KEY SHEET

PHYSICS

1	**C**	2	**B**	3	**A**	4	**C**	5	**C**
6	**A**	7	**A**	8	**C**	9	**B**	10	**A**
11	**C**	12	**A**	13	**C**	14	**C**	15	**C**
16	**D**	17	**B**	18	**D**	19	**B**	20	**C**
21	**60**	22	**5**	23	**25**	24	**6**	25	**400**
26	**210**	27	**5**	28	**1**	29	**0**	30	**363**

CHEMISTRY

31	**C**	32	**D**	33	**B**	34	**A**	35	**C**
36	**A**	37	**D**	38	**C**	39	**D**	40	**A**
41	**B**	42	**B**	43	**A**	44	**D**	45	**A**
46	**C**	47	**C**	48	**A**	49	**C**	50	**B**
51	**727**	52	**98**	53	**476**	54	**200**	55	**9960**
56	**3**	57	**3**	58	**18**	59	**7**	60	**9**

MATHEMATICS

61	**B**	62	**B**	63	**C**	64	**C**	65	**C**
66	**D**	67	**D**	68	**A**	69	**D**	70	**A**
71	**D**	72	**B**	73	**B**	74	**B**	75	**A**
76	**A**	77	**D**	78	**A**	79	**B**	80	**A**
81	**36**	82	**1120**	83	**4**	84	**21**	85	**12**
86	**80**	87	**5264**	88	**6**	89	**1100**	90	**42**

SOLUTIONS
PHYSICS

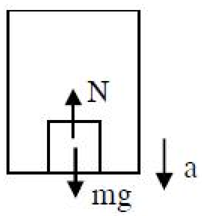
1. $P = \frac{\alpha}{\beta}\log_e\left(\frac{kt}{\beta x}\right)$

$\frac{kt}{\beta x} = 1 \Rightarrow \beta = \frac{kt}{x} = \frac{ML^2T^{-2}}{L}$

$\left(\because E = \frac{1}{2}kt\right)$

As P is dimensionless $[\alpha] = [\beta] = \left[MLT^{-2}\right]$

2.



$mg - N = ma$

$\Rightarrow N = m(g-a)$

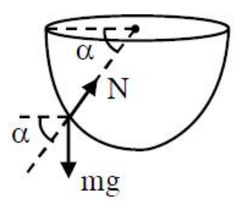
$\therefore$ Person experiences weightloss, whenacceleration of lift is downward.

3. At maximum height, V = 0

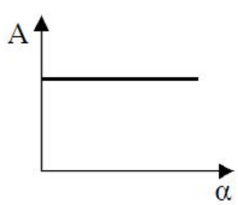
$\therefore$ Momentum of object is zero.

4. $V = \sqrt{2gR\sin\alpha}$

$N - mg\ \sin\alpha = \frac{mv^2}{R} = 2mg\sin\alpha$



$\frac{N}{2mg\sin\alpha} = \frac{1}{2} + 1 = \frac{3}{2}$



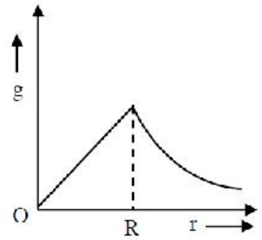
$\Rightarrow A = \text{constant}$

5. Applying conservation of angular momentum

$MR^2\omega = \left(MR^2 + 2mR^2\right)\omega'$

$\omega' = \frac{2m}{M+2m}$

6. $$g=\begin{cases}\dfrac{GMr}{R^3}, r\le R\\ \dfrac{GM}{r^2}, r\ge R\end{cases}$$

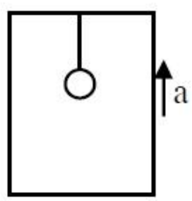


7. $$\eta=\left[1-\frac{T_L}{T_n}\right]\times 100\%$$

$$T_L=0°C=273K, T_n=373K$$

$$\therefore \eta=26.809\%$$

8. $$T=2\sqrt{\frac{\ell}{g_{eff}}}$$



$$(a)\ \text{when } a=0, T=2\pi\sqrt{\frac{\ell}{g}}$$

$$(b)\ \text{when } a=\frac{g}{6}, T'=2\pi\sqrt{\frac{\ell}{g+\frac{g}{6}}}$$

$$\therefore T'=\sqrt{\frac{6}{7}}T$$

9. $$\frac{C_P}{C_V}=1+\frac{2}{F}=1.4\Rightarrow F=5$$

By conservation of energy

$$\frac{F}{2}nR\Delta T=\frac{1}{2}[nm]v^2$$

$$\Delta T=\frac{mv^2}{FR}=\frac{Mv^2}{5R}$$

10. Charge on capacitor C_2

$$=\frac{C_2\times Q_{total}}{C_{total}}=\frac{C_2[C_1V]}{C_1+C_2}=\frac{C_1C_2V}{C_1+C_2}$$

11. **S1 :** In nonpolar molecules, centre of +ve chargecoincides with centre of –ve charge, hence netdipole moment is comes to zero.

S2 : When non polar material is placed in externalfield, centre of charges does not coincide, hencegive non zero moment in field

12. $\phi = 5t^3 + 4t^2 + 2t - 5$

$$|e| = \frac{d\phi}{dt} = 15t^2 + 8t + 2$$

At t=2, $|e| = 15 \times 2^2 + 8 \times 2 + 2$

$$\Rightarrow e = 78V \Rightarrow I = \frac{e}{R} = \frac{78}{5} = 15.60$$

13. $R = \frac{\rho \ell}{A}$

$$\frac{\Delta R}{R} = \frac{\Delta \ell}{\ell} - \frac{\Delta A}{A}$$

$\ell A = k$

$$\frac{\Delta \ell}{\ell} + \frac{\Delta A}{A} = 0$$

$$\frac{\Delta R}{R} = \frac{2\Delta \ell}{\ell}$$

$$\frac{\Delta R}{R} = 2 \times 0.4 = 0.8\%$$

14. $\frac{R_\alpha}{R_P} = \frac{M_\alpha}{M_P} \times \frac{q_P}{q_\alpha}$

$$\frac{R_\alpha}{R_P} = \frac{4}{1} \times \frac{1}{2} = 2$$

15. $\vec{E} = 301.6\sin(kz - \omega t)(-\hat{a}_x) + 452.4\sin(kz - \omega t)\hat{a}_y$

$$\vec{B} = \frac{301.6}{C}\sin(kz - \omega t)(-\hat{a}_y) + \frac{452.4}{C}\sin(kz - \omega t)(-\hat{a}_x)$$

$$\vec{H} = \frac{\vec{B}}{\mu_0} = \frac{301.6}{\mu C}\sin(kz - \omega t)(-\hat{a}_y) + \frac{452.4}{\mu C}\sin(kz - \omega t)(-\hat{a}_x)$$

$$\vec{H} = -0.8\sin(kz - \omega t)\hat{a}_y - 1.2\sin(kz - \omega t)\hat{a}_x$$

For direction $\vec{E} \times \vec{B}$ is direction of $\vec{C}$

For first part $\hat{E} = -\hat{i}, \hat{B} = ?$

$\hat{E} \times \hat{B} = \hat{k} \quad \Rightarrow \hat{B} = -\hat{j}$

Similarly for second $\hat{E} = \hat{j}, \hat{B} = ?$

$\hat{E} \times \hat{B} = \hat{k} \Rightarrow \hat{B} = -\hat{i}$

16. $\frac{a}{\lambda} = \frac{1}{100}$

For reflection size of obstacle must be much largerthan wavelength, for diffraction size should be order of wavelength.

Since the object is of size $\frac{\lambda}{100}$, much smaller thanwavelength, so scattering will occur.

17. $\lambda_e = \lambda_{photon}$

$$\frac{h}{mv} = \frac{h}{P_{photon}} \Rightarrow P_{photon} = mv$$

$$\frac{E_e}{E_{ph}} = \frac{\frac{1}{2}mv^2}{\frac{hc}{\lambda}} = \frac{1}{2}\frac{mv}{P_{ph}C} \times v = \frac{v}{2C}$$

18. $^{238}_{92}U \rightarrow \boxed{8}\,^4_2He + \boxed{6}\,^0_{-1}e + \,^{206}_{82}Pb$

8α particle and 6β particle are emitted.

19. $R = \frac{\Delta V}{\Delta i}$

$$\frac{R_1}{R_2} = \frac{\Delta v_1}{\Delta v_2}\frac{\Delta i_2}{\Delta i_1} = \frac{0.1}{0.2} \times \frac{50}{5} = 5$$

20. In amplitude modulation the amplitude of high frequency carrier wave is varied in accordance with message signal.

21. Both should have same horizontal component of velocity

$200 = 400\cos\theta$

$\theta = 60°$

22. $v^2 = u^2 + 2as$

$100 = 0 + 2(10)s$

$S = 5m$

$\text{Height from ground} = 10 - 5 = 5m$

23. $y = \frac{\text{stress}}{\text{strain}} = 2.0 \times 10^{10}$

$$\text{Energy density} = \frac{1}{2}\text{stress} \times \text{strain}$$

$$= \frac{1}{2}(\text{strain})^2\, y = \frac{1}{2}\left(5 \times 10^{-4}\right)^2 \times 20 \times 10^{10}$$

$$= 25 \times 10^2 \times 10 = 25\frac{kJ}{m^3}$$

24. $\Delta\ell \propto g$

$$\frac{\Delta\ell_{earth}}{\Delta\ell_{planet}} = \frac{g_{earth}}{g_{planet}} = \frac{10^{-4}}{6 \times 10^{-5}}$$

$g_{planet} = 6\ m/s^2$

25. $$< \varepsilon >= \frac{\int \varepsilon dt}{\int dt} = \frac{\int (L di/dt) dt}{\int dt} = \frac{L\int di}{\int dt}$$

$$< \varepsilon > = \frac{L\Delta i}{\Delta i}$$

$$i_0 = \frac{V}{R} = \frac{20}{10} = 2A,\ \text{if } i = 0A$$

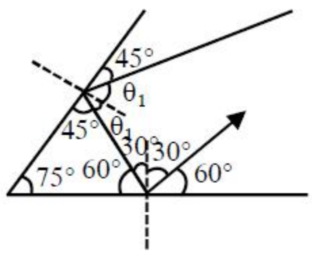
$$T = 100\ \mu s, L = 20\ mH$$

$$< \varepsilon > = \frac{20 \times 10^{-3} \times (2-0)}{100 \times 10^{-6}} = \frac{2 \times 10^3}{5}$$

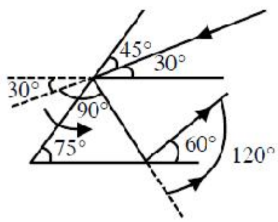
$$< \varepsilon > = 400V$$

26. $\delta_{total} = 360° - 2\theta = 360° - 2 \times 75°$

$\delta_{total} = 210°$



$\theta_1 = 45°$



$\delta = 120° + 90° = 210°$

27. $20\ MSD = 1cm$

$$1MSD = \frac{1}{20}cm$$

$$10VSD = 9MSD$$

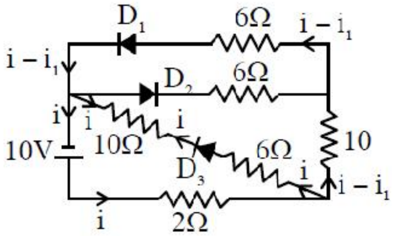
$$1VS = \frac{9}{10}MSD \qquad = \frac{9}{10} \times \frac{1}{20}cm$$

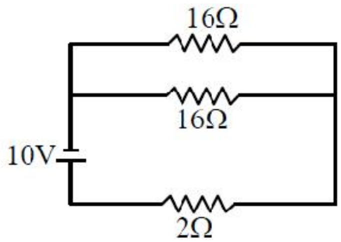
$$1VSD = \frac{9}{200}cm$$

$$VC = 1MSD - 1VSD = \frac{1}{20}cm - \frac{9}{200}cm = \frac{1}{200} \times 10mm$$

$$VC = 5 \times 10^{-2}mm = 5$$

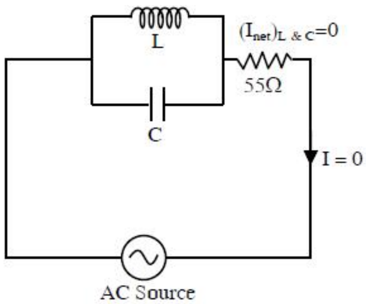
28.





$$V = IR_{net}$$
$$10 = I \times 10$$
$$I = 1A$$

29. At resonance $I_L = I_C$



Alternatively, $\frac{1}{Z} = \sqrt{\left(\frac{1}{X_L} - \frac{1}{X_C}\right)^2}$

At resoancne, $X_L = X_C$ and $Z \to \infty$

$\therefore Z_{\text{total circuit}} \to \infty$ i.e, $I = 0$

30. From continuity equation

$$av_1 = \frac{a}{2}v_2$$
$$v_2 = 2v_1$$

From Bernoulli's theorem,

$$P_1 + \rho g h_1 + \frac{1}{2}\rho v_1^2 = P_2 + \rho g h_2 + \frac{1}{2}\rho v_2^2$$
$$P_1 - P_2 = \rho\left[\left(\frac{v_2^2 - v_1^2}{2}\right) + g\left(h_2 - h_1\right)\right]$$
$$4100 = 800\left[\left(\frac{4v_1^2 - v_1^2}{2}\right) + 10 \times (0 - 1)\right]$$
$$\frac{41}{8} + 10 = \frac{3v_1^2}{2}$$
$$\frac{121}{8} \times \frac{2}{3} = v_1^2$$
$$v_1 = \sqrt{\frac{I21}{4 \times 3} \times \frac{3}{3}}$$
$$v_1 = \frac{\sqrt{363}}{6} \text{ m/s}$$

X=363

CHEMISTRY

31. Let total volume = 1000 mL = 1L
total mass of solution = 1460 g

mass of HCl $= \frac{35}{100} \times 1460$

moles of HCl $= \frac{35 \times 1460}{100 \times 36.5}$

So molarity $= \frac{35 \times 1460}{100 \times 36.5} = 14M$

32. Mass of liquid = 135 $- 40 = 95g$

Volume of liquid $= \frac{\text{mass}}{\text{density}} = \frac{95}{0.95} mL = 100\ mL = 0.1L$

mass of ideal gas $= 40.5 - 40g = 0.5g$

$PV = nRT$

$0.82 \times 0.1 = \left(\frac{0.5}{M}\right) \times 0.082 \times 250$

$M = 125$

33. $r = 0.529 \times \frac{n^2}{z} A^\circ$

$r_3 = 0.529 \times \frac{3^2}{1}$

$r_4 = 0.529 \times \frac{4^2}{1}$

$\frac{r_4}{r_3} = \frac{4^2}{3^2} = \frac{16}{9}$

$r_4 = \frac{16}{9} r_3$

34. Bond order $O_2^{2-} < O_2^{-} < O_2 < O_2^{+}$

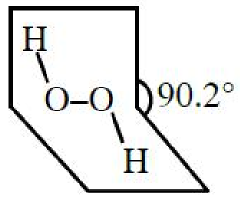
ion/molecule	Number of e^- in BMO	Number of e^- in ABMO	Bond order
O_2^+	10	5	2.5
O_2	10	6	2
O_2^-	10	7	1.5
O_2^{2-}	10	8	1

35. A cell with less variation in EMF with temperatureis preferred as reference electrode because it can beused for wider range of temperature without muchderivation from standard value so a cell with less $\left(\frac{\partial E}{\partial T}\right)_P$ is preferred.

36. Moving down the group stability of loweroxidation state increasesAl <Ga< In <Tl

37. Metal oxide with lower ΔG° is more stable
Statement II is correct

$$2HSO_4^-\ aq \xrightarrow[(2)\text{hydrolysis}]{(1)\text{Electrolysis}} 2HSO_4^- + 2H^+ + H_2O_2\ (A)$$



Solid phase.

39. MP

Be	1560 K
Mg	924 K
Ca	1124 K
Sr	1062 K

40. M.P

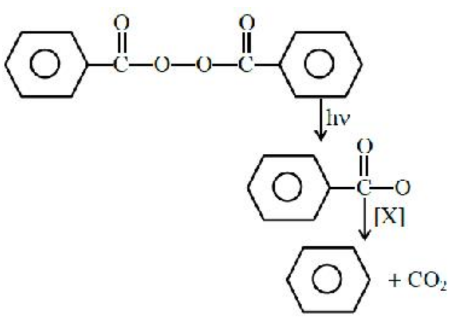
H_2O	273 K
H_2S	188 K
H_2Se	208 K
H_2Te	222 K

41. $\text{Red } P_4 + \text{Alkali} \rightarrow H_4P_2O_6\ (\text{No P} - \text{H bond})$

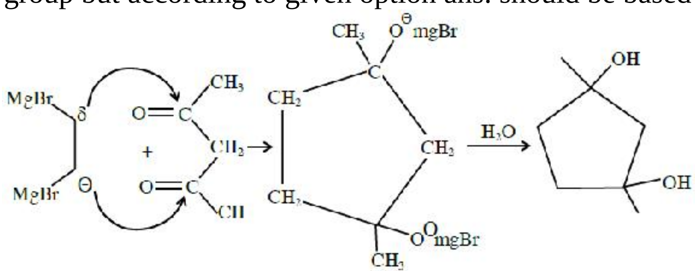
42. Polar stratospheric clouds provide surface onwhich hydrolysis of $ClONO_2$ takes place to formHOCl (Hypochlorous acid)

43. Both statement I & statement II are correct.

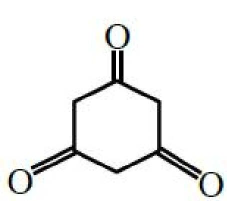
44.



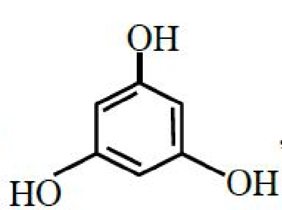
45. Although Acetyl Acetone predominantly givesAcid base reaction with G.R due to Activemethylene group but according to given option ans. should be based on nucleophilic addition reaction(NAR).



46.



is tautomer of



, Which is aromatic in nature.

47. All these enamines are interconvertible through their resonating structures. So most stable form is 'C' due to steric factor.

48. Which of the following set are correct regarding polymer.
Buna - S is copolymer of buta -1,3diene + styrene
Nylon 6.6 is condensation polymer of adipic Acid and hexamethylenediamine.
Nylon 6.6 is fiber
Terylene is fiber not thermosetting polymer
Buna-N is copolymer notHomopolymer

49. Histamine (It is use for secretion of pepsin andHCl in stomach)

50. Ring is formed due to formation of nitrosoferroussulphate

51. $\Delta U = -726$ KJ/mol

$$\Delta n_{(g)} = 1 - \frac{3}{2} = \frac{-1}{2}$$
$$\Delta H = \Delta U + \Delta n_{(g)} RT$$
$$= -726 - \frac{1}{2} \times \frac{8.3 \times 300}{1000}$$
$$= -727.245$$

52. If mass of $H_2O = 99.5$

$$m = \frac{0.5}{74.5} \times \frac{1}{0.0995}$$
$$i = \frac{0.24 \times 74.6 \times 0.0995}{0.5 \times 1.80} = 1.979$$
$$1.979 = 1 + \sim (n - 1)$$
$$1.979 = 1 + \sim$$
$$\sim = 0.979$$
$$\%\sim = 97.9\% = 98\%$$

53. Moles of $CH_3COOH = 5\text{m mole}$

moles of NaOH = 2.5m mole

NaOH	+	$CH_3COOH \longrightarrow$	$CH_3COONa + H_2O$
2.5m mole		2.5m mole	
0		2.5m mole	2.5m mole

so buffer is formed

$$pH = pKa + \log\left(\frac{\frac{2.5}{75}}{\frac{2.5}{75}}\right) = pKa$$
$$pH = 4.76 = 476 \times 10^{-2}$$

54. $k_A = \frac{\ln 2}{100}; k_B = \frac{\ln 2}{50}$

$$A_t = A_0 \times e^{-k_A t}$$
$$A_t = A_0 \times e^{\left(\frac{-\ln 2}{100} \times t\right)}$$
$$B_t = B_0 \times e^{\left(\frac{-\ln 2}{50} \times t\right)}$$
$$A_0 = B_0$$
and $A_t = 4B_t$

$$e^{-\frac{\ln 2}{100}\times t} = 4\times e^{-\frac{\ln 2}{50}\times 5}$$

$$e^{\frac{\ln 2}{100}\times t} = 4$$

$$e^{\frac{\ln 2}{100}} = 4$$

$$\frac{\ln 2}{100}\times t = \ln 4 = 2\ln 2$$

$$t = 200 \text{ sec}$$

55. Volume of $H_2 = \frac{nRT}{p} = \frac{2}{2}\times\frac{0.083\times 300}{1} = 24.92 \text{ L} = 24900 \text{ mL}$

So 1g platinum adsorb $= \frac{24900}{2.5}$ $mLH_2 = 9960$

56. Most basic oxide is V_2O_3

$$V^{+3} \rightarrow [Ar]3d^2$$

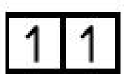
$$« = \sqrt{2(2+2)} = 2.84 \text{ BM} \approx 3$$

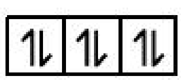
57. $CoCl_3 \cdot 4NH_3 \rightarrow \left[Co(NH_3)_4 Cl_2\right]Cl$

$$NiCl_2 \cdot 6H_2O \rightarrow \left[Ni(H_2O)_6\right]Cl_2$$

$$PtCl_4 \cdot 2HCl \rightarrow H_2\left[PtCl_6\right]$$

$$\left[Ni(H_2O)_6\right]Cl_2 \xrightarrow{2AgNO_3} 2AgCl\downarrow + \left[Ni(H_2O)_6\right](NO_3)_2$$





$$« = \sqrt{2(2+2)} \text{ B.M} = 2.84 \text{ BM} \approx 3$$

58. $C_xH_yO_z + \left(x + \frac{y}{4} - \frac{z}{2}\right)O_2 \rightarrow xCO_2 + \frac{y}{2}H_2O$

0.3g 0.2g 0.1g

$$\frac{n_{CO_2}}{n_{H_2O}} = \frac{x}{\frac{y}{2}} = \frac{\frac{0.2}{44}}{\frac{0.1}{18}} \Rightarrow \frac{2x}{y} = \frac{36}{44} = \frac{9}{11}$$

$$x = \frac{9y}{22}$$

$$\frac{n_{C_xH_yO_z}}{n_{CO_2}} = \frac{1}{x}a$$

$$\frac{0.3}{12x + y + 16z}\times\frac{44}{0.2} = \frac{1}{x}$$

$$66x = 12x + y + 16z \Rightarrow 54x = y + 16z$$

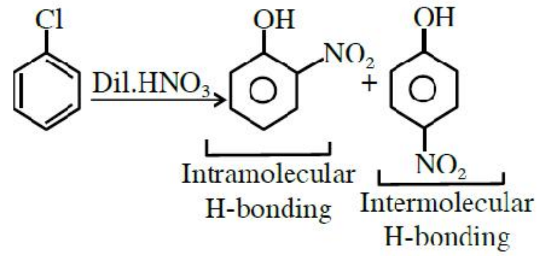
$$\frac{54\times 9y}{22}-y=16z\Rightarrow z=\frac{29y}{22}$$

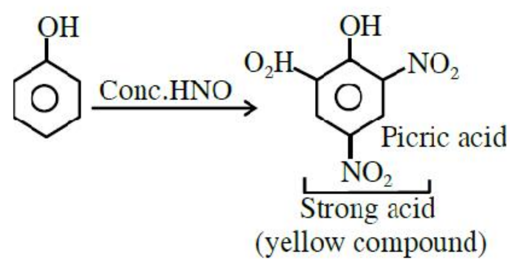
$$C_xH_yO_z=C_xH_yO_z$$

$$C_{\frac{9y}{22}}H_yO_{\frac{29y}{22}}$$

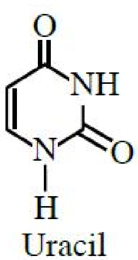
$$\% \text{ of } C=\frac{12\times 9}{(12\times 9+22+29\times 26)}\times 100=\frac{108}{594}\times 100=18.18\%$$

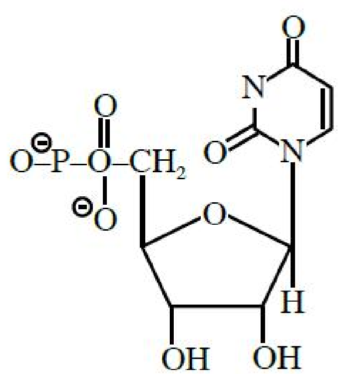
59.





60. Uracil is the base which only present is RNA.





Structure of nucleotides number of 0-9.

MATHS

61. $f(x)=\frac{x-1}{x+1}$

$$\Rightarrow f^2(x)=f(f(x))=\frac{\frac{x-1}{x+1}-1}{\frac{x-1}{x+1}+1}=-\frac{1}{x}$$

$$f^3(x)=f(f^2(x))=f\left(-\frac{1}{x}\right)=\frac{x+1}{1-x}$$

$$\Rightarrow f^4(x)=f\left(\frac{x+1}{1-x}\right)=-\frac{1}{x}$$

$$\Rightarrow f^6(x)=-\frac{1}{x}\Rightarrow f^6(6)=-\frac{1}{8}\ f^7(x)=\left(-\frac{1}{x}\right)=\frac{x+1}{1-x}$$

$$\Rightarrow f^7(7)=\frac{8}{-6}=-\frac{4}{3}$$

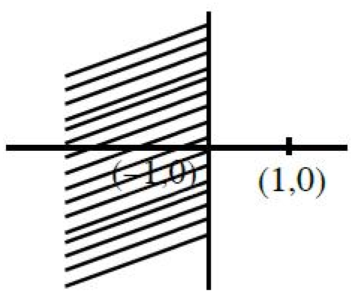
$$\therefore -\frac{1}{6}+-\frac{4}{3}=-\frac{3}{2}$$

62. Set A

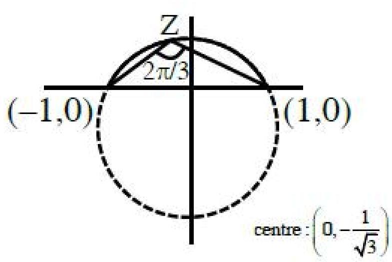
$$\Rightarrow\left|\frac{z+1}{z-1}\right|<1\Rightarrow|z+1|<|z-1|$$

$$\Rightarrow(x+1)^2+y^2<(x-1)^2+y^2$$

$$\Rightarrow x<0$$



Set B



$$\Rightarrow \arg\left(\frac{z-1}{z+1}\right)=\frac{2\pi}{3}\Rightarrow \tan^{-1}\left(\frac{y}{x-1}\right)-\tan^{-1}\left(\frac{y}{x+1}\right)=\frac{2\pi}{3}$$

$$\Rightarrow x^2+y^2+\frac{2y}{\sqrt{3}}-1=0$$

$A\cap B$

$$\Rightarrow \text{centre}\left(0,-\frac{1}{\sqrt{3}}\right)$$

63. $|\text{adj }(24A)=|\text{adj }3(\text{adj }2A)||\Rightarrow|24a|^2=(3\text{ adj}(2A))^2$

$$\Rightarrow\left(24^3|A|\right)^2=\left(3^3|\text{adj }(2A)|\right)^2=3^6\left(|2A|^2\right)^2$$

$$\Rightarrow 24^6|A|^2=\left(24^3|A|\right)^2=3^6\times2^{12}|A|^4$$

$$\Rightarrow|A|^2=\frac{24^6}{3^6\times2^{12}}=64$$

64. $\begin{vmatrix} 3 & -2 & 1 \\ 5 & -8 & 9 \\ 2 & 1 & a \end{vmatrix} = 0$

$3(-8a-9)+2(5a-18)+1(21)=0$

$\Rightarrow a=-3$

Also $\Delta_2 = \begin{vmatrix} 3 & -2 & b \\ 5 & 8 & 3 \\ 2 & 1 & -1 \end{vmatrix}^{\frac{1}{3}}$

If $b=\frac{1}{3}$

$\Delta_2 = 0$

So b must be equal to $-\frac{1}{3}$

65. $(2021)^{2023} = (7\lambda-2)^{2023}$

$= {}^{2023}C_0 (7A)^{2023} - \ldots\ldots\ldots\ {}^{2023}C_{2023}\, 2^{2023}$

$= 7t - 2^{2023}$

$\therefore -2^{2023} = -2\times 2^{2022}$

$= -2\times\left(2^3\right)^{674}$

$= -2(1+7\mu)^{674}$

$= -(7\alpha+2)$

$\Rightarrow$ remainder $=-2$ or $+5$

66. $\lim\limits_{x\to\frac{1}{\sqrt{2}}} \dfrac{\sin\left(\cos^{-1}x\right)-x}{1-\tan\left(\cos^{-1}x\right)}$

$\lim\limits_{x\to\frac{1}{\sqrt{2}}} \dfrac{\sin\left(\sin^{-1}\sqrt{1-x^2}\right)-x}{1-\tan\left(\tan^{-1}\left(\dfrac{\sqrt{1-x^2}}{x}\right)\right)}$

$\lim\limits_{x\to\frac{1}{\sqrt{2}}} \dfrac{\sqrt{1-x^2}-x}{1-\left(\dfrac{\sqrt{1-x^2}}{x}\right)}$

$\lim\limits_{x\to\frac{1}{\sqrt{2}}} (-x) = -\dfrac{1}{\sqrt{2}}$

67. $f(x)=\begin{cases} x+3 & ; \quad x<-3 \\ -(x+3) & ; \quad -3\le x<0 \\ e^x & ; \quad x\ge 0 \end{cases}$

$g(x)=\begin{cases} x^2+k_1x & ; \quad x<0 \\ 4x+k_2 & ; \quad x\ge 0 \end{cases}$

$g(f(x))=\begin{cases} f(x^2)+k_1f(x) & ; \quad f(x)<0 \\ 4f(x)+k_2 & ; \quad f(x)\ge 0 \end{cases}$

$g(f(x))=\begin{cases} (x+3)^2+k_1(x+3) & ; \quad x<-3 \\ (x+3)^2-k_1(x+3) & ; \quad -3\le x<0 \\ 4e^x+k_2 & ; \quad x\ge 0 \end{cases}$

check continuity at $x=0$

$gof(0)=g(f(0^-))=g(f(0^+))$

$4+k_2=9-3k_1=4+k_2$

$3k_1+k_2=5 \qquad \text{....(1)}$

differentiate $(g(f(x)))'=\begin{cases} 2(x+3)+k_1 & ; \quad x<-3 \\ 2(x+3)-k_1 & ; \quad -3\le x<0 \\ 4e^x & ; \quad x\ge 0 \end{cases}$

$6-k_1=4$

$k_1=2 \qquad \text{......(2)}$

$\therefore k_1=2, k_2=-1$

$gof(x)=\begin{cases} (x+3)^2+2(x+3) & ; \quad x<-3 \\ (x+3)^2-2(x+3) & ; \quad -3\le x<0 \\ 4e^x-1 & ; \quad x\ge 0 \end{cases}$

$gof(-4)+gof(4)=4e^4-2$

$\Rightarrow 2(2e^4-1)$

68. $f(x)=\begin{cases} x^2-4x-2, & \forall x\in\left(-1,\dfrac{3-\sqrt{17}}{2}\right) \\ -x^2+2x+2, & \forall x\in\left(\dfrac{3-\sqrt{17}}{2},2\right) \end{cases}$

$f'(x)$ when $x\in\left(-1,\dfrac{3-\sqrt{17}}{2}\right)$

$f'(x)=2x-4=0\Rightarrow x=2$

$f'(x) = 2(x-2) \quad \Rightarrow f'(x)$ is always $\downarrow$

$f(2) = 2$

$f(-1) = 3$

$f\left(\dfrac{3-\sqrt{7}}{2}\right) = \dfrac{\sqrt{17}-3}{2}$

$f'(x)$ when $x \in \left(\dfrac{3-\sqrt{17}}{2}, 2\right)$

$f'(x) = -2x + 2 \Rightarrow f'(x) = -2(x-1)$

$f'(x) = 0$ when $x = 1$

$f(1) = 3$

absolute minimum value $= \dfrac{\sqrt{17}-3}{2}$

absoulte maximum value $= 3$

$\text{sum} = \dfrac{\sqrt{17}-3}{2} + 3 = \dfrac{\sqrt{17}+3}{2}$

69. $\left(\dfrac{x}{a}\right)^n + \left(\dfrac{y}{b}\right)^n = 2$

Slope of tangent at (a, b)

$n \cdot \left(\dfrac{x}{a}\right)^{n-1} \cdot \dfrac{1}{a} + n\left(\dfrac{x}{b}\right)^{n-1} \cdot \dfrac{1}{b}\dfrac{dy}{dx} = 0$

$\left.\dfrac{dy}{dx}\right|_{(a,b)} = -\dfrac{b}{a}$

$\therefore$ Equation of tangent $y - b = -\dfrac{b}{a}(x-a)$

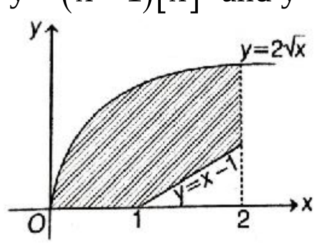
$\dfrac{x}{a} + \dfrac{y}{b} = 2 \ \forall n \in N$

70. As we know that,

$$y = (x-1)[x] = \begin{cases} 0, & 0 \le x < 1 \\ x-1, & 1 \le x < 2 \\ 2(x-1), & x = 2 \end{cases}$$

Now, on drawing the graph of given region with the help of equation of curves

$y = (x-1)[x]$ and $y = 2\sqrt{x}$

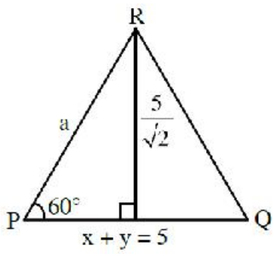


$\therefore$ Area of given region

$$= \int_0^1 2\sqrt{x}dx + \int_1^2 \left(2\sqrt{x} - x + 1\right)dx = \left[\frac{4}{3}x^{\frac{3}{2}}\right]_0^1 + \left[\frac{4}{3}x^{\frac{3}{2}} - \frac{x^2}{2} + x\right]_1^2$$

$$= \frac{4}{3} + \left[\frac{8\sqrt{2}}{3} - 2 + 2 - \frac{4}{3} + \frac{1}{2} - 1\right] = \frac{8\sqrt{2}}{3} - \frac{1}{2}\text{sq. units}$$

71.

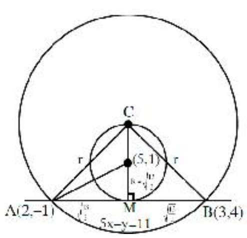


$$\sin 60° = \frac{\frac{5}{\sqrt{2}}}{a}$$

$$a = \frac{5\sqrt{2}}{3}$$

$$\text{Area of } \Delta PQR = \frac{\sqrt{3}}{4}a^2 = \frac{25}{2\sqrt{3}}$$

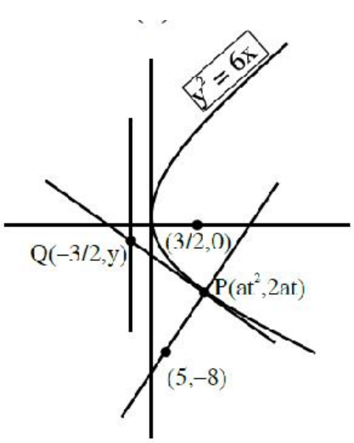
72.



$$AB = \sqrt{26}$$

$$r^2 = CM^2 + AM^2 = \left(2 \times \sqrt{\frac{13}{2}}\right)^2 + \left(\sqrt{\frac{13}{2}}\right)^2 \Rightarrow r^2 = \frac{65}{2}$$

73.



Equation of normal $y=-tx+2at+at^3\left(a=\frac{3}{2}\right)$

since passing through $(5,-8)$, we get $t=-2$

Co-oridinate of Q:$(6,-6)$

Equation of tangnet at Q: $x+2y+6=0$

Put $x=\frac{-3}{2}$ to get $R\left(\frac{-3}{2},\frac{-9}{4}\right)$

74. $l_1:\frac{x-2}{3}=\frac{y+1}{-2}=\frac{z-2}{0}$

$l_2:\frac{x-1}{1}=\frac{y+3/2}{\alpha/2}=\frac{z+5}{2}$

$l_3:\frac{x-1}{-3}=\frac{y-1/2}{-2}=\frac{z-0}{4}$

$l_1\perp l_2 \Rightarrow \frac{|3-\alpha+0|}{\sqrt{13}\sqrt{1+\frac{\alpha^2}{4}+4}}=0\Rightarrow \alpha=3$

angle between l_2 and l_3 $\cos\theta=\frac{|1\times(-3)+(-2)(\alpha/2)+2\times 4|}{\sqrt{1+4+\frac{\alpha^2}{4}}\sqrt{9+16+4}}$

$\cos\theta=\frac{|-3-\alpha+8|}{\sqrt{5+\frac{\alpha^2}{4}}\sqrt{29}}$

put $\alpha=3$

$\cos\theta=\frac{2}{\sqrt{\frac{29}{4}}\sqrt{29}}=\frac{4}{29}$

$\theta=\cos^{-1}\left(\frac{4}{29}\right)\Rightarrow \theta=\sec^{-1}\left(\frac{29}{4}\right)$

75. Let equation of rotated plane be $(2x+3y+z+20)+\lambda(x-3y+5z-8)=0$

$(2+\lambda)x+(3-3\lambda)+(1+5\lambda)z+20-8\lambda=0$

Above plane is perpendicular to $2x+3y+z+20=0$

So, $(2+\lambda)\cdot 2+(3-3\lambda)\cdot 3+(1+5\lambda)\cdot 1=0 \quad \Rightarrow \lambda=7$

$\Rightarrow$ Equation of rotated plane $x-2y+4z-4=0$

Mirror image of $A\left(2,\frac{-1}{2},2\right)$ in rotated plane is $B(a,b,c)$

Equation of AB: $\frac{x-2}{1}=\frac{y+\frac{1}{2}}{-2}=\frac{z-2}{4}=k$

Let coordinate of B be $\left(2+k,\ \frac{-1}{2},-2k,2+4k\right)$

mid point of AB is $\left(2+\frac{k}{2},\frac{-1}{2}-k,2+2k\right)$ which will lie on the plane $x-2y+4z-4=0$

Hence $k=\frac{-2}{3}$

Therefore B is $\left(\frac{4}{3},\frac{5}{6},\frac{-2}{3}\right)\equiv\left(\frac{8}{6},\frac{5}{6},\frac{-4}{6}\right)$

So, $\frac{a}{8}=\frac{b}{5}=\frac{c}{-4}$

76. $\vec{a}\times\left(\vec{b}\times\vec{c}\right)=\left(\vec{a}\cdot\vec{c}\right)\vec{b}-\left(\vec{a}\cdot\vec{b}\right)\vec{c}=3\vec{b}-\vec{c}$

$\vec{b}\times\left(\vec{c}\times\vec{a}\right)=\left(\vec{b}\cdot\vec{a}\right)\vec{c}-\left(\vec{b}\cdot\vec{c}\right)\vec{a}=\vec{c}-2\vec{a}$

$\vec{c}\times\left(\vec{b}\times\vec{a}\right)=\left(\vec{c}\cdot\vec{a}\right)\vec{b}-\left(\vec{c}\cdot\vec{b}\right)\vec{a}=3\vec{b}-2\vec{a}$

$\left[3\vec{b}-\vec{c},\vec{c}-2\vec{a},3\vec{b}-2\vec{a}\right]$

$\left(3\vec{b}-\vec{c}\right)\cdot\left[\left(\vec{c}-2\vec{a}\right)\times\left(3\vec{b}-2\vec{a}\right)\right]$

$\left(3\vec{b}-\vec{c}\right)\cdot\left[3\left(\vec{c}\times\vec{b}\right)-2\left(\vec{c}\times\vec{a}\right)-6\left(\vec{a}\times\vec{b}\right)\right]-6\left[\vec{b}\ \vec{c}\ \vec{a}\right]+6\left[\vec{c}\ \vec{a}\ \vec{b}\right]$

77. $P(H)=x,P(T)=1-x$

$P(4H,1T)=P(5H)$

${}^5C_1(x)^4(1-x)^1={}^5C_5x^5$

$5(1-x)=x$

$6x=5=0\qquad x=\frac{5}{6}$

$P(\text{atmost }2H)$

$=P(OH,5T)+P(1H,4T)+P(2H,3T)$

$={}^5C_0\left(\frac{1}{6}\right)^5+{}^5C_1\frac{5}{6}\cdot\left(\frac{1}{6}\right)^4+{}^5C_2\left(\frac{5}{6}\right)^2\left(\frac{1}{6}\right)^3$

$=\frac{1}{6^5}(1+25+250)=\frac{276}{6^5}=\frac{46}{6^4}$

78. $\sigma^2=\frac{\sum_{i=1}^{5}\left(x_i-\bar{x}\right)^2}{n}$

$\text{mean}=6$

$\frac{a+b+8+5+10}{5}=6$

$a+b=7$
$b=7-a$

$$6.8=\frac{(a-6)^2+(b-6)^2+(8-6)^2+(5-6)^2+(10-6)^2}{5}$$

$34=(a-6)^2+(7-a-6)^2+4+1+18$

$a^2-7a+12=0 \quad \Rightarrow a=4 \text{ or } a=3$

$a=4 \qquad a=3$
$b=3 \qquad b=4$

$$M=\frac{\sum_{i=1}^{5}|x_i-x|}{n}$$

$$M=\frac{|a-6|+|b-6|+|8-6|+5|5-6|+|10-6|}{5}$$

when $a=3, b=4$ when $a=4, b=3$

$M=\frac{3+2+2+1+4}{5} \qquad M=\frac{2+3+2+1+7}{5}$

$M=\frac{12}{5} \qquad M=\frac{12}{5}$

$25M=25\times\frac{12}{5}=60$

79. $f'(x)=\frac{-2}{\sqrt{1-x^2}}-\frac{4}{1+x^2}-6x-2$

$=-2\left[\frac{1}{\sqrt{1-x^2}}+\frac{2}{1+x^2}+3x+1\right]$

$f'(x)<0 \Rightarrow f(x)$ is a dec. function

$f(1)=\pi+5$

$f(-1)=5\pi+5$

$\text{Range}[a,b]\equiv[\pi+5, 5\pi+5]$

$a=\pi+5, b=5\pi+5 \Rightarrow 4a-b=11-\pi$

80. If $\Delta\equiv\nabla\equiv\wedge$

$(p\wedge q)\to((p\wedge q)\wedge r)$

it can be flase if r is false,

so not a tautology

Case-II If $\Delta\equiv\nabla\equiv\vee$

$(p\vee q)\to((p\vee q)\vee r)\equiv$ tautology

then $(p\vee q)\vee r\equiv(p\Delta r)\vee q$

Case-III if $\Delta=\vee, \nabla=\wedge$

then $(p\wedge q)\to\{(p\vee q)\wedge r\}$

Not a tautology

$\left(\text{Check } p \to T, q \to T, r \to F\right)$
Case-IV if $\Delta = \wedge, \nabla = \vee$
$(p \wedge q) \to \{(p \wedge q) \vee r\}$
Not a tautology

81. $x^4 - 3x^3 - 2x^2 + 3x + 1 = 10$
$x = 0$ is not the root of this equation so divide it by x^2
$$x^2 - 3x - 2 + \frac{3}{x} + \frac{1}{x^2} = 0$$
$$x^2 + \frac{1}{x^2} - 2 + 2 - 3\left(x - \frac{1}{x}\right) - 2 = 0$$
$$\left(x - \frac{1}{x}\right)^2 - 3\left(x - \frac{1}{x}\right) = 0$$
$$x - \frac{1}{x} = 0, \quad x - \frac{1}{x} = 3$$
$x^2 - 1 = 0 \quad x^2 - 3x - 1 = 0$
$x = \pm 1 \quad \gamma + \delta = 3$
$\alpha = 1, \beta = -1 \quad \gamma\delta = -1$
$\alpha^3 + \beta^3 + \gamma^3 + \delta^3$
$1 - 1 + (\gamma + \delta)\left((\gamma + \delta)^2 - 3\gamma\delta\right)$
$0 + 3(9 - 3(-1)) \Rightarrow 3(12) = 36$

82. $n(B) = 10$
$n(A) = 5$
The number of ways of forming a group of 3 girls of 3 boys.
$= {}^{10}C_3 \times {}^{5}C_3$
$$= \frac{10 \times 9 \times 8}{3 \times 2} \times \frac{5 \times 4}{2} = 1200$$

The number of ways when two particular boys $B_1$of B_2 be the memer of group together

$= {}^{8}C_1 \times {}^{5}C_3 = 8 \times 10 = 80$
Number of ways when boys B_1 of B_2 hot in the same group togeher
$=12000 \times 80 = 1120$

83. $x^2 + y^2 = \frac{9}{4} \qquad y = 4x$
Equation tangent in slope from
$$y = mx \pm \frac{3}{2}\sqrt{\left(1 + m^2\right)} \qquad ......(1)$$

$$y = mx + \frac{1}{m} \qquad ......(2)$$

compare $(1) \& (2)$ $\pm\frac{3}{2}\sqrt{\left(1+m^2\right)} = \frac{1}{m^2}$

$9m^2\left(1+m^2\right) = 4$

$9m^4 + 9m^2 - 4 = 0$

$9m^4 + 12m^2 - 3m^2 - 4 = 0$

$3m^2\left(3m^2 + 4\right) - \left(3m^2 + 4\right) = 0$

$m^2 = -\frac{4}{3}(\text{Rejected})$

$m^2 = \frac{1}{3} \Rightarrow m = \pm\frac{1}{\sqrt{3}}$

Equation of common tangnet $y = \frac{1}{\sqrt{3}}x + \sqrt{3}$

on x axis $y = 0$

$OQ = -3$

$b = |OQ| = 3$

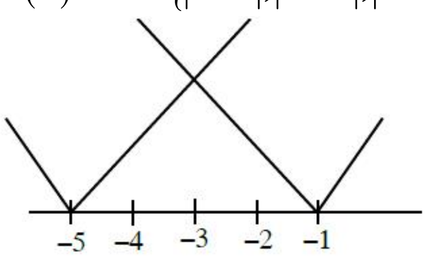
$a = 6$

$b^2 = a^2\left(1-e^2\right) \Rightarrow e^2 = 1 - \frac{9}{36} = \frac{3}{4}$

$\ell = \frac{2b^2}{a} = \frac{2\times 9}{6} = 3$

$\frac{\ell}{e^2} = \frac{3}{\frac{3}{4}} = 4$

84. $f(x) = \max\{|x+1|, |x+2|, |x+3|, |x+4|, |x+5|\}$



$\int_{-6}^{0} f(x)dx = \int_{-6}^{-3} |x+1|dx + \int_{-3}^{0} |x+5|dx$

$= -\int_{-6}^{-3} (x+1)dx + \int_{-3}^{0} (x+5)dx$

$= -\left[\frac{x^2}{2} + x\right]_{-6}^{-3} + \left[\frac{x^2}{2} + 5x\right]_{-3}^{0}$

$$= -\left[\left(\frac{9}{2}-3\right)-(18-6)\right]+\left[0-\left(\frac{9}{2}-15\right)\right]$$

$$= -\left[\frac{3}{2}-12\right]+\frac{21}{2}=\frac{21}{2}+\frac{21}{2}=21$$

85. $\left(4+x^2\right)dy-2x\left(x^2+3y+4\right)dx=0$

$$\left(x^2+4\right)\frac{dy}{dx}=2x^3+6xy+8x$$

$$\left(x^2+4\right)\frac{dy}{dx}-6xy=2x^3+8x$$

$$\frac{dy}{dx}-\frac{6x}{x^2+4}y=\frac{2x^3+3x}{x^2+y}$$

$$\text{L.I.}\frac{dy}{dx}+py=\phi$$

$$p=\frac{-6x}{x^2+4} \qquad \phi=\frac{2x^3+3x}{x^2+4}$$

$$\text{I.F.}=e^{-\int\frac{6x}{x^2+4}=e^{-3\log_e\left(x^2+4\right)}}$$

$$=e^{\log_e\left(x^2+4\right)^{-3}}=\frac{1}{\left(x^2+4\right)^3}$$

$$y\cdot\frac{1}{\left(x^2+4\right)^3}=\int\frac{2x^3+8x}{\left(x^2+4\right)^3\left(x^2+4\right)}dx$$

$$\frac{y}{\left(x^2+4\right)^3}=\int\frac{2x\left(x^2+4\right)}{\left(x^2+4\right)^3\left(x^2+4\right)}dx$$

$$x^2+4=t$$
$$2xdx=dt$$

$$\frac{y}{\left(x^2+4\right)^3}=\int\frac{dt}{t^3}$$

passes through origin$(0,0)$

$$0=\frac{-1}{2\times16}+C$$

$$\frac{y}{\left(x^2+4\right)^3}=\frac{-1}{2\left(x^2+4\right)^2}+\frac{1}{32}$$

$$y=\frac{-\left(x^2+4\right)}{2}+\frac{\left(x^2+4\right)^3}{32}$$

$$y(2)=-\frac{8}{2}+\frac{8\times8\times8}{32}=12$$

86. $\sin 10^\circ\left(\frac{1}{2}2\sin 20^\circ \sin 40^\circ\right)\cdot \sin 10^\circ \sin\left(60^\circ-10^\circ\right)\sin\left(60^\circ+10^\circ\right)$

$\sin 10^\circ \frac{1}{2}\left(\cos 20^\circ-\cos 60^\circ\right)\cdot\frac{1}{4}\sin 30^\circ$

$\frac{1}{2}\cdot\frac{1}{4}\cdot\frac{1}{2}\cdot\sin 10^\circ\left(\cos 20^\circ-\frac{1}{2}\right)$

$=\frac{1}{32}\left(2\sin 10^\circ\cos 20^\circ-\sin 10^\circ\right)$

$=\frac{1}{32}\left(\frac{1}{2}-2\sin 10^\circ\right)$

$=\frac{1}{64}\left(1-4\sin 10^\circ\right)$

$=\frac{1}{64}-\frac{1}{16}\sin 10^\circ$

Hence $\alpha=\frac{1}{64}$

$16+\alpha^{-1}=80$

87. Sum of elements in $A\cap B$

$=\underbrace{\left(2+4+6+\ldots\ldots\ldots+200\right)}_{\text{Multiple of 2}}-\underbrace{\left(6+12+6+\ldots\ldots\ldots+198\right)}_{\text{Multiple of 2 and 3 i.e.6}}$

$-\underbrace{\left(10+20+\ldots\ldots\ldots+200\right)}_{\text{Multiple of 5 \&2i.e.10}}+\underbrace{\left(30+60+\ldots\ldots\ldots+180\right)}_{\text{Multiple of 2,5\&3 i.e.30}}$

$=5264$

88. $I=\frac{48}{\pi^4}\int_0^{\pi}x^2\left(\frac{3\pi}{2}-x\right)\frac{\sin x}{1+\cos^2 x}dx \qquad \ldots.(1)$

Apply King's property

$I=\frac{48}{\pi^4}\int_0^{\pi}\left(\pi-x\right)^2\left(\frac{\pi}{2}+x\right)\frac{\sin x}{1+\cos^2 x}dx \qquad \ldots..(2)$

$(1)+(2)$

$I=\frac{12}{\pi^3}\int_0^{\pi}\frac{\sin x}{1+\cos^2 x}\left[\pi^2+\left(\pi-2\right)\cdot x\cdot\left(\pi-2x\right)\right]dx \qquad \ldots..(3)$

Apply king property again

$I=\frac{12}{\pi^3}\int_0^{\pi}\frac{\sin x}{1+\cos^2 x}\left[\pi^2+\left(\pi-2\right)\left(\pi-x\right)\left(2x-\pi\right)\right]dx\ldots..(4)$

$(3)+(4)$

$I=\frac{6}{\pi^2}\int_0^{\pi}\frac{\sin x}{1+\cos^2 x}\left[2\pi+\left(\pi-2\right)\left(\pi-2x\right)\right]dx\ldots\ldots(5)$

Applying king property

$$I=\frac{6}{\pi^2}\int_0^{\pi}\frac{\sin x}{1+\cos^2 x}\left[2\pi+(\pi-2)(2x-\pi)\right]dx......(6)$$

$(5)+(6)$

$$I=\frac{12}{\pi}\int_0^{\pi}\frac{\sin x}{1+\cos^2 x}dx$$

Let $\cos x = t \Rightarrow \sin x\, dx = -dt$

$$I=\frac{12}{\pi}\int_1^{-1}\frac{-dt}{1+t^2}=6$$

89. $$A=\sum_{i=1}^{10}\sum_{j=1}^{10}\min\{i, j\}$$

$$B=\sum_{i=1}^{10}\sum_{j=1}^{10}\max\{i, j\}$$

$$A=\sum_{j=1}^{10}\min(i,1)+\min(j,2)+.....\min(i,10)$$

$$=\underbrace{(1+1+1+.....+1)}_{19\text{ times}}+\underbrace{(2+2+2+.....+2)}_{17\text{ times}}+\underbrace{(3+3+3+.....+3)}_{15\text{ times}}+.....(1)1\text{ times}$$

$$B=\sum_{j=1}^{10}\max(i,1)+\max(j,2)+.....\max(i,10)$$

$$=\underbrace{(10+10+....+10)}_{19\text{ times}}+\underbrace{(9+9+.....+9)}_{17\text{ times}}+........+1\ 1\text{times}$$

$$A+B=20(1+2+3+......+10)$$

$$=20\times\frac{10\times 11}{2}=10\times 110=1100$$

90. $$\frac{dy}{dx}=\frac{1}{1+\sin 2x}$$

$$\int dy=\int\frac{dx}{(\sin x+\cos x)^2}$$

$$\int dy=\int\frac{\sec^2 x}{(1+\tan x)^2}$$

$$y(x)=-\frac{1}{1+\tan x}+C$$

$$y\left(\frac{\pi}{4}\right)=\frac{1}{2}=-\frac{1}{2}+C$$

$C=1$

$$y(x)=\frac{-1}{1+\tan x}+1$$

$$y(x)=\frac{-1+1+\tan x}{1+\tan x}$$

solving with $y=\sqrt{2}\sin x$

$$\frac{\tan x}{1+\tan x}=\sqrt{2}\sin x$$

$$\sin x=0, \qquad \frac{1}{\sqrt{2}}=\sin x+\cos x$$

$$x=\pi \qquad \frac{1}{2}=\sin\left(x+\frac{\pi}{4}\right)$$

$$\sin\frac{®}{6}=\sin\left(x+\frac{®}{4}\right)$$

$$x+\frac{\pi}{4}=\pi-\frac{\pi}{6},2\pi+\frac{\pi}{6}$$

$$x=\frac{5\pi}{6}-\frac{\pi}{4},x=\frac{13\pi}{6}-\frac{\pi}{4}$$

$$x=\frac{7\pi}{12},x=\frac{23\pi}{12}$$

$$\text{sum of sol.}=\pi+\frac{7\pi}{12}+\frac{23\pi}{12}$$

$$=\frac{12\pi+7\pi+23\pi}{12}=\frac{42\pi}{12}=\frac{k\pi}{12}$$

$$\Rightarrow k=42$$