PART (A) : PHYSICS

ANSWER KEY

1.	(A)	2.	(A)	3.	(B)	4.	(A)	5.	(D)
6.	(B)	7.	(B)	8.	(D)	9.	(B)	10.	(C)
11.	(B)	12.	(C)	13.	(D)	14.	(A)	15.	(A)
16.	(A)	17.	(B)	18.	(D)	19.	(A)	20.	(B)
21.	(30)	22.	(7)	23.	(7)	24.	(3)	25.	(10)
26.	(3)	27.	(7)	28.	(765)	29.	(12)	30.	(46)

SOLUTIONS

1. (A)

Internal resistance $r = \frac{R[l - l']}{l'}$

Where $l =$ balanced length with key open. $l =$ balanced length with key K closed

$\therefore r = \frac{132.40[70-60]}{60} = \frac{132.40 \times 10}{60}$

$= 21.06\Omega \approx 22.1\Omega$

2. (A)

Flux through

$S_1(\phi_1) = \frac{\text{Charge enclosed within } S_1}{e_0} = \frac{q}{\varepsilon_0}$

Since the shell is conducting therefore a charge of magnitude $-q$ will be induced at the inner surface(radius R_1) of the shell.

$\therefore$ Flux through

$S_2(\phi_2) = \frac{\text{Charge enclosed within } S_2}{e_0} = \frac{q-q}{\varepsilon_0} = 0$

3. (B)

As process us cyclic $\Rightarrow \Delta U_{ABCA} = 0$

$\Rightarrow Q_{ABCA} = W_{ABCA}$

$\Rightarrow Q_{A\to B} + Q_{B\to C} + Q_{C\to A}$

$= W_{A\to B} + W_{B\to C} + W_{C\to A}$ (1)

Given $Q_{B\to C} + Q_{C\to A} = W_{A\to B} + W_{B\to C}$ (2)

Subtracting (2) from (1),

$Q_{A\to B} = W_{C\to A} = 0$

(as process $C \to A$ is isochoric)

$\therefore$ process $A \to B$ is adiabatic

4. (A)

$$L_f = L_i\left(1+\alpha\Delta T\right)$$

$$L_{S_f} = L_{S_i}\left[1+\alpha_S\Delta T\right]$$

$$\Rightarrow \Delta L_{steel} - L_{S_i}\alpha_S\Delta T$$

$$L_{C_f} = L_{C_i}\left[1+\alpha_C\Delta T\right]$$

$$\Rightarrow \Delta L_{copper} = L_{C_i}\alpha_C\Delta T$$

For $\Delta L_{stee} = \Delta L_{copper}$

$$\Rightarrow L_{S_i}\alpha_S\Delta T = L_{C_i}\alpha_C\Delta T$$

$$\Rightarrow \frac{L_{S_i}}{L_{C_i}} = \frac{\alpha_C}{\alpha_S} = \frac{1.8\times10^{-5}}{1.2\times10^{-5}} = \frac{3}{2}$$

$$\therefore \frac{L_{S_i}}{L_{C_i}} = \frac{3}{2}\text{in}\left(1\right)\text{only}$$

5. (D)

Using $\frac{1}{\lambda} = R\left(z-1\right)^2\left[\frac{1}{n_2^{\,2}} - \frac{1}{n_1^{\,2}}\right]$

For α particle; $n_1 = 2, n_2 = 1$

For metal A; $\frac{1875R}{4} = R\left(Z_1-1\right)^2\left(\frac{3}{4}\right) \Rightarrow z_1 = 26$

For metal B; $675R = \left(Z_2-1\right)^2\left(\frac{3}{4}\right) \Rightarrow z_2 = 31$

Therefore, 4 elements lie between A and B, i.e. with $27, 28, 29, 30$

6. (B)

Current through the inductor before closing the switch $= 1A$

Current through the inductor after closing the switch(in steady state)

$$I = \frac{20}{5} = 4A$$

$$\therefore \Delta\phi = LI = 1.5Wb$$

7. (B)

$$q = \int Idt = \int -\frac{1}{r}\frac{d\phi}{dt}dt = -\frac{\Delta\phi}{r} = \frac{\mu_0 Ia}{\pi r}\ell n2$$

8. (D)

As the source is not an a.c. source, the ideal inductance will act like a short circuit (after long time), so while of the current will pass through the inductance and no current will flow through the bulb. Therefore, the bulb will not glow.

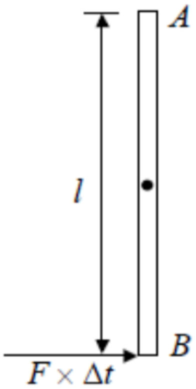
9. (B)

$K=\frac{P^2}{2m}$

From graph , $4=\frac{4^2}{2m}$

$\therefore m=2kg$

10. (C)



$F\times\Delta t=P$

Angular impulse about center of mass

$=F\times\Delta t\times\frac{l}{2}=\frac{P\times l}{2}$

Angular impulse=Change in angular momentum

$P\times\frac{l}{2}=\frac{ml^2}{12}\omega$

$\omega=\frac{6P}{ml}$

Time taken by rod to turn by 90^o is(Time period)/4.

$=\frac{1}{4}\times\frac{2\pi}{\omega}=\frac{\pi ml}{2\omega}=\frac{\pi ml}{12P}$

11. (B)

$mgl\sin\theta=\frac{1}{2}mv^2+\frac{1}{2}I\omega^2$

$=\frac{1}{2}mv^2+\frac{1}{2}\frac{mR^2}{2}\frac{v^2}{R^2}=\frac{3}{4}mv^2$

$\Rightarrow v=\sqrt{\frac{4gl\sin\theta}{3}}$

$I=I_{cm}\omega+mvR=\frac{MR^2}{2}\frac{v}{R}+MvR=\frac{3MvR}{2}$

$=\frac{3}{2}mR\sqrt{\frac{4gl\sin\theta}{3}}=\sqrt{3m^2R^2gl\sin\theta}$

12. (C)

At A: Acceleration $= g\sin\theta$ (only tangential) at B:

Acceleration $= \frac{V^2}{l} = 2g(1-\cos\theta)$ (only normal)

$\therefore g\sin\theta = 2g(1-\cos\theta)$

$\Rightarrow \cos\theta = 3/5$

13. (D)

$h = \frac{2T\cos\theta}{r\rho g} \Rightarrow \frac{T_w}{T_m}\times 13.6\times\left(-\sqrt{2}\right) = \frac{10}{-3.42}$

$\Rightarrow \frac{T_w}{T_m} \cong \frac{1}{6.5}$

14. (A)

(Wc - Wm) and (Wc + Wm)

15. (A)

$i = 6/3 = 2A$

16. (A)

$\frac{1}{v} - \frac{2}{-15} = \frac{1-2}{-10}$

$\frac{1}{v} = \frac{1}{10} - \frac{2}{15} = \frac{3-4}{30} = -\frac{1}{30}$

17. (B)

$p^x q^y c^z = \left[ML^{-1}T^{-2}\right]^x \left[\frac{ML^2T^{-2}}{L^2T}\right]^y \left(LT^{-1}\right)^2$

$= M^{x+y}L^{-x+z}T^{-2x-3y-z}$

As the quantity is dimensionless, therefore

$x + y = 0;\ \therefore x = -y$

$= x + z = 0;\ \therefore x = z$

$-2x - 3y - z = 0$, which satisfy this equation.

Hence $x = -y = z$

18. (D)

Acceleration of block $AB = \frac{3mg}{3m+m} = \frac{3}{4}g$; acceleration of block $CD = \frac{2mg}{2m+m} = \frac{2g}{3}$.

Acceleration of image in mirror AB

$=$ 2 acceleration of mirror

$= 2.\left(\frac{-3g}{4}\right) = \frac{-3}{2}g$

Acceleration of image in mirror $CD = 2.\left(\frac{2g}{3}\right) = \frac{4g}{3}$

$\therefore$ Acceleration of the two image w.r.t. each other $= \frac{4g}{3} - \left(\frac{-3g}{2}\right) = \frac{17g}{6}$

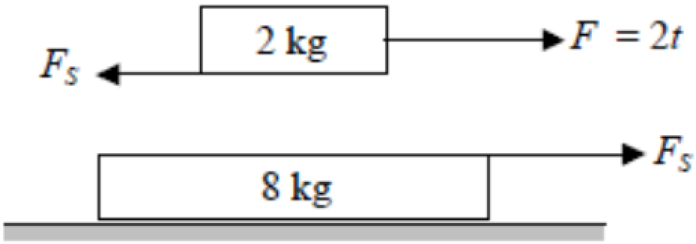
19. (A)

Let common acceleration be $a\,m/s^2$

$2t - F_S = 2a$ (i)

$F_S = 8a$ (ii)

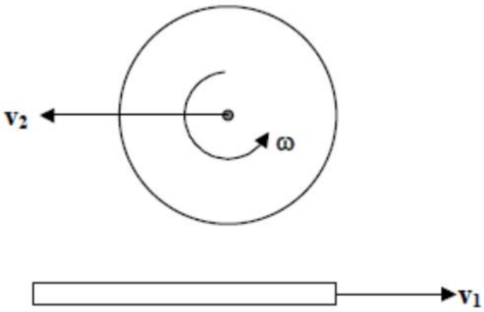
$F_S = \frac{2t}{10} \times 8 \le 0.2 \times 2 \times 10$



$t = \frac{5}{2}s, a = \frac{t}{5}$

$\frac{dv}{dt} = \frac{t}{5} \Rightarrow v = \frac{t^2}{10} = \frac{dx}{dt} \Rightarrow x = \frac{t^3}{30}m$

20. (B)



The condition for no slipping here will be

$R\omega - v_2 = v_1$

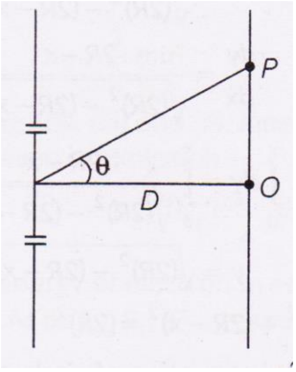
($\because$ point of contact remains at rest) In terms of displacement

$R\Delta\theta - s_2 = s_1$

$\therefore \Delta\theta = \frac{s_1 + s_2}{R} = \frac{100 + 75}{150} = \frac{7}{6}$ rad

21. (30)

For maximum, path difference $= n\lambda$



$\Rightarrow d\sin\theta = n\lambda \Rightarrow \sin\theta = \frac{n\lambda}{d}$

Substituting the given values, we get

$\sin\theta = \frac{2000\times5000\times10^{-10}}{2\times10^{-3}} = \frac{1}{2}$

$\Rightarrow \theta = 30^{\circ}$

According to the figure,

$OP = D\tan\theta = 1.\tan30^{\circ} = \frac{1}{\sqrt{3}}$

$= 0.58\text{m}$

22. (7)

$\text{Density} = \frac{\text{Mass}}{\text{Volume}}$

$\Rightarrow \rho = \frac{m}{l^3}$

$\Rightarrow \frac{d\rho}{\rho} = \frac{dm}{m} - \frac{3dl}{l}$

23. (7)

Inductive reactance,

$X_L = \omega L$

$= 100\pi\times\frac{3}{100\pi} = 3\Omega$

Capacitance reactance,

$X_C = \frac{1}{\omega C} = \frac{1}{100\pi\times\frac{1}{700\pi}} = 7\Omega$

Impedance , $Z = \sqrt{(X_C - X_L)^2 + R^2}$

$= \sqrt{(7-3)^2 + 3^2} = 5\Omega$

$i_{rms} = \frac{V_{rms}}{Z} = \frac{50\sqrt{2}}{5} = 10\sqrt{2}A$ ....(i)

So, $\tan\phi = \frac{X_C - X_L}{R} = \frac{7-3}{3} = \frac{4}{3}$

$\Rightarrow \phi = 53^\circ$ ......(ii)

$\therefore$ Wattless component

$= i_{rms}\sin\phi$

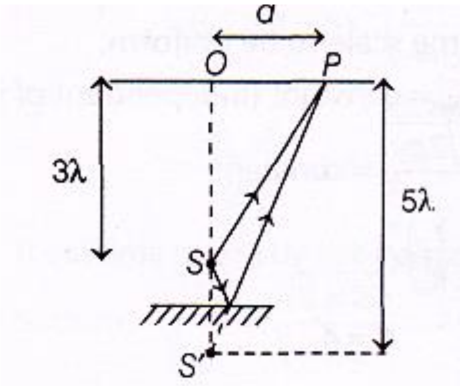
$= 10\sqrt{2} \times \sin 53^\circ$

[using Eqs. (i) and (ii)]

$= 10\sqrt{2} \times \frac{4}{5} = \left(\sqrt{2}\right)^7$ A

24. (3)

For O, $\Delta x = 2l = 2\lambda$ which will be the point of destructive interference, as due to reflection condition of destructive and constructive interferences are interchanged, so nearest point for destructive interference should have $\Delta x = \lambda$.



$\Rightarrow S'P - SP = \lambda$

$\Rightarrow \sqrt{(5\lambda)^2 + d^2} - \sqrt{(3\lambda)^2 + d^2} = \lambda$

$\Rightarrow 25\lambda^2 + d^2 = \lambda^2 + 9\lambda^2 + d^2 + 2\lambda\sqrt{9\lambda^2 + d^2}$

$\Rightarrow 15\lambda^2 = 2\lambda\sqrt{9\lambda^2 + d^2}$

$\Rightarrow d = \sqrt{\frac{189}{4}}\lambda$

$= \sqrt{\frac{189}{4}} . \frac{2}{\sqrt{21}} = 3m$

25. (10)

Energy conservation at A and P gives PE at $A = (PE + KE)$ at P

$\Rightarrow U_A = U_P + K_P$

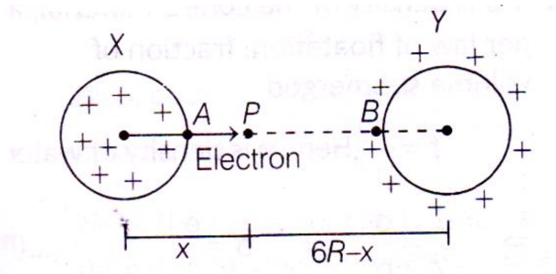
$mgh_A = mgh_P + K_P$

$1 \times 10 \times 2 = 1 \times 10 \times 1 + K_P$

$\therefore K_P = 10J$

26. (3)

Let x be the distance of null point P from sphere X.



At P, force on electron is zero.So, force due to X= force due to Y

$$\Rightarrow \frac{k.Q}{x^2} = \frac{k.4Q}{(6R-x)^2}$$

$$\Rightarrow 4x^2 = (6R-x)^2$$

$$\Rightarrow 2x = 6R - x$$

$$\Rightarrow x = 2R$$

To reach point B, the electron should just cross point P or speed of electron at P should be just greater than zero. So, by conservation of energy for motion from P to B(surface point of sphere Y)

$$KE_P + PE_P = KE_B + PE_B$$

$$0 + \frac{kQ(-e)}{2R} + \frac{k.4Q(-e)}{4R}$$

$$= KE_B + \frac{kQ(-e)}{R} + \frac{k.4Q(-e)}{5R}$$

$$\Rightarrow KE_B = \frac{0.3kQe}{R} = \frac{xkQe}{R}(\text{given})$$

27. (7)

Magnification of the object

$$m = \frac{f}{f-u} = \frac{f}{f-(-f)} = \frac{1}{2} \qquad \ldots\ldots(i)$$

As velocity component of image along axis is given by

$$v_i = -m^2 v_0 \qquad \ldots\ldots(ii)$$

$$\Rightarrow v_i = -\left(\frac{1}{2}\right)^2 4\cos 30^\circ = -\frac{\sqrt{3}}{2}\text{cm}/\text{s}$$

[using Eq. (i)]

Also velocity component of image perpendicular to axis is given by

$$v'_i = mv'_0 = \frac{1}{2}.4\sin 30^\circ = 1\text{cm}/\text{s}$$

[using Eq. (i)]

Therefore, net velocity of image is

$$v'_{net} = \sqrt{v_i^2 + v_i^2}$$

$= \sqrt{\frac{3}{4}+1} = \frac{\sqrt{7}}{2}\,\text{cm/s}$

28. (765)

From first figure, excess reading (zero error) $= 0.02\text{mm}$

From the second figure, the screw gauge gives a reading of 7.67 mm in which there is 0.02 mm excess reading, which has to be removed (substracted). So, actual reading or thickness of the wire $= 7.67 - 0.02 = 7.65\text{mm}$

29. (12)

As the mercury air (trapped) interface moves a distance of 5L in the process work done against atmosphere is

$W_i = \text{Force} \times \text{Displacement}$

$= p_0 A \times 5L$

$\Rightarrow W_1 = 5p_0 AL$ .....(i)

Also, work done against gravity is equal to increase in potential energy of the liquid.

$W_2 = 7\rho AL^2 g$ .....(ii)

Also, Pressure in horizontal part is equal to

$2p_0 = p_0 + \rho g L$

$\Rightarrow p_0 = \rho g L$ ......(iii)

From Eqs. (ii) and (iii), we get

$W_2 = 7p_0 AL$ .....(iv)

From Eqs.(i) and (iv), total work done is

$W = W_1 + W_2 = 12 p_0 AL$

$\therefore x = 12$

30. (46)

For complete cycle

$Q_{cycle} = W_{cycle} = 400 + 100 + Q_{C\to A}$

$= \frac{1}{2}\left(2\times 10^{-3}\right)\left(4\times 10^{4}\right)$

$\Rightarrow Q_{C\to A} = -460\,\text{J}$

$\Rightarrow Q_{A\to C} = +460\,\text{J}$

So, x will be 46.

PART (B) : CHEMISTRY

ANSWER KEY

31.	(B)	32.	(A)	33.	(A)	34.	(D)	35.	(B)
36.	(B)	37.	(B)	38.	(D)	39.	(B)	40.	(B)
41.	(C)	42.	(A)	43.	(D)	44.	(B)	45.	(A)
46.	(B)	47.	(B)	48.	(C)	49.	(A)	50.	(A)
51.	(2)	52.	(2)	53.	(3)	54.	(20)	55.	(3)
56.	(7)	57.	(4)	58.	(5)	59.	(3)	60.	(5)

SOLUTIONS

31. (B)

32. (A)

$$2Cu(NO_3)_2 + 4KCN \longrightarrow 4KNO_3 + 2CuCN + (CN)_2 \uparrow$$

33. (A)

Cl being more reactive displaces Br from its compound

34. (D)

Zn^{2+} dissolves in excess of $NH_3(aq)$ to form a complex $\left[Zn(NH_3)_4\right]^{2+}$.

35. (B)

MnO_4^- oxidises NO_2^- to NO_3^-

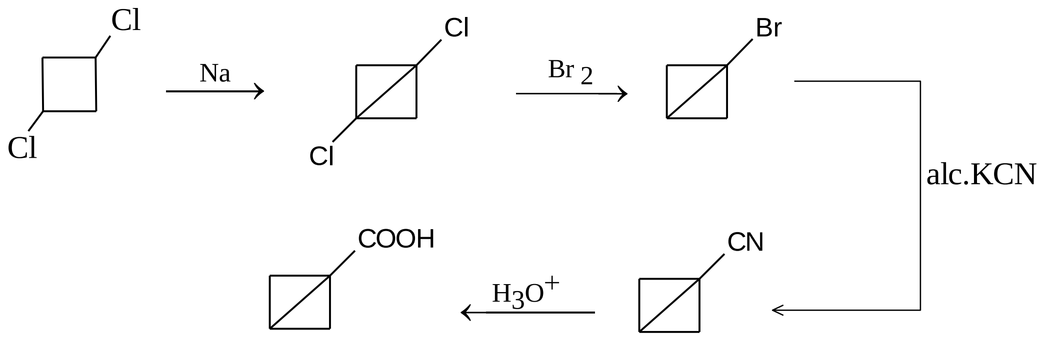
36. (B)

37. (B)

38. (D)

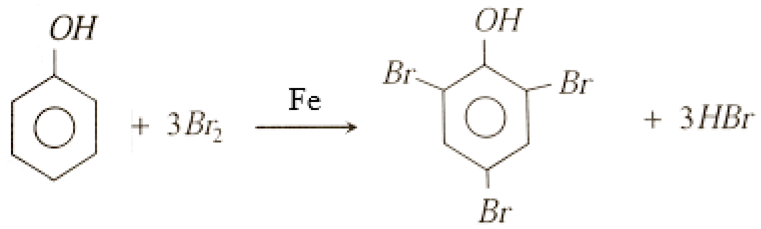
$$Na_2B_4O_7 + CuSO_4 \longrightarrow Cu(BO_2)_2 + NaBO_2 + B_2O_3$$

39. (B)



40. (B)
E_2 elimination

41. (C)



$n_{Phenol} = \frac{2}{94}$ $\therefore$ $n_{Br_2} = 3 \times \frac{2}{94} = 0.063$ moles

42. (A)

43. (D)

44. (B)
Work = area of triangle
Work is done by gas $W = -ve$
$W = -1.5$ bar m^3
$W = -1.5 \times 10^5$ Nm
$W = -150$ kJoule ; $\Delta Q = +100$ k Joule
$\Delta U = 100 - 150 = -50$ kJ

45. (A)
50 mL of 0.1 M NaOH $- \frac{50}{1000} \times 0.1 = 0.005$ moles
60 mL of 0.15 M H_2PO_4 $- \frac{60}{1000} \times 00.15 = 0.009$ moles
0.005 moles of NaOH reacts with 0.005 moles of H_3PO_4 to form 0.005 moles of NaH_2PO_4
0.009 – 0.005 = 0.004 moles of H_3PO_4 remains

$$pH = -\log K_a + \log\frac{[NaH_2PO_4]}{[H_3PO_4]}$$

$$pH = -\log 10^{-3} + \log\frac{0.005}{0.004} = 3.1$$

46. (B)

$$C_A e^{-K_A t} = C_B e^{-K_B t}$$

$$\frac{C_A}{C_B} = \frac{e^{-k_A t}}{e^{-k_A t}} \Rightarrow \frac{C_A}{C_B} = e(K_A - K_B)$$

$$4 = e\left[\frac{\ln 2}{5} - \frac{\ln 2}{15}\right]\times t$$

$$\ln 4 = \left[\frac{\ln 2}{5} - \frac{\ln 2}{15}\right]t$$

$$\ln(2)^2 = \left[\frac{\ln 2}{5} - \frac{\ln 2}{15}\right]t$$

$$2\ln 2 = \left[\frac{\ln 2}{5} - \frac{\ln 2}{15}\right]t$$

$$2\left[\frac{1}{5} - \frac{1}{15}\right]t$$

$$2 = \frac{2}{15}\times t$$

$t = 15$ minutes.

47. (B)

$$E_{cell} = E^o_{cell} - \frac{0.0592}{h}\log[Ag^+]$$

So, $E_{cell} = -0.25\text{ V}$

$E^o_{cell} = -0.799\text{ V}$

$$-0.25\text{ V} = 0.799 - \frac{0.592}{1}\log[Ag^+]$$

$$0.0592\log[Ag^+] = -0.799 + 0.25$$

$$[Ag^+] = 10^{-9.273} = 5.1\times10^{-10}$$

$$K_{sp} = [A^+][Cl^-]$$

$$K_{sp} = (5.1\times10^{-10})(0.1)$$

$$= 5.1\times10^{-11}$$

48. (C)

$$\alpha = \frac{\lambda_m}{\lambda^o_m} = \frac{10}{200} = 0.05$$

$\left[H^+\right] = C\alpha = 0.005$

$pH = -\log\left[H^+\right] = -\log[0.005]$

$= 2.3$ Ans.

49. (A)

Thermal stability α ionic character for salt having polyatomic anion.

50. (A)

51. (2)

$Na_2B_4O_7 + 2HCl \longrightarrow H_2B_4O_7 + NaCl$

$H_2B_4O_7 + 5H_2O \longrightarrow 4H_3BO_3$

52. (2)

$\underset{1\,mole}{Ca_3P_2} + 6H_2O \longrightarrow 3Ca(OH)_2 + \underset{2\,mole}{2PH_3}$

53. (3)

N_2O – Nitrous oxide

NO – Nitric oxide

NO_2 – Nitrogen dioxide

N_2O_4 – Dinitrogen tetraoxide

N_2O_5 – Dinitrogen pentaoxide

These all oxides of nitrogen when dissolved in water yields the following products :

$N_2O + H_2O \rightarrow N_2 \uparrow + \bar{O}H$

$NO + H_2O \rightarrow$ No reaction

$3NO_2 + H_2O \longrightarrow 2HNO_3 + NO$

$N_2O_4 + H_2O \rightarrow HNO_3 + HNO_2$

$N_2O_5 + H_2O \longrightarrow HNO_3$

54. (20)

55. (3)

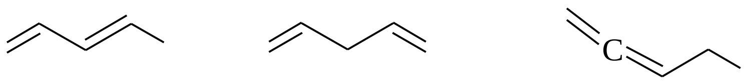
$$\left[CH_3-CH_2-\underset{\underset{CH_2-CH_3}{|}}{CH}-\right]_2 CuLi + CH_3-CH_2-Br \rightarrow$$

$$\left[CH_3-CH_2-\underset{\underset{CH_2-CH_3}{|}}{CH}-CH_2-CH_3\right] \xrightarrow{Cl_2/h\upsilon}$$

(X)

$$\underset{\text{(with one dl pair)}}{4\,\text{isomers}} \xrightarrow{\text{Fractional distillation}} 3\,\text{fractions}$$

56. (7)
7 different acyclic isomers of C_5H_8 on catalytic hydrogenation give the same n-pentane.
These are as given below:



cis & trans

57. (4)

$$CH_2=CH-\underset{\underset{Cl}{|}}{CH}-CH_3 \longrightarrow CH_2=CH-\underset{\underset{OH}{|}}{CH}-CH_3 + \underset{\underset{OH}{|}}{CH_2}-CH=CH-CH_3 \quad \text{cis \& trans}$$

58. (5)

$\alpha-D-\text{glucose(s)}+H_2O \rightarrow \alpha-D-\text{glucose(aq)}$; Heat of dissolution = 10.84kJ.....(1)

$\beta-D-\text{glucose(s)}+H_2O \rightarrow \beta-D-\text{glucose(aq)}$; Heat of dissolution = 4.68kJ .....(2)

$\alpha-D-\text{glucose(aq)} \rightarrow \beta-D-\text{glucose(aq)}$; Heat of mutarotation = -1.16 kJ ......(3)

$\alpha-D-\text{glucose(s)} \rightarrow \beta-D-\text{glucose(s)}$ .....(4)

Hence $\Delta H^0 = 10.84kJ - 4.68kJ - 1.15kJ = 5kJ$

59. (3)

60. (5)

PART (C) : MATHEMATICS

ANSWER KEY

61.	(B)	62.	(B)	63.	(D)	64.	(C)	65.	(A)
66.	(B)	67.	(C)	68.	(B)	69.	(D)	70.	(C)
71.	(D)	72.	(C)	73.	(C)	74.	(A)	75.	(A)
76.	(A)	77.	(D)	78.	(D)	79.	(B)	80.	(B)
81.	(1)	82.	(2)	83.	(7)	84.	(6)	85.	(0)
86.	(7)	87.	(1)	88.	(4)	89.	(7)	90.	(7)

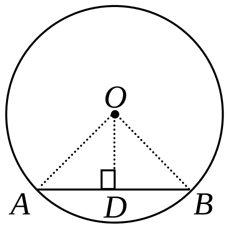
SOLUTIONS

61. **(B)**

If $AB = a$, n is side of a regular polygon.

Then area $= \frac{na^2}{4}\cot\frac{\pi}{n}$

$$\frac{\text{area of pentagon}}{\text{area of decagon}} = \frac{2\cot 36^\circ}{\cot 18^\circ} = 2:\sqrt{5}$$



62. **(B)**

$$\sum_{i=0}^{n}\sum_{j=1}^{n} {}^nC_j \cdot {}^jC_i = {}^nC_1({}^1C_0 + {}^1C_1) + {}^nC_2({}^2C_0 + {}^2C_1 + {}^2C_2) + {}^nC_3({}^3C_0 + {}^3C_1 + {}^3C_2 + {}^3C_3) + \dots$$

$$= {}^nC_1.2^1 + {}^nC_2\,2^2 + {}^nC_3.2^3 \dots \Rightarrow \left((1+x)^n - {}^nC_0\right) \Rightarrow \text{where } x = 2$$

$$= (2+1)^n - {}^nC_0 = 3^n - 1$$

63. **(D)** Using weighted A.M. and G.M.

$$\frac{3\left(\frac{11-x}{3}\right) + 5\left(\frac{x+5}{5}\right)}{3+5} \ge \left(\left(\frac{11-x}{3}\right)^3\left(\frac{x+5}{5}\right)^5\right)^{\frac{1}{3+5}}$$

$$\frac{11-x+x+5}{8} \ge \left(\frac{(11-x)^3(x+5)^5}{3^3.5^5}\right)^{1/8}$$

$$(2)^8 \ge \frac{(11-x)^3(x+5)^5}{3^3.5^5}$$

$$(11-x)^3(x+5)^5 \le 2^8.3^3.5^5 \;\Rightarrow\; (11-x)^3(x+5)^5 \le 6^3.10^5$$

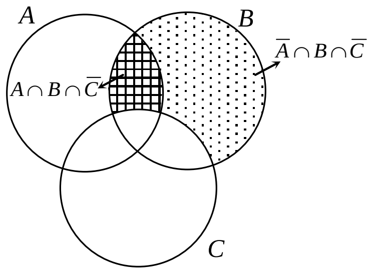
64. **(C)**

$P(B\cap C)+P(A\cap B\cap \overline{C})+P(\overline{A}\cap B\cap \overline{C})=P(B)$

$P(B\cap C)+\frac{1}{3}+\frac{1}{3}=\frac{3}{4}$

$P(B\cap C)=\frac{3}{4}-\frac{2}{3}=\frac{9-8}{12}=\frac{1}{12}$

$P(\overline{B}\cup \overline{C})=P(\overline{B\cap C})=1-P(B\cap C)=1-\frac{1}{12}=\frac{11}{12}$



65. **(A)**

Let $A=\{a_1, a_2, a_3 \ldots, a_{2n}\}$

So $P\cap Q$ can be chosen out of $2n$ is $= {}^{2n}C_3$

If $a_i (1\le i\le 2n)$

and (i) $a_i\in P$ and $a_i\in Q$

(ii) $a_i\in P$ and $a_i\notin Q$

(iii) $a_i\notin P$ and $a_i\in Q$

(iv) $a_i\notin P$ and $a_i\notin Q$

Let $a_1, a_2, a_3\in P\cap Q$

So number of ways of remaining elements = 3^{2n-3}

Hence number of ways in which $P\cap Q$ contains exactly three elements $= {}^{2n}C_3\times 3^{2n-3}$

66. **(B)**

$x^2+6x+8>0$ … (i) and $x^2+6x+8\neq 1$ … (ii)

$2x^2+2x+3>0$ … (iii) and $2x^2+2x+3\neq 1$ … (iv)

$x^2-2x>0$ … (v)

Then $\log_{2x^2+2x+3}(x^2-2x)=1 \Rightarrow x^2-2x=2x^2+2x+3$

$\Rightarrow x^2+4x+3=0 \Rightarrow x=-1, -3$

$x=-1$ satisfies the equation.

67. **(C)**

$\sqrt{D}=\sqrt{a^2+b^2+c^2}=\sqrt{a^2+b^2+a^2b^2}$

Let $b=a+1$

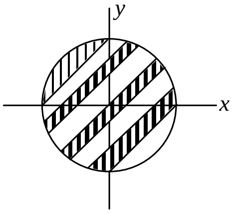
$=\sqrt{a^2+(a+1)^2+a^2(a+1)^2}=\sqrt{(a^2+a)^2+2(a^2+a)+1}=a^2+a+1=a(a+1)+1$

always an odd integer.

68. **(B)**

Area of circle is $\pi(3\pi)^2 = 9\pi^3$

By symmetry required area $= \dfrac{9\pi^3}{2}$



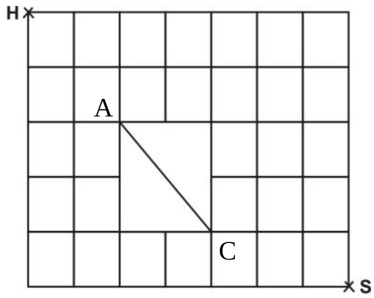
69. **(D)**

Number of ways from H to $A = \dfrac{4!}{2!2!} = 6$

Number of ways from A to $C = 1$

Number of ways from C to $S = \dfrac{4!}{1!3!} = 4$

$\therefore$ Total ways = 6 × 4 = 24



70. **(C)**

n^{th} term of 1 + 3 + 6 + 10 + ….. is $= \dfrac{n(n+1)}{2}$

$$\therefore T_n = \cot^{-1}\left(2a^{-1} + \frac{n(n+1)a}{2}\right) = \cot^{-1}\left(\frac{4+n(n+1)a^2}{2a}\right)$$

$$= \tan^{-1}\left(\frac{2a}{4+n(n+1)a^2}\right) = \tan^{-1}\left(\frac{\frac{a}{2}}{1+\frac{na}{2}\frac{(n+1)a}{2}}\right) = \tan^{-1}\left(\frac{\frac{(n+1)a}{2} - \left(\frac{na}{2}\right)}{1+\left(\frac{(n+1)a}{2}\right).\left(\frac{na}{2}\right)}\right)$$

$$= \tan^{-1}\left((n+1)\frac{a}{2}\right) - \tan^{-1}\left(\frac{na}{2}\right)$$

Put n =1, 2, 3,….., n we have,

$$S_n = \tan^{-1}\frac{(n+1)a}{2} - \tan^{-1}\frac{a}{2}$$

$$\Rightarrow \quad S_\infty = \frac{\pi}{2} - \tan^{-1}\frac{a}{2} = \cot^{-1}\left(\frac{a}{2}\right)$$

71. **(D)**

$f(x) = 1 - x - x^3 \Rightarrow f'(x) = -1 - 3x^2$, which is –ve $\forall x \in R \Rightarrow f$ is decreasing

$f(f(x)) = 1 - f(x) - f^3(x)$

$\therefore \quad f(f(x)) > f(1-5x)$

Since, $f(x)$ is decreasing hence

$\therefore \quad f(x) < 1 - 5x$

$\Rightarrow \quad 1 - x - x^3 < 1 - 5x$

$\Rightarrow \quad x \in (-2, 0) \cup (2, \infty)$

72. **(C)**

$$I=\int_0^{\pi} x\left(\sin(\cos^2 x)\cos(\sin^2 x)\right)dx$$

$$I=\int_0^{\pi}(\pi-x)\sin(\cos^2 x)\cos(\sin^2 x)dx$$

$$2I=\pi\int_0^{\pi}\sin(\cos^2 x)\cos(\sin^2 x)dx$$

$$I=\pi\int_0^{\pi/2}\sin(\cos^2 x)\cos(\sin^2 x)dx \qquad \ldots\text{(i)};$$

$$I=\pi\int_0^{\pi/2}\sin\left(\sin^2 x\right)\cos\left(\cos^2 x\right)dx \qquad \ldots\text{(ii)}$$

Add (i) and (ii) $\Rightarrow 2I=\pi\int_0^{\pi/2}(\sin 1)dx \Rightarrow I=\frac{\pi^2}{4}(\sin 1)$

73. **(C)**

Clearly $(c-a)^2\left(2b^2-2ac\right)=0$

$\Rightarrow a, b, c$ are in G.P. $(\because\ a<b<c)$

74. **(A)**

LHS achieves its maximum value at $x = 1$ (in the domain of the equation i.e., $x > 0$)
Maximum value of LHS =1 at $x = 1$
RHS achieves it minimum value at $x = 1$ (in the domain of equation i.e., $x > 0$)
Minimum value of RHS = 1 at $x = 1$
Hence only possible value of x satisfying the given equation is 1.

75. **(A)**

$$\left(x-y^2\right)dx+y\left(5x+y^2\right)dy=0$$

$$\frac{dy}{dx}=\frac{y^2-x}{y\left(5x+y^2\right)}.$$

Let $y^2=v$

$$\frac{2y\,dy}{dx}=2\left(\frac{y^2-x}{5x+y^2}\right)$$

$$\frac{dv}{dx}=2\left(\frac{v-x}{5x+v}\right) \qquad v=kx$$

$$k+x\frac{dk}{dx}=2\left(\frac{kx-x}{5x+kx}\right)$$

$$x\frac{dk}{dx}=-\frac{\left(k^2+3x+2\right)}{k+5}$$

$$\int \frac{(5+k)}{(k+1)(k+2)}dx = \int -\frac{dx}{x}$$

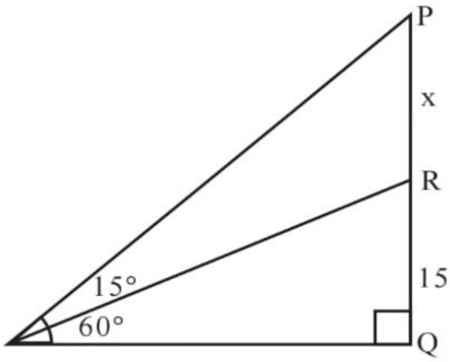
$$\int \left(\frac{4}{k+1} - \frac{3}{k+2}\right)dk = -\int \frac{dx}{x}$$

$$4\ln(k+1) - 3\ln(k+2) = -\ln x + \ln c$$

$$\ln\frac{(k+1)^4}{(k+2)^3} = -\ln x + \ln c$$

$$C(y^2+2x)^3 = (y^2+x)^4$$

76. **(A)**



$$\frac{15}{AQ} = \tan 60^{\circ} \qquad ...(1)$$

$$\frac{15+x}{AQ} = \tan 75^{\circ} \qquad ...(2)$$

$$\frac{(1)}{(2)} \Rightarrow x = 10\sqrt{3}$$

So, $PQ = 5\left(2\sqrt{3}+3\right)$m

77. **(D)**

p	q	$\sim p$	$\sim q$	$\sim p \vee q$
T	T	F	F	T
T	F	F	T	F
F	T	T	F	T
F	F	T	T	T

Options

1	2	3	4
T	T	T	T
T	F	T	T
F	T	T	T
F	T	F	T

78. **(D)**

X	0	1	2	3
***P*(*X*)**	a	b	0.3	$2b$

Let $P(X=0)=a$ and $P(X=1)=b$

Given, $P(X=3)=2P(X=1)=2b$

$P(X=2)=0.3$

$\Sigma P_i = 1 = a+b+0.3+2b$

$\Rightarrow a+3b=0.7$

$\Sigma P_i X_i = 0+b+0.6+6b = 1.3 \quad [\because \Sigma P_i X_i = 1.3]$

$7b = 0.7 \Rightarrow b = 0.1$

$\because a = 0.7-3(0.1)$

$= 0.7-0.3 = 0.4$

$\because P(X=0)=0.4$

79. **(B)**

Equation of tangent of $y=x^2$ be

$tx = y+at^2 \quad ....(1)$

$y = tx - \frac{t^2}{4}$

Solve with $y=-(x-2)^2$

$tx - \frac{t^2}{4} = -(x-2)^2$

$x^2 + x(t-4) - \frac{t^2}{4} + 4 = 0$

$D = 0$

$(t-4)^2 - 4\cdot\left(4-\frac{t^2}{4}\right) = 0$

$t^2 - 4t = 0$

$t=0$ or $t=4$

From eq. (1), required common tangent is

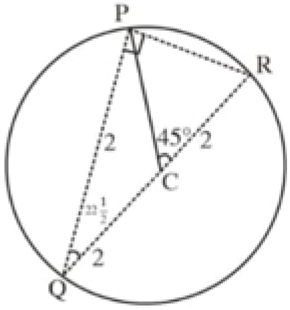
$y = 4(x-1)$

80. **(B)**

$x^2+y^2-x+2y = \frac{11}{4}$

$\left(x-\frac{1}{2}\right)^2 + (y+1)^2 = (2)^2$

Or ΔPQR



$PR = 4\sin\frac{\pi}{8}$

$PQ = 4\cos\frac{\pi}{8}$

Area $= \frac{1}{2}\times 16\times \sin\frac{\pi}{8}\cdot\cos\frac{\pi}{8}$

$= 4\sin\left(\frac{\pi}{4}\right) = 4\times\frac{1}{\sqrt{2}} = 2\sqrt{2}$

81. **(1)**

Solving the lines we get $z = -\bar{z} \Rightarrow z = i$

$|\alpha - i| = 2$; $\alpha = 2e^{i\theta} + i$, put it in the 2nd line, we get

cos θ – sin θ = 0

$\Rightarrow \quad \theta = \frac{\pi}{4}$

$\alpha = i \pm 2e^{\frac{i\pi}{4}}$

$\therefore \quad x = \pm\sqrt{2}$

$\Rightarrow \quad [|x|] = 1$

Ans. 1

82. **(2)**

Let P be (5cos θ, 4sin θ); Q be (–5sin θ, 4cos θ)

Equation of tangent at P $\frac{x}{5}\cos\theta + \frac{y}{4}\sin\theta = 1$ ..... (i)

Equation of tangent at Q $-\frac{x}{5}\sin\theta + \frac{y}{4}\cos\theta = 1$ ..... (ii)

Solving (i) and (ii) $\Rightarrow$ $R \equiv (5(\cos\theta - \sin\theta),\ 4(\sin\theta + \cos\theta))$

$\therefore \quad m : n$ is 1 : 1

$\Rightarrow \quad m + n = 2$

Alternate:

Let $P(5,\ 0)$; $Q(0,\ 4)$

$\Rightarrow \quad R(5,\ 4)$

Intersection of CR and PQ is $\left(\frac{5}{2}, 2\right)$, which is mid-point of CR

$\Rightarrow \; m:n=1:1 \;\Rightarrow\; m+n=2$

Ans. 2

83. (7)

$$I=\int_0^1 \underset{\text{I}}{\left(1-x^4\right)^7} \cdot \underset{\text{II}}{1}\, dx$$

$$=[x(1-x^4)^7]_0^1+7\times 4\int_0^1 x(1-x^4)^6 x^3 dx$$

$$=-28\int_0^1 (1-x^4)(1-x^4)^6\,dx+28\int_0^1 (1-x^4)^6 dx=-28I+28\int_0^1 (1-x^4)^6 dx$$

$$29I=28\int_0^1 (1-x^4)dx$$

$$\frac{29\int_0^1 (1-x^4)^7 dx}{4\int_0^1 (1-x^4)^6 dx}=7$$

Ans. 7

84. (6)

$$f'(x)=-a\pi\sin(\pi x)$$

$$\Rightarrow\; f'\left(\frac{1}{2}\right)=-a\pi\sin\frac{\pi}{2}=-a\pi=\pi \;\Rightarrow\; a=-1$$

$$\int(a\cos\pi x+b)dx=\left(\frac{a\sin\pi x}{\pi}+bx\right)_{1/2}^{3/2}=\left(\frac{-a}{\pi}+\frac{3b}{2}\right)-\left(\frac{a}{\pi}+\frac{b}{2}\right)$$

$$\Rightarrow\; -\frac{2a}{\pi}+b=\frac{2}{\pi}+1 \;\Rightarrow\; b=1$$

So, $\frac{-12}{\pi}\left(\sin^{-1}(-1)+\cos^{-1}1\right)=\frac{-12}{\pi}\left(-\frac{\pi}{2}+0\right)=6$

Ans. 6

85. (0)

Replace x by $-x$

$$2f(-x)+f(x)=\frac{1}{x}\sin\left(x-\frac{1}{x}\right)$$

and solving with the given equation, we get

$$3f(x)=\frac{1}{x}\sin\left(x-\frac{1}{x}\right)$$

$\Rightarrow \ I = \frac{1}{3}\int_{1/e}^{e} \frac{1}{x}\sin\left(x - \frac{1}{x}\right)dx \quad$ put $x = \frac{1}{t}$

$I = \frac{1}{3}\int_{e}^{1/e} t\sin\left(\frac{1}{t} - t\right)\left(-\frac{1}{t^2}\right)dt = -\frac{1}{3}\int_{1/e}^{e} \frac{1}{t}\sin\left(t - \frac{1}{t}\right)dt = -I$

$\Rightarrow \ I = 0$

Ans. 0

86. **(7)**

Probability of hitting the target in one fire $p = \frac{1}{5}$

Then probability of hitting the target at least once in n fires

$= 1 -$ (probability of not hitting the target)

$= 1 - \left(\frac{4}{5}\right)^n > \frac{3}{4}$

$\Rightarrow \ \left(\frac{4}{5}\right)^n < \frac{1}{4}$ as $\left(\frac{4}{5}\right)^6 > \frac{1}{4}$ and $\left(\frac{4}{5}\right)^7 < \frac{1}{4}$

The least value of $n = 7$

Ans. 7

87. **(1)**

$\sum_{r=0}^{200} \alpha_r x^r = \sum_{r=0}^{200} \beta_r (1+x)^r$

$\alpha_0 + \alpha_1 x + \alpha_2 x^2 + \ldots\ldots\ldots + \alpha_{200} x^{200}$

$= \beta_0 + \beta_1(1+x) + \ldots\ldots + \beta_{200}(1+x)^{200}$

Equating coefficient of x^{100}, we get $\alpha_{100} = {}^{100}C_{100} + {}^{101}C_{100} + \ldots\ldots\ldots {}^{200}C_{100} = {}^{201}C_{101}$

Similarly we can find $\alpha_{101}, \ldots\ldots\ldots \alpha_{200}$.

$\sum_{r=100}^{200} \alpha_r = {}^{201}C_{101} + {}^{201}C_{102} + \ldots\ldots\ldots {}^{201}C_{201}$

$A = 2^{200}$

When A is divided by 15 remainder is 1.

Ans. 1

88. **(4)**

In the first round all integers, which leave remainder 1 when divided by 15, will be marked, last number of this category is 991. Next number will be 991 + 15 – 1000 = 6. That means in second round all integers, which leave remainder 6 when divided by 15 will be marked.

In short numbers of the form $5k + 1$ will be marked.

So total no. unmarked are $\Rightarrow \frac{1000}{5} = 200 = A$

$\therefore \quad A = 200$ and $\dfrac{A}{50} = 4$.

Ans. 4

89. **(7)**

$x_1x_2x_3x_4x_5 = 1$

$$g(x_1).g(x_2)......g(x_5) = (x_1^2 - 2)(x_2^2 - 2)....(x_5^2 - 2)$$

$$= (x_1 - \sqrt{2})(x_2 - \sqrt{2}).....(x_5 - \sqrt{2})(x_1 + \sqrt{2}).....(x_1 + \sqrt{2})$$

$$= -P(-\sqrt{2}) \times -P(\sqrt{2}) = P(\sqrt{-2})P(\sqrt{2})$$

$$= (4\sqrt{2} + 3)(4\sqrt{3} - 3)$$

$$= 23$$

90. **(7)**

$1 + \alpha + \alpha^2 + ... + \alpha^7 = 0$

$\alpha = 7^{\text{th}}$ root of unity.