PART (A) : PHYSICS

ANSWER KEY

1.	(A)	2.	(B)	3.	(A)	4.	(B)	5.	(D)
6.	(D)	7.	(C)	8.	(B)	9.	(B)	10.	(B)
11.	(C)	12.	(C)	13.	(A)	14.	(D)	15.	(C)
16.	(C)	17.	(B)	18.	(A)	19.	(B)	20.	(B)
21.	(0)	22.	(6)	23.	(9)	24.	(2)	25.	(2)
26.	(11)	27.	(2)	28.	(4)	29.	(5)	30.	(35)

SOLUTIONS

1. (A)

$\tau = 0 \Rightarrow \alpha = 0 \Rightarrow \vec{\omega} = \text{constant} \,\&\, \vec{L} = I\vec{\omega}$ = constant

2. (B)

$\vec{v} = \text{constant} \therefore \vec{a} = 0 \therefore \Sigma\vec{F} = 10 - f = 0 \Rightarrow f = 10N$

3. (A)

The man must swim along the direction of shortest distance.

4. (B)

Forward biasing p-side to n-side

5. (D)

For conservative force, $F_x = \frac{-dU}{dx}; F_y = \frac{-dU}{dy}$

$$\frac{\delta F_x}{\delta y} = \frac{\delta F_y}{\delta x} = \frac{\delta^2 U}{\delta x \delta y}$$

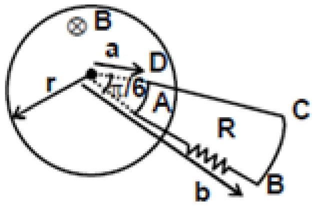
6. (D)

If $\mu_1 < \mu_2$, then also $N > 0$

7. (C)

$$V_A = V\cot 30^\circ = \frac{\omega R}{2}$$

8. (B)



$$e = \frac{d\phi}{dt} = \frac{(r^2 - a^2)}{2}\frac{\pi}{6}\frac{dB}{dt}$$

$$\therefore i = \frac{(r^2 - a^2)\pi B_0}{12R}$$

9. (B)

$$a = \frac{F_{net}}{m} = \frac{-2\rho Agx}{m} = -\omega^2 x$$

$$T = \frac{2\pi}{\omega}$$

10. (B)

$$mv^2/r = GMm/r^2$$

11. (C)

The diagram must be symmetrical about the mirror.

12. (C)

Equipotential surfaces are coaxial cylinders, therefore (4, 3, 4) and (3, 4, 0) lie on equipotential surface.

13. (A)

$$V = V_1 + V_2 \text{ and } V\gamma = V_1\gamma_1 + V_2\gamma_2$$

14. (D)

$$\frac{M\alpha}{R}\left(1 - e^{Rt/L}\right)$$

15. (C)

At first, the level of water will gradually rise to height h_0. After reaching height h_0, some of the water will get drained through the siphon. As soon as the entire cross section of the top of the siphon pipe is filled with water, the water level will begin to drop since, from given conditions, the flow–rate of the water flowing from pipe B is greater than from pipe A. As the water level drops, the pressure that drives the siphon decreases and as a result the velocity of water in pipe B decreases. Level will continue to sink until the flow rate of water through pipe B becomes equal to that through pipe A.

16. (C)

$T^2 \propto (\text{semi–major axis})^3$

17. (B)

Potential at P due to rings

$$V_P = \frac{\lambda 2\pi R}{4\pi \in_0 R} - \frac{\lambda 2\pi R}{4\pi \in_0 \sqrt{2}R} = \frac{\lambda}{2\in_0}\left(1 - \frac{1}{\sqrt{2}}\right)$$

Potential at Q due to rings is

$$V_Q = -\frac{\lambda}{2\in_0}\left(1 - \frac{1}{\sqrt{2}}\right)$$

$$\Delta(K.E.) + (V_Q - V_P)q = 0$$

$$\Rightarrow \frac{1}{2}mv^2 = \frac{\lambda}{\in_0}\left(1 - \frac{1}{\sqrt{2}}\right)q$$

18. (A)

Use $\frac{d\hat{e}_r}{dt} = \frac{d\theta}{dt}\hat{e}_e$ and $\frac{d\hat{e}_e}{dt} = -\frac{d\theta}{dt}\hat{e}_r$

19. (B)

$$T = \frac{mv^2}{R}, \text{ and } T = mg$$

20. (B)

Net field at the centre of cube = 0

$\therefore \quad E_i + E_q = 0$

$\Rightarrow \quad E_i = -\frac{q}{4\pi \in_0 d^2}$

$\therefore \quad |E_i| = \frac{q}{4\pi \in_0 d^2}$

21. (0)

No current will flow through the capacitor, so charge will be zero.

22. (6)

$A_1 v_1 = A_2 v_2$

23. (9)

For pure translatory motion of object, the force should act at the centre of mass of the object

$$y_{cm} = \frac{m \times 2\ell + 2m \times \ell}{3m} = \frac{4\ell}{3}$$

24. (2)

Energy absorbed per unit time by earth's surface, if its radius is r, is

$$Q_{abs} = \frac{\sigma 4\pi R_s^2 T_s^4}{4\pi R^2} \times \pi r^2$$

Energy radiated per unit time by earth's surface, $Q_{rad} = \sigma\left(4\pi r^2\right)T^4$

In equilibrium,

$$Q_{abs} = Q_{rad}$$

$$\Rightarrow \quad T^4 = \frac{T_s^4}{4}\frac{R_s^2}{R^2}$$

25. (2)

CM will be at rest, $m(50 - x) = 99\,m(x)$

$$\Rightarrow x = \frac{1}{2}m$$

26. (11)

$$PV = m\Sigma v_x^2$$

$$\Rightarrow m = \frac{100 \times 1}{3 \times 10^{28}} = \frac{10^{-26}}{3}\text{kg} = \frac{10^{-23}}{3}\text{g}$$

27. (2)

No e.m.f. will be induced in AC because electric lines of force are perpendicular to AC and e.m.f. induced in $AB = \text{e.m.f. induced in BC}$.

$$\therefore \text{e.m.f. induced in } AB = \frac{1}{2}\left|\frac{d\phi}{dt}\right| = \frac{1}{2}r^2\frac{dB}{dt}$$

28. (4)

Initially $\sigma 4\pi r_A^2 T_A^4 = \sigma 4\pi r_B^2 T_B^4$

$$\text{Now } \frac{Q_A}{Q_B} = \frac{\sigma(4\pi)(r_A)^2(2T_A)^4}{\sigma 4\pi(2r_B)^2(T_B)^4} = \frac{4r_A^2 T_A^4}{r_B^2 T_B^4} = 4$$

29. (5)

$$1 \times 2\% + 2 \times 1\% + \frac{1}{3} \times 3\%$$

30. (35)

$$V_R + V_C = \xi$$

$$0.5 \times 10 + V_C = 12$$

$$q = CV_C = 5 \times 7 = 35\mu C$$

PART (B) : CHEMISTRY

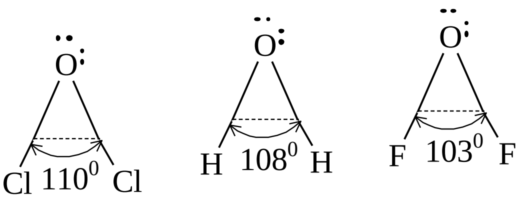
ANSWER KEY

31.	(B)	32.	(A)	33.	(C)	34.	(B)	35.	(D)
36.	(B)	37.	(B)	38.	(A)	39.	(B)	40.	(C)
41.	(C)	42.	(A)	43.	(B)	44.	(A)	45.	(C)
46.	(B)	47.	(B)	48.	(B)	49.	(D)	50.	(C)
51.	(8)	52.	(11)	53.	(270)	54.	(1)	55.	(9)
56.	(6)	57.	(5)	58.	(5)	59.	(4)	60.	(3)

SOLUTIONS

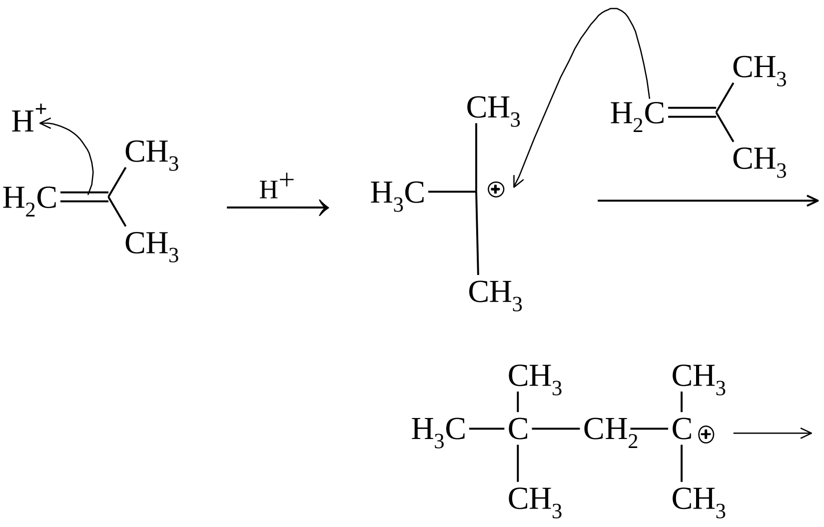
31. (B)

Because of larger size of Cl-atoms, $\angle ClOCl$ in Cl_2O is greater than $\angle HOH$ in H_2O. Whereas, because of higher electron density of F-atoms, it has strong electrostatic repulsion between F and O atoms in F_2O. The order of their bond angles is $Cl_2O > H_2O > F_2O$



32. (A)

Cationic addition polymerisation occurs as



and so on

33. (C)

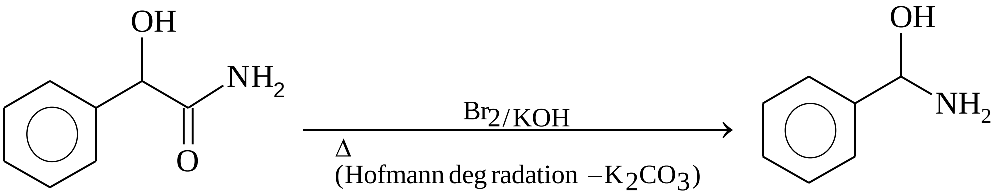
$$Zn + Cu^{2+} \rightarrow Zn^{2+} + Cu, \quad Q = \frac{[Zn^{2+}]}{[Cu^{2+}]}$$

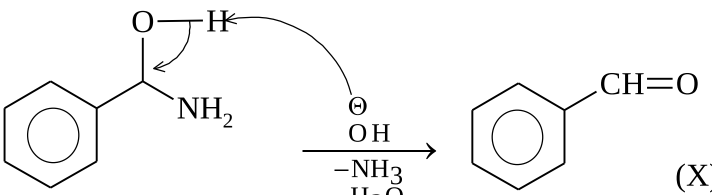
$$\Rightarrow E = E^0 - \frac{0.0591}{2}\log Q \qquad [\because n = 2]$$

$$\Rightarrow Q = 10^{2\times E^0/0.0591} = 10^{2\times 1.10/0.059}$$

$$= 10^{37.2} = \frac{[Zn^{2+}]}{[Cu^{2+}]}$$

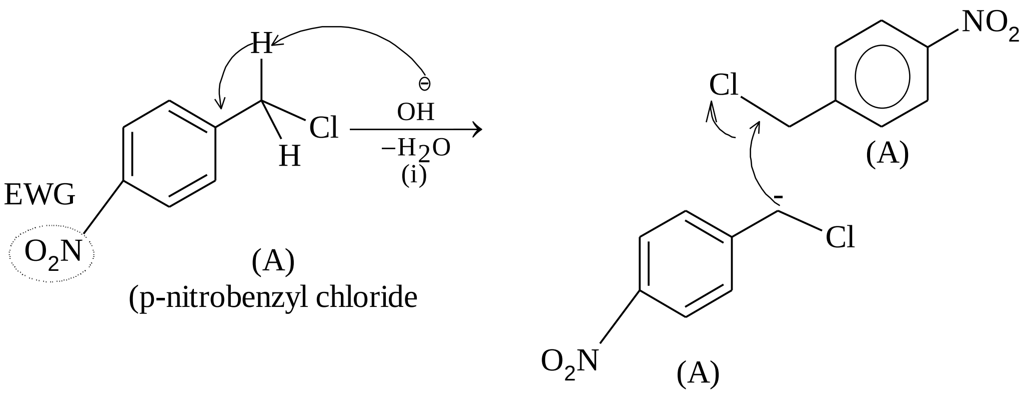
34. (B)





(X)
(Benzaldehyde)

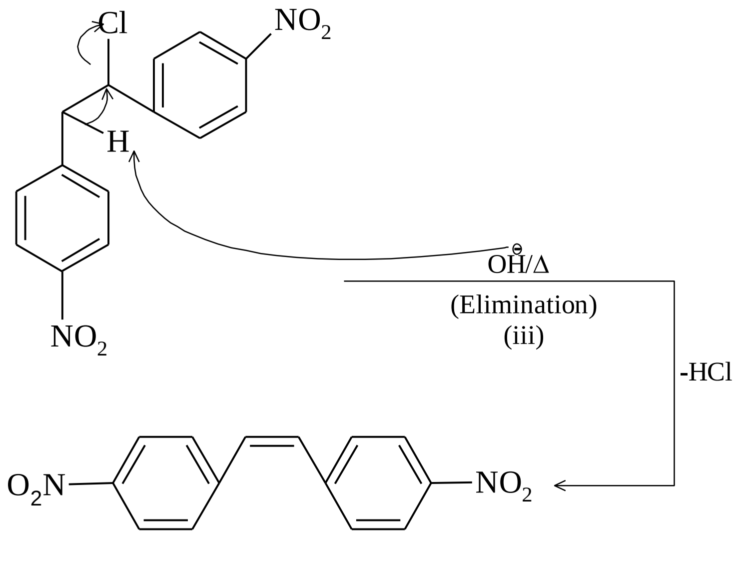
35. (D)
Let us study the mechanism of the reaction in which the steps involved are
(i) abstraction of acidic benzylic H-atom
(ii) S_N2 reaction and
(iii) elimination



(A)
(p-nitrobenzyl chloride

(A)

(A)
(Intermediate Carbanion)



(4-4'-dinitro-stibene) (P)

36. (B)

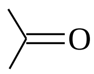
$$q_v = 18.94 \times 0.998 \times 10^3 \times 0.632$$

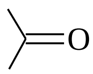
Moles of benzoic acid $= \dfrac{1.89}{122}$

$$\Delta U = \text{Heat of combustion at constant volume} = \frac{18.904 \times 10^3 \times 0.632 \times 122}{1.89}$$

$$771.2 \times 10^3 \text{cal} = 771.2 \text{ kcal}$$

37. (B)

In both (A) and (B), lone pair of N undergo resonance (+R) with the benzene ring, but in (A) lone pair of N is also delocalised with  group ® which decreases the rate of aromatic electrophilic substitution, ArS_E2 (chlorination).

In (C) and (D) –R effect of  group is operative with ring (more in D) which decrease rate of ArS_E2 reaction. So, correct order is B > A > C> D.

38. (A)

A buffer solution containing mixture of CH_3COOH and CH_3COONa is known as acidic buffer. When an equal volume of 1M HCl is added, the pH of the solution almost remains unchanged and acidic buffer is formed.

The acetate ion reacts with H^+ ion to form acetic acid thus, the solution contains CH3COOH and CH_3COONa in equal amount.

39. (B)

Pressure $(p) \propto$ number of moles (n) at constant volume and temperature

$$2NO(g) + Cl_2(g) \rightleftharpoons 2NOCl(g)$$

$t = 0$	$2p$	p	0
$t = t_{eq}$	$2p - 2x$	$p - x$	$2x$

Total pressure $p_T = 2p - 2x + p - x + 2x$

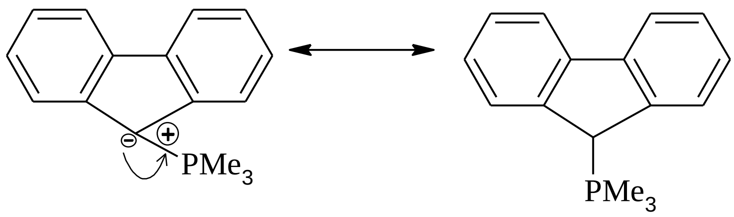
$$3p - x = 1 \text{ and } 2x = \frac{p-x}{4}\left(\because \text{Mole of NaCl} = \frac{1}{4} \times \text{mole of } Cl_2\right)$$

Solving $x = \frac{1}{26}$ and $p = \frac{9}{26}$

$$K_p = \frac{(2x^2)}{(2p-2x)^2(p-x)} = \frac{4 \times x2}{(16x)^2 \times 8x}$$

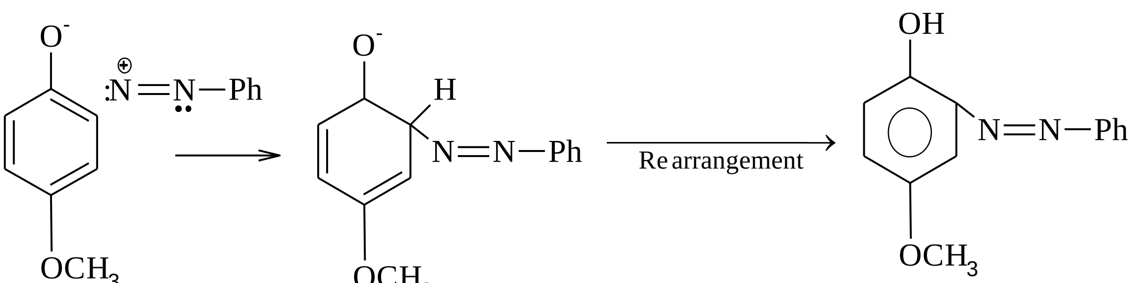
$$= \frac{4x^2}{2048 \times x^3} = \frac{1}{512x} = \frac{1}{512 \times \frac{1}{26}} = \frac{13}{256}$$

40. (C)



41. (C)

–OH converts phenol to phenoxide ion which is strongly activating than $–OCH_3$ group, so attack occurs (coupling) at ortho-position w.r.t. hydroxyl group as the para-position is blocked by $–OCH_3$ group.

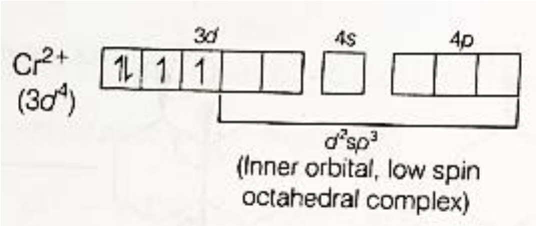


(Option C)

This reaction is called coupling reaction (ArS_E2)

42. (A)

Here NO is present as NO^+. So, oxidation state of Cr in the complex is +2.



$\therefore n = '2$

$\Rightarrow \mu = \sqrt{2(2+2)} = \sqrt{8}BM$

43. (B)

$$CO(g) + NO_2(g) \rightleftharpoons CO_2(g) + NO(g)$$

$t = 0$	1	1	1	1
$t = t_{eq}$	$1 - x$	$1 - x$	$1 + x$	$1 + x$

$\Rightarrow \underset{1mol}{CO_2} + Ba(OH) \rightarrow \underset{1mol}{BaCO_3} \downarrow + H_2O$

(At equilibrium)

$\Rightarrow 1 + x = 12 \Rightarrow x = 0.2$

$$\Rightarrow K_C = \frac{(1+x)^2}{(1-x)^2} = \frac{1.2^2}{0.8^2} = 2.25$$

44. (A)

Number of unpaired electrons in the compound $[MnCl_4]^{2-}$, $[CoCl_4]^{2-}$ and $[Fe(CN)_6]^{4-}$ are 5,3 and 0 respectively. As number of unpaired electrons increases, the magnetic moment values also increases.

45. (C)

Complete reactions are as follows

i) $Na_2SO_3 + S \xrightarrow{\Delta} Na_2S_2O_3$

ii) $\underset{(A)}{Na_2S_2O_3} + 2AgNO_3 \rightarrow 2NaNO_3 + \underset{(B)}{Ag_2S_2O_3}$

iii) $\underset{(B)}{Ag_2S_2O_3} + H_2O \rightarrow \underset{(C)}{Ag_2S} + H_2SO_4$

46. (B) Statement (b) is incorrect. Whereas all other statements are correct.

Nitrite ion $\left(NO_2^-\right)$ interferes the ring test of nitrate ion $\left(\left(NO_3^-\right)\right.$. Here NO_2^- inhibits the oxidation of NO to NO^+ which acts as a positively charged ligand in the brown ring complex.

$\overset{I}{[F}e(H_2O)5(NO+)]SO_4$

47. (B)

The statement II, III and IV are correct, whereas statement I is incorrect

i. The given structures represent the disaccharide, maltose

ii. Starch content in cereals and roots is very high

iii. $\alpha-D$ glucose and $\beta-D$ glucose are anomers as they vary in configuration at C – 1

iv Allose, altrose and talose are reducing sugars. These are monosaccharides

48. (B)

$$CuSO_4 + H_2S \rightarrow CuS + H_2SO_4$$

$$CuS + \underset{(excess)}{KCN} \rightarrow \underset{(N)}{[Cu(CN)_4]^{3-}} + \underset{(O)}{(CN)_2}$$

49. (D)

Molarity (M) = $\dfrac{\text{Number of moles of solute}}{\text{Volume of solution (in L)}}$

Number of moles of complex $= \dfrac{\text{Molarity} \times \text{Volume (in mL)}}{1000}$

$= \dfrac{0.1 \times 100}{1000} = 0.01 \text{mol}$

Number of moles of ions precipitate $\dfrac{1.2 \times 10^{22}}{6.02 \times 10^{23}} = 0.02 \text{mol}$

$\therefore$ Number of Cl^- present in ionisation sphere $= \dfrac{\text{Number of moles of ions precippated}}{\text{Number of moles of complex}}$

$= \dfrac{0.02}{0.01} = 2$

$\therefore 2Cl^-$ are present outside the square brackets, i.e. in ionisation sphere. Thus, the formula of complex is $[Co(H_2O)_5Cl]Cl_2.H_2O$

50. (C)

The van der Waals' equation of state is $\left(p + \dfrac{n^2 a}{V^2}\right)(V - nb) = nRT$

For one mole and when b = 0 the above equation condensers to

$$\left(p + \frac{a}{V^2}\right)V = RT$$

$$PV = RT - \frac{a}{V}$$

Eq. (i) is a straight line equation between pV and $\dfrac{1}{V}$ whose slope is –a. Equation with slope of the straight line given in the graph.

$$-a = \frac{20.1 - 21.6}{3 - 2} = -1.5$$

$$a = 1.5 \text{atmL}^{-2}\text{mol}^{-1}$$

51. (8)

Precipitation as hydroxide with NH_4OH in presence of NH_4Cl	Precipitation as sulphide with H_2S in presence of HCl	Precipitation as sulphide with H_2S in presence of NH_4OH
Fe^{3+}	Bi^{3+}	Mn^{2+}
Cr^{3+}	Sn^{4+}	Ni^{2+}
Al^{3+}	Y = 2	Co^{2+}
X=3		Z= 3

$\therefore [X+Y+Z]=(3+2+3)=8$

52. (11)

Solubility of Ag_2CrO_4 in 0.1M $(AgNO_3)$=xM

$$\underset{}{Ag_2CrO_4} \rightleftharpoons \underset{2x}{2Ag^+} + \underset{x}{CrO_4^{2-}}$$

$$AgNO_3 \rightleftharpoons \underset{0.1}{Ag^+} + \underset{0.1}{NO_3^-}$$

Total $[Ag^+]$ = (2x + 0.1)=0.1M

$[CrO_4^{2-}]=xM$

$[Ag^+]^2[CrO_4^{2-}]=K_{sp}(Ag_2CrO_4)$

$(0.1)^2(x)=1.1\times10^{-12}$

$x=1.1\times10^{-10}M$

$=11\times10^{-11}M$

$y\times10^{-11}M$

y = 11

53. (270)

$NaCl \rightarrow Na^+ + Cl^-$

$i = 1+\alpha(n-1)=1+\frac{60}{100}(2-1)=1.6$

Molality = 1m

$\Delta T_f = K_f \times m \times i$

$\Rightarrow 0-T_f = 1.86\times1\times1.6=2.976$

$\therefore$ Freezing point, $T_f = -2.976^0C$

=270.02K

54. (1)

According to the Freundlich equation,

$$\frac{x}{m}=kp^{1/n}$$

or $\log\frac{x}{m} = \log k + \frac{1}{n}\log p$

$\therefore$ Plot of $\log\frac{x}{m}$ *versus* log *p* is linear with slope of 45°.

Thus, slope $= \frac{1}{n} = \tan\theta = \tan 45^{\circ} = 1$ or $n = 1$ [$\because$ x = amount of gas in gram, *m* = amount of charcoal in gram]

Intercept = log k = 0.3010 or k = 2

At *p* = 0.5 atm

$\frac{x}{m} = kp^{1/n} = 2\times(0.5)^{1} = 1.00$

55. (9)

$C(s) + CO_2(g) \rightleftharpoons 2CO(g)$

$K'_{P_1} = 10^{12}\ atm^{-1}$, where $K'_{p1} = \frac{1}{K_{P_1}} = \frac{1}{10^{-12}}$

$2CO(g) + 2Cl_2(g) \rightleftharpoons 2COCl_2(g)$

$K'_{P2} = (3\times10^{-3})^{2} atm^{-1}$ $[\because K'_{P2} = (K_{P_2})^{2}]$

Adding equns (i) and (ii), we get

$\Rightarrow C(s) + CO_2(g) + 2Cl_2 \rightleftharpoons 2COCl_2(g)$

$\Rightarrow Kp = K'_{P_1} \times K'_{P_2}$

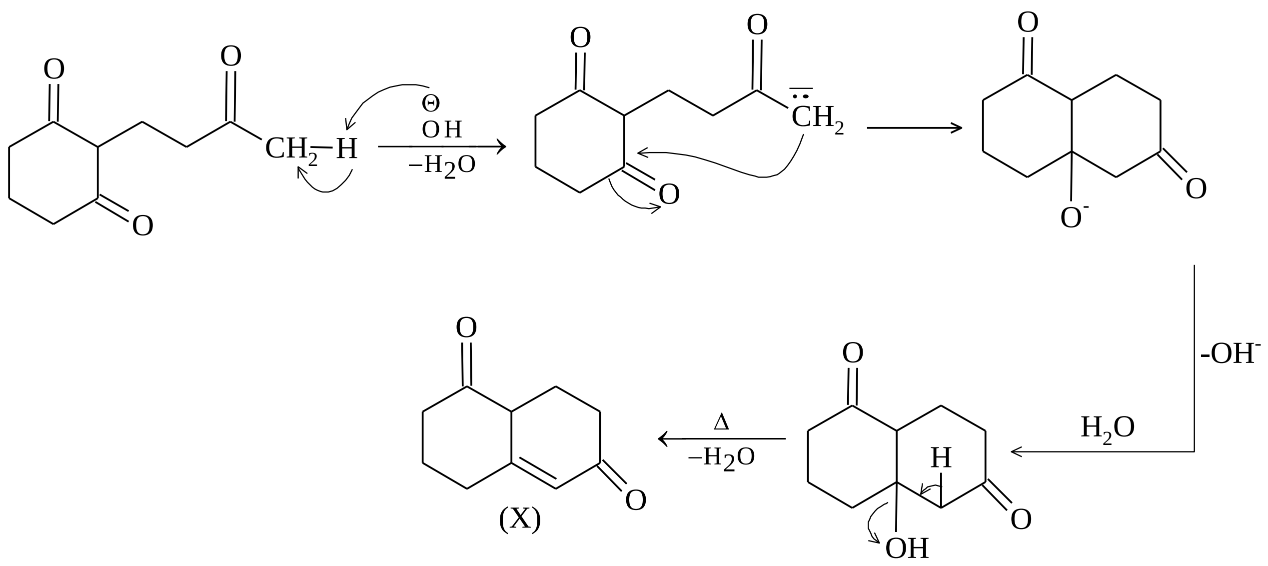
$\Rightarrow K_p = (10^{12})\times(9\times10^{-6}) = 9\times10^{6} atm^{-1}$

56. (6)

SO_3, XeO_3, H_3PO_4, ClO_4^- SO_4^{2-}, $XeOF_2$ $d\pi - p\pi$ bonding. This type of bonding is important in compounds containing third (or higher) period elements (Si, P, S, Cl, etc) These elements have vacant d-orbitals.

These vacant d-orbital form (d-p) π bonding with N, O, F which have lone pair electrons is their p-orbitals.

57. (5)



In (X), degree of unsaturation = (No of π bonds + No. of rings)
= 3 + 2 = 5

58. (5)

Let number of millimoles of $Fe_2(SO_4)_3$ = a, FeC_2O_4 =b As $Fe_2(SO_4)_3$ is not oxidisable

$$\Rightarrow b\times 3 = 40\times\frac{1}{16}\left[\because \underset{n-factor=1+2=3}{\overset{+2\quad +3}{FeC_2O_4}} \xrightarrow{[O]} Fe^{3+} + \overset{-4}{CO_2}\right]$$

$$\therefore b = \frac{40}{48} \Rightarrow \text{mass of } FeC_2O_4 = \frac{40}{48}\times 144 = 120\,mg = 0.12\,g$$

59. (4)

Statements (a), (b), (d) and (e) are correct, whereas statement (c) is incorrect

60. (3)

In $[Cu(NH_3)_4]^{2+}$, number of unpaired electron in $3d^9$ of Cu(II) is one (n =1)

$\mu_m = \sqrt{n(n+2)} = \sqrt{1\times 3}$

= 1.73BM = xBM

$\therefore x = 1.73$

$x^2 = 3$

PART (C) : MATHEMATICS

ANSWER KEY

61.	(C)	62.	(B)	63.	(A)	64.	(B)	65.	(D)
66.	(C)	67.	(C)	68.	(C)	69.	(D)	70.	(D)
71.	(B)	72.	(C)	73.	(C)	74.	(C)	75.	(D)
76.	(B)	77.	(B)	78.	(C)	79.	(A)	80.	(A)
81.	(0)	82.	(210)	83.	(4)	84.	(8)	85.	(31)
86.	(10)	87.	(2)	88.	(8)	89.	(0)	90.	(12)

SOLUTIONS

61. (C)

Given, $f(x) = x^3 - \frac{3x^2}{2} + x + \frac{1}{4}$

$= \frac{1}{4}(4x^3 - 6x^2 + 4x + 1)$

$= \frac{1}{4}(4x^3 - 6x^2 + 4x - 1 + 2)$

$= \frac{1}{4}[x^4 - (1-x^4)] + \frac{2}{4}$

$\therefore \quad f(1-x) = \frac{1}{4}[(1-x)^4 - x^4] + \frac{2}{4}$

$f(x) + f(1-x) = \frac{2}{4} + \frac{2}{4} = 1 \qquad \ldots(i)$

Replacing x by f(x), we have

$f[f(x)] + f[1 - f(x)] = 1 \qquad \ldots(ii)$

Now, $\int_{1/4}^{3/4} f(f(x))dx \qquad \ldots(iii)$

$\Rightarrow \quad I = \int_{1/4}^{3/4} f(f(1-x))dx$

$\because \int_a^b f(x)dx = \int_a^b f(a+b-x)dx]$

$= \int_{1/4}^{3/4} f(1 - f(x))dx \qquad \ldots(iv)$

[from Eq. (i)]

Adding Eqs. (iii) and (iv), we get

$2I = \int_{1/4}^{3/4} [f(f(x)) + f(1 - f(x))]dx = \int_{1/4}^{3/4} 1dx$

[from Eq. (ii)]

$= [x]_{1/4}^{3/4} = \frac{1}{2}$

$\Rightarrow \qquad I = \frac{1}{4}$

Hence, $(I)^{-3/2} = \left(\frac{1}{4}\right)^{-3/2} = 4^{3/2}$

$= (4^{1/2})^3 = 2^3 = 8$

62. (B)

Let A = event that a sum of 5 occurs

B = event that a sum of 7 occurs

C = event that neither a sum of 5 nor a sum of 7 occurs.

Then, $P(A) = \frac{4}{36} = \frac{1}{9}$

$P(B) = \frac{6}{36} = \frac{1}{6}$

$P(C) = \frac{26}{36} = \frac{13}{18}$

Thus, probability that A occurs before B is

$P[A \text{ or } (C \cap A) \text{ or } (C \cap C \cap A) \text{ or } ....]$

$P(A) + P(C \cap A) + P(C \cap C \cap A) + ...$

$P(A) + P(C) \cdot P(A) + [P(C)]^2 \cdot P(A) + ...$

$= \frac{1}{9} + \left(\frac{13}{18}\right) \times \left(\frac{1}{9}\right) + \left(\frac{13}{18}\right) \times \frac{1}{9} + ...$

$= \frac{1/9}{1 - 13/18} = \frac{2}{5}$

63. (A)

The distance between (2, 2m) and (5, 5m) is less than 5.

$\Rightarrow \quad (5 - 2)^2 + (5m - 2m)^2 < 25$

$\Rightarrow \quad 9m^2 < 16$

$\Rightarrow \quad m^2 < \frac{16}{9} \Rightarrow |m| < \frac{4}{3}$

$\Rightarrow \quad -\frac{4}{3} < m < \frac{4}{3}$

64. (B)

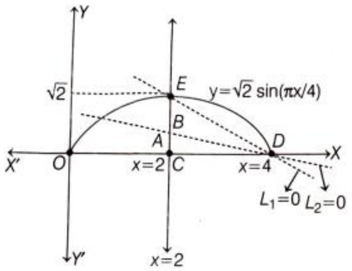
Area bounded by $y = \sqrt{2} \sin\left(\frac{\pi x}{4}\right)$ and X-axis between the lines x = 2 and x = 4 is

$\Delta = \sqrt{2} \int_2^4 \sin\left(\frac{\pi x}{4}\right) dx$

$= \frac{-4\sqrt{2}}{\pi} \cdot \cos\left(\frac{\pi x}{4}\right)\Big|_2^4$

$\Rightarrow \quad \Delta = \frac{4\sqrt{2}}{\pi} \text{ sq units} \qquad \ldots(i)$

Consider two lines L_1 and L_2 with slopes m_1 and m_2 respectively.



Then, equation of L1 passing through (4, 0) is

$$y - 0 = m_1 (x - 4)$$

$\Rightarrow$ $y - m_1(x - 4) = 0$ ...(ii)

L2 : $y - m_2 (x - 4) = 0$ ...(iii)

Similarly, let A and B be the points of intersection of L_1 and L_2 with x = 2, respectively.
Now, for finding coordinates of A and B, we put x = 2 in Eqs.
(ii) and (iii) and thus $A \equiv (2, -2m)$ and $B \equiv (2, -2m_2)$.

Since, ar $(\Delta ACD) = ar(\Delta ABD)$

$$= ar(\Delta EDBE) = \frac{\Delta}{3}$$

(given)

Now, $ar(\Delta ACD) = \frac{\Delta}{3}$

$$\frac{4\sqrt{2}}{3\pi} = \frac{1}{2} \times 2 \times (-2m_1)$$

$$\Rightarrow \quad m_1 = \frac{-2\sqrt{2}}{3\pi}$$

Also, $ar(\Delta BCD) = ar(\Delta ACD) + ar(\Delta ABD)$

$$= \frac{\Delta}{3} + \frac{\Delta}{3} = \frac{2\Delta}{3}$$

$$\Rightarrow \quad \frac{8\sqrt{2}}{3\pi} = \frac{1}{2} \times 2 \times (-2m_2)$$

$$\Rightarrow \quad m_2 = \frac{-4\sqrt{2}}{3\pi}$$

$\therefore$ Required sum = $\frac{-2\sqrt{2}}{\pi}$

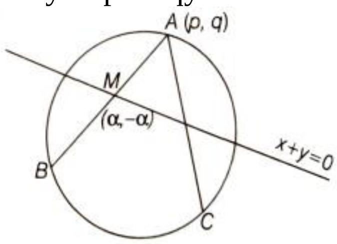
65. (D)

Let $p = \frac{1+\sqrt{2}a}{2}, q = \frac{1-\sqrt{2}a}{2}$

Now, equation of circle can be written as

$$2x^2 + 2y^2 - 2px - 2qy = 0$$

$\Rightarrow$ $x^2 + y^2 - px - qy = 0$ ...(i)



As, M lies on x + y = 0, then we may take coordinates of M as $(\alpha, -\alpha)$.

$\therefore$ M is the mid-point of AB.

$\therefore$ Coordinates of B are

$(2\alpha - p, -2\alpha - q)$

As, B lies on the circle (1).

So, $(2\alpha - p)^2 + (2\alpha - q)^2 - p(2\alpha - p) - q(-2\alpha - q) = 0$

$\Rightarrow 8\alpha^2 + 6\alpha(q - p) + 2(p^2 + q^2) = 0$

$\Rightarrow 4\alpha^2 + 3\alpha(q - p) + (p^2 + q^2) = 0$

As, there are two distinct chords.

Hence, value of α must be distinct i.e. discriminant > 0.

$\Rightarrow 9(q - p)^2 - 16(p^2 + q^2) > 0$

$\Rightarrow 9(-\sqrt{2}a)^2 - 8(1 + 2a^2) > 0$

$\Rightarrow 2a^2 - 8 > 0$

$\Rightarrow a^2 > 4$

$\Rightarrow |a| > 2$

$\Rightarrow a \in (-\infty, -2) \cup (2, \infty)$

66. (C)

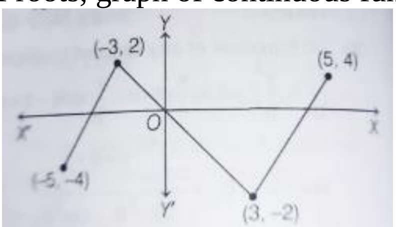
Given, f(x) + f(–x) = 0

$\Rightarrow$ f(x) is an odd function. since, points (–3, 2) and (5, 4) lies on the curve.

So, (3, –2) and (– 5, –4) will also lie on the curve because

f(x) is symmetrical about origin as f(x) is an odd function.

For minimum number of roots, graph of continuous function f(x) is as follows



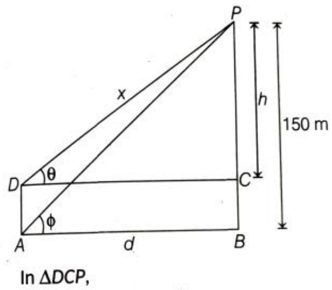
From the above graph of f(x). it is clear that equation f(x) = 0 has atleast three real roots.

67. (C)

In ΔABP,

$$\cot\phi = \frac{d}{150}$$

$$d = 150\cot\phi = 60m \left[\because \tan\phi = \frac{5}{2}\right]$$



In ΔDCP,

$$\tan\theta = \frac{h}{d}$$

$\Rightarrow$ h = 60 $\tan\theta = 80m$

$$\left[\because \tan\theta = \frac{4}{3}, given\right]$$

In ΔPDC,

$$x = \sqrt{(80)^2 + (60)^2} = 100m$$

68. (C)

$$\int \frac{1}{1+x^4}dx = \frac{1}{2}\int \frac{1+x^2+1-x^2}{1+x^4}dx$$

$$= \frac{1}{2}\int \frac{1+\frac{1}{x^2}}{x^2+\frac{1}{x^2}}dx - \frac{1}{2}\int \frac{1-\frac{1}{x^2}}{x^2+\frac{1}{x^2}}dx$$

$$= \frac{1}{2}\int \frac{d\left(x-\frac{1}{x}\right)}{\left(x-\frac{1}{x}\right)^2+2} - \frac{1}{2}\int \frac{d\left(x+\frac{1}{x}\right)}{\left(x+\frac{1}{x}\right)^2-2}$$

$$= \frac{1}{2}\cdot\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{x-\frac{1}{x}}{\sqrt{2}}\right) - \frac{1}{2}\cdot\frac{1}{2\sqrt{2}}\ln\left|\frac{x+\frac{1}{x}-\sqrt{2}}{x+\frac{1}{x}+\sqrt{2}}\right| + C$$

$$= \frac{1}{2\sqrt{2}}\tan^{-1}\left(\frac{x^2-1}{x\sqrt{2}}\right) - \frac{1}{4\sqrt{2}}\ln\left|\frac{x^2-\sqrt{2}x+1}{x^2+\sqrt{2}x+1}\right| + C$$

Hence,

$$f(x) = \frac{x^2-1}{\sqrt{2}x}, g(x) = \frac{x^2-\sqrt{2}x+1}{x^2+\sqrt{2}x+1}$$

$m = 2\sqrt{2}, n = 4\sqrt{2} \Rightarrow n = 2m$

Also, $f(2) = \frac{3}{2\sqrt{2}}$ and $g(\sqrt{2}) = \frac{1}{5}$ and (m + n) is an irrational number.

69. (D)

$$LHL = \lim_{x\to -1^-} \frac{1}{\sqrt{|x|-\{x\}}}$$

$$= \lim_{x\to -1^-} \frac{1}{\sqrt{-x-(x+2)}}$$

$$= \lim_{x\to -1^-} \frac{1}{\sqrt{-2x-2}} = \infty$$

$$RHL = \lim_{x\to -1^+} \frac{1}{\sqrt{|x|-\{x\}}}$$

$$= \lim_{x\to -1^+} \frac{1}{\sqrt{-x-(x+1)}}$$

$$= \lim_{x \to -1^+} \frac{1}{\sqrt{-2x-1}} = 1$$

LHL $\neq$ RHL

$\Rightarrow$ Limit does not exist

70. (D)

Let $A \equiv (p \to q) \wedge (q \to p)$ and $B \equiv (\sim p) \vee q$

1	2	3	4	5	6	7	8	9
p	q	$\sim p$	$p \to q$	$q \to p$	$p \leftrightarrow q$	$q \leftrightarrow p$	A	B
T	T	F	T	T	T	T	T	T
T	F	F	F	T	F	F	F	F
F	T	T	T	F	F	F	F	T
F	F	T	T	T	T	T	T	T

Columns 6 and 8 are identical

$(p \leftrightarrow q) \equiv (p \to q) \wedge (q \to p)$

Columns 4 and 9 are identical

$(p \to q) \equiv (\sim p) \vee q$

Columns 6 and 7 are identical

$(p \leftrightarrow q) \equiv (q \leftrightarrow p)$

71. (B)

An equation of normal at P is

$3x\cos\theta + 2y\cot\theta = 9 + 4$.

$\Rightarrow \quad 3x + 2y\,\text{cosec}\theta = 13\sec\theta$ ...(i)

An equation of normal at Q is

$3x + 2y\,\text{cosec}\phi = 13\sec\phi$ ...(ii)

Subtracting Eq. (ii) from Eq. (i),we have

$2y[\text{cosec}\theta - \text{cosec}\phi] = 13(\sec\theta - \sec\phi]$

$\Rightarrow \quad 2y[\text{cosec}\theta - \sec\theta] = 13(\sec\theta - \cos ec\theta]$

$$[\because \phi = \frac{\pi}{2} - \theta]$$

$\Rightarrow \quad y = \frac{-13}{2}$

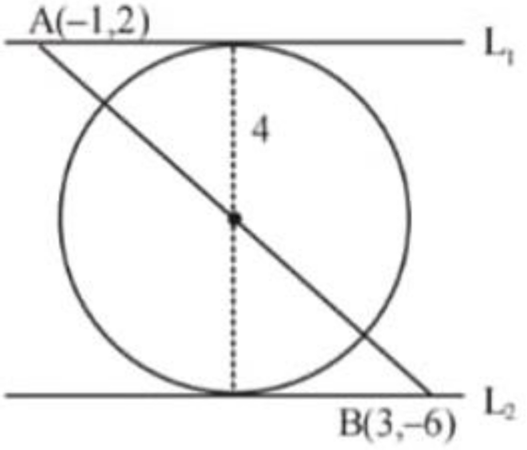
72. (C)

p	q	$\sim p$	$\sim q$	$p \to q$	$[(p \to q) \wedge \sim q]$	$[(p \to q) \wedge \sim q] \to (\sim p)$
T	T	F	T	T	F	T
T	F	F	F	F	F	T
F	T	T	T	T	F	T
F	F	T	T	T	T	T

All the entries in the last column of the above truth table is T.

So, $[(p \to q) \wedge \sim q] \to (\sim p)$ is tautology.

73. (C)



$L_1 : 4x - 3y + K_1 = 0$

$L_2 : 4x - 3y + K_2 = 0$

Now

$-4 - 6 + K_1 = 0 \Rightarrow K_1 = 10$

$12 + 18 + K_2 = 0 \Rightarrow K_2 = -30$

$\Rightarrow$ Tangent to the circle are

$4x - 3y + 10 = 0$

$4x - 3y - 30 = 0$

Length of diameter $2r = \frac{|10 + 30|}{5} = 8$

$\Rightarrow r = 4$

Now centre is mid point of A & B

$x = 1, y = -2$

Equation of circle

$(x-1)^2 + (y+2)^2 = 16$

74. (C)

Equation of the tangent at $(3\sqrt{3}\cos\theta, \sin\theta)$ is

$$\frac{x}{3\sqrt{3}}\cos\theta + y\sin\theta = 1$$

It intercepts on coordinate axes are $3\sqrt{3}\sec\theta$ and $\operatorname{cosec}\theta$

If S denotes their sum, then

$$S = 3\sqrt{3}\sec\theta + \operatorname{cosec}\theta$$

$$\Rightarrow \frac{dS}{d\theta} = 3\sqrt{3}\sec\theta\tan\theta - \operatorname{cosec}\theta\cot\theta$$

For minimum value of S, put

$$\frac{dS}{d\theta} = 0$$

$$\Rightarrow \frac{3\sqrt{3}\sin\theta}{\cos^2\theta} - \frac{\cos\theta}{\sin^2\theta} = 0$$

$$\Rightarrow \tan^3\theta = 3\sqrt{3}$$

$\Rightarrow \quad \tan\theta = \sqrt{3} = \tan\left(\frac{\pi}{3}\right)$

$\Rightarrow \quad \theta = \frac{\pi}{3}$

Also, $\frac{d^2S}{d\theta^2} = 3\sqrt{3}\sec\theta\tan^2\theta + 3\sqrt{3}\sec^3\theta$

$+\operatorname{cosec}^2\theta\cot\theta + \operatorname{cosec}^3\theta$

$\Rightarrow \quad \left.\frac{d^2S}{d\theta^2}\right|_{\theta=\pi/3} = +ve$

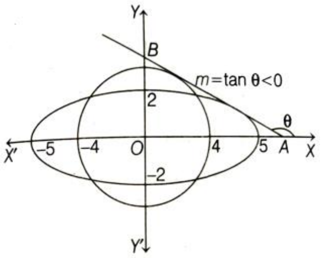
Hence, S is least when

$\theta = \frac{\pi}{3}$.

75. (D)

An equation of tangent to the ellipse

$\frac{x^2}{25} + \frac{y^2}{4} = 1$ is $y = mx + \sqrt{25m^2 + 4}$ .....(i)



As this tangent has to lie in the first quadrant.

$\therefore \quad m < 0$

The line (i) will touch the circle $x^2 + y^2 = 16$, if perpendicular distance from O(0, 0) to Eq. (i) is equal to 4.

i.e. if $\left|\frac{m \times 0 - 0 + \sqrt{25m^2 + 4}}{\sqrt{m^2 + 1}}\right| = 4$

$\Rightarrow \quad 25m^2 + 4 = 16(m^2 + 1)$

$\Rightarrow \quad 9m^2 = 12 \Rightarrow m^2 = \frac{4}{3}$

$\Rightarrow \quad m = \pm\sqrt{\frac{4}{3}}$ but $m < 0$

$\therefore \quad m = -\frac{2}{\sqrt{3}}$

From Eq. (i),

$y = \frac{-2}{\sqrt{3}}x + \sqrt{25\left(\frac{4}{3}\right) + 4}$

$\Rightarrow \quad 2x + \sqrt{3}y = 4\sqrt{7}$ …(ii)

Line (ii) meets the coordinate axes in

$A(2\sqrt{7}, 0)$ and $B\left(0, \frac{4\sqrt{7}}{\sqrt{3}}\right)$.

So, $AB^2 = 4(7) + 16\left(\frac{7}{3}\right) = (4)(7)\left(\frac{7}{3}\right)$

$\Rightarrow \quad AB = \frac{14}{\sqrt{3}}$ units.

76. (B)
Conceptual

77. (B)

Let $I = \int_5^{41} \frac{dx}{[f^{-1}(x)]^5 + 5[f^{-1}(x)]}$

Put $f^{-1}(x) = z \Rightarrow x = f(z)$

$\Rightarrow \quad dx = f'(z)\, dz$

Now, $f(x) = x^5 + 5x - 1$

$\Rightarrow \quad f(1) = 5 \Rightarrow f^{-1}(5) = 1$

And $f(2) = (2)^5 + 5 \times 2 - 1 = 41$

$\Rightarrow \quad f^{-1}(41) = 2$

$\therefore I = \int_1^2 \frac{f'(z)dz}{z^5 + 5z} = \int_1^2 \frac{5z^4 + 5}{z^5 + 5z} dz$

[$\therefore$ $f(x) = x^5 + 5x - 1$
$f(z) = z^5 + 5z - 1$

$\Rightarrow \quad f'(z) = 5z^4 + 5$]

$= \log_e (z^5 + 5z)\Big|_1^2$

$= \log_e 42 - \log_e 6 = \log_e 7$

78. (C)

Let $I = \int \frac{(\sqrt{x})^5}{(\sqrt{x})^7 + x^6} dx$

$$= \int \frac{\frac{(\sqrt{x})^5}{(\sqrt{x})^5}}{\frac{(\sqrt{x})^7}{(\sqrt{x})^5} + \frac{(\sqrt{x})^{12}}{(\sqrt{x})^5}} dx$$

$$= \int \frac{dx}{(\sqrt{x})^2 + (\sqrt{x})^7}$$

$$= \int \frac{dx}{(\sqrt{x})^7 \cdot \left[1 + \frac{1}{(\sqrt{x})^5}\right]}$$

Putting $\frac{1}{(\sqrt{x})^5} = z$

$\Rightarrow \quad dz = \frac{-5}{2(\sqrt{x})^7} dx$

$I = \int \frac{-2dz}{5(1+z)} = \frac{-2}{5} \ln|1+z| + C$

$$= \frac{2}{5}\ln\left|\frac{1}{1+z}\right| + C$$

$$= \frac{2}{5}\ln\left|\frac{1}{1+\frac{1}{(\sqrt{x})^5}}\right| + C$$

$$= \frac{2}{5}\ln\left|\frac{(\sqrt{x})^5}{(\sqrt{x})^5+1}\right| + C$$

On comparing, $a = \frac{2}{5}$ and $k = \frac{5}{2}$

79. (A)

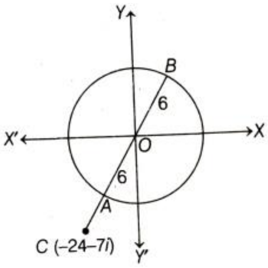
Method (1)

Since, $|z_2| = 6$ represents a circle with radius = 6 units.

Now, $|z_1 + z_2| = |z_2 - (-z_1)|$

$\Rightarrow \quad |z_2 + z_1| = |z_2 - (-24 - 7i)|$

Represents distance between point on the circle $|z_2| = 6$ and the point $C(-24 - 7i)$ $|z_1 + z_2|$ will be greatest and least at points B and A which are the end points of the diameter of the circle through C.



$\therefore \quad OC = \sqrt{(-24)^2 + (-7)^2} = 25$

$\therefore \quad CA = OC - OA = 25 - 6 = 19$ (least value)

And $CB = OC + OB = 25 + 6 = 31$ (greatest value)

80. (A)

d.r's of the line $= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & -1 \\ 1 & -2 & 3 \end{vmatrix} = \hat{i} - 4\hat{j} - 3\hat{k}$

$\therefore$ equation of line is

$\vec{r} - = \hat{i} + 2\hat{j} + 4\hat{k} + \lambda\left(\hat{i} - 4\hat{j} - 3\hat{k}\right)$

Let A(1, 2, 4) and P be $(1+\lambda, 2-4\lambda, 4-3\lambda)$

$\therefore \ \overrightarrow{PA}.\left(\hat{i} - 4\hat{j} - 3\hat{k}\right) = 0$

$\lambda = \frac{1}{2}$

$\Rightarrow \ P\left(\frac{1}{2}, 2, \frac{-5}{2}\right)$

$|AP| = \sqrt{\frac{21}{2}}$

81. (0)

Focus of $y^2 = 36x$ is (9, 0)

Let the other end point of the focal chord be B $(t_1^2, 6t_1)$, then equation of AB is

$$y - 6t = \frac{6t_1 - 6t}{t_1^2 - t^2}(x - t^2)$$

$$y - 6t = \frac{6(t_1 - t)}{(t_1 - t)(t_1 + t)}(x - t^2)$$

$\Rightarrow \quad 6(x - t^2) = (t_1 + t)(y - 6t)$

It will pass through (9, 0), if

$6(9 - t^2) = (t_1 + t)(0 - 6t)$

$$\Rightarrow \quad t_1 = \frac{-9}{t} \qquad \ldots(i)$$

Now, length of AB is given by

$$AB^2 = (t_1^2 - t^2)^2 + (6t_1 - 6t)^2$$

$[(t_1 - t)^2 (t_1 + t)^2] + [6(t_1 - t)]^2$

$(t_1 - t)^2 [t_1 + t)^2 + 36]$

$$= \left(\frac{9}{t} + t\right)^2 \left[\left(t - \frac{9}{t}\right)^2 + 36\right]$$

[using Eq. (i)]

$$\Rightarrow \quad AB = \left(t + \frac{9}{t}\right)^2 = 100 \qquad \text{(given)}$$

$$\Rightarrow \quad t + \frac{9}{t} = \pm 10$$

$\Rightarrow \quad t = 1, -1$

$\therefore$ sum of value of

$T = 1 + (-1) = 0$

82. (210)

$$\frac{x+1}{x^{2/3} - x^{1/3} + 1} - \frac{x-1}{x - x^{1/2}}$$

$$\frac{(x^{1/3})^3 + (1)^3}{x^{2/3} - x^{1/3} + 1} - \frac{x-1}{x^{1/2}(x^{1/2} - 1)}$$

$$= \frac{(x^{1/3} + 1)(x^{2/3} - x^{1/3} + 1)}{(x^{2/3} - x^{1/3} + 1)} - \frac{(x^{1/2} + 1)(x^{1/2} - 1)}{x^{1/2}(x^{1/2} - 1)}$$

$= x^{1/3} + 1 - 1 - x^{-1/2}$

$= x^{1/3} - x^{-1/2}$

$$\therefore \left(\frac{x+1}{x^{2/3} - x^{1/3} + 1} - \frac{x-1}{x - x^{1/2}}\right)^{10} = (x^{1/3} - x^{-1/2})^{10}$$

Let T_{r+1} be the general term in $(x^{1/3} - x^{-1/2})^{10}$.

Then, $T_{r+1} = {}^{10}C_r (x^{1/3})^{10-r} (-1)^r (x^{-1/2})^r$

For this term to be independent of x, we must have

$$\frac{10 - r}{3} - \frac{r}{2} = 0$$

$\Rightarrow \quad 20 - 2r - 3r = 0 \Rightarrow r = 4$

Hence, required coefficient ${}^{10}C_4(-1)^4 = 210$

83. (4)

Coordinates of any point on the ellipse are $\frac{x^2}{49}+\frac{y^2}{25}=1$ of the form $(7\cos\theta, 5\sin\theta), 0\le\theta<2\pi$.

Let $x=7\cos\theta, y=5\sin\theta$, where x and y are integers.

As $-1\le\sin\theta\le1 \Rightarrow -5\le 5\sin\theta\le 5$

$\Rightarrow \quad 5\sin\theta = -5,-4,-3,-2,-1,-,1,2,3,4,5$ (integer values)

Now, $5\sin\theta=-5 \Rightarrow \sin\theta=-1$

$\Rightarrow \quad \theta=\frac{3\pi}{2}$ and $7\cos\theta = 7\cos\left(\frac{3\pi}{2}\right)=0$

When, $5\sin\theta=\pm4 \Rightarrow \cos\theta=\pm\frac{3}{5}$

$\Rightarrow \quad 7\cos\theta=\pm\frac{21}{5}$, which are not integer.

Similarly, $5\sin\theta=\pm3,\pm2,\pm1$, then 7 cos θ is not an integer.

When, $5\sin\theta=0, \theta=0,\pi \Rightarrow 7\cos\theta=7,-7$

When, $5\sin\theta=5$, then $7\cos\theta=0$

Thus, lattice points on the ellipse are (0, –5), (7, 0), (0, 5) and (–7, 0).

84. (8)

$$L=\lim_{x\to0}\frac{(1+x^2)^{1/3}-(1-2x)^{1/4}}{x+x^2}$$

$$=\lim_{x\to0}\frac{\left(1+\frac{1}{3}x^2+....\right)-\left(1-\frac{x}{2}+....\right)}{x(1+x)}$$

$$\Rightarrow \quad L=\lim_{x\to0}\frac{\frac{1}{2}+\frac{x}{3}+.....}{1+x}=\frac{1}{2}$$

$$\therefore \quad 16L=16\times\frac{1}{2}=8$$

85. (31)

Case I if all are different, then the number of ways $={}^6C_3=20$

Case II if three each of two colours, then combination is

$3 \quad 0 \to 2!$

$2 \quad 1 \to 2!$

= 2! + 2! = 4 ways

Case III if two each of three colours, then combination is

$2 \quad 1 \quad 0 \to 3!$

$1 \quad 1 \quad 1 \to 1!$

= 3! + 1! = 7 ways

∴ required number of ways

= 20 + 4 + 7 = 31 ways

86. (10)

7th term in $\left[\sqrt[6]{3}\sqrt{2}+\frac{1}{\sqrt[3]{3}}\right]^n$

$$= {}^nC_6(2^{1/2}3^{1/6})^{n-6}\left[\frac{1}{3^{1/3}}\right]^6$$

7^{th} term from the end in

$$\left[\sqrt[6]{3}\sqrt{2}+\frac{1}{\sqrt[3]{3}}\right]^n$$

$$= {}^nC_6\cdot\left(\frac{1}{3^{1/3}}\right)^{n-6}(3^{1/6}2^{1/2})^6$$

According to the question.

$$= \frac{{}^nC_6(2^{1/2}3^{1/6})^{n-6}\left(\frac{1}{3^{1/3}}\right)^6}{{}^nC_6\left(\frac{1}{3^{1/3}}\right)^{n-6}(3^{1/6}2^{1/2})^6}$$

$$\Rightarrow \frac{2^{\frac{n-6}{2}}\cdot 3^{\frac{n-6}{6}}}{3^{\frac{6-n}{3}}\cdot 2^3\cdot 3^1}\times\frac{1}{3^2}=\frac{1}{6}$$

$$\Rightarrow 2^{\frac{n-6}{2}}\cdot 3^{\frac{n-6}{2}} = 2^2\times 3^2$$

$$\Rightarrow \frac{n-6}{2}=2$$

$$\Rightarrow n = 10$$

87. (2)

Given, $f(x)=\begin{vmatrix}\cos x & x & 1\\ 2\sin x & x^2 & 2x\\ \tan x & x & 1\end{vmatrix}$

On differentiating both sides w.r.t. x, we get

$$f'(x)=\begin{vmatrix}-\sin x & x & 1\\ 2\cos x & x^2 & 2x\\ \sec^2 x & x & 1\end{vmatrix}+\begin{vmatrix}\cos x & 1 & 1\\ 2\sin x & 2x & 2x\\ \tan x & 1 & 1\end{vmatrix}+\begin{vmatrix}\cos x & x & 0\\ 2\sin x & x^2 & 2\\ \tan x & x & 0\end{vmatrix}$$

$$\Rightarrow \frac{f'(x)}{x}=\begin{vmatrix}-\sin x & 1 & 1\\ 2\cos x & x & 2x\\ \sec^2 x & 1 & 1\end{vmatrix}+\begin{vmatrix}\cos x & 1 & 0\\ 2\sin x & x & 2\\ \tan x & 1 & 0\end{vmatrix}$$

[$\therefore$ the second determinant is zero because $C_2 = C_3$]

$$\lim_{x\to 0}\frac{f'(x)}{x}=\begin{vmatrix}0 & 1 & 1\\ 2 & 0 & 0\\ 1 & 1 & 1\end{vmatrix}+\begin{vmatrix}1 & 1 & 0\\ 0 & 0 & 2\\ 0 & 1 & 0\end{vmatrix}$$

$$= 0 - 2 = -2$$

Hence, absolute value of

$$\lim_{x\to 0}\frac{f'(x)}{x} = |-2| = 2$$

88. (8)

Let equation of ellipse E be

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$

As it passes through (3, –1)

Then $\frac{9}{a^2}+\frac{1}{b^2}=1$

$\Rightarrow$ $9b^2 + a^2 = a^2b^2$ …(i)

As $y=-x+2\sqrt{5}$ is a tangent to E.

Then, $(2\sqrt{5})^2 = a^2(-1)^2+b^2$

$\Rightarrow$ $a^2 + b^2 = 20$ …(ii)

From Eqs. (i) and (ii), we get

$a^2 = 18, 10$

For $a^2 = 10$, then $b^2 = 10$ and hence E becomes a circle.

So, $a^2 = 10$ (Rejected)

Thus, $a^2 = 18$, then $b^2 = 2$

Now, $b^2 = a^2 (1- e^2)$

$\Rightarrow$ $18e^2 = 16$ $\Rightarrow$ $9e^2 = 8$

89. (0)

We have, $\frac{dy}{dx}=\frac{y}{x}-\cos^2\left(\frac{y}{x}\right)$

Putting y = vx, so that $\frac{dy}{dx}=v+x\frac{dv}{dx}$,

We get $v+x\frac{dv}{dx}=v-\cos^2 v$

$$\frac{dv}{\cos^2 v}=-\frac{dx}{x}\Rightarrow \sec^2 v\,dv=\frac{-1}{x}dx$$

On integrating both sides, we get

tan v = – ln x + ln C

$\Rightarrow$ $\tan\left(\frac{y}{x}\right)=-\ln x+\ln C$ …(i)

Since, Eq. (i) passes through $\left(1,\frac{\pi}{4}\right)$.

So, 1 = In C $\Rightarrow$ C = e

From Eq. (i), $\tan\left(\frac{y}{x}\right)=-\ln x+\ln e$

$\Rightarrow$ $y=x\tan^{-1}\left(\ln\left(\frac{e}{x}\right)\right)=\phi(x) given$

Hence, $\phi(e)=e\cdot\tan^{-1}\left(\ln\left(\frac{e}{e}\right)\right)=e\cdot\tan^{-1}(0)=0$

90. (12)

Given,

$$f(x) = -\begin{cases} -2x + \log_{1/2}(\lambda^2 - 6\lambda + 8) & , \quad -2 \le x < -1 \\ x^3 + 3x^2 + 4x + 1 & , \quad -1 \le 3 \le x \end{cases}$$

Also, $f'(x) = 3x^2 + 6x + 4 > 0, \forall x \in [-1, 3]$

So, f(x) is increasing on [–1, 3].

And $f'(x) = -2 \times 1 + 0$

$= -2 < 0, \forall x \in [-2, -1]$

So, f(x) is decreasing on (–2, –1).

∴ if f(x) has smallest values at x = – 1, then we must have

$\lim_{h \to 0} f(-1-h) \ge f(-1)$

$\Rightarrow \lim_{h \to 0}[-2(-1-h) + \log_{1/2}(\lambda^2 - 6\lambda + 8)]$

$\ge (-1)^3 + 3(-1)^2 + 4(-1) + 1$

$\Rightarrow 2 + \log_{1/2}(\lambda^2 - 6\lambda + 8) \ge -1$

$\Rightarrow \log_{1/2}(\lambda^2 - 6\lambda + 8) \ge -3$

$\Rightarrow \log_2(\lambda^2 - 6\lambda + 8) \le 3$

$\Rightarrow \lambda^2 - 6\lambda + 8 \le 8$

$\Rightarrow \lambda^2 - 6\lambda \le 0$

$\Rightarrow \lambda(\lambda - 6) \le 0$

$\Rightarrow 0 \le \lambda \le 6$ …(i)

But in order to define $\log_{1/2}(\lambda^2 - 6\lambda + 8)$ we must have,

$\lambda^2 - 6\lambda + 8 > 0$

$\Rightarrow (\lambda - 2)(\lambda - 4) > 0$

From Eqs. (i) and (ii), we get

$\lambda \in [0, 2) \cup (4, 6]$

⇒ Possible integer (s) in the range of λ are 0, 1, 5, 6.

Hence, the sum of all possible positive integer (s) in the range of

λ = 1 + 5 + 6 = 12