PART (A) : PHYSICS

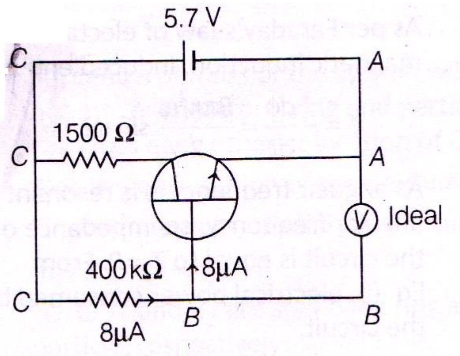
ANSWER KEY

1.	(A)	2.	(A)	3.	(D)	4.	(D)	5.	(B)
6.	(B)	7.	(B)	8.	(C)	9.	(A)	10.	(B)
11.	(A)	12.	(D)	13.	(B)	14.	(A)	15.	(A)
16.	(B)	17.	(C)	18.	(D)	19.	(B)	20.	(C)
21.	(80)	22.	(24)	23.	(12)	24.	(50)	25.	(3)
26.	(7)	27.	(6)	28.	(1)	29.	(33)	30.	(2)

SOLUTIONS

1. (A)

The circuit can be drawn as

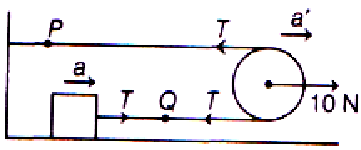


$$V_{BA} = V_B - V_A = (V_B - V_C) + (V_C - V_A)$$

$$= (-8\times10^{-6})(4\times10^{5}) + 5.7 = 2.5V$$

2. (A)

The free body diagram for the given system/ arrangement is as shown below.



Acceleration of P=0

(as attached to wall)

From the FBD of pulley,

$2T = 10$

$\Rightarrow T = 5N$

Limiting friction between block and ground

$= \mu N$

$= \mu mg$

$= 0.2\times1\times10$

$= 2N$

As tension acting on block is greater than $f_{limiting}$, block will move and experience kinetic friction as shown. So $a =$ acceleration of block

$$= \frac{5-2}{1} = 3\,m/s^2 \qquad \text{....(i)}$$

As acceleration of pulley is average of acceleration of ends of thread going over it,

$$a' = \frac{a_P + a_Q}{2} = \frac{0+a}{2} = \frac{3}{2}\,m/s^2 \qquad \text{.....(ii)}$$

Relative acceleration,

$$a_{rel} = a - a' = 3 - \frac{3}{2}$$

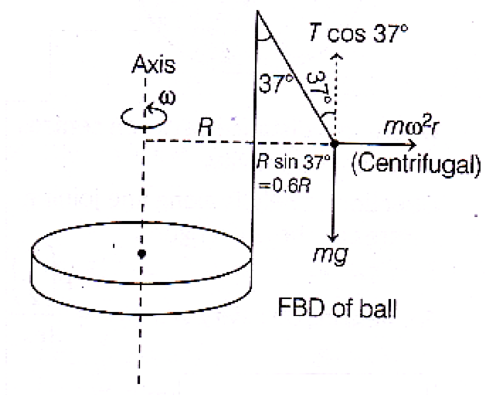
$$= 1.5\,m/s^2 \qquad \text{[from Eqs. (i) and (ii)]}$$

3. (D)

The gate shown here is an exclusive OR(XOR) gate which gives 1 as an output only when one of the inputs only when one of the inputs (i.e. either A or B) is 1.

4. (D)

The given circular platform and ball is as shown below



Distance of ball from axis of rotation,

$$r = R + R\sin 37^\circ = 1.6R = 4.8m \qquad \text{....(i)}$$

From FBD of ball, in

Horizontal: $m\omega^2 r = T\sin 37^\circ$ .....(i)

Vertical: $mg = T\cos 37^\circ$ .....(ii)

Dividing Eq. (i) by Eq. (ii), we get

$$\frac{\omega^2 R}{g} = \tan 37^\circ$$

$$\Rightarrow \omega = \sqrt{\frac{g\tan 37^\circ}{r}}$$

$$= \sqrt{\frac{10\times 0.75}{4.8}} = 1.25\,rad/s$$

5. (B)

Using lens Maker's formula,

$$\frac{1}{f_{concave}}=\left(\frac{3}{2}-1\right)\left(-\frac{1}{10-\frac{1}{15}}\right)=-\frac{1}{12}$$

Similarly,

$$\frac{1}{f_{convex}}=\left(\frac{4}{3}-1\right)\left(\frac{1}{15}+\frac{1}{15}\right)=\frac{2}{45}$$

$$\therefore \frac{1}{f_{eq}}=\frac{1}{f_{mirror}}-2\left(\frac{1}{f_{concave}}+\frac{1}{f_{convex}}\right)$$

$$=-\frac{1}{8}\Rightarrow f_{eq}=-18cm$$

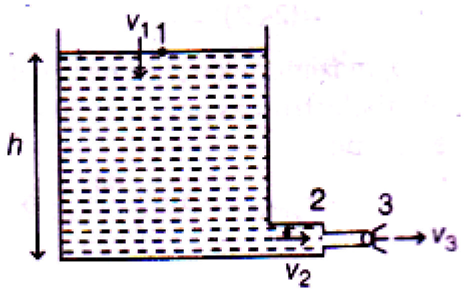
As magnification,

$$m=\frac{f}{f-u}=\frac{-18}{(-18)-(-27)}=-2$$

$$\Rightarrow v_{image}=-m^2v_{object}=-8m/s$$

6. (B)

Consider the figure shown. Let fluid velocities at points $1,2,3$ be v_1, v_2 and v_3 respectively.



Using Bernoulli's theorem between points 1 and 3, as pressures at points 1 and 3 is equal to atmospheric pressure p_0, we get

$$\Rightarrow \frac{1}{2}\rho v_3^2+\rho_0=\frac{1}{2}\rho v_1^2+p_0+\rho gh$$

$$\Rightarrow \frac{1}{2}\rho v_3^2=\frac{1}{2}\rho v_1^2+\rho gh$$

As cross-section of tank is large, v_1 is negligibly small. Therefore ignoring term $\frac{1}{2}\rho v_1^2$, we get

$$\frac{1}{2}\rho v_3^2=\rho gh\Rightarrow v_3=\sqrt{2gh} \qquad .....(i)$$

If A_2 and A_3 are cross-sectional area at points 2 and 3, by continuity equation,

$$A_2v_2=A_3v_3\Rightarrow v_2=\frac{A_3v_3}{A_2}$$

$= \frac{v_3}{\sqrt{2}}\left(\text{given}, A_2 = \sqrt{2}A_3\right)$

$\Rightarrow v_2 = \sqrt{gh}$ ……(iii) [using Eq. (i)]

By Bernoulli's theorem, between points 1 and 2.

$\Rightarrow \frac{1}{2}\rho v_2^2 + p = \frac{1}{2}\rho v_1^2 + p_0 + \rho gh$

$\Rightarrow p - p_0 = \rho gh + \frac{1}{2}\rho v_1^2 - \frac{1}{2}\rho v_2^2$ ….(iii)

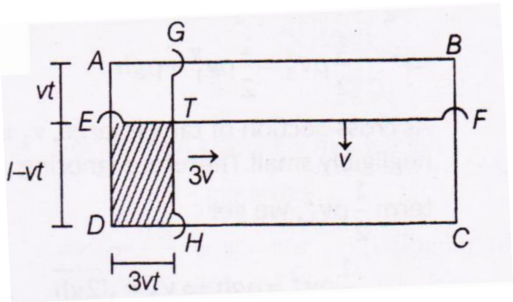
Ignoring $\frac{1}{2}\rho v_1^2$, and putting value of v_2 from Eq. (ii) into Eq. (iii), we get,

Gauge pressure,

$p - p_0 = \rho gh - \frac{1}{2} \Rightarrow \left(\sqrt{gh}\right)^2 = \frac{\rho gh}{2}$

7. (B)

At time t, distances moved by sliders GH and EF will be 3vt and vt respectively as shown



Therefore, area of loop ETHD will be

$A = 3vt(l - vt) = 3vlt = 3v^2t^2$

$\Rightarrow \frac{dA}{dt} = 3vl - 6v^2t$ ……(i)

At the instant, the loop is square, its two sides will be equal. Threfore,

$3vt = l - vt \Rightarrow t = \frac{l}{4v}$ ……(ii)

By Faraday's law, emf induced in the loop

$\varepsilon = \frac{d\phi}{dt} = \frac{d(B_0.A)}{dt} = B_0.\frac{dA}{dt}$

$= B_0\left(3vl - 6v^2t\right)$ [using Eq. (i)]

$= B_0\left[3vl - 6v^2\left(\frac{l}{4v}\right)\right]$ [using Eq. (ii)]

$= \frac{3B_0vl}{2} = 1.5B_0vl$

8. (C)

De-Broglie wavelength λ of electron having kinetic energy K is given by

$$\lambda = \frac{h}{\sqrt{2mK}} \Rightarrow K \propto \frac{1}{\lambda^2}$$

$$\frac{K_1}{K_2} = \left(\frac{\lambda_2}{\lambda_1}\right)^2$$

$$= \left(2\sqrt{2}\right)^2 = 8 \qquad \ldots.(i)$$

Also, maximum kinetic energy of photoelectron as per Einstein's equation.

$$K = E_{photon} - \phi = \frac{hc}{\lambda_{wave}} - \phi$$

$$\Rightarrow K_1 = \frac{12400}{3100} - \phi = 4 - \phi \qquad \ldots..(ii)$$

And $K_2 = \dfrac{12400}{6200} - \phi = 2 - \phi \quad \ldots..(iii)$

Dividing Eq. (ii) by Eq. (iii), we get

$$\frac{K_1}{K_2} = \frac{4-\phi}{2-\phi} = 8 \qquad \text{[using Eq. (i)]}$$

$$\Rightarrow 4 - \phi = 8(2 - \phi)$$

$$\Rightarrow \phi = \frac{12}{7}\,\text{eV} = 1.7\,\text{eV}$$

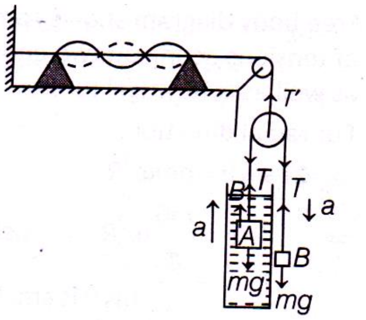
9. (A)

Buoyancy acting on A is equal to

$$B = \rho V g = \rho.\frac{m}{2\rho} - g = \frac{mg}{2} \qquad \ldots.(i)$$

(As specific gravity of block's material is 2, its density will be twice that of water, i.e. 2ρ).

Free body diagram of the two blocks will be as shown.



For block A using $F = ma$, we get

$$T + B - mg = ma$$

$$\Rightarrow T + \frac{mg}{2} - mg = ma \qquad \text{[Using Eq. (i)]}$$

$$\Rightarrow T - \frac{mg}{2} = ma \qquad \ldots\ldots(ii)$$

For block B $mg - T = ma$ …….(iii)

From Eqs. (ii) and (iii), we get

$$T - \frac{mg}{2} = mg - T$$

$$\Rightarrow T = \frac{3mg}{4} \quad …..(iv)$$

From free body diagram of pulley

$$T' = 2T = \frac{3mg}{2} \quad …..(v)$$

[using Eq. (iv)]

Frequency of nth harmonic in string wave is given by

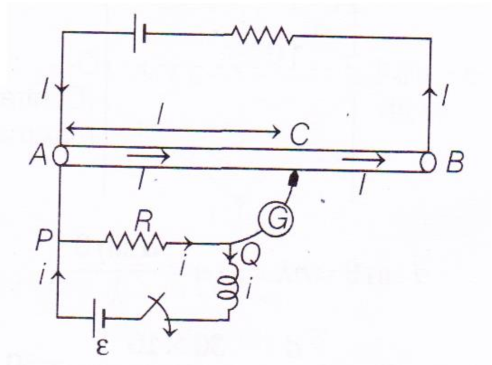
$$f = \frac{n\sqrt{\frac{T}{\mu}}}{2l} \Rightarrow l = \frac{n\sqrt{\frac{T}{\mu}}}{2f}$$

As string is vibrating in 3 loops , so $n = 3$. Also, putting value of tension from Eq. (v), we get

$$l = \frac{3\sqrt{\frac{3mg}{2\mu}}}{2f} = \sqrt{\frac{27mg}{8\mu f^2}}$$

10. (B)

In null deflection state, potential difference across PQ is same as that across AC.



$$\Rightarrow V_{AC} = V_{PQ} \Rightarrow Kl = iR \quad …..(i)$$

Where, k is potential gradient. In L-R circuit, current is given by

$$s\,i = \frac{\varepsilon}{R}\left(1 - e^{-\frac{Rt}{L}}\right) \quad ……(ii)$$

Putting i from Eq. (ii) in Eq.(i), we get

$$kl = \frac{\varepsilon}{R}\left(1 - e^{-\frac{Rt}{L}}\right)R \Rightarrow l = \frac{\varepsilon}{k}\left(1 - e^{-\frac{Rt}{L}}\right)$$

Differentiating w.r.t. time, we get

$$\frac{dl}{dt} = v = \frac{\varepsilon}{k}.\frac{R}{L}e^{-\frac{Rt}{L}}$$

Therefore, velocity is exponentially decreasing as function of time.

11. (A)

As slope of isentropic/ adiabatic is more, so process CA must be isentropic.
Obviously , as volume is constant in AB, it must be isochoric.
As for isothermal process,

$pV = nRT = \text{constant}$

$\Rightarrow p_B V_B = p_C V_C$

$\Rightarrow p_0 V_0 = 3p_0 V_1$

$\Rightarrow V_1 = \frac{V_0}{3}$

For adiabatic process CA,

$pV^\gamma = \text{constant}$

$\Rightarrow p_C V_C^\gamma = p_A V_A^\gamma$

$= 3p_0 \left(\frac{V_0}{3}\right)^\gamma = \frac{p_0}{2} V_0^\gamma$

$\Rightarrow 3^\gamma = 6$

$\Rightarrow \gamma = \log_3 6 = \frac{\ln 6}{\ln 3}$

As adiabatic exponent γ is related to degree of freedom by the relation,

$\gamma = 1 + \frac{2}{f}$

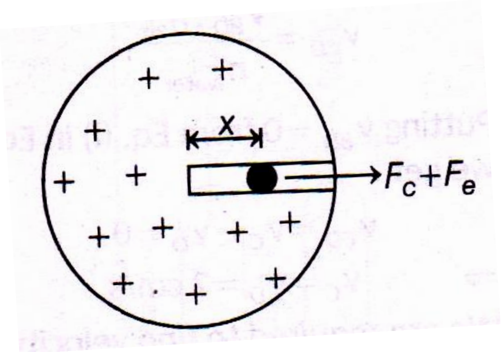
$\Rightarrow 1 + \frac{2}{f} = \frac{\ln 6}{\ln 3}$

$\Rightarrow f = 2.\frac{\ln 3}{\ln 2} \approx 3.16$

So, the answer is closest to π.

12. (D)

The ball experiences centrifugal force F_c and electric force F_e both in radially outward direction.



$\Rightarrow F = F_c + F_e = m\omega^2 x + q.\left(\frac{KQx}{R^3}\right)$

$= \left(m\omega^2 + \frac{KQq}{R^3}\right)x$

$\Rightarrow$ Acceleration relative to sphere

$$a = \left(\omega^2 + \frac{KQq}{mR^3}\right)x$$

$$\Rightarrow v_r \frac{dv_r}{dx} = \left(\omega^2 + \frac{KQq}{mR^3}\right)x$$

Where v_r = radial velocity

Integrating both sides, we get

$$\Rightarrow \int_0^{v_r} v_r\, dv_r = \int_0^R \left(\omega^2 + \frac{KQq}{mR^3}\right)x\, dx$$

$$\Rightarrow \frac{v_r^2}{2} = \frac{\omega^2 R^2}{2} + \frac{KQq}{2mR}$$

$$= \frac{\omega^2 R^2}{2} + \frac{\omega^2 R^2}{2}$$

$$\left(\because \frac{KQq}{R} = m\omega^2 R^2\right)$$

$$\Rightarrow v_r = \sqrt{2}\omega R \qquad \text{.....(i)}$$

At point of leaving, the ball will also be having tangential velocity

$$v_t = \omega R \qquad \text{......(ii)}$$

From Eqs. (i) and (ii), we get

$$v = \sqrt{v_r^2 + v_t^2} = \sqrt{3}\omega R$$

13. (B)

The given circuit can be simplified first as shown in figure-1 and finally as in figure-2.

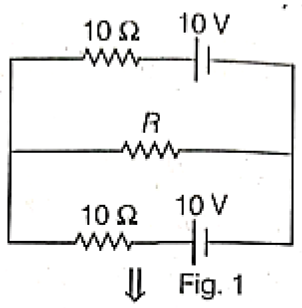


Fig. 1

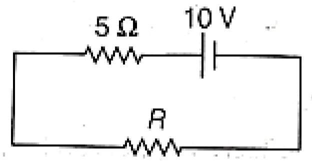


Fig. 2

As per maximum power theorem, for maximum power consumption, *R* should have value equal to resistance of remaining circuit which is 5Ω. As rate of change of resistance is $1\Omega/^{\circ}C$, therefore $\frac{\Delta R}{\Delta T} = 1$.

$$\Rightarrow \frac{5-15}{\Delta T} = 1 \Rightarrow \Delta T = -10^{\circ}\,C$$

$\Rightarrow$ Final temperature of R

$= 50 - 10 = 40^{o}$ C

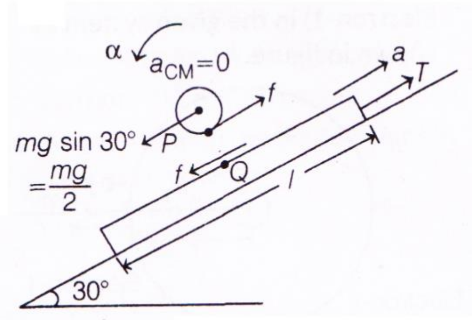
As per newton's law of cooling,

$T_f - T_{surrounding} = \left(T - T_{surrounding}\right)e^{-kt}$

$\Rightarrow 40 - 20 = \left(50 - 20\right)e^{-kt}$

$$\Rightarrow t = \frac{\ln\frac{3}{2}}{k} = \frac{\ln\frac{3}{2}}{\frac{\ln\frac{9}{4}}{100}} = \frac{\ln\frac{3}{2}}{\frac{2\cdot\ln\frac{3}{2}}{100}} = 50\,\text{s}$$

14. (A)



As for cylinder, acceleration of CM is zero,

$f = \frac{mg}{2}$ ....(i)

Using $\tau = I\alpha$ (about CM)for cylinder, we get

$fR = \frac{mR^2}{2}\alpha$

$\Rightarrow \alpha = \frac{2f}{mR} = \frac{2\left(\frac{mg}{2}\right)}{mR} = \frac{g}{R}$ ......(ii)

[using Eq. (i)]

As there is no slipping between contact point P and Q as shown in figure.

$a_P = a_Q$

$\Rightarrow \alpha R = a$

$\Rightarrow a = \frac{g}{R}.R = g$ .....(iii)

Using equation of motion for the plank, we get,

$s = ut + \frac{1}{2}at^2$

$\Rightarrow l = 0 + \frac{1}{2}gt^2$ [using Eq. (iii)]

$\Rightarrow t = \sqrt{\frac{2l}{g}}$ ......(iv)

Angle rotated by the cylinder in same time can be found using

$$\theta = 0 + \frac{1}{2}\left(\frac{g}{R}\right)\left(\sqrt{\frac{2l}{g}}\right)^2 \qquad \text{[using Eqs. (ii) and (iv)]}$$

$$\Rightarrow \theta = \frac{l}{R}$$

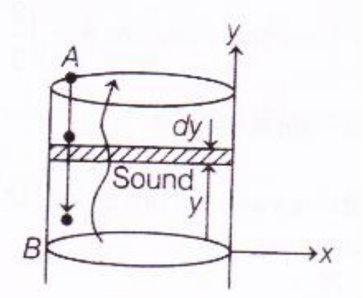
15. (A)

For motion of stone from A to B,

$$s = ut + \frac{1}{2}at^2$$

$$\Rightarrow h = 0 + \frac{1}{2}gt^2$$

$$\Rightarrow t_1 = \sqrt{\frac{2h}{g}} \qquad \text{......(i)}$$



As temperature profile is linear; temperature at height y from water's surface can be found using two point form equation.

$$\frac{T - 9T_0}{T_0 - 9T_0} = \frac{y - 0}{h - 0}$$

$$\Rightarrow T = 9T_0 - \frac{8T_0 y}{h} \qquad \text{......(ii)}$$

As speed of sound is given to be

$v = C\sqrt{T}$ time taken by sound to move through air strip of thickness dy shown is

$$dt = \frac{dy}{v} = \frac{dy}{C\sqrt{T}} = \frac{dy}{C\sqrt{9T_0 - \frac{8T_0}{h}y}}$$

$\Rightarrow$ Time for sound to move from B to A

$$t_2 = \frac{1}{C}\int_0^h \frac{dy}{\sqrt{9T_0 - \frac{8T_0}{h}y}} = \frac{h}{2C\sqrt{T_0}} \qquad \text{......(iii)}$$

From Eqs. (i) and (iii), total time

$$t_{total} = t_1 + t_2 = \sqrt{\frac{2h}{g}} + \frac{h}{2C\sqrt{T_0}}$$

16. (B)

Path difference between waves

$\Delta x = (S_2P - S_1P)n + t(n-1)$

$= \frac{dy}{D}.n + t(n-1)$

As for point $P, y = \frac{d}{2}$

$\Rightarrow \Delta x = \frac{d^2 n}{2D} + t(n-1)$

As for maxima, Δx is integral multiple of λ, therefore

$N\lambda = \frac{d^2 n}{2D} + t(n-1)$

$\Rightarrow \lambda_{max} = \frac{d^2 n}{2D} + t(n-1)$

$= \frac{(2\times10^{-3})^2 \times 1.4}{2\times10}$

$+0.2\times10^{-6}(1,4-1)$

$= 2.8\times10^{-7} + 0.8\times10^{-7}$

$= 3.6\times10^{-7}\,m$

$\therefore x = 3.6$

17. (C)

To find flux of wire through the base consider a ring element of radius r and thickness dr as shown in figure 1. Out of x and y components flux will only be due to x component of field equal to

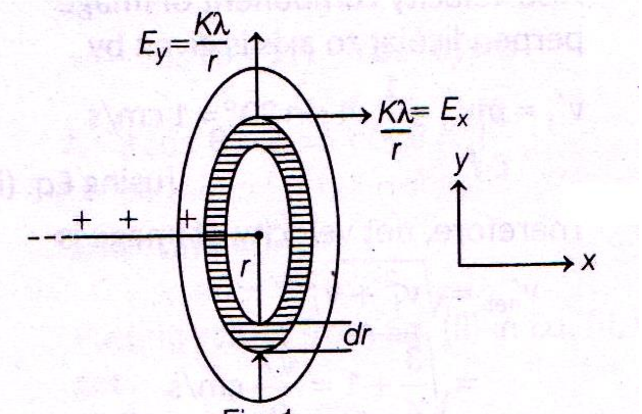


Fig. 1

$d\phi = EdA = \frac{K\lambda}{r}.2\pi r dr = 2\pi k\lambda dr$

$\therefore$ Flux through base will be

$\phi = \int EdA = 2\pi k\lambda \int_0^R dr = 2\pi k\lambda R$

$\Rightarrow \phi = \frac{\lambda R}{2\varepsilon_0}$ ......(i) $\left(\text{as}, k = \frac{1}{4\pi\varepsilon_0}\right)$

Also, flux due to point charge Q placed on axis of a disc (as in figure 2) is given by

$$\phi' = \frac{Q}{2\varepsilon_0}\left(1-\cos 60^\circ\right) = \frac{Q}{4\varepsilon_0} \quad \text{.....(ii)}$$

As, net flux through base is given zero, flux due to line charge must be cancelling the flux due to point charge

$$\Rightarrow \phi = \phi'$$

$$\Rightarrow \frac{\lambda R}{2\varepsilon_0} = \frac{Q}{4\varepsilon_0} \quad \text{[using Eqs. (i) and (ii)]}$$

$$Q = 2\lambda R$$

Let ϕ'' be flux to line charge through the curved surface. As the line charge is completely outside the cone, its flux through cone is zero as per Gauss's law

$$\Rightarrow \phi'' + \phi = 0$$

$$\Rightarrow |\phi''| = |\phi| = \frac{\lambda R}{2\varepsilon_0} \quad \text{.....(iv)}$$

[using Eq. (i)]

As in Figure, E due to point charge is perpendicular to area vector dA of element on curved surface, so flux of point charge

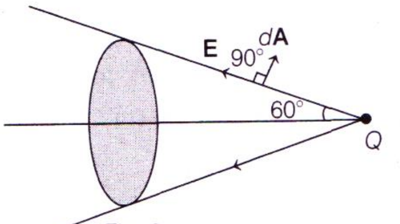


Fig. 2

$$\phi'' = \int E.dA = 0 \quad \text{......(v)}$$

$$\Rightarrow \text{Net flux} = \phi'' + \phi''' = \frac{\lambda R}{2\varepsilon_0} = \frac{Q}{4\varepsilon_0}$$

[Using Eqs. (iii), (iv) and (v)]

18. (D)

Time taken to pass one spoke

$$t = \frac{\theta}{\omega} = \frac{\pi}{4\omega} \quad \text{......(ii)}$$

$\Rightarrow$ Velocity of arrow will be

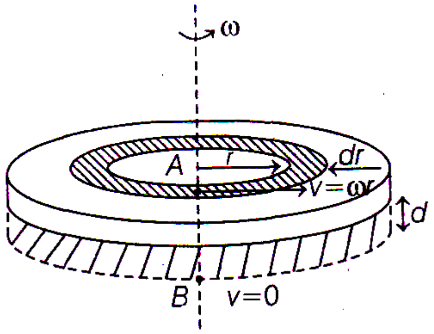
$$v = \frac{\text{Distance}}{\text{Time}} = \frac{l}{t} = \frac{l}{\frac{\pi}{4\omega}} = \frac{4\omega l}{\pi}$$

19. (B)

Viscous force on a solid in contact with liquid is given by

$$f = \eta A\frac{dv}{dy} \text{ or } \eta A\frac{\Delta y}{\Delta y} \quad \text{.......(i)}$$

So, if we consider a ring element of radius r and thickness dr as shown in the figure, for a disc paticle (as A) velocity is ωr while particle of fixed surface (as B) is at rest.



So, $\Delta v = \omega - 0 = \omega r$ .....(ii)

Also, difference in height of A and B

$\Delta y = d$ ....(iii)

And surface area of ring element

$dA = 2\pi r dr$ ....(iv)

Using Eqs. (i), (ii), (iii) and (iv),

Torque on element $= fr = \eta A \frac{\Delta v}{\Delta y} r$

$$= \eta.2\pi r dr.\frac{\omega r}{d}.r$$

$$\Rightarrow \text{Torque on disc } = \int_0^R \eta.2\pi r\, dr.\frac{\omega r}{d}.r$$

$$= \frac{2\pi\eta\omega}{d}\int_0^R r^3 dr$$

$$= \frac{\pi\eta\omega R^4}{2d}$$

20. (C)

For thin lens, object distance (u) and image distance (v) are related as

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

Putting , $u = -OA$ and $v = OB$ (as given by the figure), we get

$$\frac{1}{f} = \frac{1}{OB} - \frac{1}{-OA} = \frac{1}{OB} + \frac{1}{OA} \quad ....(i)$$

From $\Delta OCA, OA = \frac{L}{\tan\theta} \approx \frac{L}{\theta}$ .....(iii)

(as θ is very small for paraxial rays)

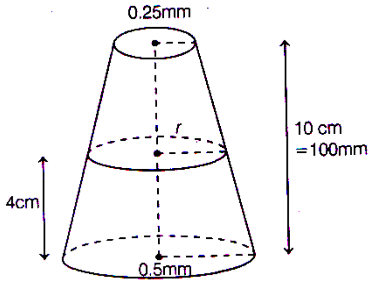
Similarly, from $\Delta OCB, OB = \frac{1}{2\theta}$ ....(iii)

Putting values of OA and OB from Eqs. (ii) and (iii) into Eq. (i), we get

$$\frac{1}{f} = \frac{2\theta}{L} + \frac{\theta}{L} = \frac{3\theta}{L} \Rightarrow f = \frac{L}{3\theta}$$

21. (80)

The situation can be shown as.



Decrease in radius(as we move up) per unit length

$$\frac{\Delta r}{\Delta l} = \frac{\text{Decrease in radius}}{\text{Change in length}}$$

$$= \frac{0.25\,\text{mm}}{100\text{mm}} = \frac{1}{400} \qquad \text{.....(i)}$$

$\therefore$ Decrease in radius as we move up by 4 cm

$$= \frac{\Delta r}{\Delta l} \times 4\text{cm} = \frac{1}{400} \times 40\,\text{mm} \text{ [using Eq. (i)]}$$

$$= 0.1\text{mm}$$

$\therefore$ Radius at height 4cm is equal to

$$r = 0.5\text{mm} - 0.1\text{mm} = 0.4\text{mm} \qquad \text{.....(ii)}$$

By ascent formula, the liquid height in capillary is given by

$$h = \frac{2T\cos\theta}{\rho r g}$$

$$\Rightarrow T = \frac{\rho r g h}{2\cos\theta}$$

$$\Rightarrow T = \frac{10^3 \times 0.4 \times 10^{-3} \times 10 \times 4 \times 10^{-2}}{2 \times \cos 0^\circ}$$

$$\Rightarrow T = 8 \times 10^{-2}\,\text{N/m} = 80\,\text{N/km}$$

22. (24)

Energy required to disassemble a solid sphere is equal to magnitude of its self energy given by

$$E = \frac{3}{5}\frac{GM^2}{R} \qquad \text{.....(i)}$$

Also, acceleration due to gravity at a height h above surface is

$$g = \frac{CM}{(R+h)^2}$$

$$\Rightarrow g = \frac{GM}{4R^2} \qquad (\because h = R) \qquad \text{......(ii)}$$

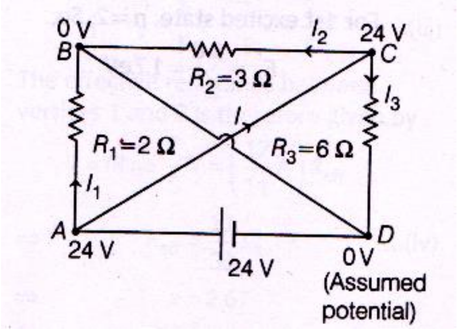
Dividing Eq. (i) by Eq. (ii), we get

$$\frac{E}{g} = \frac{12MR}{5} \Rightarrow E = 2.4MgR$$

$\therefore x = 2.4$

23. (12)

Assume potential at point D is equal to zero. As we know potential of all points on a plane wire are equal, so potential at B will also be equal to 0 V. Also potential difference across cell is 24 V, so potential at A will be 24 V. Similarly, potential at C is also 24 V.



Using Ohm's law for branch AB,

$$I_1 = \frac{V_A - V_B}{R_1}$$

$$= \frac{24-0}{2} = 12A$$

For branch CB,

$$I_2 = \frac{V_C - V_B}{R_2} = \frac{24-0}{3} = 8A$$

For branch CD,

$$I_3 = \frac{V_C - V_D}{R_3} = \frac{24-0}{6} = 4A$$

By KCL for junction C,

$$I = I_2 + I_3 = 8 + 4 = 12A$$

24. (50)

As, $dS = \frac{dQ}{T}$

$\Rightarrow dQ = T.dS$

$\Rightarrow Q = \int T.dS = \text{Area under } T - S \text{ graph}$

$= T_0S_0$

As for cyclic process, internal energy change (ΔU) is zero, so by first law of thermodynamics.

$W = Q - \Delta U = Q = T_0S_0$

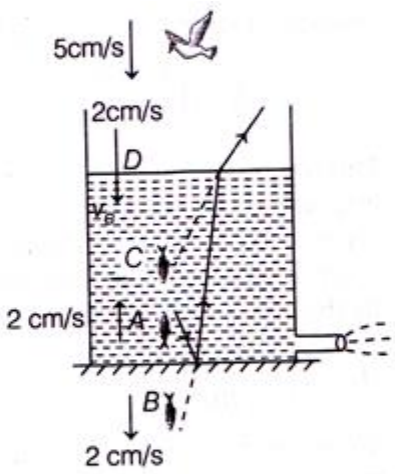
Also, heat input (H)= Sum of positive $Q_S = +2T_0S_0$

$\therefore$ Efficiency,

$$\eta = \frac{W}{H} \times 100 = \frac{T_0S_0}{2T_0S_0} \times 100 = 50\%$$

25. (3)

In the diagram shown, A is object(fish), B is image in mirror , C is final fish's image formed after refraction, D is water air interface.



So, relative velocity of B w.r.t. interface D,

$V_{BD} = V_B - V_D = 2 - 2 = 0$

As, in refraction at plane surface,

$$v_{image} = \frac{v_{object} n_r}{n_i}$$

Here velocities are relative to interface, n_r is refractive index of medium of refraction and n_i is refractive index of medium of incidence.

$\Rightarrow$ Velocity of C w.r.t. D,

$$v_{CD} = \frac{v_{BD} . n_{air}}{n_{water}}$$

Putting $v_{BD} = 0$ from Eq. (i) in Eq. (ii), we get

$v_{CD} = v_C - v_D = 0$

$\Rightarrow v_C = V_D = 2\,cm/s$ ....(iii)

We are required to find velocity of final image as observed by the bird which will be relative velocity equal to

$v_{C,Bird} = v_C - v_{Bird}$

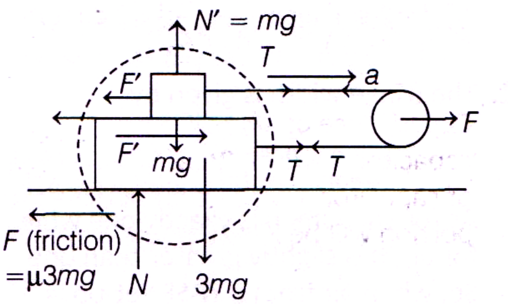
$= (-2) - (-5)$

$= 3cm/s$

26. (7)

Consider FBD of the system shown. As the blocks move together, common acceleration of the blocks is

$$a = \frac{F_{Net}}{m_{Net}} = \frac{F - \mu(3mg)}{3m} \quad .....(i)$$



From FBD of pulley, we get

$2T = F \Rightarrow T = \frac{F}{2}$ .....(ii)

As F is the maximum force for which there is no slipping between the blocks, friction between the blocks will be maximum static/ limiting friction. So, from FBD of block P, using Newton's 2 nd law, we get

$\frac{F}{2} - 5\mu mg = ma \qquad (as, F' = 5\mu mg)$

$\Rightarrow F = 2ma + 10\mu mg$ ......(iii)

Putting value of F from Eq. (iii) in Eq.(i), we get

$a = \frac{2ma + 10\mu mg - 3\mu mg}{3m}$

$\Rightarrow 3ma = 2ma + 7\mu mg$

$\Rightarrow a = 7\mu g = x\mu g(\text{given})$

$\therefore x = 7$

27. (6)

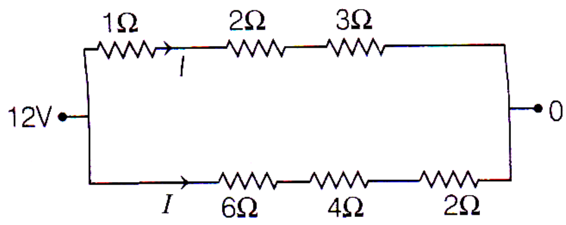


Fig. 1

In steady state, current through capacitor is zero while current in other branches will be as shown in Figure 1

As resistors in upper branch are in series, so

$I = \frac{V}{R} = \frac{12 - 0}{1 + 2 + 3} = 2A$

Similarly, current in lower branch

$I' = \frac{V}{R'} = \frac{12 - 0}{6 + 4 + 2} = 1A$

Now, using Ohm's law for 1Ω resistor (refer figure.2)

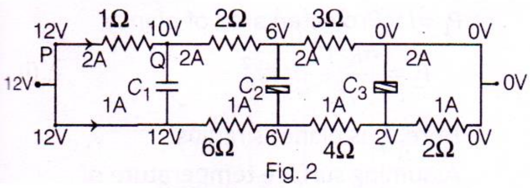


Fig. 2

$V_P - V_Q = I \times R$

$\Rightarrow 12 - V_Q = 2 \times 1 \Rightarrow V_Q = 10V$

Similarly, using Ohm's law, potentials of different points will be as shown in Figure. 2

Also, if capacitance of capacitor-1 is

$C_1 = C$ .....(i)

Capacitance of air portion of capacitor-2 will be 2C as width is half of capacitor -1, while that of dielectric portion will be $K \times 2C$ equal to 6C, as in

Figure.2 As air and dielectric are in series effective capacitance will be

$$\frac{1}{C_2} = \frac{1}{2C} + \frac{1}{6C}$$

$\Rightarrow C_2 = 1.5C$ ......(ii)

Also, for capacitor -3,

$C_3 = KC = 3C$ ......(iii)

From Figure.2, potential difference across C_1, C_2, C_3 respectively are

$V_1 = |12 - 10| = 2$ ......(iv)

$V_2 = 6 - 6 = 0$ ......(v)

$V_3 = |2 - 0| = 2$ .....(vi)

Using Eqs, (i) to (vi), ratio of charges

$Q_1 : Q_2 : Q_3 = C_1V_1 : C_2V_2 : C_3V_3$

$= C \times 2 : 1.5C \times 0 : 3C \times 2$

$= 2 : 0 : 6 \Rightarrow y = 6, x = 0$

$\therefore y - x = 6$

28. (1)

Angular frequency of source,

$\omega = 100\pi$

Inductive reactance of solenoid

$$X_L = \omega L = 100 \times \times \frac{1}{25\pi} = 4\Omega$$

Impedance of solenoid,

$$Z = \sqrt{X_L^2 + R^2} = \sqrt{4^2 + 3^2} = 5\Omega$$

Maximum current in solenoid

$$= \frac{V_{max}}{Z} = \frac{5}{5} = 1A$$

$\therefore i = 1.\sin(100\pi t + \phi)$

Emf induced in rig $\varepsilon = |M - L| \dfrac{di}{dt}$

$= \dfrac{1}{50\pi}.100\pi\cos(100\pi t + \phi)$

$= 2\cos(100\pi t + \phi)$

$\Rightarrow \varepsilon_{max} = 2$

$\therefore$ Maximum current $= \dfrac{\varepsilon_{max}}{R} = \dfrac{2}{2} = 1A$

29. (33)

It is given that , $\rho^2 U^3 =$ constant

$\Rightarrow \left(\dfrac{m}{v}\right)^2 . \left(\dfrac{f}{2}nRT\right)^3 = \text{constant}$

Here, m is mass and f is degree of freedom

$\Rightarrow \dfrac{T^3}{V^2} = \text{constant}$

$\Rightarrow \dfrac{(\rho V)^3}{V^2} = \text{constant} \left(\text{As}, T = \dfrac{pV}{nR}\right)$

$\Rightarrow pV^{\frac{1}{3}} = \text{constant}$

So, polytropic index of process $(\gamma) = \dfrac{1}{3}$

As for polytropic process

$pV^x = \text{constant}$

Molar specific heat is given by

$C = \dfrac{f}{2}R + \dfrac{R}{1-\gamma}$

Therefore, as f of CO_2 is 5, we get

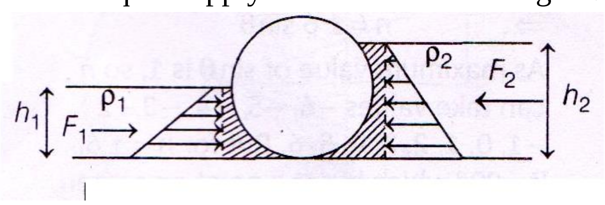
$C = \dfrac{5}{2}R + \dfrac{R}{1-\dfrac{1}{3}} = 4R$

$= 4 \times 8.31$

$= 33.24$

30. (2)

The two liquids apply forces as shown in figure.



For equilibrium, $F_1 = F_2$

$$\Rightarrow \left(\rho_{average}\right)_1 \times A_1 = \left(\rho_{average}\right)_2 \times A_2$$

$$\Rightarrow \frac{\rho_1 g h_1}{2} \times h_1 \times l = \frac{\rho_2 g h_2}{2} \times h_2 \times l$$

Here, l is length of cylinder.

$$\Rightarrow h_2 = \sqrt{\frac{\rho_1}{\rho_2}} h_1 = \sqrt{\frac{1.69}{1}} \times 1.6 = 2.08$$

PART (B) : CHEMISTRY

ANSWER KEY

31.	(B)	32.	(A)	33.	(D)	34.	(C)	35.	(C)
36.	(B)	37.	(D)	38.	(C)	39.	(D)	40.	(C)
41.	(B)	42.	(D)	43.	(B)	44.	(C)	45.	(A)
46.	(A)	47.	(D)	48.	(D)	49.	(C)	50.	(D)
51.	(3)	52.	(28)	53.	(500)	54.	(3)	55.	(5)
56.	(290)	57.	(6)	58.	(6)	59.	(5)	60.	(12)

SOLUTIONS

31. (B)

$\lambda_A = \frac{h}{m_A \upsilon_A}$ and $\lambda_B = \frac{h}{m_B \upsilon_B}$

$\frac{\lambda_A}{\lambda_B} = \frac{m_B \upsilon_B}{m_A \upsilon_A}$ ....(i)

Given $m_B = \frac{m_A}{4} \Rightarrow \upsilon_B = \frac{3}{4}\upsilon_A$

By putting there values in Eq. (i) we get

$$\frac{1\times10^{-10}}{\lambda_B} = \frac{m_A \times 3\upsilon_A}{m_A \times 4 \times \upsilon_A \times 4}$$

$$\lambda_B = \frac{16\times10^{-10}}{3} = 5.33\overset{0}{A}$$

32. (A)

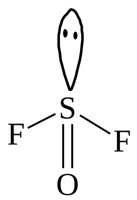
$$\text{Size} \propto \frac{1}{Z_{eff} \text{ or nuclear charge}}$$

Thus, the correct order of size is

$S^{2-} > Cl^- > Na^+ > Mg^{2+} > Be^{2+}$

33. (D)

OSF_2 has pyramidal shape.



34. (C)

$$\frac{(\upsilon_{av})_1}{(\upsilon_{av})_2}=\sqrt{\frac{T_1}{T_2}}$$

Given $T_1=150+273=423K$

$T_2=50+273=323K$

$$\therefore \frac{(\upsilon_{av})_1}{(\upsilon_{av})_2}=\sqrt{\frac{T_1}{T_2}}=\sqrt{\frac{423}{323}}=\frac{1.14}{1}$$

35. (C)

$\because \Delta G=\Delta H-T.\Delta S$

For a spontaneous reaction ΔG should be negative

$\Delta H=-238kJ,\ \Delta S=-87\ JK^{-1}$

Hence, reaction will be spontaneous when $\Delta H>T.\Delta S$. Therefore, at 1000 and 1500 K the reaction would be spontaneous.

36. (B)

pH = 5 $[H^+]=10^{-5}$ molL^{-1}

On diluting the solution 100 times

$$[H^+]=\frac{10^{-5}}{100}=10^{-7}\ molL^{-1}$$

Total H^+ ion concentration = H^+ Ions from acid + H^+ ions from water

$[H^+]=10^{-7}+10^{-7}=2\times10^{-7}M$

pH = - $\log[2\times10^{-7}]$

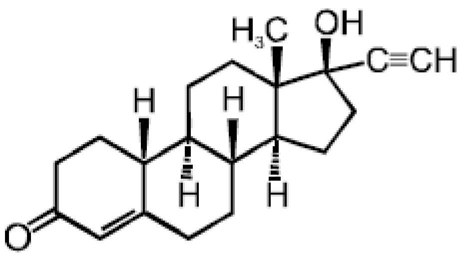
pH = 7 – 0.3010 = 6.699

37. (D)

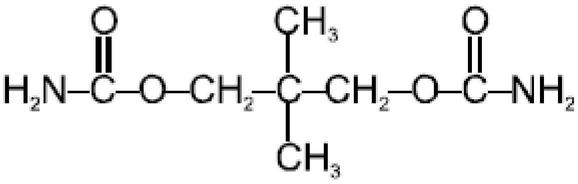
$$Na_2O_2+Dil.H_2SO_4\rightarrow Na_2SO_4+H_2O_2$$

38. (C)

Fact and information based.



Norethindrone



Equanil

39. (D)

At anode

$$Fe^{2+}(aq) \rightarrow Fe^{3+}(aq) + e^-; E^0_{oxi} = -0.77V$$

At cathode, $Ag^+ + e^- \rightarrow Ag(s); E^0_{red} = 0.80V$

Overall cell reaction

$$Fe^{2+}(aq) + Ag^+(aq) \rightarrow Fe^{3+}(aq) + Ag(s)$$

$$E^0_{cell} = E^0_{oxi} + E^0_{red} = -0.77 + 0.80 = 0.03V$$

$$Q_C = \frac{[Fe^{3+}][Ag]}{[Fe^{2+}][Ag^+]} = \frac{[Fe^{3+}]}{[Fe^{2+}][Ag^+]} = \frac{0.20}{(0.10)(1)} = 2$$

Using Nernst equation

$$E_{cell} = E^0_{cell} - \frac{0.0591}{n}\log K_C$$

$$E_{cell} = 0.03 - \frac{0.0591}{1}\log 2 = 0.0122V$$

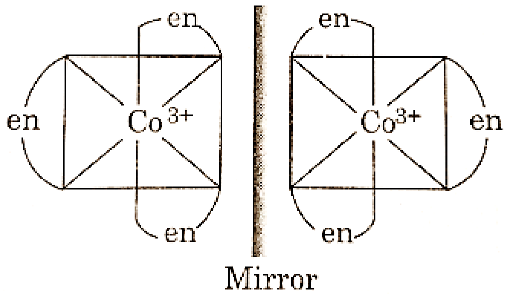
40. (C)

41. (B)

In the metallurgy of aluminium (Al) graphite anode is oxidised to carbon monoxide (CO) and carbon dioxide (CO_2)

42. (D)

$[Co(en)_3]Cl_3$, i.e. $[Co(en)_3]^{3+}$



Cis$[Co(en)_2Cl_2]Cl$, i.e. cis $[Co(en)_2Cl_2]^+$

43. (B)

D-Fructose and D-mannose give the same osazone as D-glucose.

The difference in these sugars present on the first and second carbon atoms are marked when osazone crystals are formed.

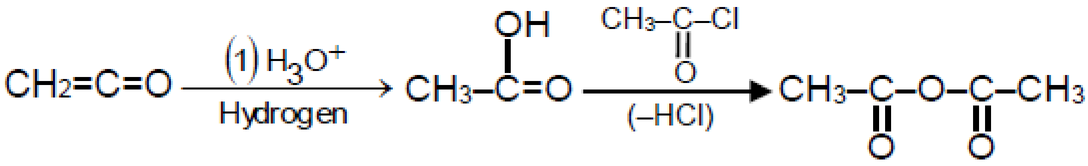
44. (C)

Group $-\overset{\overset{\large O}{||}}{C}-CH_3$ gives iodoform and 2,4-DNP test.

Esters have fruity smell and do not respond with sodium metal and $NaHCO_3$.

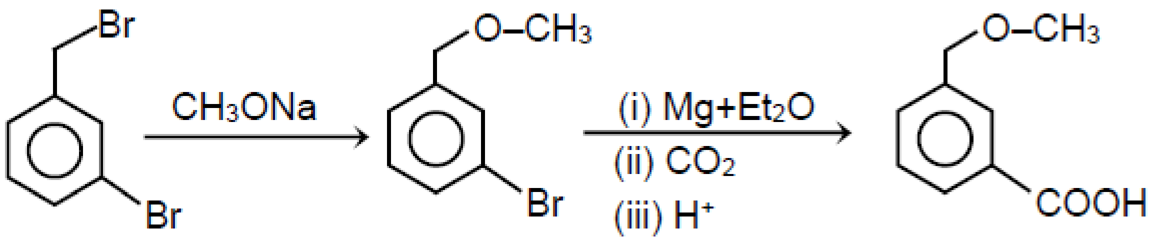
45. (A)
Na, EtOH is used to reduce ester and aldehyde group but not amide group while LiAlH4 reduces all the three groups.

46. (A)

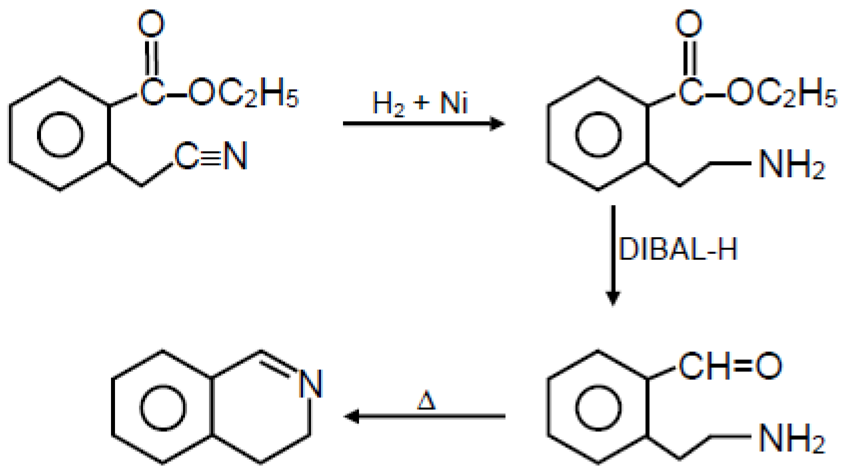


47. (D)
a = L.P is localized (sp^3 - N)
b = L.P is localized (sp^2 - N)
c, d, e = L.P. is delocalized (sp^2 - N)

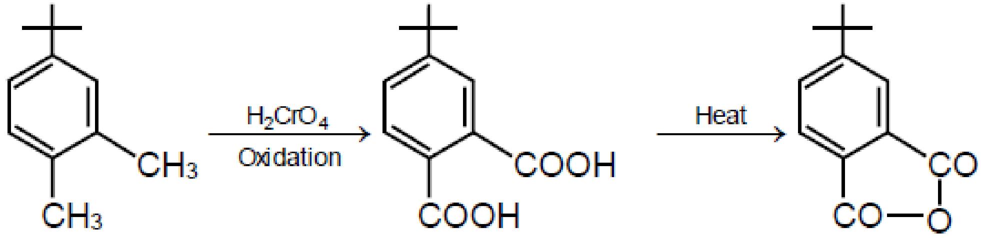
48. (D)



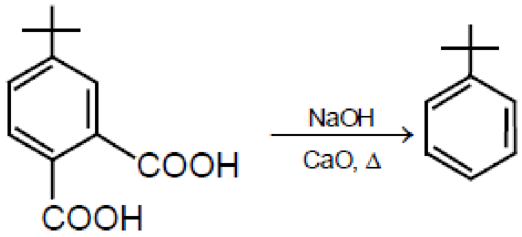
49. (C)



50. (D)



($C_{12}H_{12}O_3$)



(C_9H_{14})

51. (3)

According to first order reaction, $-d[N_2O_5]/dt = k[N_2O_5]$.

The half-life period for first-order reaction is

$$t_{1/2} = \frac{\ln 2}{k} = \frac{0.693}{k}$$

So, $t_{1/2}$ is independent of initial concentration of the reactant.

With increase in temperature, k increases (according to Arrhenius equation), $t_{1/2}$ decreases.

If the reaction completes to 99.61% completion, then,

$$t = \left(\frac{2.303}{k}\right)\log\frac{a}{a-x}$$

Where, $x = 100$ and $(a-x) = (100-99.61) = 0.39$

$$t_{99.61} = \left(\frac{2.303}{k}\right)\log\frac{100}{0.39}$$

$$= \left(\frac{2.303}{k}\right)2.41$$

$$= 8\times\frac{0.693}{k} = 8\times t_{1/2}$$

∴ Statement (a), (b) and (d) are correct.

52. (28)

$$\underset{2x=6}{AS_2^{3+}} \rightarrow \underset{2x=10}{2As^{5+}} + 4e^-$$

$$\underset{3x=-6}{S_3^{2-}} \rightarrow \underset{\substack{3x-24=-6\\3x=18}}{3SO_4^{2-}} + 24e^-$$

n-factor = 28

53. (500)

54. (3)

O_2^{2-} (Total numbers of electron =18)

$\sigma 1s^2, \sigma^* 1s^2, \sigma 2s^2, \sigma * 2s^2, \sigma 2p_z^2, \pi 2p_x^2 \approx \pi 2py^2, \overset{*}{\pi} 2px^2 \approx \overset{*}{2} py^2$

Unpaired electron = 0

B_2(Total number of electron = 10)

$\sigma 1s^2, \sigma^* 1s^2, \sigma 2s^2, \sigma^* 2s^2, \pi 2p_x^2 \approx \pi 2py^2$

Unpaired electron = 2

N_2^+ (Total number of electron = 13)

$\sigma 1s^2, \sigma^* 1s^2, \sigma 2s^2, \sigma^* 2s^2, \pi 2p_x^2 \approx \pi 2py^2, \sigma 2p_z^1$

Unpaired electron =1

O_2 (Total number of electron =16)

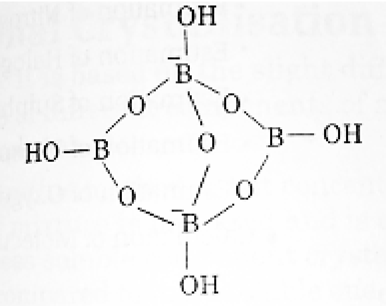
$\sigma 1s^2, \sigma^* 1s^2, \sigma 2s^2, \sigma^* 2s^2, \sigma 2p_x^2, \pi 2p_z^2 \approx \pi 2py_y^2, \overset{*}{\pi} 2p_x^1 \approx \pi \overset{*}{2} p_y^1$

Unpaired electron = 2

And hence, there are 3 ion/molecule which contain unpaired electron.

55. (5)

Borax contains the tetrahedral unit, i.e. $[B_4O_5(OH)_4]^{2-}$ and its structure can be shown as



56. (290)

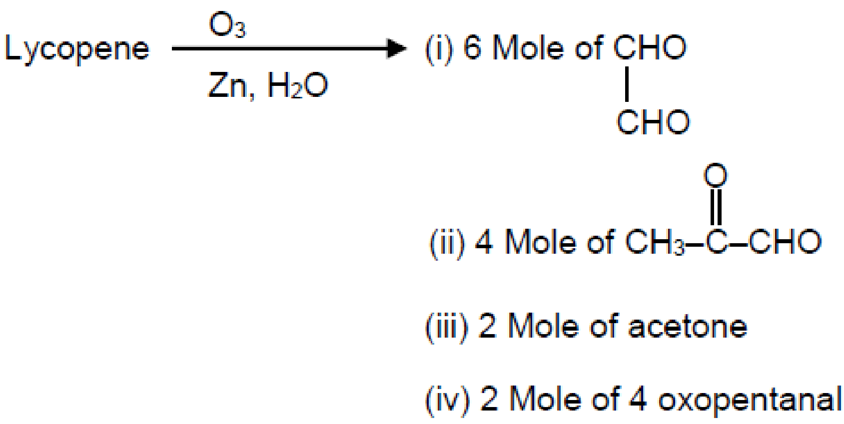
For NaCl crystal

Radius of cation/radius of anion = 0.414

$$\frac{r_{A^+}}{r_{B^-}} = 0.414$$

$$r_{B^-} = \frac{r_{A^+}}{0.414} = \frac{120}{0.414} = 289.9 \simeq 290\text{pm}$$

57. (6)



6 Mole of glyoxal gives 6 Moles of glycolic acid
Only glycol undergoes intra molecular cannizzaro reaction to produce glycolic acid

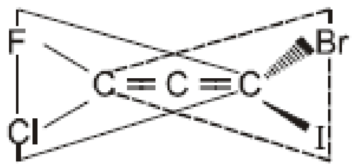
58. (6)

Possible sequences of the tetrapeptide are :

(A) Val-Ser-Gly-Ala (B) Val-Gly-Ser-Ala
(C) Ser-Val-Gly-Ala (D) Gly-Ser-Val-Ala
(E) Ser-Gly-Val-Ala (F) Gly-Val-Ser-Ala

59. (5)

Maximum five atoms can be in one plane.



60. (12)

According to Raoult's law,

$$p_t = p_X^\circ x_X^\circ + p_Y^\circ x_Y^\circ$$

In case of equimolar binary solution of X and Y,

$$50 = p_X^\circ \times \frac{1}{2} + p_Y^\circ \times \frac{1}{2}$$

$\Rightarrow \quad p_X^\circ + p_Y^\circ = 100$

$\Rightarrow \quad p_Y^\circ = 100 - p_X^\circ$

$\Rightarrow \quad p_Y^\circ = 100 - 20$

$\qquad = 80 \text{ torr}$

Now, a new solution has total vapour pressure of 25 torr.

$$25 = 20 \times x_X + 80 \times x_Y$$

As $\quad x_X + x_Y = 1$

$\Rightarrow \quad x_Y = 1 - x_X$

$\Rightarrow \quad 25 = 20 \times x_X + 80(1 - x_X)$

$\Rightarrow \quad x_X = 0.92$

$\Rightarrow \quad x_Y = 0.08$

So, $\frac{x_X}{x_Y} = \frac{0.92}{0.08} = 11.5 \simeq 12$

PART (C) : MATHEMATICS

ANSWER KEY

61.	(B)	62.	(A)	63.	(D)	64.	(D)	65.	(D)
66.	(B)	67.	(B)	68.	(A)	69.	(C)	70.	(D)
71.	(B)	72.	(B)	73.	(A)	74.	(C)	75.	(C)
76.	(A)	77.	(C)	78.	(D)	79.	(C)	80.	(C)
81.	(2)	82.	(5)	83.	(9)	84.	(210)	85.	(9)
86.	(3)	87.	(3)	88.	(5)	89.	(8)	90.	(4)

SOLUTIONS

61. (B)

$P \cap Q$ = 9 elements or 10 elements

(the elements not common in both will have 3 options. It's either in P or in Q or not in both)

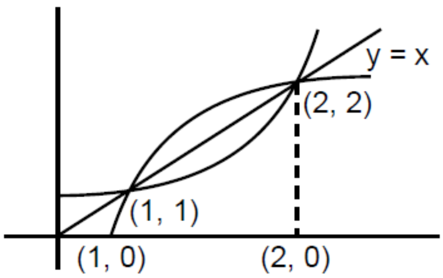
$\therefore$ n(E) = ${}^{10}C_9.(3)^1 + {}^{10}C_{10}(1)^{10} = 30 + 1 = 31$

$\therefore$ P(E) = $\frac{31}{4^{10}} = \frac{31}{2^{20}}$

62. (A)

The curve and inverse with intersect on y = x only.

Area $= 2\left|\int_1^2 x\,dx - \int_1^2 \left(x^2 - 2x + 2\right)dx\right| = \frac{1}{3}$ square.



63. (D)

Number of arrangements in which 2 are identical of one kind, two identical of another kind and one letter different from the remaining two letters is $2C_1 \times \frac{5!}{(2!)^2} = 60.$

Number of arrangements in which 2 are identical of one kind and the rest are different is

$$2C_1 \times \frac{5!}{2!} = 120$$

64. (D)

$0 < f(4a) < f(a^2 - 5)$

$\Rightarrow 4a > a^2 - 5$

$\Rightarrow (a-5)(a+1) < 0$

$\Rightarrow a \in (-1, 5)$

65. (D)

$$t=\frac{2x-3}{4x+5},\ dt=\frac{22dx}{(4x+5)^2}$$

$$\Rightarrow I=\frac{1}{22}\int t^{\frac{2}{3}}dt=\frac{3}{110}t^{\frac{5}{3}}+c$$

$$\Rightarrow \frac{3}{110}\left(\frac{2x-3}{4x+5}\right)^{\frac{5}{3}}+c\Rightarrow ab=\frac{3}{110}.\frac{5}{3}=\frac{1}{22}$$

66. (B)

$$I=\int\frac{x^2(1-\log x)}{x^4\left[\left(\frac{\log x}{x}\right)^4-1\right]}dx$$

$$I=\int\frac{(1-\log x)}{x^2\left[\left(\frac{\log x}{x}\right)^4-1\right]}dx$$

Put $\frac{\log x}{x}=t\Rightarrow\frac{1-\log x}{x^2}dx=dt$

$$I=\int\frac{dt}{t^4-1}=\frac{1}{4}\log\left|\frac{\log x-x}{\log x+x}\right|-\frac{1}{2}\tan^{-1}\left(\frac{\log x}{x}\right)+c$$

67. (B)

$$g(-x)=\int_0^{-x}f(t)dt,\ \text{but } t=-z$$

$$=-\int_0^{x}f(-z)dz=\int_0^{x}f(z)dz=g(x)$$

$\therefore\ g(x)$ is even

Non $g(x+2)=\int_0^2 f(t)dt+\int_2^{x+2}f(t)dt=g(2)+\int_0^x f(t)dt$

$\therefore\ g(x+2)=g(2)+g(x)$

$\Rightarrow\ g(x+2)=g(x)$ (as f is odd)

$g(x)$ is periodic

$\Rightarrow\ g(2)=g(4)=g(6)=......=g(2n)=0$

68. (A)

$$S=\sum_{r=1}^{16}\frac{8r}{(4r^4+1)}$$

$$\sum_{r=1}^{16}\frac{8r}{(2r^2-2r+1)(2r^2+2r+1)}=2\sum_{r=1}^{16}\left(\frac{1}{(2r^2-2r+1)}-\frac{1}{2r^2-2r+1}\right)$$

$$=2\left(\frac{1}{1}-\frac{1}{5}+\frac{1}{5}-\frac{1}{13}+\frac{1}{13}+....\frac{1}{481}-\frac{1}{545}\right)$$

$$=2\left(1-\frac{1}{545}\right)=\frac{1088}{545}$$

69. (C)

$$P\left(\frac{B}{A\cup\bar{B}}\right)=\frac{P(B\cap(A\cup\bar{B}))}{P(A\cup\bar{B})}=\frac{P(A\cap B)}{P(A\cup\bar{B})}$$

$$P(A\cup\bar{B})=PA-P(A\cap B)$$

$$\Rightarrow P(A\cap B)=P(A)-P(A\cap\bar{B})$$

$$=\frac{7}{10}-\frac{1}{2}=\frac{1}{5}$$

Also, $P(A\cup\bar{B})=P(A)+P(\bar{B})-P(A\cap\bar{B})$

$$=\frac{7}{10}+\frac{3}{5}-\frac{1}{2}=\frac{4}{5}$$

$$\therefore P\left(\frac{B}{A\cup\bar{B}}\right)=\frac{1}{4}$$

70. (D)

Let the plane $ax+by+cz=1$ & Here $\frac{x}{6}=\frac{y}{4}=\frac{z}{3}$

When $6a+4b+3c=0$

$2a-b=1$

$3a-4b+5c=1$

$a=\frac{29}{85},\ b=\frac{-27}{85},\ c=\frac{-22}{85}$

$29x-27y-22z=85$

71. (B)

Let the mid point of chord be $p(h,K)$; its equation will be

$$xh-yK=h^2-K^2$$

$$y=\frac{h}{K}x+\frac{K^2-h^2}{K}$$

It should be same as $y=mx+\frac{a}{m}$

$$\Rightarrow m=\frac{h}{K} \text{ and } \frac{a}{m}=\frac{K^2-h^2}{K}$$

$\Rightarrow\ a = \frac{K^2 - h^2}{K} \frac{h}{K} \Rightarrow\ K^2 a = K^2 h \cdot h^3$

$\Rightarrow\ h^3 = K^2 (h - a)$

Locus is $x^3 = y^2 (x - a)$

72. (B)

Let $\tan x = t$

$\therefore$ Equation reduces to

$\frac{4t^2}{1+t^2} + t^2 + \frac{2}{t^2} = 5$

$\Rightarrow\ (t^2 - 1)(t^4 + t^2 - 2) = 0$

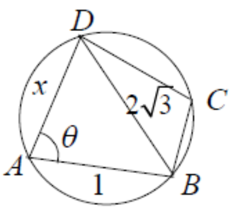
$\Rightarrow\ t^2 = 1, -2$

$x = \frac{\pi}{4}, \frac{3\pi}{4}$

73. (A)

Let $AD = x$ and $\angle BAD = \theta$

In $\Delta ABD, \frac{BD}{\sin\theta} = 2R$



$\Rightarrow\ \sin\theta = \frac{2\sqrt{3}}{2 \times 2}$ $\therefore\ \theta = 60^o$

$\cos 60^o = \frac{x^2 + 1^2 - (2\sqrt{3})^2}{2 \cdot x \cdot 1}$

$\Rightarrow\ x = \frac{1 \pm 3\sqrt{5}}{2}$

$\therefore\ x = \frac{1 + 3\sqrt{5}}{2}$ $(\because\ x > 0)$

74. (C)

Equation of the radical axis is

$4\lambda x = 9$ or $x = \frac{9}{4\lambda}$

Putting x in $x^2 + y^2 = 1$

$y = \pm\sqrt{1 - \frac{81}{16\lambda^2}}$

For the circle to have two common tangents, they must intersect at two distinct pints

$\therefore\ 1 - \frac{81}{16\lambda^2} > 0$

$\Rightarrow\ \lambda < \frac{-9}{4}$ or $\lambda > \frac{9}{4}$

75. (C)

$f(x)=x^3-3x+k \Rightarrow f'(x)=3x^2-3$

$f'(x)=3(x^2-1)<0 \ \forall \ x\in(0,1)$

Hence, $f'(x)$ is strictly decreasing.

76. (A)

Let $y=\sqrt{x+y}$

$y^2-y-x=0$

$y=\dfrac{1\pm\sqrt{1+4x}}{2}\ y>1$

$y=\dfrac{1+\sqrt{1+4x}}{2}$

77. (C)

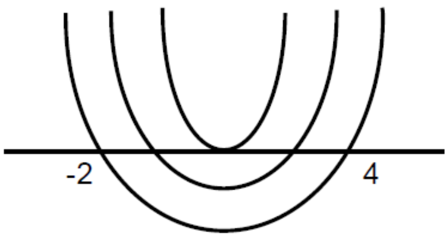
$A\cup B=A$

$f(x)=x^2-ax+4$

$D\geq0 \Rightarrow a^2-16\geq0 \Rightarrow a\leq-4$ or $a\geq4$ ..... (i)

$f(-2)\geq0 \Rightarrow 4+2a+4\geq0 \Rightarrow a\geq-4$ ..... (ii)

$f(4)\geq0 \geq 16-4a+4\geq0 \Rightarrow a\leq5$ ..... (iii)



and $-2\leq\dfrac{a}{2}\leq4;\ -4\leq a\leq8 \ \therefore a\in[4,5]$

Also, if $B=\phi$ then also $A\cup B=A$

$a^2-16<0, a\in(-4,4)$.

78. (D)

G.P. $\left\langle \begin{array}{l}1^{\text{st}} \text{ term}=a\\ \text{common ratio}=r\end{array}\right.$

$\sum_{i=1}^{201} ai=a+ar+ar^2......ar^{200}$

$625=\dfrac{a(1-r^{201})}{1-r}$ ...(i)

Now, $\sum_{i=1}^{201}\dfrac{1}{a_i}=\left(\dfrac{1}{a_1}+\dfrac{1}{a_2}+...+\dfrac{1}{a_{200}}+\dfrac{1}{a_{201}}\right)=\left(\dfrac{1}{a}+\dfrac{1}{ar}....\dfrac{1}{ar^{200}}\right)$

$=\dfrac{\frac{1}{a}\left(\left(\frac{1}{r}\right)^{201}-1\right)}{\frac{1}{r}-1}=\dfrac{1}{a}\left(\dfrac{1-r^{201}}{1-r}\right)\dfrac{1}{r^{200}}=\dfrac{1}{a}\times\dfrac{625}{a\cdot r^{200}}$

$$= \frac{625}{(a_{101})^2} = \frac{625}{625} = 1$$

79. (C)

$$\frac{2}{z_1} = \frac{1}{z_1} + \frac{1}{z_3}$$

$$\Rightarrow \frac{1}{z_1} - \frac{1}{z_2} = \frac{1}{z_3} - \frac{1}{z_1} \Rightarrow \frac{z_1 - z_1}{z_1 z_2} = \frac{z_1 - z_2}{z_1 z_3}$$

$$\Rightarrow \frac{z_2 - z_1}{z_3 - z_1} = -\frac{z_2}{z_3} \qquad \Rightarrow \arg\left(\frac{z_2 - z_1}{z_3 - z_1}\right) = \arg\left(-\frac{z_2}{z_3}\right) = \pi - \arg\left(\frac{z_2}{z_3}\right)$$

$$\Rightarrow \arg\left(\frac{z_2 - z_1}{z_3 - z_1}\right) = \pi - \arg\left(\frac{z_2 - 0}{z_3 - 0}\right)$$

$$\Rightarrow \angle CAB = \pi - \angle COB$$

$$\Rightarrow \angle CAB + \angle COB = \pi$$

$\Rightarrow$ the points *O*, *A*, *B* and *C* are concyclic.

80. (C)

Given numbers are 1, 2, 3, 4, ……, $2n + 1$

Mean of these numbers $= \bar{x} = \frac{1+2+3+.....+2n+1}{2n+1} = n+1$

$$\sigma^2 = \frac{1}{2n+1}\sum_{r=0}^{2n}\{(1+r)-(1+n)\}^2 = \frac{1}{2n+1}\sum_{r=0}^{2n}(n-r)^2 = \frac{2(1^2+2^2+....+n^2)}{2n+1}$$

$$\sigma^2 = \frac{n(n+1)}{3} \Rightarrow \sigma = \sqrt{\frac{n(n+1)}{3}}$$

81. (2)

$$\lim_{x\to 0}\frac{ax\cos x + b\sin x}{x^2\sin x} = \frac{1}{3}$$

$$\Rightarrow \lim_{x\to 0}\frac{ax\cos x + b\sin x}{x^3}.\left(\frac{x}{\sin x}\right) = \frac{1}{3}$$

$$\Rightarrow \lim_{x\to 0}\frac{(a+b)\cos x - ax\sin x}{3x^2} = \frac{1}{3}$$

$$\Rightarrow a + b = 0$$

$$\therefore \lim_{x\to 0}\frac{-ax\sin x}{3x^2} = \frac{1}{3} \Rightarrow a = -1.b = 1$$

$$b - a = 2.$$

82. (5)

For existence of non – trivial solutions,

$$\begin{vmatrix} 6 & 5 & \lambda \\ 3 & -1 & 4 \\ 1 & 2 & -3 \end{vmatrix} = 0 \Rightarrow \lambda = -5$$

If $\lambda \neq -5$, then $x=0, y=0, z=0$ is the only solution.

83. (9)

$e = 2/3$

Equation of tangent at L is $\frac{2x}{9} + \frac{y}{3} = 1$ it meets x-axis at $A\left(\frac{9}{2}, 0\right)$ & y-axis at $B(0, 3)$.

$\therefore$ Area = $4\left[\frac{1}{2}.\frac{9}{2}.3\right] = 27$

84. (210)

Consider

$$(1+x)^{10} \cdot (1-x)^{10} = \left({}^{10}C_0 + {}^{10}C_1 x + {}^{10}C_2 x^2 + {}^{10}C_3 x^3 + ...\right) \cdot \left({}^{10}C_0 - {}^{10}C_1 x + {}^{10}C_2 x^2 - {}^{10}C_3 x^3 + ...\right)$$

Coefficient of x^8

$= {}^{10}C_0 \cdot {}^{10}C_8 - {}^{10}C_1 \cdot {}^{10}C_7 + {}^{10}C_2 \cdot {}^{10}C_6 - .... + {}^{10}C_8 \cdot {}^{10}C_0$

$\therefore$ required = coefficient of x^8 in $(1-x^2)^{10}$

$= {}^{10}C_4 \cdot (-x^2)^4$

$= {}^{10}C_4 \cdot x^8$

$\therefore$ Coefficient $= {}^{10}C_4$

$= 210$

85. (9)

$(I+A)^{10} = I + (2^{10} - 1)A$

$k = 2^{10} - 1 = 2^{\lambda+1} - 1 \Rightarrow \lambda = 9$

86. (3)

$\alpha^2 - 2 = 6\alpha$ and $\beta^2 - 2 = 6\beta$

$$\frac{a_{10} - 2a_8}{2a_9} = \frac{(\alpha^{10} - \beta^{10}) - 2(\alpha^8 - \beta^8)}{2(\alpha^9 - \beta^9)}$$

$$= \frac{\alpha^8(\alpha^2 - 2) - \beta^8(\beta^2 - 2)}{2(\alpha^9 - \beta^9)}$$

$$= \frac{6(\alpha^9 - \beta^9)}{2(\alpha^9 - \beta^9)}$$

$= 3$

87. (3)

$$\tan x+\tan\left(\frac{\pi}{3}+x\right)-\tan\left(\frac{\pi}{3}-x\right)$$

$$=\tan x+\frac{\sqrt{3}+\tan x}{1-\sqrt{3}\tan x}-\frac{\sqrt{3}-\tan x}{1+\sqrt{3}\tan x}$$

$$=\tan x+\frac{8\tan x}{1-3\tan^2 x}$$

$$=\frac{9\tan x-3\tan^3 x}{1-3\tan^2 x}$$

$$=\frac{3\left(3\tan x-\tan^3 x\right)}{\left(1-3\tan^2 x\right)}$$

$$=3\tan(3x)$$

88. (5)

$y'\ln x+\dfrac{y}{x}=\ln y+\dfrac{x}{y}y'$ for the point $\left(e^2,e^2\right)$

$2y'+1=2+y'$

$y'=1$

Equation of normal is $y-e^2=-1\left(x-e^2\right)$

$y+x=2e^2$

$a=2e^2$ and $b=2e^2$

$$\left|\frac{4a+b}{b}\right|=5$$

89. (8)

Let $z=\cos x+i\sin x$

$$\left|\frac{z}{\bar{z}}+\frac{\bar{z}}{z}\right|=\frac{\left|z^2+\bar{z}^2\right|}{\left|z^2\right|}=2\left|\cos 2x\right|=1$$

$2\cos 2x=1$ or $2\cos 2x=-1$

$x=\dfrac{\pi}{6},\dfrac{5\pi}{6},\dfrac{7\pi}{6},\dfrac{11\pi}{6},\dfrac{\pi}{3},\dfrac{2\pi}{3},\dfrac{4\pi}{3}$ and $\dfrac{5\pi}{3}$

So, 8 solutions

90. (4)

$$Z=-\frac{1}{2}i\left(1+i\sqrt{3}\right)=i\,\omega^2$$

$$Z^{101}=i\,\omega$$

$$\left(Z^{101}+i^{109}\right)^{106}=\left(-i\,\omega^2\right)^{106}=-\omega^2$$

$\therefore\ -\omega^2 = \left(i\,\omega^2\right)^n = i^n \omega^{2n}$

$\omega^{2n-2}\, i^n = -1$

This is possible only when $n = 4r + 2$ and $2n - 2$ is multiple of 3 i.e., $2(4t+2) - 2$ is multiple of 3 i.e., $8r + 2$ is multiple of $3 \Rightarrow r = 2$

$\therefore\ n = 10$

$\therefore\ \dfrac{2}{5}k = 4$