PART (A) : PHYSICS

ANSWER KEY

1.	(B)	2.	(D)	3.	(D)	4.	(C)	5.	(A)
6.	(A)	7.	(D)	8.	(B)	9.	(D)	10.	(C)
11.	(D)	12.	(B)	13.	(C)	14.	(B)	15.	(D)
16.	(D)	17.	(B)	18.	(A)	19.	(B)	20.	(B)
21.	(15)	22.	(24)	23.	(2)	24.	(5)	25.	(4)
26.	(4)	27.	(20)	28.	(8)	29.	(5)	30.	(5)

SOLUTIONS

1. (B)

From the graphs

$\lambda = 9\text{cm}$

$T = 3\text{sec}.$

$\Rightarrow \quad v = \frac{\lambda}{T} = \frac{9}{3}$ cm/sec = 3 cm/sec.

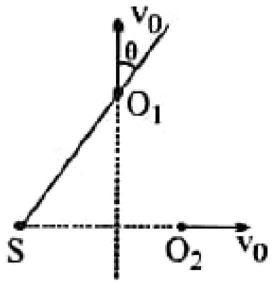
2. (D)

Combination of isobaric, isochoric & isothermal.

3. (D)

$$f_1 = f\left[\frac{v - v_0\cos\theta}{v}\right] \qquad .....(1)$$

$$f_2 = f\left[\frac{v - v_0}{v}\right] \qquad .....(2)$$

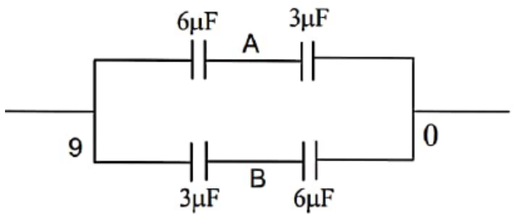


$$\therefore \frac{f_1}{f_2} = \frac{v - v_0\cos\theta}{v - v_0} > 1$$

4. (C)

$C_{eff} = 4\mu F$

Charge in each branch = $18\mu C$



$\therefore \quad 6(9 - V_A) = 18 \qquad ...(1)$

$3(9 - V_B) = 18 \qquad ...(2)$

By equation (1)

$V_A = 6$

$V_B = 3$

$\therefore \quad V_A - V_B = 3\,\text{volt}$

5. (A)

Force on each hemispherical shell due to other is

$\frac{\sigma^2}{2\in_0} \times \pi R^2$ (By electrostatic pressure)

Let surface charge density of inner shell is σ_1

Field at the surface of hemispherical shell is $= \frac{K\left(\sigma_1 \times 4\pi\left(\frac{R}{2}\right)^2\right)}{R^2}$

$= \frac{\sigma_1}{4\in_0}$

Force due top inner shell

$= \frac{\sigma_1}{4\in_0} \times \sigma g p R^2$

$\because$ Net force on hemisphere = 0

$\frac{\sigma^2}{2\in_0}\pi R^2 + \frac{\sigma_1 \sigma \pi R^2}{4\in_0} = 0$

$\Rightarrow \sigma_1 = -2\sigma$

6. (A)

$E_0 k = B_0 \omega \qquad \because E_0 = cB_0$

Also, $\left[\frac{\omega}{c} = k\right]$

7. (D)

For maximum

$\Delta P = \Delta R_1 + \Delta R_2$

$= 0.1 + 0.2 = 0.3$

$R = R_1 + R_2 = 9$

$\therefore \quad R_{max} = 9 \pm 0.03$

For minimum

$$R = \frac{R_1R_2}{R_1+R_2} = \frac{18}{3+6} = 2\Omega$$

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$= -\frac{1}{R^2}dR = -\frac{1}{R_1^2}dR_1 - \frac{1}{R_2^2}dR_2$$

$$\Rightarrow \frac{dR}{4} = \frac{0.1}{9} + \frac{0.2}{36}$$

$$= \frac{0.2}{3} = 0.06$$

$$\therefore R_{min} = 2 \pm 0.0666$$

$$\therefore R_{min} = 2 \pm 0.07$$

8. (B)

$$\vec{v}_1 = 100\cos 60^o\hat{i} + (100\sin 60^o - gt)\hat{j}$$

$$\vec{v}_2 = -100\cos 60^o\hat{i} + (100\sin 60^o - gt)\hat{j}$$

$$\vec{v}_1.\vec{v}_2 = 0$$

$$\Rightarrow -\left(100\cos 60^o\right)^2 + \left(100\sin 60^o - gt\right)^2 = 0$$

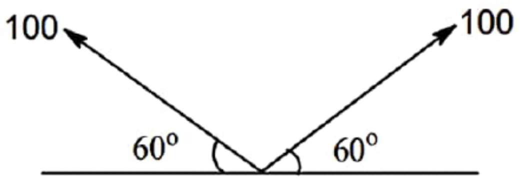
$$\Rightarrow -2500 + \left(50\sqrt{3}\right)^2 + g^2t^2 - 100gt\sqrt{3} = 0$$

$$\Rightarrow -25 + 75 + t^2 - 10\sqrt{3}\,t = 0$$

$$\Rightarrow t^2 - 10\sqrt{3}t + 50 = 0$$

$$= \frac{10\sqrt{3} \pm \sqrt{300-200}}{2} = \frac{10\sqrt{3} \pm 10}{2}$$

$$= 5\sqrt{3} \pm 5$$



9. (D)

Diffraction is obtained when the slit width is of the order of wavelength of light (or any electromagnetic wave) used.

Here, wavelength of x-rays (1-100 Å) << slit width (0.6 mm).

Therefore, no diffraction pattern will be observed.

10. (C)

Viscous force $= \eta A \frac{dv}{dy}$

$\tau = \int F_v r$

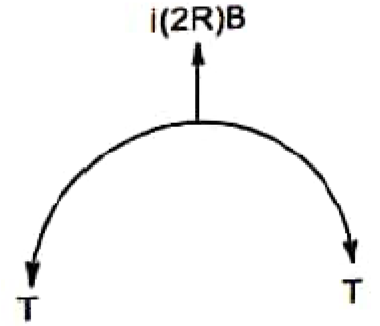
$= \int_0^R \frac{\eta(2\pi r\,dr)(r\omega)r}{h} \propto R^4$ (h – thickness of oil film)

11. (D)

FBD of half part

$2T = (100)\times(2\times0.5)0.2$

T = 10 N



12. (B)

1 half life of A = 2 half lives of B

3 half lives of A = 6 half lives of B

$\because\ N'_A = N'_B$ (after 3 half lives of A)

$\frac{N_A}{2^3} = \frac{N_B}{2^6}$

$\Rightarrow \frac{N_A}{N_B} = \frac{1}{2^3} = \frac{1}{8}$

13. (C)

$a_n = kt^4$

$\Rightarrow \frac{V^2}{R} = kt^4 \Rightarrow v^2 = kRt^4$

Average power $= \frac{\text{total work}}{\text{time}} = \frac{\Delta k}{\text{time}} = \frac{\frac{1}{2}mv^2}{t}$

$= \frac{1}{2}m\frac{(kRt^4)}{t} = \frac{1}{2}mkRt^3$

14. (B)

Let the K_1 and K_2 : P_1 and P_2 are K.E. and momentum of α-particle and remaining

$K_1 + K_2 = 5.5$ MeV ….. (1)

$P_1 = P_2$ ….. (2)

$\sqrt{2K_1\times4m} = \sqrt{2K_2\times216m}$

$= K_1 = 54K_2$

$\therefore$ by equation (1)

$K_1 = \frac{5.5\times54}{55} = 5.4\text{ MeV}$

15. (D)

$\vec{E} - y\hat{i} - x\hat{j} \qquad \frac{-\partial V}{\partial x} = -y \Rightarrow V = xy + \text{constant}$

$\frac{-\partial V}{\partial y} = -x \Rightarrow V = xy + \text{constant}$

For equipotential the equation is xy = constant.
Represents rectangular hyperbola.

16. (D)

$A = x^2$

$\varepsilon_{\text{induced}} = \frac{d\phi}{dt} = \frac{d}{dt}\left(Bx^2\right)$

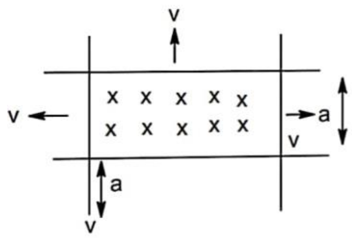
$= 2Bx\frac{dx}{dt}$

$\frac{dx}{dt} = 2v$

$\therefore \quad \varepsilon_{\text{induced}} = 2Bx(2v)$

$= 4Bva$

$\therefore$ Induced current $= \frac{\varepsilon_{\text{ind}}}{4ra} = \frac{4Bva}{4ra} = \frac{Bv}{r}$



17. (B)

$r_1 = \frac{5L}{6} \qquad r_2 = \frac{1}{6}L$

$M\omega^2 r_1 = 5M\omega^2 r_2$

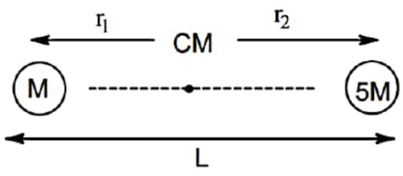
$= \frac{G\left(5M^2\right)}{L^2}$

$\therefore \quad \omega^2 r_1 = \frac{5GM}{L^2}$

$\omega^2 = \frac{6GM}{L^3}$

$$\omega = \sqrt{\frac{6GM}{L^3}}$$

Time period $= 2\pi\sqrt{\frac{L^3}{6GM}}$



18. (A)

$G' = G$

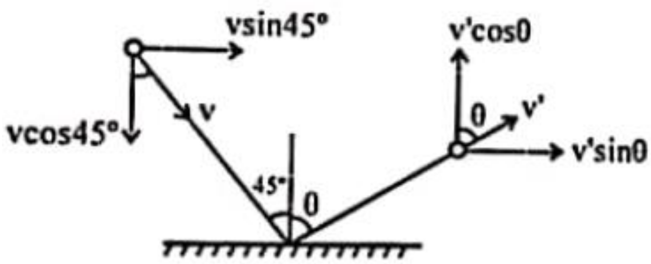
$$\frac{(G+R)S}{G+R+S} = G$$

Or $S = \frac{G^2}{R} + G$

19. (B)

From conservation of momentum

$v'\sin\theta = v\cos 45^o$ ...(1)



$v'\cos\theta = e\,v\sin 45^o$ ...(2)

$(1)+(2) \Rightarrow \sqrt{\tan\theta} = \frac{1}{e}$

$\Rightarrow \tan\theta = \sqrt{3}$

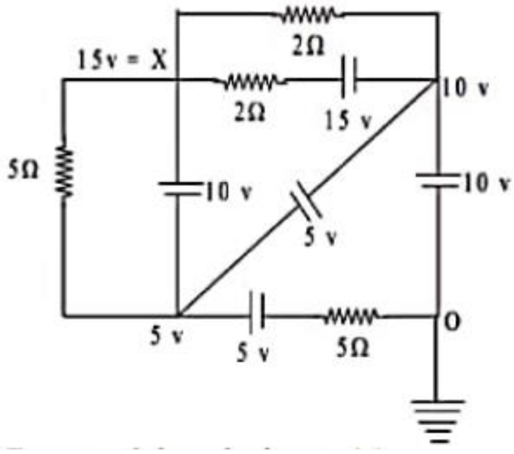
$\therefore \theta = 60^o$

20. (B)

We know that, $\lambda_m T =$ constant and the power radiated by a black body is proportional to T^4 i.e. $P \propto T^4$, Hence $P \propto (\lambda_m)^{-4}$

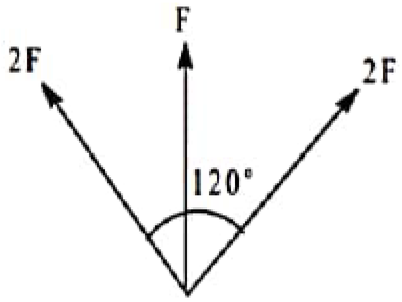
$$\Rightarrow \frac{P_2}{P_1} = \left(\frac{\lambda_{m_1}}{\lambda_{m_2}}\right)^4 = \left(\frac{\lambda_0}{3\lambda_0 4}\right)^4 = \left(\frac{4}{3}\right)^4 = \frac{256}{81}$$

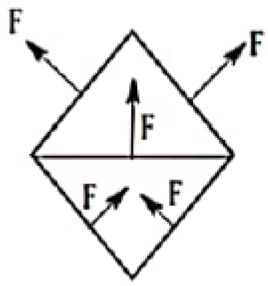
21. (15)



From nodal analysis $x = 15\,v$

22. (24)

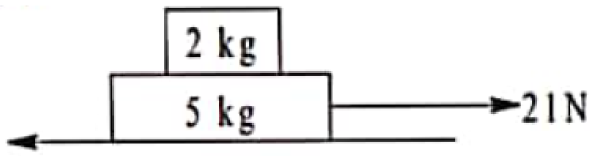




$\Rightarrow F_{net} = 3F = 3\,BiI = 3\times4\times2\times L$

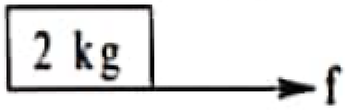
$= 24\,N$

23. (2)



$f = 0.2\times7\times10 = 14\,N$

$$a = \frac{F-f}{(M+m)} = \frac{21-14}{7} = 1\,m/s^2$$



$f = ma$

$= 2\times1 = 2\,N$

24. (5)

$$u = \frac{1}{2}Li^2 \Rightarrow \frac{dv}{dt} = \frac{Lidi}{dt} = iv_L = i(v - v_R)$$

$$= 0.4(12 - 0.4 \times 10) = 3.2 = \frac{16}{5}\text{ watt}$$

$x = 5$

25. (4)

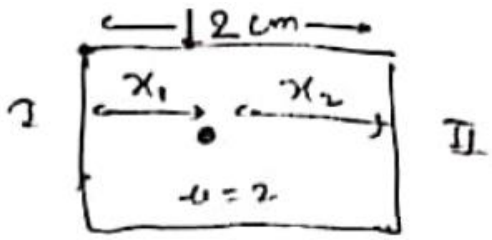
For observer note of B will not change due to zero relative motion.

Observed frequency of sound produced by A.

$$= 660\frac{330 - 30}{330} = 60\text{ Hz}$$

$\therefore$ No. of beats = 600 – 596 = 4

26. (4)



$$x_1 = 2 = \frac{d_1}{\mu} \Rightarrow d = 2\mu = 2 \times 2 = 4a$$

$$d_2 = 8\text{ cm} \ \ \& \ \ x_2 = \frac{d_2}{\mu} = \frac{8}{2} = 4\text{ cm}$$

27. (20)

$$(\mu - 1)\ell = \frac{dy}{b} = \frac{\gamma\alpha}{\beta}$$

$\Rightarrow$ as $Y = 10\beta$

$\Rightarrow$ $\ell = 20\beta$

28. (8)

Let W be the work function of metal. Then

$$eV_0 = \frac{hc}{330 \times 10^{-9}} - W \qquad ...(1)$$

$$e(2V_0) = \frac{hc}{220 \times 10^{-9}} - W \qquad ...(2)$$

Solving these two equations, we get

$$V_0 = \frac{10^9 \times h \times c}{110 \times e \times 6} = \frac{10^9 \times 6.6 \times 10^{-34} \times 3 \times 10^8}{110 \times 1.6 \times 10^{-19} \times 6} = \frac{15}{8}\text{ Volt}$$

29. (5)

$$H = \frac{Q}{t} = \frac{KA(\theta_1 - \theta_2)}{\ell}$$

or $H \propto \frac{A}{\ell}$

or $H_2 = 2H_1$

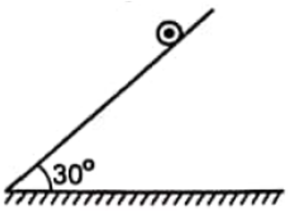
30. (5)

$\tau = \frac{mg}{2}R = \text{constant}$

Angular impulse $= \tau t$

$= \left(\frac{mgR}{2}\right)(1)$

$= \frac{m(10)R}{2} = 5mR$



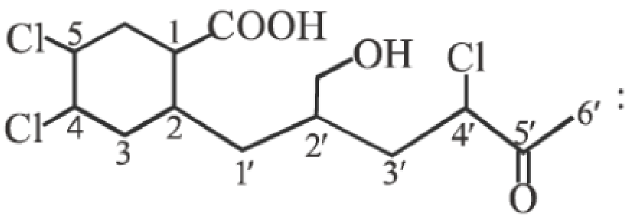
PART (B) : CHEMISTRY

ANSWER KEY

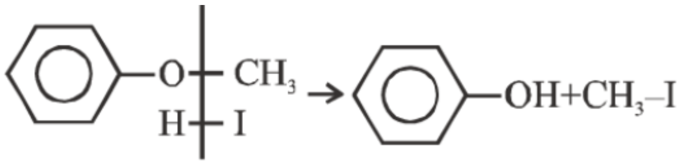
31.	(C)	32.	(C)	33.	(A)	34.	(B)	35.	(A)
36.	(C)	37.	(D)	38.	(B)	39.	(A)	40.	(C)
41.	(B)	42.	(A)	43.	(A)	44.	(C)	45.	(A)
46.	(B)	47.	(D)	48.	(B)	49.	(D)	50.	(D)
51.	(4)	52.	(4)	53.	(25)	54.	(8)	55.	(21)
56.	(2)	57.	(6)	58.	(2)	59.	(27)	60.	(6)

SOLUTIONS

31. (C)



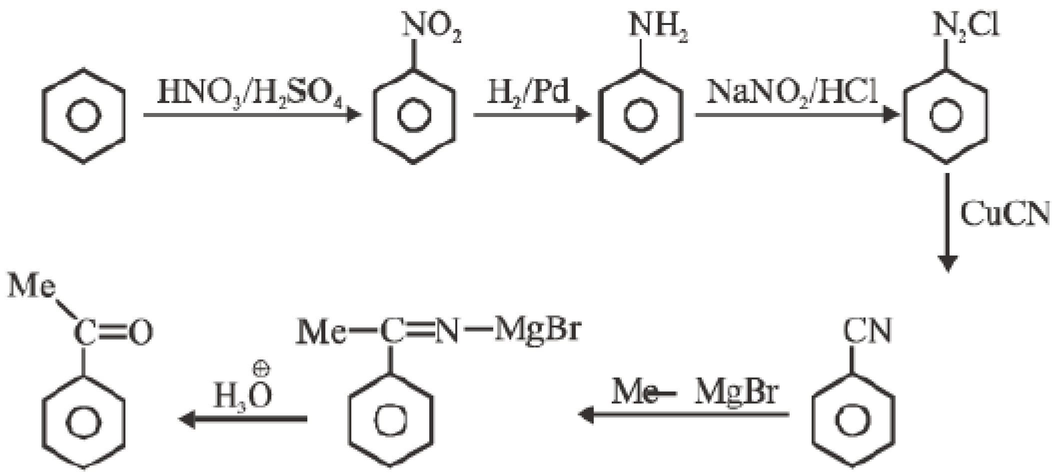
32. (C)



33. (A)

Only 1′ amine gives positive isocyanide test.

34. (B)



35. (A)

$$Cr\left|\underset{(0.04M)}{Cr^{+3}}\right|\left|\underset{(1M)}{Cr^{+3}}\right|Cr\,.$$

$$E_{cell}=E^0_{cell}-\frac{0.059}{3}\log\frac{[Cr^{+3}]_1}{[Cr^{+3}]_2}$$

$$=0-0.02\log\frac{0.04}{1}=-0.02\log\frac{4}{100}$$

$$=0.028\text{ V}$$

36. (C)

$$\frac{P_o-P_S}{P_s}=\frac{W_{solute}\times(\text{mol. wt.})_{solvent}}{(\text{mol. wt.})_{solute}\times w_{solvent}}$$

$$\frac{854-848.9}{848.9}=\frac{2\times 76}{(\text{mol wt.})_{solute}\times 100}$$

$$(\text{mol. wt.})_{solute}=253$$

$$\text{mol. wt. of Sn}=32\times n$$

$$\therefore\ 32\times n=253$$

$$n=7.9\approx 8$$

37. (D)

$$CaCO_3(s)\to CaO(s)+CO_2(s)$$

$$\text{Moles of CaO }=\frac{28}{56}=\frac{1}{2}$$

$$\text{Moles of }CaCO_3=\frac{x}{100}$$

$$\because\ \text{moles of }CaCO_3=\text{moles of CaO}$$

$$\frac{x}{100}=\frac{1}{2}\Rightarrow x=50\,\text{gm}$$

38. (B)

$$\frac{-dC_A}{dt} = K_1 \frac{C_A}{1+K_2C_A} \text{ if } C_A \text{ is small.}$$

$$1+K_2C_A = 1 \quad \therefore \frac{-dC_A}{dt} = K_1 \cdot C_A$$

39. (A)

$$CO_2(g) \rightleftharpoons CO(g) + \frac{1}{2}O_2(g)$$

$t = 0$	1 mole	0	0
$t = t_{eq}$	$(1 - \alpha)$	α	$\frac{\alpha}{2}$

$$K = \frac{[CO]\times[O_2]^{1/2}}{[CO_2]} = \frac{\alpha \times \left(\frac{\alpha}{2}\right)^{1/2}}{(1-\alpha)} = \frac{\frac{\alpha^{3/2}}{\sqrt{2}}}{(1-\alpha)}$$

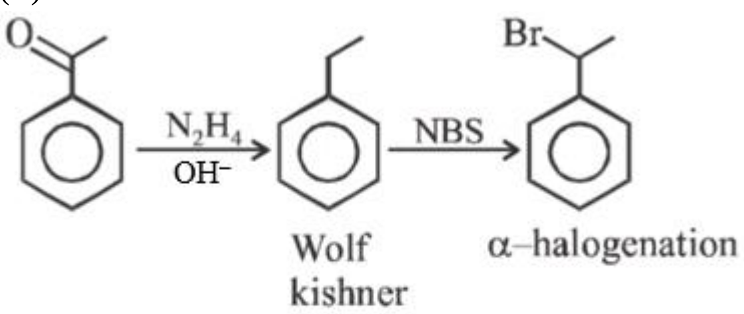
$$\because 1 >> \alpha \Rightarrow (1-\alpha) \simeq 1$$

$$K = \frac{\alpha^{3/2}}{\sqrt{2}}$$

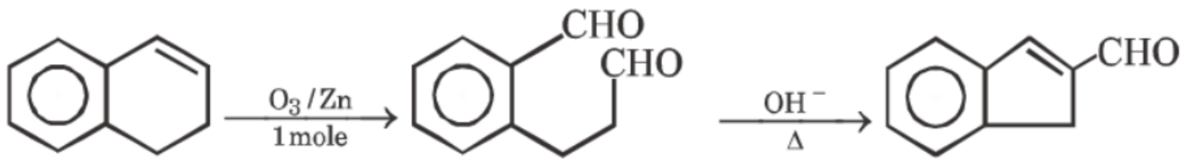
40. (C)

C — O	>	C — O	>	C — O
Pure single		Impure single		Impure double
(z)		(y)		(x)

41. (B)



42. (A)



Intramolecular aldol

43. (A)

$$2NO + O_2 \longrightarrow N_2O_4$$

Moles of NO $= \frac{PV}{RT} = \frac{800 \times 0.25}{760 \times 0.0821 \times 220}$

Moles of $O_2 = \frac{PV}{RT} = \frac{600 \times 0.1}{760 \times 0.0821 \times 220}$

∴ Limiting reagent is O_2

∵ Moles of N_2O_4 = Moles of O_2

$$= \frac{600 \times 0.1}{760 \times 0.0821 \times 220}$$

Mass of N_2O_4 = $\frac{600 \times 0.1}{760 \times 0.0821 \times 220} \times 92$

$= 0.402$ gm

44. (C)

$$\int dH = n \int C_{p,m} dT$$

$$\Delta H = 10 \int_{300\,K}^{400\,K} \left(C_{v,m} + R\right) dT$$

$$\Delta H = 10 \int_{300\,K}^{400\,K} \left(20 + 10^{-2} T + 8.314\right) dT$$

$$\Delta H = 31814\ J$$

45. (A)

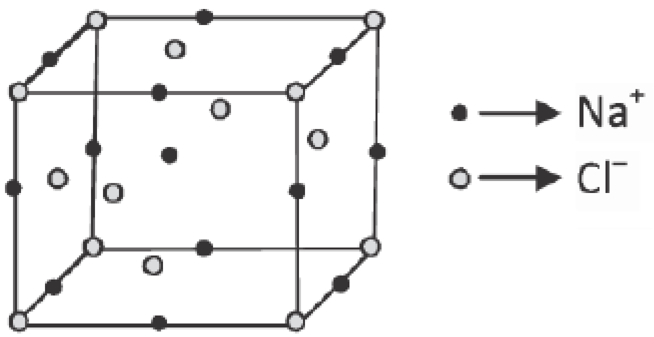
$\Delta H = Eaf - Eab$

$= 60 - 40$

$\Delta H = 20\ kJ\ mol^{-1}$

∴ Endothermic reaction

46. (B)



Rock salt structure

If all the atoms touching one face plane are removed then

No. of Na^+ left $= 8 \times \frac{1}{4} + 1 = 3$

No. of Cl^- left $= 5\times\frac{1}{2}+4\times\frac{1}{8}=3$

Hence formula is NaCl

Also since equal number of cations and anions are missing defect is schottky defect.

47. (D)

Bond order of $O_2^- = 1.5$

Bond order of $O_2 = 2$

Bond order of $O_2^+ = 2.5$

48. (B)

Conceptual

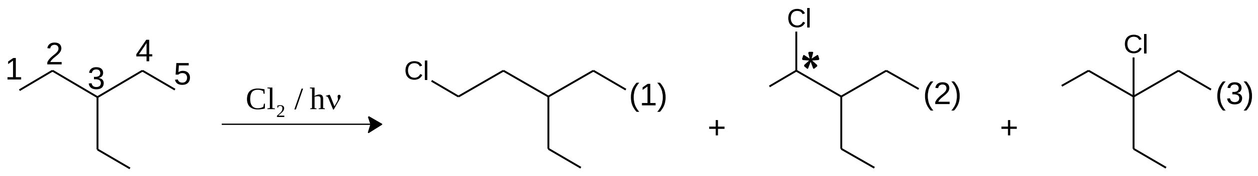
49. (D)

Conceptual

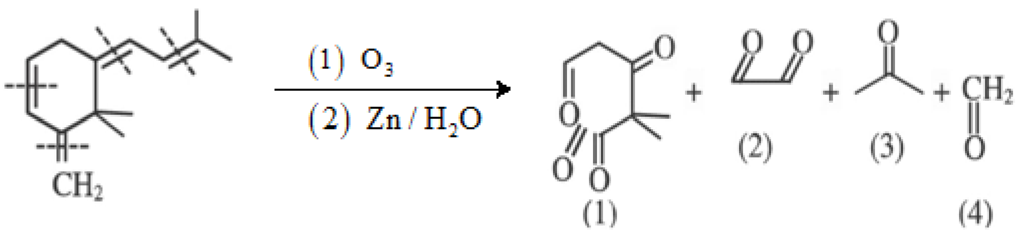
50. (D)

$NH_2-CH_2-CH_2-NH-CH_2-CH_2-NH_2$

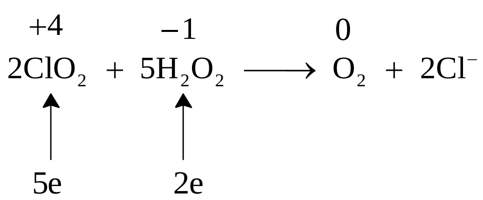
51. (4)



52. (4)



53. (25)



54. (8)

Conceptual

55. (21)
$x = 15$
$y = 6$
$x + y = 15 + 6 = 21$

56. (2)
$$\underset{}{Mg_3N_2 + 6H_2O \longrightarrow \underset{(x)}{2NH_3} + 3Mg(OH)_2}$$
$$LiH + H_2O \longrightarrow \underset{(y)}{H_2} + LiOH$$
$$\frac{x}{2} + y = \frac{2}{2} + 1 = 1 + 1 = 2$$

57. (6)
Such, Cyanide, Amide, Ketone, Ester, Alcohol, Carboxylic acid group present.

58. (2)
$$RH_2 + Ca^{2+} \longrightarrow RCa + 2H^+$$
1 L of H_2O has 20 × 0.5 M = 10 Millimol H^+
∴ Moles of $Ca^{2+} = 5$ millimoles
$$\frac{5\times10^{-3}}{1000}\times10^6\times40;\ 200 = 2\times10^2$$

59. (27)
$$A(g) \rightleftharpoons 3B(g);\ K_{P_1}$$
At equilibrium $1-\alpha$ 3α
$$K_{P_1} = \frac{(3\alpha)^3}{(1-\alpha)}\frac{P^2}{(1+2\alpha)^2}$$
$$P(g) \rightleftharpoons 2Q(g) + R(g);\ K_{P_2}$$
At equilibrium $1-\alpha$ 2α α
$$\frac{K_{P_1}}{K_{P_2}} = \frac{X}{4} = \frac{\dfrac{3^3\alpha^3P^2}{(1-\alpha)(1+2\alpha)}}{\dfrac{4\alpha^3P^2}{(1-\alpha)(1+2\alpha)^2}}$$
$$X = 27$$

60. (6)
$$\frac{W}{M}\times n = \frac{I\times t}{96500};\ \frac{0.838}{184}\times n$$

$$= \frac{40 \times 60 \times 1.0}{96500}$$

$$\Rightarrow \quad n = 6$$

PART (C) : MATHEMATICS

ANSWER KEY

61.	(D)	62.	(A)	63.	(C)	64.	(C)	65.	(D)
66.	(B)	67.	(B)	68.	(B)	69.	(D)	70.	(A)
71.	(C)	72.	(A)	73.	(B)	74.	(A)	75.	(A)
76.	(D)	77.	(B)	78.	(B)	79.	(C)	80.	(A)
81.	(315)	82.	(240)	83.	(2355)	84.	(45)	85.	(3)
86.	(1)	87.	(5)	88.	(0)	89.	(60)	90.	(4)

SOLUTIONS

61. (D)

Given that, $|z - 2 + i| \leq 2$ …(i)

$\therefore \quad |z-2+i| \geq ||z|-|2-i||$

$\Rightarrow \quad |z-2+i| \geq ||z|-\sqrt{5}|$ …(ii)

From Eqs. (i) and (ii), we have

$||z|-\sqrt{5}| \leq |z-2+i| \leq 2$

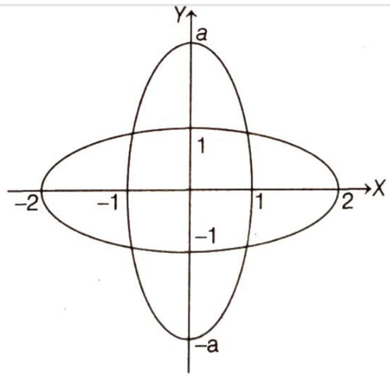
$\Rightarrow \quad ||z|-\sqrt{5}| \leq 2$

$\Rightarrow \quad -2 \leq |z|-\sqrt{5}| \leq 2$

$\Rightarrow \quad \sqrt{5}-2 \leq |z| \leq \sqrt{5}+2$

62. (A)

As the two ellipse intersect in four distinct points, a > 1.



Thus, $b^2 - 5b + 7 > 1$

$\Rightarrow \quad (b-2)(b-3) > 0$

$\Rightarrow \quad b < 2$ or $b > 3$

Hence $b \notin (2, 3)$.

63. (C)

p	q	$\sim p \vee q$	$\sim p \wedge \sim q$	$\sim (p \vee q) \wedge (\sim p \wedge \sim q)$
T	T	T	F	F
T	F	F	F	F
F	T	T	F	F
F	F	T	T	T

Thus, the resulting statement is neither tautology nor contradiction.

64. (C)

$$f(h(x)) = In\frac{1+h(x)}{1-h(x)}$$

$$= In\frac{1+\frac{x^3+3x}{3x^2+1}}{1-\frac{x^3+3x}{3x^2+1}}$$

$$= In\frac{(1+x)^3}{(1-x)^3} = In\left(\frac{1+x}{1-x}\right)^3$$

$$= 3In\left(\frac{1+x}{1-x}\right) = 3f(x)$$

65. (D)

We have, $q \leftrightarrow (\sim p \vee \sim q)$

$$\equiv q \leftrightarrow \sim (p \wedge q)$$
$$\equiv [q \rightarrow \sim (p \wedge q)] \wedge [\sim (p \wedge q) \rightarrow q]$$
$$\equiv [\sim q \vee \sim (p \wedge q)] \wedge [(p \wedge q) \vee q]$$
$$\equiv \sim [q \wedge (p \wedge q)] \wedge q$$
$$\equiv \sim (p \wedge q) \wedge q$$
$$\equiv (\sim p \vee \sim q) \wedge q$$
$$\equiv (\sim p \wedge q) \vee (\sim q \wedge q) \equiv (\sim p \wedge q]$$

66. (B)

As we know that,

$$\lim_{n\to\infty} x^{2n} = \begin{cases} 0 & , \quad x^2 < 1 \\ 1 & , \quad x^2 = 1 \\ \infty & , \quad x^2 > 1 \end{cases}$$

$$= \begin{cases} 0 & , \quad -1 < x < 1 \\ 1 & , \quad x = \pm 1 \\ \infty & , \quad x < -1 \text{ or } x > 1 \end{cases}$$

$$\therefore h(x) = \lim_{n \to \infty} \frac{x^{2n} f(x) + g(x)}{x^{2n} + 1}$$

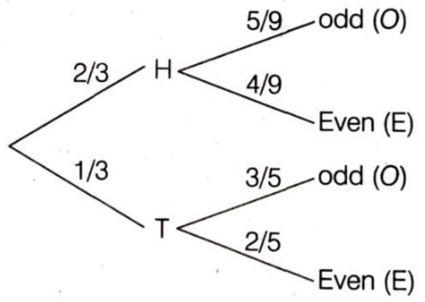
$$= \begin{cases} \dfrac{0 \cdot f(x) + g(x)}{0 + 1} & , \quad -1 < x < 1 \\ \dfrac{1 \cdot f(x) + g(x)}{1 + 1} & , \quad x = \pm 1 \\ \dfrac{f(x) + \dfrac{g(x)}{x^{2n}}}{1 + \dfrac{1}{x^{2n}}} & , \quad x < -1 \, or \, x > 1 \end{cases}$$

$$= \begin{cases} g(x) & , \quad -1 < x < 1 \\ \dfrac{f(x) + g(x)}{2} & , \quad x = \pm 1 \\ f(x) & , \quad x < -1 \, or \, x > 1 \end{cases}$$

Hence, h(x) = f(x) when $x \in R - [-1, 1]$.

67. (B)

The tree diagram with respective probabilities



The probability of selecting an even number form the numbers 1 through $9 \, is \, \frac{4}{9}$, since there are four even

Numbers out of the 9 numbers. Whereas the probability of selecting an even number from the numbers 1 through $5 \, is \, \frac{2}{5}$, since there are two even numbers out of the 5 numbers.

Hence, $p = P(E) = \frac{2}{3} \times \frac{4}{9} + \frac{1}{3} \times \frac{2}{5} = \frac{58}{135}$

68. (B)

Let $g(x) = \sin x + \cos x = \sqrt{2}\sin\left(x+\frac{\pi}{4}\right)$

$\Rightarrow \quad g(x) \in [1, \sqrt{2}]\ \forall x \in \left[0, \frac{\pi}{2}\right]$

$f(x) = \tan^{-1}(g(x)) \in \left[\frac{\pi}{4}, \tan^{-1}\sqrt{2}\right]$

$so, b = \frac{\pi}{4}, a = \tan^{-1}\sqrt{2}$

$\tan(a-b) = \tan\left(\tan^{-1}\sqrt{2} - \frac{\pi}{4}\right)$

$= \frac{\sqrt{2}-1}{1+\sqrt{2}\times 1} = \frac{\sqrt{2}-1}{\sqrt{2}+1}$

$= \frac{1}{3+2\sqrt{2}} = (3-2\sqrt{2})$

69. (D)

$\sqrt{3}\sec x + \operatorname{cosec} x + 2(\tan x - \cot x) = 0$

$\Rightarrow \quad \frac{\sqrt{3}}{\cos x} + \frac{1}{\sin x} + 2\left(\frac{\sin x}{\cos x} - \frac{\cos x}{\sin x}\right) = 0$

$\Rightarrow \quad \frac{\sqrt{3}\sin x + \cos x}{\sin x \cdot \cos x} + 2\left(\frac{\sin^2 x - \cos^2 x}{\sin x \cdot \cos x}\right) = 0$

$\Rightarrow \quad \frac{2\cos\left(x - \frac{\pi}{3}\right) - 2\cos 2x}{\sin x \cdot \cos x} = 0$

$\Rightarrow \quad \cos\left(x - \frac{\pi}{3}\right) = \cos 2x$

$\Rightarrow \quad x - \frac{\pi}{3} = 2n\pi \pm 2x$

$\Rightarrow \quad x = -\frac{\pi}{3}, -\frac{5\pi}{9}, \frac{\pi}{9}, \frac{7\pi}{9}$

So, sum $= -\frac{\pi}{3}, -\frac{5\pi}{9} + \frac{\pi}{9}, \frac{7\pi}{9} = 0$

70. (A)

Slope of tangent = 2

The tangents will be

$y = 2x \pm \sqrt{9\times 4 - 4}$

$\Rightarrow \quad 2x - y = \pm 4\sqrt{2}$

$$\Rightarrow \quad \frac{x}{2\sqrt{2}} - \frac{y}{4\sqrt{2}} = 1$$

And $\frac{x}{-2\sqrt{2}} + \frac{y}{4\sqrt{2}} = 1$

So, point of contact are

$$\left(\frac{9}{2\sqrt{2}}, \frac{1}{\sqrt{2}}\right) \text{ and } \left(\frac{-9}{2\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$$

i.e. (a, b) and (c, d) $\Rightarrow \quad |a+d| = \left(\frac{7}{2\sqrt{2}}\right)$

71. (C)

$S_n = C_0 + 3C_1 + 5C_2 + 7C_3 + \ldots + (2_n + 1)C_n$

$= (C_0 + C_1 + C_2 + C_3 + \ldots + C_n) + 2(C_1 + 2C_2 + 3C_3 + \ldots + nC_n)$

$$= 2^n + 2\left[n + n(n-1) + \frac{n(n-1)(n-2)}{2} + \ldots + n\right]$$

$$= 2^n + 2n\left[1 + (n-1) + \frac{(n-1)(n-2)}{2} + \ldots + 1\right]$$

$= 2^n + 2n(1 + 1)^{n-1}$

$= 2^n + n \cdot (2^n) = (n + 1) \cdot 2^n$

$\therefore \quad S_9 = 10(2^9) = 5(2^{10}) = 2^{10} \cdot 3^0 \cdot 5^1$

On comparing, we get

a = 10, b = 0

And c = 1

Now, abc = 0, a + c = 11,

A + b + c = 11, ab + bc + ca = 10

72. (A)

Let f(x) = A + Bx + cx^2 + Dx^3 + ...

$$i.e. = \begin{vmatrix} x & (1+\sin x)^3 & \cos x \\ 1 & In(1+x) & 2 \\ x^2 & (1+x)^2 & 0 \end{vmatrix}$$

$= A + Bx + cx^2 + Dx^3 + \ldots\ldots$

Differentiating both sides w.r.t. x, we get

$$\begin{vmatrix} 1 & (1+\sin x)^3 & \cos x \\ 0 & In(1+x) & 2 \\ 2x & (1+x)^2 & 0 \end{vmatrix} + \begin{vmatrix} x & 3(1+\sin x)^3 \cos x & \cos x \\ 1 & \frac{1}{1+x} & 2 \\ x^2 & 2(1+x) & 0 \end{vmatrix}$$

$$+ \begin{vmatrix} x & (1+\sin x)^3 & -\sin x \\ 1 & In(1+x) & 0 \\ x^2 & (1+x)^2 & 0 \end{vmatrix}$$

$= O + B + 2Cx + 3Dx^2 + \ldots\ldots\ldots\ldots$

Putting x = 0, then, we get

$$B = \begin{vmatrix} 1 & 1 & 1 \\ 0 & 0 & 2 \\ 0 & 1 & 0 \end{vmatrix} + \begin{vmatrix} 0 & 3 & 1 \\ 1 & 1 & 2 \\ 0 & 2 & 0 \end{vmatrix} + \begin{vmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{vmatrix}$$

$\Rightarrow$ B = (0 – 2) – (0 – 2) – 0 = 0

$\therefore$ Coefficient of x in f(x) = 0

73. (B)

Let $x_1, x_2, x_3, x_4, \ldots\ldots\ldots x_{30}$ be actual weights of 30 fishes

And $y_1, y_2, y_3, y_4, \ldots\ldots\ldots y_{30}$ be the weights of fishes taken from misaligned increasing scale.

Then, $y_i = x_i + 2$, i = 1, 2, 3, 4, …………, 30

$\Rightarrow \overline{Y} = \overline{X} + 2$ and $\sigma_y = \sigma_x$

[$\therefore$ standard deviation is independent of change of origin]

$\Rightarrow 30 = \overline{X} + 2 \, and \, \sigma_y = 2$

$\Rightarrow \overline{X} = 28 \, and \, \sigma_y = 2$

74. (A)

y = x touches $y = x^2 + bx + c$ at (1, 1)

$\therefore$ 1 = 1 + b + c $\Rightarrow$ b + c = 0 …(i)

And $\frac{dy}{dx} = 2x + b\Big|_{(1,1)} = 2 + b = 1$

[$\therefore$ slope of y = x is (i)] …(ii)

On solving Eqs. (i) and (ii), we get

b = – 1, c = 1

75. (A)

Given, $f\left(\frac{5x-3y}{2}\right) = \frac{5f(x)-3f(y)}{2}$

$\Rightarrow f\left(\frac{5x-3y}{5-3}\right) = \frac{5f(x)-3f(y)}{5-3}$

Which satisfies section formula for abscissa on LHS and ordinate on RHS. Hence, f(x) must be the linear function (as only straight line satisfies such section formula)

So, f(x) = ax + b

But $f(0) = 3 \, or \, a \times 0 + b = 3 \Rightarrow b = 3$ and $f'(0) = 2 \, or \, a = 2$

Thus, f(x) = 2x + 3

$\therefore$ Period of sin(f(x)) = Period of $\sin(2x+3) = \frac{2\pi}{|2|} = \pi$

76. (D)

Let C(h, k) be the centre and r is the radius of the given circle.

Since, it touches y = x at P, then we have

$$\frac{k-h}{\sqrt{2}} = \pm r \qquad \ldots\text{(i)}$$

As it makes an intercept $6\sqrt{2}$ on the line x + y = 0 i.e. y = – x, we have

$$SQ = 6\sqrt{2}, RS = 3\sqrt{2} \text{ and } CS = r$$

$\therefore$ $CR^2 = r^2 - 18$

$$\Rightarrow \left(\frac{k+h}{\sqrt{2}}\right)^2 = r^2 - 18$$

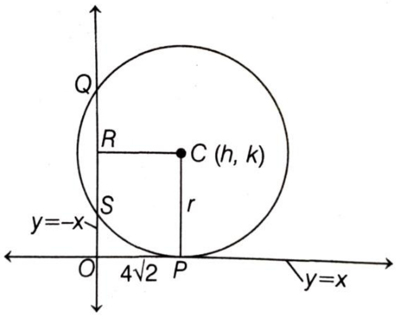
$$\Rightarrow \frac{(k+h)^2}{2} = \frac{(k-h)^2}{2} - 18$$

[from Eq. (i)]

$$\Rightarrow \frac{1}{2}(4kh) + 18 = 0$$

$\Rightarrow$ 2kh + 18 = 0

$\Rightarrow$ kh = – 9 ...(ii)



Also, $CR = \frac{|k+h|}{\sqrt{2}} = OP = 4\sqrt{2}$

$$\Rightarrow |k+h| = (4\sqrt{2})(\sqrt{2}) = 8$$

$$\Rightarrow k+h = \pm 8 \qquad \ldots\text{(iii)}$$

and then $r^2 = \frac{(k+h)^2}{2} + 18 = 50$

Also, since the given circle contains the point (– 10, 2) in its interior.

So, $(h + 10)^2 + (k - 2)^2 < 50$ ...(iv)

From Eqs. (ii) and (iii), we get

(h, k) = (– 9, 1) or (9, –1)

Since, (h, k) = (– 9, 1) only, satisfies Eq. (iv), then the required equation of the circle is

$(x + 9)^2 + (y - 1)^2 = 50$

$\Rightarrow$ $x^2 + y^2 + 18x - 2y + 32 = 0$

77. (B)

Given equation can be written as

$$\sin^{-1}\left[\cos\left(\frac{2x^2+10|x|+4}{x^2+5|x|+3}\right)\right]$$

$$=\cot\left[\cot^{-1}\left(\frac{2-18|x|}{9|x|}\right)\right]+\frac{\pi}{2}$$

$$\Rightarrow\quad \frac{\pi}{2}-\cos^{-1}\left[\cos\left(\frac{2(x^2+5|x|+3}{x^2+5|x|+3}\right)\right]$$

$$=\cot\left[\cot^{-1}\left(\frac{2}{9|x|}-2\right)\right]+\frac{\pi}{2}$$

$$-2+\frac{2}{x^2+5|x|+3}=\frac{2}{9|x|}-2$$

$$\Rightarrow\quad x^2-4|x|+3=0$$

$$\Rightarrow\quad (|x|-1)(|x|-3)=0$$

$$\Rightarrow\quad |x|=1,3$$

$$\Rightarrow\quad x=\pm1\pm3$$

$\therefore$ Product = 9

78. (B)

$$D^*(x)=\lim_{h\to0}\frac{f^2(x+h)-f^2(x)}{h}$$

$$=\lim_{h\to0}\left[\left(\frac{f(x+h)-f(x)}{h}\right)(f(x+h)+f(x))\right]$$

$$=\lim_{h\to0}\frac{f(x+h)-f(x)}{h}(f(x+0)+f(x))$$

$$=2f(x)\cdot f'(x)$$

$$\therefore\quad D^*(x\log_e x)=2x\log_e x(1+\log_e x)$$

$$\Rightarrow\quad D^*(x)\big|_{x=e}$$

$$=2\times e\times\log_e e(1+\log_e e)$$

= 4e

79. (C)

Given, $e^{|\sin x|}+e^{-|\sin x|}+4\lambda=0$ ...(i)

Let $u=e^{|\sin x|}\Rightarrow u\in[1,e]$

$$(\because|\sin x|\in[0,1].\therefore u\in[e^0,e^1])$$

From Eq. (i), $u+\frac{1}{u}+4\lambda=0$

$\Rightarrow\quad u+\frac{1}{u}=-4\lambda$ ...(ii)

Let, $f(u) = u + \dfrac{1}{u}$

$\Rightarrow \quad f'(u) = 1 - \dfrac{1}{u^2} = \dfrac{u^2 - 1}{u^2} \geq 0, \forall u \in [1, e]$

$\therefore$ f(u) is increasing function for $u \in [1, e]$

Now, $1 \leq u \leq e$

$\Rightarrow \quad f(1) \leq f(u) \leq f(e)$

[inequality remains unchanged because f is increasing function]

$\Rightarrow \quad 1 + \dfrac{1}{1} \leq f(u) \leq e + \dfrac{1}{e}$

$\Rightarrow \quad 2 \leq -4\lambda \leq \dfrac{e^2 + 1}{e}$ [from Eq. (ii)]

$\Rightarrow \quad \dfrac{-1}{2} \geq \lambda \geq \dfrac{e^2 + 1}{-4e}$

$\Rightarrow \quad \dfrac{(e^2 + 1)}{4e} \geq \lambda \geq \dfrac{-1}{2}$

$\Rightarrow \quad \lambda \in \left[\dfrac{-1 - e^2}{4e}, \dfrac{-1}{2}\right]$

80. (A)

We have,

$$\bar{x} = \frac{0 \cdot {}^{10}C_0 + 1 \cdot {}^{10}C_1 + 2 \cdot {}^{10}C_2 + \ldots + 10 \cdot {}^{10}C_{10}}{{}^{10}C_0 + {}^{10}C_1 + {}^{10}C_2 + \ldots + {}^{10}C_{10}}$$

$$= \frac{\sum_{r=0}^{10} r \cdot {}^{10}C_r}{\sum_{r=0}^{10} r \cdot {}^{10}C_r} = \frac{10 \cdot \sum_{r=0}^{10} {}^{9}C_{r-1}}{2^{10}}$$

$$= \frac{10}{2^{10}} ({}^9C_0 + {}^9C_1 + {}^9C_2 + \ldots + {}^9C_9)$$

$$= \frac{10}{2^{10}} \times 2^9 = 5$$

Now, $\dfrac{1}{N} \Sigma f_i x_i^2 = \dfrac{1}{2^{10}} \displaystyle\sum_{r=0}^{10} r^2 \cdot {}^{10}C_r$

$$= \frac{1}{2^{10}} \sum_{r=0}^{10} (r^2 - r + r)\, {}^{10}C_r$$

$$= \frac{1}{2^{10}} \left[\sum_{r=0}^{10} r(r-1)\, {}^{10}C_r + \sum_{r=0}^{10} r \cdot {}^{10}C_r \right]$$

$$= \frac{1}{2^{10}} [90 \cdot ({}^8C_0 + {}^8C_1 + {}^8C_2 + \ldots + {}^8C_8) + 10({}^9C_0 + {}^9C_1 + \ldots + {}^9C_9)]$$

$$= \frac{1}{2^{10}} [90 \times 2^8 + 10 \times 2^9)$$

$= \frac{10\times 2^8}{2^{10}}(9+2) = \frac{55}{2}$

Var $(X) = \frac{1}{N}\Sigma f_i x_i^2 - (\bar{x})^2 = \frac{55}{2} - (5)^2 = \frac{5}{2}$

81. (315)

The different possibilities of answering are as shown below

A(6)	B(5)	Number of ways
3	5	${}^6C_3 \times {}^5C_4 = 100$
4	3	${}^6C_4 \times {}^5C_3 = 150$
5	2	${}^6C_5 \times {}^5C_2 = 60$
6	1	${}^6C_6 \times {}^5C_1 = 5$
		Sum = 100 + 150 + 60 + 5 = 315

82. (240)

When a ball is dropped from the height of 48 m, the covered distance = 48 m.

The covered distance in first bounce $= \frac{2}{3}\times 48\,m$

The covered distance in second bounce $= \frac{2}{3}\times\left(\frac{2}{3}\times 48\right)m$

The covered distance in second bounce $= \frac{2}{3}\times\left(\frac{2}{3}\right)^2 \times 48\,m$

Thus, process is being infinitely.

∴ Total distance

$= 48 + 2\left[\left(\frac{2}{3}\right)48 + \left(\frac{2}{3}\right)^2\cdot 48 + \left(\frac{2}{3}\right)^3 48 + \ldots\infty\right]$

$= 48 + 2\times 48\times\frac{2}{3}\left[1 + \frac{2}{3} + \frac{2}{3} + \left(\frac{2}{3}\right)^2 + \ldots\infty\right]$

$= 48 + 64\times\frac{1}{1-\frac{2}{3}}$

$\left(\because S_\infty = \frac{a}{1-r}, for GP\right)$

$= 48 + 64\times\frac{1}{\frac{1}{3}} = 48 + 64\times 3$

= 48 + 192 = 240 m

83. (2355)

$\left(1-3x+2x^2\right)^6=\sum\frac{6!}{\alpha!\beta!\lambda!}\cdot1^{\alpha}\cdot(-3x)^{\beta}\cdot(2x^2)^{\gamma}$

Where α,β,γ are non-negative integers such that $\alpha+\beta+\gamma=6$ and $\beta+2\gamma=4$.

Possible values of α,β,γ are shown as

α	β	γ
2	4	0
3	2	1
4	0	2

$\therefore$ Coefficient of x^4 in $(1-3x+2x^2)^6$ is

$\frac{6!}{\alpha!\beta!\gamma!}\cdot1^{\alpha}\cdot(-3)^{\beta}\cdot(2)^{\gamma}$

$=\frac{6!}{2!4!0!}\cdot1^2\cdot(-3)^4\cdot(2)^0$

$+\frac{6!}{3!2!1!}\cdot1^3\cdot(-3)^2\cdot(2)^1$

$+\frac{6!}{4!0!2!}\cdot1^4\cdot(-3)^0\cdot2^2$

= 1215 + 1080 + 60 = 2355

84. (45)

Given, |a| = |b| = |2c| = 3,

$a\cdot(b+c)=a\cdot b+c\cdot a=0$

$b\cdot(c+a)=b\cdot c+a\cdot b=0$

$c\cdot(a+b)=c\cdot a+b\cdot c=0$

So, $2(a\cdot b+b\cdot c+c\cdot a)=0$

[on adding Eqs. (i), (ii) and (iii)]

Now, $|a+b+c|^2=a^2+b^2+c^2+2(a\cdot b+b\cdot c+c\cdot a)$

$=(9)+(9)+\left(\frac{9}{4}\right)+0$

$=\frac{81}{4}$

$\therefore 10|a+b+c|=10\times\frac{9}{2}=45$

85. (3)

$a\cdot p=\frac{a\cdot(b\times c)}{[a\,b\,c]}=\frac{[a\,b\,c]}{[a\,b\,c]}=1$

$$b\cdot p=\frac{b\cdot(b\times c)}{[a\,b\,c]}=\frac{0}{[a\,b\,c]}=0$$

$$b\cdot q=\frac{b\cdot(c\times a)}{[a\,b\,c]}=\frac{(b\times c)\cdot a}{[a\,b\,c]}$$

$$=\frac{a\cdot(b\times c)}{[a\,b\,c]}=\frac{[a\,b\,c]}{[a\,b\,c]}$$

$$c\cdot q=\frac{c\cdot(c\times a)}{[a\,b\,c]}=0$$

$$c\cdot r=\frac{c\cdot(a\times b)}{[a\,b\,c]}=\frac{(a\times b)\cdot c}{[a\,b\,c]}$$

$$=\frac{[a\,b\,c]}{[a\,b\,c]}=1 \text{ and } c\cdot r=0$$

$\therefore$ Given expression is equal to

1 + 0 + 1 + 0 + 1 + 0 = 3

86. (1)

Let $I=\int\sqrt[3]{\tan x}\,dx$ …(i)

Let tan x = t^3

$\Rightarrow$ $\sec^2 x\,dx = 3t^2dt$

$\Rightarrow$ $(1+\tan^2 x)\,dx = 3t^2dt$

$\Rightarrow$ $dx=\frac{3t^2}{1+t^6}dt$

$\Rightarrow$ $I=\int\frac{3t^2}{1+t^6}dt$

[from Eq. (i)]

Let $t^2 = u$ $\Rightarrow$ $tdt=\frac{1}{2}du$

$$I=\int\frac{3}{2}\left(\frac{u}{1+u^3}\right)du$$

$$=\frac{3}{2}\int\left[\frac{-1}{3}\cdot\frac{1}{u+1}+\frac{1}{3}\left(\frac{u+1}{u^2-u+1}\right)\right]du$$

$$=\frac{-1}{2}In|u+1|+\frac{3}{2}\int\frac{u+1}{u^2-u+1}du$$

$$=\frac{-1}{2}In|u+1|+\frac{1}{4}\int\frac{2u+1}{u^2-u+1}du$$

$$+\frac{3}{4}\int\frac{1}{u^2-u+1}du$$

$$=\frac{-1}{2}In|u+1|+\frac{1}{4}In|u^2-u+1|+\frac{3}{4}\int\frac{1}{\frac{3}{4}+\left(u-\frac{1}{2}\right)}du$$

$$= \frac{-1}{2} In\left|\frac{\sqrt{u^2-u+1}}{u+1}\right| + \frac{3}{4}\times\frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{u-\frac{1}{2}}{\frac{\sqrt{3}}{2}}\right)+C$$

$$= \frac{1}{2} In\left|\frac{\sqrt{u^2-u+1}}{u+1}\right| + \frac{\sqrt{3}}{2}\tan^{-1}\left(\frac{2u-1}{\sqrt{3}}\right)+C$$

Where, u = (tan x)$^{2/3}$

$\Rightarrow \quad \lambda = \frac{1}{2}$ and $\mu = \frac{\sqrt{3}}{2}$

$\therefore \quad \lambda^2 + \mu^2 = 1$

87. (5)

An equation of normal to the hyperbola

$\frac{x^2}{4} - \frac{y^2}{1} = 1$ at $(2\sec\theta, \tan\theta)$ is

$2x\sec\theta + y\cot\theta = 5$ ...(i)

Its intercepts on the axes are $\frac{5}{2\cos\theta}$ and $\frac{5}{\cot\theta}$, respectively.

The intercepts are positive and equal $0 < \theta < \frac{\pi}{2}$ *and* $\frac{5}{2\cos\theta} = \frac{5}{\cot\theta}$

$\Rightarrow \quad \sin\theta = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{6}$

From, Eq. (i) $2x\cos\frac{\pi}{6} + y\cot\frac{\pi}{6} = 5$

$\Rightarrow \quad \sqrt{3} + \sqrt{3}y = 5$

$\Rightarrow \quad y = -x + \left(\frac{5}{\sqrt{3}}\right)$

This will touch the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$,

If $\left(\frac{5}{\sqrt{3}}\right)^2 = a^2m^2 + b^2 = a^2 + b^2$

$\Rightarrow \quad \frac{3}{5}(a^2+b^2) = 5$

88. (0)

Let coordinates of P be $\left(ct_1, \frac{c}{t_1}\right)$

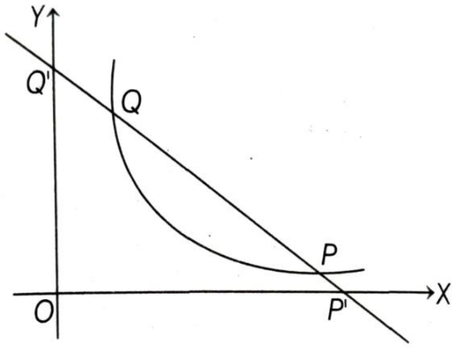
And that of Q be $\left(ct_2, \frac{c}{t_2}\right)$,

Where, $t_1t_2 > 0$

An equation of line PQ is

$$y-\frac{c}{t_1}=\frac{\frac{c}{t_2}-\frac{c}{t_1}}{ct_2-ct_1}(x-ct_1)$$

$$\Rightarrow \quad y-\frac{c}{t_1}=-\frac{1}{t_1t_2}(x-ct_1)$$



This line meets the X-axis at P'($c(t_1+t_2)$, 0) and Y-axis at

$$Q'\left(0, c\left(\frac{1}{t_1}+\frac{1}{t_2}\right)\right)$$

Now, $PP'=c\sqrt{t_2^2+\left(\frac{1}{t_1}\right)^2}$

And $QQ'=c\sqrt{t_2^2+\left(\frac{1}{t_1}\right)^2}$

$\Rightarrow$ PP' – QQ' = 0

89. (60)

1(2) 3(4) 5(6) 7(8) 9 positions four odd digits 3, 3, 5, 5 occupy four even positions $\frac{4!}{2!2!}=6$ ways

Rest five digits 2, 2, 8, 8, 8 can occupy rest positions is $\frac{5!}{2!3!}=6$ ways

$\therefore$ Required number of numbers

$$=6\times 10=60$$

90. (4)

We are given that

$$\frac{at_1^2}{at_2^2}=\mu^2 \Rightarrow t_1=\mu t_2$$

Then, $h=a\mu t_2^2, k=a(\mu t_1+t_2)$

Eliminating t_2, we get

$$k^2=a^2(\mu+1)^2t_2^2=a^2(\mu+1)^2$$

$$\left(\frac{h}{a\mu}\right)=a\left(\sqrt{\mu}+\frac{1}{\sqrt{\mu}}\right)^2 h$$

We are given, $\left(\sqrt{\mu}+\frac{1}{\sqrt{\mu}}\right)^2=\frac{25}{4}$

$\Rightarrow \quad \sqrt{\mu}+\frac{1}{\sqrt{\mu}}=\frac{5}{2}$

$\left[\therefore \sqrt{\mu}+\frac{1}{\sqrt{\mu}}=-\frac{5}{2}\right.$ Rejected because $\mu>1$ given i.e. $\sqrt{\mu}$ taken positive only]

$\Rightarrow \quad \sqrt{\mu}=2 \Rightarrow \mu=4$