PART-1 : PHYSICS

SECTION-I(i) : (Maximum Marks: 32)

- This section contains **EIGHT** questions.
- Each question has **FOUR** options for correct answer(s). **ONE OR MORE THAN ONE** of these four option(s) is (are) correct option(s).
- For each question, choose the correct option(s) to answer the question.
- Answer to each question will be evaluated according to the following marking scheme:

  Full Marks : +4 If only (all) the correct option(s) is (are) chosen.

  Partial Marks : +3 If all the four options are correct but ONLY three options are chosen.

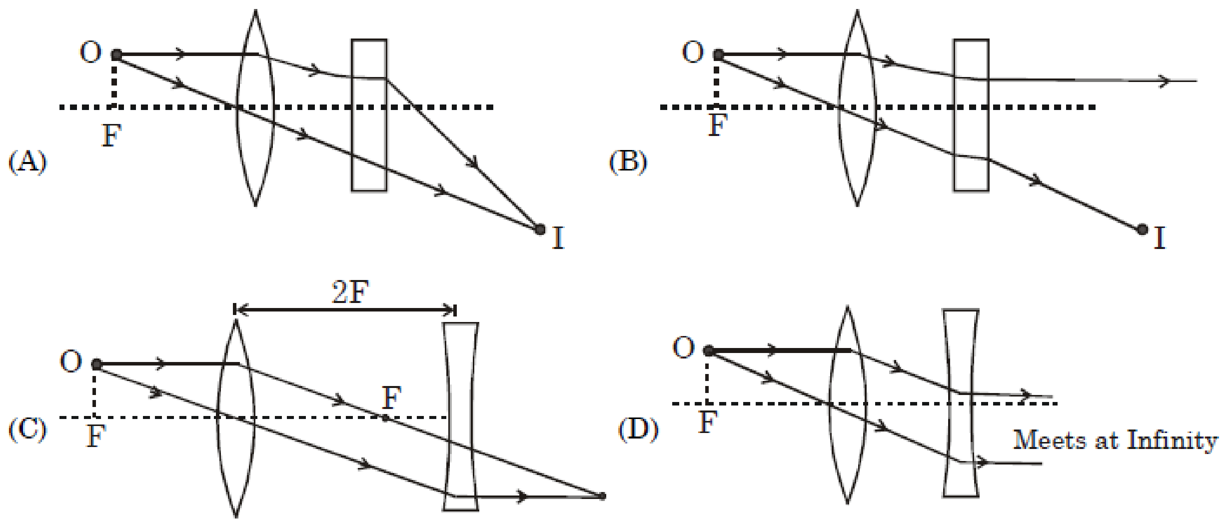
  Partial Marks : +2 If three or more options are correct but ONLY two options are chosen, both of which are correct options.

  Partial Marks : +1 If two or more options are correct but ONLY one option is chosen and it is a correct option.

  Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered).

  Negative Marks : –2 In all other cases.
- **For Example :** If first, third and fourth are the **ONLY** three correct options for a question with second option being an incorrect option; selecting only all the three correct options will result in +4 marks. Selecting only two of the three correct options (e.g. the first and fourth options), without selecting any incorrect option (second option in this case), will result in +2 marks. Selecting only one of the three correct options (either first or third or fourth option), without selecting any incorrect option (second option in this case), will result in +1 marks. Selecting any incorrect option(s) (second option in this case), with or without selection of any correct option(s) will result in –2 marks.

1\. Choose **INCORRECT** ray diagram. All symbols have their usual meaning and all the rays shown are paraxial. (focal length of each lens is F)



Ans. (A, B, C, D)

(A) Light rays should come out parallel as object as focus and it should also go out as parallel beam as slab does not converge or diverge the rays.

(B) Same as (A)

(C) Second lens is a diverging lens.

(D) Parallel beam through diverging lens should meat at focal plane virtually.

2.

In presence of a uniform magnetic field $B\hat{j}$ and a uniform electric field $(-E)\hat{k}$, a particle moves undeflectcd. Which of the following statements is (are) ***CORRECT*** ?

(A) The particle can have positive charge, velocity $=-\frac{E}{B}\hat{i}$

(B) The particle can have positive charge, velocity $=\frac{E}{B}\hat{i}$

(C) The particle can have negative charge, velocity $=-\frac{E}{B}\hat{i}$

(D) The particle can have negative charge, velocity $=\frac{E}{B}\hat{i}$

Ans. (B, D)

For underflected motion,

$$q\vec{E}+q\left(\vec{v}\times\vec{B}\right)=0$$

So,

$$q\left(-E\hat{k}\right)+q\left(\vec{v}\times B\hat{j}\right)=0$$

(B) $q>0$

$$=q\left[-E\hat{k}+\frac{E}{B}\hat{i}\times B\hat{j}\right]=q\left[-E\hat{k}+E\hat{k}\right]=0$$

It is clear it does not depend on positive or negative nature of charge.

3.

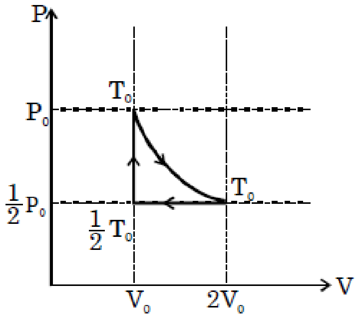
One mole of monoatomic ideal gas is initially at pressure P_0 and volume V_0. The gas then undergoes a three-stage cycle consisting of the following processes:

(i) An isothermal expansion till it reaches volume $2V_0$, and heat Q flows into the gas

(ii) An isobaric compression back to the original volume V_0

(iii) An isochoric increase in pressure till the original pressure P_0 is regained.

The efficiency of this cycle can be expressed as



The corresponding P-V diagram is shown above.

(A) $\in = \frac{4Q - 2RT_0}{4Q + 3RT_0}$ (B) $\in = \frac{4Q + 2RT_0}{4Q - 3RT_0}$ (C) $\in = \frac{4Q - 2RT_0}{4Q + RT_0}$ (D) $\in = \frac{4Q + 2RT_0}{4Q + RT_0}$

Ans. (A)

The efficiency of cyclic process is

$$\in = \frac{\text{Work done in complete process}}{\text{heat absorbed}}$$

For isothermal process

$\Delta Q = \Delta U + \Delta W$

$Q = \Delta + W$

$Q = W$

For isochoric process work done is

$$= -\frac{P_0}{2}(2V_0 - V_0) = -\frac{P_0V_0}{2}$$

$$= -\frac{1}{2}(RT_0) = -\frac{RT_0}{2}$$

(using $P_0V_0 = 1 \times RT_0$)

for iso basic process work done is zero.

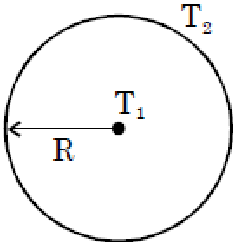
and heat absorbed is

$$Q = nCv\Delta T = 1 \times \frac{3R}{2} \times \left(\frac{T_0}{2}\right) = \frac{3RT_0}{4}$$

$$\in = \frac{Q + \left(-\frac{RT_0}{2}\right)}{Q + \frac{3RT_0}{4}} = \frac{4Q - 2RT_0}{4Q + 3RT_0}$$

4.

A sphere of radius R is having conductivity K with temperature T_1 at the centre and T_2 at the surface. Assume $T_1 > T_2$. Thermal conductivity K of the sphere varies with radius r as $K = \frac{K_0}{r^2}$. Then select **CORRECT** option(s) for steady state heat flow.



(A) Heat flows radial outwards.

(B) The heat current is $\frac{4\pi K_0(T_1 - T_2)}{R}$.

(C) Heat can flow in any random manner not necessary in radial direction.

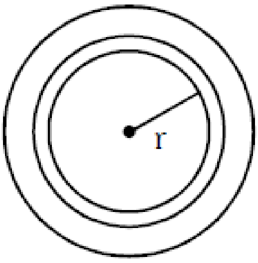
(D) The heat current is $\frac{2\pi K_0(T_1 - T_2)}{R}$.

Ans. (A, B)

For radial flow conductivity should be spherical symmetrical as given.

In steady state

$$i_H = -\left(\frac{K_0}{r^2}\right)(4\pi r^2)\left(\frac{dT}{dr}\right)$$

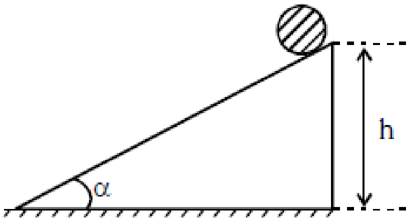


$$i_H \int_0^R dr = -4\pi k_0 \int_{T_1}^{T_2} dt$$

$$i_H = \frac{4\pi K_0(T_1 - T_2)}{R}$$

5.

A disc of mass 'm' is released from top of inclined plane of slope 'α' as shown in fig. If 'μ' is the coefficient of friction between disc and plane surface, choose the *CORRECT* statement(s)



(A) If $\mu < \frac{1}{3}\tan\alpha$, the acceleration of disc along plane is (g sinα).

(B) If $\mu > \frac{1}{3}\tan\alpha$, the net force acting on disc is $\frac{2mg\sin\alpha}{3}$.

(C) If $\mu > \frac{1}{3}\tan\alpha$, then velocity of disc at the bottom of inclined plane is $\sqrt{\frac{4gh}{3}}$.

(D) If disc rolls on inclined plane without slipping, then frictional force is $\frac{1}{3}mg\sin\alpha$.

Ans. (B,C,D)

For pure rolling

$$mg\sin\theta - f_s = ma$$

$$f_s r = \frac{mr^2}{2}\alpha$$

$a = \alpha\, r$

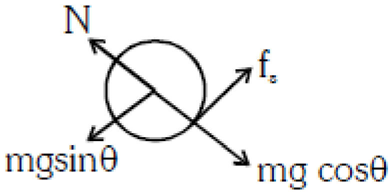
solve these question then

$$a = \frac{2g\sin\theta}{3}$$

$$f_s = \frac{mg\sin\theta}{3}$$

for no slipping

$$\frac{mg\sin\theta}{3} \le \mu mg\cos\theta$$



(A) if $\mu < \frac{\tan\theta}{3}$, slipping will be there

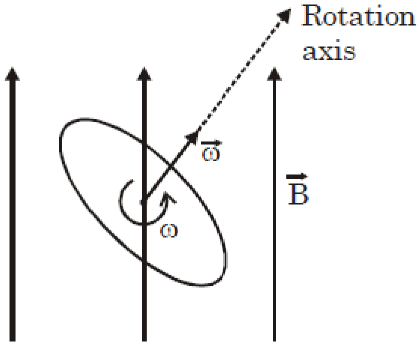
so, $a = g\sin\theta - \frac{f_k}{m} < g\sin\theta$

(C) $mgh = \frac{1}{2}mv^2 + \frac{1}{2}\frac{mv^2}{2}\left(\frac{v}{r}\right)^2$

$$v = \sqrt{\frac{4gh}{3}}$$

6.

A uniform, thin, uniformly charged disk of mass m radius R and uniform surface charge density σ rotates with angular speed ω about an axis through its centre and perpendicular to disc. The disk is in region with a uniform magnetic field B that makes angle θ with rotation axis. Mark the CORRECT statement :-



(A) Torque exerted on the disk by magnetic field is $\frac{1}{4}\pi\sigma R^4\omega B\sin\theta$

(B) Frequency with which angular velocity vector rotates is given by $\frac{\pi\sigma R^2 B}{4m}\sin\theta$.

(C) For an observer looking from above the angular velocity vector begins to rotate anticlockwise sense.

(D) For an observer looking from above the angular velocity vector begins to rotate clockwise sense.

Ans. (A, D)

$T = MB\sin\theta = \frac{d\vec{L}}{dt} = \frac{d}{dt}(L_{Horizontal})$

MB $\sin\theta = L\sin\theta\Omega$ [Ω angular velocity of $\vec{\omega}$]

$$\Omega = \frac{MB}{L}$$

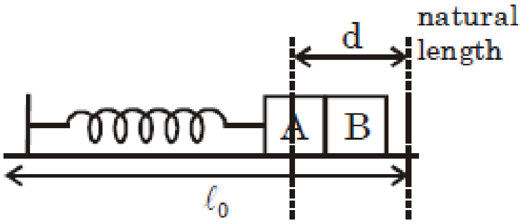
$T = MB\sin\theta = \frac{1}{4}\pi\sigma R^4\omega B\sin\theta$ [M is magnetic moment]

Frequency $= \frac{\Omega}{2\pi} = \frac{MB}{2\pi L} = \frac{\pi\sigma R^2 B}{2M}$

and Ω is in clockwise direction as seen from above

7.

Block A has mass m_A and is attached to a spring. Another block B, having mass m_B is pressed against A so that the spring deforms a distance d. Coefficient of friction between blocks and the ground is μ. x represents the elongation or comprension of spring.



(A) If blocks get separated then x must be the comprension of spring.

(B) If blocks get separated then x = 0.

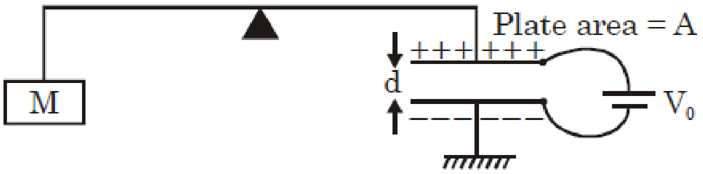
(C) For the blocks to get separated, $d < \frac{2\mu g(m_A + m_B)}{k}$

(D) For the blocks to get separated, $d > \frac{2\mu g(m_A + m_B)}{k}$

Ans. (B,D)

8.

A capacitance balance is shown in figure. The balance has a weight attached on one side and a capacitor that has a variable gap width on other side. Assume the upper plate of the capacitor has negligibles mass. When the capacitor potential difference between plates is V_0. The attractive force between the plates balances the weight of the hanging mass :-



(A) Equilibrium of weight is stable

(B) Equilibrium of weight is unstable

(C) Value of V_0 required to balance weight is given by $V_0 = d\sqrt{\frac{2Mg}{\epsilon_0 A}}$

(D) For a small displacement block of mass M executes simple harmonic motion.

Ans. (B, C)

$$F = \left(\frac{Q}{2A\epsilon_0}\right)d = \frac{Q^2}{2A\epsilon_0} = \frac{C^2V^2}{2A\epsilon_0} = \frac{V_0^2\epsilon_0^2}{2A\epsilon_0}\frac{A^2}{d^2}$$

$$F = \frac{V_0^2\epsilon_0 A}{2d^2}$$

If d increases, F decreases $\Rightarrow$ unstable equilibrium.

$$\frac{V_0^2\epsilon_0 A}{2d^2} = mg \quad \Rightarrow V_0 = d\sqrt{\frac{2mg}{\epsilon_0 A}}$$

SECTION-II : (Maximum Marks : 16)

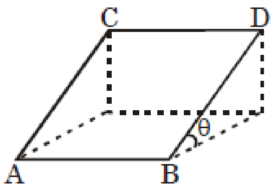
- This section contains TWO paragraphs.
- Based on each paragraph, there will be TWO questions
- Each question has FOUR options (A), (B), (C) and (D) ONE OR MORE THAN ONE of these four option(s) is(are) correct.
- For each question, darken the bubble(s) corresponding to all the correct option(s) in the ORS.
- For each question, marks will be awarded in one of the following categories :

Full Marks : +4 If only the bubble(s) corresponding to all the correct option(s) is (are) darkened.

Zero Marks : 0 If none of the bubbles is darkened.

Negative Marks : –2 In all other cases.

According to quantum mechanics a particle of momentum p can be regarded as a plane matter wave with wavelength $\lambda = \frac{h}{p}$, where h is planck constant. As shown in figure ABCD is a flat square of side length L at an inclination angle θ to the horizontal plane. A neutron beam of initial kinetic energy E_0 is divided into two beams at point A. One beam moves along path ACD and the other beam along path ABD. When the two beams meet at point D they interfere. The mass of neutron is m.



9.

Phase difference for the matter waves in taking two paths is :-

(A) $\frac{2\pi L}{h}\sqrt{2m(E_0 - mgL\sin\theta)}$

(B) $\frac{2\pi L}{h}\left(\sqrt{2mE_0} - \sqrt{2m(E_0 - mgL\sin\theta)}\right)$

(C) $\frac{2\pi L}{h}\sqrt{2mE_0}$

(D) No phase difference occurs between two beams reaching point D

Ans. (B)

10.

How many times can one get maximum neutron number reading at point D when θ changes from 0° to 90° ?

(A) $\frac{L}{h}\left(\sqrt{2m(E_0 - mgL)} - \sqrt{2mE_0}\right)$

(B) $\frac{L}{h}\sqrt{2m(E_0 - mgL)}$

(C) $\frac{L}{h}\sqrt{2mE_0}$

(D) There is no variation in reading of neutron number at point D

Ans. (A)

PARAGRAPH 11 AND 12

Photons from the surface of the Sun and neutrinos from its core can tell us about solar temperature and also confirm that the Sun shines because of nuclear reactions.

Throughout this problem, take the mass of the Sun to be $M_{\odot} = 2.00 \times 10^{30}$ kg, its radius, $R_{\odot} = 7.00 \times 10^{8}$ m, its luminosity (radiation energy emitted per unit time), $L_{\odot} = 3.85 \times 10^{26}$ W, and the Earth-Sun distance, $d_{\odot} = 1.50 \times 10^{11}$ m.

In 1938, Hans Bethe proposed that nuclear fusion of hydrogen into helium in the core of the Sun is the source of its energy. The net nuclear reaction is :

$$4\,^{1}H \rightarrow {}^{4}He + 2e^{+} + 2\upsilon_{e}$$

[Energy released in this reaction is $\Delta E = 4.0 \times 10^{-12}$ J]

The "electron neutrinos", υ_e, produced in this reaction may be taken to be massless. They escape the Sun and their detection on the Earth confirms the occurrence of nuclear reactions inside the Sun. Energy carried away by the neutrinos can be neglected in this problems.

Travelling from the core of the Sun to the Earth, some of the electron neutrinos, υ_e, are converted to other types of neutrinos, υ_x. The efficiency of the detector for detecting υ_x is 1/6 of its efficiency for detecting υ_e. If there is no neutrino conversion, we expect to detect an average of N_1 neutrinos in a year. However, due to the conversion, an average of N_2 neutrinos (υ_e and υ_x combined) are actually detected per year.

11. Choose the **CORRECT** statement(s) :

(A) Assuming Sun radiates like a perfect black body, temperature of sun's surface is approximately 5.76×10^{3} K

(B) Assuming Sun radiates like a perfect black body, temperature of sun's surface is approximately 5.76×10^{4} K

(C) Assuming Sun radiates like a perfect black body, momentum of photons with maximum spectral emissive power radiated from surface of sun is 1.32×10^{-27} kg m/s.

(D) Assuming Sun radiates like a perfect black body, momentum of photons with maximum spectral emissive power radiated from surface of sun is 1.32×10^{-28} kg m/s.

Ans. (A,C)

12. Choose the CORRECT statement(s) :

(A) The flux density, Φ_{υ}, of the number of neutrinos arriving at the Earth is 6.8×10^{14} m^{-2} s^{-1}

(B) The flux density, Φ_{υ}, of the number of neutrinos arriving at the Earth is 3.4×10^{14} m^{-2} s^{-1}

(C) The fraction f of υ_e converted to υ_x is given by $\frac{6}{5}\left(1 - \frac{N_2}{N_1}\right)$

(D) The fraction f of υ_e converted to υ_x is given by $\frac{5}{6}\left(1 - \frac{N_1}{N_2}\right)$

Ans. (A,C)

SECTION-III: (Maximum Marks: 18)

- This section contains SIX questions.
- The answer to each question is a NUMERICAL VALUE.
- For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the **second decimal place**; e.g. 6.25, 7.00, –0.33, –.30, 30.27, –127.30, if answer is 11.36777..... then both 11.36 and 11.37 will be correct)
- Answer to each question will be evaluated according to the following marking scheme:
  Full Marks : +3 If ONLY the correct numerical value is entered as answer.
  Zero Marks : 0 If none of the bubbles is darkened.
  Negative Marks : –1 In all other cases

13.

Two travelling waves $3\cos(kx+\omega t)$ and $5\cos(kx-\omega t)$ are travelling in -ve x-axis and +ve x-axis respectively on a string. The magnitude of average energy passing through a point on string per second for resulting wave is of the form $\left(\frac{A_0\rho\omega^3 S}{k}\right)$. Calculate the value of A_0. Density of string is ρ and cross-sectional area is S.

Ans. 8.00

Resulting wave is

$y = 3\cos(kx-\omega t) + 5\cos(kx + \omega t)$

The instanteneous power for a wave on string

$$P = -T\left(\frac{\partial y}{\partial t}\right)\left(\frac{\partial y}{\partial x}\right)$$

$P = -\mu v^2\ (+3\omega\sin(kx-\omega t)-5\omega\sin(kx+\omega t))x$
$(-3k\ \sin(kx-\omega t)-5k\ \sin(kx + \omega t)$

$$P = \mu v^2\omega k\left[9\sin^2(kx-\omega t)-25\sin^2(kx+\omega t)\right]$$

$$\langle P\rangle = \mu v^2\omega k\left[9\langle\sin^2(kx-\omega t)\rangle-25\langle\sin^2(kx+\omega t)\rangle\right]$$

$$= \mu v^2\omega k\left[\frac{9}{2}-\frac{25}{2}\right] = -8\rho v^2\omega k$$

$$= 8\rho S\left(\frac{\omega}{k}\right)^2\omega k = -\frac{8\rho S w^3}{k}$$

14.

In Young's double slit experiment set-up with light of wavelength $\lambda = 6000$Å, distance between the two slits is 2mm and distance between the plane of slits and the screen is 2m. The slits are of equal width. When a sheet of glass of refractive index 1.5 (which permits only a fraction η of the incident light to pass through) and thickness 8000Å is placed in front of the lower slit, it is observed that the intensity at a point P, 0.15 mm above the initial central maxima does not change. Find the value of η.

Ans. 0.20 to 0.22

Intensity at point 0.15 mm above central maxima in original system.

for that point

$$\Delta x = \frac{dy}{D} = \frac{2\times10^{-3}\times0.15\times10^{-3}}{2}$$

$= 1.5 \times 10^{-7}$ m

phase difference

$$\Delta Q = \frac{2\pi}{\lambda}\Delta x = \frac{2\pi}{6000\times10^{-10}}\times\frac{3}{2}\times10^{-7} = \frac{\pi}{2}$$

Initially intensity is same so intensity

$I_R = 2I_0\,(1 + \cos\Delta\theta) = 2I_0$

Now if a sheet is placed the Δx of that point

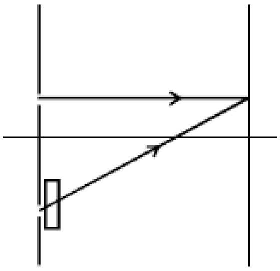
$$\Delta x = \frac{dy}{D} + (\mu - 1)t$$

$$= 1.5\times10^{-7} + \left(\frac{3}{2}-1\right)8000\times10^{-10}$$

$$= 1.5 \times 10^{-7} + 4 \times 10^{-7} = 5.5 \times 10^{-7}\text{ m}$$

$$\text{phase diff. } \Delta Q = \frac{2\pi}{6000\times10^{-10}}\times\frac{11}{2}\times10^{-7}$$

$$= \frac{11\pi}{6}$$



no sheet allow only η fraction so to get same intensity

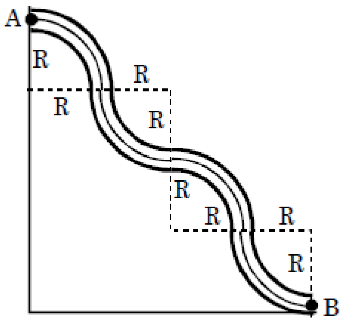
$$I + \eta I + 2\sqrt{\eta}\,I\cos\left(\frac{11\pi}{6}\right) = 2I_0$$

$$\eta^2 - 5\eta + 1 = 0$$

$$\eta = \frac{5-\sqrt{21}}{2} = 0.21$$

15.

A smooth fixed pipe is made-up of four quarter circular segments attached as shown in the figure. A rope of length equal to the length of the pipe is passed through the pipe and is hinged at point A. If the initial acceleration of the rope when released from rest by removing hinge at A is Xg, find the value of 2X.

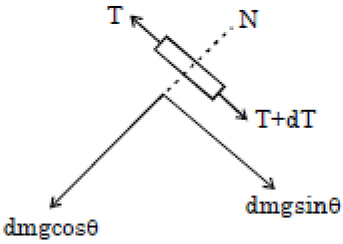


Ans. 1.27

For any small element force analysis just after release.

dmgsinθ + dT = dma

mass of element can be written as



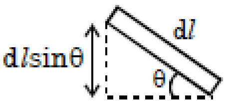
$dm = \lambda dl$

$\lambda g\, dl \sin\theta + dT = dma$

$\lambda g\, dy + dT = dma$

$$\lambda g \int_0^{4R} dy + \int_0^{0} dT = a \int_0^{M} dm$$

$\lambda g(4R) = Ma$



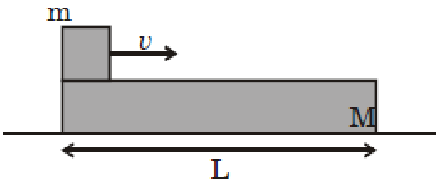
$\lambda g(4R) = \lambda(2\pi R)a$

$$\therefore \quad x = \frac{2}{\pi}$$

$$2 \times \frac{2}{\pi} = 1.27$$

12.

As shown in the figure below, a rectangular block of mass M and length L is placed on a smooth table. A small cube of mass m with negligible length is placed on the left end of the block with horizontal initial velocity v sliding to the right. The coefficient of kinetic friction between the block and the cube is μ, and the cube does not fall from the right end of the block. What is the displacement (in m) of the rectangular block when the cube stops sliding on the rectangular block exactly at its right most end ? (Take m = 2kg , M = 6kg and L = 10m)



Ans. 2.50

Initially, the velocity of the cube and rectangle block are v and 0 respectively. The kinetic friction is $f_k = \mu mg$ and the acceleration on the cube and the block are

$$a_m = -\mu g \text{ and } a_M = \frac{\mu mg}{M}$$

$$v_m = -\mu gt$$

$$v_M = \frac{\mu mgt}{M}$$

They will accelerate until they have the same velocity when the cube is at the right end, i.e.,

$$v - \mu gt = \frac{\mu mgt}{M} \Rightarrow \left(\frac{m}{M} + 1\right)\mu gt = v$$

$$\Rightarrow t = \frac{v}{\mu g}\left(\frac{1}{1 + \frac{m}{M}}\right)$$

At this time the position of the cube and the right end of the block are:

$$x_m = vt + \frac{1}{2}a_m t^2 = \frac{v^2}{\mu g}\left(\frac{1}{1+\frac{m}{M}}\right) - \frac{1}{2}\frac{v^2}{\mu g}\left(\frac{1}{1+\frac{m}{M}}\right)^2$$

$$= \frac{v^2}{\mu g}\left(\frac{1}{1+\frac{m}{M}}\right)\left(1 - \frac{1}{2}\left(\frac{1}{1+\frac{m}{M}}\right)\right)$$

$$x_m = L + \frac{1}{2}a_M t^2 = L + \frac{1}{2}\frac{m}{M}\frac{v^2}{\mu g}\left(\frac{1}{1+\frac{m}{M}}\right)^2$$

If they coincide at time t,

$x_m = x_M$

$$\Rightarrow \frac{v^2}{\mu g}\frac{1}{\left(1+\frac{m}{M}\right)^2}\left(\frac{1}{2}+\frac{m}{M}-\frac{1}{2}\frac{m}{M}\right) = L$$

$$\Rightarrow \frac{v^2}{2\mu g} = L\left(1+\frac{m}{M}\right) \Rightarrow v = \sqrt{2\mu g L\left(1+\frac{m}{M}\right)}$$

The displacement of the block is

$$d = \frac{1}{2}a_M t^2 = \frac{1}{2}\frac{m}{M}\frac{v^2}{\mu g}\left(\frac{1}{1+\frac{m}{M}}\right)^2$$

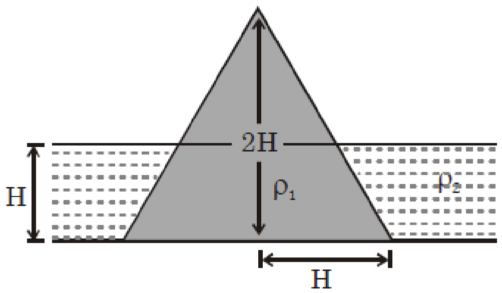
$$= \frac{m}{2M\mu g}\left(\frac{1}{1+\frac{m}{M}}\right)^2 \times 2\mu g L\left(1+\frac{m}{M}\right)$$

$$d = \frac{L\frac{m}{M}}{1+\frac{m}{M}} = \left(\frac{m}{m+M}\right)L$$

$$= \frac{2\times 10}{2+6} = \frac{20}{8} = 2.50\text{m}$$

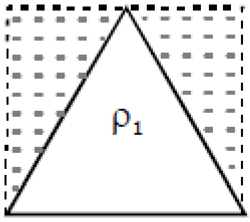
13.

A Liquid ρ_1 is filled inside the cone, liquid ρ_2 is filled outside upto height H as shown in the figure. The light cone is in equilibrium under the action of hydrostatic forces of two liquids of densities ρ_1 and ρ_2. Find $\frac{\rho_2}{\rho_1}$.



Ans. 3.20

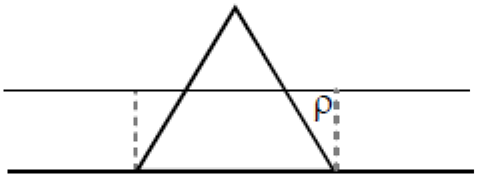
For equilibrium force on cone due to ρ_1 and ρ_2 must be equal for force of ρ_1



upward force due to ρ_1 is equal to weight of liquid in shaded region.

$$F_{up} = \frac{2}{3}\pi H^2(2H)\rho_1 g$$

Downward force of ρ_2 is equal to weight of ρ_2 above cone



$F_{down} = \rho_2$ vg where v is volume of ρ_2

Volume ρ_2 is

$v = \pi H^2 \times H - v_{cone}$

Here V_{cone} is volume of cone up to height H from base.

$v_{cone} = v_{total} - v'_{cone}$

v_{total} = total volume of cone $= \frac{1}{3}\pi H^2(2H)$

v'_{cone} = volume of cone above

$$\rho_2 = \frac{1}{3}\pi\left(\frac{H}{2}\right)^2 H = \frac{1}{3}\pi\left(\frac{H}{2}\right)^2 H$$

$$v_{cone} = \frac{2}{3}\pi H^3 - \frac{\pi H^3}{12} = \frac{7}{12}\pi H^3$$

Now $v = \pi H^3 - \frac{7}{12}\pi H^3 = \frac{5}{12}\pi H^3$

$$F_{down} = \rho_2\left(\frac{5}{12}\pi H^3\right)g$$

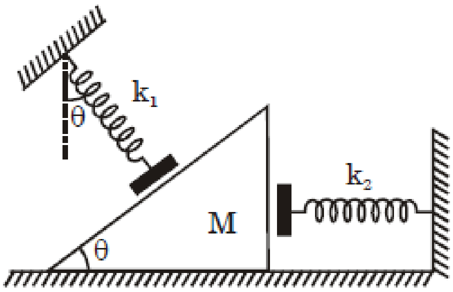
for equilibrium

$F_{up} = F_{down}$

$$\frac{\rho_2}{\rho_1} = \frac{16}{5}$$

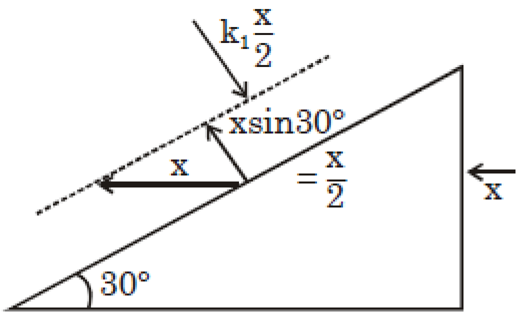
14.

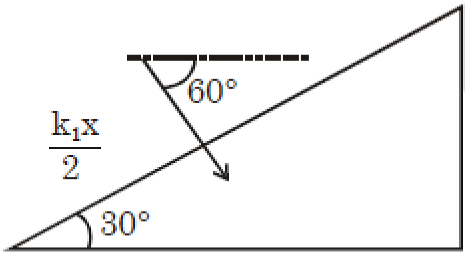
A wedge block of mass M is placed on a smooth surface and kept in contact of two unattached springs of force constants k_1 and k_2. If springs are constrained to deform along length only. Find the time period of small oscillation of wedge on horizontal plane. (given : M = 4 kg, $k_1 = 16\pi^2$ N/m, $k_2 = \pi^2$ N/m & $\theta = 30°$)



Ans. 3.00

(A) On displacing the wedge by distance x towards left

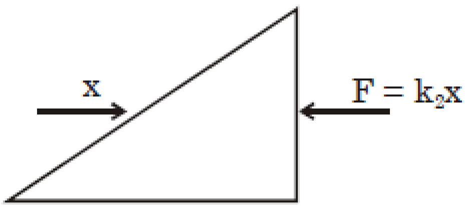




$$\therefore F_{net} = \left(\frac{k_1 \cos 60°}{2}\right)x = \left(\frac{k_1}{4}x\right)$$

$$\text{Half time period } = \pi\sqrt{\frac{m}{\frac{k_1}{4}}} = \pi\sqrt{\frac{4m}{k_1}}$$

(B) On displacing towards right



$F_{net} = k_2x \Rightarrow$ Half time period $= \pi\sqrt{\frac{m}{k_2}}$

(C) So, total time period

$$\pi\sqrt{\frac{4m}{k_1}} + \pi\sqrt{\frac{m}{k_2}}$$

$$= \pi\sqrt{\frac{4 \times 4}{16\pi^2}} + \pi\sqrt{\frac{4}{\pi^2}}$$

$= 1 + 2 = 3$ seconds

PART-2 : CHEMISTRY

SECTION-I : (Maximum Marks: 32)

- This section contains **EIGHT** questions.
- Each question has **FOUR** options for correct answer(s). **ONE OR MORE THAN ONE** of these four option(s) is (are) correct option(s).
- For each question, choose the correct option(s) to answer the question.
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks : +4 If only (all) the correct option(s) is (are) chosen.
Partial Marks : +3 If all the four options are correct but ONLY three options are chosen.
Partial Marks : +2 If three or more options are correct but ONLY two options are chosen, both of which are correct options.
Partial Marks : +1 If two or more options are correct but ONLY one option is chosen and it is a correct option.
Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered).
Negative Marks : –2 In all other cases.

- **For Example :** If first, third and fourth are the **ONLY** three correct options for a question with second option being an incorrect option; selecting only all the three correct options will result in +4 marks. Selecting only two of the three correct options (e.g. the first and fourth options), without selecting any incorrect option (second option in this case), will result in +2 marks. Selecting only one of the three correct options (either first or third or fourth option), without selecting any incorrect option (second option in this case), will result in +1 marks. Selecting any incorrect option(s) (second option in this case), with or without selection of any correct option(s) will result in –2 marks.

---

1. Select the correct statments (s)

(A) End centred tetragonal (a × a × c) is equivalent to primitive tetragonal $\left(\frac{a}{\sqrt{2}} \times \frac{a}{\sqrt{2}} \times c\right)$

(B) Face centred tetragonal (a × a × c) is equivalent to body centre tetragonal $\left(\frac{a}{\sqrt{2}} \times \frac{a}{\sqrt{2}} \times c\right)$

(C) End centred (base-centred) cubic unit cell (a × a × a) is equivalent to simple tetragonal uunit cell $\left(\frac{a}{\sqrt{2}} \times \frac{a}{\sqrt{2}} \times a\right)$.

(D) Additional lattice point at body centre of hexagonal unit cells destroys 6-fold axis of symmetry.

Ans. (A,B,C,D)

2. Select the correct statement(s) -

(A) Over a boundary surface, value of probability density $|\Psi|^2$ is constant for an orbital.

(B) Probability of finding an electron is 100 % in an orbital

(C) Number of angular nodes are n - l –1

(D) For 2s orbital the radial probality density is maximum at the nucleus.

Ans. (A,D)

(A) Fact

(B) Probability of finding an electron is nearly 90% in an orbital

(C) No of angular nodes are *l*

(D) For 1s $|\Psi|^2$ is maximum at nucleus

3.

Select ore(s) in which at least **three ions** are present out of CO_3^{2-} , S^{2-}, OH^- , Copper ion , Iron ion

(A) Malachite (B) Chalcopyrite (C) Siderite (D) Cuprite

Ans. (A, B)

4.

Select which is/are Amphotheric oxides :-

(A) Chromium(III) oxide

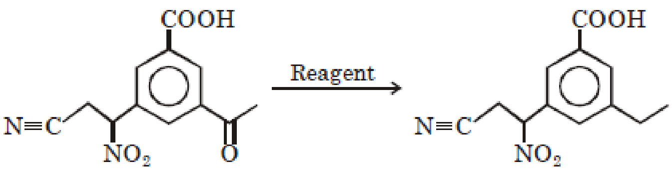
(B) Manganese(VII) oxide

(C) Lead(II) oxide

(D) Iron(III) oxide

Ans. (A, C)

5.



Above conversion is carried out with the help of :

(A) $LiAlH_4$

(B) $NaBH_4$

(C) $NH_2–NH_2$ / $OH^{\ominus}$ / Δ , then $H^{\oplus}$

(D) Zn–Hg / HCl

Ans. (C)

6.

1 litre aq. solution containing sufficient sodium salt of 2,3 dimethyl butan-1,4dioicacid is electrolysed in an electrolytic cell using 9.65 mA current for 2×10^6 sec using inert Pt electrode with 100% current efficiency. Select the correct option(s) -

(A) Mass of hydrocarbon formed at anode is 5.6 gm

(B) Mass of hydrocarbon formed at anode is 11.2 gm

(C) Gas evolved at cathode occupies 2.27 litre at STP

(D) Gas evolved at cathode occupies 1.135 litre at STP

Ans.(A,C)

7. For the reaction : A(g) +heat $\rightleftharpoons$ B(g) + C(g) ; $K_p^0 = 0.1$ at 298 K. Select the correct option(s)

(A) ΔG for the reaction at 1 atm & 298K is zero.

(B) ΔG for the reaction at 1 atm & 298K may be zero, postive or negative

(C) ΔG^0 for the reaction at 1 atm & 298K is postive

(D) $(\Delta G^0)_{1atm,298K} < (\Delta G^0)_{1atm,398K}$

Ans.(B,C)

$\Delta G^0 = RTlnKp$

$\Delta G^0 = \Delta G^0 + RTlnQ$

8.

Select reaction in which chemical composition of product precipitate is changed when exposed to air for some time :-

(A) $BaCl_2(aq.) + Na_2SO_3\ (aq.) \rightarrow$ **ppt - 1**

(B) $FeSO_4\ (aq.) + NaOH\ (aq.) \rightarrow$ **ppt - 2**

(C) $BaS + ZnSO_4\ (aq.) \rightarrow$ **ppt - 3**

(D) $MnSO_4\ (aq.) + NaOH\ (aq.) \rightarrow$ **ppt - 4**

Ans.(A, B, D)

SECTION–II : (Maximum Marks : 16)

- This section contains TWO paragraphs.
- Based on each paragraph, there will be TWO questions
- Each question has FOUR options (A), (B), (C) and (D) ONE OR MORE THAN ONE of these four option(s) is(are) correct.
- For each question, darken the bubble(s) corresponding to all the correct option(s) in the ORS.
- For each question, marks will be awarded in one of the following categories :

Full Marks : +4 If only the bubble(s) corresponding to all the correct option(s) is (are) darkened.

Zero Marks : 0 If none of the bubbles is darkened.

Negative Marks : –2 In all other cases.

PARAGRAPH FOR QUESTIONS 15 AND 16

- **Salt - S** $\xrightarrow{\text{cold and dil. HCl}}$ **Gas (G) + Transparent clear solution**
- **Gas (G)** $\longrightarrow$ Colourless, odourless which produces insoluble salt when passed in lime water but no redox reaction with acidic $KMnO_4$
- **Transparent clear solution** $\xrightarrow{H_2S}$ **Black ppt**

9.

Black ppt can be :-

(A) Ag_2S (B) PbS (C) CuS (D) NiS

Ans. (C)

10. Select **CORRECT** statment(s) :-

(A) Black ppt is insoluble in YAS (yellow ammonium sulphide)

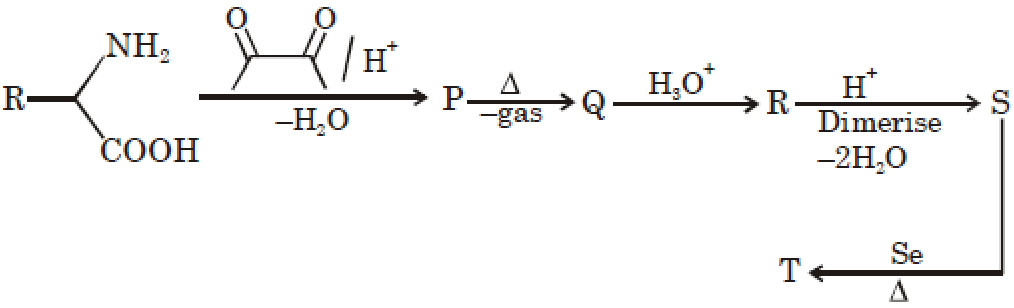
(B) Salt-S have IInd group cation

(C) Gas - G produce soluble salt in KOH (Aq.)

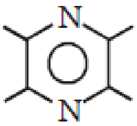
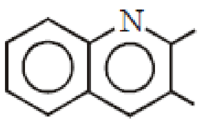
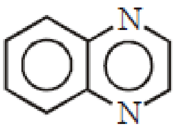
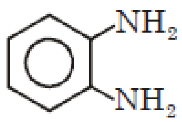
(D) Gas - G turns red litmus paper blue

Ans. (A, B, C)

PARAGRAPH FOR QUESTIONS 11 AND 12

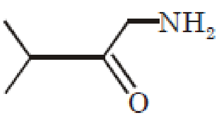
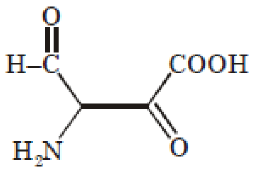
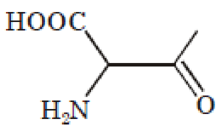
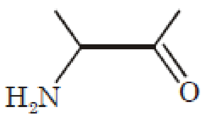


11. Identify 'T' product is above sequence ?

(A) 
(B) 
(C) 
(D) 

Ans. (A)

12. Identify 'R' product in above sequence ?

(A) 
(B) 
(C) 
(D) 

Ans. (D)

SECTION-III: (Maximum Marks: 18)

- This section contains **SIX** questions.
- The answer to each question is a **NUMERICAL VALUE.**
- For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the **second decimal place**; e.g. 6.25, 7.00, –0.33, –.30, 30.27, –127.30, if answer is 11.36777..... then both 11.36 and 11.37 will be correct)
- Answer to each question will be evaluated according to the following marking scheme:
  Full Marks : +3 If ONLY the correct numerical value is entered as answer.
  Zero Marks : 0 If none of the bubbles is darkened.
  Negative Marks : –1 In all other cases

13. The enthalpy of atomisation of $P_4O_6(s)$ is 3500 kJ/mol. If P–O bond enthalpy is 250 kJ/mol and enthalpy of sublimation of $P_4O_6(s)$ is 'x' kJ/mol, the value of $\frac{x}{500}$ is

Ans. 1

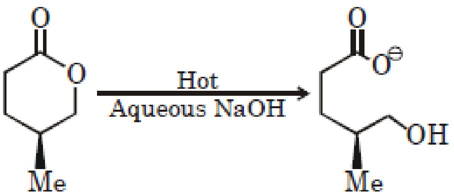
14.

Find the number of complexes/ion which can show atleast two kind of isomerism.

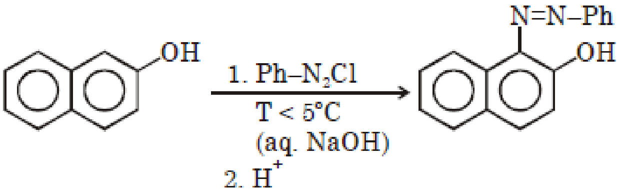
$[Pt(en)_2Cl_2]^{+2}$, $[Pt(en)Cl_2]^0$, $[Co(H_2O)_4Cl_2]Cl\cdot 2H_2O$, $[Fe(NH_3)_4(NO_2)_2]^+$, $[Cl(PPh_3)PtCl_2PtCl(PPh_3)]$, $[Co(en)_3][Co(NH_2)_6]$

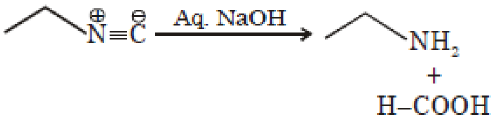
Ans. 5

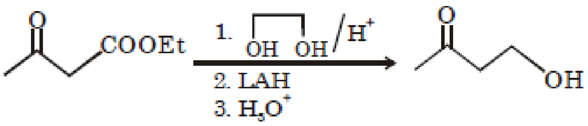
15. Identify reactions correctly matched with their major product ?

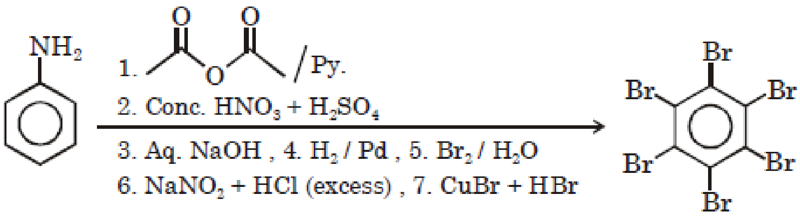
(a) 

(b) 

(c) 

(d) 

(e) 

(f) 

Ans. 4

16.

20 ml NaOH solution is needed for complete reaction with 1.25 millimoles Cl_2 in cold conditions.

The concentration of NaOH solution (in g/L) is :

Ans. 5

17.

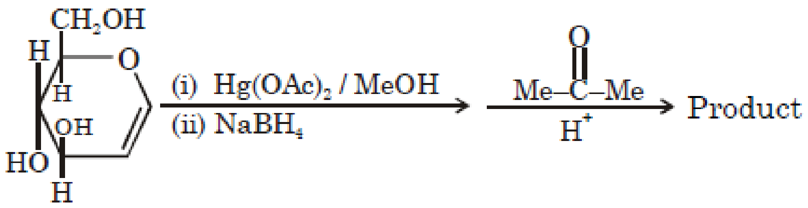
Which among the following can show more than three types of positive oxidation states.

Sc , Zn , Fe , Cr , Mn , V , Pb , Cu

Ans. 4

18.

Identify total number of chiral centre present in product of following reaction :



Ans. 4

PART-3 : MATHEMATICS

SECTION-I : (Maximum Marks: 32)

- This section contains **EIGHT** questions.
- Each question has **FOUR** options for correct answer(s). **ONE OR MORE THAN ONE** of these four option(s) is (are) correct option(s).
- For each question, choose the correct option(s) to answer the question.
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks : +4 If only (all) the correct option(s) is (are) chosen.
Partial Marks : +3 If all the four options are correct but ONLY three options are chosen.
Partial Marks : +2 If three or more options are correct but ONLY two options are chosen, both of which are correct options.
Partial Marks : +1 If two or more options are correct but ONLY one option is chosen and it is a correct option.
Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered).
Negative Marks : –2 In all other cases.

- **For Example :** If first, third and fourth are the **ONLY** three correct options for a question with second option being an incorrect option; selecting only all the three correct options will result in +4 marks. Selecting only two of the three correct options (e.g. the first and fourth options), without selecting any incorrect option (second option in this case), will result in +2 marks. Selecting only one of the three correct options (either first or third or fourth option), without selecting any incorrect option (second option in this case), will result in +1 marks. Selecting any incorrect option(s) (second option in this case), with or without selection of any correct option(s) will result in –2 marks.

1.

Consider a plane P : $6x + 3y + 2z = 6$ and P_1 is an other plane parallel with P and passing through centre of inscribed sphere of tetrahedron formed by plane P and co-ordinate planes, then

(A) point $\left(\frac{1}{3},\frac{1}{3},\frac{1}{3}\right)$ can be centre of a sphere which touches plane P and all three co-ordinate plane is 1^{st} octant $(x, y, z > 0)$.

(B) point $\left(\frac{3}{2},\frac{3}{2},\frac{3}{2}\right)$ can be centre of a sphere which touches plane P and all three co-ordinate plane is 1^{st} octant $(x, y, z > 0)$.

(C) volume bounded between P and P_1 in 1^{st} Octant is $\frac{4501}{5832}$ unit^3.

(D) distance between P and P_1 is $\frac{3}{2}$ units

Ans. (A,B,C)

Centre of sphere will be (α, α, α) and at distance α units from P

$\Rightarrow \quad 11\alpha - 6 = \pm 7\alpha \Rightarrow \alpha = \frac{1}{3}, \frac{3}{2}$

centre of inscribed sphere is $\left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right)$

$\Rightarrow$ equation of P_1 $\quad 6x + 3y + 2z = \frac{11}{3}$

Distance between P & $P_1 = \frac{6 - \frac{11}{3}}{7} = \frac{1}{3}$

Volume $= \frac{1}{6}\left(\frac{6}{7}\sqrt{36+9+4} - \frac{11}{21}.\frac{121}{18.9.6}\sqrt{36+9+4}\right)$

$= \frac{4501}{5832}$

2.

If three 3×3 invertible matrices A, B, C are Idempotent, Involutary and Orthogonal matrices respectively, then -

(A) $(ABC)^{-1} = (AB^TC)^T$

(B) $|adj(2AB^{-1}C)| = 8$

(C) If $(ABC)^{-1} = (CBA)^{-1}$, then $BC = CB$

(D) $adj(3A^{-1}BC^{-1}) = 9adj((C(B^T)^{-1}A)^T)$

Ans. (A,C,D)

$A^2 = A$; $B^2 = I$ and $CC^T = I = C^TC$

$\Rightarrow \quad A = I = A^{-1} = A^T$; $B = B^{-1}$ and $C^T = C^{-1}$

(A) $(ABC)^{-1} = C^{-1}B^{-1} = C^TBA^T = (AB^TC)^T$

(B) $|adj(2AB^{-1}C)|$

$= 64|adjC|\,|adjB|\,|adjA| = \pm 64$

(C) $C^{-1}B^{-1}A^{-1} = A^{-1}B^{-1}C^{-1}$

$\Rightarrow C^TB = BC^T = BC = CB$

(D) $adj(3A^{-1}BC^{-1}) = 9adj(A^TB^{-1}C^T)$

3.

Consider series $f(x) = 1 + 3\sin^2 x + 5\sin^4 x + \ldots\ldots\ \infty$, $g(x) = \cos x + \cos 3x + \cos 5x + \ldots\ldots$ n terms, then

(A) $\int f(x)dx = \tan x + \frac{2\tan^3 x}{3} + C$ (where C is constant of integration, $x \neq (2n+1)\frac{\pi}{2}$)

(B) $\lim_{x \to 0}(f(x))^{\frac{\operatorname{cosec}^2 x}{g^2(x)}} = e^{\frac{3}{n^2}}$

(C) If $I_{2n} = \int_0^{\pi/2} g(x)dx$, then $\frac{1}{5}$ is maximum value of $(I_{2n+2} - I_{2n})$ for $n \in [1, 6]$, $n \in N$

(D) If $x \in [0, \pi]$ and $n = 4$, then equation $g(x) = 0$ has 9 solutions.

Ans. (A,B,C)

$f(x) = \sec^2x + 2\sec^2x\tan^2x$

$$g(x) = \frac{\sin(2nx)}{2\sin x}$$

(A) $\int(1+2\tan^2 x)\sec^2 x dx = \tan x + \frac{2}{3}\tan^3 x + C$

(B) $\lim_{n\to 0}\frac{(\sec^2 x + 2\sec^2 x\tan^2 x - 1).4}{\sin^2 2nx} = e^{\frac{3}{n^2}}$

(C) $I_{2n+2} - I_{2n} = \frac{1}{2}\int_0^{\pi/2}\frac{\sin(2n+2)x - \sin 2nx}{\sin x}$

$$= \int_0^{\pi/2}\cos(2n+1)x dx = \frac{1}{(2n+1)}\sin(2n+1)\frac{\pi}{2}$$

$$= -\frac{1}{3}, \frac{1}{5}, -\frac{1}{7}, \frac{1}{9}, \frac{-1}{11}, \frac{1}{13} \quad \text{for } n \in [1, 6]$$

(D) $g(x) = 0 \Rightarrow x = \frac{\pi}{2}$

or $\sin 8x = 0$ and $\sin x \neq 0$

$\Rightarrow \quad x = \frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}, \frac{5\pi}{8}, \frac{3\pi}{4}, \frac{7\pi}{8}$

4.

If $\alpha, \beta, \gamma \in R$ and satisfy the relations $\alpha^2 + 2\beta^2 + 3\alpha = 1 + \gamma^2$ and $2\alpha^2 + 4\beta^2 = 2\gamma^2 + 5\beta$, then

(A) Minimum value of $\alpha^2 + \beta^2$ is $\frac{4}{61}$.

(B) Minimum value of $\alpha^2 + \beta^2$ is $\frac{2}{17}$

(C) Minimum value of $\alpha^2 + \beta^2 - 2\alpha + 1$ is $\frac{16}{61}$

(D) Minimum value of $\alpha^2 + \beta^2 - 2\alpha + 1$ is $\frac{1}{17}$

Ans. (A,C)

$$\left.\begin{array}{l}\alpha^2 + 3\alpha + 2\beta^2 - \gamma^2 = 1 \\ 2\alpha^2 + 4\beta^2 - 5\beta - 2\gamma^2 = 0\end{array}\right\} \Rightarrow 6\alpha + 5\beta = 2$$

5.

$f : [0, 2] \to R$ is continuous on $[0, 2]$ and twice derivable in $(0, 2)$. If $f(0) = 0$; $f(1) = 1$ and $f(2) = 1$, then

(A) $f'(c) = \frac{1}{3}$ for atleast one $c \in (0, 2)$

(B) $2f'(c) + 2c = 3$ for atleast one $c \in (0, 1)$

(C) $2f'(c) + 2c = 3$ for atleast one $c \in (1, 2)$

(D) $f''(c) = -1$ for atleast one $c \in (0, 2)$.

Ans. (A,B,C,D)

By LMVT $\frac{f(1)-f(0)}{1}=f'(c_1)=1$

similarly $\frac{f(2)-f(1)}{1}=f'(c_2)=0$

$\because \frac{1}{3}\in(f'(c_2).f'(c_1))$ and f' is continuous.

$\Rightarrow f'(c)=\frac{1}{3}$ for atleast one 'c' by IVT.

also consider a quadratic

$g(x) = a\left(x-\frac{3}{2}\right)^2+b$

such that g(0) = 0, g(1) = 1, g(2) = 1

which gives $g(x)=\frac{9-(2x-3)^2}{8}$

Let H(x) = f(x) – g(x)

$\Rightarrow$ H(0) = H(1) = H(2) = 0

Applying Rolle's twice on H(x) gives elmaining options.

6.

If $I(a)=\int_0^{\infty}\frac{\tan^{-1}(ax)-\tan^{-1}x}{x}dx$, then

(A) I'(1), I'(2), I'(3) are in H.P. (B) $I'(2)=\frac{\pi}{4}$

(C) $I(\pi)=\frac{\pi \ell n\pi}{2}$ (D) $I'(3)=\frac{\pi}{6}$

Ans. (A,B,C,D)

$$I(a)=\int_0^{\infty}\frac{\tan^{-1}(ax)-\tan^{2}x}{x}dx$$

$$I'(a)=\int_0^{\infty}\frac{dx}{1+x^2a^2}=\frac{\pi}{2a}\Rightarrow I(a)=\frac{\pi}{2}\ell na\{\because I(1)=0\}$$

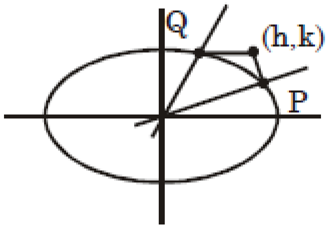
7.

If a pair of variable straight lines $x^2 + 9y^2 + txy = 0$ (t is a real parameter) cut the curve $x^2 + 9y^2 = 9$ at P and Q (P,Q and origin are non-collinear), then locus of the point of intersection of tangents at P and Q is -

(A) (9x – y)(9x + y) = 0 (B) (x – 9y)(x + 9y) = 0

(C) (3x – y)(3x + y) = 0 (D) (x – 3y)(x + 3y) = 0

Ans. (D)



Let intersection point of tangents be (h,k)

equation of $PQ : \frac{xh}{9} + \frac{yk}{1} = 1$

joint equation of OP and OQ is

$$\left.\begin{array}{l} \frac{x^2}{9} + \frac{y^2}{1} = \left(\frac{xh}{9} + \frac{yk}{1}\right)^2 \\ x^2 + 9y^2 + txy = 0 \end{array}\right\} \text{same}$$

$$\frac{\frac{1}{9} - \frac{h^2}{81}}{1} = \frac{1-k^2}{9} \Rightarrow 9 - h^2 = 9 - 9k^2$$

$\Rightarrow x^2 - 9y^2 = 0 \Rightarrow (x + 3y)(x - 3y) = 0$

8.

If A and B are square matrices such that AB = B and BA = A, then which of the following is/are always true ?

(A) A is an idempotent matrix

(B) B is an involutary matrix

(C) $A^6 + B^6 = A^8 + B^8$

(D) $A^2 + B^2 = A + I$

Ans. (A,C)

$A^2 = A.(BA) = (AB)A = BA = A$ similarly $B^2 = B$.

SECTION-II : (Maximum Marks : 16)

- This section contains TWO paragraphs.
- Based on each paragraph, there will be TWO questions
- Each question has FOUR options (A), (B), (C) and (D) ONE OR MORE THAN ONE of these four option(s) is(are) correct.
- For each question, darken the bubble(s) corresponding to all the correct option(s) in the ORS.
- For each question, marks will be awarded in one of the following categories :

Full Marks	: +4	If only the bubble(s) corresponding to all the correct option(s) is (are) darkened.
Zero Marks	: 0	If none of the bubbles is darkened.
Negative Marks	: –2	In all other cases.

PARAGRAPH FOR QUESTIONS 9 AND 10

Let a circle C : $x^2 + y^2 = 1$ and two parabolas $P_1 : y^2 = 4(x - 1)$ and $P_2 : y^2 = -4(x + 1)$. Both parabolas slides on the circle towards each other with same angular velocity such that they are tangent to circle and their axis are normal to circle and after 9 min. they get coincident first time, then

9.

If P_1 & P_2 intersect each other during their rotation then their point of intersection lies on $(x \neq 0)$ -

(A) $4x^2 + y^4 + 8y^2 = 0$ (B) $4x^2 + y^4 + 4y^2 = 0$

(C) $8x^2 + y^4 + 8y^2 = 0$ (D) $8x^2 + y^4 + 4y^2 = 0$

Ans. (A)

Focii of P_1 & P_2 are

$(2\cos\theta, 2\sin\theta)$ & $(-2\cos\theta, 2\sin\theta)$

Directrix of P_1 & P_2 are

$(\cos\theta)x + (\sin\theta)y = 0$ & $(\cos\theta)x - (\sin\theta)y = 0$,

then equation of parabolas are

$(x - 2\cot\theta)^2 + (y - 2\sin\theta)^2 = (x\cos\theta + y\sin\theta)^2$ ....(1)

$(x + 2\cos\theta)^2 + (y - 2\sin\theta)^2 = (x\cos\theta - y\sin\theta)^2$ ....(2)

(1) + (2)

$2x^2 + 2y^2 + 8 - 8(\sin\theta)y = 2x^2\cos^2\theta + 2y^2\sin^2\theta$ ....(3)

$(\sin^2\theta)x^2 + (\cos^2\theta)y^2 - (4\sin\theta)y + 4 = 0$

(1) – (2) $\Rightarrow$ $y(\sin\theta) = -2$ ....(4)

from (3) & (4)

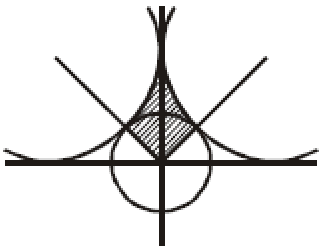
$4x^2 + y^4 + 8y^2 = 0$

10.

After 4.5 min. of starting the rotation, a point is selected at random inside the area bounded by parabolas and their axes then probability that selected point also lies inside the circle 'C' is -

(A) $\frac{3\pi}{8}$ (B) $\frac{3\pi}{16}$ (C) $\frac{3\pi}{32}$ (D) $\frac{\pi}{16}$

Ans. (B)



After 4.5 min. axes of parabolas are $y = \pm x$

required area $= 2\left\{2 - \int_1^2 2\sqrt{x-1}\,dx\right\} = \frac{4}{3}$

Probability $= \frac{3\pi}{16}$

PARAGRAPH FOR QUESTIONS 11 AND 12

Let $A(z_a)$, $B(z_b)$, $C(z_c)$ are three non-collinear points where $z_a = i$, $z_b = \frac{1}{2} + 2i$, $z_c = 1 + 4i$ and a curve is $z = z_a \cos^4 t + 2z_b \cos^2 t.\sin^2 t + z_c \sin^4 t (t \in R)$

11.

Equation of curve in cartesian form is-

(A) $y = x^2 + x + 1$ (B) $y = (x+1)^2$ (C) $y = (x-1)^2$ (D) $y = -x^2 + x - 1$

Ans. (B)

$$z = i\cos^4 t + \left(\frac{1}{2} + 2i\right)2.\cos^2 t \sin^2 t + (1 + 4i)\sin^4 t$$

$\Rightarrow h = \cos^2 t \sin^2 t + \sin^4 t = \sin^2 t$

$k = \cos^4 t + 4\cos^2 t \sin^2 t + 4\sin^4 t$

$k = (1-h)^2 + 4(1-h)h + 4h^2$

$k = (h+1)^2 \Rightarrow y = (x+1)^2$

12.

A line bisecting AB and parallel to AC intersects the given curve at-

(A) two distinct points (B) two co-incident points

(C) only one point (D) No point

Ans. (B)

Line bisecting AB & parallel to AC is

$$y - \frac{3}{2} = 3\left(x - \frac{1}{4}\right)$$

$\Rightarrow y = 3x + \frac{3}{4}$ solve with curve $\Rightarrow \left(x - \frac{1}{2}\right)^2 = 0$

$\Rightarrow$ 2 coincident points.

SECTION-III: (Maximum Marks: 18)

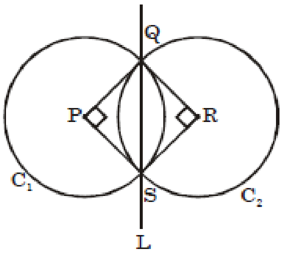
- This section contains **SIX** questions.
- The answer to each question is a **NUMERICAL VALUE.**
- For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the **second decimal place**; e.g. 6.25, 7.00, –0.33, –.30, 30.27, –127.30, if answer is 11.36777..... then both 11.36 and 11.37 will be correct)
- Answer to each question will be evaluated according to the following marking scheme:
  Full Marks : +3 If ONLY the correct numerical value is entered as answer.
  Zero Marks : 0 If none of the bubbles is darkened.
  Negative Marks : –1 In all other cases

13.

$C_1 = 0$ and $L = 0$ are equations of circle and its chord of contact corresponding to point A and $C_2 \equiv C_1 + \lambda L = 0 : (\lambda \in R)$ is an other circle, whose centre A lies on director circle of C_1. If r_1, r_2 are radii of C_1 and C_2 respectively and $\frac{\pi r_1^2}{r_1^2 + r_2^2}$ is angle between C_1 and C_2, then minimum integral value of $2r_1^2 - r_2^2$ is

Ans. 1

PQRS is square $\Rightarrow$ $r_1 = r_2$



14.

Area of triangle formed by any tangent to the curve $y = \frac{2x^2 - x - 1}{2x^2 + 5x + 2}\left(x \neq -2, -\frac{1}{2}\right)$ and pair of lines $xy - x + 2y = 2$ is

Ans. 6

Curve is rectangular Hyperbola

$(x + 2)(y - 1) = -3$

and pair of lines $(x + 2)(y - 1) = 0$ are pair of asymptodes.

$\therefore$ Area $= 2c^2 = 6$.

15.

Consider four distinct circles with radii 3, r_1, r_2, r_3 ($r_1, r_2, r_3 > 3$) touches all three fixed distinct lines in a plane, where r_1, r_2, r_3 are roots of equation $x^3 - mx^2 + 243x - n = 0$, m,n $\in$ R, then value of $\frac{m+n}{7(9 + r_1 + r_2 + r_3)}$ is

Ans. 3

3, r_1, r_2, r_3 are inradius and three exradii of triangle formed by three lines

$$\because \quad \frac{1}{r_1}+\frac{1}{r_2}+\frac{1}{r_3}=\frac{1}{r}=\frac{1}{3} \Rightarrow \frac{3}{\frac{1}{r_1}+\frac{1}{r_2}+\frac{1}{r_3}}=9$$

and from given equation

$r_1r_2 + r_2r_3 + r_3r_1 = 243$

$$\Rightarrow \quad \frac{1}{r_1}+\frac{1}{r_2}+\frac{1}{r_3}=\frac{243}{r_1r_2r_3} \Rightarrow r_1r_2r_3 = 3^6$$

$\Rightarrow \quad (r_1r_2r_3)^{1/3} = 9 \quad \Rightarrow \quad r_1 = r_2 = 9$

($\because$ HM = GM)

$m = 27$, $n = 729$

16.

Let f be a derivable function satisfying $f(x + y) = f(x) + f(y) + 2xy - 2$, $\forall$ $x, y \in$ R and $f'(0) = -2$, then the number of real roots of $f(x) = 0$ is(are)

Ans. 0

$f(x + y) = f(x) + f(y) + 2xy - 2$

differentiate wrt 'y'

$f'(x + y) = f'(y) + 2x$

put $y = 0$ to get $f'(x) = 2x - 2$

$f(x) = x^2 - 2x + c$

using $f(0) = 2$ we get $c = 2$

$f(x) = x^2 - 2x + 2$.

17.

Let $f(x) = \frac{1}{e^x + 8e^{-x} + 4e^{-3x}}$, $g(x) = \frac{1}{e^{3x} + 8e^x + 4e^{-x}}$ and $\int (f(x) - 2g(x))dx = h(x) + c$ (where 'c' is constant of integration) and $\lim\limits_{x \to \infty} h(x) = \frac{\pi}{4}$. If $h(0) = \frac{1}{a}\tan^{-1}\left(\frac{b}{c}\right)$ (where $a \in$ N, b and c are coprime), then the value of (a + b + c) is

Ans. 7

$$\int (f(x) - 2g(x))dx = \int \frac{e^{3x} - 2e^x}{e^{4x} + 8e^{2x} + 4}dx$$

Let $e^x = t$

$$I = \int \frac{1 - \frac{2}{t^2}}{t^2 + 8 + \frac{1}{t^2}}dt = \frac{1}{2}\tan^{-1}\left(\frac{e^x + 2e^{-x}}{2}\right) + c$$

Hence, $h(x) = \frac{1}{2}\tan^{-1}\left(\frac{e^x + 2e^{-x}}{2}\right)$

$$h(0) = \frac{1}{2}\tan^{-1}\left(\frac{3}{2}\right)$$

$\therefore$ a = 2, b = 3, c = 2.

18.

Let $f : [3,4] \to [3,4]$ be a bijective decreasing function $\forall$ x $\in$ [3,4], then the value of $\int_3^4 \left(f(x) - f^{-1}(x)\right)dx$ is

Ans. 0

$f(3) = 4$ and $f(4) = 3$

Given integral $= \int_3^4 f(x)dx - \int_3^4 f^{-1}(x)dx$

$= \int_3^4 f(x)dx + \int_4^3 f^{-1}(x)dx$

$= 4 \times 3 - 3 \times 4 = 0$

PART-1 : PHYSICS

SECTION-I : (Maximum Marks: 32)

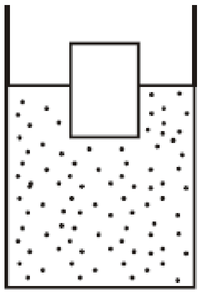
- This section contains **EIGHT** questions.
- Each question has **FOUR** options for correct answer(s). **ONE OR MORE THAN ONE** of these four option(s) is (are) correct option(s).
- For each question, choose the correct option(s) to answer the question.
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks : +4 If only (all) the correct option(s) is (are) chosen.
Partial Marks : +3 If all the four options are correct but ONLY three options are chosen.
Partial Marks : +2 If three or more options are correct but ONLY two options are chosen, both of which are correct options.
Partial Marks : +1 If two or more options are correct but ONLY one option is chosen and it is a correct option.
Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered).
Negative Marks : –2 In all other cases.

- **For Example :** If first, third and fourth are the **ONLY** three correct options for a question with second option being an incorrect option; selecting only all the three correct options will result in +4 marks. Selecting only two of the three correct options (e.g. the first and fourth options), without selecting any incorrect option (second option in this case), will result in +2 marks. Selecting only one of the three correct options (either first or third or fourth option), without selecting any incorrect option (second option in this case), will result in +1 marks. Selecting any incorrect option(s) (second option in this case), with or without selection of any correct option(s) will result in –2 marks.

1

Figure shows a cylindrical piece of ice floating in water that has cross sectional area A and its height is h. The density of ice and water are ρ_{Ice} and ρ_w respectively. The cross section area of container is 4A. The ice is displaced from equilibrium by z in the vertical direction. During the push ice moves at velocity v. Assume that water below the bottom level of the ice does not move and the water above the bottom level of the ice moves with constant velocity (d is initially submerging depth). Mark the correct statements :-



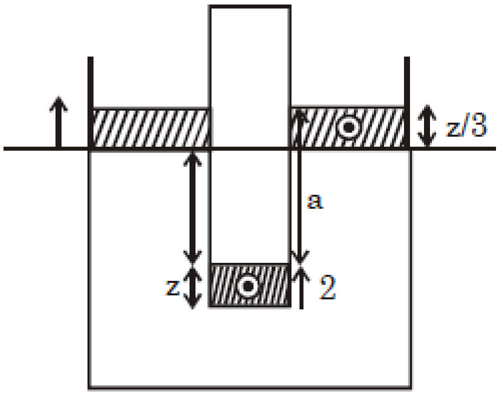
(A) Potential energy change of water is $\rho_w Ag(zd + \frac{2}{3}z^2)$

(B) Potential energy change of water is $\rho_w Ag\left(\frac{zd}{2} + \frac{1}{3}z^2\right)$

(C) Kinetic energy of the ice is $\frac{2}{3}\rho_w Adv^2$

(D) Total kinetic energy of the system is $\frac{2}{3}\rho_w Av^2\left(d + \frac{z^2}{3}\right)$

Ans. (A,C, D)

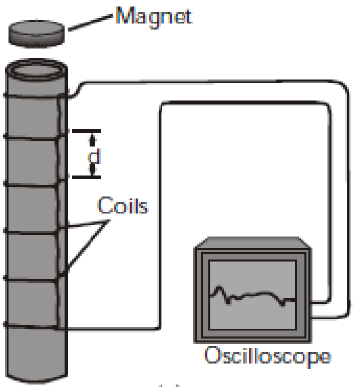


Volume of water displaced will be water rise
AZ = 3Ah where h is water rise in container
Change in potential energy = mgh where m is mass of water displaced and h is hieght of centre of mass rised

$$\left(d + \frac{2z}{3}\right)\rho_\omega Azg$$

2.

Figure shows an experiment designed to measure the acceleration due to gravity. A large plastic tube is encircled by a wire, which is arranged in loops separated by d as shown in figure. The oscilloscope registers change in voltage. Mark the correct statement(s) :-



(a)

(A) When magnet passes through tube an emf is induced in the plastic tube.
(B) If tube is made of conducting material the falling magnet will slow down.
(C) As the magnet passes through tube initially induced emf is positive then zero and then negative.
(D) At the instant magnet is at the centre of the loop induced emf is zero.

Ans. (A,B,C,D)

3.

Figure (a) shows a listener standing at a perpendicular distance d from a railway track. A train crosses point A at a constant speed of v emitting sound of frequency f. Figure (b) shows graphs of frequency versus time heard by listener in three different situations (labelled as (1), (2) and (3)). In each one of them only one of the parameters i.e. velocity of train (v), frequency of source (f), perpendicular distance d and speed of sound (v_S) can be changed. (Train crosses point A at t = 0)

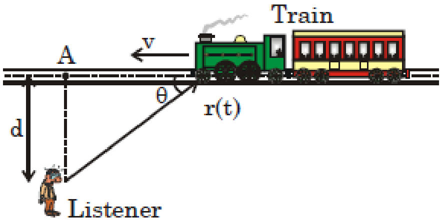


Figure (a)

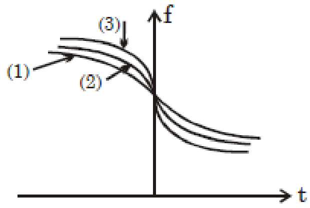


Figure (b)

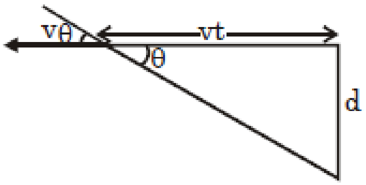
(A) If velocity of train is changed then $v_3 > v_2 > v_1$

(B) If perpendicular distance is changed then $d_1 > d_2 > d_3$

(C) If frequency of source is changed then $f_3 > f_2 > f_1$

(D) If speed of sound is different then $(v_s)_3 < (v_s)_2 < (v_s)_1$

Ans. (A,B,D)



for t > 0

$$f' = \frac{v_s}{v_s + v\cos\theta} f$$

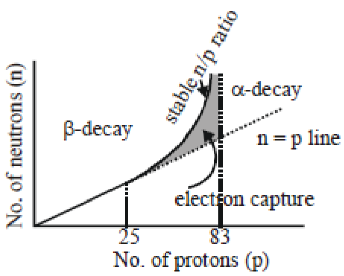
$$= \frac{v_s}{v_s + \frac{v.vt}{\sqrt{d^2 + v^2t^2}}} f = \frac{v_s}{v_s + \frac{v}{\sqrt{1 + \frac{d^2}{v^2t^2}}}}$$

⇒ Solve for each part.

If frequency of source is changed then points will not be coinciding at t = 0 for the 3 situations.

4.

If all the known isotopes of the elements are plotted on a graph of number of neutrons (n) versus number of protons (p), it is observed that all isotopes lying outside of a "stable" n/p ratio region are radioactive as shown in figure. The graph exhibits its straight line behavior with unit slope up to p = 25. Above p = 25 those isotopes with an n/p ratio lying below the stable region usually undergo electron capture while those with n/p ratios lying above the stable region usually undergo beta-decay. Very heavy isotopes p > 83 are unstable because of their relatively large nuclei and they undergo alpha decay.



(A) The radioisotope of magnesium with mass number 27 and atomic number 12 may undergo Beta decay.

(B) The radioisotope of magnesium with mass number 27 and atomic number 12 may undergo Alpha decay.

(C) For a hypothetical isotope of an element $^{178}_{88}X$ we may expect α-decay

(D) For a hypothetical isotope of an element $^{178}_{88}X$ we may expect β^+–decay

Ans. (A,C)

A = 27 P = 12

z = 12 n = 15

n > p

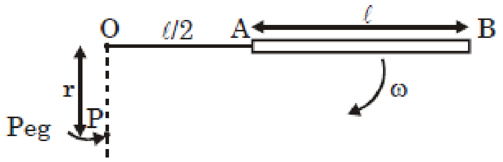
$^{178}_{88}x \rightarrow P = 88$

n = 178 – 88 = 90

n > p & z > 83 α decay will take place

5.

A uniform thin rod AB mass M of length ℓ attached to a string OA of length $\frac{\ell}{2}$ is supported by a smooth horizontal plane and rotates with angular velocity ω around a vertical axis through O. A peg P is inserted in the plane in order that on striking it the bar will come exactly to rest :-



(A) Magnitude of angular momentum of rod about O is $\frac{4}{3}M\ell^2\omega$

(B) Magnitude of tension in string is $M\ell\omega^2$

(C) Location of peg for rod coming to rest is $r = \frac{13}{12}\ell$

(D) Magnitude of angular impulse by peg on the rod is $\frac{4}{3}M\ell^2\omega$

Ans. (B,C)

(A) $MV_{cm}L + I_{cm}\omega$

$$M\omega L^2 + \frac{ML^2}{12}\omega$$

$$L_0 = \frac{13}{12}ML^2\omega$$

(B) $T = ma_{cm}$

$$T = m\omega^2 r_{cm} = m\omega^2 \ell_{cm}$$

(C) By impulse momentum theorem

$$\int Ndt = mV$$

By angular Impulse

$$r\int Ndt = mV\ell + \frac{M\ell^2}{12}\frac{V}{L}$$

$$rMV = \frac{13}{12}MVL$$

(D) Angular impulse by peg

$$= \frac{13}{12}M\omega L^2$$

6.

n mol of helium gas undergoes a thermodynamic process in which the molar heat capacity C of the gas can be described as a function of the absolute temperature $C = \frac{3RT}{4T_0}$, where R is gas constant and T_0 is the initial temperature of the helium gas :-

(A) Volume of gas as a function of temperature is given by $V = V_0\left(\frac{T_0}{T}\right)^{3/2} \exp\left[\frac{3(T-T_0)}{4T_0}\right]$

(B) Volume of gas as a function of temperature is given by $V = V_0\left(\frac{T_0}{T}\right)^{1/2} \exp\left[\frac{2(T-T_0)}{3T_0}\right]$

(C) Workdone on the system until the point at which the helium gas reaches the minimal volume is $\frac{3}{8}nRT_0$

(D) Workdone on the system until the point at which the helium gas reaches the minimal volume is $-\frac{3}{8}nRT_0$

Ans. (A, C)

$dQ = dU + dW$

$ncdT = nC_VdT + PdV$

$$n\frac{3RT}{4T_0} = n\frac{3R}{2} + \frac{nRT}{V}\frac{dV}{dT}$$

On rearranging and intergration we get

$$V = V_0\left(\frac{T_0}{T}\right)^{3/2} e^{\frac{3(T-T_0)}{4T_0}}$$

For minimum volume, $\frac{dV}{dT} = 0$

$T = 2T_0$

$dW = PdV = \int \frac{nRT}{V}dV$

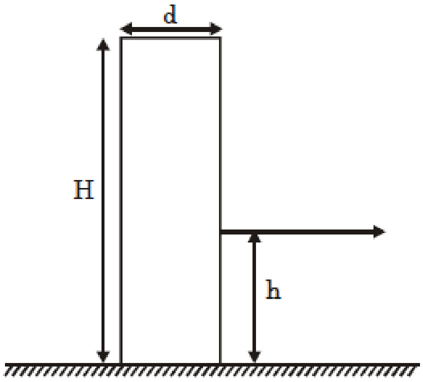
on solveing

$$W = \frac{-3}{8}nRT_0$$

i.e. work is done on the system.

7.

A curved wooden sculpture that can be taken as cylinder of height H and diameter d. A thread is attached at a point that is h height above the base, and the sculpture is placed in the middle of a rough horizontal table-top. The coefficient of friction between table top and the sculpture is $\mu_{kinetic} = \mu_{static} = \mu$. sculpture is to be pulled horizontally by the thread and drag it in one continuous movement to the edge of the table without toppling. sculpture acceleration a must satisfy :-

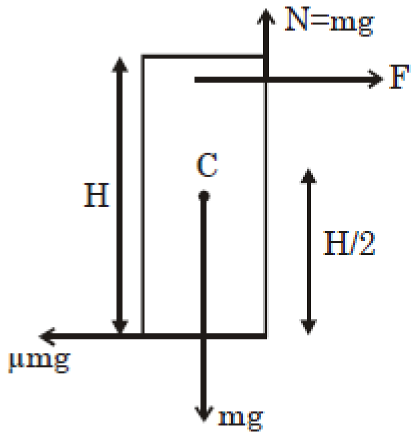


(A) $a \geq \left(\frac{2\mu h - d}{H - 2h}\right)g$ (B) $a \leq \left(\frac{2\mu h - d}{H - 2h}\right)g$ (C) $a \leq \left(\frac{2\mu h + d}{H - 2h}\right)g$ (D) $a \geq \left(\frac{2\mu h + d}{H - 2h}\right)g$

Ans. (B, C)

Case-I

When force is above COM



$T_c = 0$

$$\mu mg\frac{H}{2} + F\left(h - \frac{H}{2}\right) = mg\frac{d}{2}$$

$\mu mgH + F(2h - H) = mg\,d$

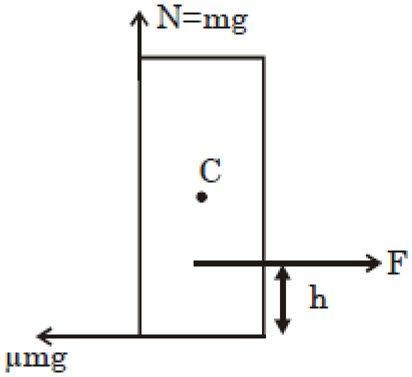
On Solving

$$a = \frac{F - \mu mg}{m} = \frac{g(2uh - d)}{H - 2h}$$

$$a \leq g\left(\frac{2uh - d}{H - 2h}\right)$$

Case-II

When force is below COM



$T_c = 0$

$$\mu mg\frac{H}{2} + mg\frac{d}{2} = F\left(\frac{H}{2} - h\right)$$

On solving

$$a = \frac{F - \mu mg}{m} = \left(\frac{2uh + d}{H - 2h}\right)g$$

$$a \leq \left(\frac{2\mu h + d}{H - 2h}\right)g$$

8.

One method for determining the compressibility of a dielectric material uses a driven LC circuit that has a parallel plate capacitor. The dielectric is inserted between the plates and the change in resonance frequency is determined as the plates are subjected to a compressive stress.

d = dielectric thickness, Δd = change in thickness

k = dielectric constant, Y = young's modulus

ΔP = Pressure, ω_C = Resonance frequency after compression,

ω_0 = Resonance frequency before compression

Assume that dielectric constant of dielectric remains constant under compression. Mark the CORRECT options :-

(A) $\frac{\omega_c}{\omega_0} = 1 - \frac{\Delta d}{2d}$ for $\Delta d << d$

(B) $\frac{\omega_c}{\omega_0} = 1 - \frac{2\Delta d}{d}$ for $\Delta d << d$

(C) $Y = \frac{2\Delta P}{\left(1 - \frac{\omega_c}{\omega_0}\right)}$

(D) $Y = \frac{\Delta P}{2\left(1 - \frac{\omega_c}{\omega_0}\right)}$

Ans. (A,D)

SECTION–II : (Maximum Marks : 16)

- This section contains TWO question.
- Question contains two columns, Column-I and Column-II.
- Column-I has four entries (A), (B), (C) and (D)
- Column-II has five entries (P), (Q), (R), (S) and (T)
- Match the entries in Column-I with the entries in column-II.
- One or more entries in Column-I may match with one or more entries in Column-II.
- The ORS contains a 4 × 5 matrix whose layout will be similar to the one shown below :

(A)	(P)	(Q)	(R)	(S)	(T)
(B)	(P)	(Q)	(R)	(S)	(T)
(C)	(P)	(Q)	(R)	(S)	(T)
(D)	(P)	(Q)	(R)	(S)	(T)

- For each entry in column-I, darken the bubbles of all the matching entries. For example, if entry (A) in Column-I matches with entries (Q), (R) and (T), then darken these three bubbles in the ORS. Similarly, for entries (B), (C) and (D).
- For each question, marks will be awarded in one of the following categories :

For each entry in Column-I

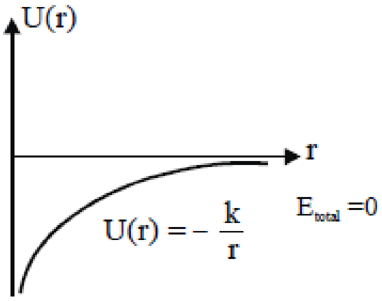
Full Marks	: +2	If only the bubble(s) corresponding to all the correct match(es) is (are) darkened
Zero Marks	: 0	In none of the bubbles is darkened
Negative Marks	: –1	In all other cases

9. Column I describes about motion of a particle under a single conservative force. Column II shows graph of potential energy v/s position or potential energy v/s time. E_{total} denotes total energy. Potential energy at infinity is assumed to be zero.

Column-I | Column-II

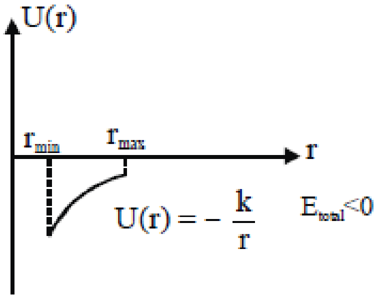
(A) Periodic motion

(P)

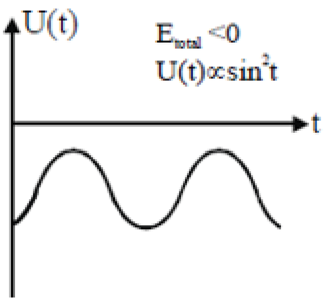


(B) Oscillatory

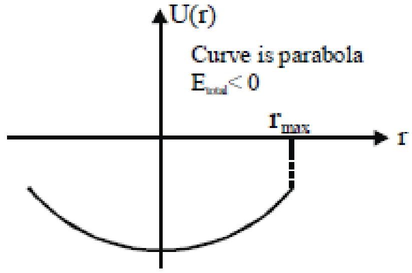
(Q)



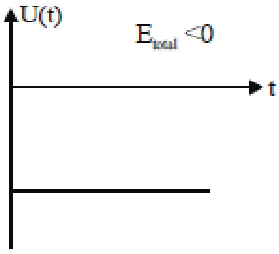
(C) Unbound trajectory (R)



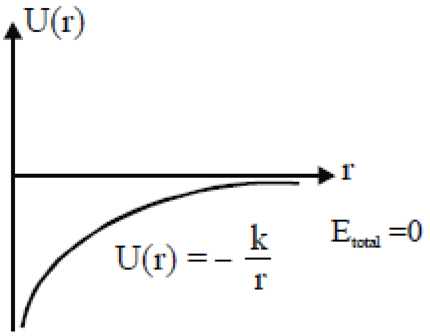
(D) Trajectory may be ellipse or circle (S)



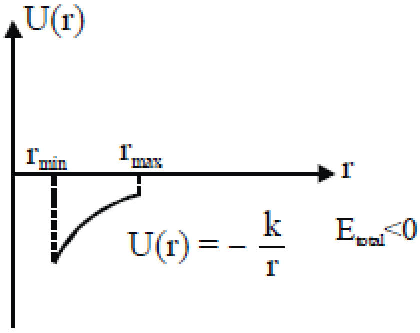
(T)



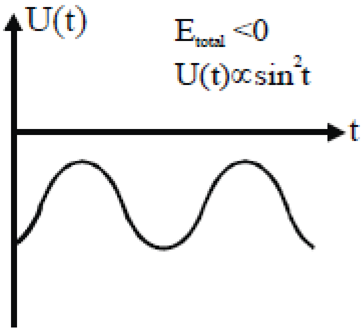
Ans. (A)-(Q,R,S,T); (B)-(R,S) (C)-(P); (D)-(Q,T)



..... motion of a particle released from infinity under attraction force.



..... motion of a particle moving in an elliptical path.

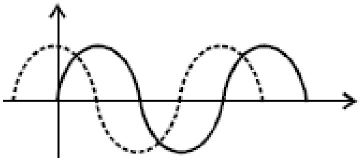
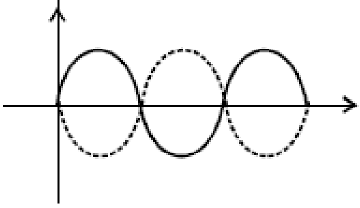
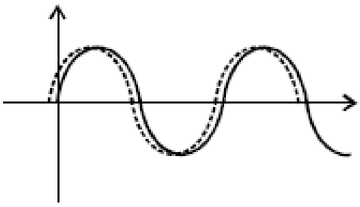
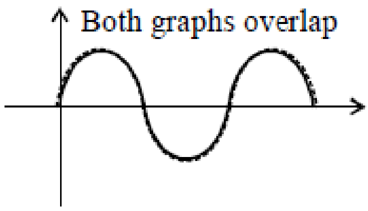


..... motion of a particle

performing SHM.

10.

Column I shows some graphs. Each graph has two different sinusoidal curve that are plotted with dark segment represented by ____ and a dotted segment represented by ------- Column II shows variation of certain parameters represented by corresponding dotted or dark segment.

Column-I		Column-II
(A) 	(P)	In a series LCR circuit of AC source of frequency less than the resonance frequency, current 'i' v/s time 't'------- AND source voltage 'V_S'v/s 't' _____
(B) 	(Q)	In an LC oscillations with capacitor initially charged and inductor with no current. Instantaneous voltage across capacitor 'V_C' v/s time 't'-------AND instantaneous current in inductor 'i_L' v/s time 't'_____
(C) 	(R)	In a longitudinal wave, particle displacement 's' v/s time 't' at x = 0_____ AND excess pressure 'p' v/s time 't' at x = 0 -------
(D) 	(S)	In transverse wave on a string particle displacement is 's'. Then consider displacement 's' v/s time 't' graph for two positions x_1 and x_2 on string. 's' v/s 't' at position-1 (x_1) _____AND 's' v/s 't' at position-2 (x_2) -------Consider all possible values of x_1 and x_2
	(T)	Two particles are performing SHM about same mean position and same frequency. One (1) is starting from mean position and other (2) is starting from somewhere between mean and positive extreme with going towards extreme position. Particle displacement is x. Consider 'x' v/s time 't' for the two particles. 'x' v/s 't' at particle-1_____AND 'x' v/s 't' for particle-2-------

Ans.(A)-(Q,R,S) ; (B)-(S) ; (C)-(P,S,T) ; (D)-(S)

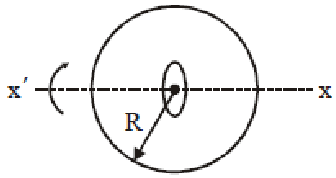
SECTION-III: (Maximum Marks: 18)

- This section contains **SIX** questions.
- The answer to each question is a **NUMERICAL VALUE.**
- For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the **second decimal place**; e.g. 6.25, 7.00, –0.33, –.30, 30.27, –127.30, if answer is 11.36777..... then both 11.36 and 11.37 will be correct)
- Answer to each question will be evaluated according to the following marking scheme:
  Full Marks : +3 If ONLY the correct numerical value is entered as answer.
  Zero Marks : 0 If none of the bubbles is darkened.
  Negative Marks : –1 In all other cases

---

11. A non–conducting ring of radius R having uniformly distributed charge Q starts rotating about x–x′ axis passing through diameter with an angular acceleration α as shown in the figure. Another small conducting ring having radius a (a << R) is kept fixed at the centre of bigger ring in such a way that axis xx′ is passing through its centre and perpendicular to its plane. If the resistance of small ring is r = 1Ω, find the induced current in it in ampere.

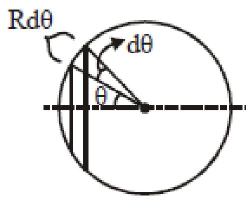
(Given $q = \frac{16\times10^2}{\mu_0}$C, R = 1 m, a = 0.1 m, α = 8 rad/s^2)



Ans. 8

$dq = \frac{q}{2\pi R}.Rd\theta = \frac{q}{2\pi}.d\theta$

$di = \frac{dq}{T} = \frac{qd\theta\omega}{2\pi 2\pi}$



$di = \frac{q\omega}{4\pi^2}.d\theta$

$dB = \frac{\mu_0 di(R\sin\theta)^2}{2R^3}$

$\int dB = \int_0^{\pi}\frac{\mu_0 \sin^2\theta}{2R}\left(\frac{q\omega}{4\pi^2}\right)d\theta$

$B = \frac{\mu_0 q\omega}{16\pi R}$

$\phi = B\pi a^2$

$\phi = \pi a^2.\frac{\mu_0 q\omega}{16\pi R}$

$\phi = \frac{\mu_0 q\omega a^2}{16R}$

$|\varepsilon| = \left|\frac{d\phi}{dt}\right|$

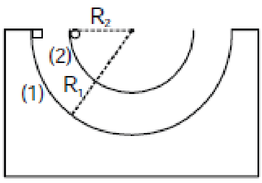
$|\varepsilon| = \frac{\mu_0 q a^2}{16R}\alpha$.

= 8 volt

$i = \frac{8}{1} = 8$ A.

12.

A fixed wedge has two semicircular tracks of radii R_1 and R_2 respectively the track (1) is friction less. Track 2 is rough. A small block is kept at top of track (1) and a ball of radius r at top of track (2), ($r << R_2$). Both ball and block are released simultaueously, ball performs pure rolling. If both of them reach the bottom of tracks simultaneously, then ratio of $\frac{R_1}{R_2}$ is n find value of 5n.

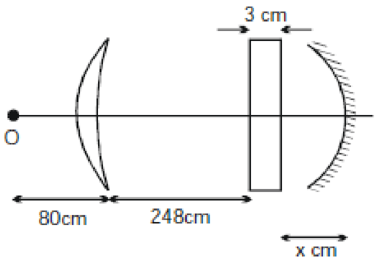


Ans. 7

$$\frac{R_1}{R_2} = \frac{a_1}{a_2} \text{ (as time taken is same) } = \frac{7}{5}$$

13.

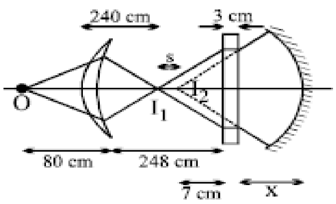
A concave-convex lens made of glass of refractive index 1.5 has surfaces of radii 20 cm and 60 cm. A concave mirror of radius of curvature 20 cm is placed co-axially to the lens. A glass slab of thickness 3 cm and refractive index 1.5 is placed closed to the mirror in the space between the mirror and lens as shown in figure. The distance of the nearest surface of the slab from lens is 248 cm. An object is placed 80 cm to the left of the lens. If final position of the image formed after refraction from lens, refraction from slab, reflection from mirror, refraction from the slab and lens is at the object O, find the distance x (in cm) of the mirror from nearer surface of slab. Fill value of x/2.



Ans. 5

For lens, $\frac{1}{f} = (\mu - 1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$

Here, $R_2 = +60$ cm and $R_1 = +20$ cm



$$\therefore \frac{1}{f} = (1.5 - 1)\left[\frac{1}{20} - \frac{1}{60}\right]$$

$$\therefore \frac{1}{f} = 0.5 \times \frac{(3-1)}{60} \Rightarrow f = 60 \text{ cm}$$

Also $\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$

$$\Rightarrow \frac{1}{60} = \frac{1}{v} + \frac{1}{80}$$

$$\therefore \frac{1}{v} = \frac{1}{60} - \frac{1}{80} = \frac{4-3}{240} = \frac{1}{240}$$

$\Rightarrow v = 240$ cm

Shift due to slab, $s = 3\left[1 - \frac{1}{1.5}\right] = 1$ cm

$\therefore$ distance of I_2 from slab = 7 cm

for final image to form at O, ray should retrace its path after reflection from mirror. So I_2 should be at centre of curvature of mirror i.e. $7 + 3 + x = 20$

$\Rightarrow x = 10$ cm

14.

The period of oscillation of a simple pendulum is $T = 2\pi\sqrt{L/g}$. Measured value of L is 20.0 cm known to 1 mm accuracy and time for 100 oscillations of the pendulum is found to be 90 s using a wrist watch of 1s resolution. What is the accuracy in the determination of g ? If answer is n% fill $\frac{18}{7}n$.

Ans. 7

$$g = \frac{4\pi^2 L}{T^2}$$

Here, $T = \frac{t}{n}$ and $\Delta T = \frac{\Delta t}{n}$.

Therefore, $\frac{\Delta T}{T} = \frac{\Delta t}{t}$.

The errors in both L and t are the least count errors.
Therefore,

$$\left(\frac{\Delta g}{g}\right) = \left(\frac{\Delta L}{L}\right) + 2\left(\frac{\Delta T}{T}\right)$$

$$= \frac{0.1}{20.0} + 2\left(\frac{1}{90}\right) = 0.027$$

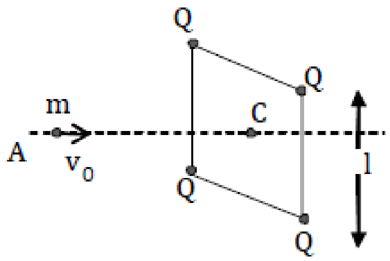
Thus, the percentage error in g is

$$100\left(\frac{\Delta g}{g}\right) = 100\left(\frac{\Delta L}{L}\right) + 2 \times 100\left(\frac{\Delta T}{T}\right)$$

15.

Four identical positive point charges Q are fixed at the four corners of a square of a side length ℓ. Another charged particle of mass m and charge +q is projected towards centre of square from a large distance along the line perpendicular to plane of square. The minimum value of initial velocity v_0 required to cross the square is?

(m = 1 gm, l = $4\sqrt{2}$ m, Q = 1 μc, q = 0.5 μc)



Ans. 3

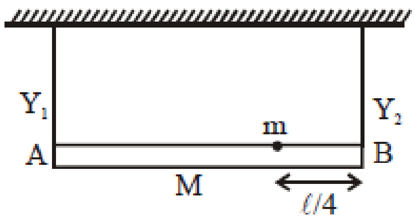
Particle will cross the square if it crosses the point of maximum potential between A and C–which is C itself.

$$\frac{1}{2}mv_0^2 = K + \frac{1}{4\pi\varepsilon_0}\frac{4Q^2q}{(l/\sqrt{2})}$$

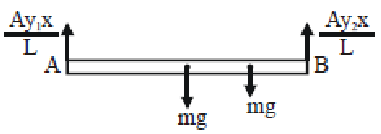
$$K > 0 \Rightarrow V_0^2 = 8\sqrt{2}\left(\frac{1}{4\pi\varepsilon_0}\right)\left(\frac{Qq}{ml}\right)$$

16.

A rigid rod AB of mass M and length ℓ is suspended horizontally from two vertical wires having same length and same area of cross-section. When a mass 'm' is placed at a distance $\frac{\ell}{4}$ from end B, rod remains horizontal. If the ratio of Young modulus of two wires $\frac{Y_1}{Y_2}$ is $\left(\frac{xM+m}{2M+ym}\right)$ then find the value of x+y.



Ans. 5



$$\frac{Ax}{L}(Y_1 + Y_2) = (M+m)g$$

$$\left(\frac{Ay_1x}{L}\right)\frac{\ell}{2} + mg\times\frac{\ell}{4} = \left(\frac{Ay_2x}{L}\right)\frac{\ell}{2}$$

$$\frac{Y_1}{Y_2} = \frac{2M+m}{2M+3m}$$

PART-2 : CHEMISTRY

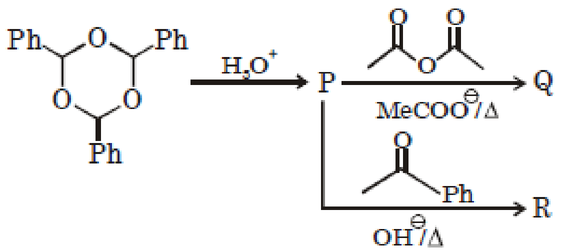
SECTION-I : (Maximum Marks: 32)

- This section contains **EIGHT** questions.
- Each question has **FOUR** options for correct answer(s). **ONE OR MORE THAN ONE** of these four option(s) is (are) correct option(s).
- For each question, choose the correct option(s) to answer the question.
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks : +4 If only (all) the correct option(s) is (are) chosen.
Partial Marks : +3 If all the four options are correct but ONLY three options are chosen.
Partial Marks : +2 If three or more options are correct but ONLY two options are chosen, both of which are correct options.
Partial Marks : +1 If two or more options are correct but ONLY one option is chosen and it is a correct option.
Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered).
Negative Marks : –2 In all other cases.

- **For Example :** If first, third and fourth are the **ONLY** three correct options for a question with second option being an incorrect option; selecting only all the three correct options will result in +4 marks. Selecting only two of the three correct options (e.g. the first and fourth options), without selecting any incorrect option (second option in this case), will result in +2 marks. Selecting only one of the three correct options (either first or third or fourth option), without selecting any incorrect option (second option in this case), will result in +1 marks. Selecting any incorrect option(s) (second option in this case), with or without selection of any correct option(s) will result in –2 marks.

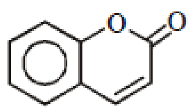
1.

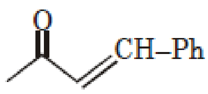


Identify correct statement for above sequence ?

(A) Ph–C(=O)–H is 'P' product

(B) Ph–CH=CH–COOH is 'Q' product

(C)  is 'Q' product

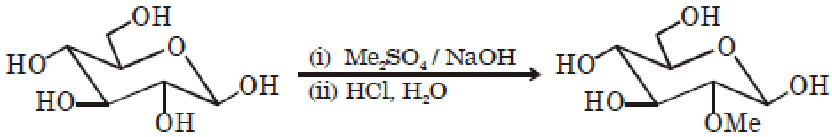
(D) 

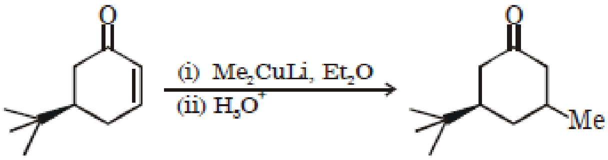
is 'R' product

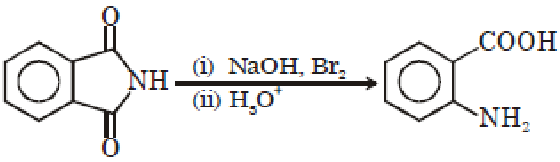
Ans. (A,B)

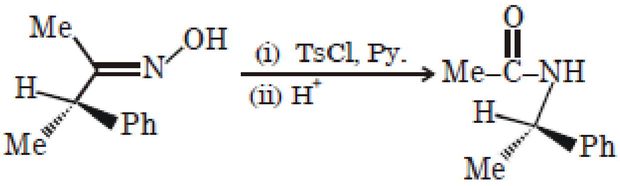
2.

Identify reaction correctly matched with their major product ?

(A) 

(B) 

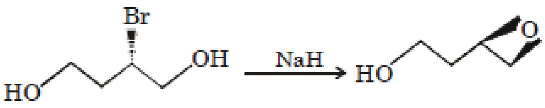
(C) 

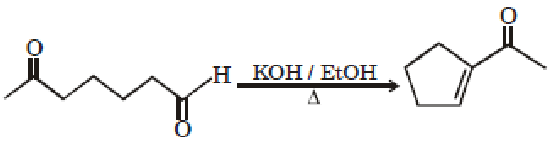
(D) 

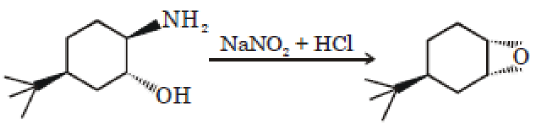
Ans. (B,C,D)

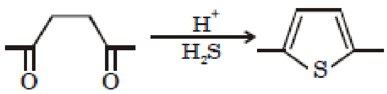
3.

Identify major product formed are correctly matched in following reactions ?

(A) 

(B) 

(C) 

(D) 

Ans. (A,B,C,D)

4.

Select the **CORRECT** statement(s) regarding silicates :-

(A) SiO_4^{4-} units polymerize to form silicates because Si atom has more tendency to form pπ–pπ bonds with 'O' atom.

(B) $(Si_2O_5)_n^{2n-}$ is general formula for sheet silicates.

(C) Silica exist in form of O = Si = O (linear shape)

(D) $Be_3Al_2Si_6O_{18}$ is an orthosilicate.

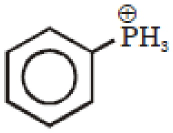
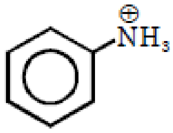
Ans. (B)

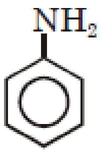
5.

Mark the **correct** statements,reactions quoted below ?

(A) p-Toluidine, aniline and α-napthyl amine react with hinsberg reagent to form alka soluble species.

(B) Number of valid resonating structure(s) of (I) and (II) are 6.

(I)  (II) 

(C) 

(1) Ac_2O → Mono substitution at meta in final product.

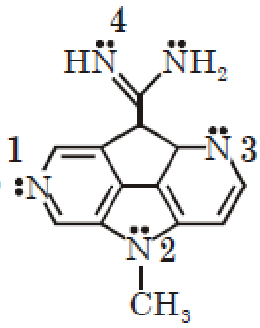
(2) $HNO_3/H^+/288K$

(3) NaOH(aq)

(4) HNO_2/Δ

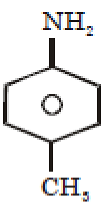
(5) 

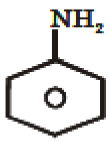
(6) CH_3Cl/Anhy Fe

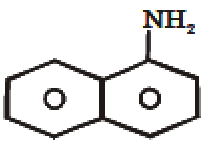
(D) 

K_b order is 4 > 1 > 3 > 2

Ans. (A,D)

(A)  – p – Toluidine (1° amine)

 – Benzene Amine (1° amine)

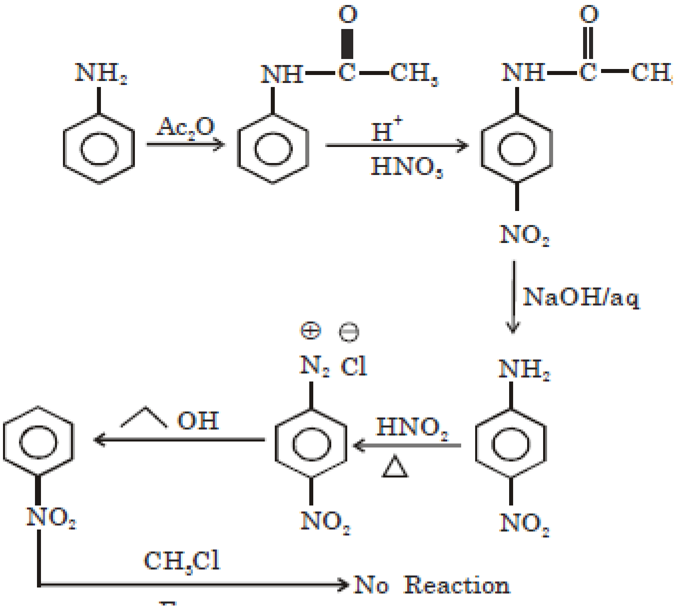
 – α– napthyl amine (1° amine)

All 1° amine give Hinsberg test

(B)  Resonating structure = 5

 Resonating structure = 2

Ans = 7

(C) 

6. Identify the option(s) of **CORRECT** statement(s) regarding synergic bonding in molecules-

(I) The C–O bond order among iso electronic and iso-structural species
$[Ni(CO)_4] > [Co(CO)_4]^- > [Fe(CO)_4]^{2-}$

(II) Metal carbonyls have large tendency to follow Sidgwick E.A.N. rule

(III) The metal-carbon bond order in $[Me_3P\ Ni(CO)_3]$ is less than in $[F_3P\ Ni(CO)_3]$

(IV) The paramagnetic character of the molecule/ion remains constant during synergic bonding in metal carbonyls.

(V) If CO is replaced by NO in $[Cr(CO)_6]$, then it is better represented as $[Cr(NO)_4]$, rather than $[Cr(NO)_6]$

(A) I, II (B) I, III (C) III, IV (D) IV, V

Ans. (A,D)

7.

When H_2S gas is passed through a hot acidic aqueous solution containing Cr^{+3}, Cd^{+2}, Hg^{+2} and Co^{+2}, a precipitate is formed which can not have the composition of

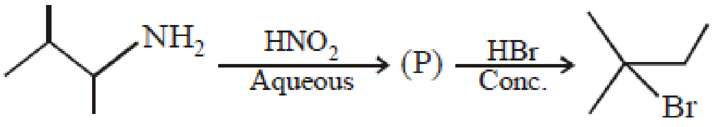
(A) CdS and Cr_2S_3 (B) HgS and CoS (C) CdS and CoS (D) CdS and HgS

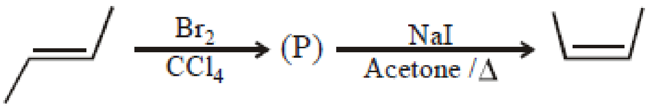
Ans. (A, B, C)

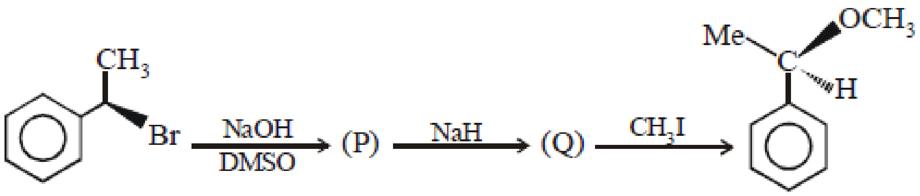
Group II radical will precipitate out.

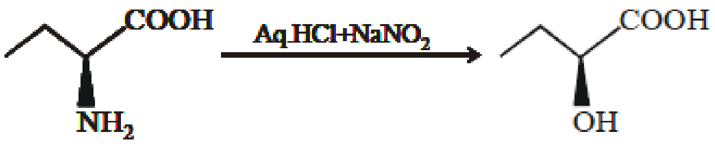
8.

Which reaction sequence(s) represent correct major product :

(A) 

(B) 

(C) 

(D) 

Ans. (A, D)

S_N2 causes inversion of configuration. NGP causes retention of configuration.

SECTION–II : (Maximum Marks : 16)

- This section contains TWO question.
- Question contains two columns, Column-I and Column-II.
- Column-I has four entries (A), (B), (C) and (D)
- Column-II has five entries (P), (Q), (R), (S) and (T)
- Match the entries in Column-I with the entries in column-II.
- One or more entries in Column-I may match with one or more entries in Column-II.
- The ORS contains a 4 × 5 matrix whose layout will be similar to the one shown below :

(A)	(P)	(Q)	(R)	(S)	(T)
(B)	(P)	(Q)	(R)	(S)	(T)
(C)	(P)	(Q)	(R)	(S)	(T)
(D)	(P)	(Q)	(R)	(S)	(T)

- For each entry in column-I, darken the bubbles of all the matching entries. For example, if entry (A) in Column-I matches with entries (Q), (R) and (T), then darken these three bubbles in the ORS. Similarly, for entries (B), (C) and (D).
- For each question, marks will be awarded in one of the following categories :

For each entry in Column-I

Full Marks	: +2	If only the bubble(s) corresponding to all the correct match(es) is (are) darkened
Zero Marks	: 0	In none of the bubbles is darkened
Negative Marks	: –1	In all other cases

9.

Column I Electrolysis	Column II pH at 298 K and products at anode and cathode
(A) Electrolysis of 100 L aqueous solution of CH_3COOK by passing 2F of electricity	(P) pH = 12.3 Anode = Ethane(g) + CO_2(g) Cathode = H_2(g)
(B) Electrolysis of 10 L aqueous solution of HCOOK by passing 1F of electricity	(Q) pH = 13.0 Anode = H_2 (g) + CO_2(g) Cathode = H_2(g)
(C) Electrolysis of 10 L aqueous solution of K_2SO_4 by passing 1F of electricity	(R) pH = 7.0 Anode = O_2 (g) Cathode = H_2(g)
(D) Electrolysis of 10 L aqueous solution of CuF_2 by passing 1F of electricity	(S) pH = 1.0 Anode = O_2 (g) Cathode = Cu
	(T) pH = 2.0 Anode = H_2 (g) + CO_2(g) Cathode = H_2(g)

Ans.(A)-(P); (B)-(Q); (C)-(R); (D)-(S)

(A) $2F = 2Eq\ of\ \overset{\ominus}{O}H$

$$[\overset{\ominus}{O}H] = \frac{2Eq}{100L} = 2\times10^{-2}N$$

pOH = 1.7, pH = 14–1.7 = 12.3

At anode : Ethane(g) + CO_2(g)

At Cathode : H_2(g)

(B) $1F = 1Eq\ of\ \overset{\ominus}{O}H$

$$[\overset{\ominus}{O}H] = \frac{1Eq}{10L} = 10^{-1}N$$

pOH = 1, pH = 14–1 = 13

At anode : H_2(g) + CO_2(g)

At Cathode : H_2(g)

(C) $1F = 1Eq\ of\ \overset{\ominus}{O}H = 1Eq\ H^{\oplus}$

So solution is neutral

pH = 7

At anode : O_2(g)

At Cathode : H_2(g)

(D) $Cu_2 \xrightarrow{\text{Electrolysis}} Cu^{2+} + 2F^{\ominus}$

$$\text{At cathode}: Cu^{2+} + 2e^- \longrightarrow Cu(s)$$

$$\text{At anode}: H_2O \longrightarrow \frac{1}{2}O_2(g) + 2H^{\oplus} + 2e^-$$

$$\left(\text{since } E^{\ominus}_{\text{oxid of } H_2O} > E^{\ominus}_{\text{oxide of } F^{\ominus}}\right)$$

At anode :

$$1F = 1Eq\,H^{\oplus}; [H^{\oplus}] = \frac{Eq}{V_L} = \frac{1Eq}{10L} = 10^{-1}N$$

pH = 1

10.

Match the following :-

Column-I	Column-II
(A) $NO[BF_4]$	(P) Paramagnetic cation
(B) $(NH_4)_2\,[NiCl_4]$	(Q) Paramagnetic anion
(C) CsO_2	(R) Tetrahedral cation
(D) $[Co(H_2O)_4][CoCl_4]$	(S) Tetrahedral anion
	(T) π-bond(s) in cation

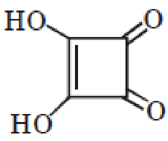
Ans. (A)-(S,T); (B)-(Q,R,S); (C)-(Q); (D)-(P,Q,R,S)

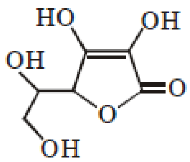
SECTION-III: (Maximum Marks: 18)

- This section contains **SIX** questions.
- The answer to each question is a **NUMERICAL VALUE.**
- For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the **second decimal place**; e.g. 6.25, 7.00, –0.33, –.30, 30.27, –127.30, if answer is 11.36777..... then both 11.36 and 11.37 will be correct)
- Answer to each question will be evaluated according to the following marking scheme:
  Full Marks : +3 If ONLY the correct numerical value is entered as answer.
  Zero Marks : 0 If none of the bubbles is darkened.
  Negative Marks : –1 In all other cases

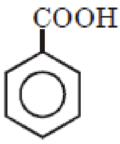
11.

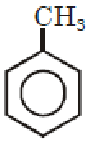
Identify total number of compounds which librate $CO_2\uparrow$ gas on reaction with aqueous $NaHCO_3$?

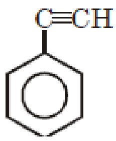
(a) 

(b) 

(c) 

(d) 

(e) 

(f)  ;

(g) H_2SO_4

(h) MeCOOH

Ans. 5

12.

The half-life period of I^{125}_{53} is 60 days. What percent of the original radioactivity of its sample will be present after 180 days ?

Ans. (12.50)

13.

On the basis of the following observations made with aqueous solutions of given complexes, assigning secondary valency of each metal, write the sum of secondary valencies of all given complexes as your answer.

Formula	Moles of AgCl precipitated per mole of compound with excess $AgNO_3$ solution
(i) $PdCl_2.4NH_3$	2 moles
(ii) $NiCl_2.6NH_3$	2 moles
(iii) $CoCl_3.4NH_3$	1 mole
(iv) $PtCl_2.2NH_3$	Zero mole

Ans.(20.00)

14.

How many of the following processes may involve coagulation of colloids ?

(i) Tanning of Leather

(ii) Persistent dialysis

(iii) Electrophoresis

(iv) Peptization

(v) Mixing of two lyophobic sols

(v) Mixing of two lyophobic sols

(vi) Mixing sol-I ($AgNO_3$ with excess KI) and sol-II (KI with excess $AgNO_3$)

(vii) Centrifugation

(viii) Freezing

Ans.(7.00)

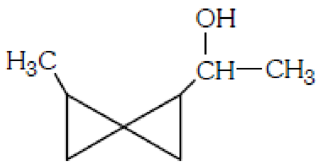
15.

6×10^{-3} mol $K_2Cr_2O_7$ reacts completely with 9×10^{-3} mol X^{n+} to give XO_3^- and Cr^{+3} then value of n will be.

Ans. 1

Eq. of $K_2Cr_2O_7$ = Eq. of X^{n+}.

16.



Total number of optical isomers of above comopound are :

Ans. 4

$2^2 = 4$.

MATHEMATICS

SECTION–I(i) : (Maximum Marks : 32)

- This section contains **EIGHT** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is (are) correct.
- For each question, darken the bubble(s) corresponding to all the correct option(s) in the ORS
- For each question, marks will be awarded in one of the following categories :

Full Marks : +4 If only the bubble(s) corresponding to all the correct option(s) is (are) darkened.

Zero Marks : 0 If none of the bubbles is darkened.

Negative Marks : –2 In all other cases.

1.

The solution of differential equation $3\frac{dx}{dy}=\frac{x}{x^3-y}$ is $x^{\ell}=mx^n y+c$, then which of the following is/are correct ?

(c is any arbitrary constant)

(A) $\ell + m + n = 11$ (B) $\ell + n = 9$ (C) $\ell + 2m = 10$ (D) $m + n = 4$

Ans. (A,B,C)

$$3x^2\frac{dx}{dy}=\frac{x^3}{x^3-y} \qquad x^3 = t$$

$$\Rightarrow \frac{dt}{dy}=\frac{t}{t-y} \qquad 3x^2\,dx = dt$$

$$\Rightarrow t\,dt = y\,dt + t\,dy \Rightarrow \frac{t^2}{2}=t.y+\frac{c}{2}$$

$$\Rightarrow x^6 = 2x^3y + c$$

2.

Let 8 persons are sitting in a row, If probability that two particular person A and B must be sitting next to each other and C must be somewhere behind D is equal to K, then K lies in the interval :

(A) $\left[\frac{1}{10},\frac{1}{5}\right]$ (B) $\left(\frac{1}{8},\frac{1}{4}\right)$ (C) $\left(\frac{1}{9},\frac{1}{3}\right)$ (D) $\left[\frac{1}{8},\frac{1}{2}\right]$

Ans. (A,C,D)

x x [AB] x x x x

C D

[AB] and 6 otherr is 7! but A and B can be arranged in 2! ways

Total ways = 7! . 2!

when C is behind D

∴ Required number of ways =

$$\frac{7!.2!}{2!}=5040 \text{ ways}$$

$$\text{So probability} = \frac{5040}{8!}=\frac{1}{8}$$

3.

The co-efficient of x^5 in the expansion of $(1+5x)^5 + (1+5x)^6 + \ldots\ldots + (1+5x)^{19}$ is

(A) ${}^{20}C_{14} . 5^5$ (B) ${}^{20}C_6 . 5^5$ (C) $\left({}^{21}C_7 - {}^{20}C_7\right)5^5$ (D) ${}^{20}C_5 . 5^6$

Ans. (A,B,C)

Coefficient of $x^5 = {}^5C_5 . 5^5 + {}^6C_5 . 5^5 + \ldots .. +$

${}^{19}C_5 . 5^5 \; [{}^5C_5 + {}^6C_5 + \ldots . + {}^{19}C_5]$

$= {}^{20}C_6 . 5^5$

4.

If [x] denotes the greatest integer less than or equal to x then $\int_0^{\infty}\left[\frac{2}{e^x}\right]dx$ is equal to :

(A) $\ell n\, 2$ (B) e^2 (C) $\log_2 e$ (D) $\lim_{x\to1} \ell n\left(\frac{x^3-x^2+x-1}{x-1}\right)$

Ans. (A,D)

$$I = \int_0^{\ell n 2}\left[\frac{2}{e^x}\right]dx + \int_{\ell n 2}^{\infty}\left[\frac{2}{e^x}\right]dx = \int_0^{\ell n 2} 1\,dx + \int_{\ell n 2}^{\infty} 0\,dx$$

$= \ell n\, 2$

5.

Let f be a real valued function such that $f(x)+3xf\left(\frac{1}{x}\right)=2(x+1)$ for all real x >0 then which of the following is/are correct

(A) $f(2020) = 2021$ (B) $f'(2020) = \frac{1}{2}$

(C) $f'(2021) = \frac{1}{2}$ (D) Number of solutions of $f(x) = x^2$ is 2

Ans. (B,C)

$f(x) + 3x\,f\left(\frac{1}{x}\right) = 2(x+1) \quad \ldots .(1)$

Replace x by $\frac{1}{x}$

$f\left(\frac{1}{x}\right)+\frac{3}{x}f(x)=2\left(\frac{1}{x}+1\right) \ldots .(2)$

Solving (1) and (2), we get

$f(x) = \frac{x+1}{2}$ $\quad f'(x) = \frac{1}{2}$

$f(x)=x^2 \Rightarrow 2x^2-x-1=0$

$x = 1,\ x = \frac{-1}{2}$ (Rejected)

6.

If a line x + y – 2 = 0 is tangent to an ellipse at a point P (a, b) with foci $S_1(3,2)$ and $S_2(4,7)$ then which of the following is/are correct

(A) 2a – b = 1

(B) a + b = 2

(C) equation of auxiliary circle is $\left(x-\frac{7}{2}\right)^2+\left(y-\frac{9}{2}\right)^2=20$

(D) eccentricity of ellipse is $\frac{\sqrt{26}}{4\sqrt{5}}$

Ans. (A,B,C,D)

Let S_1' be the reflection of S_1 (3, 2)

$$\frac{x-3}{1}=\frac{y-2}{1}=-3$$

$S_1'(0,-1)$ and $S_1'S_2 : y = 2x - 1$

solve with x + y – 2 = 0

a = 1, b = 1

$PS_1+PS_2=\sqrt{5}+\sqrt{45}$

$S_1S_2=\sqrt{26}$

7.

Let g(x) = $\begin{cases} 2(x+1) & -\infty < x \le -1 \\ \sqrt{1-x^2} & -1 < x < 1 \\ \left|\left|\left|x\right|-1\right|-1\right|, & 1 \le x < \infty \end{cases}$

then

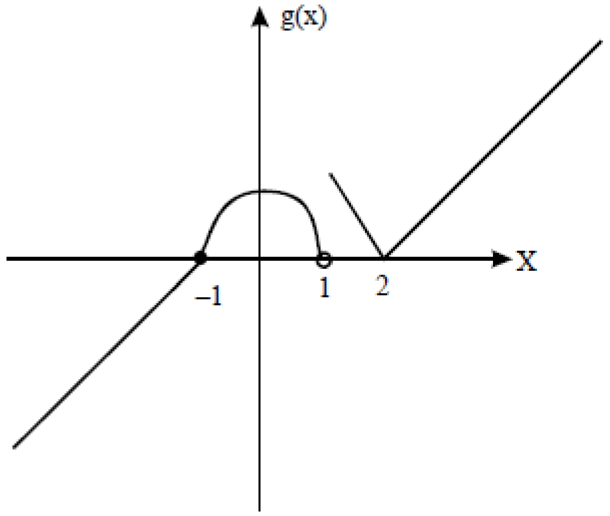
(A) g(x) is non–differentiable at exactly three points.

(B) g(x) is continuous in (–∞,1]

(C) g(x) is differentiable in (–∞,–1)

(D) g(x) has finite type of discontinuity at x =1 but continuous at x = –1

Ans. (A,C,D)



g(x) is discontinuous at x=1 and non-differentiable at x= –1,1,2

8.

The value of the expression $\frac{2}{(n+1)}\sum_{r=0}^{n}\left({}^{n+1}C_{r+1}(-1)^r\left(\sqrt{k}\right)^{r+1}\right)$, where k is a constant, is equal to : (given k > 0)

(A) $\frac{2}{(n+1)}$ when k =1

(B) $\int_0^k x^{-1/2}\left(1-x^{1/2}\right)^n$ dx for any k

(C) $\int_0^1 \frac{4tdt}{\left(1+t^2\right)^{n+2}}$ when $k=\frac{1}{4}$

(D) $2\sqrt{k}\,{}^nC_0-\frac{2k}{2}\,{}^nC_1+\frac{2k\sqrt{k}\,{}^nC_2}{3}+.....+\frac{(-1)^n 2\left(\sqrt{k}\right)^{n+1}}{(n+1)}$

Ans. (A,B,C,D)

(A),(B), (C), (D) Given expression

$$E=-\frac{2}{(n+1)}\sum_{r=0}^{n}{}^{n+1}C_{r+1}\left(-\sqrt{k}\right)^{r+1}$$

$$=\frac{2}{n+1}\left(-\left(1-\sqrt{k}\right)^{n+1}+1\right).$$

Evaluate (B) and (C) by putting $\sqrt{x}=y$ and $t^2=y$ respectively, and then integrating

In (D)

$$t_r=(-1)^r.2\frac{{}^nC_r\left(\sqrt{k}\right)^{r+1}}{r+1}=\frac{2}{n+1}\,{}^{n+1}C_{r+1}(-1)^r\left(\sqrt{k}\right)^{r+1}$$

= general term of E.

SECTION–II : (Maximum Marks : 16)

- This section contains TWO question.
- Question contains two columns, Column-I and Column-II.
- Column-I has four entries (A), (B), (C) and (D)
- Column-II has five entries (P), (Q), (R), (S) and (T)
- Match the entries in Column-I with the entries in column-II.
- One or more entries in Column-I may match with one or more entries in Column-II.
- The ORS contains a 4 × 5 matrix whose layout will be similar to the one shown below :

(A) (P) (Q) (R) (S) (T)

(B) (P) (Q) (R) (S) (T)

(C) (P) (Q) (R) (S) (T)

(D) (P) (Q) (R) (S) (T)

- For each entry in column-I, darken the bubbles of all the matching entries. For example, if entry (A) in Column-I matches with entries (Q), (R) and (T), then darken these three bubbles in the ORS. Similarly, for entries (B), (C) and (D).
- For each question, marks will be awarded in one of the following categories :

For each entry in Column-I

Full Marks	: +2	If only the bubble(s) corresponding to all the correct match(es) is (are) darkened
Zero Marks	: 0	In none of the bubbles is darkened
Negative Marks	: –1	In all other cases

9.

	Column-I		Column-II
(A)	If each roots of the equation $x^2 + bx + a = 0$ is one less than the roots of the equation $x^2 + ax + b = 0$, then $\|a + b\|$ is less than $(a, b \in R)$	(P)	1
(B)	If $2016^x + 2016^{-x} = 3$, then $\frac{1}{4}\sqrt{\frac{2016^{6x} - 2016^{-6x}}{2016^x - 2016^{-x}}}$ is greater than or equal to	(Q)	3
(C)	If the third & sixth term of a geometric progression are 10 and 1000 respectively, then $\frac{\sqrt{g_1 g_8}}{25}$ is less than or equal to	(R)	4
(D)	If $\|x + 2\| + \|x + 3\| + \|x + 4\| \geq k \ \forall \ x \in R$, then k can be	(S)	5
		(T)	6

Ans.(A)→(S,T); (B)→(P,Q); (C)→(R,S,T); (D)→(P)

(A) $x^2 + bx + a = 0 \begin{cases} \alpha - 1 \\ \beta - 1 \end{cases}$

$\alpha + \beta - 2 = -b, \ (\alpha-1)(\beta-1) = a$

$x^2 + ax + b = 0 \begin{cases} \alpha \\ \beta \end{cases}$

$\alpha + \beta = -a, \ \alpha\beta = b$

$\therefore \ -a - 2 = -b$ and $b = -1$

$\Rightarrow a = -3$

(B) Let $(2016)^x = t$

given $t + \frac{1}{t} = 3$

$$\frac{1}{4}\sqrt{\frac{(2016)^{6x} - (2016)^{-6x}}{(2016)^x - (2016)^{-x}}} = \frac{1}{4}\sqrt{\frac{t^6 - t^{-6}}{t - t^{-1}}}$$

$$= \frac{1}{4}\sqrt{t^5 + \frac{1}{t^5} + t^3 + \frac{1}{t^3} + \frac{1}{t} + t}$$

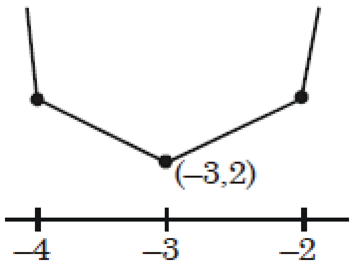
$$= \frac{1}{4}\sqrt{123 + 18 + 3} = 3$$

(C) $T_3 = ar^2 = 10$

$T_6 = ar^5 = 1000$

$$\therefore \ \frac{\sqrt{g_1 g_8}}{25} = \frac{\sqrt{a.ar^7}}{25}$$

$$= \frac{\sqrt{10000}}{25} = 4$$

(D) 

minimum value of

$|x + 2| + |x + 3| + |x + 4|$ is 2

$\therefore k \le 2$

10.

In the parabola $y^2 + 4 = 4x$, a chord passing through the point (2, 0) cuts the parabola at P and Q. If P = (5, 4) and the tangents at P and Q meet at R then match the following .

	Column-I		**Column-II**
(A)	The focus is	(P)	$\left(0, \frac{3}{2}\right)$
(B)	The centroid of the ΔPQR is	(Q)	(2, 0)
(C)	The circumcentre of the ΔPQR is	(R)	$\left(\frac{25}{12}, \frac{3}{2}\right)$
(D)	The orthocentre of the ΔPQR is	(S)	$\left(\frac{25}{8}, \frac{3}{2}\right)$
		(T)	None of these

Ans. A→ Q, B→ R, C→ S, D→ P

A. $y^2 = 4(x-1)$. Putting $x - 1 = X$ and $y = Y$, we get

$Y^2 = 4X = 4 . 1. X$

$\therefore$ focus $= (1, 0)_{X, Y} = (2, 0)$

B. PQ has the equation $y - 0 = \frac{4-0}{5-2}(x-2)$ or

$4x - 3y - 8 = 0$

If $R = (\alpha, \beta)$, the polar line PQ should be

$y\beta = 2(x + \alpha) - 4$

or $2x - \beta y + 2\alpha - 4 = 0$

$\therefore \frac{4}{2} = \frac{-3}{-2} = \frac{-8}{2\alpha - 4} \Rightarrow \beta = \frac{3}{2}$ and $\alpha = 0$,

So, $R = \left(0, \frac{3}{2}\right)$

Solving $4x - 3y - 8 = 0$ and $y^2 = 4(x-1)$,
we get

$$\left(\frac{4x-8}{3}\right)^2 = 4(x-1) \quad \text{or} \quad x = 5,\ \frac{5}{4}. \text{ So}$$

$$Q = \left(\frac{5}{4}, -1\right)$$

$$\therefore \text{ centroid, } G = \left(\frac{5+\frac{5}{4}+0}{3}, \frac{4-1+\frac{3}{2}}{3}\right) = \left(\frac{25}{12}, \frac{3}{2}\right)$$

C. (α, β) is the circumcentre if

$$(\alpha-0)^2 + \left(\beta - \frac{3}{2}\right)^2 = (\alpha-5)^2 + (\beta-4)^2$$

$$= \left(\alpha - \frac{5}{4}\right)^2 + (\beta+1)^2$$

$$\Rightarrow \quad -3\beta + \frac{9}{4} = -10\alpha - 8\beta + 41 = -\frac{5}{2}\alpha + 2\beta + \frac{41}{16}$$

Solving $\alpha = \frac{25}{8}$, $\beta = \frac{3}{2}$, So circumference

$$M = \left(\frac{25}{8}, \frac{3}{2}\right)$$

(D) (α, β) is the orthocentre if

$$\frac{\alpha + \frac{25}{4}}{3} = \frac{25}{12} \text{ and } \frac{\beta+3}{3} = \frac{3}{2}$$

$$\therefore \quad \alpha = 0,\ \beta = \frac{3}{2}. \text{ So orthocentre } = \left(0, \frac{3}{2}\right)$$

SECTION-III: (Maximum Marks: 18)

- This section contains **SIX** questions.
- The answer to each question is a **NUMERICAL VALUE**.
- For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the **second decimal place**; e.g. 6.25, 7.00, –0.33, –.30, 30.27, –127.30, if answer is 11.36777..... then both 11.36 and 11.37 will be correct)
- Answer to each question will be evaluated according to the following marking scheme:
  Full Marks : +3 If ONLY the correct numerical value is entered as answer.
  Zero Marks : 0 If none of the bubbles is darkened.
  Negative Marks : –1 In all other cases

11.

Let $A=\begin{bmatrix}-3 & 0 & 2\\ 1 & x & 5\\ -2 & 0 & x^2\end{bmatrix}, B=\begin{bmatrix}2\\ b\\ -1\end{bmatrix}$ and C = [3 5 1], then the number of integral value(s) of 'b' for which $Tr(ABC) \le -18\ \forall\ x \in R$ is (are)

Ans. 5

$$ABC=\begin{bmatrix}-3 & 0 & 2\\ 1 & x & 5\\ -2 & 0 & x^2\end{bmatrix}\begin{bmatrix}2\\ b\\ -1\end{bmatrix}\begin{bmatrix}3 & 5 & 1\end{bmatrix}$$

$Tr(ABC) = -x^2 + 5bx - 43$

$-x^2 + 5bx - 43 \le -18\ \forall\ x \in R$

$-x^2 + 5bx - 25 \le 0\ \forall\ x \in R$

$D \le 0 \Rightarrow b \in [-2,2]$

12.

The plane x + 2y + 3z = 7 is rotated about the line where it cut yz – plane by an angle θ. In the new position the plane contains the point (–1,0,2). If $|\cos\theta| = \frac{p}{q}$ (where p and q are coprime) then the absolute value of (p – q) is

Ans. 1

Let equation of plane be $x + 2y + 3z - 7 + \lambda x = 0$

since (–1,0,2) lies on it $\therefore \lambda = -2$

equation of plane is $-x + 2y + 3z - 7 = 0$

$$|\cos\theta| = \frac{6}{7} = \frac{p}{q}$$

13.

$\int e^{e^x+x^3+x}\left[\left(xe^x + 3x^3 + x + 1\right)\sin x + x\cos x\right]dx$ is equal to $f(x)\,e^{e^x+x^3+x} + c$ then $\left[f\left(\frac{5\pi}{2}\right)\right]$ is equal to (where[.] represent G.I.F.)

Ans. 7.00

$$= \int \underbrace{x\sin x}_{I}\ \underbrace{e^{e^x+x^3+x}\left(e^x + 3x^2 + 1\right)}_{II}dx$$

$$+\int e^{e^x+x^3+x}\left(\sin x + x\cos x\right)dx$$

$$= x\sin x\, e^{e^x+x^3+x} - \int\left(x\cos x + \sin x\right)e^{e^x+x^3+x}dx$$

$$+\int e^{e^x+x^3+x}\left(\sin x + x\cos x\right)dx$$

$$= x\sin .e^{e^x+x^3+x} + c$$

14.

Let z_1 and z_2 be two complex numbers such that $|z_1| = 1$ and $|z_2| = 10$. If $\theta = \arg\left(\frac{z_1 - z_2}{z_2}\right)$ then maximum value of $\tan^2\theta$ can be expressed as $\frac{m}{n}$ (where m and n are coprime). the value of (100m −n) is

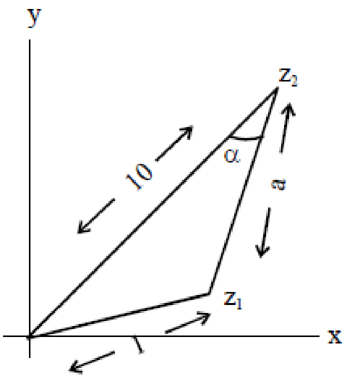
Ans. 1.00

$$\theta = \arg\left(\frac{z_1 - z_2}{z_2}\right)$$

$$\theta = \pi + \arg\left(\frac{z_2 - z_1}{z_2}\right) = \pi + \alpha$$

let $a = |z_1 - z_2|$

$$\cos\alpha = \frac{a^2 + 100 - 1}{20a}$$



$$\cos\alpha = \frac{\left(a + \frac{99}{a}\right)}{20}$$

$$\tan^2\theta = \sec^2\theta - 1 = \sec^2(\pi + \alpha) - 1 = \sec^2\alpha - 1$$

$$\tan^2\theta = \frac{400}{\left(a + \frac{99}{a}\right)^2} - 1$$

$$= \frac{400}{4 \times 99} - 1 \qquad (\because \text{A.M} \geq \text{G.M.})$$

$$= \frac{1}{99}$$

15.

Circle with centre (0,0) is circumscribing the ellipse $5x^2 + 4y^2 + xy - 2 = 0$ then square of the radius of circle is of the form $\frac{a+\sqrt{b}}{c}$ (where $a,b,c \in N$ and HCF of (a, c) is 1) then value of $\frac{c+b-3a}{2}$ is

Ans : 1.50

Centre is (0, 0)

Let $P(r\cos\theta, r\sin\theta)$ be any point on ellipse.

$$\Rightarrow \quad r^2 = \frac{4}{9+\cos 2\theta + \sin 2\theta}$$

$$\Rightarrow \quad r_{max} = \frac{2}{\sqrt{9-\sqrt{2}}}$$

$$\therefore r^2 = \frac{36+\sqrt{32}}{79}$$

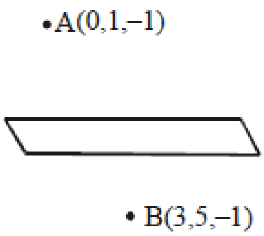
a= 36, b =32, c=79

16.

In the plane $x + y - 2z = 5$, P(a,b,c) is a point such that the sum of its distances from the point A (0, 1, –1) and B (3, 5, –1) is least then the value of (a + b + c) is

Ans. 2.00

$$\frac{x-0}{3} = \frac{y-1}{4} = \frac{z+1}{0} = \lambda$$



General Point on line is $(3\lambda, 4\lambda+1, -1)$

This point will satisfy plane $x + y - 2z = 5$

$$3\lambda + 4\lambda + 1 + 2 = 5$$

$$\lambda = \frac{2}{7}$$

$$P\left(\frac{6}{7}, \frac{15}{7}, -1\right)$$