PART (A) : PHYSICS

ANSWER KEY

1.	(A)	2.	(D)	3.	(D)	4.	(A)	5.	(D)
6.	(D)	7.	(A)	8.	(D)	9.	(A)	10.	(D)
11.	(B)	12.	(A)	13.	(C)	14.	(D)	15.	(B)
16.	(B)	17.	(A)	18.	(D)	19.	(A)	20.	(A)
21.	(16)	22.	(150)	23.	(240)	24.	(3)	25.	(5)
26.	(4)	27.	(5)	28.	(30)	29.	(1)	30.	(20)

SOLUTIONS

1. (A)

$$r=\frac{v_2}{v_1}=\sqrt{\frac{0.8H}{H}}=\sqrt{0.8}$$

$$\vec{v}_b=150\,\hat{n}\,km/hr$$

$$\vec{v}_r=-100\,\hat{n}\,km/hr$$

$$\vec{v}_{br}=250\,\hat{n}\,km/hr$$

$$\gamma=\sqrt{0.8}=\left|\frac{\vec{v}'_{br}}{\vec{v}_{br}}\right|$$

$$\vec{v}'_{br}=\vec{v}'_{br}+\vec{v}'_r$$

$$=-223.6\,\hat{n}+(-100)\hat{n}$$

$$v'_b=-323.6\hat{n}$$

2. (D)

$$a_t=\frac{2v_2}{v_1+v_2}a_i\Rightarrow\frac{a_t}{a_i}=\frac{2\times100}{200+100}=\frac{2}{3}$$

3. (D)

$$L=\frac{nv}{4f}=25n\ \text{cm with}\ n=1,3,5,\ldots\ldots\text{i.e.}$$

$$L=25cm,75cm,125cm,\ldots\ldots\ldots..$$

Now $L_{min}=120-L_{max}=120-75=45cm$

4. (A)

$$13.6Z^2\left(\frac{1}{2^2}-\frac{1}{\infty^2}\right)=13.6\left(\frac{1}{1^2}-\frac{1}{\infty^2}\right)$$

$$\Rightarrow Z=2$$

5. (D)

Range on the inclined plane

$$\sqrt{h_{max}^2 + \left(\frac{R}{2}\right)^2} = \frac{u^2}{g}\frac{\sqrt{21}}{8}$$

6. (D)

$$F_{net} = F_b - mg = \rho Vg - mg = \rho\pi r^2 hg - mg$$

7. (A)

$$L = mvr \propto \sqrt{r}$$

$$r_A > (r_B)_{mean} > r_C$$

$$L_A > L_B > L_C$$

8. (D)

$$\lambda_m T = \text{constant}$$

9. (A)

Here $a_1 : a_2 :: 2 : 1$

$$\therefore \frac{I_{max}}{I_{min}} = \frac{(a_1 + a_2)^2}{(a_1 - a_2)^2} = \frac{9}{1}$$

10. (D)

Required power $= 16 \times 2$

$= 32MW = 32 \times 10^6 W$

Hence number of fission of Uranium nuclei per second

$$= \frac{32 \times 10^6}{200 \times 1.6 \times 10^{-13}} = 1 \times 10^{18}$$

11. (B)

235 g contains 6 × 10^{23} atoms

2000 g contains $\frac{6 \times 10^{23}}{235} \times 2000$ atoms

E = total energy released

$$= \frac{6 \times 10^{23}}{235} \times 2000 \times 185 \text{ MeV}$$

Power output $= \frac{E}{t_0}$

Where $t_0 = 30$ days

$= 30 \times 24 \times 60 \times 60 \sec$

12. (A)

$$\oint B \cdot d\ell = \mu_0 i_{enclosed}$$

$$i_{enclosed} = 0$$

13. (C)

Truth table for NAND gate

x	y	z
0	0	1
0	1	1
1	0	1
0	0	0

14. (D)

Initial kinetic energy.

$$K_i = E$$

De-broglie wavelength

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mK_i}} = \frac{h}{\sqrt{2mE}}$$

Final kinetic energy :

$$K_f = E - V$$

$$\lambda' = \frac{h}{p} = \frac{h}{\sqrt{2mK_f}}$$

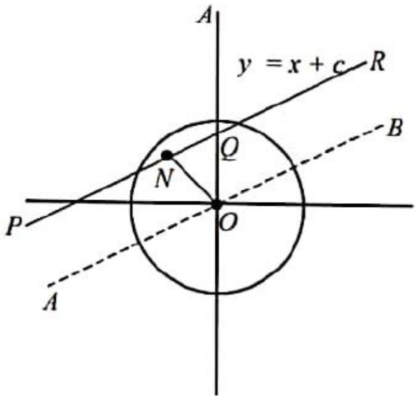
$$\lambda' = \frac{h}{\sqrt{2m(E-V)}} = \frac{h}{\sqrt{2mE}\sqrt{1-\frac{V}{E}}}$$

$$\lambda' = \frac{\lambda}{\sqrt{1-\frac{V}{E}}}$$

15. (B)

We can consider a rolling ring as a rod of length 2R rotating with angular velocity ω. Drawing the circuit

16. (B)



$$I_{PQR} = I_{AOB} + M(ON)^2$$

$$I_{PQR} = \frac{1}{4}MR^2 + M\left(\frac{C}{\sqrt{2}}\right)^2 \qquad (1)$$

But $I_{PQR} = \frac{1}{2}MR^2$ (2)

From (1) and (2),

$$C = \frac{R}{\sqrt{2}}$$

17. (A)

$$\frac{\text{Volume submerged}}{\text{Volume}} = \frac{\rho_s}{\rho_L}$$

and is independent of g_{eff}

18. (D)

$$mg\cos\theta - qE\sin\theta = \frac{mv^2}{R} \qquad (1)$$

Applying work energy theorem

$$\frac{1}{2}mv^2 = mgR(1-\cos\theta) + qER\sin\theta \qquad (2)$$

Solving (1) and (2)

$$\frac{qE}{mg} = \frac{3-2\sqrt{2}}{3}$$

19. (A)

$$F_B = W - W' = 1 \pm (0.1)$$

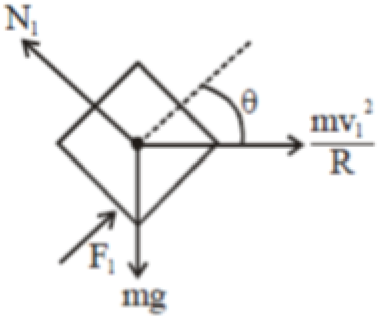
$$\frac{\rho_0 V_g}{\rho_w V_g} = \frac{5 \pm 0.05}{1 \pm 0.1}$$

$$\frac{\rho_0}{\rho_w} = \frac{5 \pm 0.05}{1 \pm 0.1} = (5) \pm (\Delta\rho)$$

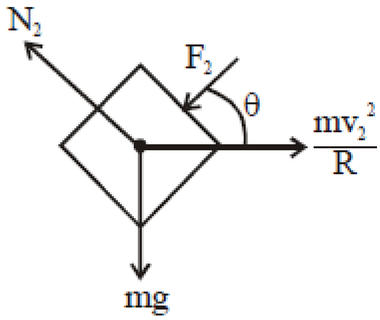
$$\frac{\Delta\rho}{5} = \frac{0.05}{5} + \frac{0.1}{1} = 0.01 + 0.1 = 0.11$$

20. (A)

In the frame of train



$$F_1 + \frac{mv_1^2}{R}\cos\theta = mg\sin\theta$$



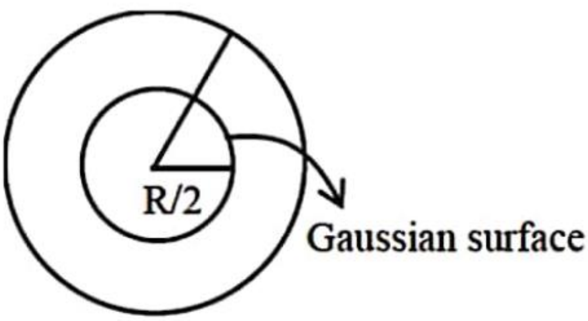
$$\frac{mv_2^2}{R}\cos\theta = mg\sin\theta + F_2$$

$F_1 = F_2$ given

$$g\sin\theta - \frac{v_1^2}{R}\cos\theta = \frac{v_2^2}{R}\cos\theta - g\sin\theta$$

$$\tan\theta = \frac{v_2^2 + v_1^2}{2gR} = \frac{100 + 400}{2 \times g \times 1000} = \frac{1}{4g}$$

21. (16)



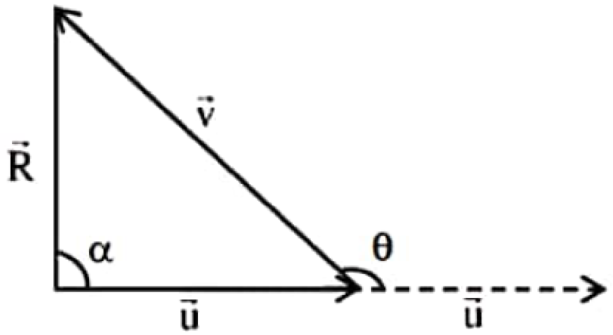
$$\oint \vec{E}.d\vec{s} = \frac{q_{in}}{\varepsilon_0}$$

$$E4\pi\left(\frac{R}{2}\right)^2 = \frac{\int_0^{R/2} \rho 4\pi r^2 dr}{\varepsilon_0}$$

$$\Rightarrow E\pi R^2 = \frac{\int_0^{R/2} Kr4\pi r^2 dr}{\varepsilon_0}$$

22. (150)

$$\tan\alpha = \frac{Q\sin\theta}{P+Q\cos\theta}\left\{\text{here } \alpha = 90^\circ\right\}$$



$$\tan 90^\circ = \infty = \frac{1}{0} = \frac{v\sin\theta}{u+v\cos\theta}$$

$$u + v\cos\theta = 0 \qquad \ldots\ldots.(i) \to \cos\theta = -\frac{u}{v}$$

$$R = \sqrt{u^2+v^2+2uv\cos\theta} \text{ here } R = \frac{v}{2} \to \text{Given}$$

$$\frac{v^2}{2} = u^2+v^2+2uv\cos\theta \frac{-3v^2}{4} = u^2 2uv \times \frac{-u}{v}$$

$$\frac{3v^2}{4} = u^2$$

$$u = 4\frac{\sqrt{3}v}{2}$$

$$\cos\theta = -\frac{u}{v} = -\frac{1}{v}\times\frac{\sqrt{3}u}{v}$$

$$\cos\theta = -\frac{\sqrt{3}}{2}$$

$$\theta = 150^\circ$$

23. (240)

Given $x^2 = v+1$

we know that $a = \frac{vdv}{dx}$

$$v = x^2 - 1$$

On diff. $a = \frac{vdv}{dx}$ & $\frac{dv}{dx} = 2x$

So $a = (x^2 - 1) \times 2x$

Hence $x = 5$

$= 24 \times 10$

$= 240\,m/s^2$

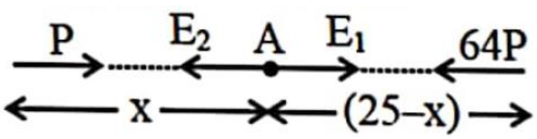
24. (3)

Current through R_1 and R_2 come out to be zero(Potential difference $= 0$)

Current through $R_3 = \frac{\text{Net emf}}{\text{Total Resistance}}$

$i = \frac{2}{3}A$

25. (5)



$E_1 = E_2$

$\frac{k2P}{x^3} = \frac{k2(64P)}{(25-x)^3}$

$\frac{1}{x} = \frac{4}{25-x}$

$x = 5cm$

26. (4)

As we know,

$T_1 = mg\cos\theta = \frac{mg}{2}$

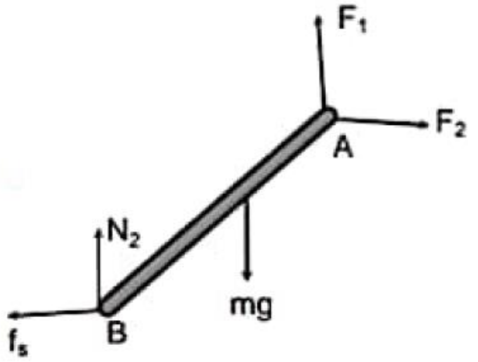
Similarly, $T_2 = mg + \frac{mv^2}{l}$

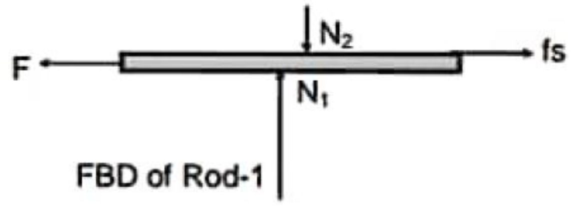
$= mg + \frac{mv}{l} \quad \left\{\sqrt{2gl(1-\cos 60^o)}\right\}^2 = 2mg$

$\therefore \quad \frac{T_2}{T_1} = \frac{2mg}{mg/2} = 4$

27. (5)

Taking torque about A $mg\frac{\ell}{2}\cos\alpha = f_s\ell\sin\alpha + N_2\ell\cos\alpha$





FBD of Rod – 2

$$\Rightarrow N_2 = \frac{mg}{2(1+\mu\tan\alpha)}$$

$$F \geq f_s \Rightarrow F_{min} = f_{s\,max} = \mu N_2 \Rightarrow F_{min} = F_2 = \frac{\mu mg}{2(1+\mu\tan\alpha)} = \frac{(0.5\times3\times10)}{2(1+0.5\times1)} = 5N$$

28. (30)

Time of flight $= \frac{2\times10}{10} = 2s$;

Range $= 2\times15 = 30m$

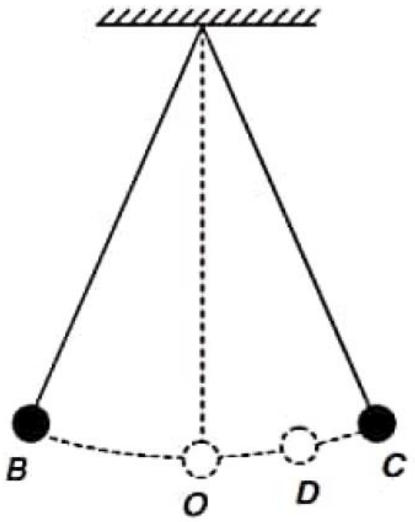
29. (1)

Given time period $T = 6s$

Amplitude $= OC = OB = \frac{1}{2}BC = \frac{10}{2}$

$= 5cm(\text{Figure})$

$\therefore OD = 2.5cm$



Let the displacement of the pendulum be given by

$x = A\sin(\omega t + \phi)$

Where $A = 5cm$

And $\omega = \frac{2\pi}{T} = \frac{2\pi}{6} = \frac{\pi}{3}\,\text{rad}\,s^{-1}$

Let us suppose that at $t = 0$, the pendulum is at C, i.e., at $t = 0$, $x = A$, so that

$A = A\sin(\omega \times 0 + \phi)$

Or $A = A\sin\phi$ or $\sin\phi = 1$ or $\phi = \frac{\pi}{2}$

Thus the motion of the pendulum is given by $\left(\text{putting}\,\phi = \frac{\pi}{2}\right)$

$x = A\sin\left(\omega t + \frac{\pi}{2}\right) = A\cos\omega t = 5\cos\omega t$

The value of t for which $x = 2.5\text{cm}$ is given by

$2.5 = 5\cos\omega t$

Or $\cos\omega t = \frac{1}{2}$ or $\omega t = \frac{\pi}{3}$

Since $\omega = \frac{\pi}{3}$, we have $t = 1$ second

30. (20)

$V_0 = \sqrt{2}V_{rms} = \sqrt{2} \times 20V$

$X_C = \frac{1}{\omega C} = \frac{1}{100 \times (100 \times 10^{-6})} = 100\Omega$

Let I_1 be the peak value of the current in the upper branch of the circuit. Then

$I_1 = \frac{V_0}{Z_1} = \frac{V_0}{(x_c^2 + R_1^2)^{1/2}}$

$= \frac{20\sqrt{2}}{\left[(100)^2 + (100)^2\right]^{1/2}}$

$= \frac{1}{5}A$

$\therefore$ Peak value of voltage across 100Ω resistor is

$V_1 = I_1R_1 = \frac{1}{5} \times 100 = 20V$

PART (B) : CHEMISTRY

ANSWER KEY

31.	(C)	32.	(B)	33.	(A)	34.	(C)	35.	(A)
36.	(B)	37.	(C)	38.	(D)	39.	(D)	40.	(C)
41.	(B)	42.	(D)	43.	(D)	44.	(A)	45.	(B)
46.	(D)	47.	(A)	48.	(A)	49.	(C)	50.	(B)
51.	(8)	52.	(3)	53.	(3)	54.	(5)	55.	(966)
56	(9)	57.	(4)	58.	(15)	59.	(3)	60.	(5)

SOLUTIONS

31. (C)

Use the energy-level diagram. $(n+\ell)$

(Aufbau's principle)

32. (B)

$K_{sp} = [Mg^{+2}][OH^-]^2$

$10^{-12} = [10^{-2}][OH^-]^2$

$[OH^-]^2 = 10^{-10}$

$[OH^-] = 10^{-5}$

$p_{OH} = 5$

$p_H = 9$

33. (A)

34. (C)

35. (A)

$G\frac{\ell}{a} = k$ ....(1)

$\Lambda_m = k \times \frac{1000}{nl(\ell \times a) \times 10^{-3}}$ ....(2)

From (1) and (2) $\Lambda_m = 2\Omega^{-1} - cm^2 - mol^{-1}$

36. (B)

Sucrose D(+) glucose +D(-) Fructose

$[\alpha] = +66.5^0 \quad [\alpha] = +52.5^0 \quad [\alpha] = -92.8^0$

Thus rotation change from positive to negative after hydrolysis. Due to this reason hydrolysis of sucrose is known as inversion and mixture after hydrolysis is known as invert sugar. Therefore option (B) is correct.

37. (C)

In graph (I) – two radial nodes are present

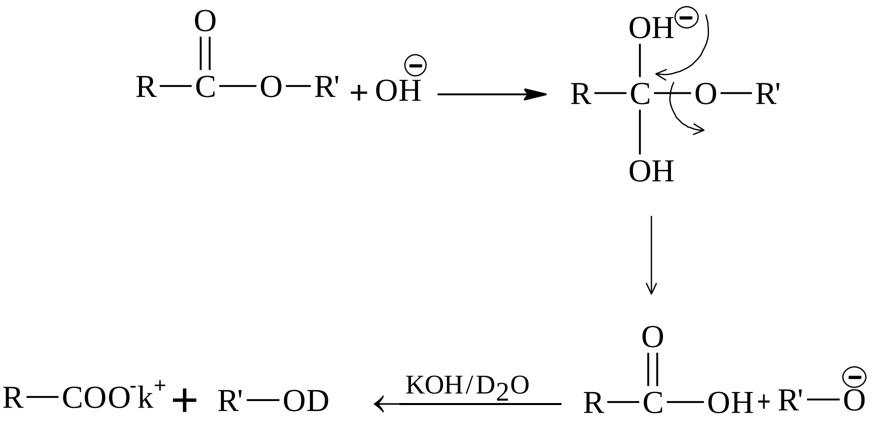
$\therefore n - l - 1 = 2 \; n = 5 \text{ for } l = 2$

In Graph (II) one radial node is present

$\therefore n - l - 1 = 2 \; n = 4 \text{ for } l = 2$

Graph (IV) does not represents any radial distribution curve

38. (D)



39. (D)

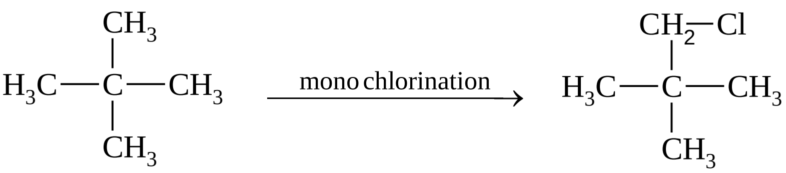
For BCC, $\sqrt{3}a = 4r$

$$r = \frac{\sqrt{3} \times 351}{4} = 152\text{pm}$$

40. (C)

At high pressure Z = $1 + \frac{Pb}{RT}$

41. (B)



Neopentane only one compound

Mol. wt = 72u

42. (D)

$$2CCl_3CHO \xrightarrow{OH^{(-)}} CCl_3COONa + CCl_3CH_2OH$$

Cannizaro reaction is a disproportionation reaction

One aldehyde molecule is oxidize to salt of the carboxylic Acid, other one is reduced to Alcohol. So the compound is CCl_3CH_2OH

IUPAC Name is 2, 2, 2 – Trichloro ethanol

43. (D)

Across a period metallic strength decreases and down the group it increases.

44. A

Vinyl group

$(CH_2 = CH-)$

On ozonolosys give formaldehyde

45. (B)

46. (D)

Stronger the conjugate acid better is the leaving ability.

47. (A)

$$CoCl_3.6NH_3 \rightarrow xCl^- \xrightarrow{AgNO_3} x\,AgCl\downarrow$$

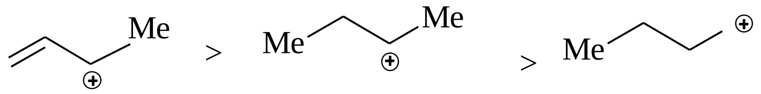
n(AgCl) = xn($CoCl_3.6NH_3$)

$$\frac{4.78}{143.5} = x\frac{2.675}{267.5} \quad \therefore x = 3$$

$\therefore$ The complex is $[Co(NH_3)_6]Cl_3$

48. (A)

S_N1 proceeds via carbocation intermediate, the most stable one forming the product faster. Hence reactivity order for A, B, C depends on stability of carbocation created



49. (C)

$$\Delta G = -nFE \qquad \Rightarrow E = \frac{-\Delta G}{nF}$$

$$E = -\frac{966\times 10^3}{4\times 96500}$$

= -2.5V

$\therefore$ The potential difference needed for the reduction = 2.5V

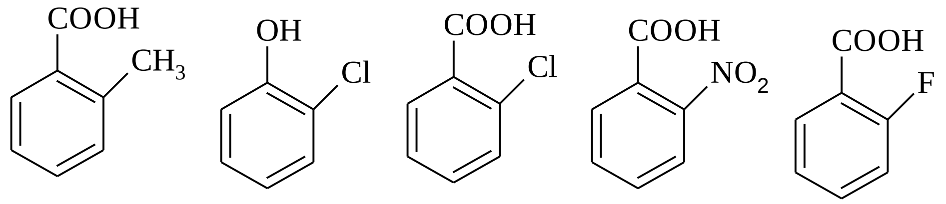
50. (B)
In presence of peroxide HBr follows anti Markownikoff's addition.

51. (8)

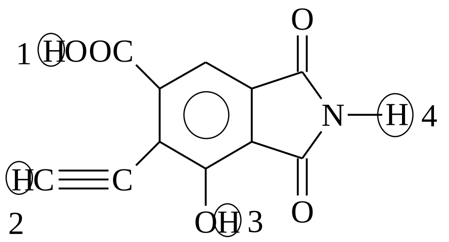
52. (3)
3 double bond equivalents

53. (3)
Ibuprofen, Aspirin and Phenacitin are antipyretics

54. (5)



55. (966)



$\Rightarrow$Three acidic H-atoms (1, 3, and 4) will react with 3 moles of NaOH(x = 3)
$\Rightarrow$Four acidic H-atoms (1, 2,3 and 4) will react with 4 moles of volume of 4 moles of CH(y=4)
Volume of 4 moles of CH_4 at STP = $4\times 22.4L = \Delta 89.6L = zL$
$\Rightarrow 10(x+y+z) = 3+4+89.6 = 966$

56 (9)
The reaction of the elements with NaOH can be computed as

Be^0	B^0	C^0	Al^0	Si^0	P^0	S^0	Sn^0	Pb^0
$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$
$\overset{+2}{Na_2BeO_2}$	$\overset{+3}{Na_3BO_3}$	$\overset{+4}{Na_2CO_3}$	$\overset{+3}{NaAlO_2}$	$\overset{+4}{Na_2SiO_3}$	$\overset{+1}{NaH_2PO_2}$	$\overset{+2}{Na_2S_2O_2}$	$\overset{+4}{Na_2SnO_3}$	$\overset{+4}{Na_2PbO_3}$
$+H_2$	$+H_2$	$+Na$ $+H_2$	$+H_2$	$+H_2$	$+PH_3$	$+Na_2S$	$+H_2$	$+H_2$

$$\overset{+3}{AlCl_3} + 3NaOH \rightarrow \underset{\text{White gelatineous}}{\overset{+3}{Al(OH)_3}} + 3NaCl$$

$$\overset{+3}{FeCl_3} + 3NaOH \rightarrow \underset{\text{Reddish brown}}{\overset{+3}{Fe(OH)_3}} + 3NaCl$$

$$\overset{+2}{Sn}Cl_2 + 2NaOH \rightarrow \underset{\text{White}}{\overset{+2}{Sn}(OH)_2} + 2NaCl$$

57. (4)

$H_4P_2O_5$ contains 6 bridging oxo-groups

N_2O_3 contains 1 bridging oxo-groups

$H_2S_2O_3$ does not contain any bridging oxo-group

$H_5P_3O_{10}$ contains 2 bridging oxo-groups.

58. (15)

$A \rightarrow B; w_1 = 0, \Delta U1 = 1 \times C_V(T), q1 = C_V(T)$

$B \rightarrow C; w_2 = -2p_0v_0, \Delta U_2 = 1 \times C_v(2T), q_2 = C_V(2T) + 2p_0v_0$

$C \rightarrow D; w_3 = 0, \Delta U_3 = 1 \times C_V(-2T), q_3 = -CV(2T)$

$D \rightarrow A; w_4 = p_0v_0, \Delta U_4 = -1 \times C_V(T), q_4 = -C_V(T) - p_0v_0$

$$\text{Efficiency} = \frac{|w|}{q_{absorbed}} = \frac{p_0V_0}{C_VT + C_V2T + 2p_0v_0}$$

$$= \frac{RT}{3 \times C_V \times T + 2RT} = \frac{RT}{3 \times \frac{3R}{2} \times T + 2RT}$$

$$= \frac{2}{13} = 0.1538 = 15.38\%$$

Note: (i) For one mole mono-atomic gas, $C_V = \frac{3R}{2}$

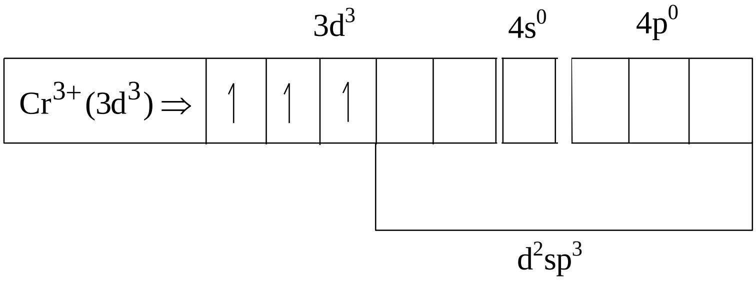
(ii) Heat is absorbed in $A \rightarrow B$ and $B \rightarrow C$ processes.

59. (3)

$Cr(NH_3)_4Cl_2Br$ –

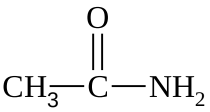
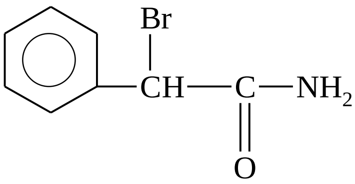
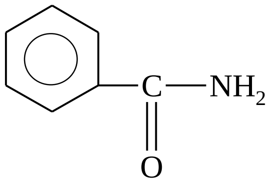
- $[Cr(NH_3)_4(Br)Cl]Cl \xrightarrow{AgNO_3} AgCl(\text{White ppt})$
- $[Cr(NH_3)_4Cl_2]Br \xrightarrow{AgNO_3} AgBr(\text{Pale yellow ppt.})$

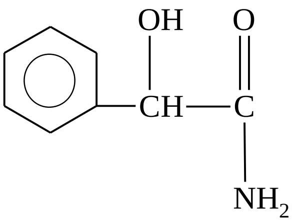
In (X) or (Y), Cr (III) is d^2sp^3-hybridised having 3 unpaired electrons (n).

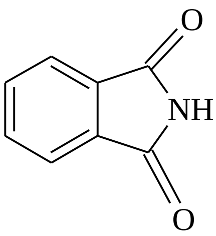


60. (5)

The following amides respond to Hafmann bromamide reaction.


,

,

,





PART (C) : MATHEMATICS

ANSWER KEY

61.	(C)	62.	(C)	63.	(A)	64.	(B)	65.	(C)
66.	(D)	67.	(A)	68.	(A)	69.	(A)	70.	(D)
71.	(B)	72.	(D)	73.	(C)	74.	(B)	75.	(A)
76.	(B)	77.	(A)	78.	(C)	79.	(D)	80.	(B)
81.	(2)	82.	(4)	83.	(6)	84.	(5)	85.	(0)
86.	(6)	87.	(2)	88.	(2)	89.	(6)	90.	(2)

SOLUTIONS

61. (C)

$a_1 = 1$

$\Rightarrow \ a_2 = 1 - \frac{1}{2}$

$\Rightarrow \ a_3 = 1 - \frac{1}{2} + \frac{1}{3}$

....

$a_\infty = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \ldots\ldots\ldots\ldots$

$= \ln 2$

$\ln(1+x) = x - \frac{x}{2} + \frac{x}{3} - \frac{x}{4} + \ldots\ldots$

Put $x = 1$

$\ln 2 = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \ldots\ldots$

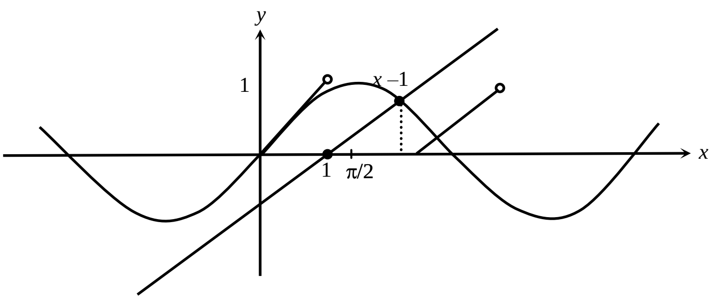
62. (C)

In the neighborhood of $x = \alpha$

$\min(\sin x, \{x\}) = \begin{cases} x - 1; & x < \alpha \\ \sin x; & x > \alpha \end{cases}$

LHL $\lim_{x \to \alpha^-} \left[\frac{x-1}{x-1} \right] = 1$

RHL $\lim_{x \to \alpha^+} \left[\frac{\sin x}{x-1} \right] = 0$



63. (A)

As domain of first function $= \left[-\frac{1}{2}, 0 \right) \cup \left\{ \frac{1}{2} \right\}$

Domain of 2nd function $[-3, -2]$

$\Rightarrow$ there will not be any point in common.

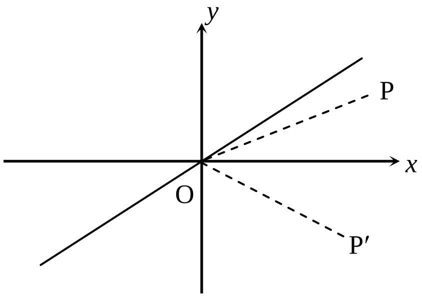
64. (B)

Coefficient of x^6 in $= 1 \times {}^6C_1 + 1 \times {}^5C_1 + 1 \times {}^2C_1 \times {}^3C_1 + {}^2C_1 \times {}^4C_1 + {}^3C_2 = 28$

65. **(C)**

The variable line is passing through a fixed point (2, 1) is point of intersection of given lines.

So image of (4, 7) is at a constant distance from (2, 1) hence locus is a circle.



66. **(D)**

Inequality is defined if $\tan x \in [-1, 1]$

So, options (A) and (B) are incorrect.

If $x \in \left[-\frac{\pi}{4}, \frac{\pi}{4}\right]$, then $f = \tan^{-1}(\sin x) - \sin^{-1}(\tan x)$

$$f'(x) = \frac{\cos x}{1+\sin^2 x} - \frac{\sec^2 x}{\sqrt{1-\tan^2 x}} = 0$$

$$\Rightarrow \quad \frac{\cos^2 x}{(1+\sin^2 x)^2} = \frac{\sec^4 x}{1-\tan^2 x} \quad \Rightarrow \quad (\cos^4 x)(\cos^2 x - \sin^2 x) = (1+\sin^2 x)^2$$

Substitute $\sin^2 x = t$

$(1-t)^2(1-2t) = (1+t)^2$

$\Rightarrow (1+t^2-2t)(1-2t) = 1+t^2+2t$

$\Rightarrow \cancel{1} + \cancel{t^2} - 2t - 2t - 2t^3 + 4t^2 = \cancel{1} + \cancel{t^2} + 2t$

$\Rightarrow 2t^3 - 4t^2 - 6t = 0$

$2t(t^2 - 2t + 3) = 0$

So, t = 0

For $x > 0$, $f'(x) < 0$ (by investigation)

$f'\left(\frac{\pi}{4}\right) = \frac{1/\sqrt{2}}{1+\frac{1}{2}} - \frac{2}{\sqrt{1-1}} < 0$. Hence $\left[-\frac{\pi}{4}, 0\right]$ is correct.

Alternatively: Put $x = \frac{\pi}{4}$ to check between (c) and (d) i.e., $\tan^{-1}\left(\frac{1}{\sqrt{2}}\right) < \sin^{-1}(1)$.

Hence (C) is incorrect.

67. **(A)**

$$f'(x) = \frac{1}{2+x^4}$$

By LMVT $f'(c) = \frac{f(2)-f(1)}{2-1}$ for some $c \in (1, 2)$

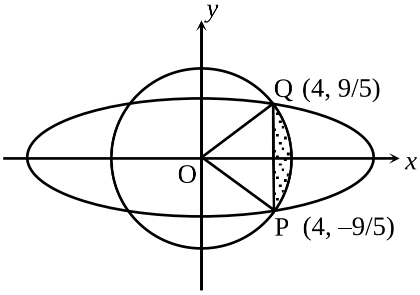
$\Rightarrow \quad f(2)=\dfrac{1}{2+c^4}$

$\Rightarrow \quad 1<c<2$

$\Rightarrow \quad 3<2+c^4<18$

$\Rightarrow \quad f(2)<\dfrac{1}{3}$

68. **(A)**



As eccentricity of ellipse $=\dfrac{4}{5}$

Co-ordinate of foci $=(4,\ 0),\ (-4,\ 0)$

$\Rightarrow \left(4,\ -\dfrac{9}{5}\right)$ is one of the end point of latus-rectum

$\Rightarrow$ Required area is

$\dfrac{1}{2\pi}\times\pi\times\left(4^2+\dfrac{9^2}{5^2}\right)\times 2\tan^{-1}\left(\dfrac{9}{20}\right)-$ area of ΔPOQ

$=\dfrac{481}{25}\tan^{-1}\left(\dfrac{9}{20}\right)-\dfrac{1}{2}\times 4\times\left(\dfrac{18}{5}\right)=\dfrac{481}{25}\tan^{-1}\left(\dfrac{9}{20}\right)-\dfrac{36}{5}$

69. **(A)**

Given limit reduces to $\lim\limits_{z\to\infty}\dfrac{\cot 1-\dfrac{1}{2}}{z}=0$

70. **(D)**

$a^2\left(70-\dfrac{12b}{a}+\dfrac{2c}{a}\right)>0$

$\Rightarrow \quad a^2[70+12(\alpha+\beta)+2\alpha\beta]>0 \quad\Rightarrow\quad (\alpha+7)(\beta+5)+(\beta+7)(\alpha+5)>0$

71. **(B)**

Let A be the event of drawing a black ball

$$P\left(\frac{E_{iB}}{A}\right)=\frac{P\left(\frac{A}{E_{iB}}\right).P(E_{iB})}{\sum_{i=1}^{3}P\left(\frac{A}{E_{iB}}\right).P(E_{iB})}=\frac{{}^1C_1.\frac{1}{3}}{\frac{1}{3}[{}^1C_1+{}^2C_1+{}^3C_1]}=\frac{1}{6}$$

72. **(D)**

$(z^3)=(-(\overline{\omega}^7)) \quad\Rightarrow\quad |z|^3=|\overline{\omega}|^7=|\omega|^7$ or $|z|^{15}=|\omega|^{35}$ …(i)

Again $z^5.\omega^{11}=1 \quad\Rightarrow\quad |z|^5.|\omega|^{11}=1$ or $|z|^{15}\ |\omega|^{33}=1$ …(ii)

From (i) and (ii) $\Rightarrow |z|=|\omega|=1$

Again $-(\overline{\omega})^{35} = \dfrac{1}{\omega^{33}} \Rightarrow (\overline{\omega})^2 = -1 = i^2 \Rightarrow \omega = i$ or $-i$

73. (C)

Let centre be $(0, \alpha)$

Equation of circle is $x^2 + (y-\alpha)^2 = k^2$ ...(i)

Equation of chord of contact for $R(x_1, y_1)$ is

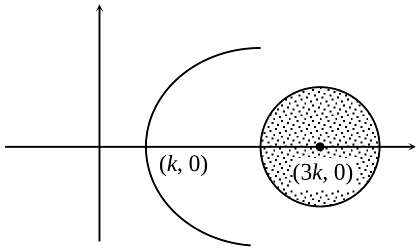
$xx_1 + yy_1 - \alpha(y + y_1) + \alpha^2 - k^2 = 0$ which passes through $(k, 0)$

i.e., $\alpha^2 - \alpha y_1 + kx_1 - k^2 = 0$ as α is real

$\Rightarrow y_1^2 - 4k(x_1 - k) \geq 0 \Rightarrow y^2 \geq 4k(x-k)$

Now there is no real intersection for

$y^2 \geq 4k(x-k)$ and $(x-3k)^2 + y^2 \leq k^2$



74. (B)

$\cos(\alpha+\beta) = \dfrac{2\sin\alpha \pm \sqrt{-4\sin^2\beta}}{2} \Rightarrow \sin\beta = 0 \Rightarrow \beta = \pi$

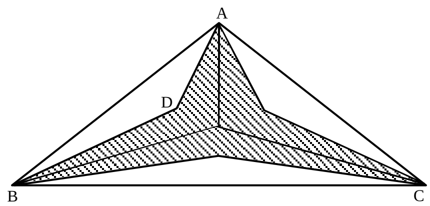
i.e., $-\cos\alpha = \sin\alpha$

$\Rightarrow \tan\alpha = -1 \Rightarrow \alpha = \dfrac{3\pi}{4}, \dfrac{7\pi}{4}$

75. (A)

Required area

$= \Delta ABC - 3\Delta ABD = \dfrac{\sqrt{3}.3^2}{4} - \dfrac{3}{2}.3.\dfrac{3}{2}\tan 15°$



76. (B)

$\dfrac{dy}{dx} = \begin{cases} 12x^2 - 2x + 2 & ; \quad 2 < x \leq 3 \\ 12x^2 + 2x - 2 & ; \quad 0 \leq x < 2 \end{cases}$; i.e., $2(2x+1)(3x-1)$ for $0 \leq x < 2$

$f(x)$ is continuous but not differentiable at x = 2 and $\dfrac{dy}{dx}$ = 0 at $x = \dfrac{1}{3}$ decreasing in $\left(0, \dfrac{1}{3}\right)$ and increasing for $\left(\dfrac{1}{3}, 2\right) \cup (2, 3)$.

Minima occurs at $x = \dfrac{1}{3}$

77. (A)

$\dfrac{dy}{dx} + 2y\tan x = \sin x$

$\text{I.F} = e^{\int 2\tan x\, dx} = e^{\ln(\sec x)^2} = \sec^2 x$

$$y\left(\sec^2 x\right)=\int \sin x \sec^2 x\, dx + C$$

$$y\cdot\sec^2 x = \sec x + C$$

Put $x=\frac{\pi}{3},\ y=0$

$$y=\cos x - 2\cos^2 x$$

$$=\frac{1}{8}-2\left(\cos x-\frac{1}{4}\right)^2$$

$$\therefore\ y_{\max}=\frac{1}{8}$$

78. **(C)**

$$\frac{x^2}{6/k}-\frac{y^2}{6}=1 \qquad ....(1)$$

$$e^2=1+\frac{6}{6/k}$$

$$a=\sqrt{\frac{6}{k}}$$

Equation of directrix $x=\frac{a}{e} \Rightarrow x=\sqrt{\frac{6}{k(k+1)}}$

$$\frac{6}{k(k+1)}=1$$

$$k=2$$

From eq. (1), we get $2x^2-y^2=6$

Check options

79. **(D)**

$$-(p\leftrightarrow(q\to p))$$

$$-(p\leftrightarrow q)=(p\wedge\sim(q\to p))\vee((q\to p)\wedge\sim p)$$

$$\sim(p\leftrightarrow(q\to p))=(p\wedge\sim(q\to p)\vee\sim p)$$

$$(p\wedge\sim(q\to p))=p\wedge(q\wedge\sim p)=(p\wedge\sim p)\wedge q=c$$

$$(q\to p)\wedge\sim p=(\sim q\vee p)\wedge\sim p=\sim p\wedge(\sim q\vee p)$$

$$=(\sim p\wedge\sim q)\vee(\sim p\wedge p)=\sim p\wedge\sim q$$

$$\sim(p\leftrightarrow(q\to p))=c\vee(\sim p\wedge\sim q)=\sim p\wedge\sim q$$

80. **(B)**

$|\vec{a}|=9$ & $\left(x\vec{a}+y\vec{b}\right)\cdot\left(6y\vec{a}-18x\vec{b}\right)=0$

$\Rightarrow\ 6xy|a|^2-18x^2\left(\vec{a}\cdot\vec{b}\right)+6y^2\left(\vec{a}\cdot\vec{b}\right)-18xy|\vec{b}|^2=0$

$\Rightarrow\ 6xy\left(|\vec{a}|^2-3|\vec{b}|^2\right)+6\left(\vec{a}\cdot\vec{b}\right)\left(y^2-3x^2\right)=0$

This should hold $\forall\ x, y\in R\times R$

$\therefore\ |\vec{a}|^{-2}=3|\vec{b}|^2$ & $\left(\vec{a}\cdot\vec{b}\right)=0$

Now, $|\vec{a}\times\vec{b}|^2=|\vec{a}|^2|\vec{b}|^2-\left(\vec{a}\cdot\vec{b}\right)^2$

$=|\vec{a}|^2\cdot\dfrac{|\vec{a}|^2}{3}$

$\therefore\ |\vec{a}\times\vec{b}|=\dfrac{|\vec{a}|^2}{\sqrt{3}}=\dfrac{81}{\sqrt{3}}=27\sqrt{3}$

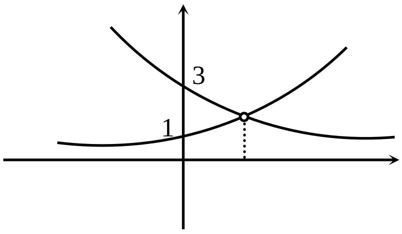
81. **(2)**

Let $f^{-1}(x)$ be $g\ \Rightarrow \ln\left(g+\sqrt{g^2+1}\right)=x$

$\Rightarrow\ g+\sqrt{g^2+1}=e^x$ ...(i)

and $-g+\sqrt{g^2+1}=e^{-x}$ ....(ii)

Now, $\left|e^x-e^{-x}\right|=2e^{-|x|}$



Case I: $x>0;\ e^{-|x|}=e^{-x}$ and $e^x>e^{-x}$

$\Rightarrow\ e^x-e^{-x}=2e^{-x}\ \Rightarrow\ e^x=3e^{-x}$

Case II: $x<0;\ e^{-|x|}=e^x$ and $e^x<e^{-x}$

$\Rightarrow\ e^{-x}-e^x=2e^x\ \Rightarrow\ e^{-x}=3e^x$

$\therefore$ Two solutions.

Ans. 2

82. **(4)**

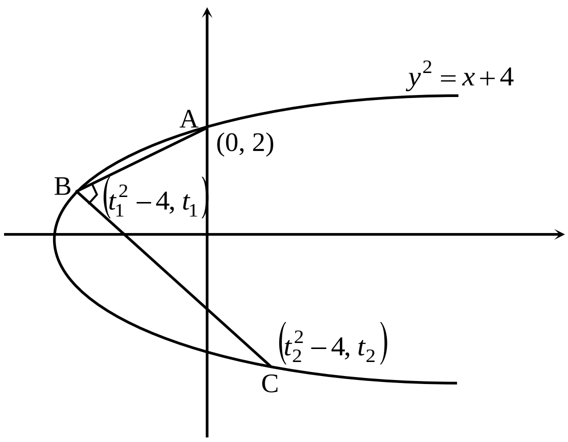
Now $\left(\dfrac{t_2-t_1}{t_2^2-t_1^2}\right)\left(\dfrac{t_1-2}{t_1^2-4}\right)=-1$

put $t_1\neq t_2$ and $t_1\neq 2$

$\Rightarrow\ \dfrac{1}{(t_1+t_2)}\dfrac{1}{\left(\dfrac{1}{t_1+2}\right)}=-1$

$\Rightarrow\ t_1^2+t_1t_2+2t_1+2t_2+1=0$

Quadratic in t_1 should have real roots.



$\Rightarrow\ (t_2+2)^2-4(2t_2+1)\geq 0\ \Rightarrow\ t_2^2-4t_2\geq 0$

$\Rightarrow\ t_2 \leftarrow (-\infty,\ 0] \cup [4,\ \infty)$

There least positive value of t_2 is 4.

Ans. 4

83. **(6)**

Since it is an identity the value of L.H.S. and R.H.S. are equal for all values of x

Put $x = 0 \Rightarrow \alpha_1 \alpha_2 \alpha_3 \alpha_4 = 6$

Ans. 6

84. **(5)**

Given inequality can be written as:

$f''(x) - 2f'(x) \geq 3(f'(x) - 2f(x))$

Let $f'(x) - 2f(x) = g(x)$

$\Rightarrow\ g'(x) - 3g(x) \geq 0$ Multiply e^{-3x}

$\Rightarrow\ \left(g(x)e^{-3x}\right)' \geq 0 \Rightarrow g(x)e^{-3x}$ is non-decreasing.

Now $g(0) = f'(0) - 2f(0) = -2$

$g(x)e^{-3x} \geq -2,\ \forall x \geq 0$

$f'(x) - 2f(x) \geq -2e^{3x},\ \forall x \geq 0$ Multiply e^{-2x}

$\Rightarrow\ \left(f(x)e^{-2x}\right)' \geq -2e^{x},\ \forall x \geq 0$

$\Rightarrow\ \left(f(x)e^{-2x} + 2e^{x}\right)' \geq 0$

$\Rightarrow\ f(x)e^{-2x} + 2e^{x} \geq 3$

$\Rightarrow\ f(x) \geq 3e^{2x} - 2e^{3x},\ \forall x \geq 0$

Comparing $ah(bx) - bh(ax)$ with $3e^{2x} - 2e^{3x}$ we get $h(x) = e^{x},\ a = 3,\ b = 2$

$\Rightarrow\ (a+b)h(0) = 5$

Ans. 5

85. **(0)**

Since $[\sqrt{2046} = [\sqrt{2047}] = [\sqrt{2048}] = [\sqrt{2049} = 45$

$\therefore$ 2003rd term is $2003 + 45 = 2048$

Hence remainder is 0

Ans. 0

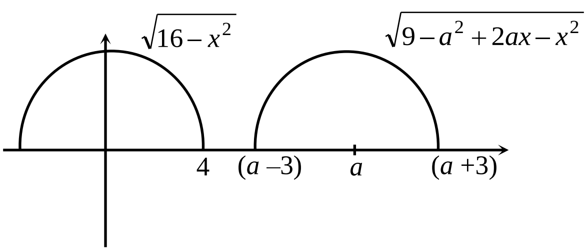
86. **(6)**

$y = \sqrt{9 - a^2 + 2ax - x^2} \Rightarrow$

$(x-a)^2 + y^2 = 9$

For given inequality to hold for positive x.

$a - 3 < 4$



$\Rightarrow \ a<7 \ \Rightarrow \ a=6$

Ans. 6

87. **(2)**

For sum of coefficient substitute replace $x_1 = x_2 = \ldots. x_n = 1$

Sum of coefficient $\left({}^nC_1 + 2^2\ {}^nC_2 + \ldots..n^2\ {}^nC_n\right)^n = \left(n(n+1)2^{n-2}\right)^n$

$$= \left(n(n+1)\right)^n\ 2^{n^2-2n}$$

$\Rightarrow \ k = 2$

Ans. 2

88. **(2)**

$$\text{RHL} = \lim_{x \to 0^+} \frac{\left(1+a^3\right)+8e^{\frac{1}{x}}}{1+\left(2+b+b^2\right)e^{\frac{1}{x}}} = 2$$

$\Rightarrow \ 2+b+b^2 = 4 \ \Rightarrow \ b^2+b-2=0 \ \Rightarrow \ b=1$

$$\text{LHL} = \lim_{x \to 0^-} \frac{\left(1+a^3\right)+8e^{\frac{1}{x}}}{1+\left(2+b+b^2\right)e^{\frac{1}{x}}} = 2$$

$\Rightarrow \ 1+a^3 = 2 \ \Rightarrow \ a=1$

Ans. 2

89. **(6)**

$y = a - bx^2$

when $x = 2$, $y = 1$

$1 = a - 4b \quad \Rightarrow \quad a = 1+4b$

$$A = 2\int_0^{\sqrt{a/b}} (a-bx^2)dx = 2\left[ax - \frac{bx^3}{3}\right]_0^{\sqrt{a/b}} = 2\left[\frac{a\sqrt{a}}{\sqrt{b}} - \frac{b}{3}.\frac{a\sqrt{a}}{b\sqrt{b}}\right]$$

$$= \frac{2}{3}\left[\frac{3a\sqrt{a}}{\sqrt{b}} - \frac{a\sqrt{a}}{\sqrt{b}}\right] = \frac{2}{3}\left[\frac{2a\sqrt{a}}{\sqrt{b}}\right] = \frac{4}{3}\cdot\frac{(1+4b)\sqrt{1+4b}}{\sqrt{b}}$$

$$\frac{dA}{db} = \frac{4}{3}\left[\frac{b^{1/2}\frac{3}{2}\sqrt{1+4b}.4 - (1+4b)^{3/2}\frac{1}{2}\frac{1}{\sqrt{b}}}{b}\right] = 0$$

$\Rightarrow \ 12b = 1+4b \ \Rightarrow \ 8b = 1 \ \Rightarrow \qquad b = \frac{1}{8},\ a = \frac{3}{2}$

$\therefore\ A_{\min} = \frac{4}{3}.\left(\frac{3}{2}\right).\sqrt{\frac{3}{2}}.2\sqrt{2} = \frac{4\sqrt{3}\sqrt{2}}{\sqrt{2}}$

$\therefore$ Minimum area $= \sqrt{48}$

$\Rightarrow\ A = 48 \Rightarrow \frac{A}{8} = 6$

Ans. 6

90. **(2)**

$r \le \frac{R}{2} \le \frac{1}{2}$

$r = \frac{\Delta}{s} \Rightarrow \frac{\Delta}{s} \le \frac{1}{2} \Rightarrow \frac{s}{\Delta} \ge 2$

Ans. 2