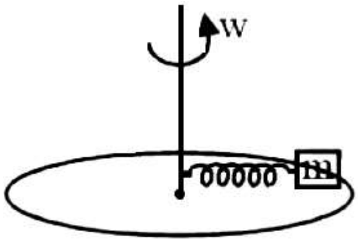
PART (A) : PHYSICS

Answer Key & Solution

1. (D)



FBD of m in frame of disc/–

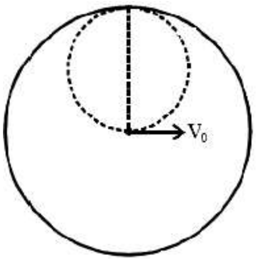
$$k\Delta \ell = m\omega^2 \left(\ell_0 + \Delta \ell\right)$$

$$\Delta \ell = \frac{m\omega^2 \ell_0}{k - m\omega^2} \approx \frac{m\omega^2 \ell_0}{k}$$

$$\frac{\Delta \ell}{\ell_0} = \text{Relative change } = \frac{m\omega^2}{k}$$

2. (B)

Top vies of solenoid

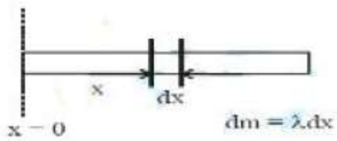


Maximum possible radius of electron $= \frac{R}{2}$

$$\therefore \frac{R}{2} = \frac{mv}{qB} = \frac{mv_{max}}{e(\mu ni)}$$

$$V_{max} = \frac{R}{2}\frac{e\mu_0 ni}{m}$$

3. (D)



$$x_{cm} = \frac{\int x dm}{\int dm} = \frac{\int (\lambda dx) x}{\int dm}$$

$$= \frac{\int_0^L \left(a + \frac{bx^2}{L^2}\right) x\,dx}{\int_0^L \left(a + \frac{bx^2}{L^2}\right) dx}$$

$$= \frac{\frac{aL^2}{2} + \frac{b}{L^2}\cdot\frac{L^4}{4}}{aL + \frac{b}{L^2}\cdot\frac{L^3}{3}}$$

$$= \frac{\left(\frac{4a+2b}{8}\right)L}{\frac{(3a+b)}{3}} = \frac{3}{4}\frac{(2a+b)L}{(3a+b)}$$

4. (A)

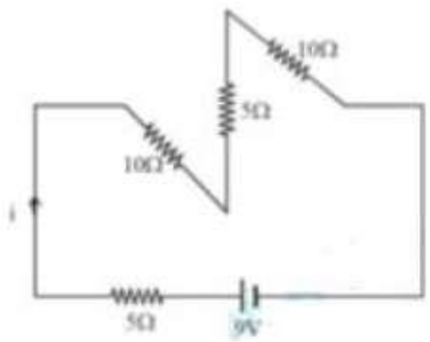
Direction of polarization $= E = \hat{k}$

Direction of propagation $= E \times B = \frac{\hat{i}+\hat{j}}{\sqrt{2}}$

$\therefore E \times B = \frac{\hat{i}+\hat{j}}{\sqrt{2}}$

$B = \frac{\hat{i}-\hat{j}}{\sqrt{2}}$

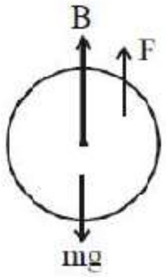
5. (C)



$i = \frac{9}{(5+10+5+10)} = \frac{9}{30}A$

6. (B)

FBD of droplet



$$B + F = mg$$

$$B = \left(\frac{2}{3}\pi R^3\right)\rho g$$

$$F = T(2\pi R)$$

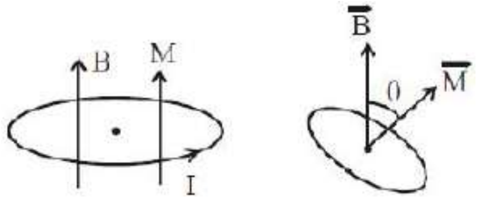
$$m = d\left(\frac{4}{3}\pi R^3\right)$$

$$\left(\frac{2}{3}\pi R^3\right)\rho g + T(2\pi R) = d\left(\frac{4}{3}\pi R^3\right)g$$

$$T(2\pi R) = \left(\frac{2}{3}\pi R^3\right)g[2d-\rho]$$

$$R = \sqrt{\frac{3T}{(2d-\rho)g}}$$

7. (B)



$$\vec{T} = \vec{M}\times\vec{B} = -MB\sin\theta$$

$$I\alpha = -MB\sin\theta$$

For small θ,

$$\alpha = -\frac{MB}{I}\theta$$

$$\omega = \sqrt{\frac{MB}{I}} = \sqrt{\frac{(i)(\pi R^2)B}{\left(\frac{mR^2}{2}\right)}}$$

$$\omega = \sqrt{\frac{2i\pi B}{m}}$$

$$\therefore = T = \frac{2\pi}{\omega} = \sqrt{\frac{2\pi m}{iB}}$$

8. (D)

$$\frac{nv}{2\ell}=420$$

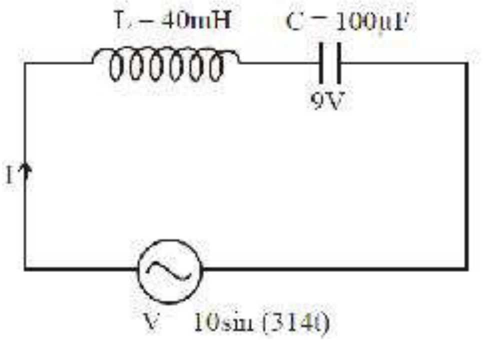
$$\frac{(n+1)v}{2\ell}=490$$

$$\frac{v}{2\ell}=70$$

$$\ell=\frac{v}{140}=\frac{1}{140}\sqrt{\frac{540}{6\times10^{-3}}}=\frac{1}{140}\sqrt{90\times10^{3}}$$

$$\ell=\frac{300}{140}=2.142$$

9. (A)

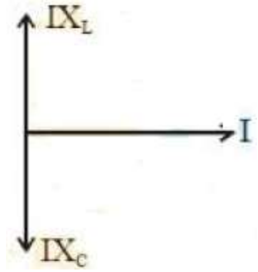


$$X_L=\omega L=314\times40\times10^{-3}=12.56\Omega$$

$$X_C=\frac{1}{\omega C}=\frac{1}{314\times100\times10^{-6}}$$

$$=\frac{10^4}{314}=31.84\Omega$$

Phasor

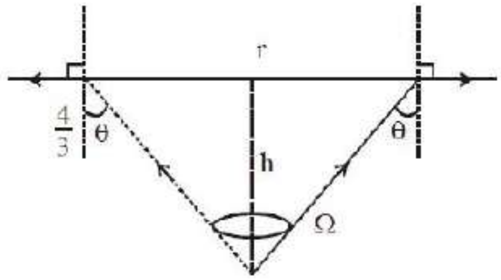


$$V_m=I_m\left(X_C-X_L\right)$$

$$10=I_m\left(31.84-12.56\right)$$

$$I_m=\frac{10}{19.28}=0.52A$$

10. (A)



$$\frac{4}{3}\sin\theta = 1\sin 90^{\circ}$$

$$\sin\theta = \frac{3}{4}$$

Area of sphere in which light spread $= 4\pi R^2$

$$\Omega = 2\pi(1-\cos\theta)$$

$$\Omega = 2\pi\left(1-\frac{\sqrt{7}}{4}\right)$$

$P \rightarrow 4\pi$ steradians

$$P^1 \rightarrow \frac{P}{4\pi}(1-\cos\theta)$$

$$\text{Ratio } = \frac{P^1}{P} = \frac{2\pi(1-\cos\theta)}{4\pi} = \frac{(1-\cos\theta)}{2} = \frac{1-\frac{\sqrt{7}}{4}}{2}$$

$$= \frac{0.33}{2} = 0.17$$

11. (A)

$$\lambda = \frac{1}{\sqrt{2}\pi n_v d^2}$$

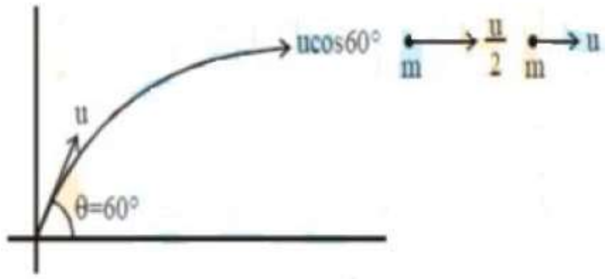
$$\tau = \frac{\lambda}{v} = \frac{1}{\sqrt{2}\pi n_v d^2 v} = \frac{1}{\sqrt{2}\pi n_b d^2}\sqrt{\frac{M}{3RT}}$$

$$\frac{\tau_1}{\tau_2} = \sqrt{\frac{M_1}{M_2}}\frac{d_2^2}{d_1^2}$$

$$= \sqrt{\frac{40}{140}}\frac{(0.1)^2}{(0.07)^2}$$

$$= 1.09$$

12. (C)



$$x = u_x t + \frac{1}{2} a_x t^2$$

$$y = u_t t + \frac{1}{2} a_y t^2$$

$$32 = 0 \times t + \frac{1}{2}(4)(t)^2$$

$$t^2 = 16$$

$$t = 4\,\text{sec}$$

$$x = 3 \times 4 + \frac{1}{2} \times 6 \times 4^2$$

$$= 12 + 48 = 60\,\text{m}$$

13. (C)

By momentum conservation,

$$\frac{mu}{2} + mu = 2mv^1$$

$$v^1 = \frac{3V}{4}$$

Range after collision $= \frac{3v}{4}\sqrt{\frac{2H}{g}}$

$$= \frac{3v}{4}\sqrt{\frac{2.u^2 \sin^2 60^o}{g^2}}$$

$$= \frac{3}{4}\frac{\sqrt{3}}{2}.\frac{u^2}{g} = \frac{3\sqrt{3}u^2}{8g}$$

14. (D)

1 Rydberg energy $= 13.6\,\text{eV}$

So, ionization energy $= (136Z^2)\,\text{eV}$

$$= 9 \times 13.6\,\text{eV}$$

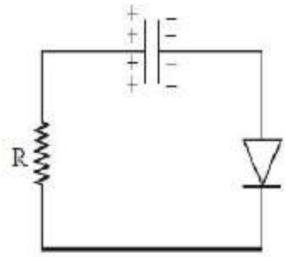
$$Z = 3$$

$$\frac{1}{\lambda} = RZ^2\left(\frac{1}{1^2} - \frac{1}{3^2}\right) = 1.09 \times 10^7 \times 9 \times \frac{8}{9}$$

$\lambda = 11.4\,\text{nm}$

15. (A)

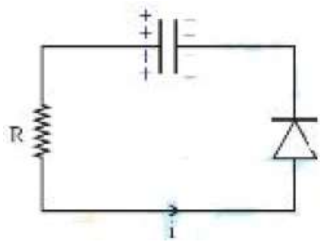
For (A)



No current flows

Hence $Q_A = CV$

For (B)



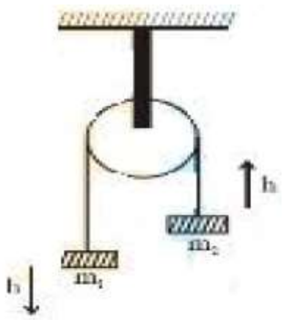
$$i = \frac{V}{R} e^{-\frac{t}{RC}}$$

$$q = CVe^{\frac{t}{RC}}$$

At $t = CR$

$$Q_B = CVe^{-1} = \frac{CV}{e}$$

16. (B)



By sing work energy theorem

$$Wg = \Delta KE$$

$$(m_1 - m_2)gh = \frac{1}{2}(m_1 + m_2)V^2 + \frac{1}{2}I\omega^2$$

$$(m_1 - m_2)gh = \frac{1}{2}(m_1 + m_2)(\omega R)^2 + \frac{1}{2}I\omega^2$$

$$(m_1 - m_2)gh = \frac{\omega^2}{2}\left[(m_1 + m_2)R^2 + I\right]$$

$$\omega = \sqrt{\frac{2(m_1 - m_2)gh}{(m_1 + m_2)R^2 + I}}$$

17. (A)

$$V_e = \sqrt{\frac{2GM}{R}} \text{ (Escape velocity)}$$

$$V_A = \sqrt{\frac{2GM}{R}}$$

$$V_B = \sqrt{\frac{2G[M/2]}{R/2}} = \sqrt{\frac{2GM}{R}}$$

$$\frac{V_A}{V_B} = 1 = \frac{n}{4} \Rightarrow n = 4$$

18. (D)

$$\frac{\text{Energy stored}}{\text{Volume}} = \frac{1}{2}\frac{(\text{stress})^2}{Y}$$

$$\frac{u_1}{u_2} = \frac{1}{4} \Rightarrow 4u_1 = u_2$$

$$4\frac{1}{2Y}\left[\frac{W.4}{\pi d_1^2}\right]^2 = \frac{1}{2Y}\left[\frac{W.4}{\pi d_2^2}\right]^2$$

$$4 = \left(\frac{d_1}{d_2}\right)^4$$

$$\Rightarrow \frac{d_1}{d_2} = \sqrt{2} : 1$$

19. (D)

Given: $z = \frac{(a)^2 (b)^{2/3}}{(c)^{1/2} (d)^3}$

Percentage error in Z_1

$$=\frac{\Delta Z}{Z}=\frac{Z\Delta\alpha}{\alpha}+\frac{2}{3}\frac{\Delta b}{b}+\frac{1}{2}\frac{\Delta c}{c}+\frac{3\Delta d}{d}$$

$$=(2\times 2)+\left(\frac{2}{3}\times 1.5\right)+\left(\frac{1}{2}\times 4\right)+(3\times 2.5)=14.5\%$$

20. (A)

$$a=\frac{eE}{m}$$

$$v+u+at=\left(\frac{eE}{m}\right)t$$

$$\lambda=\frac{h}{mv}$$

$$\frac{d\lambda}{dt}=\frac{-(hm).\frac{dv}{dt}}{(mv)^2}=-\frac{ah}{mv^2}-\frac{h}{|e|Et^2}$$

21. (1819)

$$(PV)^{\gamma}=\text{constant}$$

$$(TV)^{\gamma-1}=C$$

$$300\times V^{\frac{7}{5}-1}=T_2\left(\frac{V}{16}\right)^{\frac{7}{5}-1}$$

$$300\times 2^{4\times\frac{2}{5}}=T_2$$

Isobaric process

$$V=\frac{nRT}{P}$$

$$V_2=kT_2 \quad \text{............ (1)}$$

$$2V_2=KT_f \quad \text{........... (2)}$$

$$\frac{1}{2}=\frac{T_2}{T_f}\Rightarrow T_f=2T_2$$

$$T_f=2\times 300\times 2^{\frac{8}{5}}=1819$$

22. (40.00)

In balancing

$$\frac{R}{S}=\frac{25}{75}$$

New resistance $R^1=\frac{\rho\,\ell}{A}$

$$= \frac{\rho \times \frac{\ell}{2}}{\frac{A}{4}} = \frac{\rho\,\ell}{2} \times 4A$$

$R^1 = 2R$

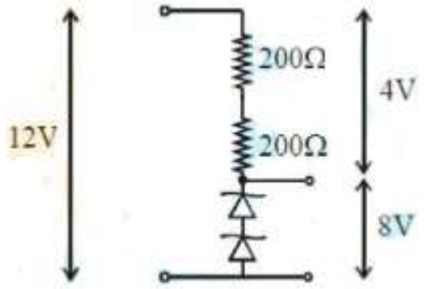
$$\frac{2R}{S} = \frac{\ell'}{100-\ell'}$$

$$2 \times \frac{1}{3} = \frac{\ell'}{100-\ell'} = 3\ell' = 200 - 2\ell'$$

$5\ell' = 200$

$\ell' = 40$

23. (12.00)



Current in circuit $\frac{4}{400} = \frac{1}{100}$ A

So power dissipated in each diode = VI

$= 4 \times \frac{1}{100}$ W

$= 40 \times 10^{-3}$ mW

24. (750.00)

The length of the screen used portion for 15 fringes, and also for ten fringes

$$15 \times 500 \times \frac{D}{d} = 10 \times \frac{\lambda D}{d}$$

$15 \times 50 = \lambda$

$\lambda = 750\,\text{nm}$

25. (48.00)

The flux passes through ABCD (x – y) plane is zero, because electric field parallel to surface. Flux of the electric field through surface BCGF (y - z)

At BCGF (electric field) $\Rightarrow \vec{E} = 12\hat{i} - (y^2 + 1)\hat{j}\,(x = 3m)$

Flux $\phi_{II} = 12 \times 4 = 48 Nm^2 / C$

So $\phi_I - \phi_{II} = 0 - 48 = -48\, Nm^2/C$

26. (3.00)

Magnetic field in solenoid $B = \mu_0 ni$

$$\Rightarrow \frac{B}{\mu_0} = ni$$

(where $n =$ number of turns per unit length)

$$\Rightarrow \frac{B}{\mu_0} = \frac{Ni}{L} \Rightarrow 3\times10^3 = \frac{100i}{10\times10^{-2}} \qquad \Rightarrow i = 3A$$

27. (20.00)

Number of undecayed atom after time t_2; $\frac{N_0}{3} = N_0 e^{-\lambda t_2}$ ……….. (i)

Number of undecayed atom after time t_1 : $\frac{2N_0}{3} = N_0 e^{-\lambda t_1}$ ……….. (ii)

Dividing (ii) by (i), we get

$$2 = e^{\lambda(t_2 - t_1)}$$

$$\Rightarrow In2 = \lambda(t_2 - t_1)$$

$$\Rightarrow t_2 - t_1 =; \ln 2/\lambda$$

28. (2.00)

$$\upsilon = \alpha\sqrt{x},$$

$$\Rightarrow \frac{dx}{dt} = \alpha\sqrt{x} \Rightarrow \frac{dx}{\sqrt{x}} = \alpha dt$$

Integrating both sides,

$$\int_0^x \frac{dx}{\sqrt{x}} = \alpha\int_0^t dt; \left[\frac{2\sqrt{x}}{1}\right]_0^x = \alpha[t]_0^t$$

$$\Rightarrow 2\sqrt{x} = \alpha t \Rightarrow x = \frac{\alpha^2}{4}t^2$$

29. (5.00)

Surface tension, $T = \frac{F}{\ell} = \frac{F}{\ell}.\frac{\ell}{\ell}.\frac{T^2}{T^2}$

(As, $F.\ell = K$ (energy). $\frac{T^2}{\ell^2} = V^{-2}$)

Therefore, surface tension $= \left[KV^{-2}T^{-2}\right]$

30. (22.00)

Apply work energy Theorem

$$W_{hand} + W_{gravity} = \Delta K$$

$$F(0.2) - (0.2)(10)(2.2) = 0 \Rightarrow F = 22N$$

PART (B) : CHEMISTRY

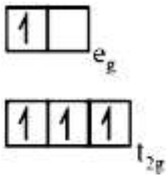
Answer Key & Solution

31. (B)

I $\left[Cr\left(H_2O\right)_6\right]^{2+}$

$Cr^{+2} \Rightarrow [Ar]3d^4$

$H_2O \rightarrow$ Weak field ligand



Unpaired $e^- = 4$

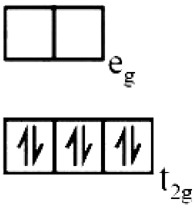
Magnetic moment $= \sqrt{24}BM$

$= 4.89BM$

II $\left[Fe\left(CN\right)_6\right]^{4-}$

$Fe^{+2} \Rightarrow [Ar]3d^6$

$CN^- \rightarrow$ Strong field ligand



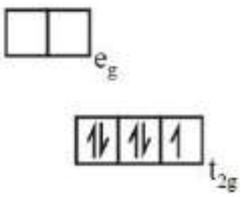
Unpaired $e^- = 0$

Magnetic moment $= 0\,BM$

III $\left[Fe\left(C_2O_4\right)_3\right]^{3-}$

$Fe^{+3} \Rightarrow [Ar]3d^5$

As $\Delta_0 > P$



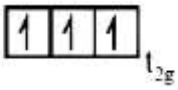
Unpaired $e^- = 1$

Magnetic moment $= \sqrt{3}\,BM$

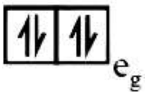
$= 1.73BM$

$IV \left(Et_4N\right)^+ \left[CoCl_4\right]^{2-}$

$Co^{+2} \Rightarrow [Ar]3d^7$



$\left[CoCl_4\right]^{-2} =$



Unpaired electrons = 3

Magnetic moment $= \sqrt{15}\,BM$

$= 3.87\,BM$

Hence order of magnetic moment is

$I > IV > III > II$

32. (A)

IE values indicate, that the metal belongs to *I st* group since second IE is very high

($\because$ only one valence electron)

Metal hydroxide will be of type, MOH.

$MOH + HCl \rightarrow MCl + H_2O$

$(1 mol)(1 mol)$

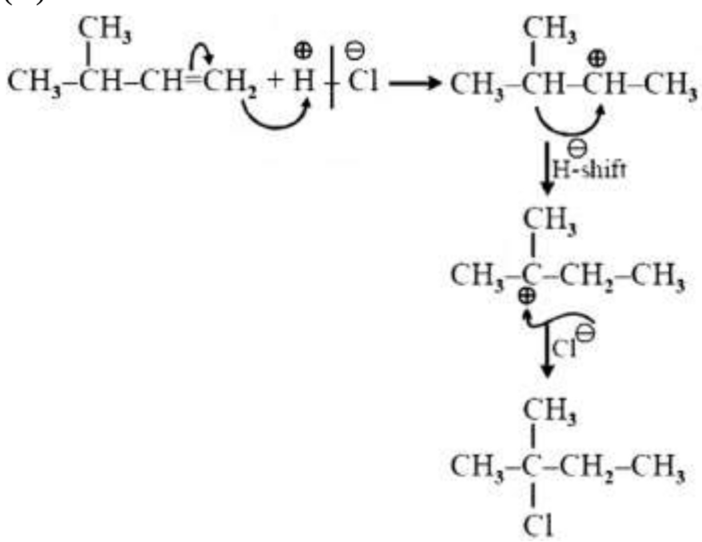
$MOH + \frac{1}{2}H_2SO_4 \rightarrow \frac{1}{2}M_2SO_4 + H_2O$

$(1 mol)\left(\frac{1}{2} mol\right)$

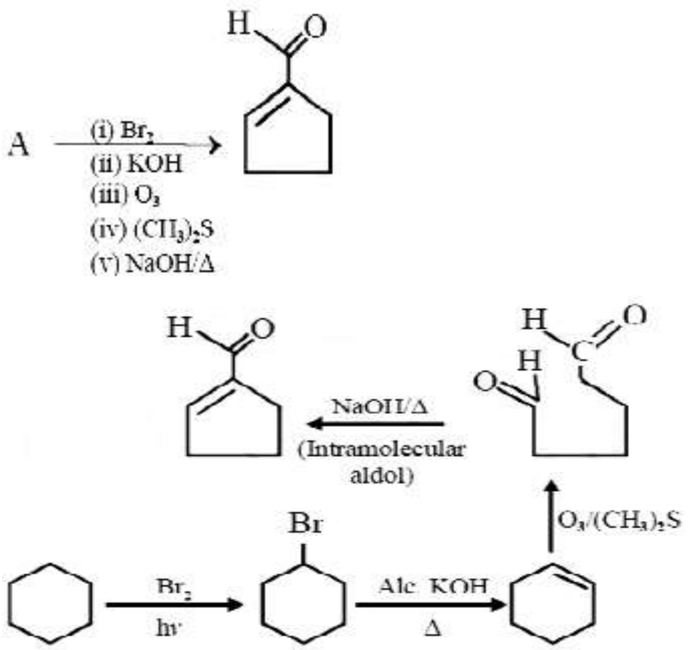
So one mole of HCl required to react with one mole MOH.

So $\frac{1}{2}$ mole of H_2SO_4 required to react with one mole MOH.

33. (A)



34. (C)



35. (D)

Adsorption of Gases will decreases

36. (D)

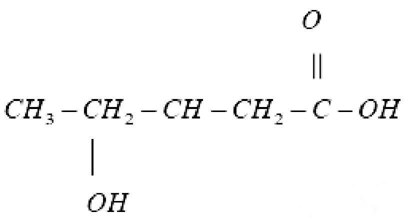
PHBV

Poly β – hydroxy butyrate – co– β – hydroxy valerate

$$\underset{\underset{OH}{|}}{CH_3 - CH} - CH_2 - COOH$$

(3- hydroxy butanoic acid)

+



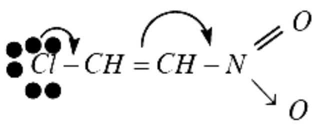
(3-hydroxy pentanoic acid)

37. (C)
Biochemical oxygen demand (BOD) is amount of oxygen required by bacteria to break downorganic waste in a certain volume of water sample

38. (C)
Lithium has highest hydration enthalpy among alkali metals due to its small size.
LiCl is soluble in pyridine because LiCl have more covalent character.
Li does not form ethynide with ethyne.
Both Li and Mg reacts slowly with H_2O

39. (A)
Distilled water have lowest ionic conductance.

40. (D)



Due to –Meffect of $-NO_2$ and +Meffect of Cl more D.B. character between $C-Cl$ bond. So shortest bond length.

41. (A)

$$A \rightleftharpoons B$$

$$K = \frac{[B]}{[A]} = \frac{11}{6}$$

42. (C)

43. (A)

$$Cr(OH)_3(s) \rightleftharpoons Cr^{3+}(aq.) + 3OH^-(aq)$$

(s) (3s)

$$k_{sp} = 27(s)^4 = 6\times10^{-31}$$

$$\Rightarrow [3(s)]^4 = 18\times10^{-31}$$

$\left[OH^-\right] = 3(s) = \left[18 \times 10^{-31}\right]^{1/4}$

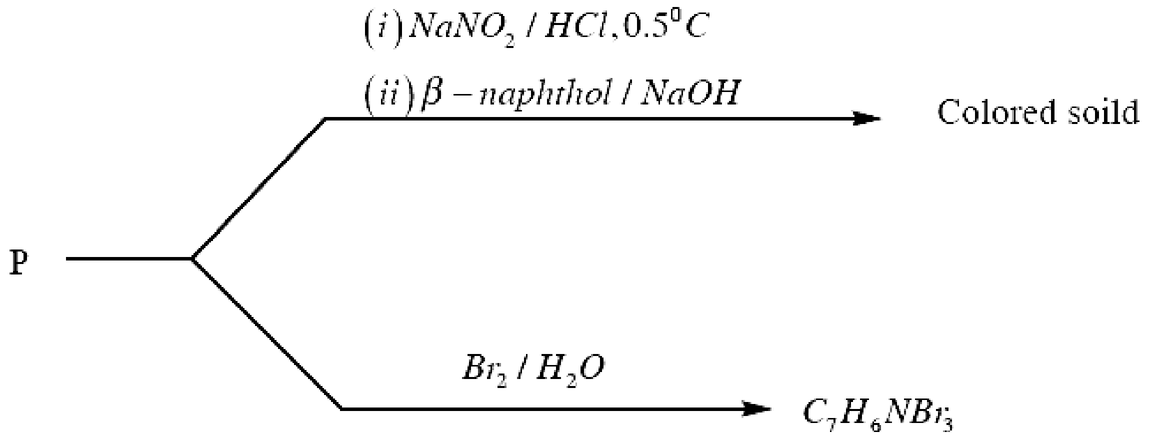
44. (D)

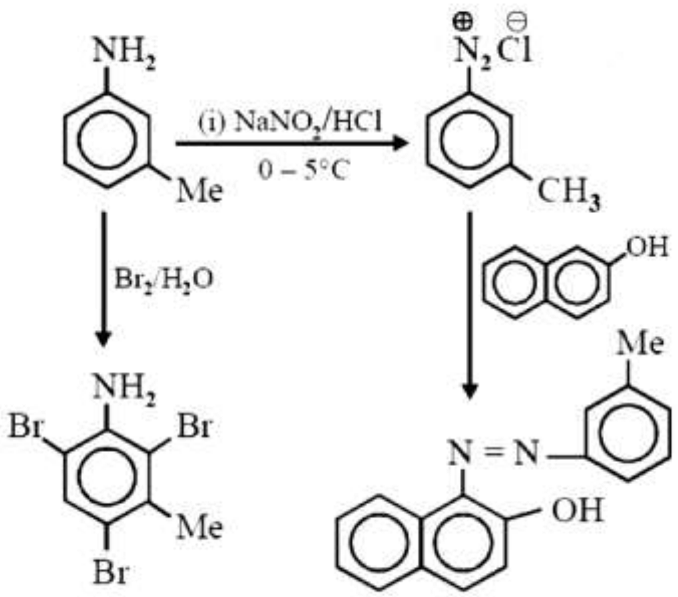
$Zn + 2HCl \rightarrow ZnCl_2 + H_2$

$Zn + 2NaOH \rightarrow Na_2ZnO_2 + H_2$

The ratio of the volume of H_2 is 1 : 1

45. (B)





(colored dye)

46. (D)

Alanine does not show **Biuret test** because **Biuret test** is used for deduction of peptidelinkage & alanine is amino acid. Albumine is protein so have paptide linkage so it gives positive**Biuret test**. Positive **Barfoed test** is shown by monosaccharide but not disaccharide. Positive**Molisch's test** is shown by glucose

47. (B)

$$H_3N_3B_3Cl_3 + 3LiBH_4 \xrightarrow[\text{(T.H.F)}]{\text{In tetrahydrofurame}} H_3N_3B_3H_3 + 3LiCl + 3BH_3.THF$$

(A) (B)

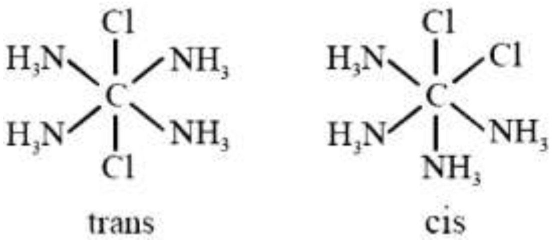
Inorganic Benzene
(Borazine)

$$H_3N_3B_3Cl_3 + 3CH_3MgBr \xrightarrow{\rightarrow} H_3N_3B_3(CH_3)_3 + 3MgBrCl$$

(A) (C)

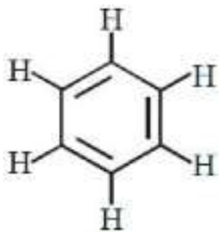
48. (D)

$[Co(NH_3)_4Cl_2]$



cis isomer has $Cl-Co-Cl$ angle of 90^o

49. (D)



Each carbon atom is sp^2 hybridized Therefore each carbon has $3sp^2$ hybrid orbitals. Hence total sp^2 hybrid orbitals are 18.

50. (A)

$$ds = \int \frac{q_{rev}}{T}$$

51. (2.18 to 2.23)

$0 - T_f' = 2 \times 0.5 = 1$

$T_f' = -1^o C = 272\,K$

For gas $P = \dfrac{0.1 \times 0.08 \times 272}{1}$

$P = 2.176\,atm$

$P_1V_1 = P_2V_2$

$2.176 \times 1 = 1 \times V_2$

$V_2 = 2.176 \text{ litre}$

52. (10)

$$\text{ppm} = \frac{10.3 \times 10^{-3}}{1030} \times 10^6 = 10$$

53. (3.98 to 3.99 or –3.98 to –3.99)

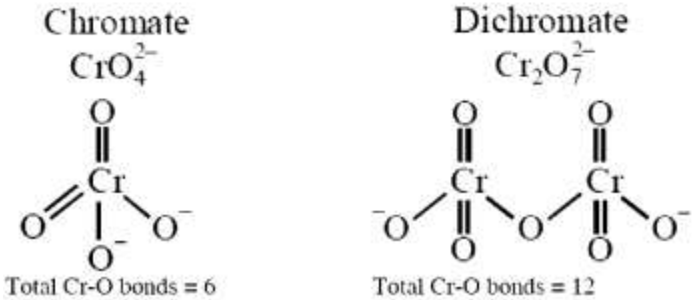
$$\ln\left(\frac{t_1}{t_2}\right) = \frac{-Ea}{R}\left[\frac{1}{T_2} - \frac{1}{T_1}\right]$$

$$\ln\left(\frac{60}{40}\right) = \frac{-Ea}{8.3}\left[\frac{1}{400} - \frac{1}{300}\right]$$

$$E = 0.4 \times 1200 \times 8.3$$

$$= 3.984 \text{kJ / mole}$$

54. (18)



$(4\sigma + 2\pi)$ $(8\sigma + 4\pi)$

Total number of bonds between chromium and oxygen in both structures are 18.

55. (66.66 to 66.67)

56. (84.29)

According to Arrhenius equation : $k = -Ae^{E_a/RT}$ or $\ln\left(\frac{k_2}{k_1}\right) = \frac{E_0}{R}\left(\frac{1}{T_1} - \frac{1}{T_2}\right)$

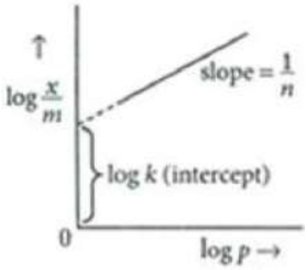
Given : $K_2 / K_1 = 5, T_1 = 27 + 273 = 300\,K$ $T_2 = 42 + 273 = 315\,K$ $R = 8.314\,J\,mol^{-1}$

Putting the value of eq. (i) we get $\ln(5) = \frac{E_a}{8.314}\left(\frac{1}{300} - \frac{1}{315}\right)$ or $1.6094 = \frac{E_a}{8.314}\left(\frac{15}{300 \times 315}\right)$

$$E_a = \frac{8.314 \times 300 \times 315 \times 1.6094}{15} = 84297\,J / \text{mole}$$

57. (48)

Given; slope = 2, intercept $= 0.4771, \log 3 = 0.4771, \frac{x}{m} = ?$



According to Freundlich adsorption isotherm, $\log\frac{x}{m} = \log k + \frac{1}{n}\log p$

Slope $= \frac{1}{n} = 2 \Rightarrow n = 0.5$ Intercept $(\log k) = 0.477 \Rightarrow k = 3,\ \frac{x}{m} = kp^{1/n} = 3\times(4)^2 = 48$

58. (0)

Oxidation state of Ru in $\left[Ru\left(H_2O\right)_6\right]^{2+}$ is $+2 \therefore Ru^{2+}$ in $\left[Ru\left(H_2O\right)_6\right]^{2+}: 4d^6 \Rightarrow t_{2g}^6 e_g^0$, since $\Delta_0 > P$ As number of unpaired electron is zero, therefore, magnetic moment is zero.

59. (18)

Combustion of propane, $CH_3CH_2CH_{3(G)} + 5O_{2(g)} \rightarrow 3CO_{2(g)} + 4H_2O_{(l)}$ combustion of butane,

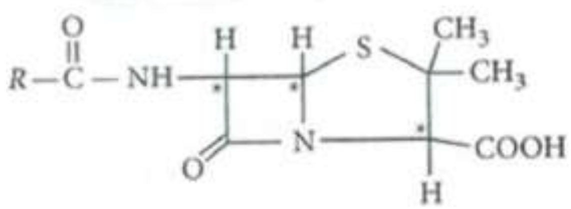
$$CH_3CH_2CH_2CH_{3(g)} + \frac{13}{2}O_{2(g)} \rightarrow 3CO_{2(g)} + 5H_2O_{(l)}$$

Hence, the total number of moles of O_2 required for complete combustion of 1moles of propane and 2 moles of butane $= 5 + 13 = 18$ moles.

60. (3)

Penicillin has 3 chiral centres.

Penicillin has 3 chiral centres.



PART (C) : MATHEMATICS

Answer Key & Solution

61. (B)

$$A=\lim_{x\to 0} x\left[\frac{4}{x}\right]=\lim_{x\to 0} x\left(\frac{4}{x}\right)-x\left\{\frac{4}{x}\right\}=4$$

$f(x)=\left[x^2\right]\sin(\pi x)$ will be discontinuous at nonintegers

$\therefore x=\sqrt{A+1}$ i.e $\sqrt{5}$

62. (A)

$7x+6y-2z=0$ .... (1)

$3x+4y+2z=0$ .... (2)

$x-2y-6z=0$ .... (3)

$$\Delta\begin{vmatrix}7 & 6 & -2\\ 3 & 4 & 2\\ 1 & -2 & -6\end{vmatrix}=0 \Rightarrow \text{Infinite solutions}$$

Now $(1)+(2)\, y=-x$ put in (1), (2) & (3) all will lead to $x=2z$

63. (D)

$x=2\sin\theta-\sin 2\theta$

$$\Rightarrow \frac{dx}{d\theta}=2\cos\theta-2\cos 2\theta=4\sin\left(\frac{\theta}{2}\right)\sin\left(\frac{3\theta}{2}\right)$$

$y=2\cos\theta-\cos 2\theta$

$$\Rightarrow \frac{dy}{d\theta}=-2\sin\theta+2\sin 2\theta=4\sin\frac{\theta}{2}\cos\frac{\theta}{2}$$

$$\Rightarrow \frac{dy}{dx}=\cot\left(\frac{3\theta}{2}\right)\Rightarrow \frac{d^2y}{dx^2}=\frac{-\frac{3}{2}\operatorname{cosec}^2\left(\frac{3\theta}{2}\right)}{4\sin\left(\frac{\theta}{2}\right)\sin\frac{3\theta}{2}}$$

$$\Rightarrow \left(\frac{d^2y}{dx^2}\right)_{\theta=\pi}=\frac{3}{8}$$

64. (B)

Let $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1; a>b;$

$$2b=\frac{4}{\sqrt{3}}\Rightarrow b=\frac{2}{\sqrt{3}}\Rightarrow b^2=\frac{4}{3}$$

Tangent $y = \frac{-x}{6} + \frac{4}{3}$ compare with

$$y = mx \pm \sqrt{a^2m^2 + b^2}$$

$$\Rightarrow m = \frac{-1}{6} \Rightarrow \sqrt{\frac{a^2}{36} + \frac{4}{3}} = \frac{4}{3} \Rightarrow a = 4$$

$$e = \sqrt{1 - \frac{b^2}{a^2}} = \frac{1}{2}\sqrt{\frac{11}{3}}$$

65. (2)

$$ax^2 - 2bx + 5 = 0$$

$$\Rightarrow \alpha = \frac{b}{a}; \alpha^2 = \frac{5}{a} \Rightarrow b^2 = 5a$$

$$x^2 - 2bx - 10 = 0$$

$$\Rightarrow \alpha^2 - 2b\alpha - 10 = 0$$

$$\Rightarrow a = \frac{1}{4} \Rightarrow \alpha^2 = 20; \alpha\beta = -10 \Rightarrow \beta^2 = 5$$

$$\Rightarrow \alpha^2 + \beta^2 = 25$$

66. (B)

Required area = Area of trepezium ABCD –

Area of parabola between $x = \frac{1}{2}$ & $x = \frac{\sqrt{3}}{2}$

$$A = \frac{1}{2}\left(\frac{\sqrt{3}}{2} - \frac{1}{2}\right)\left(\frac{1}{2} + 1 - \frac{\sqrt{3}}{2}\right) - \int_{1/2}^{\sqrt{3}/2}\left(x - \frac{1}{2}\right)^2 dx = \frac{\sqrt{3}}{4} - \frac{1}{3}$$

67. (B)

$$\Sigma P(X) = 1 \Rightarrow K^2 + 2K + K + 2K + 5K^2 = 1$$

$$\Rightarrow 6K^2 + 5K - 1 = 0 \Rightarrow (6K - 1)(K + 1) = 0$$

$$\Rightarrow K = -1(\text{rejected}) \Rightarrow K = \frac{1}{6}$$

$$P(X > 2) = K + 2K + 5K^2 = \frac{23}{36}$$

68. (C)

$$x = \sum_{n=0}^{\infty}(-1)^n \tan^{2n}\theta = 1 - \tan^2\theta + \tan^4\theta + ...$$

$$\Rightarrow x = \cos^2\theta$$

$$y=\sum_{n=0}^{\infty}\cos^{2n}\theta \Rightarrow y=1+\cos^2\theta+\cos^4\theta+.....$$

$$\Rightarrow y=\frac{1}{\sin^2\theta}\Rightarrow y=\frac{1}{1-x}$$

$$\Rightarrow y(1-x)=1$$

69. (A)

$$F'(x)=x^2g(x)=x^2\int_1^x f(u)du \Rightarrow F'(1)=0$$

$$F''(x)=x^2f(x)-2x\int_1^x f(u)du$$

$$F''(1)=1.f(1)-2\times 0$$

$$F''(1)=3$$

$F'(1)=0$ and $F''(1)=3<0$ So, Minimum

70. (B)

$$y^2=8x$$

$$4t_1=-2\Rightarrow t_1=-\frac{1}{2}$$

$$t_1.t_2=-1$$

$$t_2=-\frac{1}{t_1}$$

$$\Rightarrow t_2=2$$

So coordinate of B is (8, 8)

$\therefore$ Equation of tangent at B is

$$8y=4(x+8)\Rightarrow 2y=x+8$$

71. (C)

10 different balls in 4 different boxes.

$$\frac{1}{4^{10}}\left(4!\times\frac{10!}{2!\times 3!\times 0!\times 5!}+4!\times\frac{10!}{2!\times 3!\times 4!}+4!\times\frac{10!}{(2!)^2\times 2!\times (3!)^2\times 2!}\right)$$

$$=\frac{945}{2^{15}}$$

72. (C)

$$A:x\in(-2,2):B:x\in(-\infty,-1]\cup[5,\infty)$$

$\Rightarrow B-A=R-(-2,5)$

73. (D)

$$\frac{dy}{dx}=\frac{xy}{x^2+y^2}$$

Let $y=vx$

$$\frac{dy}{dx}=v+x,\frac{dv}{dx}$$

$$v+x\frac{dv}{dx}=\frac{xvx}{x^2+v^2x^2}=\frac{v}{2+v^2}$$

$$x\frac{dv}{dx}=\frac{v}{1+v^2}-v=\frac{v-v-v^3}{1+v^2}=-\frac{v^3}{1+v^2}$$

$$\int\frac{1+v^2}{v^3}.dv=\int-\frac{dx}{x}$$

$$\Rightarrow\int v^{-3}.dv+\int\frac{1}{v}dv=-\int\frac{dx}{x}$$

$$\Rightarrow\frac{v^{-2}}{-2}+\ell nv=\ell nx+\lambda$$

$$\Rightarrow-\frac{1}{2v^2}+\ell n\left(\frac{y}{x}\right)=-\ell nx+\lambda$$

$$\Rightarrow-\frac{1}{2}\frac{x^2}{y^2}+\ell ny-\ell nx=-\ell nx+\lambda$$

$$\Rightarrow-\frac{1}{2}+0=\lambda\Rightarrow\lambda=-\frac{1}{2}$$

$$\Rightarrow-\frac{1}{2}\frac{x^2}{y^2}+\ell ny+\frac{1}{2}=0\ \text{ at }\ y=e$$

$$\Rightarrow-\frac{1}{2}\frac{x^2}{e^2}+1+\frac{1}{2}=0\Rightarrow\frac{x^2}{2e^2}=\frac{3}{2}\Rightarrow x^2=3e^2$$

74. (A)

$$I=\int\frac{d\theta}{\cos^2\theta(\tan 2\theta+\sec 20\theta)}$$

$$=\int\frac{\sec^2\theta d\theta}{\frac{2\tan\theta}{1-\tan^2\theta}+\frac{1+\tan^2\theta}{1-\tan^2\theta}}=\int\frac{(1-\tan^2\theta)\sec^2\theta d\theta}{(1+\tan\theta)^2}$$

$$\tan\theta=t\Rightarrow\sec^2\theta d\theta=dt$$

$$I=\int\frac{1-t^2}{(1+t)^2}dt=\int\frac{(1-t)(1+t)}{(1+t)^2}dt$$

$= \int \frac{1}{1+t} - \frac{t}{1+t} dt$

$= \ell n|1+t| - \int \left(\frac{1+t}{1+t} - \frac{1}{1+t}\right) dt$

$= \ell n|1+t| - t + \ell n|1+t|$

$= 2\ell n|1+t| - t + C$

$= 2\ell n|1+t| - t + C$

$= 2\ell n|1+t| - t + C$

$= 2\ell n|1+\tan\theta| - \tan\theta + C$

$\lambda = -1, f(\theta) = 1+\tan\theta$

75. (D)

$z = x + iy$ $\qquad |x| + |y| = 4$

$|z| = \sqrt{x^2+y^2} \Rightarrow |z|_{min} = \sqrt{8} \,\&\, |z|_{max} = 4 = \sqrt{16}$

So $|z|$ cannot be $\sqrt{7}$

76. (B)

$p \to (p \wedge \sim q)$ is F $\Rightarrow p$ is T & $p \wedge \sim q$ is F $\Rightarrow q$ is T

$\therefore$ p is T , q is T

77. (C)

$R_1 \to R_1 + R_3 - 2R_2$

$$f(x) = \begin{vmatrix} a+c-2b & 0 & 0 \\ x+b & x+3 & x+2 \\ x+c & x+4 & x+3 \end{vmatrix}$$

$= (a+c-2b)\left((x+3)^2 - (x+2)(x+4)\right)$

$= x^2 + 6x + 9 - x^2 - 6x - 8 = 1$

$\Rightarrow f(x) = 1 \Rightarrow f(50) = 1$

78. (D)

$T_{r+1} = {}^{16}C_r \left(\frac{x}{\cos\theta}\right)^{16-r} \left(\frac{1}{x\sin\theta}\right)^r$

$= {}^{16}C_r (x)^{16-2r} \times \frac{1}{(\cos\theta)^{16-r}(\sin\theta)^r}$

For independent of x; $16 - 2r = 0 \Rightarrow r = 8$

$$\Rightarrow T_9 = {}^{16}C_8 \frac{1}{\cos^8\theta \sin^8\theta}$$

$$= {}^{16}C_8 \frac{2^8}{(\sin 2\theta)^8}$$

For $\theta \in \left[\frac{\pi}{8}, \frac{\pi}{4}\right] \ell_1$ is least for $\theta_1 = \frac{\pi}{4}$

For $\theta \in \left[\frac{\pi}{16}, \frac{\pi}{8}\right] \ell_2$ is least for $\theta_2 = \frac{\pi}{8}$

$$\frac{\ell_2}{\ell_1} = \frac{(\sin 2\theta_1)^8}{(\sin 2\theta_2)^8} = \left(\sqrt{2}\right)^8 = \frac{16}{1}$$

79. (D)

$$\sum_{n=1}^{100} a_{2n+1} = 200 \Rightarrow a_3 + a_5 + a_7 + ... + a_{201} = 200$$

$$\Rightarrow ar^2 \frac{\left(r^{200} - 1\right)}{\left(r^2 - 1\right)} = 200$$

$$\sum_{n=1}^{100} a_{2n} = 100 \Rightarrow a_2 + a_4 + a_6 + ... + a_{200} = 100$$

$$\Rightarrow \frac{ar\left(r^{200} - 1\right)}{\left(r^2 - 1\right)} = 100$$

On dividing $r = 2$

On adding $a_2 + a_3 + a_4 + a_5 + ... + a_{200} + a_{201} = 300$

$$\Rightarrow r\left(a_1 + a_2 + a_3 + ... + a_{200}\right) = 300$$

$$\Rightarrow \sum_{n=1}^{200} a_n = 150$$

80. (C)

$$f\left(g(x)\right) = x$$

$$f'\left(g(x)\right) g'(x) = 1$$

Put $x = a$

$$\Rightarrow f'(b) g'(a) = 1$$

$$f'(b) = \frac{1}{5}$$

81. (14)

Common term are : 23, 51, 79, ………….. $T_n T_n \le 407 \Rightarrow 23 + (n-1)28 \le 407$

$\Rightarrow n \le 14.71$

$n = 14$

82. (30)

$\vec{b}.\vec{c} = 10 \Rightarrow 5|\vec{c}|\cos\frac{\pi}{3} = 10 \Rightarrow |\vec{c}| = 4$

$\left|\vec{a}\times\left(\vec{b}\times\vec{c}\right)\right| = |\vec{a}|\left|\vec{b}\times\vec{c}\right|$

$= \sqrt{3}.5.4.\sin\frac{\pi}{8} = 30$

83. (3)

If $\lambda = -7$, then planes will be parallel & distance between them will be $\frac{3}{\sqrt{633}} \Rightarrow k = 3$ But

If $\lambda \ne -7$, then plane will be intersecting & distance between then will be 0

84. (51)

$S = 1.\,{}^{25}C_0 + 5.\,{}^{25}C_1 + 9.\,{}^{25}C_2 + ... + (101)^{25}C_{25}$

$S = 101\,{}^{25}C_{25} + 97\,{}^{25}C_1 + ... + 1^{25}C_{25}$

$2S = (102)\left(2^{25}\right)$

$S = 51\left(2^{25}\right)$

85. (36)

Common tangent is $S_1 - S_2 = 0$

$\Rightarrow -6x + 8y - 8 + k = 0$

Use p = r for 1^{st} circle

$\Rightarrow \frac{|-18 - 8 + k|}{10} = 1$

$\Rightarrow k = 36 \text{ or } 16 \Rightarrow k_{max} = 36$

86. (2)

Let $x = 5\cos\theta, y = 5\sin\theta$, So give expression $= \log_5\{5(3\cos\theta + 4\sin\theta)\}$

As $3\cos\theta + 4\sin\theta \le 5$

$\Rightarrow$ Maximum value of $= \log_5(25) = 2$

87. (360)

Required coefficient of x^6 is given by

$2\left\{\frac{1}{1!\times 5!}+\frac{1}{2!\times 4!}\right\}+\frac{1}{(3!)^2}$

$=\frac{1}{6!}\left\{{}^6C_1+{}^6C_2+{}^6C_3+{}^6C_5\right\}=\frac{2^6-2}{6!}=\frac{31}{360}$

88. (3)

Given $f^{-1}(2a-4)=\frac{1}{2}$

$\therefore f\left(\frac{1}{2}\right)=2a-4$

Put $x=1/2$ in $f(x)=3\sqrt{\frac{9}{\log_2(3-2x)}}-1$

We have, $f\left(\frac{1}{2}\right)=\sqrt[3]{9}-1=2$

$\therefore 2a-4=2\Rightarrow a=3$

89. (1)

The required condition are

$3-15-2\alpha=0$ .....(i)

And

$2+3+2\alpha+\beta=0$ .....(ii)

$\Rightarrow \alpha=-6$ and $\beta=7$

$\therefore \alpha+\beta=1$

90. (2)

Equation of tangent of $y^2=8x$ is

$y=mx+\frac{2}{m}$

And $xy+1=0$

From (i) and (ii)

$x\left(mx+\frac{2}{m}\right)+1=0$ or $mx^2+\frac{2}{m}x+1=0$

$\Rightarrow D=0$

$\Rightarrow \frac{4}{m^2}=4m\Rightarrow m^3=1$

$\therefore m=1$

$\therefore$ Common tangent is $y=x+2$

$\therefore$ y – intercept = 2