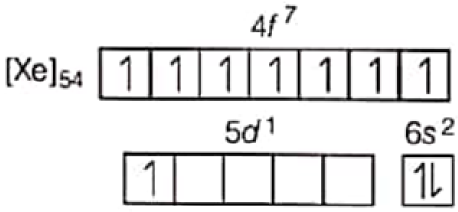
Number of atoms (B) per unit cell = $(6-1) \times \frac{1}{2} = \frac{5}{2}$

(one atom B is missing)

Thus, formula is $A_1B_{5/2} = A_2B_5$

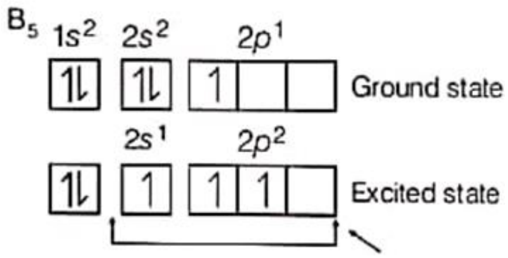
Therefore, correct option is (D)

35. **(D)**
Gd (64)



All the electrons in 4f-orbital are unpaired, hence stable.
Thus ,Gd (64) has EC as
$[Xe]_{54}\ 4f^7\ 5d^1\ 6s^2$
Instead of $[Xe]_{54}\ 4f^8\ 6s^2$

36. **(A)**

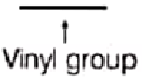


Due to absence of d-orbital, maximum covalency is four. Thus, BF_6^{3-} is not formed,
Thus, option (A) is nor formed

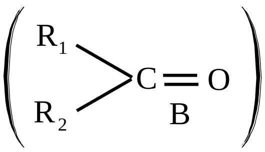
37. **(B)**

Alkenes give carbonyl compounds on ozonolysis

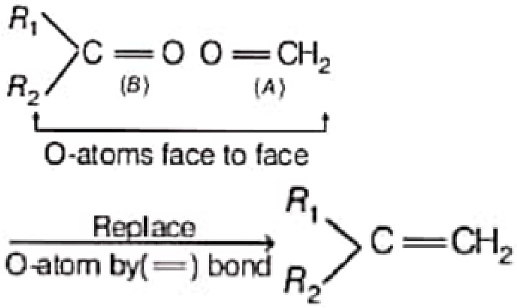
$$\text{Alkene} \xrightarrow{O_3/Zn/H_2O}$$



$\underset{(A)}{HCHO}$+ other carbonyl compound

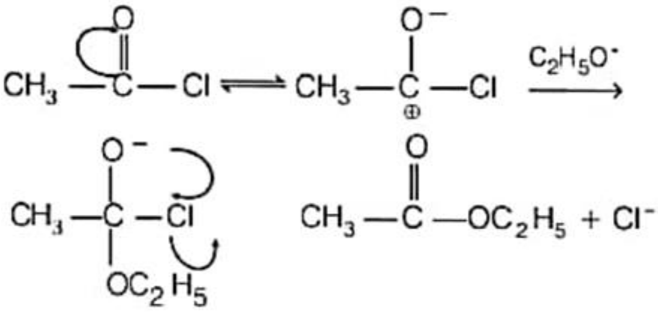


To determine alkene, place carbonyl compounds with their O-atom face to face. Replace O-atom by a double bond



38. **(D)**

Ethoxide ion is a better nucleophile and gives nucleophilic substitution reaction with ethanoyl chloride. Therefore, write the S_N reaction and find out the final product



This is by S_N reaction. Cl^- is a better leaving group than $C_2H_5O^-$ and the ethyl ethanonate is formed.

39. **(A)**

$$A_xB_y \rightleftharpoons xA^{y+} + yB^{x-}$$

After dissociation, $(1-\alpha)$ $\quad x\alpha \quad y\alpha$

$$i = n(A_xB_y) + n(A^{y+}) + n(B^{x-})$$
$$= 1 - \alpha + x\alpha + y\alpha$$
$$= 1 + \alpha(x + y - 1)$$
$$\therefore \quad \alpha = \frac{i-1}{(x+y-1)}$$

40. **(A)**

All aldehydes including reducing sugar (as glucose, fructose) give **Silver-mirror test** (with Tollen's reagent)

$$RCHO + 2[Ag(NH_3)_2]^+ + 3OH^- \xrightarrow{\Delta} RCOO^- + 2Ag\downarrow + 4NH_3 + 2H_2O$$

Silver mirror

$(R = H,\ CH_3)$

Thus, (A) acetaldehyde gives silver – mirror test

41. **(A)**

Most of the d–block metal compounds are coloured in solid or in solution states. The colour of transition metal ions is due to the presence of unpaired or incomplete or d–orbitals. So, find the metal which has completely filled d–orbitals.

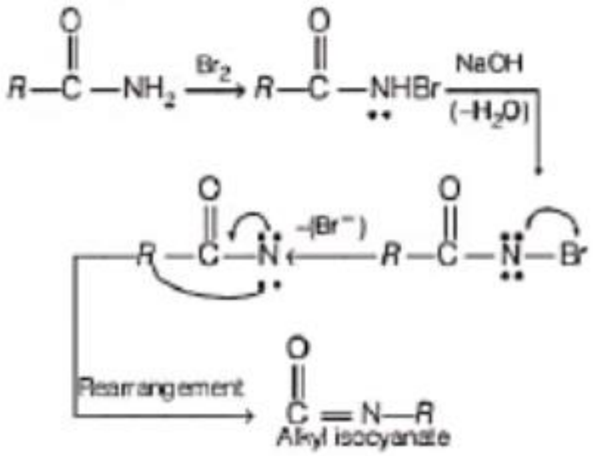
The electronic configuration of

$Cd(Z = 48) = [Kr]5s^2, 4d^{10}$ and $Cd^{2+} = [Kr]5s^0 4d^{10}$

Hence, Cd^{2+} it has completely filled d–orbital is due to which it is colourless.

42. **(A)**

Reaction is Hofmann–bromide reaction, The mechanism is as



In all cases, alkyl isocyanates are converted into amines by reaction with alkali.

$$O = C = N - R \xrightarrow[H_2O]{OH^-} RNH_2 + CO_3^{2-}$$

43. **(D)**

Viscosity is an intensity property because it does not depend on the amount of the substances.

44. **(C)**

Rate $= k(C_x)^{1.5}(C_y)^{-1}(C_z)^0$

Adding all exponential term $= 1.5 + (-1) + 0$

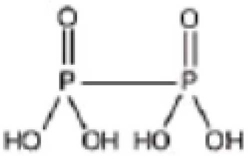
$= 0.5$

45. **(D)**

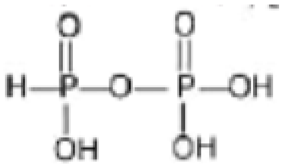
In ortho hydrogen, both protonshave the same spin while in case of para hydrogen, both protons have opposite spin.

46. **(D)**

Hypophosphoric acid $(H_4P_2O_6)$

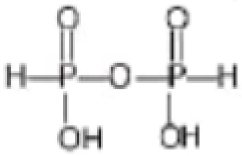


Isohypophosphoric acid $(H_4P_2O_6)$

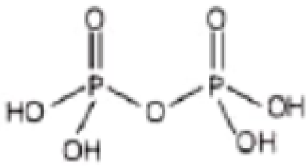


Diphosphorous acid

(pyrophosphorous acid, $H_4P_2O_5$)



Diphosphoric acid (pyrophosphoric acid, $H_2P_2O_7$)



Hence, hypophosphoric acid contain P–P bond instead of P–O–P bond.

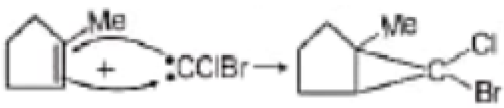
47. **(D)**

Solubility of I_2 is greatly increased by adding its salt KI in water solution.

48. **(B)**

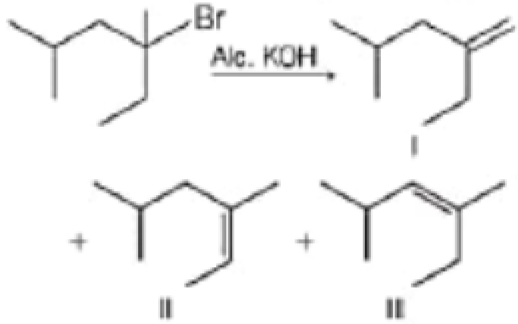
$CHClBr_2 + Me_3COK \rightarrow CClBr_2 \rightarrow :CClBr$

Chlorobromocarbene $(CClBr)$ undergoes addition.



49. **(D)**

3° halide in alcoholic KOH prefers E_1 path



II (more stable) product is the major product.

Reaction is stereoselective not stereospecific

$\text{Rate} = k[\text{Bromide}]^1[\text{KOH}]^{-1}$

50. **(C)**

$\underset{3}{CH_2}-\underset{2}{CH}-\underset{1}{C}\equiv N$

Prop–2–ene–1–nitrile

51. **(4)**

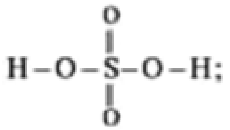
$PCl_5 + SO_2 \rightarrow POCl_3 + SOCl_2$

$PCl_5 + H_2O \rightarrow POCl_3 + 2HCl$

$PCl_5 + H_2SO_4 \rightarrow POCl_3 + SO_2Cl_2 + 2HCl$

$6PCl_5 + P_4O_{10} \rightarrow 10POCl_3$

52. **(8)**



$6\sigma \,\&\, 2\pi$

53. **(3)**

Magnetic moment $\mu = \sqrt{n(n+2)}$ where n = number of unpaired electrons $\sqrt{15} = \sqrt{n(n+2)}$ $\therefore n = 3$

54. **(5)**

$FeSO_4.(NH_4)_2SO_4.6H_2O$ in solution will give $Fe^{2+}, 2(SO_4^{2-}), 2(NH_4^+)$ hence total number of ions is 5.

55. **(4)**

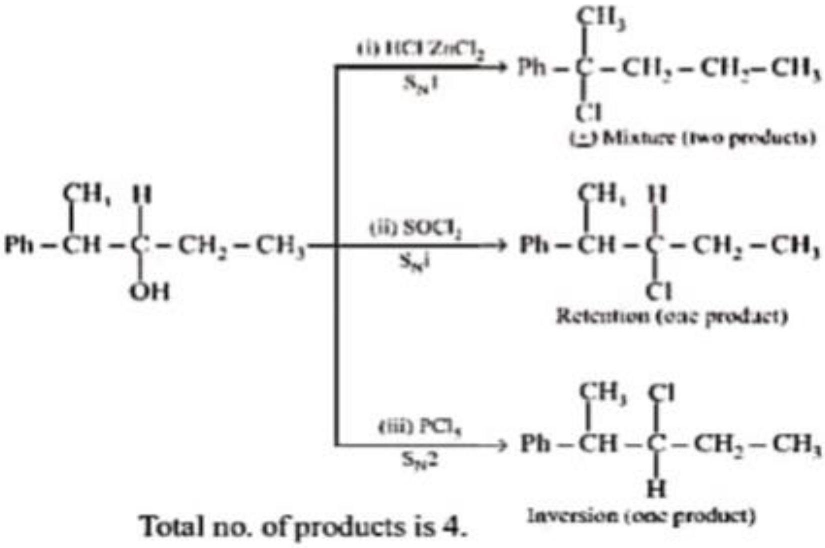
Addition of HBr to 2–pentyne gives two structural isomers (I) and (II)

$CH_3-C\equiv C-CH_2CH_3 \xrightarrow{HBr}$

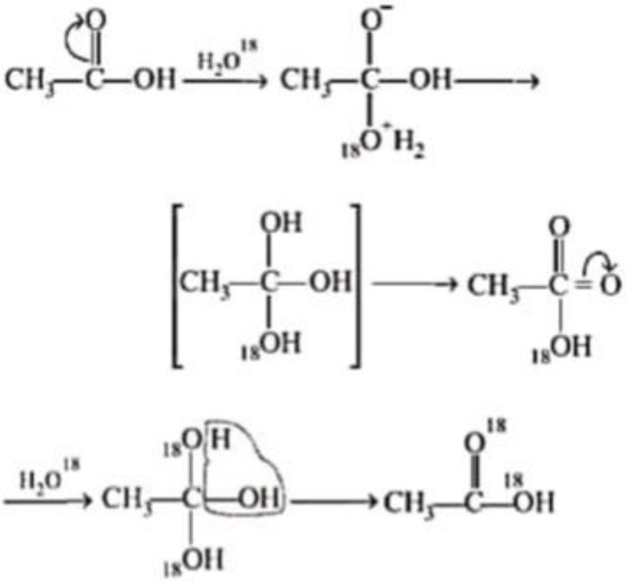
$\underset{(I)}{CH_3C(Br)=CHCH_2CH_3} + \underset{(II)}{CH_3CH=C(Br)CH_2CH_3}$

Each one of these will exist as a pair of geometrical isomers. Thus, total number of isomers is 4.

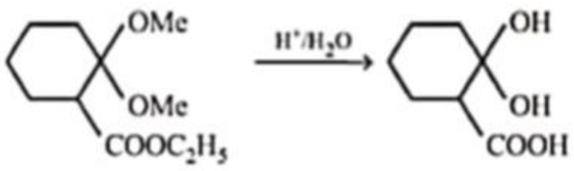
56. **(4)**

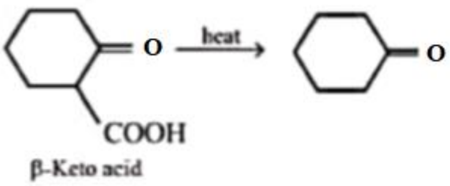


57. **(2)**



58. **(1)**





Remember that $\beta-$keto acids undergo decarboxylation on heating.

59. **(4)**

Amide, lactam, carboxylic and thioetherlinakge are present.

60. **(6.02)**

$$\frac{1.48/32}{0.43/56} = \frac{1.48 \times 56}{0.43 \times 32} = 6.02$$

PART (C) : MATHEMATICS

SOLUTION

61. (C)

$\frac{2}{\cos 2\alpha} = \frac{1}{\sin\beta\cos\beta}$

$\Rightarrow \sin 2\beta = \sin(90-2\alpha) \Rightarrow \alpha + \beta$

$= \frac{\pi}{4}$

62. (B)

$\frac{r\,{}^{n}C_{r}}{{}^{n}C_{r-1}} = \frac{r.\underline{|n}}{\underline{|r}.\underline{|n-r}} \cdot \frac{\underline{|r-1}.\underline{|n-r+1}}{\underline{|n}} = \frac{\underline{|n-r+1}}{\underline{|n-r}}$

$= n - r + 1$

so required sum = $n + (n-1) + (n-2) + (n-3) + (n-4)$

= 5n – 10 = 5 (n – 2)

63. (B)

${}^{14}C_7 + \sum_{i=1}^{3} {}^{17-i}C_6$

$= {}^{14}C_7 + {}^{14}C_6 + {}^{15}C_6 + {}^{16}C_6$

$= {}^{15}C_7 + {}^{15}C_6 + {}^{16}C_6$

$= {}^{16}C_7 + {}^{16}C_6 = {}^{17}C_7$

64. (D)

Y = A$\sin\omega$t

$\therefore \frac{dy}{dx} = A\omega\cos\omega t$

$\frac{d^2y}{dx^2} = -A\omega^2\sin\omega t$

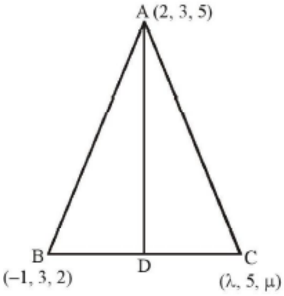
$\frac{d^3y}{dx^3} = -A\omega^3\cos\omega t$

$\frac{d^4y}{dx^4} = +A\omega^4\sin\omega t$

$\therefore \frac{d^5y}{dx^5} = A\omega^5\cos\omega t = A\omega^5\sin\left(\omega t + \frac{\pi}{2}\right)$

65. (C)

Since AD is the median



$\therefore D = \left(\frac{\lambda - 1}{2}, 4, \frac{\mu + 2}{2}\right)$

Now, dR's of AD is

$a = \left(\frac{\lambda - 1}{2} - 2\right) = \frac{\lambda - 5}{2}$

$b = 4 - 3 = 1, c = \frac{\mu + 2}{2} - 5 = \frac{\mu - 8}{2}$

Also a,b, c are dP's

$\therefore$ a = kl. b – km, c – kn where $l = m = n$ and $l^2 + m^2 + n^2 = 1$

$\Rightarrow l = m = n = \frac{1}{\sqrt{3}}$

Now, a = 1, b =1, and c = 1

$\Rightarrow \lambda = 7$ and $\mu = 10$

66. (A)

$a_1 = 2, b_1 = 2, c_1 = -1$ and $a_2 = 1, b_2 = 2, \ c_2 = 2$

$$\cos\theta = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}}$$

$$= \frac{2 + 4 - 2}{\sqrt{4 + 4 + 1}\sqrt{1 + 4 + 4}} = \pm\frac{4}{9}$$

67. (C)

Contrapositive of $p \Rightarrow q$ is $\sim q \Rightarrow \sim p$

$\therefore$ contrapositive of $(p \vee q) \Rightarrow r$ is

$\sim r \Rightarrow \sim (p \vee q)$ i.e. $\sim r \Rightarrow (\sim p \wedge \sim q)$

68. (A)

Let the co-ordinate of other ends are (x, y, z). The centre of sphere is C(3, 6, 1)

Therefore, $\frac{x + 2}{2} = 3 \Rightarrow x = 4$

$\frac{y + 2}{2} = 6 \Rightarrow y = 9$ and $\frac{z + 5}{2} = 1 \Rightarrow z = -3$

69. (D)

$P(\bar{E}E) + P(\bar{E}\bar{E}\bar{E}\bar{E}E) + ...$

$= \frac{5}{6} \times \frac{1}{6} + \left(\frac{5}{6}\right)^4 \times \frac{1}{6} + \left(\frac{5}{6}\right)^8 \frac{1}{6} ....\infty$

$= \frac{5}{36}\left[1 + \left(\frac{5}{6}\right)^3 ......\right] = \frac{30}{91}$

70. (B)

A, b, c in A.P $\Rightarrow a + c = 2b$; b, c, d in G.P.

$\Rightarrow bd = c^2$; c, d, e in H.P. $\Rightarrow d = \frac{2ce}{c + e}$

$\therefore \frac{a + c}{2} \times \frac{2ce}{c + e} = c^2 \Rightarrow (a + c)e = (c + e)c \Rightarrow c^2 = ae$

Therefore in G.P.

71. (B)

$\left(3x^2 - \frac{1}{x^2}\right)^{15}$

$T_{r+1} = {}^{15}C_r (3x^2)^{15-r} \left(-\frac{1}{x^2}\right)^r$

$= {}^{15}C_r 3^{15-r} (-1)^r \; x^{30-2r-2r}$

$\Rightarrow r = 5$

Coefficient of $T_6 = -{}^{15}C_5 3^{10} = \frac{-15!}{10!5!} 3^{10}$

72. (A)

$I = \int \frac{dx}{(x - \beta)\sqrt{(x - \alpha)(\beta - x)}}$

Put $x = \alpha \sin^2\theta + \beta\cos^2\theta$

[see the standard substitutions]

$dx = 2(\alpha - \beta)\sin\theta\cos\theta d\theta$

$(x - \alpha) = (\beta - \alpha)\cos^2\theta$

$(x - \beta) = (\alpha - \beta)\sin^2\theta$

$\therefore I = \int \frac{2(\alpha - \beta)\sin\theta\cos\theta d\theta}{(\alpha - \beta)\sin^2\theta(\beta - \alpha)\sin\theta\cos\theta}$

$= \frac{2}{\beta - \alpha}\int \frac{d\theta}{\sin^2\theta} = \frac{2}{\beta - \alpha}\int \operatorname{cosec}^2\theta d\theta$

$= \frac{2}{\beta - \alpha}(-\cot\theta) + C = \frac{2}{\alpha - \beta}\cot\theta + C$

$x = \alpha\sin^2\theta + \beta\cos^2\theta$

$\Rightarrow x \cos ec^2\theta + \alpha + \beta \cot^2\theta$

$\Rightarrow x(1+\cot^2\theta) = \alpha + \beta\cot^2\theta$

$\therefore \cot\theta = \sqrt{\dfrac{x-\alpha}{\beta - x}}$

$\therefore I = \dfrac{2}{\alpha - \beta}\sqrt{\dfrac{x-\alpha}{\beta - x}} + C$

73. (D)

Given planes are

P : x + y – 2z + 7 = 0

Q : x + y + 2z + 2 = 0

And R : 3x + 3y – 6z – 11 = 0

Consider Plane P and R

Here $a_1 = 1, b_1 = 1, c_1 = -2$

and $a_2 = 3$, $b_2 = 3$, $c_2 = -6$

Since $\dfrac{a_1}{a_2} = \dfrac{b_1}{b_2} = \dfrac{c_1}{c_2} = \dfrac{1}{3}$

Therefore P and R are parallel

74. (A)

$\dfrac{1}{1^4} + \dfrac{1}{2^4} + \dfrac{1}{3^4} + \dfrac{1}{4^4} + ...\infty = \dfrac{\pi^4}{90}$

$\dfrac{1}{1^4} + \dfrac{1}{3^4} + \dfrac{1}{5^4} + ...+\infty + \dfrac{1}{2^4}\left(\dfrac{1}{1^4} + \dfrac{1}{2^4} + \dfrac{1}{3^4} + \dfrac{1}{4^4} + ...\infty\right) = \dfrac{\pi^4}{90}$

$\dfrac{1}{1^4} + \dfrac{1}{3^4} + \dfrac{1}{5^4} + \dfrac{1}{7^4} + ...+\infty + \dfrac{1}{16}\times\dfrac{\pi^4}{90} = \dfrac{\pi^4}{90}$

$\therefore \dfrac{1}{1^4} + \dfrac{1}{3^4} + \dfrac{1}{5^4} + \dfrac{1}{7^4} + ...+\infty = \dfrac{\pi^4}{90} - \dfrac{1}{16}\left(\dfrac{\pi^4}{90}\right)$

$= \dfrac{15}{16}\left(\dfrac{\pi^4}{90}\right) = \dfrac{\pi^4}{90}$

75. (B)

We have f(x) =exp$\left(\sqrt{5x-3-2x^2}\right)$

i.e., $f(x) = e^{\sqrt{5x-3-2x^2}}$

For domain of f(x), $5x - 3 - 2x^2$ should be non-negative.

i.e. $5x - 3 - 2x^2 \geq 0$

$\Rightarrow 2x^2 - 5x + 3 \leq 0$ (By taking -ve sign common)

$\Rightarrow 2x(x-1) - 3(x-1) \leq 0$

$\Rightarrow (2x-3)(x-1) \le 0$

$\therefore 1 \le x \le \frac{3}{2}$ i.e. $x \in \left[1, \frac{3}{2}\right]$

Hence, domain of the given function is $\left[1, \frac{3}{2}\right]$

76. (A)

$\int \frac{dx}{\cos^3 x\sqrt{4\sin x\cos x}}$

$= \int \frac{dx}{2\cos^4 x\sqrt{\tan x}}$

Let tan x = $t^2 \Rightarrow \sec^2 x = 1 + t^4$

$\sec^2 xdx = 2tdt$

$= \int \frac{\sec^4 xdx}{2\sqrt{\tan x}} = \int \frac{\sec^2 x(\sec^2 xdx)}{2\sqrt{\tan x}}$

$= \int \frac{(1+t^4)2tdt}{2t} = \int (1+t^4)dt$

$= t + \frac{t^5}{5} + k$

$= \sqrt{\tan x} + \frac{1}{5}\tan^{5/2} x + k[t = \sqrt{\tan x}]$

$A = \frac{1}{2}, B = \frac{5}{2}, C = \frac{1}{5}$

$A + B + C = \frac{16}{5}$

77. (D)

We have, $a = \cos\left(\frac{2\pi}{7}\right) + i\sin\left(\frac{2\pi}{7}\right)$

$\Rightarrow a^7 = \left[\cos\left(\frac{2\pi}{7}\right) + i\sin\left(\frac{2\pi}{7}\right)\right]^7 = \cos 2\pi + i\sin 2\pi = 1$ ....(i)

Let $S = \alpha + \beta = (a + a^2 + a^4) + (a^3 + a^5 + a^6)$ $[\because \alpha = a + a^2 + a^4, \beta = a^3 + a^5 + a^6]$

$\Rightarrow S = a + a^2 + a^3 + a^4 + a^5 + a^6$

$= \frac{a(1-a^6)}{1-a}$

$\Rightarrow S = \frac{a-a^7}{1-a} = \frac{a-1}{1-a} = -1$ ....(ii)

Let $P = \alpha\beta = (a + a^2 + a^4)(a^3 + a^5 + a^6)$

$= a^4 + a^6 + a^7 + a^5 + a^7 + a^8 + a^7 + a^9 + a^{10}$

$= a^4 + a^6 + 1 + a^5 + 1 + a + 1 + a^2 + a^3$ [From Eq. (i)]

$= 3 + (a + a^2 + a^3 + a^4 + a^5 + a^6) = 3 + S$

$= 3 - 1 = 2$ [From Eq. (ii)]

Required equation is $x^2 \; Sx + P = 0$

$\Rightarrow x^2 + x + 2 = 0$

78. (A)

We have

$$AB = \begin{bmatrix} \cos^2\theta & \cos\theta\sin\theta \\ \cos\theta\sin\theta & \sin^2\theta \end{bmatrix}\begin{bmatrix} \cos^2\phi & \cos\phi\sin\phi \\ \cos\phi\sin\phi & \sin^2\phi \end{bmatrix}$$

$$= \begin{bmatrix} \cos^2\theta\cos^2\phi + \cos\theta\cos\phi\sin\theta\sin\phi & \cos^2\theta\cos\phi\sin\phi + \cos\theta\sin\theta\sin^2\phi \\ \cos\theta\sin\theta\cos^2\phi + \sin^2\theta\cos\phi\sin\phi & \cos\theta\cos\phi\sin\theta\sin\phi + \sin^2\theta\sin^2\phi \end{bmatrix}$$

$$= \cos(\theta - \phi)\begin{bmatrix} \cos\theta\cos\phi & \cos\theta\sin\phi \\ \sin\theta\cos\phi & \sin\theta\sin\phi \end{bmatrix}$$

Since, AB = 0 $\therefore \cos(\theta - \phi) = 0$

$\therefore \theta - \phi$ is an odd multiple of $\frac{\pi}{2}$

79. (C)

$f(x) = xe^{x(1-x)}, x \in R$

$f'(x) = e^{x(1-x)} \cdot \left[1 + x - 2x^2\right] = -e^{x(1-x)} \cdot \left[2x^2 - x - 1\right]$

$= -2e^{x(1-x)}\left[\left(x + \frac{1}{2}\right)(x - 1)\right]$

$f'(x) = -2e^{x(1-x)} \cdot A$

where $A = \left(x + \frac{1}{2}\right)(x - 1)$

Now, exponential function is always +ve and $f'(x)$ will be opposite to the sign of A which is -ve in $\left(-\frac{1}{2}, 1\right)$

Hence, $f'(x)$ is +ve in $\left(-\frac{1}{2}, 1\right)$

$\therefore$ f(x) is increasing on $\left(-\frac{1}{2}, 1\right)$

80. (D)

Slope of the equations $4x^2 - 9y^2 = 36$

$$8x - 18y\frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = \frac{4x}{9y} \; or \; m_1 = \frac{4x}{9y}$$

Slope of the straight line 5x + 2y – 10 = 0 is

$m_2 = -\frac{5}{2}$

Therefore, for the perpendicular $m_1 m_2 = -1$

Now, $\frac{4x}{9y} \times \frac{-5}{2} = -1 \Rightarrow y = \frac{10x}{9}$

Putting $y = \frac{10x}{9}$ in $4x^2 - 9y^2 = 36$ gives imaginary roots resulting in no tangents.

81. (0)

If the GP be a, ar, ar, ar^2,...... then $a_n = ar^{n-1}$

$$D = \begin{vmatrix} \log a + (n-1)\log r & \log a + n\log r & \log a + (n+1)\log r \\ \log a + n\log r & \log a + (n+1)\log r & \log a + (n+2)\log r \\ \log a + (n+1)\log r & \log a + (n+2)\log r & \log a + (n+3)\log r \end{vmatrix}$$

$R_3 \to R_3 - R_2$ and $R_2 \to R_2 - R_1$

$$\begin{vmatrix} \log a + (n-1)\log r & \log a + n\log r & \log a + (n+1)\log r \\ \log r & \log r & \log r \\ \log r & \log r & l\,og\, r \end{vmatrix}$$

= 0, since R_2 and R_3 are identical.

82. (7)

Given equation of ellipse is

$\frac{x^2}{16} + \frac{y^2}{b^2} = 1$

Eccentricity = e = $\sqrt{1 - \frac{b^2}{16}}$

Foci: $\pm ae = \pm 4\sqrt{1 - \frac{b^2}{16}}$

Equation of hyperbola is $\frac{x^2}{144} - \frac{y^2}{81} = \frac{1}{25}$

$\Rightarrow \frac{x^2}{\frac{144}{25}} - \frac{y^2}{\frac{81}{25}} = 1$

Eccentricity = e = $\sqrt{1 + \frac{81}{25} \times \frac{25}{144}} = \sqrt{1 + \frac{81}{144}}$

$\sqrt{\frac{225}{144}} = \frac{15}{12}$

Foci : $\pm ae = \pm \frac{12}{5} \times \frac{15}{12} = \pm 3$

Since, foci of ellipse and hyperbola coincide

$$\pm 4\sqrt{1-\frac{b^2}{16}} = \pm 3 \Rightarrow b^2 = 7$$

83. (1)
For x > 10, f(x) = x – 2
Therefore g (x) = x – 2 – 2 = x – 4
$\therefore g'(x) = 1$

84. (14)
Since 0 < y < x < 2y
$\therefore y > \frac{x}{2} \Rightarrow x - y < \frac{x}{2}$
$\therefore x - y < y < x < 2x + y$
Hence median = $\frac{y+x}{2} = 10$
$\Rightarrow x + y = 20$ …(i)
And range = (2x + y) – (x – y) = x + 2y
But range = 28
$\therefore x + 2y = 28$ ….(ii)
From equations (i) and (ii)
x- 12, y= 8
∴ Mean

$$= \frac{(x-y)+y+x+(2x+y)}{4} = \frac{4x+y}{4}$$

$$= x + \frac{y}{4} = 12 + \frac{8}{4} = 14$$

85. (0)
Given determinant

$$\begin{vmatrix} 1 & a & a^2 \\ \cos(n-1)x & \cos nx & \cos(n+1)x \\ \sin(n-1)x & \sin nx & \sin(n+1)x \end{vmatrix}$$

$$\Rightarrow \begin{vmatrix} 1+a^2-2a\cos x & a & a^2 \\ 0 & \cos nx & \cos(n+1)x \\ 0 & \sin nx & \sin(n+1)x \end{vmatrix} = 0$$

By applying
$C_1 \to C_1 + C_3 - 2\cos x C_2$
By expanding
$(1 + a^2 - 2a\cos x)[\cos nx \sin(n+1)x - \sin nx \cos(n+1)x] = 0$

Now, $(1 + a^2 - 2a\cos x)\sin(n+1-n)x = 0$

$\Rightarrow (1 + a^2 - 2a\cos x)\sin x = 0$

sinx = 0 or cosx = $\dfrac{1+a^2}{2a}$

As $a \neq 1 \therefore \left(\dfrac{1+a^2}{2a}\right) > 1$

$\Rightarrow \cos x > 1$ It is not possible.

$\therefore$ sinx = 0

86. (1)

Required area

$\int_{-1}^{1} |y|\,dx = \int_{-1}^{1} |x^5|\,dx = 2\int_{0}^{1} |x^5|\,dx$

$= 2\int_{0}^{1} x^5 dx = 2\left[\dfrac{x^6}{6}\right]_0^1 = \dfrac{2}{6} = \dfrac{1}{3}$

87. (3)

$\int_{\log\sqrt{\pi/2}}^{\log\sqrt{\pi}} e^{2x}\sec^2\left(\dfrac{1}{3}e^{2x}\right)dx$

Put $e^{2x} = t \Rightarrow 2e^{2x}dx = dt$

When $x = \log\sqrt{\pi/2}, t = e^{2\log\sqrt{\pi/2}}$

$= e^{\log \pi/2} = \dfrac{\pi}{2}$

When $x = \log\sqrt{\pi}, t = e^{2\log\sqrt{\pi}} = \pi$

$\therefore \int_{\frac{\pi}{2}}^{\pi} \dfrac{1}{2}\sec^2\left(\dfrac{1}{3}t\right)dt$

$= \dfrac{1}{2}\cdot\dfrac{1}{\frac{1}{3}}\left[\tan\dfrac{t}{3}.\right]_{\pi/2}^{\pi}$

$= \dfrac{3}{2}\left[\tan\dfrac{\pi}{3} - \tan\dfrac{\pi}{6}\right]$

$= \dfrac{3}{2}\left[\sqrt{3} - \dfrac{1}{\sqrt{3}}\right] = \sqrt{3}$

88. (9)

Total no. of arrangements of the letters of the word UNIVERSITY is $\dfrac{10!}{2!}$. No. of arrangements when both I's are together = 9!

So, the no. of ways in which 2I's do not together = $\frac{10!}{2!} - 9!$

$\therefore$ Required probability

$$\frac{\frac{10!}{2!} - 9!}{\frac{10!}{2!}} = \frac{10! - 9!2!}{10!}$$

$$= \frac{\frac{10!}{2!} - 9!}{\frac{10!}{2!}} = \frac{10! - 9!2!}{10!}$$

$$= \frac{10 \times 9! - 9!.2!}{10!} = \frac{9![10 - 2]}{10 \times 9!}$$

$$= \frac{8}{10} = \frac{4}{5}$$

89. (17)

Sum of more than 7 can be obtained when (2, 6), (3, 5)(3, 6), (4, 4), (4, 5)(4, 6), (5, 3) (5, 4) ((5, 5) (5, 6)(6, 2) (6,3) (6, 4) (6, 5)(6, 6)

$\therefore$ Probability of sum > 7 = $\frac{15}{36} = \frac{5}{12}$

90. (3)

Ratio $= \frac{c+a}{b} = \frac{4+6}{5} = 2:1$