PART (A) : PHYSICS

SOLUTION

1. (20.00)

Time taken by pulse to reach from P to $Q(t_0) = \frac{F_0}{k}$, where $F_0 = 3$

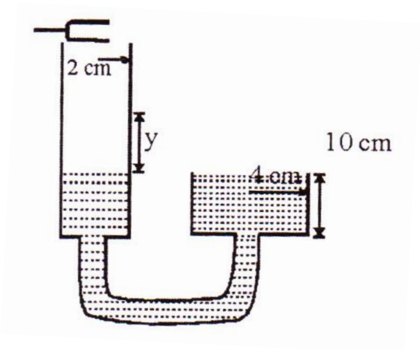
Now, $V = \sqrt{\frac{T}{m}}$ $\Rightarrow \frac{dX}{dt} = \sqrt{\frac{F_0 - kt}{m}}$

$\therefore \int_0^L dX = \frac{1}{\sqrt{m}}\int_0^{t_0}(F_0 - kt)^{1/2}dt$

$L = \frac{2}{3k\sqrt{m}}F_0^{3/2}$

$\therefore k = \frac{2}{3L}\sqrt{\frac{F_0^3}{m}} = \frac{2}{3\times1}\sqrt{\frac{3\times3\times3}{3\times10^{-2}}} = \frac{2\times3}{3\times0.1} = 20$

2. (336.00)



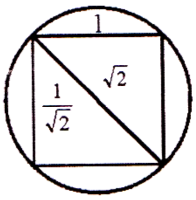
$\pi\times2^2\times y = \pi\times4^2\times10$

$y = 40cm = \lambda/2$

$\pi = 80cm$

$0.8\times420 = 336m/s$

3. (1716.00)



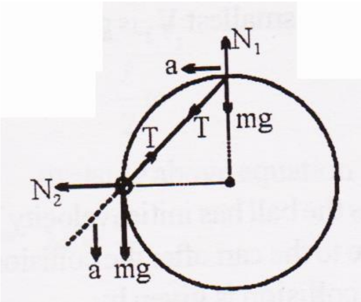
$$\frac{dQ}{dt} = \frac{k_1A_1}{L}\Delta T + \frac{k_2A_2}{L}\Delta T = \frac{80\times1\times78}{0.5} + \frac{14\times\left(\pi\times\left(\frac{1}{\sqrt{2}}\right)^2 - 1^2\right)\times78}{0.5}$$

$$=156\left[80+7(\pi-2)\right]=156\left[80+7\left(\frac{22-14}{7}\right)\right]=156\times 88$$

$$\Delta m=\frac{156\times 88\times 7\times 60}{80\times 4200}=11\times 156\times 10^{-2}\,kg$$

$$=17160\ gm$$

4. (50.00)



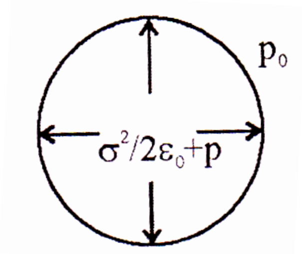
$$N_1=\frac{T}{\sqrt{2}}+mg \qquad \frac{T}{\sqrt{2}}=ma \qquad ....(ii)$$

$$\frac{T}{\sqrt{2}}=N_2 \qquad mg-\frac{T}{\sqrt{2}}=ma \qquad ....(i)$$

$$\therefore mg=\frac{T}{\sqrt{2}}+\frac{T}{\sqrt{2}}\Rightarrow mg=T\sqrt{2}\ \therefore T=\frac{mg}{\sqrt{2}}=\frac{(10^{-1})(10)}{\sqrt{2}}=\frac{1}{\sqrt{2}}N$$

$$\therefore 100T^2=100\left(\frac{1}{2}\right)=50$$

5. (96.00)



$$V=\frac{4}{3}\pi r^3 \qquad .....(1)$$

$$\sigma=\frac{Q}{4\pi r^2} \qquad ....(2)$$

$$\frac{\sigma^2}{2\epsilon_0}+p-p_0=\frac{4s}{r} \qquad ....(3)$$

$$p-p_0=0 \qquad ....(4)$$

$$\Rightarrow \frac{1}{2\varepsilon_0}\cdot\frac{Q^2}{16\pi^2 r^4}=\frac{4s}{r}$$

$$\Rightarrow \frac{1}{2\varepsilon_0}.\frac{n\pi\varepsilon_0 rs}{16\pi^2 r^4}.\frac{4}{3}\pi r^3 = 4s$$

$n = 96$

6. (1250.00)

Induced EMF $= \frac{1}{2}B\omega l^2$

At any time t

$$L\frac{di}{dt} + iR = \frac{B\omega l^2}{2}$$

Solving for i, we get

$$i = \frac{B\omega l^2}{2R}\left[1 - e^{-Rt/L}\right]$$

Torque about the hinge P is

$$\tau = \int_0^l i(dx)B.x = \frac{1}{2}iBl^2 = \frac{Bl^2}{2}\frac{B\omega l^2}{2R}\left[1 - e^{-Rt/L}\right] = \frac{B^2\omega l^4}{4R}\left[1 - e^{-Rt/L}\right]$$

Max. value occur at $t = \infty$ and half of this is equal to

$$i_1 = \frac{B\omega l^2}{4R} \text{ when } 1 - e^{-Rt/L} = \frac{1}{2}$$

$\therefore$ Torque at this instant $= \frac{B^2\omega l^4}{8R} = 1.25$

7. (40.00)

Let us denote the elastic constant(spring constant) of the rope by k and its unstretched length ℓ_0. The maximum length of the rope is $\ell_1 - h - h_0 = 23m$, whilst in equilibrium it is $\ell_2 = (23-8)m = 15m$. Initially, and at the jumper's lowest position, the kinetic energy is zero. If we ignore the mass of the rope and assume that the jumper's centre of mass is half-way up his body, we can use conservation of energy to write.

$$mgh = \frac{1}{2}k(\ell_1 - \ell_0)^2$$

In addition, in equilibrium,

$$mg = k(\ell_2 - \ell_0)$$

Dividing the two equations by each other we obtain a quadratic equation for ℓ_0.

$$\ell_0^2 + 2(h - \ell_1)\ell_0 + (\ell_1^2 - 2h\ell_2) = \ell_0^2 + 4\ell_0 - 221 = 0$$

Which gives $\ell_0 = 13m$

When the falling jumper attains his highest speed, his acceleration must be zero, and so this must occur at the same level as the final equilibrium position $(\ell = \ell_2)$

Again applying the law of conservation of energy,

$$\frac{1}{2}mv^2 + \frac{1}{2}k(\ell_2 - \ell_0)^2 = mg(\ell_2 + h_0)$$

Where the ratio m/k is the same as that obtained from the equilibrium condition, namely,

$$\frac{m}{k}=\frac{\ell_2-\ell_0}{g}$$

Substituting this into the energy equation, shows that the maximum speed of the jumper is $v=18\,ms^{-1}\approx 65\,kmh^{-1}$. It is easy to see that his maximum acceleration occurs at the lowest point of the jump. Since the largest extension of the rope(10m) is five that at the equilibrium position (2m), the greatest tension in the rope is $5mg$. So the highest net force exerted on the jumper is 4mg, and his maximum acceleration is 4g.

8. (40.00)

Both block moves downward with acceleration $g\left(=10\,m/s^2\right)$ by using string constraint pulley p will move upward with acceleration $40\,m/s^2$.

9. (A, B, D)

$\Delta U_{cycle}=0$

$Q_{net}=W_{net}$

$-40-130+400=W_{cb}+W_{ba}+W_{ac}$

$230=0+(-80J)+W_{ac}$

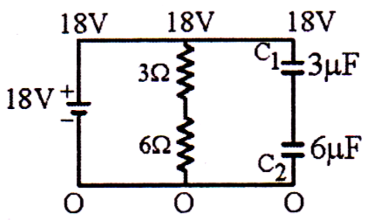
$W_{ac}=310J$

Thermal efficiency $=\dfrac{\text{Work output}}{\text{Heat input}}$

$=\dfrac{230}{400}\times 100=11.5\times 5=57.5$

10. (A, B, D)

When S is open:

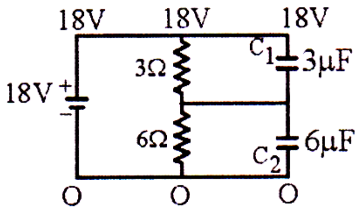


$C_1\,\&\,C_2$ are in series

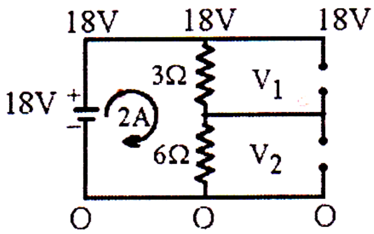
$\therefore C_{eq}=\dfrac{C_1C_2}{C_1+C_2}=2\mu F$

Charge (Q) $=18\times 2=36\mu C$

When S is closed



At steady state:



$V_1 = 2\times 3 = 6V$

$V_2 = 2\times 6 = 12V$

$Q_1 = C_1V_1 = 18\mu C \neq 36\mu C$

$Q_2 = C_2V_2 = 72\mu C \neq 36\mu C$

$\therefore$ Charges on both $C_1 \,\&\, C_2$ has changed.

11. (A, C, D)

Magnetic field is given by $\vec{B} = \dfrac{\mu_0\left(\vec{j}\times\vec{a}\right)}{2}$

12. (A, B, D)

Perimeter is decreasing at a rate of $2v$

$\therefore \dfrac{d}{dt}(2\pi r) = 2v \Rightarrow \dfrac{dr}{dt} = \dfrac{v}{\pi}$

$\therefore r = \left(r_0 - \dfrac{v}{\pi}t\right)$

$\phi = B.\pi r^2 \Rightarrow \varepsilon\left|\dfrac{-d\phi}{dt}\right| = B.2\pi.r\dfrac{dr}{dt}$

$\therefore \varepsilon = 2B\pi\left(r_0 - \dfrac{v}{\pi}t\right).\dfrac{v}{\pi} = 2Bv\left(r_0 - \dfrac{v}{\pi}t\right)$

$I = \dfrac{\phi}{R} = \dfrac{2Bvr}{\pi.2\pi r} = \dfrac{Bv}{\pi\lambda}$

13. (B, C, D)

Let maximum intensity be I_0.

$I = I_0\omega t\dfrac{\Delta\phi}{2}$

$$I_{av} = \frac{I_0}{2}$$

$$I < I_{av} \Rightarrow \cos^2\left(\frac{\Delta\phi}{2}\right) < \frac{1}{2}$$

$$\Rightarrow \cos\left(\frac{\Delta\phi}{2}\right) < \frac{1}{\sqrt{2}}$$

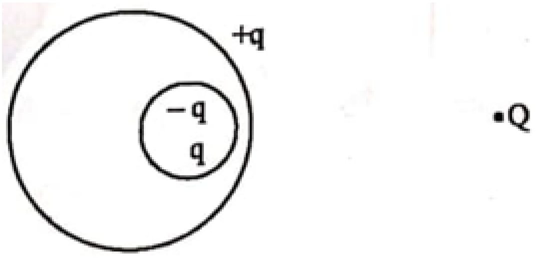
$$\Rightarrow \frac{\pi}{2} < \Delta\phi < \frac{3\pi}{2}$$

14. (B, D)

The probability of decay in 1 sec is the decay constant λ, which remains constant.
After average life 37% of nuclei remains undecayed.
Part of energy of disintegration is taken by the recoiling daughter nucleus.

15. (A)

16. (C)



(A) If outside charge is shifted to other position, distribution of +q on outer surface changes. Inside cavity charge is unaffected.
(B) If inside charges is shifted, distribution of (–q) on inner surface will changes so that net effect of (+q) & (–q) inside cavity becomes zero outside cavity.
(C) When magnitude of charge inside cavity is changed (–q) inside cavity and (+q) outside cavity also change.
(D) When conductor is earthed charge on outer surface changes so that potential of conductor becomes zero.

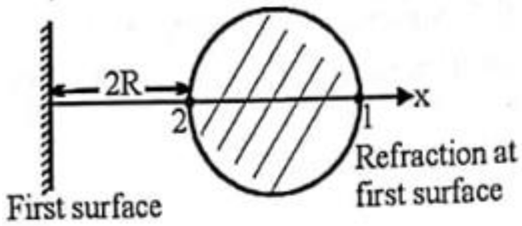
17. (D)

(A) $\frac{\phi}{i} = \frac{\vec{E}\cdot\vec{A}}{jA} = \frac{\vec{E}}{-j} = \rho$

(C) $\sigma\phi V = \frac{\phi V}{\rho} = \frac{EAV}{\rho} = \frac{V^2 A}{\rho L} = \frac{V^2}{R}$

(D) $\frac{V}{\sigma\phi} = \frac{V\rho}{\phi} = \frac{EL\rho}{EA} = R$

18. (A)



$$\frac{\mu_r}{v}-\frac{\mu_i}{u}=\frac{\mu_r-\mu_i}{R}$$

$$\frac{1}{v}-\frac{4/3}{(-2R)}=\frac{1-4/3}{(-R)}$$

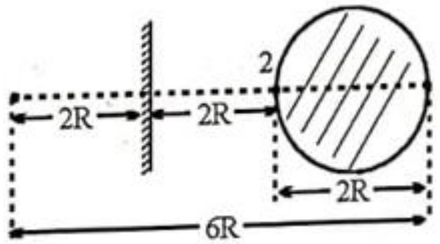
$$\frac{1}{v}+\frac{2}{3R}=\frac{1}{3R}$$

$$v=-3R \quad \text{(virtual)}$$

$$m=\frac{\mu_i v}{\mu_r u}=\frac{4/3}{(1)}\frac{(-3R)}{(-2R)}$$

$m=2$ (magnitude)

Reflection from mirror & refraction from second surface

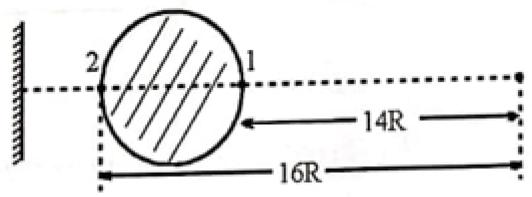


$$\frac{4/3}{v}-\frac{1}{(-4R)}=\frac{4/3-1}{R}$$

$$v=16R \qquad \text{(Real)}$$

$$m=\frac{\mu_i}{\mu_r}\frac{v}{u}=\frac{1}{-3} \Rightarrow m=-3(\text{magnitude})$$

Refraction a first surface after reflection from mirror and refraction at second surface.



$$\frac{\mu_r}{v}-\frac{\mu_i}{u}=\frac{\mu_r-\mu_i}{R}$$

$$\frac{1}{v}-\frac{4/3}{14R}=\frac{1-4/3}{-R}$$

$$v=\frac{7R}{3} \quad \text{(Real)}$$

$$m = \frac{\mu_i}{\mu_r}\frac{v}{u}$$

$$m = \frac{4}{3}\times\frac{7/3R}{+14R} = \frac{4}{3}\times\frac{7}{3\times 14} \quad \Rightarrow \quad m = \frac{2}{9}\text{(diminished)}$$

PART (B) : CHEMISTRY

SOLUTION

1. (6.1)

2. (0.625)

3. (0.667)

4. (00.32)

Number of moles of $Cu(NO_3)_2 = \frac{3.74}{187} = 0.02$

$2Cu\left(NO_3\right)_2 + 4KI \rightarrow Cu_2I_2 \downarrow + I_2 + 4KNO_3$

Number of moles of Cu_2I_2 precipitated = 0.01

Number of moles of Cu_2S precipitated = 0.01

Mass of Cu_2S precipitates $= \left(0.01 \times 158\right) g = 1.58\, g$

5. (1.71)

6. (42.46)

$$E = E^0 - \frac{0.0591}{2} \log\left\{\frac{1}{\left[H^+\right]^4\left[SO_4^{2-}\right]^2}\right\}$$

$$= 2.01 - \frac{0.0591}{2} \log\left\{\frac{1}{(20)^4(10)^2}\right\}$$

As 0.1 mole of Pb is consumed,

Number of faraday = no. of eq. of Pb consumed = 0.2 F.

$\therefore$ Charge = 0.2 × 96500 coulombs.

Energy = (0.2 × 96500) × 2.20 = 42460 J

$= 42.46\, kJ$.

7. (0.40)

As mole ratio represents pressure ration in gases, for the following equilibria, we have:

Initially pressure:	$3p$ atm (say)		$5p$ atm (say)		0 atm
	NO	+	$NO_2(g)$	$\rightleftharpoons$	$N_2O_3(g)$; Kp_1
Equilibrium pressure:	p_1 (say)		0.5 atm		p_2(say)

And, $2NO_2(g) \rightleftharpoons N_2O_4(g)$; $Kp_2 = 8$

Equilibrium pressure: 0.5 atm 2 atm

$\because \quad Kp_2 = \frac{p_{N_2O_4}}{p^2_{NO_2}} = \frac{p_{N_2O_4}}{0.5^2} = 8; \; p_{N_2O_4} = 2\text{ atm}.$

Now, out of 3*p* of NO, p_2 atm of it converts to p_2 atm of N_2O_3,

$\therefore \quad 3p - p_2 = p_1 \qquad ...(1)$

As NO_2 converts to both N_2O_3 and N_2O_4, out of 5*p* atm of NO_2, p_2 atm converted to p_2 atm of N_2O_3 and 4 atm of it gave 2 atm of N_2O_4 and 0.5 atm of NO_2 remained at equilibrium.

$\therefore \quad 5p - p_2 - 4 = 0.5 \qquad ...(2)$

Further, at equilibrium, the total pressure is given by

$$p_{NO} + p_{NO_2} + p_{N_2O_3} + p_{N_2O_4} = 5.5$$

or $\quad p_1 + 0.5 + p_2 + 2 = 5.5$

Solving equations (1), (2) and (3),

$p_1 = 2.5$ and $p_2 = 0.5$.

$$\therefore \quad Kp_1 = \frac{p_{N_2O_3}}{p_{NO} \times p_{NO_2}} = \frac{p_2}{p_1 \times 0.5} = \frac{0.5}{2.5 \times 0.5} = 0.40\text{ atm}^{-1}$$

8. (27.27)

First Method : m.e. method

m.e. of MgO + m.e. of Mg_3N_2 = m.e. of HCl reacted

= m.e. of total HCl – m.e. of NaOH

= 60 – 12 = 48.

In the dissolution of ash, HCl reacts with total Mg in Mg_3N_2 and in MgO and also with N in Mg_3N_2

$\therefore$ m.e. of total Mg + m.e. of N = 48

or m.e. of total Mg ++ m.e. of NH_3 = 48

m.e. of total Mg = 84 – 4 = 44.

Further, Mg converted to Mg_3N_2 whose N converts to NH_4Cl (or NH_3),

$\therefore$ m.e. of Mg converted to $Mg_3N_2 = 3 \times$ m.e. of NH_3

$= 3 \times (10 - 6)$

= 12.

$\therefore$ Percentage of Mg converted ot $Mg_3N_2 = \frac{12}{44} \times 100$

Second Method : Mole Method

The reactions involved are

$$\underset{x\text{ mmol(say)}}{Mg} \rightarrow \underset{x\text{ mmol}}{MgO}; \; \underset{x\text{ mmol}}{MgO} + 2HCl = MgCl_2 + H_2O$$

$$\underset{y\text{ mmol (say)}}{Mg} \rightarrow \underset{\frac{y}{3}\text{mmol}}{Mg_3N_2}; \; \underset{\frac{y}{3}\text{mmol}}{Mg_3N_2} + 8HCl = 3MgCl_2 + \underset{\frac{2y}{3}\text{mmol}}{2NH_4Cl}$$

$\therefore$ 2*x* mmol of HCl + $\frac{8y}{3}$ mmol of HCl

= total mmol of HCl – mmol of NaOH

$= 60 - 12 = 48$

$2x + \frac{8y}{3} = 48 \qquad ...(1)$

Further, mmol of NH_4Cl = mmol of NH_3 = (10 – 6)

or $\frac{2y}{3} = 4 \qquad ...(2)$.

From eqns. (1) and (2), one can calculate : $x = 16$ and $y = 6$.

$\therefore$ Percentage of Mg converted to $Mg_3N_2 = \frac{y}{x+y} = 27.27\%$

(**Note:** Mole method is more convenient to equivalent method.)

9. (A, B, C, D)

10. (C, D)

11. (A, B, D)

12. (A, B, C)

13. (A, B, C)

14. (ABD)

Pressure $\rightarrow$ y $\quad T = \frac{x \times y}{nR}$

Volume $\rightarrow$ x $\quad y = \frac{nRT}{x}$

$$\frac{y-P}{x-V} = \frac{\frac{P}{2} - P}{2V - V} = \frac{-P}{2V}$$

$$\frac{nRT}{x} - P = \frac{-P}{2V}(x - V)$$

$$\Rightarrow T = \frac{x}{nR}\left[\frac{-P}{2V}(x-V) + P\right]$$

T is max. at $x = 1.5\,V$

15. (A)

(I) rate $= \frac{k[X]}{X_s + [X]}$

Case-1: $[X] >> X_s; [X] + X_s \approx [X]$

Rate $= \frac{k[X]}{[X]} = k$ (Zero order w.r.t. X)

$I \rightarrow P, S$

Case-2: $[X] << X_S; [X] + X_S \approx X_S$

$\therefore$ rate $= \dfrac{k[X]}{X_S} = k'[X]$ (1^{st} order w.r.t. X)

$\therefore$ I $\rightarrow$ Q, T

Case-3: $[X] \approx X_S$

Rate $= \dfrac{k[X]}{X_S + [X]}$

In this case curve-R given in List-II will match.

$\therefore$ I $\rightarrow$ P, Q, R, S, T (The graph of half-life should start from origin)

(II) rate $= \dfrac{k[X]}{X_S + [X]}$

$\because$ $[X] << X_S$

$\therefore$ $X_S + [X] \approx X_S$

$\therefore$ rate $= \dfrac{k[X]}{X_S} = k'[X]$ (1^{st} order w.r.t. X)

(III) rate $= \dfrac{k[X]}{X_S + [X]}$

$\because$ $[X] >> X_s$

$\therefore$ $X_S + [X] \approx [X]$

$\therefore$ rate $= \dfrac{k[X]}{[X]} = k$ (Zero order w.r.t. X)

$\therefore$ III $\rightarrow$ P, S

(IV) rate $= \dfrac{k[X]^2}{X_S + [X]}$

$\because$ $[X] >> X_S$

$\therefore$ $X_S + [X] \approx [X]$

$\therefore$ rate $= \dfrac{k[X]^2}{[X]} = k[X]$ (1^{st} order w.r.t. X)

16. (D)

(P) $Mg(HCO_3)_2 + Ca(OH)_2 \rightarrow 2CaCO_3 \downarrow + MgCO_3 + 2H_2O$

(Q) $BaO_2 + H_2SO_4 \rightarrow BaSO_4 + H_2O_2$

(R) $Ca(OH)_2 + MgCl_2 \rightarrow Mg(OH)_2 + CaCl_2$

(S) $BaO_2 + 2HCl \rightarrow BaCl_2 + H_2O_2$

(T) $Ca(HCO_3)_2 + Ca(OH)_2 \rightarrow 2CaCO_3 \downarrow + 2H_2O$

17. (A)

(I) $\left[Cr(CN)_6\right]^{4-}$: $Cr^{2+}[Ar]3d^4\,4s^0$

It is d^2sp^3 hybridised as CN^- is a strong field ligand.

(II) $\left[RuCl_6\right]^{2-}$: $Ru^{4+} = [Kr]4d^4\,5s^0$

t_{2g} set contains four electron.

(III) $\left[Cr(H_2O)_6\right]^{2+}$: $Cr^{2+} = [Ar]3d^4\,4s^0$

It has four unpaired electrons as H_2Ok is weak field ligand.
So, its $\mu = 4.9$ B.M.

(IV) $\left[Fe(H_2O)_6\right]^{2+}$: $Fe^{2+} = [Ar]3d^6\,4s^0$

$= t_{2g}^4\,e_g^2$

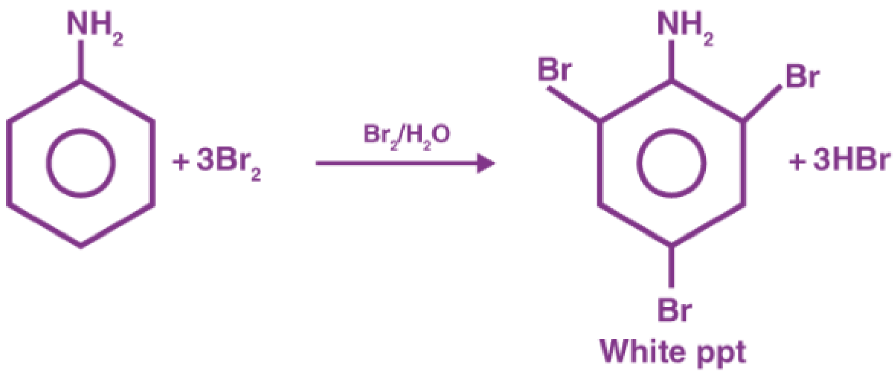
It has four unpaired electrons, its $\mu = 4.9$ B.M.

18. (D)

(I) Aniline:

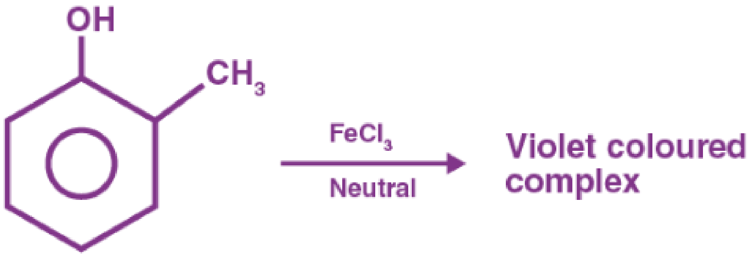
Since it contains both carbon and nitrogen so its sodium fusion extract with boiling $FeSO_4$, followed by acidification with conc. H_2SO_4 gives Prussian blue colour.

$Fe^{2+} + 6CN^- \rightarrow [Fe(CN)_6]^{4-}$

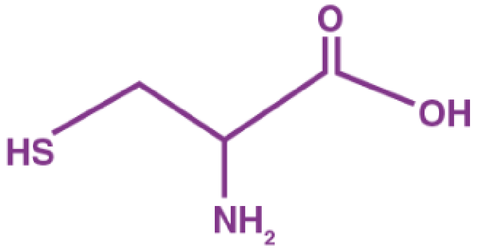


I – (P, S)

(II) o-Cresol:

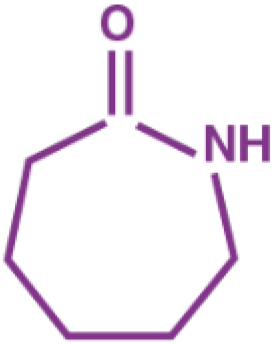


(III) Cysteine:



Since it has both, sulphur and nitrogen, so its sodium fusion extract will give blood red colour with Fe^{3+} and it has carboxylic group so it will give effervescence with $NaHCO_3$.

(IV) Caprolactam:



Its sodium fusion extract will give Prussian blue colour on boiling with $FeSO_4$ followed by acidification with conc. H_2SO_4.

PART (C) : MATHEMATICS

SOLUTIONS

1. (3)

$p = 3!\times {}^4C_4 \times 4!;\ q = {}^3C_2 \times 4! \times {}^5C_2 \times 2! \times 2!$

$r = 4! \times 3!$

$\frac{q}{3(p+r)} = \frac{10}{3}$

2. (1)

Every element has 3 choices either subset P or subset Q or neither of both.

Total case = 3^5 = 243

Total number of unordered pair $= \frac{243+1}{2} = 122$

3. (3)

$L_1 : \frac{x-2}{1} = \frac{y-9}{2} = \frac{z-13}{3} = \lambda$

$\Rightarrow\ x = 2+\lambda,\ y = 2\lambda+9,\ z = 3\lambda+13$

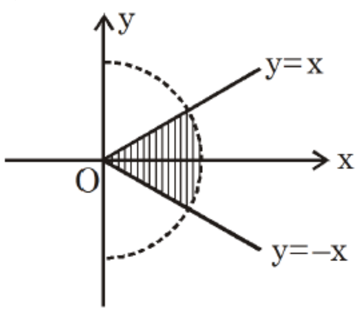
This lies on L_2

$\therefore\ \underbrace{\frac{2+\lambda-a}{1}}_{p} = \underbrace{\frac{2\lambda+9-7}{-2}}_{q} = \underbrace{\frac{3\lambda+13+2}{3}}_{r}$

by q and r : $\lambda = -3$

put in q and solve p and q, a = –3

4. (2)



$\therefore$ Area $= \frac{1}{4}$(area of circle)

$\lambda = \frac{\pi}{2}$

5. (40.00)

$\ln(x+y) = 4xy$ (At x = 0, y = 1)

$x+y=e^{4xy}$

$\Rightarrow 1+\frac{dy}{dx}=e^{4xy}\left(4x\frac{dy}{dx}+4y\right)$

At $x=0$ $\boxed{\frac{dy}{dx}=3}$

$\frac{d^2y}{dx^2}=e^{4xy}\left(4x\frac{dy}{dx}+4y\right)^2+e^{4xy}\left(4x\frac{d^2y}{dx^2}+4y\right)$

At x = 0, $\frac{d^2y}{dx^2}=e^0(4)^2+e^0(24)$

$\Rightarrow \frac{d^2y}{dx^2}=40$

6. (7.00)

At, $x=\pm\frac{3}{2},\pm 1,\pm\frac{1}{2},0$

$f(x)=\text{diff}+\text{non}-\text{diff}=\text{non}-\text{diff}$

7. (4.00)

$\underbrace{\left(4\cos^2 x+\sec^2 x\right)}_{\geq 4}+\underbrace{\left(\tan^2 x+\cot^2 x\right)}_{\geq 2}=6$

$\therefore \cos^2 x=\frac{1}{2}$ and $\tan^2 x=1$

Hence 4 solutions

8. (3.00)

$\tan^2 x+\tan^2\left(60^\circ-x\right)+\tan^2\left(60^\circ+x\right)$

$=6+9\tan^2 3x$

So required sum is $6+9\tan^2 60=33$

9. (A, B)

$\left|\frac{2^k}{\left((1+i)^2\right)^k}+\frac{\left((1+i)^2\right)^k}{2^k}\right|=\left|\frac{2^k}{(2i)^k}+\frac{(2i)^k}{2^k}\right|$

$=\left|\frac{1}{i^k}+i^k\right|=\left|\frac{i^k}{i^{2k}}+i^k\right|=i^k\left(\frac{1}{(-1)^k}+1\right)$

$=\left|i^k\left((-1)^k+1\right)\right|$

If k is odd, 0

If k is even, 2

10. (A, B, C)

If L_1 & L_2 are coplanar then

$$\begin{vmatrix} x_1-x_2 & y_1-y_2 & z_1-z_2 \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} = 0$$

$$\begin{vmatrix} 3 & -2 & 9 \\ 2 & -3 & 8 \\ 1 & -4 & 7 \end{vmatrix}$$

$$3(11)+2(6)+9(-5)$$

$$33+12-45=0$$

Angle $\cos\theta = \dfrac{2+12+56}{\sqrt{77}\sqrt{66}} = \dfrac{70}{11\sqrt{42}}$

11. (B, D)

$$\text{tr}\left((A-AB)(A^T-B^TA^T)\right)=0$$

$$\Rightarrow\ A-A=0_n$$

Since, $\text{tr}\left(CC^T\right)=0$

$$\Rightarrow\ C=0_n$$

12. (A, D)

Let p is number of factors of m which are less than equal to 6, out of these k are odd factors

$P(A \cap B) = P(A)\cdot P(B)$ (as A, B are independent)

$$\frac{k}{6}=\frac{1}{2}\cdot\frac{p}{6}\ \Rightarrow\ p=2k$$

Case-I : k = 1, p = 2, possible numbers 2, 2 × 7, 2 × 11, 2 × 13, 2 × 17, 2 × 19, 2 × 23, 2 × 29

Case-II : k = 2, p = 4, possible numbers 6, 6 × 7, 20

Case-III : k = 3, p = 6, possible number 60

13. (A, B, C)

Let ⊥ distance from H to BC, BD and CD be p, 2p, 3p

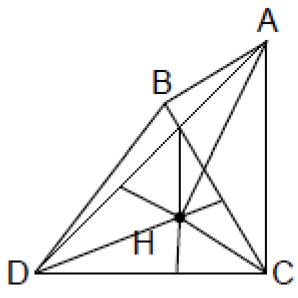
$$\frac{1}{2}\times 12(p+2p+3p)=\frac{1}{2}12^2\sin 60^\circ$$

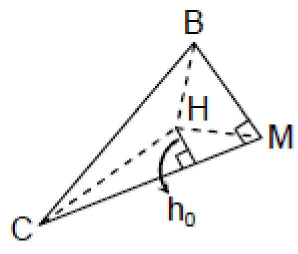
$$p=\sqrt{3}$$

$$\frac{1}{2}\times 6\times 2\sqrt{3}+\frac{1}{2}\times 12\sqrt{3}+\frac{1}{2}\times h_0 6\sqrt{3}$$

$$=\frac{1}{2}\left(\frac{1}{2}\times 12^2\sin 60^\circ\right)$$

$$\therefore\ h_0=2.$$





So ⊥ distance of A from CM $=\sqrt{2^2+3^2}=\sqrt{13}$

14. (A, C, D)

Let f(x) = 1   0 < x < a

–1   a < x < b

1   b < x < 1

As $\int_0^1 f(x)dx = 0 \Rightarrow a-(b-a)+(1-b)=0; 2a+1=2b$

$$\int_0^1 x f(x)dx=\int_0^a x\,fdx+\int_a^b x f(x)dx+\int_b^1 x f(x)dx=\frac{a^2}{2}-\left(\frac{b^2-a^2}{2}\right)+\frac{1-b^2}{2}=\frac{2a^2-2b^2+1}{2}$$

$$I=\frac{2a^2-2\left(\frac{2a+1}{2}\right)^2+1}{2}=\frac{1-4a}{4};\ I_{max}=\frac{1}{4}$$

Similarly minimum is $-\frac{1}{4}$

15. (B)

(I) $(\vec{a}.\vec{c})\vec{b}-(\vec{b}.\vec{c})\vec{a}=-5\vec{a}+4\vec{b}$

$\vec{b}.\vec{c}=5, \vec{a}.\vec{c}=4\ \&\ \vec{a}.\vec{b}=3$

$\vec{a}\times(\vec{b}\times\vec{c})=(\vec{a}.\vec{c})\vec{b}-(\vec{a}.\vec{b})\vec{c}=4\vec{b}-3\vec{c}$ -

(II) Total number of squares

$=(8\times8)+(7\times7)+(6\times6)+(5\times5)+(4\times4)+(3\times3)+(2\times2)+(1\times1)$

= 204

sum of digits $=2+0+4\Rightarrow k=6$

then k – 1 = 5

(III) Point is (4, 2, k)

8 – 8 + k = 9

k = 9

(IV) $\int_{-\pi/4}^{\pi/4}\frac{1}{\cos^2 x}dx=\int_{-\pi/4}^{\pi/4}\sec^2 x\,dx=2[\tan x]_0^{\pi/4}=2$

16. (D)

(I) $2+11\hat{i}=x^3-3xy^2+i(3x^2y-y^3)$

$x^3-3xy^2=2$

$3x^2y-y^3=11$

$\Rightarrow$ $x=2, y=1$ (by inspection)

$\Rightarrow$ $x^2+y^2=5$

(II) $\left(\cos\frac{\pi}{6}+i\sin\frac{\pi}{6}\right)^{20007}+\left(\cos\frac{\pi}{6}-i\sin\frac{\pi}{6}\right)^{20007}$

$=2\cos\left(\frac{2007\times\pi}{6}\right)=2\cos\left(\frac{669\pi}{2}\right)=0$

(III) $\frac{2\lambda+1}{\sqrt{\lambda^2+2}\sqrt{\lambda^2+2}}=\frac{1}{2}$

$\Rightarrow\ \lambda=0,4 \quad \Rightarrow \lambda=4$

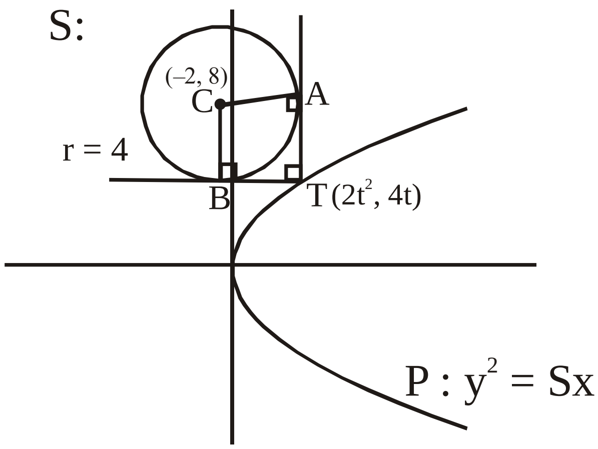
(IV) $\sqrt{5+12i}=\sqrt{(3+2i)^2}=3+2i$

$\therefore\ \lambda=3^4+2^4=97$

So, $\frac{\lambda+3}{10}=\frac{100}{10}=10$

17. (D)

For TA + TB to be minimum should be along the common normal



$\Rightarrow\ \frac{1}{t}\times\frac{4t-8}{2t^2+2}=-1$

$2t - 4 = -t(t^2 + 1)$

$t^3 + 3t - 4 = 0$

$t = 1$

$T \equiv (2, 4) \Rightarrow a + b = 6$

T lies on the director circle

$(x + 2)^2 + (y - 8)^2 = 32$

$\therefore$ TA $\perp$ TB

$\Rightarrow$ figure CATB is a square now tangent to circle is

$y-8=m(x-2)\pm4\sqrt{1+m^2}$

$\Rightarrow$ (2, 4)

$-4=4m\pm4\sqrt{1+m^2}$

$(m + 1)^2 = 1 + m^2$

$m = \infty$ or $m = 0$

$\Rightarrow$ tangent is x = 2 & y = 4

$\therefore$ A ≡ (2, 8) & B ≡ (–2, 4).]

18. (B)

$f'(x) = 2x - x f(x)$

$f'(x) + x f(x) = 2x$

solving above differential equation, we get

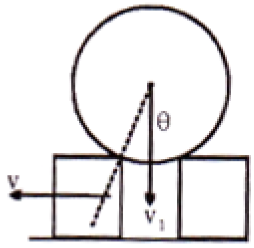
$f(x) = 2$

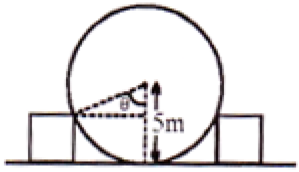
Now, verify the options.]

PART (A) : PHYSICS

SOLUTION

1. (5)





Along normal their velocity are same

$v_1 \cos\theta = v\sin\theta$

At instant of touching ground.

$\cos\theta = \frac{2.5}{5} = \frac{1}{2} \Rightarrow \theta = 60^\circ \Rightarrow \frac{v_1}{2} = \frac{v\sqrt{3}}{2}$

$w_g = \Delta k \Rightarrow Mg \times 2.5 = 25M$

$\Rightarrow 25 = \frac{v_1^2}{2} + \frac{3}{2} \times \frac{v_1^2}{3} \Rightarrow v_1 = 5m/s$

2. (4)

R.M.S. value of supply voltage, $V = 200\,\text{volts}$

Across resistance, rms voltage, $V_R = \sqrt{(200)^2 - (120)^2} = 160\,\text{Volts}$

$\therefore \text{Current}, i = \frac{160}{40} = 4\,\text{Amperes}$

3. (6)

$\frac{\lambda}{2\pi \in_0 r} = E_{break}$

$r = \frac{\lambda}{2\pi \in_0 E_{break}}$

$= \frac{10^{-3}}{2\times 3.14 \times 8.85 \times 10^{-12} \times 3 \times 10^6}$

$= \frac{1}{2\times 3.14 \times 8.85 \times 3} \times 10^3$

$= 5.99m \approx 6m$

4. (1)

$\frac{2mg}{k} = 5 \times 10^{-2}$

$\frac{1}{v} = \frac{1}{f} - \frac{1}{u}$

$\frac{1}{u} = \frac{1}{-10} - \frac{1}{-20} = \frac{-2+1}{20}$

$v = 20cm$

$d_1 = 20cm$ (initial distance of image from mirror)

$\frac{1}{v} = \frac{1}{-10} - \frac{1}{-15} = \frac{-3+2}{30}$

$v = -30cm$

$d_2 = 30cm$ (final distance of image from mirror)

$d_2 - d_1 = 10cm$ (distance in which the image oscillates)

5. (3)

As $\mu << m, M$, momentum conservation

$MV = (M+m)V'$

Gives for the velocity of the two carts after collision,

$V' = \frac{MV}{M+m}$

Consider the circular motion of the ball atop the cart M if it were stationary. If at the lowest and highest points the ball has speeds V_1 and V_2 respectively, we have

$\frac{1}{2}\mu V_1^2 = \frac{1}{2}\mu V_2^2 + 2\mu gR$,

$\frac{\mu V_2^2}{R} = T + \mu g$

Where T is the tension in the string when the ball is at the highest point. The smallest V_2 is given by $T = 0$. Hence the smallest V_1 is given by

$\frac{1}{2}\mu V_1^2 = \frac{1}{2}\mu gR + 2\mu gR$ i.e. $V_1 = \sqrt{5gR}$

With the cart moving , V_1 is the velocity of the ball relative to the cart. As the ball has initial velocity V and the cart has velocity V' after the collision, the velocity of the ball relative to the cart after the collisions is $V - V'$. Hence the smallest V for the ball to go round in a circle after the collision is given by

$V - V' = V - \frac{MV}{M+m} = \sqrt{5gR}$ i.e. $V = \frac{M+m}{m}\sqrt{5gR}$

6. (6)

$P = \frac{h}{\lambda} = \frac{6.6 \times 10^{-34}}{5 \times 10^{-10}} = 1.32 \times 10^{-24}$

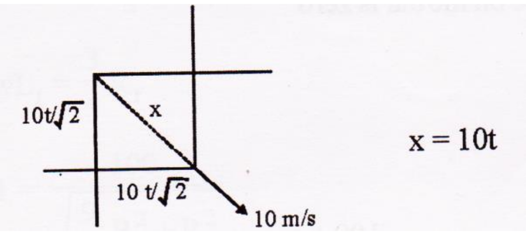
$$E=\frac{p^2}{2m}=qV$$

$$=\frac{\left(1.32\times10^{-24}1\right)^2}{2\times9\times10^{-31}\times1.6\times10^{-19}}=V$$

$$=\frac{1.32\times1.32\times10^{-48}}{18\times1.60\times10^{-50}}=\frac{132\times132}{18\times160}=\frac{121}{60}=6.05V\simeq6V$$

7. (5)

$$\phi=B\left[\frac{10t}{\sqrt{2}}\right]^2$$



$$\phi=B\left[\frac{10t}{\sqrt{2}}\right]^2$$

$$\frac{d\phi}{dt}=100Bt=100\times(.10)\times(.10)=1V$$

$$R=(.01)\times4\left(\frac{10t}{\sqrt{2}}\right)$$

$$i=\frac{1}{R}\frac{d\phi}{dt}=35.35\approx35A$$

8. (3)

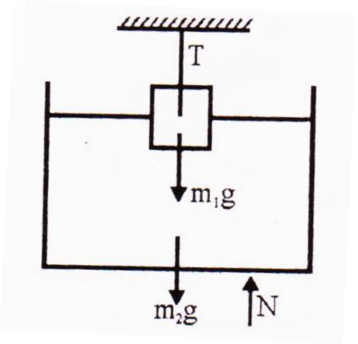
Beam is parallel to base $\Rightarrow$ mm deviation

$$\mu=\frac{\sin\left(\frac{\delta+y}{2}\right)}{\sin\left(\frac{\delta}{2}\right)}\Rightarrow\sqrt{3}=\frac{\sin\left(\frac{60+\gamma}{2}\right)}{\sin\left(\frac{60}{2}\right)}$$

$$\sin\left(\frac{60+\gamma}{2}\right)=\frac{\sqrt{3}}{2}\Rightarrow\frac{60+\gamma}{2}=60$$

$$\gamma=60^\circ$$

9. (A, C, D)



$N = (m_1g + m_2g) - T$

if $T = 0 \Rightarrow N = (m_1g + m_2g)$

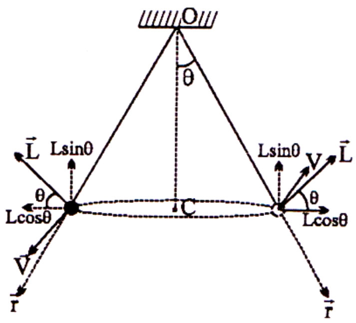
if $T > 0 \Rightarrow N < m_1g + m_2g$ and T cannot be negative.

10. (A, C, D)

Orbital speed $(v_0) = \sqrt{\frac{GM}{R}}$

Escapes speed $(v_e) = \sqrt{\frac{2GM}{R}}$

11. (A, B, C, D)



$\tau_{av} \times \Delta t = L_2 - L_1 = \Delta L$

$\vec{L}_1 = -L\cos\theta\hat{i} + L\sin\theta\hat{j}$, $\quad \vec{L}_2 = L\cos\theta\hat{i} + L\sin\theta\hat{j}$

$\vec{L}_2 - \vec{L}_1 = (2L\cos\theta\hat{i})$

Magnitude is same but direction of angular momentum is continuously changing about point of suspension.

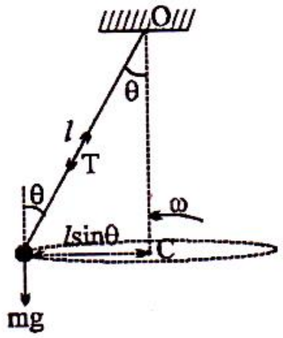
Magnitude as well as direction of angular momentum remains constants with respect to centre. So, option (B) is correct.

(C) $T\cos\theta = mg$ …..(1)

$T\sin\theta = m\omega^2 l\sin\theta$ …….(2)

$m\omega^2 l\cos\theta = mg$

$$\omega^2 = \frac{g}{l\cos\theta}, \omega = \sqrt{\frac{g}{l\cos\theta}}$$



$$V = \omega(l\sin\theta) = \sqrt{\frac{gl}{\cos\theta}}\sin\theta$$

$$F_{av} \times \Delta t = \Delta P = P_2 - P_1$$

$$mV = m\sin\theta\sqrt{\frac{gl}{\cos\theta}}$$

$$F_{av} = \frac{P_2 - P_1}{\Delta t} = \frac{2m\sin\theta\sqrt{\frac{gl}{\cos\theta}}}{\pi/\omega}$$

$$F_{av} = \frac{2m\sin\theta}{\pi}\sqrt{\frac{gl}{\cos\theta}} \times \sqrt{\frac{g}{l\cos\theta}} \Rightarrow F_{av} = \frac{2mg\tan\theta}{\pi}$$

L will be constant about line OC as shown in diagram

12. (A, D)

Final momentum in y-axis is zero.

So $MV_1 = \left(\frac{5}{12}V_0\sin\theta\right)3M$

$$V_1 = \frac{3}{4}V_0$$

Along x-axis.

Initial momentum =final momentum $\left(\frac{5}{12}V_0\cos\theta\right)3M = MV_1'$

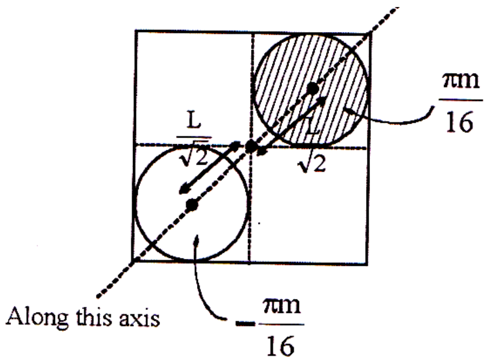
$$V_1' = V_0$$

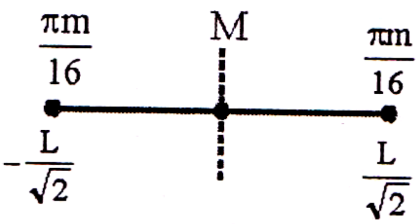
13. (B, C, D)

Intensity increases no. of photon coming on plates therefore it wouldn't change stopping potential work function maximum K.E.

14. (A, B, C, D)
Note that the quantities specific heat and latent heat both contain a term 'energy per unit mass'. However, energy itself contains mass and hence the dimensions of both these quantities do not contain mass.

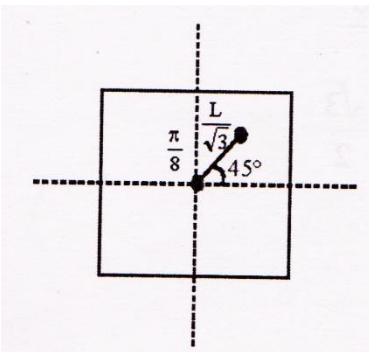
15. (D)





$$X_{Cr} = \frac{\left(\frac{\pi m}{16}\right)\frac{L}{\sqrt{2}} - \frac{\pi m}{16}\left(-\frac{L}{\sqrt{2}}\right) + m}{m}$$

$$\Rightarrow \frac{\pi}{8}\left(\frac{L}{\sqrt{2}}\right) \text{ from origin}$$



$$\vec{r} = \frac{\pi}{8}\left(\frac{L}{\sqrt{2}}\right)\left(\cos 45\hat{i} + \sin 45\hat{j}\right) = \frac{\pi L}{16}\left(\hat{i} + \hat{j}\right)$$

$$= +\frac{mL}{M} = +\frac{\sigma\pi L^2}{4\sigma L^2}.L = \frac{\pi}{4}.L$$

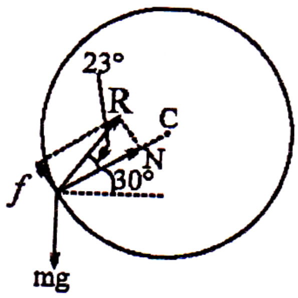
16. (B)
R is resultant of friction and normal reaction

$$R\sin 53^{\circ} = mg$$

$R\cos 53^\circ = ma$

$\Rightarrow g\left(\frac{\cos 53^\circ}{\sin 53^\circ}\right) = a$

$\Rightarrow a = g \times \cot 53^\circ = 3g/4$



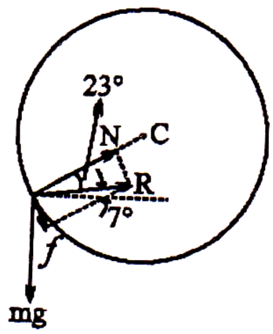
Other possibility

R is resultant of friction and normal reaction

$R\sin 7^\circ = mg$

$R\cos 7^\circ = ma$

$\Rightarrow g\left(\frac{\cos 7^\circ}{\sin 7^\circ}\right) = a$



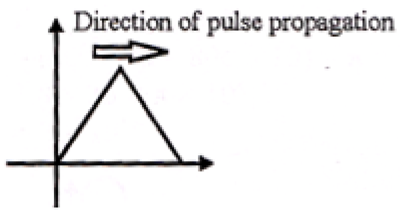
$\Rightarrow a = g \times \cot 7^\circ$ (This option is not given)

17. (B)

From the figure, it is clear that the level of water in the right pan remains the same thus pressure (& force) at the bottom of right vessel is constant but that on the left pan increases.

Thus, water overflows & the left side of the pan tips down.

18. (C)



Velocity of wave, $v = \sqrt{\dfrac{T}{\mu}}$

Velocity (v_p) of particles are in y direction

$$v_p = \frac{dy}{dt} = \frac{dy}{(dx/v)} - = v\frac{dy}{dx} = mv$$

total kinetic energy, $k = \dfrac{1}{2}m_T v^2 = \dfrac{1}{2}\mu \times a v_P^2 = \dfrac{1}{2}\mu a m^2 v^2$

$$= \frac{m^2 Ta}{2\mu}$$

$\therefore$ total energy = 2 K.E.

i.e. $= \dfrac{m^2 Ta}{\mu}$

PART (B) : CHEMISTRY

ANSWER KEY

1. (8)
2. (3)
3. (5)
4. (4)
5. (3)
6. (0)
7. (9)
8. (2)
9. (A, B, D)
10. (A, C)
11. (A, B, C, D)
12. (A, B, C)
13. (A, B, D)
14. (C, D)
15. (C)
16. (D)
17. (A)
18. (A)

PART (C) : MATHEMATICS

SOLUTIONS

1. (7)

E_1 : There is a gust of wind.

E_2 : There is not gust of wind.

A : archer misses the target

$$P(A)=P(E_1).P(A/E_1)+P(E_2).P(A/E_2)$$

$$=\frac{3}{10}\cdot\frac{3}{5}+\frac{7}{10}\cdot\frac{3}{10}=\frac{39}{100}$$

Now, required probability

$$=P(E_2/A)=\frac{P(E_2\cap A)}{P(A)}$$

$$=\frac{P(E_2)\cdot P(A/E_2)}{P(A)}=\frac{\frac{7}{10}\cdot\frac{3}{10}}{\frac{39}{100}}=\frac{7}{13}$$

2. (3)

Let $z_1=\cos A+i\sin A$

$z_2=\cos B+i\sin B$

$z_3=\cos C+i\sin C$

$\Rightarrow\ z_1+z_2+z_3=0$

$$\frac{z_1^2}{z_2z_3}+\frac{z_2^2}{z_1z_3}+\frac{z_3^2}{z_1z_2}=3$$

$$e^{i(2A-B-C)}+e^{i(2B-A-C)}+e^{i(2C-A-B)}=3$$

$\Rightarrow\ \sum\cos(2A-B-C)=3$

3. (1)

$$\cos^{-1}\beta_1+\cos^{-1}\beta_2+.....+\cos^{-1}\beta_k=k\frac{\pi}{2}$$

Since, $0\le\beta_r\le1\Rightarrow\beta_1=\beta_2=\beta_3=.....=\beta_k=0$

$$\therefore\ A=\sum_{r=1}^{k}(\beta_r)^r=0$$

$$\therefore\ \lim_{x\to0}\frac{2\left[(1+x^2)^{1/3}-(1-2x)^{1/4}\right]}{x+x^2}=1$$

4. (6)

Since, p, q, r are cube roots of unity

$\Rightarrow\ p = 1, q = \omega, r = \omega^2$

$$z = \begin{vmatrix} 1 & p & p^2 \\ 1 & q & q^2 \\ 1 & r & r^2 \end{vmatrix} = (p-q)(q-r)(r-p)$$

$\Rightarrow\ z = (1-\omega)(\omega-\omega^2)(\omega^2-1)$

$\Rightarrow\ z = -3\omega + 3\omega^2$

$\Rightarrow\ z + 3 = -3\omega + 3\omega^2 + 3 = -6\omega$

$\Rightarrow\ |z+3| = |-6\omega| = 6$

5. (4)

$N = 2^3.3^4.5^2$

$a = (3+1).(4+1).(2+1) = 60$

$b = 3.(4+1).2 = 30$

c = Total divisor – divisor divisible by 15

= (3 + 1) (4 + 1) (2 + 1) – (3 + 1).4.2 = 28

d = 3.(4 + 1).(2 + 1) = 45

6. (9)

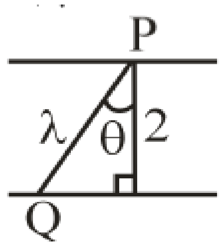
Line L is the shortest distance line of given lines.

$$\text{S.D.} = \frac{\begin{vmatrix} +4 & -13 & +27 \\ 4 & -3 & -2 \\ 2 & 3 & -10 \end{vmatrix}}{|\vec{b}_1 \times \vec{b}_2|} = \left|\frac{4(36)+13(-36)+27(18)}{\sqrt{36^2+(-36)^2+18^2}}\right| = 3$$

7. (7)

Distance between planes

$$= \left|\frac{12-(-2)}{\sqrt{9+4+36}}\right| = \frac{14}{7} = 2$$



$$\cos\theta = \frac{(1,2,2).(3,-2,6)}{3\times 7} = \frac{3-4+12}{21}$$

$$\cos\theta = \frac{11}{21} = \frac{2}{\lambda}$$

$$\lambda = \frac{42}{11}$$

$$[2\lambda] = \left[\frac{84}{11}\right] = 7$$

8. (4)

$$\frac{2+3z+4z^2}{2-3z+4z^2} = \frac{2+3\bar{z}+4\bar{z}^2}{2-3\bar{z}+4\bar{z}^2}$$

$$1+\frac{6z}{2-3z+4z^2} = 1+\frac{6\bar{z}}{2-3\bar{z}+4\bar{z}^2}$$

$$2z-3z\bar{z}+4z\bar{z}^2 = 2\bar{z}-3z\bar{z}+4z^2\bar{z}$$

$$2(z-\bar{z})+4z\bar{z}(\bar{z}-z)=0$$

$$(z-\bar{z})(2-4z\bar{z})=0$$

$$z\bar{z}=\frac{1}{2} \text{ and } 8|z|^2=4$$

9. (C, D)

$$\because \begin{vmatrix} 2 & 3 & -1 \\ 1 & 2 & 3 \\ -1 & 2 & -2 \end{vmatrix} = 2(-4-6)-3(-2+3)-1.(2+2)$$

$$= -20-3-4 = -27 \neq 0$$

$\therefore$ Lines are neither parallel nor intersecting i.e. skew lines.

$$\vec{n}_1\times\vec{n}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 3 \\ -1 & 2 & -2 \end{vmatrix} = -10\hat{i}-\hat{j}+4\hat{k} = \vec{n} \text{ (say)}$$

Then, $\text{S.D.} = \left|\text{projection of } (2\hat{i}+3\hat{j}-\hat{k}) \text{ on n}\right|$

$$= \left|\frac{(2\hat{i}+3\hat{j}-\hat{k}).\vec{n}}{|\vec{n}|}\right| = \frac{27}{\sqrt{117}}$$

10. (A, C)

$$\vec{a}.\vec{b} < 0$$

$$(\sin^2 x - \sin x) - \cos^2 x + 3 - 4\sin x < 0$$

$\Rightarrow \quad 2\sin^2 x - 5\sin x + 2 < 0$

$\Rightarrow \quad \underbrace{(\sin x - 2)}_{-ve}(2\sin x - 1) < 0$

$\Rightarrow \quad \sin x > \frac{1}{2}$

$\Rightarrow \quad x \in \left(\frac{\pi}{6}, \frac{5\pi}{6}\right)$

11. (B, C, D)

Case-I :

Only two non-adjacent digits are same

$= {}^5C_2 \times 6 \times 5 \times 4 \times 3 - 4 \times 6 \times 5 \times 4 \times 3$

$= 3600 - 1440 = 2160$

Case-II :

Only two adjacent digits are same

$= 4 \times 6 \times 5 \times 4 \times 3 = 1440$

Case-III :

Only two digits are same

$= {}^5C_2 \times 6 \times 5 \times 4 \times 3 = 3600$

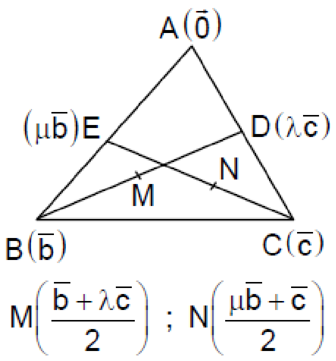
12. (A, B, C)

$\Delta = \frac{1}{2}|\bar{b} \times \bar{c}|; \quad \Delta_1 = \frac{(1-\mu\lambda)|\bar{b} \times \bar{c}|}{8}$

$\Delta_2 = \frac{1}{2}\mu\lambda|\bar{b} \times \bar{c}|$

$\Delta_3 = \frac{1}{2}(1-\mu\lambda)|\bar{b} \times \bar{c}|$

$\Delta_4 = \frac{(1-\mu\lambda)|\bar{b} \times \bar{c}|}{8}$

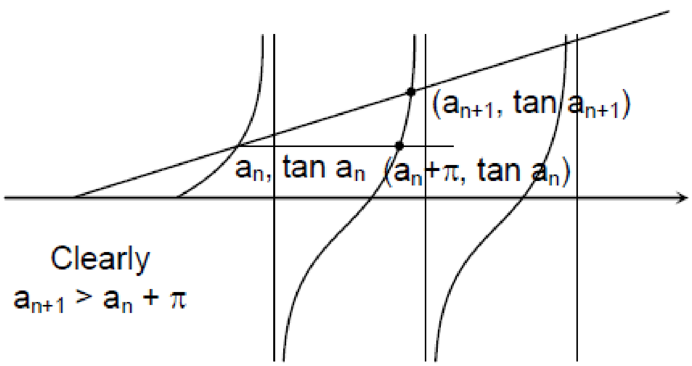


13. (A, B, D)

$\frac{dy}{dx} = \cos x\Big|_{x=a_n} = \frac{\sin a_n}{a_n + \frac{\pi}{2}}$

Taking intersection of $y = \tan x$

and $y = x + \frac{\pi}{2}$



14. (A, B, C)

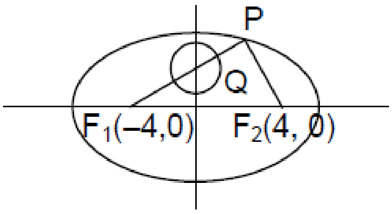
$PQ + PF_2 = 2a - QF_1$

$(PQ + PF_2)_{max} = 12 - (QF_1)_{min} = 12 - (4) = 8$

$(PQ + PF_2)_{min} = 12 - (QF_1)_{max} = 12 - 6 = 6$

For option C, PF_1 is tangent to circle

So, $QF_1 = \sqrt{24}$ so $PQ + PF_2 = 12 - \sqrt{24}$



15. (B)

For each element of set A, there are 4 possibilities i.e.,

(i) element of B but not element of C

(ii) element of C but not element of B

(iii) neither element of B nor element of C

(iv) element of B and C

Total = 4^5 and if B ∩ C is empty then ways = 3^5

favourable = $4^5 - 3^5$

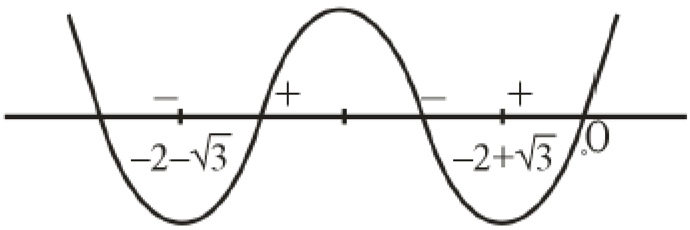
16. (C)

$-a = x^4 + 8x^3 + 18x^2 + 8x$

Let $f(x) = x^4 + 8x^3 + 18x^2 + 8x$

$f'(x) = 4x^3 + 24x^2 + 16x + 8$

$= 4(x+2)(x^2+4x+1)$



$f(-2) = 8$

$f(x) = x^2(x^2+4x+1) + 4x(x^2+4x+1) + (x^2+4x+1) - 1$

$f(2+\sqrt{3}) = f(-2+\sqrt{3}) = -1$

So, $-a \in (-1, 8)$

$a \in (-8, 1)$

17. (D)

$$\begin{bmatrix} 3 & 2 \\ 4 & 3 \end{bmatrix}^{2022} = \left(3I + 2\begin{bmatrix} 0 & 1 \\ 2 & 0 \end{bmatrix}\right)^n = 3^n I + {}^nC_1 3^{n-1} 2A + {}^nC_2 3^{n-2} 2^3 I + {}^nC_3 3^{n-3} (2A)^3 + {}^nC_4 3^{n-4} 2^6 I.....$$

So, $\begin{bmatrix} 3 & 2 \\ 4 & 3 \end{bmatrix}^n + \begin{bmatrix} 3 & -2 \\ -4 & 3 \end{bmatrix}^n = 2\left(3^n + {}^nC_2 3^{n-2}\, 2^3 + {}^nC_4 3^{n-4}\, 2^6 + {}^nC_6 3^{n-6}\, 2^9 + ...\right)I$

$\left(2\sum {}^nC_{2k}\, 3^{n-2k}\, 2^{3k}\right)I = \left(\left(3+2\sqrt{2}\right)^n + \left(3-2\sqrt{2}\right)^n\right)I$

18. (C)

If $\lim\limits_{n\to\infty} \dfrac{\left((n+1)!\right)^{\frac{1}{n+1}} - (n!)^{\frac{1}{n}}}{(n+1)-n} = \ell \Rightarrow \lim\limits_{n\to\infty} \dfrac{(n!)^{\frac{1}{n}}}{n} = \ell = \dfrac{1}{e}$

$\lim\limits_{n\to\infty} \left((n+2)!\right)^{\frac{1}{n+2}} - (n+1)^{\frac{1}{n+1}} + \left((n+1)!\right)^{\frac{1}{n+1}} - (n!)^{\frac{1}{n}} = \dfrac{2}{e}$