IIT – JEE: 2023 **AITS – 6** **DATE: 30/04/23**

ADVANCED (ANSWER KEY)

PAPER - I

Que.	1	2	3	4	5	6	7	8	9	10
Ans.	BC	BD	ABC	ACD	ABD	ABCD	BC	ABD	ABC	ABC
Que.	11	12	13	14	15	16	17	18	19	20
Ans.	B	D	D	B	A	C	A	A	B	C
Que.	21	22	23	24	25	26	27	28	29	30
Ans.	C	B	A	BC	ABC	ABCD	A	BCD	AB	BCD
Que.	31	32	33	34	35	36	37	38	39	40
Ans.	B	B	A	C	D	D	A	A	A	A
Que.	41	42	43	44	45	46	47	48	49	50
Ans.	BCD	ABCD	AC	BCD	ACD	ABC	BC	AC	ABD	AB
Que.	51	52	53	54	55	56	57	58	59	60
Ans.	B	C	C	A	A	B	D	C	B	C

PAPER - II

Que.	1	2	3	4	5	6	7	8	9	10
Ans.	C	D	C	A	B	D	BCD	BCD	ACD	C
Que.	11	12	13	14	15	16	17	18	19	20
Ans.	A	B	8	1	5	8	3	2	7	5
Que.	21	22	23	24	25	26	27	28	29	30
Ans.	A	A	C	A	C	AB	BCD	ABC	ABCD	AB
Que.	31	32	33	34	35	36	37	38	39	40
Ans.	C	D	5	4	6	6	5	8	6	3
Que.	41	42	43	44	45	46	47	48	49	50
Ans.	A	A	D	C	A	BC	AB	ABCD	BC	ABCD
Que.	51	52	53	54	55	56	57	58	59	60
Ans.	B	D	1	2	5	2	3	5	9	3

Note : **Detailed solution to this test is available on Today after 05.00 pm on our website.:www.iitianspace.com**

PAPER-1

SOLUTIONS

1. **Sol:** $\vec{F} = -\frac{dU}{dr} = -k\vec{r}$

$kr = \frac{mv^2}{r}$ ....(1)

$mvr = \frac{nh}{2\pi}$ ....(2)

$r^2 k = \frac{mn^2h^2}{4\pi^2m^2r^2}$; $r^4 = \left(\frac{h^2}{4\pi^2mk}\right)n^2$

3. **Sol:** From work energy theorem
$W_{cell} = \Delta H + U_f - U_i$, where $U_f = U_i$

4. **Sol:** In the process AC, temperature of the system first increases and then decreases.

5. **Sol:** The acceleration of centre of mass of the whole system will be zero.

6. **Sol:** Applying impulse momentum equation for the ball

$\Rightarrow \int_0^{0.1} (N - mg)\,dt = mv_{yf} - my_{yi}$

$\Rightarrow \int_0^{0.1} Ndt = 10 \times \frac{10}{2} + 10 \times 10 + 10 \times 0.1$

$\Rightarrow \int_0^{0.1} Ndt = 160 \Rightarrow \langle N \rangle = \frac{\int Ndt}{0.1} = 1600$

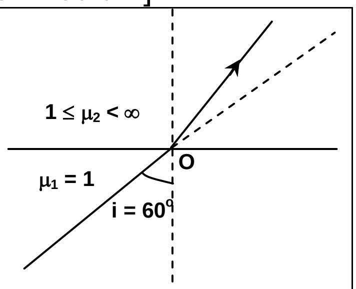
7. **Sol:** $E_{inside} 4\pi x^2 = \frac{\int_0^x \left(\frac{5}{2\pi}\right)\left(1 - \frac{r}{30}\right) 4\pi r^2\, dr}{\varepsilon_0}$

$\frac{dE_{inside}}{dx} = 0$

14- 16. **Sol:** $1 \le \mu_1 < \infty$ and $1 \le \mu_2 < \infty$.

Case – I

$\mu_2 \ge 1$ [Refraction in which rays comes from rarer to denser medium]

$\mu_1 \sin i = \mu_2 \sin r$ $\sin r = (\mu_1/ \mu_2) \sin i$ $r = \sin^{-1}\left[\frac{\mu_1}{\mu_2}\sin i\right]$ $\delta = i - \sin^{-1}\left[\frac{\mu_1}{\mu_2}\sin i\right]$ $0 \le \delta\, i \Rightarrow 0 \le \delta \le 60$ **Case – II** $\mu_2 \le \mu_1$	

<table>
<tr>
<td>
If refraction takes place $\delta = r - i = \sin^{-1}\left[\frac{\mu_1}{\mu_2}\sin i\right] - i$
$0 \le \delta \le (\pi/2) - i \Rightarrow 0 \le \delta \le 30^o$
If reflection takes place
$\delta = \pi - 2i = 60^o$
Alternate:
$\theta_0 \Rightarrow$ highest value of deflection
$\Rightarrow$ when μ_2 is high $\Rightarrow$ the ray will be normal,

$\Rightarrow \theta_0 = 60^o$
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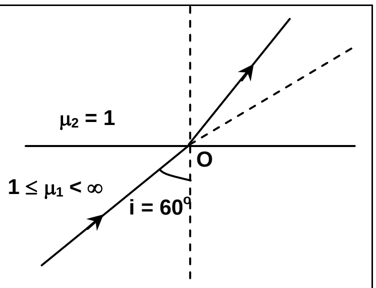

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$\theta_1 \Rightarrow$ value of deflection just before total internal reflection, $\theta_1 = 30^o$

$k_0 \Rightarrow$ when deflection is zero $\Rightarrow \mu_1 = \mu_2$

17. Sol: $\Delta f_A = 0$

$\Delta f_B = \frac{2v_0}{c - v_s} f_0 = \frac{2 \times 10}{300} \times 3000 = 200\,\text{Hz}$

$\Delta f_C = \frac{2cv_s}{c^2 - v_s^2} f_0 = \frac{2 \times 350 \times 50}{400 \times 300} \times 3000 = 875\,\text{Hz}$

$\Delta f_D = \frac{360 \times 2 \times 50}{400 \times 300} \times 3000 = 900\,\text{Hz}$

18. Sol: $\omega_n = \omega_0 \frac{z^2}{n^3}$; $v_n = \frac{z}{n}$; $I_n = I_0 \frac{z^2}{n^3}; E_n = E_0 \frac{z^2}{n^2} f.$

CHEMISTRY PAPER – I (SOLUTION)

21. (C)

$$R-CH(Cl)-CH_2(Cl) + NaNH_2 \longrightarrow R-CH=CH-Cl + R-C(Cl)=CH_2-Cl + NH_3 + NaCl$$

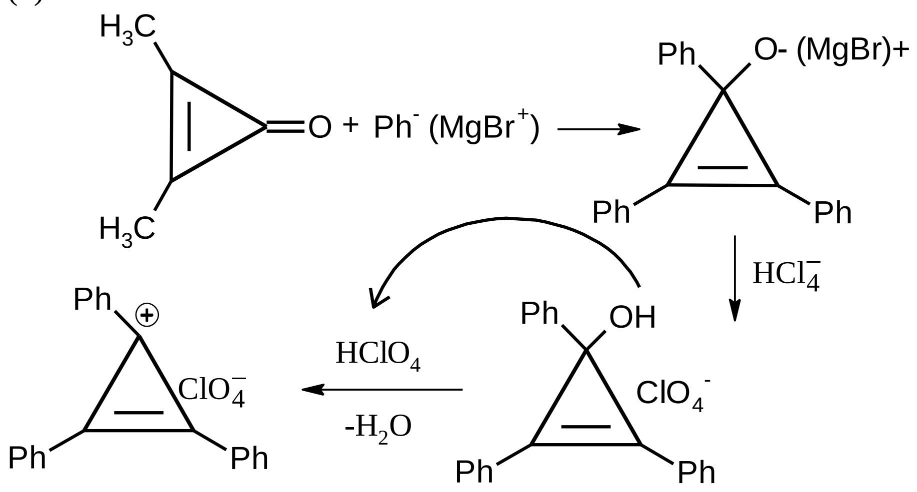
1 mole (R–CH(Cl)–CH₂Cl) 1 mole ($NaNH_2$) 1 mole (R–CH=CH–Cl + R–C(Cl)=CH₂)

$$R-CH=CH-Cl + R-C(Cl)=CH_2 + 2\,NaNH_2 \longrightarrow R-C\equiv C^- Na^+ + 2\,NH_3 + NaCl$$

	R–CH=CH–Cl + R–C(Cl)=CH₂	2 NaNH₂	R–C≡C⁻Na⁺
	1 mole	1 mole	0
Final mixture	1/2 mole	0	1/2 mole

$$R-C\equiv C^- Na^+ \xrightarrow{H_3O^+} RC\equiv CH$$

22. (b)



23. (a)

$$\Delta x . m \Delta V \geq \frac{h}{4\pi}$$

$$\Delta V \geq V$$

$$\Delta x . mV \geq \frac{h}{4\pi}$$

$$\Delta x \geq \frac{h}{4\pi mV}$$

$$\Delta x \geq \frac{\lambda}{4\pi}$$

24. (bc)

25. (abc)
26. (abcd)

27. (a)

$E = 0.4108 + 0.003\,T$

$$\frac{\partial E}{\partial T} = 0.003\,vk^{-1}$$

$$nF\left(\frac{\partial E}{\partial T}\right) = \Delta s$$

$$\Delta S = 2\times 96500\,d \times 0.003\ vk^{-1}$$

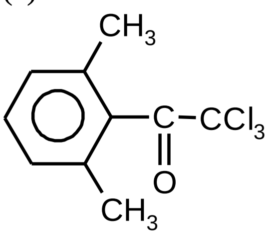
$$= 2\times 96.5\times 3 = 579\,Jk^{-1}$$

28. (b)

$C_6H_5CH_2-\underset{\underset{O}{\|}}{C}-CH_3$

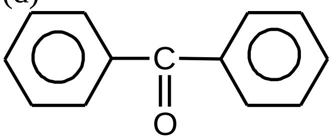
More acidic benzylic hydrogen involved in enolate formation

(c)



Can't undergo nucleophilic addition with $\overline{O}H$ due to steric repulsion.

(d)



Non-methyl ketone

29. (ab)

CdS and ZnS are soluble in conc. HCl

30. (bcd)

In pH_3 phosphors uses pure P orbital's for bond functions.

$$4H_3PO_3 \xrightarrow[\Delta]{} 3H_3PO_4 + pH_3$$

31. (b)

32. (b)

33. (a)

34. (c)

$$Na_2S + Na_2SO_3 + I_2 \longrightarrow Na_2S_2O_3 + 2NaI$$

$$2\,Na_2S_2O_3 + I_2 \longrightarrow Na_2S_4O_6 + 2\,NaI$$

35. (d)

Total moles of $Cu^{2+} = 0.2 \times \frac{100}{1000} = 0.02$

Mass of $Cu^{2+} = 0.02 \times 63.5 = 1.27\,g$

Mass of cu left = 0.01×63.5

In solution = 0.635

(Here E.w. of Cu is 63.5 because Cu^{2+} reduces to Cu^{1+} by I^-)

Mass of deposited = 1.27 – 0.635

= 0.635 g

m = d × Area × Thickness

$0.635 = q \times 2\pi r\left(r+h\right) \times \text{Thickness}$

$0.635 = 9 \times 2 \times \pi \times \frac{1}{\pi}\left(\frac{1}{\pi} + 6.03\right) \times \text{Thickness}$

$0.635 = 18 \times \left(\frac{7}{22} + 6.03\right) \times \text{Thickness}$

$\text{Thickness} = \frac{0.635}{18 \times 6.35} = \frac{1}{180}\,\text{Cm} = \frac{1}{18}\,\text{mm}$

= 0.055 mm

36. (d)

$AgBr_{(s)} \rightleftharpoons Ag^+_{(aq)} + Br^-_{(aq)}$ $\qquad K_{sp} = 3.5 \times 10^{-13}$

$Ag^+_{(aq)} + 2S_2O_3^{2-} \rightleftharpoons \left[Ag\left(S_2O_3\right)\right]^{3-}_{conc}$ $\qquad K_f = 2 \times 10^{-13}$

$AgBr_{(s)} + 2S_2O_3^{2-}{}_{(aq)} \rightleftharpoons \left[Ag\left(S_2O_3\right)_2\right]^{3-} + Br^-_{(aq)}$

Total required $S_2O_3^{2-}$ = $S_2O_3^{2-}$ required for formation of complex + $S_2O_3^{2-}$ maintained at equilibrium

$S_2O_3^{2-}$ required for formaion of complex = $2 \times 5.3 \times 10^{-3} = 10.6 \times 10^{-3}$

$$K_{aq} = \frac{\left[Ag\left(S_2O_3\right)_2\right]^{3} - \left[Br^-\right]}{\left[S_2O_3^{2-}\right]^{-2}} = K_{sp} \times K_f = 7$$

$$\left[S_2O_3^{2-}\right]^2 = \frac{5.3 \times 10^{-3} \times 5.3 \times 10^{-3}}{7} = \frac{28 \times 10^{-6}}{7} = 4 \times 10^{-6}$$

$$\left[S_2O_3^{2-}\right] = 2 \times 10^{-3}\,M$$

Total moles of $Na_2S_2O_3$ = $10.6 \times 10^{-3} + 2 \times 10^{-3}$

$= 12.6 \times 10^{-3}$

37. (a)

38. (a)

39. (a)

I. $P_{ideal} = 150 \times \frac{1}{3} + 75 \times \frac{2}{3} = 100\,mm$ $\quad P_{solution} > P_{ideal}$ + ve deviation

II. $\frac{1}{P_{ideal}} = \frac{0.6}{120} + \frac{0.4}{80} = \frac{1}{100}$ $\quad$ Obeys Raoult's law

III. $P_{ideal} = 100 \times \frac{3}{5} + 150 \times \frac{2}{5} = 60 + 60 = 120$

$P_{solution} < P_{ideal}$ $\quad$ (– ve deviation)

40. (a)

PAPER – I (SOLUTIONS)

41. (BCD)

$$(x-y)f(x+y)-(x+y)f(x-y)=(x^2-y^2)\{(x+y)^2-(x-y)^2\}$$

$$=(x-y)\left[(x+y)^3+c(x+y)\right]-(x+y)\left[(x-y)^3+c(x-y)\right]$$

$\therefore f(x)=x^3+cx$

But f (1) = 1

$\therefore$ f (x) = x^3

42. (ABCD)

$$\left[\vec{a}\vec{b}\vec{c}\right]=\frac{1}{\left[\vec{p}\vec{q}\vec{r}\right]}$$

AM $\geq$ GM

43. (AC)

$$(1-y)^m(1+y)^n=1+(n-m)y+\left\{\frac{m(m-1)}{2}+\frac{n(n-1)}{2}-mn\right\}y^2+$$

Given $n-m=10$

$$\frac{m^2+n^2-2mn-m-n}{2}=10$$

Solving $m=35$

$n=45$

44. (BCD)

$$p(n)=\left(\frac{1}{2}\right)\left(\frac{2}{3}\right)\left(\frac{3}{4}\right)......\left(\frac{n-2}{n-1}\right)\left(\frac{n-1}{n}\right)\left(\frac{1}{n+1}\right)$$

$$=\frac{1}{n(n+1)}<\frac{1}{2010}$$

$$\frac{n(n+1)}{2}>1005$$

$n\geq 45$

45. (ACD)

$$\int x^{16}\left(\frac{x^2(x^6-1)}{x^2-1}\right)^4(9x^4+7x^2+5)dx$$

$$\int x^{16}(x^6+x^4+x^2)^4(9x^4+7x^2+5)$$

$$\int(x^9+x^7+x^5)^4(9x^8+7x^6+5x^4)dx$$

$x^9+x^7+x^5=t$

We get

$$g(x)=\frac{\left(x^9+x^7+x^5\right)^5}{5}\ (\because c=0)$$

g (1) and g (–1) are not defined

$g(5) = 5^{24}(651)^5$

46. (ABC)

Let $g(x)=x-x^2-f(x)$

$g(1)=g(2)=.......=g(8)=0$

$\therefore g'(x)=0$ has at least 7 roots

Let $h(x)=1-2x-f'(x)$

$h'(x)=0$ has at least 6 roots

Let $v(x)=-2-f''(x)$

$v'(x)=0$ has at least 5 roots

$\Rightarrow f'''(x)=0$ for at least 5 values of x

47. (BC)

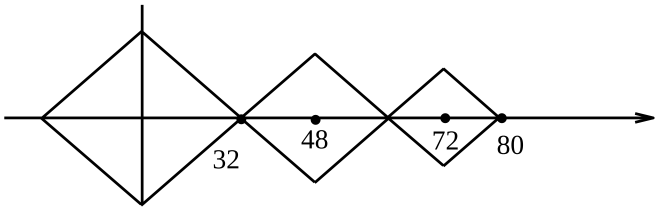
$f'(x)=3x^2-2x+100>0$

$\therefore f(x)$ is increasing function

48. (AC)

$a_1=0,\ b_1=32,\ a_2=0+\frac{3}{2}b_1=48,$

$b_2=\frac{b_1}{2}=16;\quad a_3=48+\frac{3}{2}\times16=72;\quad b_3=8$



So the 3 loops from $i=1$ to $i=3$ are alike area of i^{th} loop (square) $=\frac{1}{2}(\text{diagonal})^2$

$A_i=\frac{1}{2}(2b_i)^2=2b_i^2$

$\frac{A_{i+1}}{A_i}=\frac{1}{4}$

Area will form G.P.

49. (ABD)

$\overrightarrow{V_1} = \ell\vec{a} + m\vec{b} + n\vec{c}$

$\overrightarrow{V_2} = \ell\vec{b} + m\vec{c} + n\vec{a}$

$\overrightarrow{V_3} = \ell\vec{c} + m\vec{a} + n\vec{b}$

$\left[\overrightarrow{V_1}\,\overrightarrow{V_2}\,\overrightarrow{V_3}\right] = 0$

i.e. $\begin{vmatrix} \ell & m & n \\ n & \ell & m \\ m & n & \ell \end{vmatrix}\left[\vec{a}\vec{b}\vec{c}\right] = 0$

Where $\left[\vec{a}\,\vec{b}\,\vec{c}\right] \neq 0$

50. (AB)

By $AM \geq GM$

Solution for Que. Nos. 51 to 52

51. (B) 52. (C)

(i) & (ii) give

$\left(\vec{x}\times\vec{y}\right)\times\left(\vec{y}\times\vec{z}\right) = \left(\vec{a}\times\vec{b}\right) \quad \Rightarrow \left[\vec{x}\vec{y}\vec{z}\right]\vec{y} = \vec{a}\times\vec{b}$

Also (ii) gives $\vec{x}.\left(\vec{y}\times\vec{z}\right) = \vec{x}.\vec{b} \quad \Rightarrow \left[\vec{x}\vec{y}\vec{z}\right] = r \Rightarrow \vec{y} = \dfrac{\vec{a}\times\vec{b}}{r}$

Thus, (i) gives $\left(\vec{x}\times\vec{y}\right)\times\vec{y} = \dfrac{\vec{a}\times\left(\vec{a}\times\vec{b}\right)}{r} \Rightarrow \vec{y} - \left|\vec{y}\right|^2\vec{x} = \dfrac{\vec{a}\times\left(\vec{a}\times\vec{b}\right)}{r}$

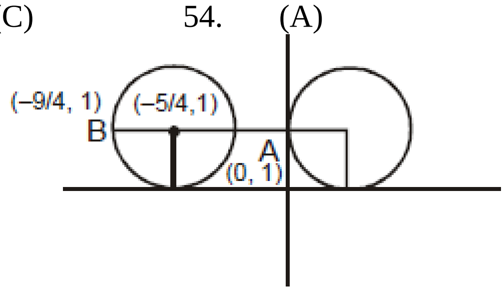
$\Rightarrow \dfrac{\vec{a}\times\vec{b}}{r} - \dfrac{\left|\vec{a}\times\vec{b}\right|^2}{r^2}\vec{x} = \dfrac{\vec{a}\times\left(\vec{a}\times\vec{b}\right)}{r}$

Similarly, take cross with $\vec{y}$ in equation (ii)

$\Rightarrow \left(\vec{y}\times\vec{z}\right)\times\vec{y} = \left|\vec{y}\right|^2\vec{z}\left(\vec{y}.\vec{z}\right)\vec{y} \quad \Rightarrow \dfrac{\vec{b}\times\left(\vec{a}\times\vec{b}\right)}{r} = \dfrac{\left|\vec{a}\times\vec{b}\right|^2}{r^2}\vec{z} - \dfrac{\vec{a}\times\vec{b}}{r}$

Solution for Que. Nos. 53 to 54

53. (C) 54. (A)



$\alpha^2 - 7\alpha + 11 \leq 1$

For some part to be in 2nd quadrant

$\Rightarrow \quad \alpha \in [2,5]$

$\therefore \quad \left(\alpha^2 - 7\alpha + 11\right)_{min} = -5/4$

$AB = \frac{9}{4}$ & $\arg z = \pi - \tan^{-1}\left(\frac{4}{9}\right)$

55. (A)

$\phi(x) - \phi\left(\frac{x}{2}\right) = \phi\left(\frac{x}{2}\right) - \phi\left(\frac{x}{4}\right) + x^2$

$\phi\left(\frac{x}{2}\right) - \phi\left(\frac{x}{4}\right) = \phi\left(\frac{x}{4}\right) - \left(\frac{x}{8}\right) + \frac{x^2}{4}$

$\phi\left(\frac{x}{4}\right) - \phi\left(\frac{x}{8}\right) = \phi\left(\frac{x}{8}\right) - \phi\left(\frac{x}{16}\right) + \frac{x^2}{16}$

....

$\phi\left(\frac{x}{2^{n-1}}\right) - \phi\left(\frac{x}{2^n}\right) = \phi\left(\frac{x}{2^n}\right) - \phi\left(\frac{x}{2^{n+1}}\right) + \frac{x^2}{\left(2^{n-1}\right)^2}$

Adding all, we get

$\phi(x) - \phi\left(\frac{x}{2^n}\right) = \phi\left(\frac{x}{2}\right) - \phi\left(\frac{x}{2^{n+1}}\right) + \frac{4}{3}x^2$

Take limits as $n \to \infty$

$\Rightarrow \phi(x) - \phi(0) = \phi\left(\frac{x}{2}\right) - \phi(0) + \frac{4}{3}x^2 \qquad \Rightarrow \phi(x) - \phi\left(\frac{x}{2}\right) = \frac{4}{3}x^2$

Repeating the same process

$\Rightarrow \phi(x) - \phi\left(\frac{x}{2}\right) = \frac{4}{3}x^2$

Repeating the same process,

$\Rightarrow \phi(x) - \phi(0) = \frac{16}{9}x^2 \qquad \Rightarrow y - 1 = \frac{16}{9}x^2$

56. (B)

vertex is $(0,1)$

57. (D)

(P) $e_1 < 1 < e_2 \qquad f(1) < 0$

(Q) both roots > 1

(i) $D \geq 0$

(ii) $f(1) > 0 \qquad a \in \left[2\sqrt{2}, 3\right)$

(iii) $a/2 > 1$

(R) $\frac{1}{e_1^2} + \frac{1}{e_2^2} = 1 \Rightarrow a = \pm 2\sqrt{2}$

(S) $1 < e_1 < \sqrt{2} \leq e_2 \qquad \Rightarrow f\left(\sqrt{2}\right) < 0$ and $f(1) > 0 \Rightarrow a \in \left(2\sqrt{2}, 3\right)$

58. (C)

(P) $\arg(z) = \arg(-z) = \arg\left(\frac{z}{-z}\right) = \arg(-1) = \pi$

(Q) $|z-4| = \text{Re}(z) \Rightarrow y^2 = 8(x-2)$

Tangent $y = m(x-2) + \frac{2}{m} \Rightarrow 0 = -2m + \frac{2}{m}$

$m = \pm 1$

(R) $z - 1 = e^{ia} \Rightarrow z = 2\cos(a/2)e^{ia/2}$

$\tan\left(\arg\frac{(z-1)}{2}\right) - \frac{2i}{z}$

$= \tan a/2 - \frac{i}{\cos a/2}(\cos a/2 - i\sin a/2) = -i$

(S) $\theta_1 - \frac{\pi}{4} = \theta_2 \;\&\; \theta_1 + \theta_2 = \frac{\pi}{2}$

59. (B)

(P) possible point of discontinuously

$4x = 0, 1, 2......, 20$

$3x = 1, 2, ..., 15$

But $f(x)$ is continuous (a) $x = 0, 1, 2, 3, 4, 5$

Total 30

(Q) $f(-x) = f(x)$

$-\alpha x^3 - \beta x - \tan x \,\text{sgn}(x) = \alpha x^3 + \beta x - \tan x \,\text{sgn}(x)$

$\alpha x^3 + \beta x = 0 \Rightarrow \alpha = \beta = 0$

$\alpha = [a]^2 - 5[a] + 4 \;\&\; \beta = 6\{a\}^2 - 5\{a\} + 1$

(R) $f(x) + f(1-x) = 1$

(S) $P(x) = (50 + x)(1000 - 10x)$

$P'(x) = 0$

$\Rightarrow \quad x = 25$

60. (C)

(P) Let $x - 0.4 = t$

$\int_{0.6}^{3.6} \{t\}\,dt = 3\int_0^1 \{t\}\,dt = 3/2$

(Q) Let $x = \tan a$

(R) $\int_2^{-1} f(x)\,dx = -\int_{-1}^{2} f(x)\,dx = -\left[\int_{-1}^{4} f(x)\,dx + \int_4^2 f(x)\,dx\right]$

(S) $\lim_{x \to 0} \frac{x\int_0^x e^{t^2}\,dt}{1 + x - e^x}$

$$\lim_{x\to 0}\frac{\int_0^x e^{t^2}dt}{x}\times\frac{x^2}{1+x-e^x}$$

$$\left(\lim_{x\to 0}\int_0^x\frac{e^{t^2}dt}{x}\right)\times(-2)$$

$$\lim_{x\to 0}e^{x^2}\times(-2)=-2$$

PAPER-2

SOLUTIONS

1. **Sol:** $y = A\sin\left(\frac{2\pi t}{T}\right)$

$t_2 - t_1 = \frac{T}{2\pi}\left(\sin^{-1}\frac{y_1}{A} - \sin^{-1}\frac{y_2}{A}\right) = 0.17s$

2. **Sol:** $v^2 = \omega^2(A^2 - x^2) = 3\omega^2 a^2 \Rightarrow \omega = \frac{v}{\sqrt{3}a} = 1$ rad/sec

$x = 2a\sin(\omega t + \phi)$, $v = 2a\omega\cos(\omega t + \phi)$

At t = 0, x = a and v = +ve

$\frac{1}{2} = \sin\phi$

$\Rightarrow \phi = n\pi + (-1)^n \frac{\pi}{6} = \begin{cases} \pi/6 \\ 5\pi/6 \end{cases}$

v at $\phi = \pi/6$ = +ve and v at $\phi = 5\pi/6$ = –ve

So, $\phi = \pi/6$

$x = 2a\sin(\omega t + \phi) = 2a[\sin\omega t\cos\phi + \cos\omega t\sin\phi]$

$x = a(\sqrt{3}\sin t + \cos t)$

3. **Sol:** $m\omega^2 R\left(\frac{h}{4}\right) = mgr \Rightarrow \sqrt{\frac{4gr}{Rh}}$

4. **Sol:** $mv_0 = 2mv \therefore v = v_0/2$

$\frac{1}{2}mv_0^2 = \frac{q^2}{4\pi\varepsilon_0\sqrt{R^2 + x^2}} + 2.\frac{1}{2}m.\frac{v_0^2}{4}$

$\Rightarrow \frac{1}{4}mv_0^2 = \frac{q^2}{4\pi\varepsilon_0\sqrt{R^2 + x^2}} \Rightarrow x = \sqrt{\frac{q^4}{\pi^2\varepsilon_0^2 m^2 v_0^4} - R^2}$

5. **Sol:** $\sin\theta_C = \mu_2/\mu_1$

$1 - \cos^2\theta_C = \left(\frac{\mu_2}{\mu_1}\right)^2$

$$1-(\hat{n}.\hat{p})^2 = \left(\frac{\mu_2}{\mu_1}\right)^2$$

$$1-\left[\frac{4}{\sqrt{25}}\right] = \left(\frac{\mu_2}{\mu_1}\right)^2 \Rightarrow \frac{\mu_2}{\mu_1} = \frac{3}{5} \Rightarrow \mu_2 = \frac{3\sqrt{3}}{5}$$

6. **Sol:** $F = -\frac{dU}{dx}$

$$\int dU = -\int F dx$$

$$U = \frac{kx^2}{2} - \frac{\alpha x^4}{4}$$

9. **Sol:** Velocity is given by slope of x-t graph, hence slope is zero at A and at peak of region CO and bottom most point of EF.

For AB and EF acceleration is positive for these region graph is concave up.

For BC and OE acceleration is zero as velocity is constant.

For CD acceleration is negative as graph is concave down for this region.

10. **Sol:** $\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} + \frac{x}{f_1 f_2}$

$$\frac{1}{f} = \frac{1}{f_1} - \frac{1}{f_1} + \frac{x}{f_1^2}$$

$$\frac{1}{f_1} = \frac{x}{f_1^2}.$$

$\Rightarrow$ f > 0 for every x.

for x = 0 = ∞

Hence, for x = 0, system will behave like a glass.

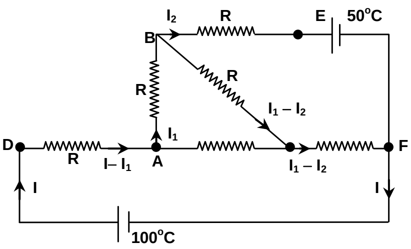
11. **Sol:** Hint: $\vec{L}_0 = \vec{r}_{cm} \times m(\vec{v}_{cm} - \vec{v}_0) + I_{cm}\vec{\omega}$

15. **Sol:** Here R = $\frac{\ell}{KA}$

Using Kirchhoff's law

$3RI - RI_1 - RI_2 = 100$ (1)

$RI + RI_1 - 3RI_2 = 50$ (2)



$-RI + 3R_1 - RI_2 = 0$ (3)

Solving (1), (2) & (3), we have

$I_2 = 0,\ I_1 = \frac{25}{2R}, I = \frac{75}{2R}$

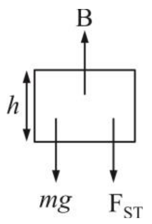
$\Rightarrow T_B = 50^oC.$

16. Sol: $E = 2 \times \frac{60}{75} \times \left(\frac{1000}{100}\right) = 160$ volt; $E = E_0 \frac{\ell_1}{\ell_2}\left(\frac{R_1+R_2}{R_2}\right)$

$\frac{\Delta E}{E} = \left|\frac{\Delta \ell_1}{\ell_1}\right| + \left|\frac{\Delta \ell_2}{\ell_2}\right| + \left|\frac{\Delta(R_1+R_2)}{R_1+R_2}\right| + \left|\frac{\Delta R_1}{R_1}\right| \Rightarrow \Delta E = 0.64$ volts.

17. Sol: $\frac{dN}{dt} = -\lambda N + \frac{9\lambda N_0^2}{N} \Rightarrow N = 3N_0 \text{ if } t \to \infty$

18. Sol: $mg + F_{ST} = B$



$mg + 4aT = a^2 h \rho_w g$

Putting values $h = 2m$

19. Sol: For cylinder A. For cylinder B

$dQ = nC_p dT$ $dQ = nC_v dT'$

$nC_P dT = nC_V dT'$

$\therefore \quad dT = \frac{C_P \times 30}{C_v}$ = 30 × 1.4 = 42 K

20. **Sol:** T cos 37° + T cos 53° = mg

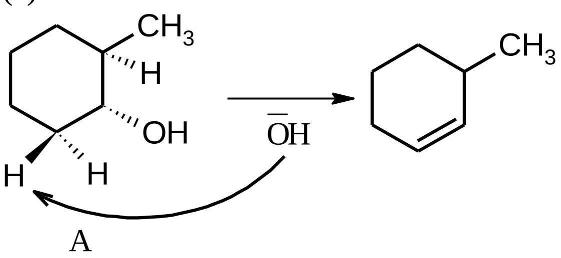
T sin 53° + T sin 37° = $\frac{mv^2}{R}$

$\Rightarrow$ v = $\sqrt{Rg} = 5 m/s$

CHEMISTRY PAPER – II (SOLUTION)

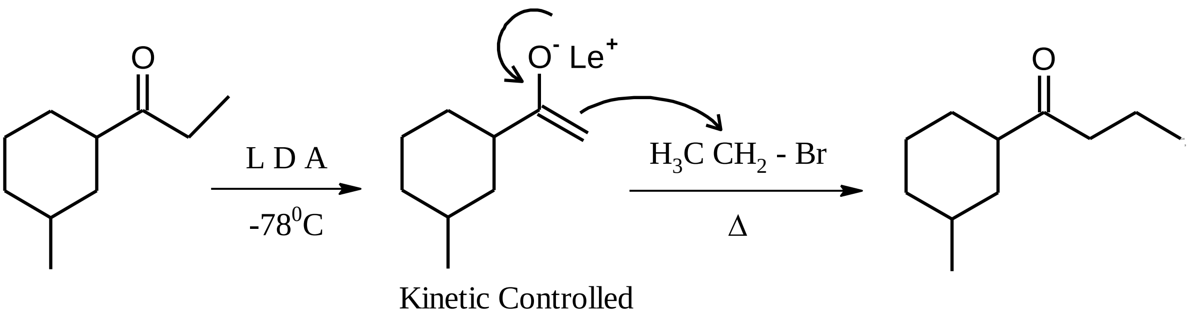
21. (a)

22. (a)



23. (c)

24. (a)



25. (c)

26. (ab)
Main factor that effect electro proton mobility is the magnitude of charge.

Serine (A) $\longrightarrow$ neutral amino acid

Glutamic acid (B) $\longrightarrow$ acidic amino acid

Lysine (C) $\longrightarrow$ Basic amino acid
At pH = 1 A & B posses a net charge of +1 ;
C passes a net charge of +2
C moves faster than A & B towards cathode
At pH = 12 A & C passes a net charge of –1
B passes a net charge of –2

27. (bcd)
At same T, P, short hired α-emitters
Emit α-rays with greater ranges.

28. (abc)
In F-centre crystals an electron is trapped in the anion vacancy. So it becomes n-type conductor.

29. (abcd)

30. (ab)
Amines couples in slightly acidic medium and phenols couples in slightly alkaline medium.
Diazo coupling does not takes place either is strongly acidic or strongly alkaline.

31. (c)

32. (d)

33. (5)

Volume of one protein $=\frac{4}{3}\pi(5\text{nm})^3$

$=\frac{4}{3}\pi\times125\times\text{nm}^3$

Volume of 1000 proteins $=\frac{4}{3}\pi\times125\times10^3\text{ nm}^3$

Volume of mycoplasma $=\frac{4}{3}\pi\times\left(\frac{0.33\times10^3}{2}\right)^3\text{ nm}^3$

$=\frac{4}{3}\pi\times\frac{10^9}{27\times8}\text{ nm}^3$

Volume available to proteins $=\frac{4}{3}\pi\times\frac{10^9}{27\times8}\times0.27\times0.5$

$=\frac{4}{3}\pi\times\frac{10^9}{8}\times5$

No. of molecules of each kind $=\frac{10^6\times5}{8\times125\times10^3}=5$

34. (4)

Degree of disassociation (α_1) of 0.01 N solution = $\frac{5}{100}=0.05$

Degree of disassociation of 0.04 N acid is α_2

$$\frac{\alpha_2}{\alpha_1}=\sqrt{\frac{C_1}{C_2}}\Rightarrow\frac{\alpha_2}{0.05}=\sqrt{\frac{0.01}{0.04}}=\frac{1}{2}$$

$$\alpha_2=0.025$$

$$\alpha_2=\frac{\alpha}{\lambda^0}\Rightarrow\quad\lambda\text{ of }0.04=0.025\times4\times10^{-2}$$

acid = 10^{-3} sm^2 eq^{-1}

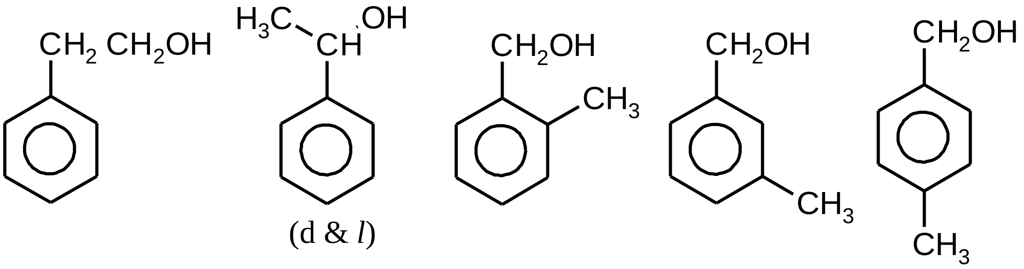
$$\lambda=\frac{K}{C}$$

$$k=\lambda C=10^{-3}\text{s m}^2\text{ eq}^{-1}\times40\text{ eq.m}^{-1}$$

$$=4\times10^{-2}\text{ s m}^{-1}$$

35. (6)
$C_8H_{10}O$ has ether, alcohol & phenols.

Since compound react with Na metal at room temperature and not dissolve in NaOH, it should be alcohol.



Total : 6

36. (6)
Benzyl is $Be_3 Al_2 Si_6O_{18}$
n = 6

37. (5)
NH_4Cl, $Al_2(SO_4)_3$, $BeSO_4$, $CuSO_4$ & $FeSO_4$ under go cationic hydrolysis. So their aq. Solutions are acidic. Hence red to litmus.

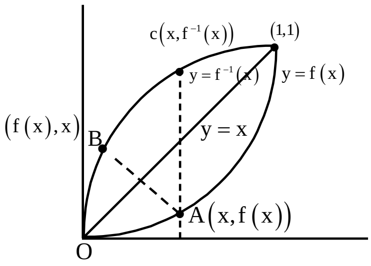
38. (8)
Aldehydes α–hydroxy acid, α-hydroxy ketones HCOOH, $H_2C_2O_4$ and oximes gives reduces Tollen's reagent and gives silver mirror.

39. (6)
In (III), (V), (VI), (VII), (VIII) & IX reactions O_2 is formed as one of the product.

40. (3)
The carbon atoms may be at adjacent positions, may have one boron atom between them, or may be on opposite sides of icosahedron. There are only three because the 12 positions of icosahedron are all inherently equivalent

MATHS PAPER - II (SOLUTION)

41. **(A)**



Slope of OC < Slope of OB

$$\frac{f^{-1}(x)-0}{x-0}<\frac{x-0}{f(x)-0}$$

$$f(x)f^{-1}(x)<x^2$$

42. **(A)**

Z lies inside $\Delta \therefore \ell, m, n$ all are of same sign

43. **(D)**

$$\lim_{x\to\infty}\frac{x^{n+1}f(x)}{x}=P \quad (\infty/\infty \text{ form})$$

Using L – hospitals

$$\lim_{x\to\infty}(n+1)x^n f(x)+x^{n+1}f'(x)=P$$

$$(n+1)P+\lim_{x\to\infty}x^{n+1}f'(x)=P$$

44. **(C)**

Note:- If more than 2 rational points exist on circle then centre will be rational. Hence p, q can be not be rational together.

$$P(p\,\&\,q\text{ rational})=\frac{2}{6}\times\frac{2}{6}=\frac{1}{9}$$

Ans: $1-\frac{1}{9}=\frac{8}{9}$

45. **(A)**

$$\int_0^t \left(f(x)-\left(x^4-4x^2\right)\right)dx=k\int_0^t\left(\left(2x^2-x^3\right)-f(x)\right)dx$$

On differentiating w.r.t. t we get

$$f(t)-\left(t^4-4t^2\right)=k\left\{2t^2-t^3-f(t)\right\}$$

46. **(BC)**

Let $\frac{b-x}{1-bx}=t \Rightarrow x=\frac{t-b}{bt-1} \Rightarrow dx=\frac{b^2-1}{(bt-1)^2}dt$

and $1-x^2=\frac{(b^2-1)(t^2-1)}{(bt-1)^2}$ and

$\therefore \int_0^b \frac{1-(f(x))^6}{1-x^2}dx=\int_0^b \frac{t^6-1}{t^2-1}dt=\frac{b^5}{5}+\frac{b^3}{3}+b$

$\rightarrow f(b)$ is an odd function .... (B)

$\rightarrow f(1)=\frac{1}{5}+\frac{1}{3}+1=\frac{3+5+15}{15}=\frac{23}{15}$ .... (C)

47. (AB)

Any tangent to the parabola $y^2=4x$ is $y=mx+\frac{1}{m}$

If this line is tangent to the ellipse, then $\frac{1}{m^2}=8m^2+2$

$\Rightarrow 8m^4+2m^2-1=0 \Rightarrow m^2=\frac{1}{4} \Rightarrow m=\pm\frac{1}{2}$

Equations are

$y=\frac{x}{2}+2$ or $y=-\frac{x}{2}-2$

$2y=x+4$ or $2y=-x-4$

$x-2y+4=0$ or $x+2y+4=0$

48. (ABCD)

$AA^T=A^T\ A=I$. Also $A^T=A$ so $A^2=I \Rightarrow I$ in involutory matrix.

$\Rightarrow |A^2|=|A^2|=I$ or $|A|=\pm 1$

But $|A|=\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}=(a+b+c)\begin{vmatrix} 1 & b & c \\ 1 & c & a \\ 1 & a & b \end{vmatrix}=(a+b+c)(ab+bc+ca-a^2-b^2-c^2)$

$|A|=(ab+bc+ca-a^2-b^2-c^2)$ $(\because a+b+c=1)$

$\therefore a^2+b^2+c^2-ab-bc-ca \geq 0$

So, $|A|=-1$, Hence $a^3+b^3+c^3-3abc=1$

Again, $a^2+b^2+c^2-ab-bc-ca=1$

$\Rightarrow 1-3(ab+bc+ca)=1$ so $ab+bc+ca=0$

$\Rightarrow$ at least one of a, b, c is negative.

49. (BC)

Here, $f'(x)=\frac{1}{x}+\sqrt{1+\sin x}, x>0$ but $f(x)$ is not differentiable in $(0,\infty)$ as $\sin x$ may be-1 and then $f''(x)=\frac{-1}{x^2}+\frac{\cos x}{2\sqrt{1+\sin x}}$ will not exists.

$\Rightarrow f'(x)$ is continuous for all $x\in(0,\infty)$ but $f'(x)$ is not differentiable on $(0,\infty)$.

Also $f'(x)\le 3$ if $x>1$ and $f(x)>3$, if $x>e^3$.

Let $\alpha=e^3 \Rightarrow$ options (C) is true.

(D) is not possible as $f(x)\to\infty$ where $x\to\infty$

50. (ABCD)

$$\frac{dy}{dx}=\frac{-2y\cot x\pm\sqrt{4y^2\cot^2 x+4y^2}}{2}$$

$$\therefore \frac{dy}{dx}=y(-\cot x\pm\operatorname{cosec} x)\Rightarrow \log y=-\log(\sin x)+\log\left(\tan\frac{x}{2}\right)+,=\log C$$

$$\Rightarrow y=\frac{C\tan\frac{x}{2}}{\sin x}=\frac{C}{2\cos^2\frac{x}{2}}=\frac{C}{1+\cos x} \qquad ....(D)$$

$$\Rightarrow x=2\cos^{-1}\sqrt{\frac{C}{2y}} \qquad ....(B)$$

$$\text{Also} \Rightarrow y=\frac{c}{1-\cos x}=\frac{C}{2\sin^2\frac{x}{2}} \qquad ....(C)$$

$$x=2\sin^{-1}\sqrt{\frac{C}{2y}} \qquad ....(A)$$

51. **(C)**

(P)

12 → 4, 4, 4

$$n=\left(\frac{12!}{(4!)^3}\times\frac{1}{3!}\right)\times 3!=\frac{12!}{(4!)^3} \text{ So, } \frac{(4!)^4}{12!}n=4!=24.$$

(Q) $x\ge 1,\ y\ge 2,\ z\ge 3$

$a=x-1\ge 0$ $\qquad a+b+c=(x+y+z)-6=9$

$b=y-2\ge 0$ $\qquad$ So, $\lambda={}^{9+3-1}C_2={}^{11}C_2=55$

$c=z-3\ge 0$

(R) ${}^8C_2=28$

(S) Case – I only one driver selected

${}^2C_1\times{}^4C_2\times 2!=24$

Case – II both driver selected

$^{2}C_1 \times 2! \times 2! \times {}^{4}C_1 = 24$

Ans. (40)

52. **(D)**

(P) $|M_r| = \dfrac{1}{r-1} - \dfrac{1}{r} \Rightarrow l = \lim\limits_{n\to\infty}\left(1-\dfrac{1}{n}\right) = 1$

(Q) $\left(1-\cos^2\gamma\right) - \cos^2\alpha + 2\cos\alpha\cos\beta\cos\gamma - \cos^2\beta = 2\cos\alpha\cos\beta\cos\gamma \Rightarrow \sin^2\alpha + \sin^2\beta + \sin^2\gamma = 2$

(R) $|C| = |A^2| = 16$

(S) $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} = I + B$

$\Rightarrow B^2 = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} = -I \therefore A^4 = (I+B)^4 = -4I$

53. **(1)**

For $n > 6$, $S_n = 873 + \text{multiple of } 7$

$S_n - 7\left[\dfrac{S_n}{7}\right] = 873 + 7\lambda - 7\left[\dfrac{873+7\lambda}{7}\right]; \lambda \in N$

$= 873 - 868 = 5$

$\left[\cot^{-1}\cot 5\right] = 1$

54. **(2)**

$Z_1Z_2 + Z_2Z_3 + Z_3Z_1$

$= Z_1Z_2Z_3\left(\overline{Z}_1 + \overline{Z}_2 + \overline{Z}_3\right)$

$\therefore Z_1Z_2 + Z_2Z_3 + Z_3Z_1 = 1$

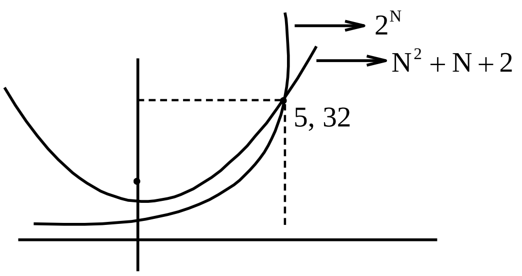
$\therefore Z_1, Z_2, Z_3$ are root of

$Z^3 - Z^2 + Z - 1 = 0$ root of will be

$-i, 1, i$

Hence $Z_1 = -i;\ Z_2 = 1,\ Z_3 = i$

55. **(5)**



$P(\text{O heads}) = \left(\dfrac{1}{2}\right)^N$

$$P(1\text{head})=\frac{1}{2}\left(\frac{1}{2}\right)^{N-1}\left({}^{N}C_1\right)=N\left(\frac{1}{2}\right)^{N}$$

$$P(2\text{ head})=\left(\frac{1}{2}\right)^{2}\left(\frac{1}{2}\right)^{N-2}\left({}^{N}C_2\right)=\frac{N(N-1)}{2}\left(\frac{1}{2}\right)^{N}$$

$$\left(\frac{1}{2}\right)^{N}+N\left(\frac{1}{2}\right)^{N}+\frac{N(N-1)}{2}\left(\frac{1}{2}\right)^{N}=\frac{1}{2}$$

$$N^2+N+2=2^N\Rightarrow N=5$$

56. **(2)**

$$f(0)=f(4)$$

$$f(0)=f(10)$$

$$f(10-2-(-x))=f(2-x)=f(2+x)$$

$$f(8+x)=f(2+x);\ \text{period } 6$$

$$\therefore f(0)=f(6)=f(12)=f(18)=f(24)=f(30)=f(36)$$

$$f(4)=f(10)=f(16)=f(22)=f(28)=f(34)=f(40)$$

$$a=14$$

57. **(3)**

If $y=\frac{x^3+1}{2}=f(x)$

Then $f^{-1}(x)=\sqrt[3]{2x-1}$

$$\therefore f(x)=f^{-1}(x)=x$$

$$\frac{x^3+1}{2}=x$$

$$x\in\left\{1,\frac{-1\pm\sqrt{5}}{2}\right\}$$

58. **(5)**

$$I_n=2\int_0^1 x\left(1+\frac{x^2}{2}+\frac{x^4}{4}+\ldots\ldots+\frac{x^{2n}}{2^n}\right)dx$$

$$\left\{\int_{-1}^{1}(\text{odd})=0\right\}$$

$$I_n=2\left[\frac{x^2}{1.2}+\frac{x^4}{2.4}+\frac{x^6}{4.6}+\ldots..+\frac{x^{2n+2}}{2n(2n+2)}\right]_0^1$$

$$=2\left[\frac{1}{1.2}+\frac{1}{2.4}+\ldots..+\frac{1}{2n(2n+2)}\right]$$

$$=1+\frac{1}{2}\left[\left(1-\frac{1}{2}\right)+\left(\frac{1}{2}-\frac{1}{3}\right)+\ldots..+\left(\frac{1}{n}-\frac{1}{n+1}\right)\right]$$

$$I_n = 1 + \frac{1}{2}\left(1 - \frac{1}{n+1}\right)$$

$$\lim_{n\to\infty} I_n = \frac{3}{2}$$

59. **(9)**

$$I = \int_0^1 \frac{\sin^{-1}\sqrt{x}}{x^2 - x + 1}dx = \int_0^1 \frac{\sin^{-1}\sqrt{1-x}}{(1-x)^2 - (1-x) + 1}dx$$

$$= \int_0^1 \frac{\sin^{-1}\left(\sqrt{1-x}\right)}{x^2 - x + 1}dx = \int_0^1 \frac{\cos^{-1}\sqrt{x}}{x^2 - x + 1}\,dx$$

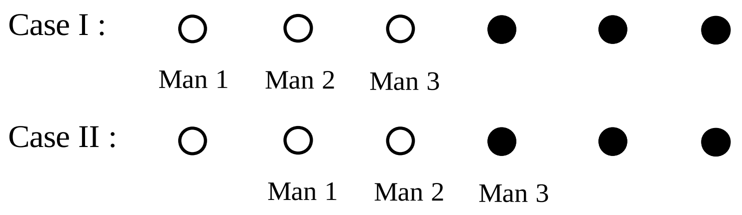
$$2I = \int_0^1 \frac{\pi/2}{x^2 - x + 1}dx$$

On simplifying

$$I = \frac{\pi}{\sqrt{108}}$$

60. **(3)**

Since the chamber contains 3 bullets, the two possible cases for the shot are



Clearly in 1st case the 3rd man survives

$$P(\text{survive}) = \frac{1}{2}$$