PART (A) : PHYSICS

SOLUTION

1. (D)

$$\frac{1}{2} K \times \frac{4}{100} = 1$$

K = 50

$$\therefore T\ 2\pi \sqrt{\frac{m}{k}} = 2\pi \sqrt{\frac{4}{50}}$$

2. (A)

P = 2x

FV = 2x

$$mV\ \frac{dV}{dx} V = 2x \Rightarrow \int_{O}^{x} \frac{2}{m} \text{xdx}$$

$$\frac{V^3}{3} = \frac{2}{m} \frac{x^2}{2}$$

$$V = \left[\frac{3}{m} x^2\right]^{1/3}$$

3. (C)

Impulse-momentum theorem

For translatory motion

$$MV_{cm} = I \qquad \ldots(i)$$

For rotatory motion

$$\left(\frac{2}{5} Mr^2\right)\left(\frac{V_{cm}}{r}\right) = Ih \qquad \ldots..(ii)$$

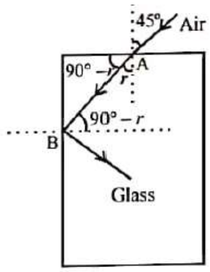
By solving above equations $\frac{h}{r}\ \frac{2}{5}$

4. (A)

For point A, $_a\mu_g = \frac{\sin 45^0}{\sin r}$

$$\Rightarrow \sin r = \frac{1}{\sqrt{2}\ _a\mu_g}$$

For point B, $\sin((90^0 - r)\ _a\mu_g$ where, $(90^0 - r)$ is critical angle.



$$\therefore \cos r = {}_a\mu_g = \frac{1}{{}_a\mu_g}$$

$$\Rightarrow {}_a\mu_g = \frac{1}{\cos r}$$

$$= \frac{1}{\sqrt{1-\sin^2 r}} = \frac{1}{\sqrt{1-\frac{1}{2\,{}_a\mu_g^{\,2}}}}$$

$$\Rightarrow {}_a\mu_g^{\,2} = \frac{1}{1-\frac{1}{2\,{}_a\mu_g^{\,2}}} = \frac{2\,{}_a\mu_g^{\,2}}{2\,{}_a\mu_g^{\,2}-1}$$

$$\Rightarrow 2\,{}_a\mu_g^{\,2} - 1 = 2 \Rightarrow {}_a\mu_g = \sqrt{\frac{3}{2}}$$

5. (A)

Given wave equation is y (x, t)

$$= e^{-\left(ax^2 + bt^2 + 2\sqrt{ab}xt\right)}$$

$$= e^{-\left[\left(\sqrt{ax}\right)^2 + \left(\sqrt{bt}\right)^2 + 2\sqrt{ax}.\sqrt{bt}\right]}$$

$$= e^{-\left(\sqrt{ax}\right)^2 + \left(\sqrt{bt}\right)^2}$$

$$= e^{-\left(x+\sqrt{\frac{b}{a}}t\right)^2}$$

It is a function of type y = f(x + vt)

$\Rightarrow$ speed of wave = $\sqrt{\frac{b}{a}}$

6. (D)

$s = t^3 + 5$

$\Rightarrow$ velocity v = $\frac{ds}{dt} = 3t^2$

Tangential acceleration $a_t = \frac{dv}{dt} = 6t$

Radial acceleration $a_c = \frac{v^2}{R} = \frac{9t^4}{R}$

At t = 2s, $a_t = 6 \times 2 = 12$ m/s^2

$a_c = \frac{9 \times 16}{20} = 7.2\ m/s^2$

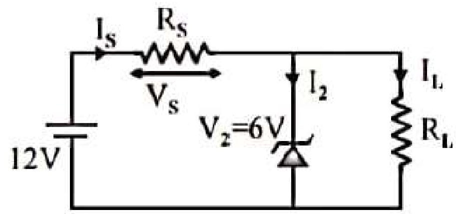
$\therefore$ Resultant acceration

$= \sqrt{a_t^2 + a_c^2} = \sqrt{(12)^2 + (7.2)^2} = \sqrt{144 + 51.84}$

$= \sqrt{195.84} = 14\, m/s^2$

7. (B)

$P_Z = V_Z\ I_Z \Rightarrow I_Z = \frac{2.4\,\text{mW}}{6\text{V}} = 0.4\ \text{mA}$



$V_s = 12 - 6 = 6\text{V}$

$I_S R_S = V_S = 6\text{V}$

$R_s = \frac{V_S}{I_S}\ \frac{6\text{V}}{(I_Z + I_L)}$

$(R_S)_{\max} = \frac{6}{(I_S)_{\min}}\ \frac{6\text{V}}{0.4\text{mA}} = 15\text{k}\Omega$

8. (D)

$\frac{R}{R_0}\ \frac{1}{128} = \frac{1}{2^7} = \frac{1}{2^{t/T_H}}$

$\Rightarrow t = 7T_H = 7 \times 18 = 126$ days

9. (B)

For given transmission band 88-108 MHz $(\Delta f)_{\max} = 75\,\text{kHz}$

Given $(\Delta f)_{\text{actual}} = 18.75\,\text{kHz}$

$\therefore$ % modulation m $= \frac{(\Delta f)_{actual}}{(\Delta f)_{\max}} \times 100 = \frac{18.75}{75} = 25\%$

10. (B)

Efficiency of Cannot engine, $n = 1 - \frac{T_2}{T_1}$ where T_1 and T_2 be the temperature of source and sink respectively.

$\therefore \frac{T_2}{T_1} = 1 - \eta = 1 - \frac{40}{100} = \frac{60}{100} = \frac{3}{5}$ $\quad (\because \eta = 40\%)$

$T_2 = \frac{3}{5}T_1 = \frac{3}{5} \times 500\ K = 300K$ ....(i)

$(\because T_1 = 500K)$

Let T_1' be the temperature of the source for the same sink temperature when efficiency $\eta' = 50\%$

$\therefore \frac{T_2}{T_1} = 1 - \eta' = 1 - \frac{50}{100} = \frac{1}{2}$

$T_1' = 2T_2 = 2 \times 300\text{K} = 600\text{ K}$ (Using eq. (i))

11. (B)

According to Einstein's photoelectric equation

$h\nu = \phi_0 + K_{max}$ we have

$h\nu = \phi_0 + 0.5$ ....(i)

And $1.2h\nu = \phi_0 + 0.8$ ....(ii)

Therefore, from above two equations $\phi_0 = 1.0$ eV

12. (D)

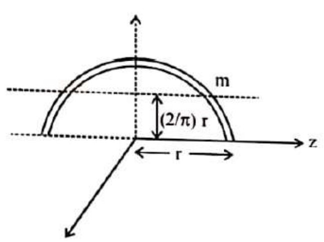
$p_1 = p_2$

$m_1 v_1 = m_2 v_2$

$m_1 = m_2$

$2\rho \frac{4}{3}\pi R_1^3 = \rho . \frac{4}{3}\pi R_2^3 ; \frac{R_1^3}{R_2^3} = 1:2; R_1 : R_2 = 1:2^{1/3}$

13. (C)



Moment of inertia about z-axis, $I_z = mr^2$

(about centre of mass)

Applying parallel axes theorem

$I_z = I_{cm} + mk^2$

$I_{cm} = I_z - m\left(\frac{2}{\pi}r\right)^2 = mr^2 - \frac{m4r^2}{\pi^2} = mr^2\left(1 - \frac{4}{\pi^2}\right)$

i.e. k = 4

14. (D)

Initially centre of mass is at the centre. When sand is poured it will fall and again after a limit, centre of mass will rise.

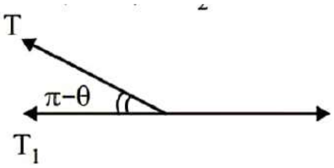
15. (B)

$g \propto \frac{1}{R^2}$

R decreasing g increase hence, curve b represents correct variation.

16. (D)

$T_1 + T\cos(\pi - \theta) = T_2$



$\therefore \cos(\pi - \theta) = \frac{T_2 - T_1}{T}$

$\therefore -\cos\theta = \frac{T_2 - T_1}{T}$

$\therefore \cos\theta = \frac{T_2 - T_1}{T}$

17. (A)

$M = 60 Am^2$

$\vec{\tau} = 1.2 \times 10^{-3}$ Nm, $B_H = 40 \times 10^{-6}$ Wb/m^2

$\vec{\tau} = \vec{M} \times \vec{B}_H \Rightarrow \tau = MB_H \sin\theta$

$\Rightarrow 1.2 \times 10^{-3} = 60 \times 40 \times 10^{-6} \sin\theta$

$\Rightarrow \sin\theta = \frac{1.2 \times 10^{-3}}{60 \times 40 \times 10^{-6}} = \frac{1}{2} = \sin 30^0$

$\Rightarrow \theta = 30^0$

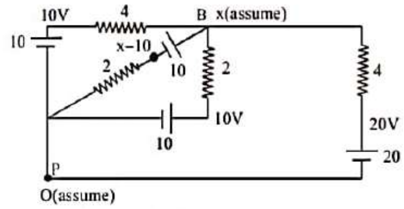
18. (D)

$V = \frac{V_0}{T/4} t \Rightarrow V = \frac{4V_0}{T} t$

$$\Rightarrow V_{rms} = \sqrt{<V^2>} = \frac{4V_0}{T}\sqrt{<t^2>} = \frac{4V_0}{T}\left\{\frac{\int_0^{T/4} t^2 dt}{\int_0^{T/4} dt}\right\}^{1/2} = \frac{V_0}{\sqrt{3}}$$

19. (A)

The simplified circuit is



We have to find I.

Let potential of point P be 0. Potential at other points are shown in the figure apply Kirchoff's current law at B where potential is assume to be x volt.

$$\frac{x-10}{4}+\frac{x-10}{2}+\frac{x-20}{4}+\frac{(x-10)-0}{2}=0$$

$$\Rightarrow x-10+2x-20+x-20+2x-20=0$$

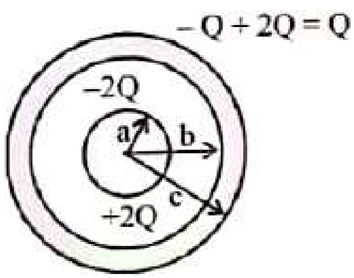
$$\Rightarrow 6x=70 \Rightarrow x=\frac{35}{3}\text{ volt}$$

$$\therefore \text{I}=\frac{20-\frac{35}{3}}{4}=\frac{25}{12}A$$

20. (A)

Surface charge density $(\sigma)=\frac{\text{Charge}}{\text{Surface area}}$

So, $\sigma_{\text{inner}}=\frac{-2Q}{4\pi b^2}$



And $\sigma_{\text{outer}}=\frac{Q}{4\pi c^2}$

21. (2.68)

Wavelength of monochromatic green light = 5.5×10^{-5} cm

Intensity $\text{I}=\frac{\text{Power}}{\text{Area}}$

$$=\frac{100\times(3/100)}{4\pi(5)^2}=\frac{3}{100\pi}Wm^{-2}$$

Now, half of this intensity (I) belongs to electrical field and half of that to magnetic field, therefore

$$\frac{\text{I}}{2}=\frac{1}{4}\varepsilon_0E_0^2C$$

Or $E_0=\sqrt{\frac{2I}{\varepsilon_0 C}}$

$$= \sqrt{\frac{2 \times \left(\frac{3}{100}\pi\right)}{\left(\frac{1}{4\pi \times 9 \times 10^9}\right) \times (3 \times 10^8)}} = \sqrt{\frac{6}{25} \times 30} = \sqrt{7.2}$$

$\therefore E_0 = 2.68 V/m$

22. (10)

$$\lambda_g = \frac{12280\text{eV Å}}{100\text{keV}}$$

$$= \frac{1228}{10000}\text{Å}$$

Now, $h_g = \lambda_e$

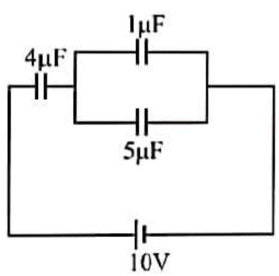
$$\Rightarrow \frac{1228\text{Å}}{1000} = \frac{12.28\text{Å}}{\sqrt{V}}$$

$\Rightarrow \sqrt{V} = 100$

$\Rightarrow V = 10^4 \text{ V}$

$\Rightarrow V = 10 \text{ kV}$

23. (24.00)



$$q = C_{eq} V = \left(\frac{4 \times 6}{4+6}\right) \times 10\mu C = 24\mu C$$

(in steady state, no current slow through the battery, so p.d. at 2Ω and 5Ω will be zero)

24. (25.00)

In double-slit interference, the distance y of a bright fringe from the centre (zero-order bright fringe)

$$y = n\lambda \frac{D}{d}$$

Where n = 0, 1, 2, 3,… etc

Thus at a given point, we have

$n\lambda$ = constant

$n_1\lambda_1 = n_2\lambda_2$

$$n_2 = \frac{n_1\lambda_1}{\lambda 2} = \frac{12 \times 6000}{4800} = 15$$

25. (6.40)

$$\Delta v = -\left[\int E_x dx + \int E_y dy\right]$$

$$= -\left[\int_1^3 2x dx + \int_2^4 3y^2 dy\right]$$

$|\Delta v| = 64$

26. (9.50)

At point P,

$$v = \sqrt{2gl}$$

$$\Rightarrow a_c = \frac{v^2}{R} = 2g$$

$$a_t = g\sin 60 = \frac{g\sqrt{3}}{2}$$

$$a_{net} = \sqrt{a_c^2 + a_t^2}$$

$$= g\,\frac{\sqrt{19}}{2} \Rightarrow \alpha = 19,\ \beta = 2$$

27. (4.00)

$$dU = \mu C_V dT$$

$$\therefore \mu C_V = \frac{dU}{dT} = \frac{80}{20} = 4 J/K$$

28. (5.00)

$$K = \frac{\pi}{2} = \frac{2\pi}{\lambda} \Rightarrow \lambda = 4$$

Second overtone $\Rightarrow \frac{5\lambda}{4} = L \Rightarrow L = 5m$

29. (3.00)

$$W = \frac{\Delta Q}{u}$$

$$\Rightarrow \Delta u = \frac{3}{4}\Delta Q$$

$$nC_v\Delta T = \frac{3}{4}nC\Delta T$$

$$\Rightarrow C = \frac{4C_v}{3} = C_v + \frac{R}{1-x}$$

$$\frac{R}{1-x} = \frac{C_v}{3} = \frac{R}{2}$$

$$\Rightarrow x = -1$$

$\Rightarrow PV^{-1}$ = constant

$\Rightarrow P^3V^x = K$

30. (25.00)

$\varepsilon = \left(\vec{v} \times \vec{B}\right) . \vec{\ell}$

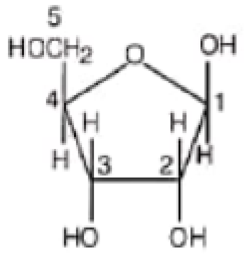
$= \left[1\hat{i} \times (3\hat{i} + 4\hat{j} + 5\hat{k})\right] . 5\hat{j}$

$\varepsilon = 25v$

PART (C) : CHEMISTRY

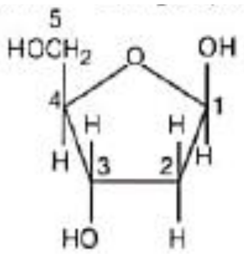
SOLUTION

31. (B)



β-D-ribose used in RNA;

At 2nd carbon –OH group is present.



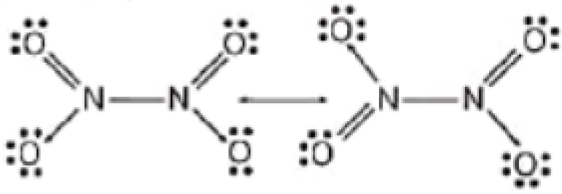
At 2nd carbon –OH group is missing.
Thus, correct answer is (B).

32. (C)
Covalent character is favoured by Fajan's rule
(i) larger the charge on the ions,
(ii) smaller the size of cations,
(iii) larger the size of cations,
(iv) cation with 18-electron structure (e.g., Cu^+), then larger the polarizing power and the covalent character is favoured.
On all the given compounds, anion is same (Cl^-), hence polarizing power is decided by size and charge of cation.
Al^{3+} with maximum charge and smallest size has maximum polarizing power hence, $AlCl_3$ is maximum covalent.
Thus, correct answer is (C).

33. (A)
Thermal stability of the hydrides decreases as we go down the group in periodic table for group 15 (N-family)
BiH_3 < SbH_3 < AsH_3 < PH_3 < NH_3
Least stable Most stable
M-H – 255 247 322 391
Bond energy
kJ mol^{-1}
(B) Due to absence of *d*-orbital, nitrogen can't form $d\pi - p\pi$ bond, thus it is correct.

(C) The N–N bond (Be 160 kJ mol^{-1}) is weaker than P–P bond (BE 209 kJ mol^{-1}). Thus, it is correct.

(D) N_2O_4 can form two resonance structures



Thus, it is correct.

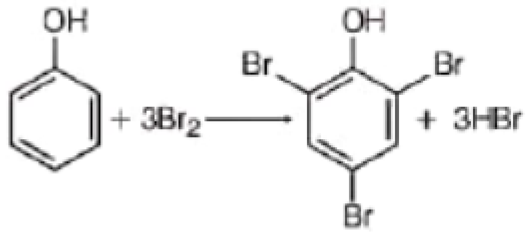
Thus, only (A) is wrong.

34. (D)

Br_2 is formed by a redox reaction

$5Br^- + BrO_3^- + 6H^- \rightarrow 3Br_2 + 3H_2O$

–OH group is the activating group and there is S_E at *o* and *p*-positions giving yellowish white precipitate of 2,4,6-tribromophenol.



Thus, correct answer is (D).

35. (C)

$$\text{Molality} = \frac{\text{Moles of solute}}{\text{Mass of solvent (in kg)}}$$

$$= \frac{5.2\ \text{mol}\ CH_3OH}{1\ \text{kg}\ (=100\ \text{g})\ H_2O}$$

$$n_1(CH_3OH) = 5.2$$

$$n_2(H_2O) = \frac{1000}{18} = 55.56$$

$$\therefore\ n_1 + n_2 = 5.20 + 55.56 = 60.76\ \text{mol}$$

$$\therefore\ X_{CH_3OH} = \text{Mole fraction of } CH_3OH$$

$$= \frac{n_1}{n_1 + n_2} = \frac{52}{60.76} = 0.086$$

Thus, correct answer is (C).

36. (B)

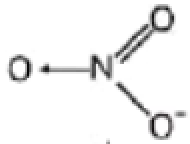
Count σ-bond, lone pairs and unpaired electron or count number of atoms directly attached, lone pairs and unpaired electrons.

This is = 2, then sp = 5, then sp^3d

= 3, then sp^2 = 6, then sp^3d^2

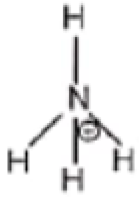
= 4, then sp^3

NO_3^- (I)



NO_2^+ (II)

NH_4^+ (III)



	σ - bond	Lone pair	Unpaired electron	Total
I.	3	x	x	$3(sp^2)$
II.	2	x	x	$2(sp)$
III.	4	x	x	$4(sp^3)$

Thus, correct answer is (B) sp^2, sp, sp^3.

Alternate solution

Count the sum of outermost shell electrons of all the atoms and charge present at molecule.

Now, divide the sum by eight and get the hydridisation. If sum is eight or less than eight then it is divide by two.

I. $NO_3^- \Rightarrow \frac{5+3\times6+1}{8} = \frac{5+18+1}{8} = \frac{24}{8} = 3(sp^2)$

II. $NO_2^+ \Rightarrow \frac{5+2\times6-1}{8} = \frac{5+12-1}{8} = \frac{16}{8} = 2(sp)$

III. $NH_4^+ \Rightarrow \frac{5+4\times1-1}{2} = \frac{5+4-1}{2} = \frac{8}{2} = 4(sp^3)$

Thus, correct answer is (B) sp^2, sp, sp^3.

37. (A)

Depression in freezing point (ΔT_ℓ) can be given as $\Delta T_f = i \cdot K_f \cdot m$. To solve this problem, we will have to first calculate the value of ΔT_f and i. finally, put all the given valued and calculated value in the expression of

$$\Delta T_\ell = i \cdot K_f \cdot \frac{\text{mass of solute}}{\text{molar mass of solute} \times \text{mass of solvent}}$$

To calculate the mass of solute.

ΔT_f = Freezing point of H_2O – freezing point of ethylene glycol solution = 0 – (–6°) = 6°

$K_f = 1.86^\circ$ kg mol^{-1}

w_1 = mass of ethylene glycol in grams

w_2 = mass of solvent (H_2O) in grams = 4000 g

m_1 = Molar mass of ethylene glycol = 62 g mol^{-1}

i = van't Hoff factor = 1 (ethylene glycol being non-electrolyte)

$$\Delta T_f = \frac{1000\,K_f\ w_i\,(i)}{m_1 m_2}$$

$$\therefore\ 6 = \frac{1000\times1.86\times w_1\times1}{62\times4000}$$

$$w_1 = 800\,g$$

Thus, correct answer is (A).

38. (C)

Reduction hydrogen half-cell is

$H^+(xM)|Pt(H_2)$

+ Pressure (p)

Half-cell reaction is

$2H^+(aq) + 2e^- \rightarrow H_2(g)$

Reaction quotient $= Q = p_{H_2} \Big/ \left[H^+\right]^2$, $n = 2$

$$E_{red} = E^0_{red} - \frac{0.0591}{n}\log Q$$

$$= 0 - \frac{0.0591}{2}\log Q$$

	p_{H_2}	$[H^+]$	Q	E_{red}
(a)	1 atm	2.0 M	0.25	+ve
(b)	1 atm	1.0 M	1.0	0
(c)	2 atm	1.0 M	2.0	–ve
(d)	2 atm	2.0 M	0.50	+ve

$E^0_{red} = 0.00\ V$ for standard hydrogen electrode

If $Q > 1$, then $E_{red} = -ve$.

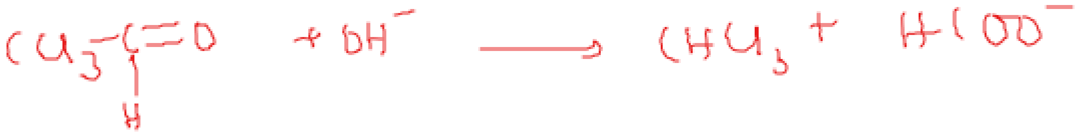
Thus, correct answer is (C).

39. (D)

	Reagent	**Phenol**	**Benzoic acid**	**Conclusion**
(a)	Aqueous NaOH	Salt formation	Salt formation	No specific colour change
(b)	Tollen's reagent	No effect	No effect	
(c)	Molisch reagent	No effect	No effect	
(d)	Neutral $FeCl_3$	Voilet colour	Buff-coloured precipitate	Thus, $FeCl_3$ can be used to make distinction

Thus, correct answer is (D).

40. (D)



41. (A)

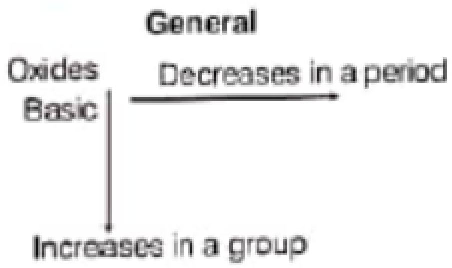
Oxides when dissolved in water form hydroxides.

$Na_2O + H_2O \rightarrow 2NaOH$

$MgO + H_2O \rightarrow Mg(OH)_2$

$Al_2O_3 + 3H_2O \rightarrow 2Al(OH)_3$

Smaller the size of cation, larger the charge, greater the polarizing power of cation, hence grater the covalent character, Hence smaller the basic nature.



Charge $Al^{3+} > Mg^{2+} > Na^{+} = K^{+}$

Size $Al^{3+} > Mg^{2+} > Na^{+} > K^{+}$

Polarising power $Al^{3+} > Mg^{2+} > Na^{+} > K^{+}$

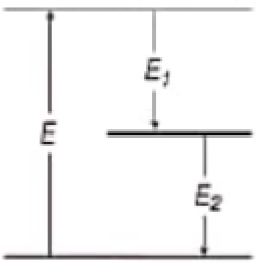
Basic nature of oxides

$Al_2O_3 < MgO < Na_2O < K_2O$

Thus, correct answer is (A).

42. (C)

Energy values are always additive.



$$E_{total} = E_1 + E_2$$

$$\frac{hc}{\lambda} = \frac{hc}{\lambda_1} + \frac{hc}{\lambda_2}$$

$$\therefore \frac{1}{\lambda} = \frac{1}{\lambda_1} + \frac{1}{\lambda_2}$$

$$\frac{1}{355} = \frac{1}{680} + \frac{1}{\lambda_2}$$

$$\lambda_2 = 742.77 \text{ nm} \simeq 743 \text{ nm}$$

Thus, correct answer is (C).

43. (D)

(A) S_2 molecules is paramagnetic due to unpaired electrons in MO and is blue-coloured compound, this true.

(B) The vapour at 200°C consists mostly of S_8 ring, thus correct.

(C) At 600°C, the gas mainly consists of S_2 molecules, thus correct.

(D) Oxidation states of Sulphur are

-2 in H_2S

0 in S_8

+2 in $S_2O_3^{2-}$

+4 in SO_2

+6 in SO_3

Thus, incorrect. (Valency cannot be less then 4)

Hence, correct option is (D).

44. (A)

Entropy change for *n* moles of isothermal expansion of an ideal gas from volume V_1 to volume V_2 is

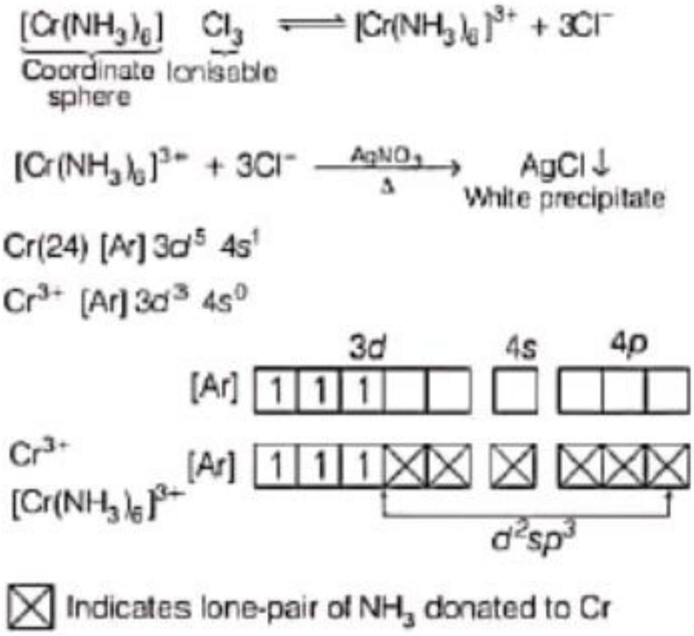
$$\Delta S = 2.303\, nR \log \frac{V_2}{V_1}$$

$$= 2.303 \times 2 \times 8.3143 \log \frac{100}{10}$$

$$= 38.296\ J\ mol^{-1}K^{-}$$

Thus, correct option is (A).

45. (C)



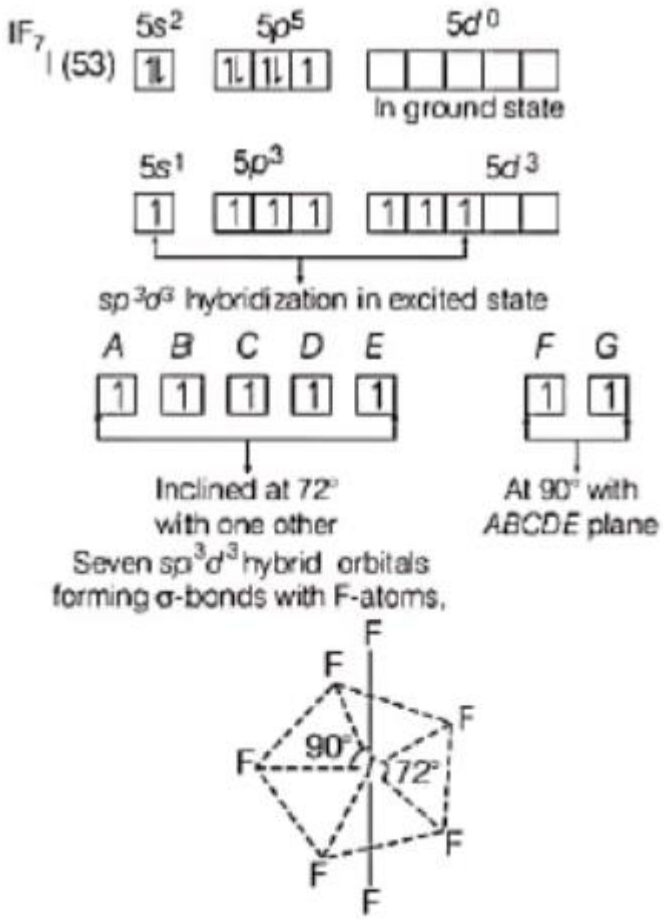
(A) d^2sp^3-hybridization, octahedral. Thus correct.

(B) There are three unpaired electrons, hence paramagnetic. Thus, correct

(C) d^2sp^3-inner orbital complex, this incorrect.

(D) Due to ionizable Cl^{-} ions, white precipitate with $AgNO_3$, this correct.
Therefore, (C) is wrong.

46. (D)



Pentagonal-bipyramidal structure.
Thus, correct answer is (D).

47. (C)
For every 10°C rise of temperature, rate is doubled.
Thus, temperature coefficient of the reaction = 2
When temperature is increased by 50°, rate becomes

$= 2^{(50/50)}$

$= 2^{5}$ times = 32 times

Thus, correct answer is (C).

48. (C)
Electron-withdrawing group (–I effect) stabilizes the anion, and thus increases acidic nature.
Thus, (C), (D) > (A), (B) acidic
Further the electron withdrawing group from the –COOH group, its effect in increasing acid strength decreases, thus (C) with Cl at α-position is stronger than (D) with Cl at γ-position.

CH_3COOH $K_a = 1.75 \times 10^{-5}$

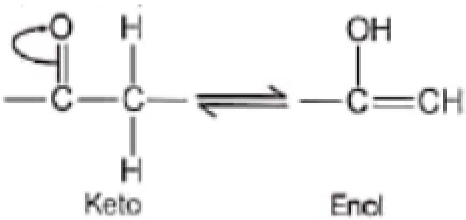
HCOOH $K_a = 17.7 \times 10^{-5}$

$CH_3CH_2\overset{\alpha}{C}HCO_2H$ (with Cl on the α-carbon) $K_a = 1.4 \times 10^{-3}$

$\overset{\gamma}{C}H_2CH_2CH_2CO_2H$ (with Cl on the γ-carbon) $K_a = 2.96 \times 10^{-5}$

49. (C)

Tautomerism is due to spontaneous interconversion of two isomeric forms with difference functional groups into one other. The term tautomer means constitutional isomers that undergo means constitutional isomers that undergo such rapid interconversion that can't be independently isolated.



Thus, (C) 2-pentanone exhibit tautomerism.

50. (A)

K_p depends upon the partial pressure of reactants and products so first calculate their partial pressure and then, calculate K_p

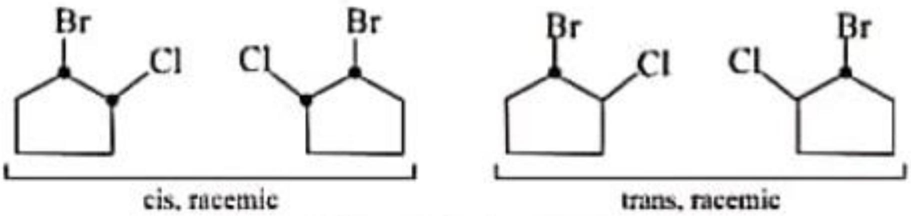
$$CO_2(g) + C(s) \rightleftharpoons 2CO(g)$$

initial 0.5 atm –

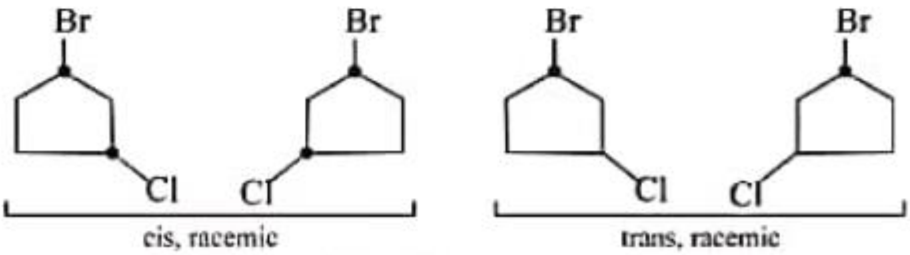
at equilibrium (0.5 – p) 2p atm

This is a case of heterogeneous equilibrium.

51. (4)



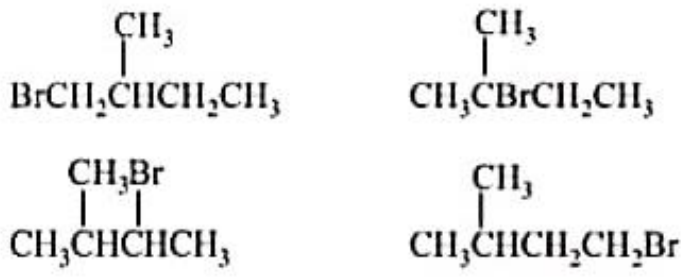
1-Bromo-2-cyclopentane



1-Bromo-3-cyclopentane

52. (4)

All alkyl bromides having carbon skeleton of isopentane (2-methylbutane $(CH_3)_2CHCH_2CH_3$) will give isopentane via Grignard reagent.



53. (5)

$$d=\frac{n\times M}{N_A\times a^3}\ \text{or}\ n=\frac{d\times N_A\times a^3}{M}$$

$$\Rightarrow\ n=\frac{2\times 6\times 10^{23}\left(5\times 10^{-8}\right)^3}{75}=2$$

Therefore metal crystallizes in *bcc* structure and for a *bcc* lattice $\sqrt{3}a=4r$

$$r=\frac{\sqrt{3}}{4}a=\frac{\sqrt{3}\times 5}{4}=2.165\ \text{Å}=216.5\ \text{pm}$$

$$x=\frac{216.5}{43.3}$$

$$x=5$$

54. (4)

For an octahedral void $a=2(r+R)$

In fcc lattice the largest void present is octahedral void. If the radius of void sphere is R and of lattice sphere is r.

Then, $r=\frac{\sqrt{2}\times 400}{4}=141.42\ \text{pm}\ \ (a=400\ \text{pm})$

Applying condition for octahedral void,

$$2(r+R)=a$$

$\therefore\ 2R=a-2r=400-2\times 141.42$

$\therefore$ Diameter of greatest sphere = 117.16 pm

$$d=\frac{117.16}{29.29}=4$$

55. (3)

Let the solubility of $Al(OH)_3$ in 0.2 M NaOH solution be s.

$$\underset{}{Al(OH)_3}\rightleftharpoons\underset{s}{Al^{3+}}+\underset{3s}{3OH^-}\ \text{and}\ \underset{0.2\,M}{NaOH}\rightleftharpoons\underset{0.2\,M}{Na^+}+\underset{0.2\,M}{OH^-}$$

$$[Al^{3+}]=s\ \text{and}\ [OH^-]=3s+0.2\approx 0.2\ []$$

$$K_{sp}=2.4\times 10^{-2}=[Al^{3+}][OH]^3$$

$2.4\times10^{-2} = s(0.2)^3$

$s = \frac{2.4\times10^{-2}}{8\times10^{-3}} = 3\ \text{mol/L}$

56. (5)

For initial solution,

$\because\ \pi = \frac{500}{760}\ \text{atm},\quad T = 283\ K$

$\frac{500}{760}\times V_1 = n\times R\times 283 \qquad ...(i)$

After dilution, let volume becomes V_2 and temperature is raised to 25^oC, i.e., 298 K.

$\pi = \frac{105.3}{760}\ \text{atm}$

$\frac{105.3}{760}\times V_2 = n\times R\times 298 \qquad ...(ii)$

$\therefore$ By Eqs. (i) and (ii), we get $\frac{V_1}{V_2} = \frac{283}{298}\times\frac{105.3}{500}$

$\frac{V_1}{V_2} = \frac{1}{5}$

$\therefore\ V_2 = 5V_1$

i.e., solution was diluted to 5 times.

57. (2)

Startch molecules have colloidal dimentions. Blood is a –vely charged colloidal system. Rest of the compounds, i.e., NaCl, urea and cane sugar form true solution in water.

58. (2)

$Fe(OH)_3$ particles absorb Fe^{3+} ions and get peptized to give a positively charged sol. Similarly AgCl particles absorbs Ag^+ ions to give a positively charged solution.

59. (3)

Total cationic charge = Total anionic charge

$2n + 6 + 24 = 36$

$n = 3$

60. (5)

$3Br_2 + 3Na_2CO_3 \rightarrow 5NaBr + NaBrO_3 + 3CO_2$

PART (C) : MATHEMATICS

Answer Key & Solution

61. (D)

The 5 lines intersect in $\binom{5}{2} = 10$ points

The 10 points when joined gives $\binom{10}{2} = 45$ lines.

Each of the old line contains 4 points giving $\binom{4}{2} = 6$ lines

Therefore the new lines are $45 - 5 \times 6 = 15$

The probability of a new line $= \dfrac{15}{15+5} = \dfrac{3}{4}$.

62. (D)

$\because \quad f(x) = 2\sqrt{x}$

$\therefore \quad f'(x) = \dfrac{1}{\sqrt{x}} > 0$

$\Rightarrow f(x)$ is an increasing function

$\therefore \quad f_1(x) = f(x) = 2\sqrt{x}$

or $\quad f_1(x) = 2\sqrt{x}, x \in [0,1]$ …(i)

and $\quad g(x) = 2\sqrt{(1-x)}$

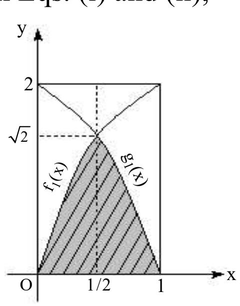
$\therefore \quad g'(x) = -\dfrac{1}{\sqrt{(1-x)}} < 0$

$\Rightarrow g(x)$ is decreasing function

$\therefore \quad g_1(x) = g(x) = 2\sqrt{(1-x)}$

or $\quad g_1(x) = 2\sqrt{(1-x)}, x \in [0,1]$ …(ii)

From Eqs. (i) and (ii),



$2\sqrt{x} = 2\sqrt{(1-x)}$

or $\quad x = 1 - x$

or $\quad x=\dfrac{1}{2}$

Then, $f_1\left(\dfrac{1}{2}\right)=g_1\left(\dfrac{1}{2}\right)=\sqrt{2}$

$\therefore$ Required area = $\int_0^{1/2} f_1(x)dx+\int_{1/2}^{1} g_1(x)dx$

$$=\int_0^{1/2} 2\sqrt{x}dx+\int_{1/2}^{1} 2\sqrt{(1-x)}dx$$

$$=\frac{4}{3}\{x^{3/2}\}_0^{\frac{1}{2}}+\frac{4}{3}\{-(1-x)^{3/2}\}_{\frac{1}{2}}^{1}$$

$$=\frac{4}{3}\left(\frac{1}{2\sqrt{2}}\right)+\frac{4}{3}\left(\frac{1}{2\sqrt{2}}\right)=\frac{4}{3\sqrt{2}}\,sq\,unit$$

63. (B)

Domain of $\sqrt{x(x-3)}$ is $x\in(-\infty,0]\cup[3,\infty)$

Now $(3|x|-3)^2=|x|+7\Rightarrow|x|=\dfrac{1}{9}$ or $|x|=2$

$\therefore$ No. of solutions are 2.

64. (B)

By observation we find that in the above product no odd exponent of 'x' occurs

$\therefore$ Sum of coefficients of all odd expansion of $x=0$

65. (A)

$$\sum_{j=1}^{k} a_j a_{j+1}=k\left(a_1\ a_{k+1}\right)$$

Here, $k=4$ $\quad\therefore$ It is a root of $x^2-6x+8=0$

66. (C)

We know that limiting case of Binomial distribution is poisson distribution using

$$\underset{n\to\infty}{Lt}\left(\left(1-\frac{m}{n}\right)^{-\frac{n}{m}}\right)^{\left(1-\frac{r}{n}\right)(-m)}=e^{-m}$$

67. (C)

$$\frac{\int_0^n \{x\}\,dx}{\int_0^n [x]\,dx}=\frac{\frac{n}{2}}{\frac{n(n-1)}{2}}=\frac{1}{n-1}$$

So reciprocal $=n-1$

68. (B)

$f(x) = 3[x] + 5 = 5[x] - 3 \Rightarrow [x] = 4$

Now $[x + f(x)] = 21$ $\quad \therefore f(x) = 17$

$$\int_1^2 x[x + f(x)]\, dx = 2\left|\frac{x^2}{2}\right|_1^2 = \frac{63}{2}$$

69. (C)

Area $= \frac{x^2}{2}\sin\theta$, $A_{\max}$ at $\theta = \frac{\pi}{2}$

$\therefore$ Area $= \frac{x^2}{2}$

70. (D)

Given $\sum_{i=1}^{n} \overrightarrow{a_i} = 0$

$$\left(\sum_{i=1}^{n}\overrightarrow{a_i}\right).\left(\sum_{i=1}^{n}\overrightarrow{a_j}\right) = \sum_{i=1}^{n}|a_i|^2 + 2\sum_{1\le i\le j\le n}\sum \overrightarrow{a_i}\,\overrightarrow{a_j} \Rightarrow 0 = n + 2\sum_{1\le i\le j\le n}\sum \overrightarrow{a_i}.\overrightarrow{a_j}$$

$$\therefore \sum_{1\le i\le j\le n}\sum \overrightarrow{a_i}.\overrightarrow{a_j} = -\frac{n}{2}$$

71. (D)

$\vec{a} = 8\vec{b},\ \vec{c} = -7\vec{b}$

Now, $\vec{a}.\vec{c} = |\vec{a}||\vec{c}|\cos\theta$

$(8\vec{b}).(-7\vec{b}) = 8 \times 7b^2\cos\theta \Rightarrow \cos\theta = -1 \Rightarrow \theta = \pi$

72. (B)

$$\int_0^{\pi}[\cot x]dx = \int_0^{\pi}[\cot(\pi - x)]dx = \int_0^{\pi}[-\cot x]dx$$

We know that $[x] + [-x] = -1$ if $x \notin z$

$= 0$ if $x \in z$

$$\therefore 2I = \int_0^{\pi}[\cot x]\,dx + \int_0^{\pi}[-\cot x]dx$$

$$2I = \int_0^{\pi}(-1)\,dx \Rightarrow I = -\frac{\pi}{2}$$

73. (B)

$n(s) = {}^{20}C_4$

Statement – 1 common difference 1 total case = 17
common difference 2 total case = 14
common difference 3 total case = 11
common difference 4 total case = 8
common difference 5 total case = 5
common difference 6 total case = 2

Probability $= \dfrac{17+14+11+8+5+2}{{}^{20}C_4} = \dfrac{1}{85}$

74. (A)

$\sim\left(\left(P \wedge \sim R\right) \leftrightarrow Q\right) = \sim\left(Q \leftrightarrow \left(P \wedge \sim R\right)\right)$

75. (A)

$3\sin P + 4\cos Q = 6 \quad ....(1)$

$4\sin Q + 3\cos P = 1 \quad ....(2)$

From the above, $\underline{|P}$ is obtuse

Now squaring and adding gives

$\sin\left(P+Q\right) = \dfrac{1}{2} \Rightarrow P+Q = \dfrac{5\pi}{6} \Rightarrow R = \dfrac{\pi}{6}$

76. (D)

No. of ways $= (10+1)(9+1)(7+1) - 1 = 879$

77. (A)

$f(x) = [x]\cos\left(\dfrac{2x-1}{2}\right)\pi \; = [x]\cos\left(x - \dfrac{1}{2}\right)\pi \; = [x]\sin \pi x$ is continuous $\forall\ x \in R$

78. (C)

We have

$$1 + 2\left(\frac{\cos\theta}{\cos\frac{\theta}{2} - \sin\frac{\theta}{2}}\right)^2 = 1 + \frac{1}{2}\left(\frac{1-\sin^2\theta}{1-\sin\theta}\right) \qquad = 1 + \frac{1}{2}\left(1+\sin\theta\right) = \frac{3}{2} + \frac{1}{2}\sin\theta$$

Since, $-\dfrac{\pi}{2} \le \theta \le \dfrac{\pi}{2}$

$\Rightarrow\ -1 \le \sin\theta < 1 \qquad \Rightarrow\ -\dfrac{1}{2} \le \dfrac{1}{2}\sin\theta < \dfrac{1}{2} \quad \Rightarrow\ \dfrac{3}{2} - \dfrac{1}{2} \le \dfrac{3}{2} + \dfrac{1}{2}\sin\theta < \dfrac{3}{2} + \dfrac{1}{2}$

$\Rightarrow\ 1 \le 1+\frac{1}{2}\left(\frac{\cos\theta}{\cos\frac{\theta}{2}-\sin\frac{\theta}{2}}\right)<2 \qquad \Rightarrow \left[1+\frac{1}{2}\left(\frac{\cos\theta}{\cos\frac{\theta}{2}-\sin\frac{\theta}{2}}\right)\right]<2$

$\Rightarrow \left[1+\frac{1}{2}\left(\frac{\cos\theta}{\cos\frac{\theta}{2}-\sin\frac{\theta}{2}}\right)\right]=1$

Thus, the given inequality reduces to

$\left|2x^2-4x-7\right|<1$

$\Rightarrow\ -1<2x^2-4x-7$ and $2x^2-4x-7<1$

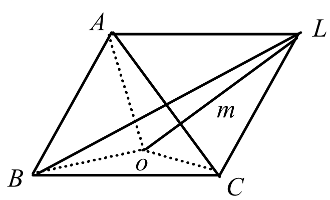
$\Rightarrow\ 2x^2-4x-6>0$ and $2x^2-4x-8<0$

$\Rightarrow\ x^2-2x-3>0$ and $x^2-2x-4<0$

$\Rightarrow\ x<-1\ \ or\ \ x>3$ and $\Rightarrow\ 1-\sqrt{5}<x<1+\sqrt{5}$

$\Rightarrow\ x\in\left(1-\sqrt{5},-1\right)\cup\left(3,\,1+\sqrt{5}\right)$

79. (A)



Let OL be the pole and angle of elevation of top of pole at each of A, B, C be α

$\frac{OL}{OA}=\tan\alpha \Rightarrow OA=OL\ \cot\alpha$, $OB=OL\ \cot\alpha$, $OC=OL\ \cot\alpha$

$\Rightarrow OA=OB=OC$

$\therefore$ O is circumcentre

80. (B)

Given roots of $f(x)=0$ are $3^{\sqrt{\log_3 7}}$ and $7^{\sqrt{\log_7 3}}$ which are equal

$\therefore\ D=0\Rightarrow ab=16$

Set of numbers whose product is 16 are $(1,16,)$ $(4,4)$ $(2,8)$ $(8,2)$ $(16,1)$. Here suitable sets for $f(1)$ to be maximum are $(1,16)$

$\therefore\ f(1)=1+16-8=9$ which is maximum of $f(1)$

81. (8)

$\int_0^x f\left(\sqrt{t}\right)dt=x^2+\int_x^1 t\ f\left(\sqrt{t}\right)dt$

Differentiating both sides we get

$f\left(\sqrt{x}\right)=2x+f\left(\sqrt{t}\right)0-x\,f\left(\sqrt{x}\right)$ $\Rightarrow (x+1)f\left(\sqrt{x}\right)=2x \Rightarrow f(2)=\frac{8}{5}$

82. (8)

Required area $=4\int_0^{\pi}(x+\sin x-x)\,dx=8\ sq.units$

83. (4)

$x+y=3 \Rightarrow 2.\frac{x}{2}+y=3$

$A.M \geq G.M \Rightarrow \frac{2.\frac{x}{2}+y}{3} \geq \left[\left(\frac{x}{2}\right)^2 y\right]^{1/3} \Rightarrow 1 \geq \left(\frac{x^2y}{4}\right)^{1/3} \Rightarrow x^2y \leq 4$

84. (10)

No. of odd divisors of 360 are 6 (1, 3, 5, 9, 15, 45)

Their sum $=78$

Mean of odd divisors $=13$

Median $=\frac{5+9}{2}=7$

Mean of median and mean $=\frac{13+7}{2}=10$

85. (1)

Let the coordinates of C be $(1,c)$

Here, $m_2=\frac{c-y}{1-x} \Rightarrow m_2=\frac{c-m_1x}{1-x}$ (Let $y=mx$)

$\Rightarrow \quad m_2-m_2x=c-m_1x \qquad \Rightarrow \quad c=(m_1-m_2)x+m_2 \qquad ....(i)$

Now, area of $\Delta\ ABC=\frac{1}{2}\begin{vmatrix}0 & 0 & 1\\ x & m_1x & 1\\ 1 & c & 1\end{vmatrix}=\frac{1}{2}(cx-m_1x)$

$=\frac{1}{2}\left\|\left[\left((m_1-m_2)x+m_2\right)x-m_1x\right]\right\|=\frac{1}{2}\left\|\left[(m_1-m_2)x^2+m_2x-m_1x\right]\right\|$

$=\frac{1}{2}(m_1-m_2)(x-x^2) \qquad \left[\because\ x>x^2 \text{ in } (0,1)\right]$

$\therefore\ f(x)=4(x-x^2) \quad \Rightarrow \quad \max\ f(x)=1 \text{ when } x=\frac{1}{2}$

86. (5)

If (h,k) be the centre

$(h-1)^2+(k-0)^2=k^2 \Rightarrow h=1$

$(h-2)^2+(k-3)^2=k^2 \Rightarrow k=\frac{5}{3}$

$\therefore$ Radius $=\frac{5}{3}$

87. (0)
By graph No.of solutions = zero

88. (0)

89. (5)
Since lines are perpendicular
$\Rightarrow 2\times 2+\frac{2}{3}\times\frac{3}{2}-\alpha=0 \Rightarrow \alpha=5$

90. (7)

1	2	3	4	5	6	…….
200	200	200	240	280	………	…….

Sum $=11040$

$120+80+160+40+200+240+....=11040$

$\frac{n}{2}\left[2a+(n-1)d\right]+80+40=11040$

$\frac{n}{2}\left[240+(n-1)40\right]=10920$

$n\left[6+n-1\right]=546$

$n(n+5)=546$

$n=21$