PACE-IIT & MEDICAL

ANDHERI / BORIVALI / DADAR / CHEMBUR / THANE / MULUND/ NERUL / POWAI

IIT – JEE 2023 **A.I.T.S – 5 (ADVANCED)** **DATE: 23/04/23**

PAPER I

Que.	1	2	3	4	5	6	7	8	9	10
Ans.	A	D	A	A	A	C	A	B	A	A
Que.	11	12	13	14	15	16	17	18	19	20
Ans.	AD	D	ACD	ABD	ABCD	5	2	5	6	1
Que.	21	22	23	24	25	26	27	28	29	30
Ans.	C	C	B	C	C	C	C	C	C	B
Que.	31	32	33	34	35	36	37	38	39	40
Ans.	AB	AB	BD	ACD	BC	4	2	2	3	4
Que.	41	42	43	44	45	46	47	48	49	50
Ans.	D	A	D	B	D	C	C	B	C	B
Que.	51	52	53	54	55	56	57	58	59	60
Ans.	BCD	BC	ABD	BCD	BCD	1	1	8	3	9

PAPER II

Que.	1	2	3	4	5	6	7	8	9	10
Ans.	AB	ACD	BC	ABD	BC	ABD	ABCD	ABCD	C	B
Que.	11	12	13	14	15	16	17	18	19	20
Ans.	B	B	D	C	D	C	A	C	D	B
Que.	21	22	23	24	25	26	27	28	29	30
Ans.	BD	BC	BCD	BC	AC	ACD	AB	ABC	A	C
Que.	31	32	33	34	35	36	37	38	39	40
Ans.	B	C	D	C	B	C	D	B	A	B
Que.	41	42	43	44	45	46	47	48	49	50
Ans.	BCD	ABCD	BC	ABCD	A	BD	ABD	ABCD	B	A
Que.	51	52	53	54	55	56	57	58	59	60
Ans.	D	C	D	B	A	D	C	D	C	C

Note : Detailed solution to this test is available on Tuesday after 02.00 pm on our website.:www.iitianspace.com

PHYSICS PAPER – I (SOLUTION)

1. (A)

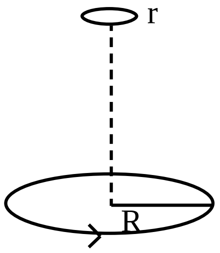
$$\frac{V}{R_2}e^{-t/R_2C}+\frac{V}{R_1}\left(1-e^{-R_1t/L}\right)=\text{const.}$$

$$\Rightarrow \frac{V}{R_2}e^{-t/R_2c}=\frac{V}{R_1}e^{-R_1t}/L \qquad \Rightarrow R_1=R_2$$

$$\frac{R_1}{L}=\frac{1}{R_2c} \qquad R_1R_2=\frac{L}{C}$$

$$R=\sqrt{\frac{L}{C}}=\sqrt{\frac{90\times10^{-3}}{10^{-6}}}=300\,\Omega$$

2. (D)

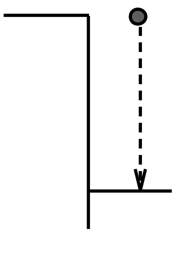


$$B=\frac{\mu_0 iR^2}{2\left(R^2+x^2\right)^{3/2}}$$

$$\phi=B\pi r^2=\frac{\mu_0 i\pi r^2R^2}{2\left(R^2+x^2\right)^{3/2}}$$

$$M=\frac{\phi}{i}=\frac{\mu_0\pi R^2r^2}{2\left(R^2+x^2\right)^{3/2}}$$

3. (A)



$$S_1=\frac{1}{2}\times10\left(3+t\right)^2$$

$$S_2=40t+\frac{1}{2}\times10t^2$$

$$5t^2+40t=5\left(9+t^2+6t\right)$$

$$40t=45+30t$$

$$t=4.5\text{sec.}$$

$$S=5\times\left(7.5\right)^2=\frac{1125}{4}\text{m}$$

4. (A)

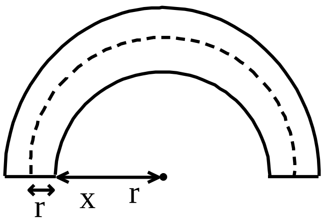
$$MR^2=\frac{2}{5}MR^2+Mx^2$$

$$x=\sqrt{\frac{3}{5}}R=\sqrt{0.6}\ R$$

5. (A)

$$dR = \frac{\rho \times \pi x}{a\,dx}$$

$$\frac{1}{R} = \int \frac{1}{dr} = \int_{r}^{r+a} \frac{a\,dx}{\rho \times \pi x} = \frac{a}{\pi\rho} \ln\left(1 + \frac{a}{r}\right)$$

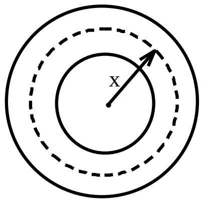


$$R = \frac{\pi\rho}{a \ln\left(1 + \frac{a}{r}\right)}$$

6. (C)

$$B \times 2\pi x = \mu_0 J \pi \left(x^2 - a^2\right)$$

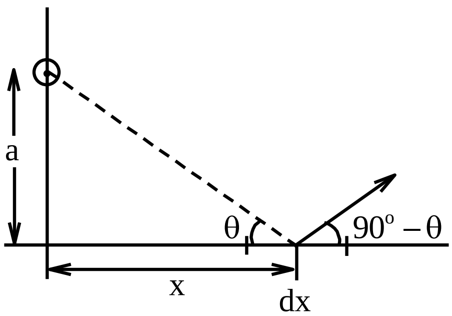
$$B = \frac{\mu_0 J}{2}\left(x - \frac{a^2}{x}\right)$$



7. (A)

$$B = \frac{\mu_0 i}{2\pi\sqrt{x^2 + a^2}}$$

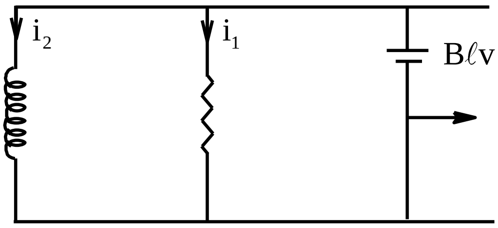
$$\int \vec{B} \cdot \vec{d}\ell = \int \frac{\mu_0 i}{2\pi\left(\sqrt{x^2 + a^2}\right)} dx \cos\left(90^o - \theta\right)$$



$$= \frac{\mu_0 i}{2\pi} \int_{a/\sqrt{3}}^{a} \frac{a}{x^2 + a^2} = \frac{\mu_0 i a}{2\pi} \times \frac{1}{a} \tan^{-1}\left(\frac{x}{a}\right)^2$$

$$\frac{\mu_0 i}{2\pi} \times \left[\frac{\pi}{4} - \frac{\pi}{6}\right] = \frac{\mu_0 i}{24}$$

8. **(B)**



$$i_1 = \frac{B\ell v}{R} \qquad B\ell v = \frac{Ldi}{dt}$$

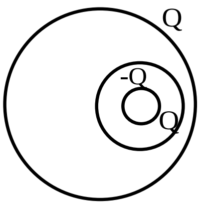
$$i_2 = \frac{B\ell vt}{L} \qquad i = i_1 + i_2$$

$$F = \left(\frac{B\ell v}{R} + \frac{B\ell vt}{L}\right)B\ell \qquad P = Fv = \frac{B^2\ell^2v^2}{R} + \frac{B^2\ell^2v^2t}{L}$$

$$w = \int_0^2 \rho dt = \frac{B^2\ell^2v^2}{R}\times 2 + \frac{B^2\ell^2v^2}{L}\times\frac{2^2}{2} \qquad = \frac{2^2\times 2^2\times 1^2}{2}\times 2 + \frac{2^2\times 2^2\times 1^2}{2}\times\frac{2^2}{2} = 32$$

9. **(A)**

$$U = \frac{Q^2}{\frac{2\times 4\pi\in_0 rr_1}{r_1 - r}} + \frac{Q^2}{4\pi\in_0 r_2}\times\frac{1}{2}$$



$$kQ^2\left(\frac{1}{2r_2} - \frac{1}{2r_1} + \frac{1}{2r}\right)$$

10. **(A)**

$$mv_0\ell = mv\ell + \left(\frac{m\ell^2}{3}\right)\omega$$

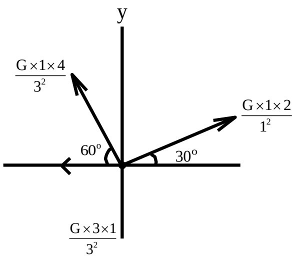
$$\frac{\omega\ell - v}{v_0} = 1$$

11. **(AD)**

$$F_x = 2G\cos 30^o - \frac{4G}{9}\cos 60^o - \frac{G}{3}$$

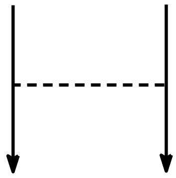
$$= G\left[\sqrt{3} - \frac{2}{9} - \frac{1}{3}\right] = C\left[\sqrt{3} - \frac{5}{9}\right]$$

$$F_y = \frac{4G}{9}\cos 30^o + 2G\sin 30^o$$



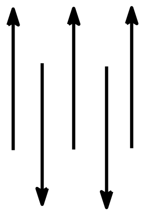
$$G\left[\frac{4}{3}\times\frac{\sqrt{3}}{2}+\frac{2\times1}{2}\right]=\left(\frac{2\sqrt{3}}{9}+1\right)G$$

12. **(D)**



$E_{middle}=\frac{kp}{x^3}+\frac{kp}{x^3}$ $\qquad U=-P.E$

$v-ve$



$E_{middle}=\frac{-kp}{x^3}-\frac{kp}{x^3}$

$v+ve$.

$\tau=p\times E=0$ in both cases.

13. **(ACD)**

$$\omega_1=\frac{1}{2}\times2\alpha\times T=-ve$$

$$\omega_{3T/2}=\frac{1}{2}\times-2\alpha\times T+\frac{1}{2}\times\alpha\times\frac{T}{2}=-ve$$

$$\omega_{5T/2}=\frac{1}{2}\times-2\alpha\times T+\frac{1}{2}\times\alpha\times\frac{T}{2}+\alpha\times T=+ve$$

$$\omega_{3T}=\frac{1}{2}\times2\alpha\times T+\frac{1}{2}\alpha\times\frac{T}{2}+\alpha T-\frac{1}{2}\times\alpha\times\frac{T}{2}=+ve$$

14. **(ABD)**

(A) So i become ………….

(B) $\left(k_{max}\right)_1=h\nu-\phi$ $\qquad$ .....(1)

$\left(k_{max}\right)_2=\frac{h\nu}{2}-\phi$ $\qquad$ .....(2)

$2\left(k_{max}\right)_2=h\nu-2\phi$ $\qquad$ .....(3)

(1) – (2)

$\left(k_{max}\right)_1-2\left(k_{max}\right)_2=\phi$

$$\left(k_{max}\right)_2 = \frac{\left(k_{max}\right)_1}{2} - \frac{\phi}{2}$$

(C) & (D)

If $E_{incidenet} < h\nu$, no current is there.

If $E_{incident} > h\nu$, option (A)

15. **(ABCD)**

HCF of 50, 150, 250 is 50 odd multiples of 50 Hz is possible, So this may be closed organ pipe producing odd multiple of 50 Hz fundamental frequency.

Or it may be odd harmonics of open organ pipe with again 50 Hz as its fundamental frequency.

16. **(5)**

$$\varepsilon = \frac{B\omega\left(2R\right)^2}{2}$$

$$\Rightarrow B = \frac{\varepsilon}{2R^2\omega} = \frac{10\times10^{-3}}{2\times0.5^2\times4} = 5 \text{ mT}$$

17. **(2)**

$$v_A - iR + 2 - \frac{Ldi}{dt} = v_B$$

$$v_A - v_B = 8 = iR - 2 + L\frac{di}{dt} = 2R - 2 + L\times1$$

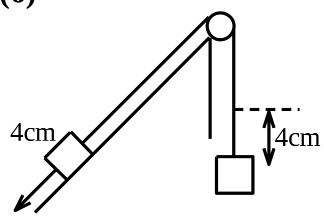
$$v_A{}' - v_B{}' = 4 = 2R - 2 - L\times1$$

$$L = 2H,\ R = 4\Omega \Rightarrow \frac{R}{L} = 2$$

18. **(5)**

$$R = \frac{V}{I} = \frac{6}{1.2} = 5\ \Omega$$

19. **(6)**



$$Wg = 2g\times\frac{40}{100}\ 1g\times\frac{40}{100}\sin30^o$$

$$= \frac{800}{100}\ \frac{200}{100} = \frac{600}{100} = 6\,J$$

20. **(1)**

$$F = u_{rel}\ \frac{dm}{dt} = 5\times\rho\frac{dv}{dt}$$

$$= 10^3\times5\times200\times10^{-6}\ N$$

$$= 1\,N$$

CHEMISTRY (PAPER – I)
SOLUTIONS

21. **(c)** $E_2 - E_1 = \left[\frac{-E_1}{4} + E_1\right] = \frac{+3E_1}{4}$, $E_4 - E_3 = \frac{-E_1}{16} + \frac{E_1}{9} = \frac{7E_1}{16\times 9}$

$\frac{E_2 - E_1}{E_4 - E_3} = \frac{3}{4}\times\frac{144}{7} = \frac{108}{7} = \frac{108}{7} \simeq$ **15**.

22. **(c)** $Z_A = \frac{1}{8}\times 8 = 1$; $Z_B = \frac{1}{2}\times 5 = \frac{5}{2}$ $AB_{5/2}$ i.e. A_2B_5

23. **(b)** $\Delta G = \Delta H - T\Delta S$. At equilibrium $\Delta G = 0$, so $\Delta H = T\Delta S$

$\Delta H = 273 \times (60.01 - 38.20) = 5954.13$ J $mole^{-1}$.

24. **(c)** $k = \frac{2.303}{2\times 10^4}\log\frac{800}{50}$

$= \mathbf{1.386 \times 10^{-4}}$.

25. **(c)** At pH = 3, $[H^+] = 10^{-3}$ M

At pH = 4, $[H^+] = 10^{-4}$ M

When in equal volume of the two solutions are mixed, the $[H^+]$

$= \frac{10^{-3} + 10^{-4}}{2}$. $= \frac{10^{-3}[1+0.1]}{2} = \frac{1.1}{2}\times 10^{-3}$

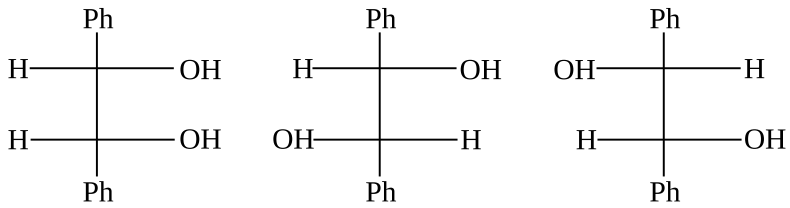
$[H^+] = 5.5 \times 10^{-4}$

$-\log [H^+] = -\log (5.5) - \log 10^{-4}$

pH = –0.7404 + 4 = **3.26**

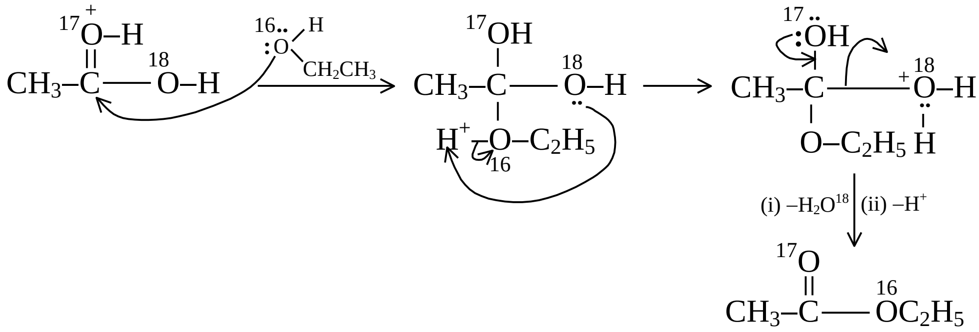
26. **(c)** Because it contains 4π electrons which disobeys Huckel's rule.

27. **(c)** $PhCOCOPh \xrightarrow{LiAlH_4} Ph - \overset{*}{C}H(OH) - \overset{*}{C}H(OH) - Ph$. The number of possible stereoisomers in the product is 3.



28. **(a)** Due to neighbouring group participation of –OEt group.

29. **(c)**



30. **(b)** The blue colouration is due to solvated electrons.

31. **(a, b)** In the given reaction 'X' can be Au and Ag_2S.

$$4Au + 8CN^- + 2H_2O + O_2 \longrightarrow 4Au(CN)_2^- + 4OH^-$$

$$2Ag_2S + 8CN^- + 2H_2O + O_2 \longrightarrow 4Ag(CN)_2^- + 4OH^- + 2S$$

32. **(a, b)**

33. **(b, d)** For $[Cu(NH_3)_4]^{2+}$ $Cu^{2+} = (Ar)3d^9$

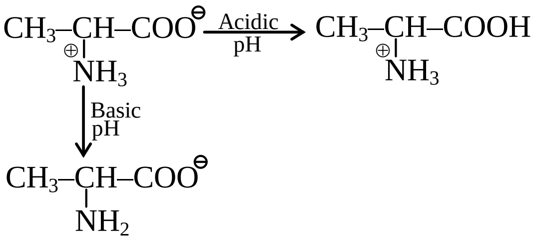
For $[Fe(CN)_6]^{3-}$ $Fe^{3+} = (Ar)3d^5$

Since, they have unpaired electrons they will be paramagnetic.

34. **(a, c, d)**

Both Ag^+ and Hg_2^{2+} give white precipitate of AgCl and Hg_2Cl_2 respectively.

35. **(b, c)**



36. **(4)** We know, $P = \frac{dRT}{M}$; $\frac{P_A}{P_B} = \frac{d_A M_B}{d_B M_A} = \frac{3 \times 2}{1.5 \times 1} = \mathbf{\frac{4}{1}}$

37. **(2)** $0.2046 = (1 + \alpha) \times 1.86 \times 0.1$

$\Rightarrow$ $1 + \alpha = 1.1$ $\Rightarrow$ $\alpha = 0.1$

$[H^+] = C\alpha = 0.1 \times (0.1) = 10^{-2}$ (M)

pH = 2

38. **(2)** Number of equivalent of MnO_4^- = number of equivalent of M^{x+}

$1 \times 5 = 1.67 \times$ n.f. n.f. $= 5 - x$

$$5 - x = \frac{5}{1.67} = 2.994$$

$x = \mathbf{2}$.

39. **(3)** NO^+ and CN^- both have 14 electrons and also both will have the following configuration.

$$\sigma 1s^2 \sigma^* 1s^2 \sigma 2s^2 \sigma^* 2s^2 \begin{pmatrix} \pi_{2p_y}^2 \\ \pi_{2p_z}^2 \end{pmatrix} \sigma 2p_x^2$$

$\therefore$ Bond order of NO^+ = Bond order of CN^- = 3.

40. **(4)**

	A(g)	+ 2B(g) ⇌	2C(g)	+ D(g)
Initial conc.	x	$1.5x$	0	0
At equilibrium	$x - y$	$1.5x - 2y$	2y	y

Given: $x - y = 1.5x - 2y$; $y = 0.5x$. Thus, at equilibrium $[A] = 0.5x$ $[B] = 0.5x$ $[C] = x$ $[D] = 0.5x$

$$\therefore K_c = K_p = \frac{(x^2)(0.5x)}{(0.5x)(0.5x)^2} = \mathbf{4}.$$

MATHS PAPER – I (SOLUTION)

41. (D)

$\omega^{\omega-\omega^2} = e^{-2\pi/\sqrt{3}}$

so $\ln(\omega)^{\omega-\omega^2} = -\frac{2\pi}{\sqrt{3}}$.

42. (A)

Given, $f(x) = \int_{\frac{\pi}{2}}^{x} (\cos t - 2\sin t)dt \Rightarrow \quad f'(x) = (\cos x - 2\sin x) < 0$ in $x \in \left[\frac{\pi}{2}, \pi\right]$

$\therefore \quad f(x)$ is decreasing $(\downarrow)$ function in $\left[\frac{\pi}{2}, \pi\right]$

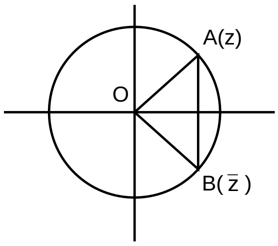
Hence, $f\left(x = \frac{\pi}{2}\right)_{\text{greatest}} = \int_{\frac{\pi}{2}}^{\frac{\pi}{2}} (\cos t - 2\sin t)dx = 0.$

43. (A)

$|z - \bar{z}|$ = straight line AB length

while $|z|(\arg z - \arg \bar{z})$ = Arc AB

$\therefore |z - \bar{z}| \le |z|(\arg z - \arg \bar{z})$



44. (B)

(AB) . (AB) = A(BA)B

$= A^3B^2$

(AB)(AB)(AB)

$= A^7B^3$

so $(AB)^n = A^{2^n-1}B^n$

so $k = 2^{10} - 1 = 1023$.

45. (D)

We must have, $-1 \le \left|\frac{x^2-4}{3x}\right| \le 1 \quad \Rightarrow -1 \le \frac{x^2-4}{3x} \le 1 \quad \Rightarrow \frac{x^2-4}{3x} + 1 \ge 0$ and $\frac{x^2-4}{3x} - 1 \le 0$

$\Rightarrow \frac{(x+4)(x-1)}{3x} \ge 0$ and $\frac{(x-4)(x+1)}{3x} \le 0$

$\Rightarrow \quad x = -4, -3, -2, -1$, 1, 2, 3, 4. Hence, number of integral values is 8.

46. (C)

$D < 0 \Rightarrow 4b^2 - 4ac < 0$

(i) if $b = \frac{a+c}{2} \Rightarrow (a + c)^2 - 4ac < 0 \Rightarrow (a - c)^2 < 0$ which is not possible

(ii) if $b^2 = ac$, $0 < 0$ (not possible)

(iii) if $b = \frac{2ac}{a+c} \Rightarrow \frac{4a^2c^2 - ac(a+c)^2}{(a+c)^2} < 0 \Rightarrow \frac{ac(a-c)^2}{(a+c)^2} > 0$

which is true as $ac - b^2 > 0 \Rightarrow ac > b^2 \ge 0$.

47. (C)
$P^3 = P(I - P) = P - P^2 = P - (I - P) = 2P - I$
$P^4 = P(2P - I) = 2(I - P) - P = 2I - 3P$
$P^5 = P(2I - 3P) = 2P - 3(I - P) = 5P - 3I$
$P^6 = P(5P - 3I) = 5(I - P) - 3P = 5I - 8P$
So n = 6.

48. (b)
Team totals must be 0, 1, 2, …, 39.
Let the teams be $T_1, T_2, \ldots, T_{40}$, so that T_i losses to T_j for i < j.
In other words, this order uniquely determines the result of every game.
There are 40! such orders and 780 games, so 2^{780} possible outcomes for the games.
Hence the probability is $\frac{40!}{2^{780}}$.

49. (C)
$$f(n) = \sum_{r=1}^{n}\left[r^2\left({}^nC_r - {}^nC_{r-1}\right) + (2r)\,{}^nC_r + {}^nC_r\right] = \sum_{r=1}^{n}\left[\left(r^2 + 2r + 1\right){}^nC_r - r^2\,{}^nC_{r-1}\right]$$
$$= \sum_{r=1}^{n}\left[(r+1)^2\,{}^nC_r - r^2\,{}^nC_{r-1}\right] = \sum_{r=1}^{n}\left[V_{r+1} - V_r\right]$$
$= V_2 - V_1 + V_3 - V_2 + \ldots + V_{n+1} - V_n$
$= V_{n+1} - V_1 = (n + 1)^2\,{}^nC_n - 1 = n^2 + 2n.$

50. (B)
$(2345)^{678} = (2344 + 1)^{678} = 4k + 1$
and $(1234)^{567}$ is divisible by 4
Hence the remainder is 1.

51. B, C, D
$\overrightarrow{PQ} = \hat{i}\left(\lambda_2 - \lambda_1 - 1\right) + \hat{j}\lambda_2 + k\left(1 + \lambda_1\right)$
$\overrightarrow{PQ}\cdot\left(\hat{i} - \hat{k}\right) = 0$ and $\overrightarrow{PQ}\cdot\left(\hat{i} + \hat{j}\right) = 0$
$\Rightarrow \lambda_2 = 0$ and $\lambda_1 = -1$.
Therefore $\overrightarrow{PQ} = \overline{0}$
$\vec{P} = \vec{Q} = \hat{j} + \hat{k}$
projection of $\overrightarrow{OP}$ on $\hat{i} + \hat{j} + \hat{k}$ is
$$= \left(\hat{j} + \hat{k}\right)\cdot\frac{\hat{i} + \hat{j} + \hat{k}}{\sqrt{3}} = \frac{2}{\sqrt{3}}.$$

52. B, C
$n = 3^{52}\,7^{52}$
so number of divisor = 53 × 53 = 2809
$21^{52} = (20 + 1)^{52} = {}^{52}C_0\,20^{52} + {}^{52}C_1\,20^{51} + \ldots + {}^{52}C_{51}\,20 + 1$
$= 100k + 52 \times 20 + 1 = 100k' + 41.$
Number of divisor which is multiple of 9 = 53 × 51 = 2703.

53. A, B, D
$$\arg\left(\frac{z - (3 + 4i)}{z - (1 - 2i)}\right) = \frac{\pi}{2}$$

so $z_c = \frac{3+4i+1-2i}{2} = 2 + i = z_1$

$\Rightarrow |z_1^8| = 625 \Rightarrow \arg(z_1 - 2) = \frac{\pi}{2}$.

(D) bc(1 – a), ac(1 – b), ab(1 – c) are in A.P.

54. C, D

b is H.M. of a and c

$b < \frac{a+c}{2}$

$b - a < c - b$

$a - b > b - c$

$\frac{1}{a-b} - \frac{1}{b-c} < 0$

also $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in A.P.

so bc(1 – a), ac(1 – b), ab(1 – c) are in A.P.

55. (B, C, D)

Given, $f(x) = \begin{cases} |2x-1|^{\alpha} \cdot \sin(2\pi x), & 0 < x \le 1 \\ 0, & x = 0 \end{cases}$

Note : for $\alpha = 0$, f (x) is not defined at $x = \frac{1}{2} \in [0, 1]$ ...... (1)

$\therefore$ $f(x)$ is continuous at x = 0, 1 but for continuous at $x = \frac{1}{2}$,

$f\left(\frac{1}{2}\right)^{-} = f\left(\frac{1}{2}\right)^{+} = f\left(\frac{1}{2}\right)$

$\Rightarrow \lim_{x \to \frac{1}{2}} |2x-1|^{\alpha} \sin(2\pi x) = 0$; put $x = \frac{1}{2} - h$

$\Rightarrow \lim_{h \to 0} (2h)^{\alpha} \sin 2\pi h = 0 \Rightarrow \alpha \ge 0$ ...... (2)

Hence, $(1) \cap (2) \Rightarrow \alpha > 0$.]

Note : f (x) is differentiable at $x = \frac{1}{2}\ \forall\ \alpha > 0$ and hence differentiable in (1, 0).

56. 5

Total number of cases = coefficient of x^n in $(1 + x + x^2 + \ldots + x^n)^3 = {}^{n+2}C_2$

So required probability = $\frac{3}{{}^{n+2}C_2} = \frac{6}{(n+1)(n+2)} \ge \frac{1}{6}$

$\Rightarrow n \ge \frac{-3+\sqrt{145}}{2}$

$n_{min} = 5$

so minimum number of red balls = 5.

57. 1.

$\log_6(abc) = 6$

$\Rightarrow$ (abc) = 6^6

let $a = \frac{b}{r}$ and c = br

$\Rightarrow$ b = 36 and a = $\frac{36}{r}$ $\Rightarrow$ r = 2 , 3, 4, 6, 9, 12, 18

also, $36\left(1-\frac{1}{r}\right)$ is a perfect cube.

$\Rightarrow$ $36\left(1-\frac{1}{r}\right)$ is a perfect cube for r = 4.

$\Rightarrow$ a + b + c = 36 + 9 + 144 = 189.

58. 8

By A.M./G.M.

$x^4 + y^4 \ge 2x^2y^2$

and $2x^2y^2 + z^2 \ge \sqrt{8}$ xyz

$\Rightarrow \frac{x^4+y^4+z^2}{xyz} \ge \sqrt{8}$.

59. 3.

Let the circumcentre of the triangle be the origin.

$\Rightarrow$ orthocentre is $\vec{a}+\vec{b}+\vec{c}$ and the centroid is $\frac{\vec{a}+\vec{b}+\vec{c}}{3}$

$\Rightarrow$ F = $\frac{2}{3}(\vec{a}+\vec{b}+\vec{c})$

$\Rightarrow (\vec{a}-\vec{F})^2+(\vec{b}-\vec{F})^2+(\vec{c}-\vec{F})^2 = \frac{1}{9}\left[(\vec{a}-2(\vec{b}+\vec{c}))^2+(\vec{b}-2(\vec{a}+\vec{c}))^2+(\vec{c}-2(\vec{a}+\vec{b}))^2\right]$

$= \frac{1}{9}(27) = 3$.

60. 9

Given, area $(\Delta O_1O_3O_5) = 12\sqrt{3} \Rightarrow \frac{\sqrt{3}}{4}(\text{side})^2 = 12\sqrt{3}$

[**Note :** $\Delta O_1O_2O_3$ is equilateral with length of each side equal 4r]

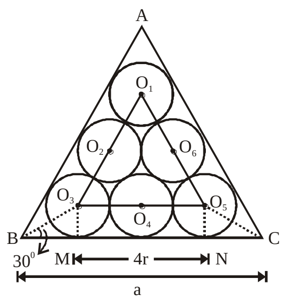
$\Rightarrow \quad (O_1O_3) = 4\sqrt{3}$

Also, $O_1O_3 = 4r = 4\sqrt{3}$

$\Rightarrow \quad r = \sqrt{3}$ (radius of circle)

In ΔBMO_3, $\frac{MO_3}{BM} = \tan 30°$

$\Rightarrow \quad \frac{\sqrt{3}}{BM} = \frac{1}{\sqrt{3}} \Rightarrow BM = 3$



Also, $BC = BM + MN + CN$

$\Rightarrow \quad a = 3 + 4\sqrt{3} + 3$ or $a = 6 + 4\sqrt{3}$

So, area $(\Delta ABC) = \frac{\sqrt{3}}{4}a^2 = \frac{\sqrt{3}}{4}\left(6+4\sqrt{3}\right)^2 = \sqrt{3}\left(3+2\sqrt{3}\right)^2 = \sqrt{3}\left(21+12\sqrt{3}\right)$

$= 36 + 21\sqrt{3} = p + 21\sqrt{q}$ (Given) $\Rightarrow p = 36, q = 3$.

PHYSICS PAPER – II (SOLUTION)

1. **(AB)**

2. **(ACD)**

Fringe width $= \frac{\lambda D}{d}$

If D increases then fringe width increases and fringes inflate λ decreases when the whole system is immersed in a liquid of refractive index μ then fringe width decreases fringes increases.

3. **(BC)**

$$\frac{v_A}{v_B} = \sqrt{\frac{\gamma_A}{\gamma_B}\frac{M_B}{M_A}} = \sqrt{\frac{42}{25}}$$

(M_A and M_B are molecular weight A and B respectively)

$$\frac{3v_A}{2l_A} = \frac{3v_B}{4l_B} \Rightarrow \frac{l_A}{l_B} = \frac{2v_A}{v_B} = \sqrt{\frac{168}{25}}$$

4. **(ABD)**

5. **(BC)**

Applying Kirchholl's rule

$V = 4 + 3 = 7$ while i increasing.

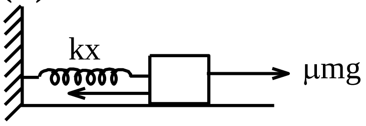
$V = -3 + 4 = 1$ while i is decreasing

6. **(ABD)**

7. **(ABCD)**

8. **(ABCD)**

9. **(C)**



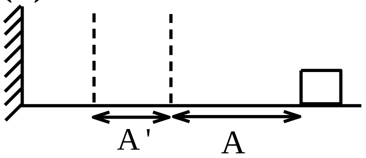
$F = -kx + \mu mg$ $\qquad$ $F = 0 \Rightarrow kx_0 = \mu mg$

$F = \left(-k(x_0 + y)\right) + \mu mg$ $\qquad$ $ma = -ky$

$$a = \frac{k}{my}$$

$\omega\sqrt{\frac{k}{m}}$ $\qquad$ $T = 2\pi\sqrt{\frac{m}{k}}$

10. **(B)**



$$\mu mg\left(A + A'\right) + \frac{1}{2}k\left(A^2 - A'^2\right) = 0$$

$$\Rightarrow \quad \mu mg\left(A+A'\right)-\frac{1}{2}k\left(A^2-A'^2\right)=0$$

$$\frac{1}{2}k\left(A-A'\right)=\mu mg$$

$$A-A'=\frac{2\mu mg}{k} \qquad A'=A-\frac{2\mu mg}{k}$$

11. **(B)**

12. **(B)**

13. **(D)**

14. **(C)**

15. **(D)**

$$V=-\int_0^d \frac{aQ}{\epsilon_0}\left(1+\frac{x^2}{d^2}\right)dx$$

$$=-\frac{aQ}{\epsilon_0}\left[d+\frac{d}{3}\right]=\frac{4aQd}{3\epsilon_0}$$

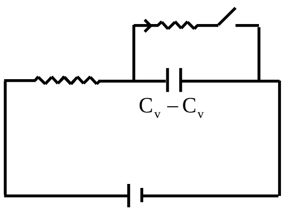
16. **(C)**

$$C=\frac{Q}{V}=\frac{3\epsilon_0}{4ad}$$

17. (A)

18. **(C)**

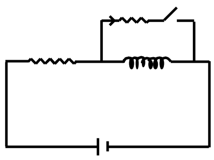
(A) $i_0=\frac{V}{R}\ V_0-V$



$i_\infty=\frac{V}{2R}$

$V_\infty=\frac{V}{2}$

(B) $i_0=0,\ V_0=0$

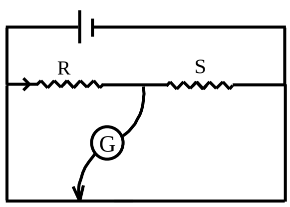


$i_\infty=0,\ V_\infty=0$

(C) i_0

$$=\frac{V}{RG}\times\frac{G}{R+G}$$

$$\frac{RG}{R+G}+S$$



$$=\frac{VG}{RG+RS+GS}=\frac{VG}{RG+R^2+RG}$$

$$=\frac{VG}{R\left[2G+R\right]}=\frac{V\times R}{R\times 3R}i_0\frac{V}{3R}$$

$$i_\infty = \frac{V}{2R}$$

$$i_\infty < i_0 \Rightarrow i \downarrow$$

(D) $i = \frac{B\ell v}{R}$ $v \uparrow \Rightarrow i \uparrow$ and $V_R \uparrow$

19. (D)

20. (B)

CHEMISTRY PAPER – II (SOLUTION)

21. **(b, d)** Only structures given in option (b) and (d) will form cations after reacting with aqueous $AgNO_3$ because they will be more stable due to having aromatic character.

22. **(b, c)**

More is value of *i* lesser will be freezing point of solution

and $i = \dfrac{\text{Molecular wt. theoretical}}{\text{Molecular wt. experimental}}$

23. **(b, c, d)**

24. **(b, c)** Alum and Borax reduce the rate of setting of plaster of paris. NaCl accelerates the rate of setting while $CaSO_4$ has no effect on the rate of setting.

25. **(a, c)**

(a) Zn^{2+} (Z = 30) $1s^22s^22p^63s^23p^63d^{10}$
Cu^{+} (Z = 29) $1s^22s^22p^63s^23p^63d^{10}$
Both the species are diamagnetic.

(b) Co^{2+} (Z = 27) $1s^22s^22p^63s^23p^63d^7$ (3 unpaired electrons)
Ni^{2+} (Z = 28) $1s^22s^22p^63s^23p^63d^8$ (2 unpaired electrons)

(c) Mn^{4+} (Z = 25) $1s^22s^22p^63s^23p^63d^3$ (3 unpaired electrons)
Co^{2+} (Z = 27) $1s^22s^22p^63s^23p^63d^7$ (3 unpaired electrons)
Both the species are paramagnetic and their magnetic moments are same.

(d) Mg^{2+} (Z = 12) $1s^22s^22p^6$ (no unpaired electrons)
Sc^{+} (Z = 21) $1s^22s^22p^63s^23p^63d^14s^1$ (2 unpaired electrons)

26. **(a, c, d)** Peroxy acid give Anti addition of diol and $KMnO_4$ gives syn addition.

27. **(a, b)**

$$\overset{+4}{MnO_2} + \overset{-1}{HCl} \longrightarrow MnCl_2 + 2H_2O + \overset{0}{Cl_2} \quad \ldots(i)$$
n = 2 (for MnO_2), n = 2 (for Cl_2)

$$\overset{0}{Cl_2} + \overset{-1}{KI} \longrightarrow \overset{0}{I_2} + \overset{-1}{KCl} \quad \ldots(ii)$$
n = 2 (for Cl_2), n = 2 (for I_2)

$\therefore$ Eq. of I_2 = eq. of Cl_2 reacted in equation (ii)
= eq. of Cl_2 produced in equation (i)
= eq. of MnO_2 reacted in equation (i)
$= 2 \times \dfrac{8.7}{87}$

$\therefore\ 2 \times \dfrac{\text{wt. of } I_2}{\text{Molecular wt. of } I_2} = 2 \times \dfrac{8.7}{87}$

$\therefore$ weight of I_2 = **25.4 g**.

28. **(a, b, c)**

No. of equivalent of $KMnO_4 = 0.1 \times 5 = 0.5$
No. of equivalent of $Fe^{+2} = 0.5$
No. of equivalent of $FeCO_4 = 0.166 \times 3 = 0.498$
No. of equivalent of $C_2O_4^{2-} = 0.25 \times 2 = 0.5$

29. **(a)** $A \longrightarrow 2B + C$

t = 0	a	0	0
t = 40	a – x	2x	x
t = ∞	0	2a	a

$(a - x) \times 60 + 2x \times 50 + x(-80) = 26$

$60a - 40x = 26$ …(1)

$2a \times 50 + a(-80) = 10$ …(2)

By (1) and (2)

$a = \frac{1}{2}$; $x = \frac{1}{10}$

$$K_{27^o C} = \frac{2.303}{40} \log \frac{1/2}{1/2 - 1/10}$$

$K_{27^o C} = 5.57 \times 10^{-3}\ min^{-1}$.

30. **(c)** $\log \frac{k_2}{k_1} = \frac{E_a}{2.303R}\left[\frac{1}{T_1} - \frac{1}{T_2}\right]$

$$\log \frac{k_2}{5.57 \times 10^{-3}} = \frac{27 \times 10^3}{2.303 \times 8.314}\left[\frac{1}{300} - \frac{1}{400}\right]$$

$$\log \frac{k_2}{5.57 \times 10^{-3}} = 1.175$$

$$\frac{k_2}{5.57 \times 10^{-3}} = 14.97$$

$(k_2)_{127} = 83.36 \times 10^{-3}$

$(t_{1/2})_{127} = \frac{0.693}{k_2}$ = **8.4 min**.

31. **(b)** $(CH_3)_2CH-CHO \xrightarrow{\overset{\ominus}{OH}} (CH_3)_2CH-CH_2OH + (CH_3)_2CH-COO^{\ominus}\overset{\oplus}{Na}$

32. **(c)** $\begin{matrix} CHO \\ | \\ CHO \end{matrix} \xrightarrow{\overset{\ominus}{OH}} \begin{matrix} CH_2OH \\ | \\ CO\overset{\ominus}{O}\overset{\oplus}{Na} \end{matrix}$ [the carbonyl group from which $H^{\ominus}$ shifts, converts into salt and the other one converts into alcohol]

33. **(d)** Chlorofluorocarbons break up into Cl° (free radical) by UV radiation in the atmosphere and instantly reacts with O_3 decomposing it into O_2 and O. ClO converts 'O' in to O_2, which stops the formation of O_3 according to equation $O + O_2 \longrightarrow O_3$.

34. **(c)**
35. **(b)**
36. **(c)**

37. **(d)**

$[Zn(NH_3)_4]^{2+} \rightarrow d^{10}$ configuration $\rightarrow sp^3$ hybridisation $\rightarrow$ tetrahedral.

$[Cu(NH_3)_4]^{2+} \rightarrow d^9$ configuration $\rightarrow dsp^2$ hybridisation $\rightarrow$ square planar.

$[Fe(CN)_6]^{3-} \rightarrow d^5$ configuration $\rightarrow d^2sp^3$ hybridisation $\rightarrow$ octahedral.

$Fe(CO)_5 \rightarrow d^8$ configuration $\rightarrow dsp^3$ hybridisation $\rightarrow$ trigonal bipyramidal.

38. **(b)**
39. **(a)**
40. **(b)**

MATHS PAPER – II (SOLUTION)

41. (BCD)

$$f(x)=\left(\frac{\pi}{2}-\cos^{-1}(\cos x)\right)\left(\frac{\pi}{2}\sin^{-1}(\sin x)\right)$$

$$=\begin{cases}\left(\frac{\pi}{2}-x\right)^2, & 0\le x\le\frac{\pi}{2}\\ \left(\frac{\pi}{2}-x\right)\left(\frac{\pi}{2}-(x-\pi)\right)=-\left(\frac{\pi}{2}-x\right)^2, & \frac{\pi}{2}<x\le\pi\\ \left(\frac{\pi}{2}-(2\pi-x)\right)\cdot\left(\frac{\pi}{2}-(\pi-x)\right)=\left(x-\frac{3\pi}{2}\right)\left(x-\frac{\pi}{2}\right)=(x-\pi)^2-\frac{\pi^2}{4}, & \pi<x\le\frac{3\pi}{2}\\ \left(\frac{\pi}{2}-(2\pi-x)\right)\cdot\left(\frac{\pi}{2}-(x-2\pi)\right)=\left(x-\frac{3\pi}{2}\right)\left(\frac{5\pi}{2}-x\right)=\frac{\pi^2}{4}-(x-2\pi)^2, & \frac{3\pi}{2}<x\le 2\pi\end{cases}$$

$$f(x)=\left(\frac{\pi}{2}-\cos^{-1}(\cos x)\right)\left(\frac{\pi}{2}-\sin^{-1}(\sin x)\right)$$

$$f(x)=\left(\frac{\pi}{2}-x\right)^2\ \forall x\in\left[0,\frac{\pi}{2}\right],$$

$$f(x)=\left(\frac{\pi}{2}-x\right)\left(\frac{\pi}{2}-(\pi-x)\right)$$

$$f(x)=-\left(\frac{\pi}{2}-x\right)^2;\ \forall x\in\left(\frac{\pi}{2},\pi\right]$$

$$f(x)=(\pi-x)^2-\frac{\pi}{4},\pi<x\le\frac{3\pi}{2},$$

$$f(x)=\frac{\pi^2}{4}-(2\pi-x)^2\ \forall\frac{3\pi}{2}<x\le 2\pi$$

At $x=\pi$, $f(x)$ is not differentiable

At $x=\pi$, it is local as well as global minimum.

Range : $\left[\frac{-\pi^2}{4},\frac{\pi^2}{4}\right]$,

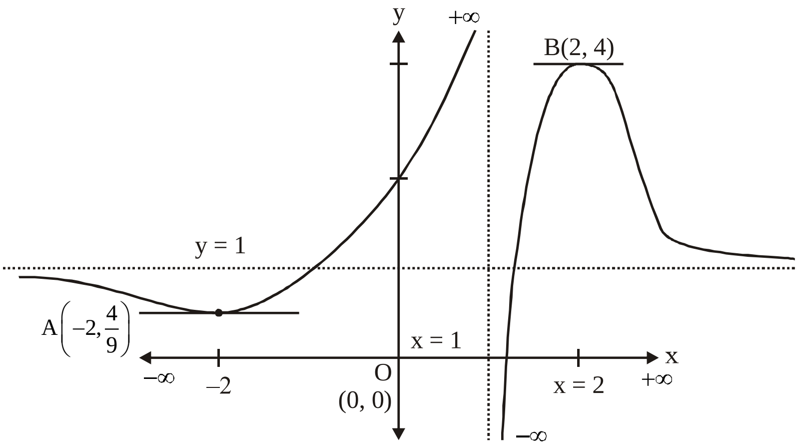
$$\int_0^{\frac{\pi}{2}} f(x)\,dx=\int_0^{\frac{\pi}{2}}\left(\frac{\pi}{2}-x\right)^2 dx$$

Using king

$$I=\int_0^{\frac{\pi}{2}} x^2dx=\left.\frac{x^3}{3}\right]_0^{\frac{\pi}{2}}=\frac{\pi^3}{24}$$

42. (B)

43. (BC)



Graph of $f(x)=\frac{(x^3-4)}{(x-1)3}$

Now, verify alternatives.

44. (ABCD)

Obviously, $f(x)=\sin^{-1}x$ and $f'(x)=\frac{1}{\sqrt{1-x^2}}\Rightarrow f'\left(\frac{\sqrt{3}}{2}\right)=2$

Note : $\sin^{-1}x>x \quad \forall x\in(0,1)\Rightarrow f\left(\frac{2}{3}\right)>\frac{2}{3}$

Also, $\lim_{x\to 0}\frac{\sin^{-1}x}{x}=1$

Again, $f(x)+f\left(\sqrt{1+x^2}\right)-\sin^{-1}x+\sin^{-1}\left(\sqrt{1-x^2}\right)=\sin^{-1}x+\cos^{-1}x=\frac{\pi}{2}$

45. (A)

$\vec{\alpha}\cdot\vec{\beta}=\frac{b}{a}+\frac{4a}{b}+1$

as $\frac{b}{a}+\frac{4a}{b}+1\geq 5$

so $\left(\frac{10}{5+\vec{\alpha}\cdot\vec{\beta}}\right)_{max}=1.$

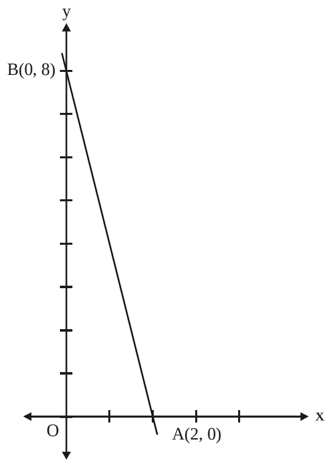
46. (BD)

Let the line be

$\frac{x}{a}+\frac{y}{b}=1, a, b>0$

Where $\frac{1}{a}+\frac{4}{b}=1 \quad\Rightarrow\quad \frac{4}{b}=1-\frac{1}{a} \quad\Rightarrow\quad \frac{4}{b}=\frac{a-1}{a}$

$b=\frac{4a}{a-1}$



Now,

$$2A = ab = a\left(\frac{4a}{a-1}\right) = \frac{4\left(a^2-1+1\right)}{a-1} = 4\left[\left(a+1\right)+\frac{1}{a-1}\right]$$

$$2\frac{dA}{da} = 4\left[1-\frac{1}{\left(a-1\right)^2}\right] = 0$$

$a – 1 = 1$ or $–1$

$a = 2;\ b = 8,\ m = – 4$

$$A\big]_{min} = \frac{16}{2} = 8$$

Equation is $\left(y-4\right) = -4\left(x-1\right) \Rightarrow y-4 = -4x+4 \Rightarrow 4x+y = 8$ Ans.

Also, $r = \dfrac{\Delta}{s} = \dfrac{\frac{1}{2}\times 2\times 8}{\frac{2+8+\sqrt{68}}{2}} = \dfrac{8}{5+\sqrt{17}} = 5-\sqrt{17}$

Now verify alternatives.]

47. (ABD)

(A) We know that every continuous function in closed interval [a, b] is bounded.
So, Given statement is explained by extreme value theorem.

(B) Given $f\left(x\right) = x^2-2x+3 = \left(x-1\right)^2+2$

vertex of $f\left(x\right) = \left(1, 2\right)$

Clearly, f (x) has neither a maximum nor a minimum in interval .

(C) Let $f\left(x\right) = \tan\frac{\pi x}{4}+\left(x-1\right)\sin x+5\cos \pi x$

Clearly, f (x) is continuous function in [0, 1].

Also, f (0) = 5 and $f\left(1\right) = 1+0-5 = -4 \Rightarrow f\left(0\right) f\left(1\right) < 0$

So, f (x) = 0 has atleast one root in (1, 0) (Using intermediate value theorem)

(D) Given, $12a+6b+4c+3d = 0$

$$\Rightarrow \quad a+\frac{b}{2}+\frac{c}{3}+\frac{d}{4} = 0 \qquad \text{...... (1)}$$

Now, $I = \int_0^1 \left(a+bx+cx^2+dx^3\right)dx$

$$= \left(ax+\frac{bx^2}{2}+\frac{cx^3}{3}+\frac{dx^4}{3}\right)_0^1 = a+\frac{b}{2}+\frac{c}{3}+\frac{d}{4} = 0 \qquad \text{(Using equation (1))}$$

$\therefore$ $f\left(x\right)$ must have atleast one roots in (0, 1). (Using Property of definite integral.)

So, the given statement is true.

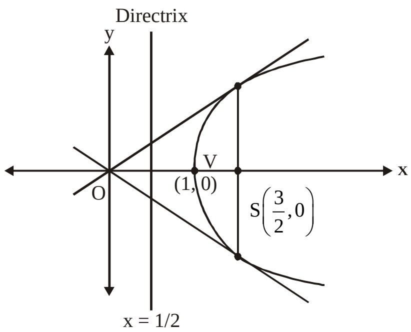
48. (ABCD)

To find locus of the middle points of all focal chords

$T = S_1$

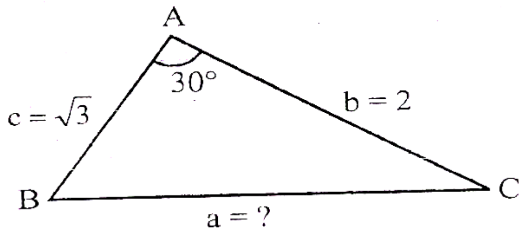
$yk-2\left(x+h\right) = k^2-4h$ passes through (1, 0)

Hence, locus is $y^2 = 2x-2 = 2\left(x-1\right)$.



Now, verify alternatives.

Sol. $\cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$



$\Rightarrow \quad \cos 30^o = \dfrac{(2)^2 + (\sqrt{3})^2 - a^2}{2(2)(\sqrt{3})} = \dfrac{\sqrt{3}}{2}$

$\Rightarrow \quad 7 - a^2 = 6 \Rightarrow a = 1$

49. (b)

$\tan\left(\dfrac{B-C}{2}\right) = \left(\dfrac{b-c}{b+c}\right)\cot\dfrac{A}{2}$

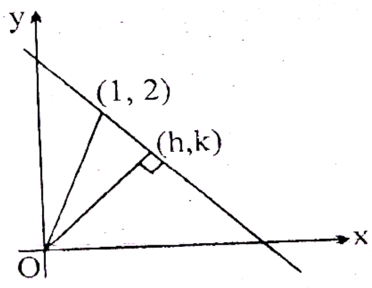
$= \left(\dfrac{2-\sqrt{3}}{2+\sqrt{3}}\right)\cot 15^o = \left(\dfrac{2-\sqrt{3}}{2+\sqrt{3}}\right)(2+\sqrt{3}) = (2-\sqrt{3})$ Ans.

50. (a)

Area of triangle $ABC = \dfrac{1}{2}bc \sin A = \dfrac{1}{2}(2)\ (\sqrt{3})\ \sin 30^o = \dfrac{\sqrt{3}}{2}$

$r = \dfrac{\Delta}{s} = \dfrac{\sqrt{3}/2}{\dfrac{3+\sqrt{3}}{2}} = \dfrac{\sqrt{3}-1}{2}$

Sol. Given $(4a+3)x - (a+1)y - (2a+1) = 0$



$(3x - y - 1) + a(4x - y - 2) = 0$

family of lines passes through the fixed point P which is the intersection of

$3x - y = 1 \qquad \text{and} \qquad 4x - y = 2$

solving P(1, 2),

now

51. (d)

$\dfrac{k}{h}\cdot\dfrac{k-2}{h-1} = -1$

$\therefore$ locus is $x(x-1) + y(y-2) = 0$

$x^2 + y^2 - 2y - x = 0$

$$\left(x-\frac{1}{2}\right)^2+(y-1)^2=\frac{5}{4}$$

$$(2x-1)^2+4(y-1)^2=5$$

52. (c)

Again $y-2=m(x-1)$

$$x=0;\ y=2-m;\quad y=0,\ x=1-\frac{2}{m}$$

$$\therefore\quad 2A=(2-m)\left(1-\frac{2}{m}\right)\ (m<0)$$

$$2A=2-m-\frac{4}{m}+2=4+\left(-m-\frac{4}{m}\right)$$

let $-m=M$ $(M>0)$

$$2A=4+M+\frac{4}{M}$$

$$=4+\left(\sqrt{M}-\frac{2}{\sqrt{M}}\right)^2+4$$

$$=8+\left(\sqrt{M}-\frac{2}{\sqrt{M}}\right)^2$$

area is minimum if $M=2 \Rightarrow m=-2$

$$2A|_{min}=8 \Rightarrow A|_{min}=4$$

53. (D)

54. (B)

Sol. $g(x)=x-k$ where $k=\int_0^1 f(t)dt$ …(1)

$$f(x)=\frac{x^3}{2}+1-x\int_0^x g(t)dt=\frac{x^3}{2}+1-x\int_0^x (t-k)dt=\frac{x^3}{2}+1-x\left[\left(\frac{t^2}{2}-kt\right)_0^x\right]$$

$$=\frac{x^3}{2}+1-x\left(\frac{x^2}{2}-kx\right)=1+kx^2$$

$f(x)=1+kx^2$ …(2)

from (1)

$$k=\int_0^1(1+kt^2)dt=t+\frac{kt^3}{3}\Big|_0^1=1+\frac{k}{3}\Rightarrow \frac{2k}{3}=1\Rightarrow k=\frac{3}{2}$$

$\therefore$ $g(x)=x-\frac{3}{2} \Rightarrow$ g is linear with slope 1 and cuts the y-axis at $\frac{3}{2}$

Also $f(x)=1+\frac{3x^2}{2}$ which is a quadratic polynomial

55. (A)

56. (D)

Sol. **(i)** E_1: door gets opened on the 5^{th} trial when unsuccessful keys are discarded

$P(E) = P(F\ F\ F\ F\ S)$ D < 1, 9

$= \frac{9}{10} \cdot \frac{8}{9} \cdot \frac{7}{8} \cdot \frac{6}{7} \cdot \frac{1}{6} = \frac{1}{10}$.

(ii) E_2 : doors gets opened unsuccessful keys are not discarded

Here events are independent with $p = \frac{1}{10}$ and $q = \frac{9}{10}$

$P(E_2) = P(FFFFS) = \left(\frac{9}{10}\right)^4 \cdot \frac{1}{10}$ Ans.

57. (C)

58. (D)

59. (C)

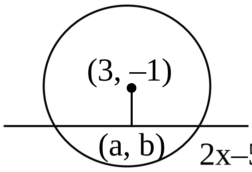
(A) $e^{x^2} > 0$ for all x hence $\int_{\alpha}^{\alpha^3} e^{x^2} dx > 0$ or < 0 (think!)

Hence, $\int_{\alpha}^{\alpha^3} e^{x^2} dx = 0$ is possible only if $\alpha = \alpha^3$

$\therefore \quad \alpha = 0, 1, -1$

$\therefore$ 3 values. Ans.

(B) $2a - 3b + 18 = 0$ …(1)



$\left(\frac{b+1}{a-3}\right)\left(\frac{2}{5}\right) = -1$

$2b + 2 = 15 - 5a$

$5a + 2b = 13$ …(2)

Solving (1) and (2) a = 1 and b = 4

Hence, (a + b) = 5, which is twin prime with 3 and 7.

(C) $x^2 - \frac{10}{9}x + c = 0$ < α, α^2

$\therefore \quad \alpha + \alpha^2 = \frac{10}{9}$

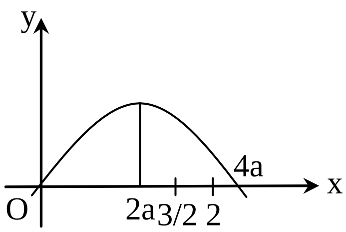
$9\alpha^2 + 9\alpha - 10 = 0 \Rightarrow (3\alpha + 5)(3\alpha - 2) = 0$

$\alpha = \frac{2}{3} \Rightarrow \alpha = \frac{-5}{3}$

If $\alpha = \frac{2}{3} \Rightarrow \frac{4}{9} - \frac{10}{9}\left(\frac{2}{3}\right) + c = 0$

$c = \frac{8}{27} \equiv \frac{m}{n} \Rightarrow (m + n) = 35$. Ans

If $\alpha = \frac{-5}{3}$, then c < 0 hence rejected.



(D) Case-I

If 0 < a < 1 (obviously ‘a’ can not be <0)

Then for f(x) to be increasing

$4ax - x^2$ should be decreasing in $\left(\frac{3}{2}, 2\right)$

$\Rightarrow \quad \frac{3}{2} \geq 2a \qquad$ and $\qquad 2 < 4a$

$\therefore \quad a \leq \frac{3}{4}$ and $a > \frac{1}{2} \quad \Rightarrow \quad a \in \left(\frac{1}{2}, \frac{3}{4}\right]$

Case – II If a > 1 then for f(x) to be increasing

$4ax - x^2$ increasing in $\left(\frac{3}{2}, 2\right)$

$\therefore \quad 2a \geq 2 \quad \Rightarrow \quad a \geq 1$ but $a \neq 1$; $\quad \therefore \quad a > 1$

$\therefore$ final answer is $\left(\frac{1}{2}, \frac{3}{4}\right] \cup (1, \infty)$. Ans.

60. (C)

(A) Let $z = x + iy$, $i = \sqrt{-1}$

$\therefore (x+iy)^2 = 1 - \frac{3i}{4} \Rightarrow x^2 - y^2 = 1$ and $2xy = \frac{-3}{4}$

Now, $(x^2+y^2)^2 = (x^2-y^2) + 4x^2y^2 = 1 + \frac{9}{16} = \frac{25}{16}$

$$x^2 + y^2 = \frac{5}{4}$$

Also, $x^2 - y^2 = 1$

$$2x^2 = \frac{9}{4} \Rightarrow x = \frac{3}{2\sqrt{2}} \; or \; \frac{-3}{2\sqrt{2}}$$

If $x = \frac{3}{2\sqrt{2}}$ then $y = \frac{-\sqrt{2}}{4}$

If $x = \frac{-3}{2\sqrt{2}}$ then $y = \frac{\sqrt{2}}{4}$

$\therefore \quad (\operatorname{Re} z)(\operatorname{Im} z) = \frac{-3}{2\sqrt{2}}\left(\frac{\sqrt{2}}{4}\right) = \frac{-3}{8}$

(B) $n(S) = 6^3 = 216$

Let a, b and c are the throws on 1st, 2nd and 3rd roll, hence we must have $a \leq b \leq c$

Let $b - a = x_1$ and

$C - b = x_2$

Add,

$c - a = x_1 + x_2$

$c = x_1 + x_2 + a \leq 6$ (As, $a \leq 6$)

but a ≥ 1, giving a = 1 we have

$x_1 + x_2 + a \le 5.$

To solve this inequality we introduce a false beggar say T

$x_1 + x_2 + a + T = 5$

$\therefore$ O O O O O∅∅∅

n(A) = $^8C_3 = 56$

$\therefore p = \frac{56}{216} = \frac{7}{27} \Rightarrow p = 7.$

Aliternatively:

$a \le b \le c$

we have 4 cases

(i) a < b < c $\rightarrow {}^6C_3 \cdot 1 = 20$.

(ii) a = b = c $\rightarrow {}^6C_1 = 6$

(iii) a = b < c $\rightarrow {}^6C_2 = 15$

(iv) a < b = c $\rightarrow {}^6C_2 = 15$

n(A) = 56.

(C) Domian x > 0

for x > 1, $x^2 > 1 \Rightarrow x^2 + x > x + 1$

$\therefore \quad 2^{x^2+2} > 2^{x+1}$ and $\log_2 x > 0$

$\therefore \quad 2^{x^2+x} + \log_2 x > 2^{x+1}$ for all x > 1

Again for 0 < x < 1

for x < 1

$x^2 + x < x + 1$

$2^{x^2+x} < 2^{x+1}$ and $\log_2 x$ is –ve

$\therefore \quad 2^{x^2+x} + \log_2 x < 2^{x+1}$

for all 0 < x < 1

$\therefore \quad x = 1$ satisfies

(D)

$\cos^3 x = 1 - \left(\sin^2 x\right)^3 = \left(1 - \sin^2 x\right)\left(1 + \sin^4 x + \sin^2 x\right)$

$\cos^2 x = 0 \Rightarrow x = \frac{\pi}{2}\ or\ x = \frac{3\pi}{2}$

$\cos x = 1 + \sin^4 x + \sin^2 x$

$\left(1 - \cos x\right) + \sin^2 x\left(1 + \sin^4 x\right) = 0$

$$\left(1-\cos x\right)+\left(1-\cos x\right)\left(1+\cos x\right)\left(1+\sin^4 x\right)=0$$

$$\underbrace{1+\left(1+\cos x\right)\left(1+\sin^4 x\right)}_{\text{always positive}}=0$$

Hence, x = 0 or $\frac{\pi}{2}\, or\, \frac{3\pi}{2} \Rightarrow 3$