PART (A) : PHYSICS

ANSWER KEY

1.	(A)	2.	(D)	3.	(B)	4.	(B)	5.	(D)
6.	(C)	7.	(A)	8.	(A)	9.	(B)	10.	(B)
11.	(B)	12.	(B)	13.	(B)	14.	(B)	15.	(B)
16.	(A)	17.	(B)	18.	(A)	19.	(B)	20.	(B)
21.	(6)	22.	(1)	23.	(25)	24.	(60)	25.	(2)
26.	(3)	27.	(10)	28.	(14)	29.	(4)	30.	(1)

SOLUTIONS

1. (A)

Given that $T^3V^2 = \text{constant}$

$\Rightarrow \left(\frac{pV}{nR}\right)^3 V^2 = \text{constant}$ (∵ using ideal gas equation $pV = nRT$ or $T = \frac{pV}{nR}$)

$\Rightarrow p^3V^5 = \text{constant} \Rightarrow pV^{\frac{5}{3}} = \text{constant}$ .....(i)

Differentiating Eq. (i) with respect to volume V on both sides, we get

$$\Rightarrow \frac{dp}{dV}.V^{\frac{5}{3}} + p.V^{\frac{2}{3}} = 0$$

$$\Rightarrow \frac{dp}{dV} = -\frac{5}{3}\frac{p}{V} \quad ....(ii)$$

Bulk modulus is defined as

$$B = \frac{\text{Volumetric stress}}{\text{Volumetric strain}}$$

$$= \frac{dp}{-\frac{dV}{V}} = -V\frac{dp}{dV}$$

$$= -V\left(-\frac{5}{3}\frac{p}{V}\right) \quad \text{[using Eq. (ii)]}$$

$$= \frac{5}{3}p$$

2. (D)

Given, potential at point C is zero, this means potential at O is zero as surface of metal bodies is equipotential in nature

$$\Rightarrow V_0 = 0 \Rightarrow V_{dipole} + V_{point} = 0$$

$$\Rightarrow \frac{Kp}{(2R)^2}\cos 180^0 + \frac{Kq}{(2R)} = 0$$

$\Rightarrow p = 2qR$ ……(i)

Now, as net field inside metal ball =0

So at O

$\Rightarrow E_{dipole} + E_{point} + E_{induced} = 0$

$\Rightarrow -\frac{2Kp}{(2R)3}\hat{i} - \frac{Kq}{(2R)2}\hat{i} + E_{induced} = 0$ …….(ii)

$\Rightarrow E_{induced} = \frac{3}{8}\frac{Kp}{R^3}\hat{i} = \frac{3}{4}\frac{Kq}{R^2}\hat{i}$

(Using equation (i) and (ii))

3. (B)

Since, force applied is same

$\Rightarrow k_1x_1 = k_2x_2 = k_{eq}, x_0$, where

$k_{eq} = \frac{k_1k_2}{k_1+k_2}$

$\Rightarrow x_2 = \frac{k_1x_1}{k_2}$ and $x_0 = \left(\frac{k_1+k_2}{k_2}\right)x_1$

W_1 = Work done by F on block,

$W_1 = \frac{1}{2}k_{eq}x_0^2$

$= \frac{1}{2}\frac{k_1k_2}{k_1+k_2} \times \left(\frac{k_1+k_2}{k_2}\right)^2 x_1^2$

$= \frac{1}{2}\frac{k_1(k_1+k_2)x_1^2}{k_2}$

As change in KE of block is zero, work done by S_2 on block

$= -W_1 = -\frac{1}{2}\frac{k_1(k_1+k_2)x_2^1}{k_2}$

Work done by S_2 on S_1 = change in PE of S_1 = $\frac{1}{2}k_1x_1^2$

As displacement of wall is zero, work done by S_1 on wall = 0

4. (B)

Given $f_0 = 2cm, f_e = 5cm$

$|v_0| + |u_e| = 20cm,$

$v_e = -25cm$

From lens formula $\frac{1}{f_e} = \frac{1}{v_e} - \frac{1}{f_e} = -\frac{1}{25} - \frac{1}{5}$

$\therefore u_e = -\frac{25}{6}cm$

Distance of real image from objective,

$$v_0 = 20 - |u_e| = 20 - \frac{25}{6}$$

$$= \frac{120-25}{6} = \frac{95}{6}\,\text{cm}$$

Now, $\frac{1}{f_0} = \frac{1}{v_0} - \frac{1}{u_0}$

$$\Rightarrow \frac{1}{u_0} = \frac{1}{v_0} - \frac{1}{f_0} = \frac{1}{(95/6)} = \frac{1}{2}$$

$$= \frac{6}{95} - \frac{1}{2} = \frac{12-95}{190} = -\frac{83}{190}$$

$$\therefore u_0 = -\frac{190}{83} = -2.3\text{cm}$$

Magnifying power

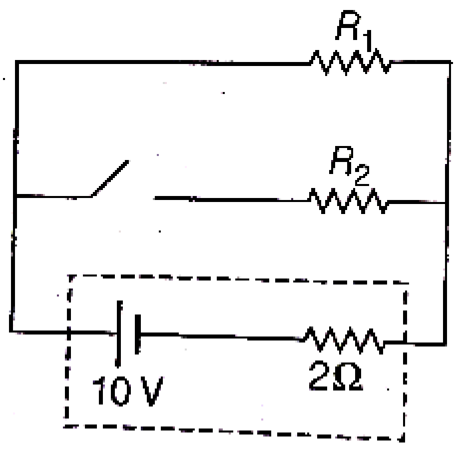
$$M = \frac{v_0}{u_0}\left(1 + \frac{D}{f_e}\right)$$

$$= -\frac{95/6}{2.3}\left(1 + \frac{25}{5}\right) = -41.5$$

It is close to 40

5. (D)

Diode 1 is forward biased as its p-side is connected to higher potential terminal of cell. So, it behaves as place wire while Diode 2 is reverse biased. So, it will act like an open switch. Therefore, equivalent circuit is as shown



So, effective resistance of external circuit is R1. For maximum power consumption $R_{external}$ = $R_{intenal}$ as as maximum power theorem

$$\Rightarrow R_1 = 2\Omega$$

And R_2 may be any value, hence option (d) is correct

6. (C)

At any general time t, potential difference across inductor is equal to emf of cell. So, for inductor

$$\frac{Ldi}{dt} = E$$

$$\Rightarrow di = \frac{E}{L} dt$$

Integrating both side, we get

$$\Rightarrow i = \frac{E}{L} t \qquad \text{......(i)}$$

So, magnetic energy stored in an inductor

$$U = \frac{1}{2} Li^2 = \frac{1}{2} L \left(\frac{E}{L} t \right)^2 \text{ [using Eq. (i)]}$$

$$\Rightarrow U \propto t^2$$

$\Rightarrow$Graph between U and t will be upward concave parabola

7. (A)

$$a = \frac{F}{m} = \frac{eE}{m} = \frac{\rho de}{6m\varepsilon_0}$$

$$t = \sqrt{\frac{2s}{a}}$$

$$t = \sqrt{\frac{12\varepsilon_0 m}{\rho e}}$$

8. (A)

(i) λ_1 is of radio waves

(ii) λ_2 is of UV rays

(iii) λ_3 is of X-ray

(iv) λ_4 is of infrared rays

$\therefore \lambda_3 < \lambda_2 < \lambda_4 < \lambda_1$

9. (B)

Considering that to find mass of parent and daughter nuclei mass of electron needs to be subtracted from atomic masses.

$Q = (m_{reactant} - m_{product})c^2$

$=[(m_1 - Zm_3) - \{(m_2 - Zm_3 + m_3) + m_3\}]c^2$

$=(m_1 - m_2 - 2m_3)c^2$

$\therefore k + 1 = 2 + 1 = 3$

10. (B)

For disc to be at rest, contact force F by the insect on the disc should pass through centre O of the disc. From free body diagram shown, for horizontal direction.

$F\cos\theta = ma$ ....(i)

For vertical direction,

$F\sin\theta = mg$ ....(ii)

Dividing Eq. (i) by Eq.(ii) we get

$$\cot\theta = \frac{a}{g} \Rightarrow a = g\cot\theta = \frac{gx}{h}$$

Where, x = Distance of insect from the centre of groove.

As acceleration is comparable to $a = \omega^2 x$ and is directed towards centre of the groove, so motion of the insect is SHM, with angular frequency

$$\omega = \sqrt{\frac{g}{h}} \quad .....(iii)$$

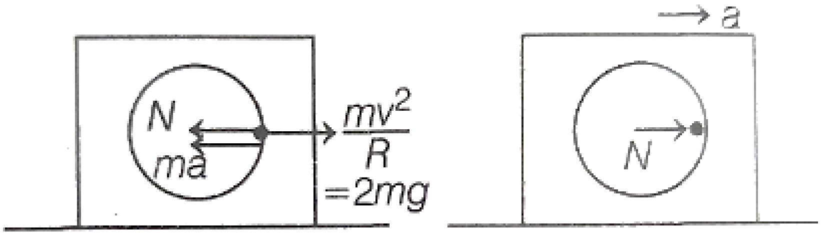
Also, amplitude of SHM is equal to maximum distance of the insect from centre R of the groove, so amplitude is A = RQ = h (using trigonometry) .....(iv)

Maximum speed in SHM is at mean position equal to

$$v = A\omega = h\sqrt{\frac{g}{h}} = \sqrt{gh} \quad \text{[using eqs. (iii) and (iv)]}$$

11. (B)

Free body diagram of ball (in block's frame) and block (showing only horizontal forces) are as shown in figure



From free body diagram of the ball,

N + ma = 2mg ...(i)

From free body diagram of the block using

$F_{net} = ma$

N = 2ma .....(ii)

From Eqs (i) and (ii) we get

$$2ma + ma = 2mg \Rightarrow a = \frac{2g}{3}$$

12. (B)

The final output of the arrangement as shown below is

$$Y = \overline{Z.B} = \overline{(A+\overline{B}).B} = (\overline{A+\overline{B}}) + \overline{B}$$

$$= \overline{A}.B + \overline{B} \quad \text{(using de Morgan's theorem)}$$

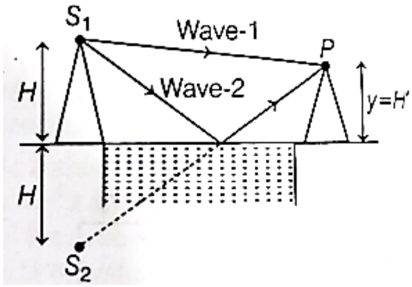
Truth table for the output Y is

A	B	$\overline{A}$	$\overline{B}$	$\overline{A}.B$	Y
0	0	1	1	0	1
1	0	0	1	0	1
0	1	1	0	1	1
1	1	0	0	0	0

As the output corresponds to NAND gate, so the arrangement is equivalent to NAND gate.

13. (B)

Signal will reach directly (Wave-1) and after reflection (Wave -2) as shown



The reflected signal can be assumed to be coming from image S2 of transmitter S_1 as shown in the figure. So, the geometry is similar to Young's double slit experiment (YDSE).

Also, due to reflection of wave -2 at water surface, it undergoes phase change of π which is equivalent to path difference of $\frac{\lambda}{2}$. Therefore, net path difference $=\frac{2HH'}{D}+\frac{\lambda}{2}$

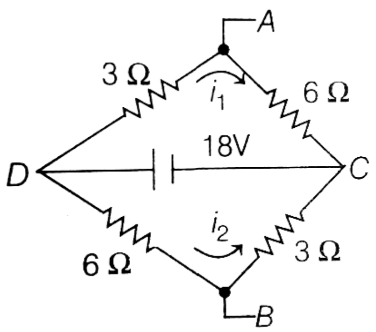
For maxima, $\frac{2HH'}{D}+\frac{\lambda}{2}=n\lambda$

$$\Rightarrow \frac{2HH'}{D}=\left(n-\frac{1}{2}\right)\lambda$$

Only option satisfying above equation is option (b) for n = 2

$$\Rightarrow H'=\left(2-\frac{1}{2}\right)\lambda\times\frac{D}{2H}=\frac{3\lambda D}{4H}$$

14. (B)



Applying KCL in CDAC

$18-3i_1-6i_1=0$

$\therefore i_1=2A$

Applying KCL in CDBC,

$18 - 6i_2 - 3i_2 = 0$

$i_2 = 2A$

Now

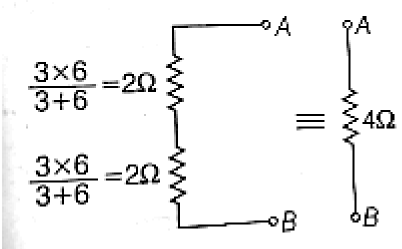
$V_D - V_A = 3 \times 2 = 6V$ ……(i)

$V_D - V_B = 6 \times 2 = 12V$ ……(ii)

Subtracting Eq. (i) from Eq. (ii), we get

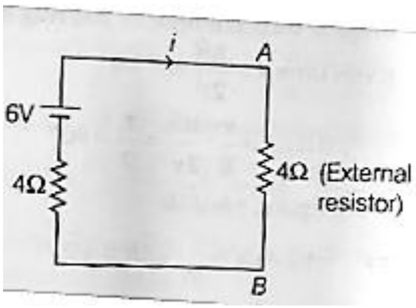
$V_A - V_B = 6V$

Also, as ideal cell has zero resistance, to find effective resistance between A and B replace cell by plain (resistance less) wire, we get



Thus, given circuit is equivalent to a cell of emf $V_{AB} = 6V$ and internal resistance $R_{AB} = 4\Omega$ as shown. For maximum power

$R_{external} = R_{internal}$



Current i supplied to external resistor, by Ohm's law

$$i = \frac{6}{4+4} = \frac{3}{4}A$$

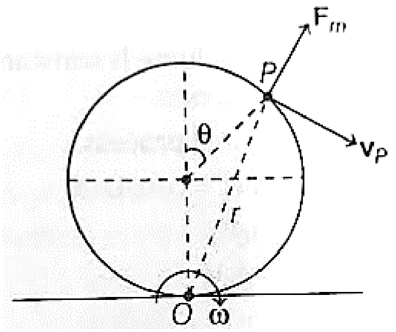
Therefore, power consumed

$$= i^2R = \left(\frac{3}{4}\right)2 \times 4 = 2.25W$$

15. (B)

In pure rolling, bottommost point (O in figure) is instantly at rest and so, we can assume an instantaneous axis. Assume distance between bottommost point O and point charge at P equal to r. Considering pure rolling as pure rotation about instantaneous axis through O, velocity of point charge V_P is perpendicular to line joining point charge and O, is given by

$V_P = \omega r$ …..(i)



Now, magnetic force on point charge,

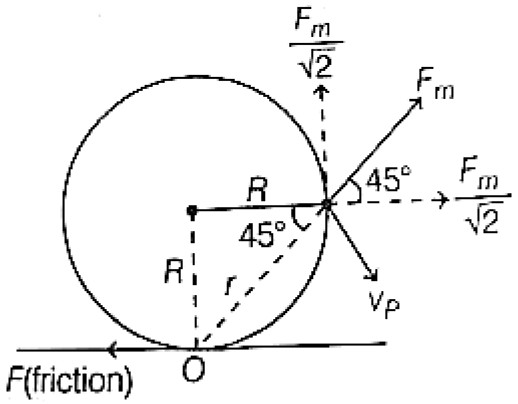
$F_m = qv_pB \sin 90^0 = qV_pB$ (as shown) ....(ii)

$\Rightarrow$ About O, $\alpha = \frac{\tau}{I} = \frac{F_m \times 0}{I} = 0$

$\Rightarrow \omega = \text{constant}$

As in pure rolling $V_{CM} = \omega R$ and ω is constant, so V_{CM} is also constant and acceleration of CM is zero. So, ring rolls with constant speed v and angular speed,

$\omega = \frac{V_{CM}}{R} = \frac{V}{R}$ . ....(iii)



Angular displacement of the ring in given time, $t = \frac{\pi R}{2v}$ will be

$\theta = \omega t = \frac{v}{R}, \frac{\pi R}{2v} = \frac{\pi}{2} = 90^0$

From figure $r = \sqrt{2}R$ .....(iv)

$\Rightarrow Vp = \omega r = \frac{v}{R}.\sqrt{2}R = \sqrt{2}v$ .....(v)

[Using eqs (i), (iii) and (iv)]

By Newton's law

$\Sigma f_x = ma_x$

$\Rightarrow -f + \frac{F_m}{\sqrt{2}}$ $[\because a_x = a_{CM} = 0]$

$\Rightarrow f = \frac{F_M}{\sqrt{2}}$

$= \frac{qv_P B}{\sqrt{2}}$

$= q\sqrt{2}vB / \sqrt{2}$

=qvB [using eqs. (ii) and (v)]

16. (A)

Let extension in spring at time t is equal to x. Free body diagram of block will be as shown

By $F_{net} = ma$,

mg – kx = ma

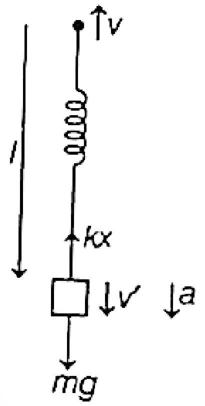
$\Rightarrow 3.6 \times 10 - 0.2x$

=3.6(10-t)

$\Rightarrow x = 18t$

$\Rightarrow I - I_0 = 18t$

Here l is the length of spring at shown time t and l_0 is natural length of spring



Differentiating Eq (i) w.r.t. time, we get

$\frac{dl}{dt} = v_{relative} = 18 m/s$

(Here $V_{relative}$ is relative velocity of block w.r.t. upper end of spring)

$\Rightarrow v + v' = 18$ ……(ii)

(Here v and v' are velocities of upper end of spring and block respectively). Now, as acceleration of block is given to be

$a = 10 - t$

$\Rightarrow dv' = (10 - t)dt$

Integrating both sides, we get

$\Rightarrow v' = 10t - \frac{t^2}{2}$ …….(iii)

Putting value of v' from eq (iii) in Eq. (iii) we ger

$\Rightarrow v = \frac{t^2}{2} - 10t + 18$ …..(iv)

At rest, v =0

$\Rightarrow v = \frac{t^2}{2} - 10t + 18$

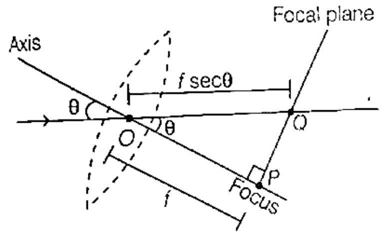
$\Rightarrow t^2 - 20t + 36 = 0$

$\Rightarrow t = 2s$ or $18s$

At t =2s, velocity of end of spring becomes zero for the first time.

17. (B)

When the lens is at mean position image is formed at focus (point P) at a distance f from the lens. When the lens is at extreme position as shown in figure, axs of the lens tilts by θ, so incident wave makes angle θ with the axis. As per rules of ray tracing this beam will be focussed in focal plane at point which can be located using the incident ray passing through optical centre of the lens



So, distance travelled by point of convergence as lens moves from mean to extreme position $\left(\frac{1}{4}\text{th oscillation}\right)$ is $OQ - OP = f\sec\theta - f = f(\sec\theta - 1)$

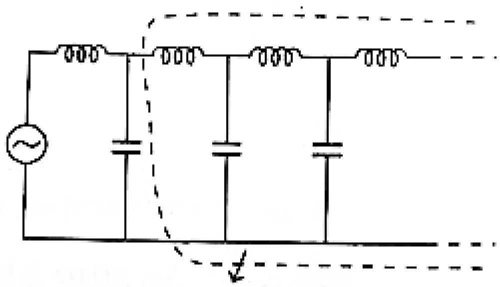
Therefore, in one oscillation, the convergence or image point moves 4 times this distance

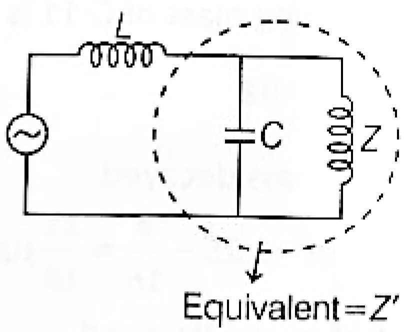
$\therefore$ Required distance $= 4f(\sec\theta - 1)$

$= 4f\left(\frac{1-\cos\theta}{\cos\theta}\right)$

18. (A)

Let equivalent impedance of the circuit is Z (inductive) as shown below





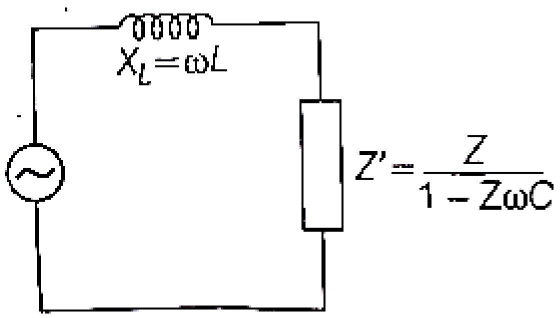
As the circuit portion in boundary is same as original circuit. SO, it impedance will also be Z. Therefore, portion in boundary can be replaced by element of impedance Z(inductive) as shown in figure.

Now as capacitor and the equivalent inductive element are in parallel, their equivalent impedance Z' can be calculated by formula for parallel combination. So

$$\frac{1}{Z'}=\frac{1}{Z}+\frac{1}{X_c}=\frac{1}{Z\hat{j}}+\frac{1}{\left(-\frac{1}{\omega C}\right)\hat{j}}$$

$$\Rightarrow Z'=\frac{Z}{1-Z\omega C}\hat{j} \qquad .....(i)$$

Circuit now reduces to



Therefore as the element are in series equivalent impedance will be

$$Z=X_L+Z'$$

$$\Rightarrow Z\hat{j}=\omega L\hat{j}+\frac{Z}{1+Z\omega C}\hat{j} \text{ [using eq (i)]}$$

$$\Rightarrow C.Z^2-\omega LC.Z+L=0$$

Solving the above quadratic equation

We get

$$Z=\frac{\omega LC\pm\sqrt{\omega^2L^2C^2-4LC}}{2C}$$

For circuit to be purely inductive impedance of the circuit should be independent of C. This will happed when term inside square root is zero. Therefore

$$\omega^2L^2C^2-4LC=0$$

And $Z=\frac{\omega LC}{2C}=\frac{\omega L}{2}=\omega L_{eq}$

$$\Rightarrow L_{eq}=\frac{L}{2}$$

19. (B)

Work done in closed path is zero for conservative and constant forces but not for non-conservative and variable force. As at higher temperatures, extre degrees of freedom (modes of energy) get activated internal energy can become more than doubled even though thermodynamic temperature is doubled.

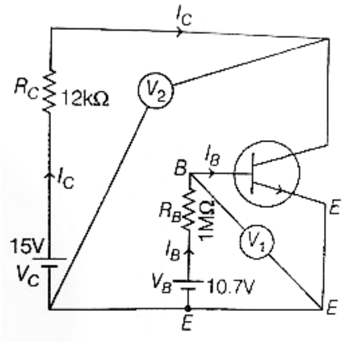
20. (B)

The current gain for CE configuration using the relation

$$\beta = \frac{\alpha}{1-\alpha}$$

Here α gain is for CB and β gain is for CE.

$$\Rightarrow \beta = \frac{\frac{100}{101}}{1-\frac{100}{101}} = 100 \qquad \ldots.(i)$$



From the circuit,

$V_1 = V_B - I_BR_B$

$$\Rightarrow 0.7 = 10.7 - I_B \times 10^6$$

$$\Rightarrow I_B = 10^{-5}\,A$$

As $I_C = \beta I_B$, therefore using Eq (i) we get

$$I_C = 100 \times 10^{-5} = 10^{-3}\,A$$

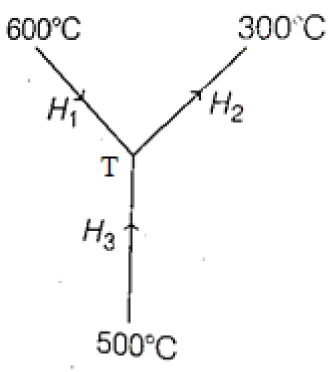
Also, form the circuit

$$V_2 = V_C - I_CR_C$$

$$= 15 - (10^{-3} \times 12 \times 10^2) = 3V$$

21. (6)

Let temperature of junction is T is shown in the figure



By conservation of thermal current

$$H_1 + H_3 = H_2$$

$$\Rightarrow \frac{KA(600-T)}{I}+\frac{3KA(500-T)}{I}=\frac{2KA(T-300)}{I}$$

$$\Rightarrow 600-T+3(500-T)=2(T-300)$$

$$\Rightarrow T=450^{0}C$$

$$\therefore \frac{H_1+H_2}{H_3}$$

$$=\frac{\frac{KA(600-450)}{I}+\frac{2KA(450-300)}{I}}{\frac{KA(500-450)}{I}}=\frac{150+150}{50}=\frac{300}{50}=6$$

22. (1)

Given that $P_{capacitor} = P_{resistor}$

$$\Rightarrow V_C i=V_R i\Rightarrow V_C=V_R$$

$$\Rightarrow \varepsilon\left(1-e^{-\frac{1}{RC}}\right)=\varepsilon e^{-\frac{t}{RC}}\Rightarrow e^{\frac{t}{RC}}=2$$

$$\Rightarrow t=RC\ln 2=0.69RC$$

23. (25)

Give $f'=f_{fundamental}$

$$\Rightarrow \frac{3u}{4l}\left\{\frac{u-v}{u}\right\}=\frac{u}{4l}$$

$$\Rightarrow \frac{v}{u}=\frac{2}{3}=\frac{a}{b}$$

$$\Rightarrow a+b=5$$

$$\Rightarrow (a+b)^2=25$$

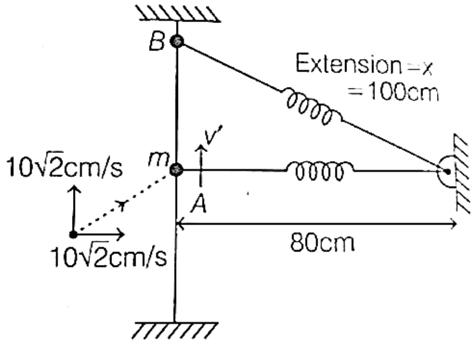
24. (60)

Velocity components of bead before collision are as shown in figure. Let velocity of (bead+particle) equal to v’ after collision. By conservation o momentum along rod,

$$2mv'=m.10\sqrt{2}$$

$$\Rightarrow v'=5\sqrt{2}m/s$$

At position of maximum displacement (B), speed is zero. So, all kinetic energy converts to spring potential energy.



$\Rightarrow$ Gain in PE = Loss in KE

$\Rightarrow \frac{1}{2}kx^2 = \frac{1}{2}2m.v'^2$

$\Rightarrow x = \sqrt{\frac{2m}{k}}v' = \sqrt{\frac{2\times1}{100}}\times5\sqrt{2}$

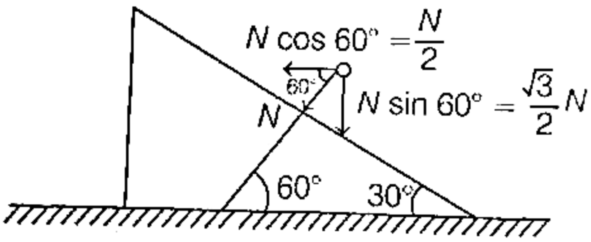
=1m = 100cm

$\therefore$ Max displacement

AB = $\sqrt{100^2 - 80^2} = 60\text{cm}$

25. (2)

Normal impulsive force between the ball and the wedge acts as shown during collision.



Horizontal component of the normal force = $N\cos60^0 = \frac{N}{2}$

If time of collision is Δt, for horizontal direction, by impulse momentum theorem, for wedge, that is, suing impulse = change in momentum, we get

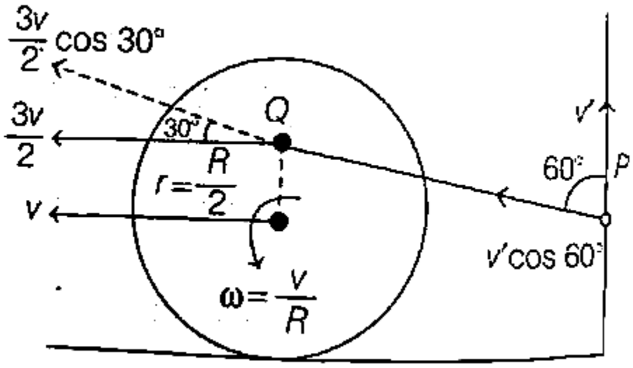
$\frac{N}{2}\Delta t = 0.4\times2$

$\therefore$ Impulse of normal force

$= N\Delta t = 1.6N - s$

26. (3)

Velocity of Q due to translation = v



Velocity of Q due to rotation

$= \omega r = \frac{v}{2}$

So, net velocity of Q = $\frac{3V}{2}$

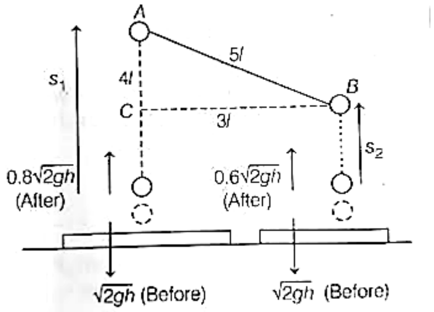
If velocity of ring is v', equating velocity components of P and Q along the rod, as it is rigid, we get

$v' \cos 60^0 = \frac{3v}{2} \cos 30^0$

$\Rightarrow v' = \frac{3\sqrt{3}}{2} = \frac{3\sqrt{3} \times \sqrt{3}}{2} = 4.5 m/s$

27. (10)

After releasing, the situation can be shown as



As velocity just after collision is e times the incident velocity, therefore velocities of balls after collision will be as shown, Let the string becomes taut at time t in position AB.

From ΔABC,

$AC = \sqrt{(5l)^2 - (3l)^2} = 4l$

$\Rightarrow$Displacement of ball-1 will be 4l greater than ball -2

$\Rightarrow s_1 = s_2 + 4l$

$\Rightarrow 0.8\sqrt{2gh}t - \frac{1}{2}gt^2 = 0.6\sqrt{2gh}t - \frac{1}{2}gt^2 + 4l$

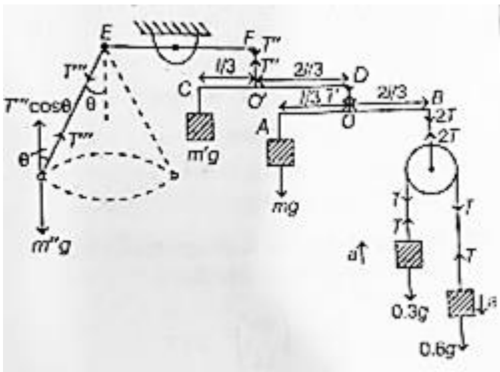
$\Rightarrow 0.2\sqrt{2ghl} = 4l$

$$\Rightarrow t = \frac{20l}{2gh}$$

$$\therefore x = 20, y = 2 \Rightarrow \frac{x}{y} = \frac{20}{2} = 10$$

28. (14)

The various forces acting in the arrangement are shown below



For 0.6 kg block, by F = ma, we get

$0.6g - T = 0.6a$ ……(i)

For 0.3 kg block, we get,

$T - 0.3g = 0.3a$ …….(ii)

Solving Eqs. (i) and (ii), we get

$T = 0.4g$

For rod AB to be equilibrium

τ of mg = τ of tension

$\Rightarrow mg\frac{1}{3} = 0.8g \times \frac{2l}{3}$ [using Eq. (iii)]

$\Rightarrow m = 1.6kg$ ….(iv)

Similarly, by considering equilibrium of CD, we get $m'g\frac{1}{3} = (mg + 2)\frac{2l}{3}$

$$\Rightarrow m'g\frac{1}{3} = (mg + 2T)\frac{2l}{3}$$

$\Rightarrow m'g\frac{1}{3} = (1.6g + 2 \times 0.4g)\frac{2l}{3}$ [using Eq. (iii) and (iv)]

$$\Rightarrow m' = 4.8kg$$

For equilibrium of EF, we get

$$T'''\cos\theta\frac{1}{3} = T'' \times \frac{2l}{3}$$

$$\Rightarrow m''g\frac{1}{3} = (m'g + T')\frac{2l}{3}$$

$$\Rightarrow m''g = 2(4.8 \times g + 2.4g)$$

$$\Rightarrow m'' = 14.4kg$$

29. (4)

For maxima at point P of screen in Arragement-1

$d\sin\theta = \pm n\lambda$

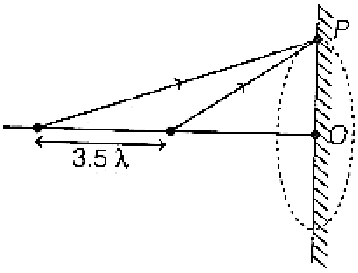
$\Rightarrow 6\lambda\sin\theta = \pm n\lambda$

$\Rightarrow n = \pm 6\sin\theta$

As maximum value of $\sin\theta$ is 1, son n can take values -6,-5,-4,-3,-2, -1,0 1,2,3,4,5,6,. But for n $n = \pm 6, \theta = 90^0$ which is not a point on screen, so, should be excluded. So total of 11 maximas will be obtained

$\therefore n_1 = 11$

In Arrangement -2 owing to symmetry about x-axis, circular fringes are obtained, as shown in figure. Path difference at O (centre of screen) is 3.5 λ. As we move from O to ∞ on screen, path differences of $3\lambda, 2\lambda$ and λ are obtained. Corresponding to theses, three bright circular fringes will be observed



$\Rightarrow n_2 = 3$

Therefore, $\frac{n_1}{n_2} = \frac{11}{3} = 3.67$

30. (1)

For equilibrium of wire AB shown in fig. 1

Surface tension force = weight

$\Rightarrow F = mg$

$\Rightarrow 2T.2l\tan\theta = mg$

$\Rightarrow \tan\theta = \frac{mg}{4Tl}$ ……(i)

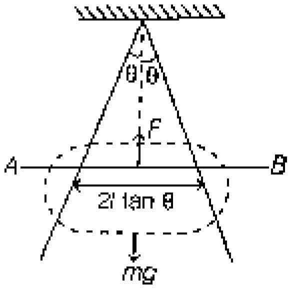


Fig. 1

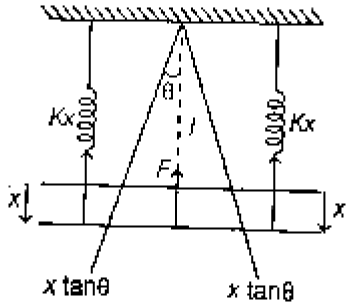


Fig. 2

Now, if the wire is displaced by amount x as shown in fig 2. Increase in length of wire in contact with soap film $= 2x\tan\theta$

Also, increase in length of spring = x

Therefore, restoring force is

$F' = 2T.2x\tan\theta + 2Kx$

$= (4T\tan\theta + 2K)x$ [Using Eq. (i)]

$= \dfrac{3mg}{l}x = Cx$

$\Rightarrow$ Force constant = $C = \dfrac{3mg}{l}$

As time period is given $2\pi\sqrt{\dfrac{m}{c}}$

Therefore, we get

$$T = 2\pi\sqrt{\dfrac{m}{\dfrac{3mg}{l}}} = \pi\sqrt{\dfrac{4l}{3g}}$$

$\therefore x = \dfrac{4}{3} = 1.33$

PART (B) : CHEMISTRY

ANSWER KEY

31.	(D)	32.	(D)	33.	(B)	34.	(A)	35.	(B)
36.	(D)	37.	(C)	38.	(B)	39.	(A)	40.	(B)
41	(B)	42.	(D)	43.	(C)	44.	(B)	45.	(B)
46.	(D)	47.	(B)	48.	(D)	49.	(B)	50.	(C)
51.	(21)	52.	(3)	53.	(4)	54.	(3)	55.	(2)
56.	(5)	57.	(6)	58.	(3)	59.	(4)	60.	(10)

SOLUTIONS

31. (D)

Rate (r) = $k[A]^n[B]^m \Rightarrow$ initial rate $r_1 = k[A]^n[B]^m$

Final rate, $r_2 = k[2A]^n\left[\frac{B}{2}\right]^m = 2^{n-m}k[A]^n[B]^m$

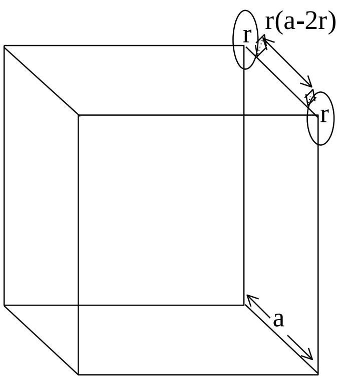
$$\Rightarrow \frac{r_2}{r_1} = \frac{2^{n-m}\times k[A]^n[B]^m}{k[A]^n[B]^m} = 2^{n-m}$$

32. (D)

Statement (d) is incorrect, whereas all other statements are correct.
The corrected form is the electric arc is struck between the electrodes of metals immersed in the dispersion medium. (concentrated alkali).

33. (B)

For bcc, $r = \frac{\sqrt{3}}{4}$ a where, r = radius of sphere (atom); a = edge length of unit cell



Edge length not covered by atom = a – 2r

$$a - 2\left(\frac{\sqrt{3}}{4}\right)a = a\left(\frac{2-\sqrt{3}}{2}\right)$$

$\Rightarrow$% of edge length not covered

$$= \frac{a\left(\frac{2-\sqrt{3}}{2}\right)}{a} \times 100 = 0.134 \times 100 = 13.4\%$$

34. (A)

Total strength of all H-bonds = 30.8 – 14.4 = 16.4 kJ mol^{-1}

There are six nearest neighbours, but each hydrogen bond involves 2 molecules

Hence, effective neighbours = 3

Hence, strength of H-bond = $\frac{16.4}{3} = 5.47 \text{kJmol}^{-1}$

35. (B)

Osmotic pressure $\pi = iCRT$

$$4.92 = \left\{\frac{x}{200} + 2 \times 0.05\right\} 0.0821 \times 300 \text{atm}$$

$[\because \text{for NaCl}, \alpha = 1 \Rightarrow i = 2]$

$$\Rightarrow \frac{x}{200} + 0.1 = \frac{4.92}{0.0821 \times 300}$$

$$\therefore \frac{x}{200} + 0.1 = 0.2$$

$$\Rightarrow x = 0.1 \times 200 = 20g$$

36. (D)

Maximum wavelength radiation means minimum energy radiation which is emitted when transition occurs from $4 \rightarrow 2$ (visible or Balmer region)

$$\Rightarrow \frac{1}{\lambda} = R\left(\frac{1}{4} - \frac{1}{16}\right) \Rightarrow \frac{1}{\lambda} = R \times \frac{12}{4 \times 16} = \frac{3R}{16} \text{ or } \lambda = \frac{16}{3R}$$

37. (C)

(i) $Cu^{2+} + e^- \rightarrow Cu^+;$

$\Delta G_1^0 = -1 \times F \times E_1^0 = -Fx_1$

(ii) $ln^{3+} + 2e^- \rightarrow ln^+; \Delta G_2^0 = -2 \times F \times E_2^0 = -2Fx_2$

(iii) $ln^{2+} + e^- \rightarrow ln^+; \Delta G_3^0 = -1 \times F \times E_3^0 = -Fx_3$

For the reaction

$$ln^{2+} + Cu^{2+} \rightarrow ln^{3+} + Cu^+ (n = 1)$$

$$\Delta G^0 = \Delta G_1^0 + \Delta G_3^0 - \Delta G_2^0$$

$$\Rightarrow -1 \times F \times E_3^0 = -Fx_1 - Fx_3 - (-2Fx_2)$$

$$\Rightarrow E_3^0 = (x_1 + x_3 - 2x_2)V$$

38. (B)

General formula for cyclic silicates is $(SiO_3)_n^{-2n}$

If $n = 6$, then $(SiO_3)_6^{-12} = Si_6O_{18}^{-12}$

39. (A)

Inorganic salt like aluminium bromide reacts with sodium hydroxide to give Al $(OH)_3$.

$$AlBr_3 + 3NaOH \rightarrow \underset{\text{(White gelatinous ppt)}}{Al(OH)_3} + 3NaBr \xrightarrow[\text{(excess)}]{NaOH} H_2O + \underset{\text{(Soluble)}}{NaAlO_2}$$

The white precipitate of $Al(OH)_3$ gets dissolved in excess of NaOH to give a water soluble salt, sodium aluminate, $NaAlO_2$. $3AgNO_3 + AlBr_3 \rightarrow \underset{\text{(Light yellow ppt)}}{3AgBr} + Al(NO_3)_3$

Thus, the probable salt is $AlBr_3$.

40. (B)

Correct order is present in only option (B).

(A) For group 13 elements, atomic size varies as B< Ga < Al < In < Tl.

(C) Water solubility of group -1 metal hydroxides follow the order
LiOH < NaOH <KOH <RbOH < CsOH

(D) Water solubility of group-2 metal sulphates decreases down the group
$BeSO_4 > MgSO_4 > CsSO_4 > SrSO_4 > BaSO_4$

41 (B)

The reaction involved are

$$\underset{\text{Sodium}}{2Na} + \underset{\text{(Air)excess}}{O_2} \rightarrow \underset{X}{Na_2O_2} \text{ (sodium peroxide)}$$

$$\underset{X}{2Na_2O_2} + 2CO_2 \rightarrow 2Na_2CO_3 + \underset{Y}{O_2}$$

42. (D)

The complete hydrolysis equation of XeF_4 is $6XeF_4 + 12H_2O \rightarrow 4Xe + 2XeO_3 + 24HF + 3O_2$

So, complete hydrolysis of XeF_4 does not produce XeO_2F_2.

43. (C)

H_2O is a weak field ligand, so cannot force pairing of electron (s)

$[Zn(H_2O)_6]^{2+} \Rightarrow (d^{10})t_{2g}^6e_g^4 \Rightarrow n = 0$

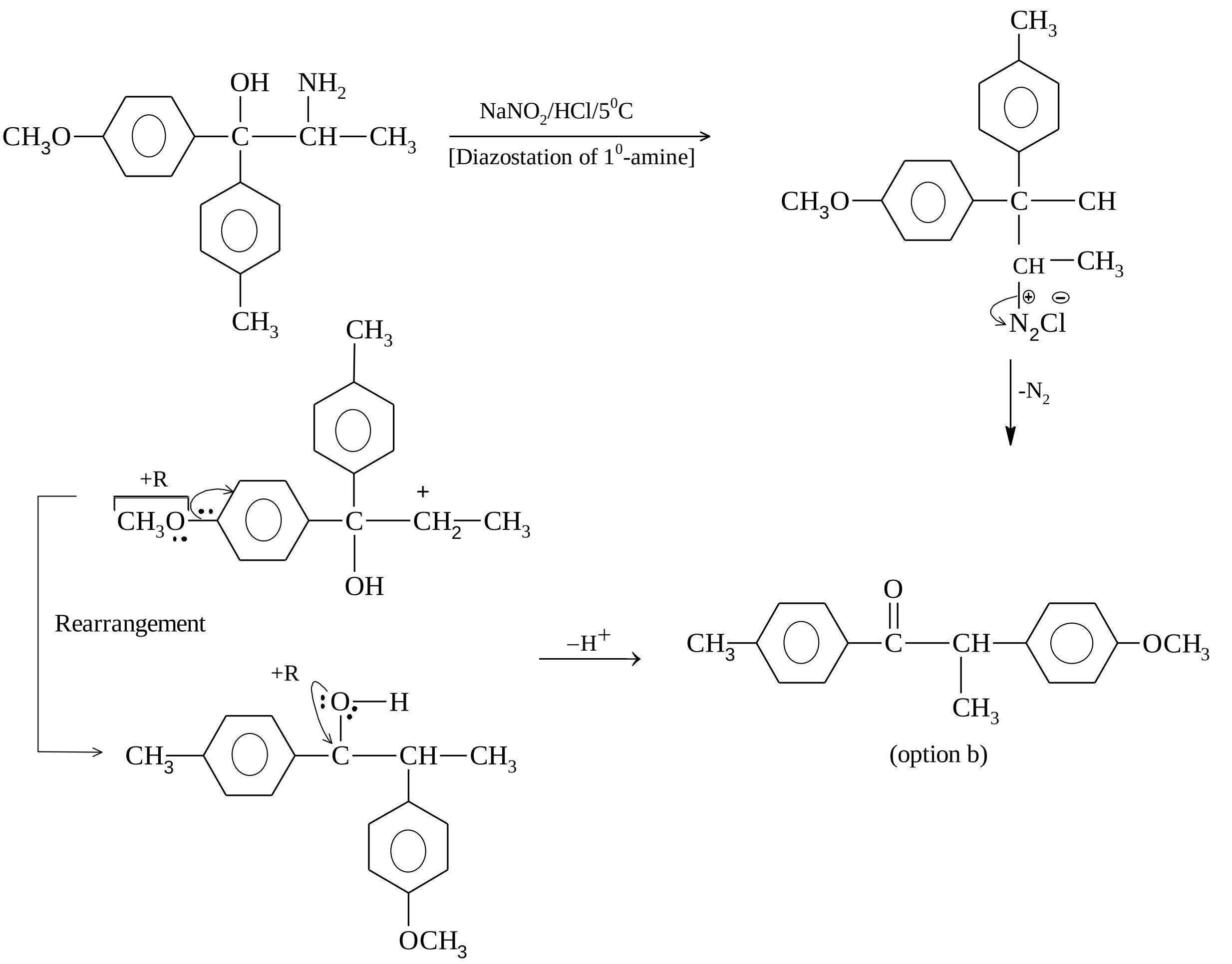
$[Cr(H_2O)_6]^{2+} \Rightarrow (d^4)t_{2g}^3e_g^1 \Rightarrow n = 4$

$\left[Mn(H_2O)_6\right]^{2+} \Rightarrow (d^5)t_{2g}^3e_g^2 \Rightarrow n = 5$

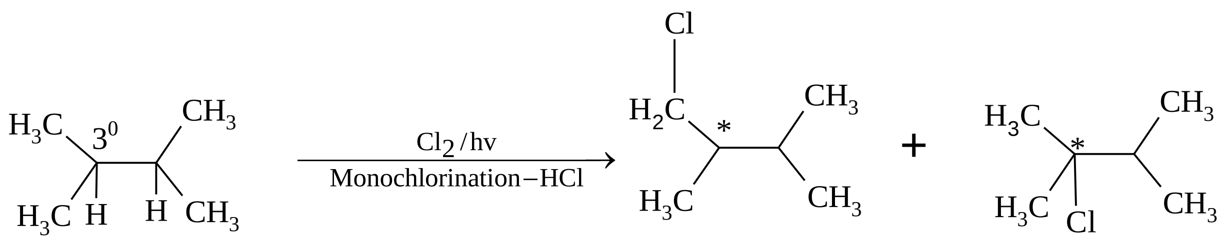
$\left[Cu(H_2O)_6\right]^{2+} \Rightarrow (d^9)t_{2g}^6e_g^3 \Rightarrow n = 1$

Spin only magnetic moment $\mu = \sqrt{n(n+2)}BM$

44. (B)

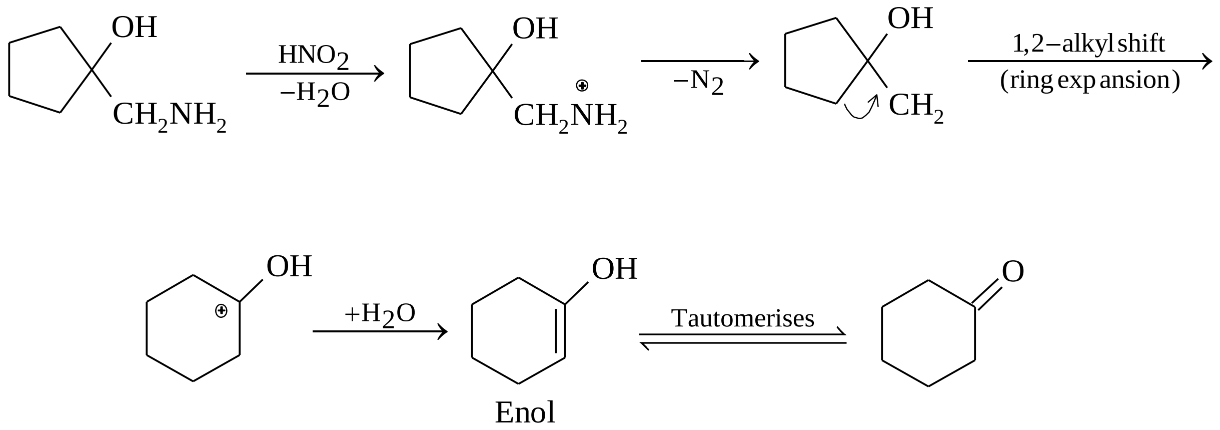


45. (B)

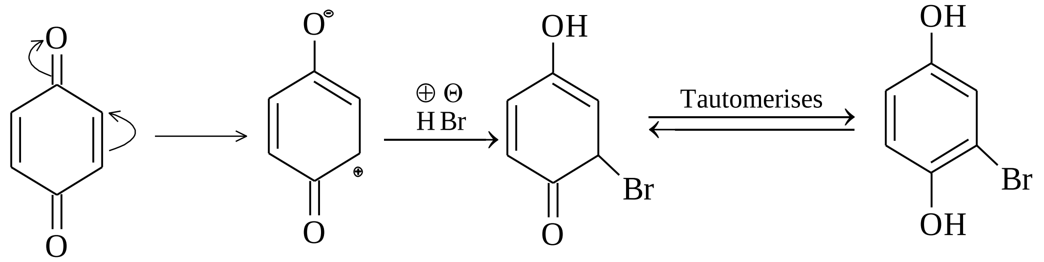


(2,3-dimethylbutane) 1-chloro-2,3-dimethyl butane

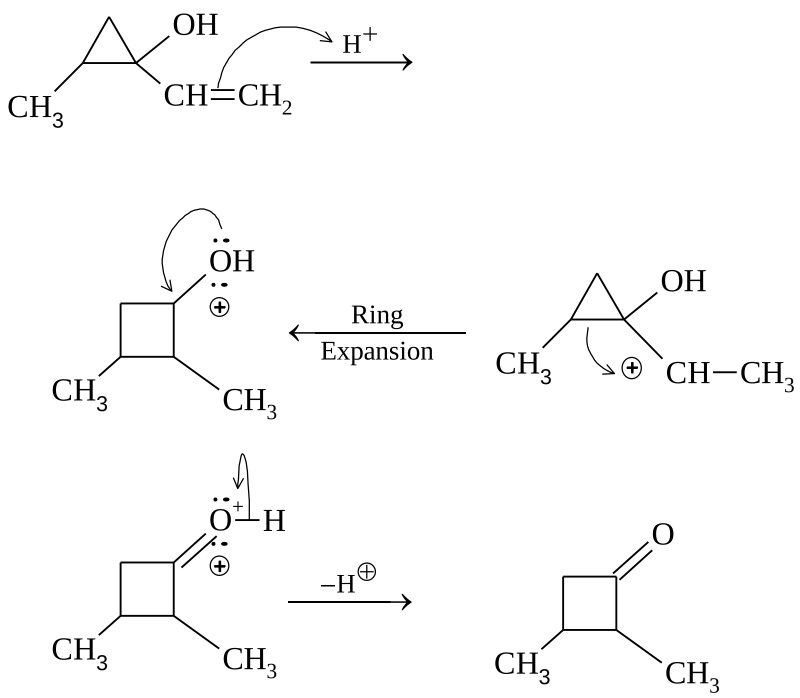
46. (D)


OH
CH_2NH_2
HNO_2
$-H_2O$
OH
$CH_2\overset{\oplus}{N}H_2$
$-N_2$
OH
CH_2
1,2–alkyl shift
(ring expansion)
OH
$+H_2O$
OH
Enol
Tautomerises
O

47. (B)

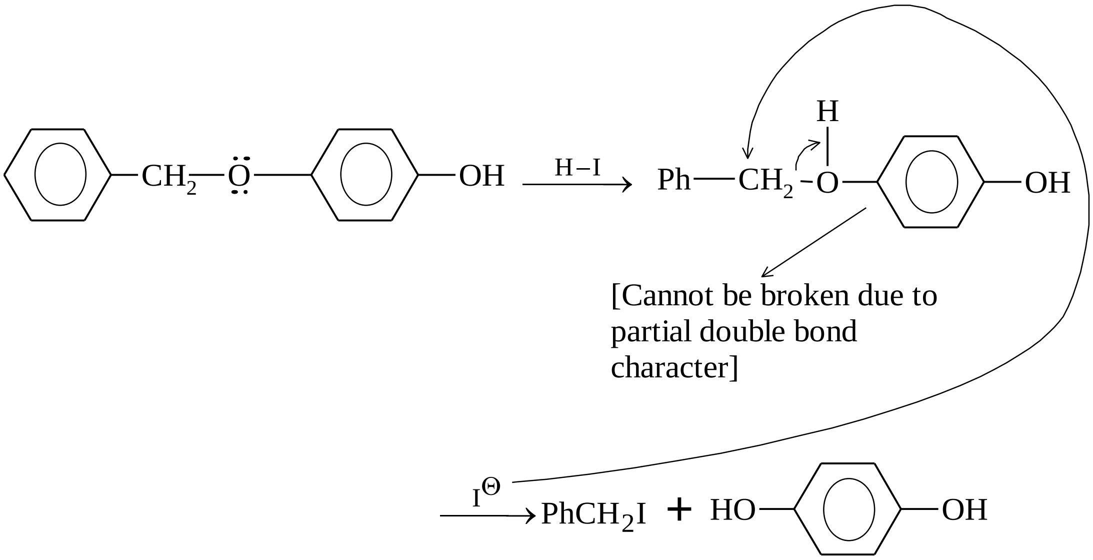

O
O
$O^{\ominus}$
O
H Br
OH
Br
O
Tautomerises
OH
Br
OH

48. (D)

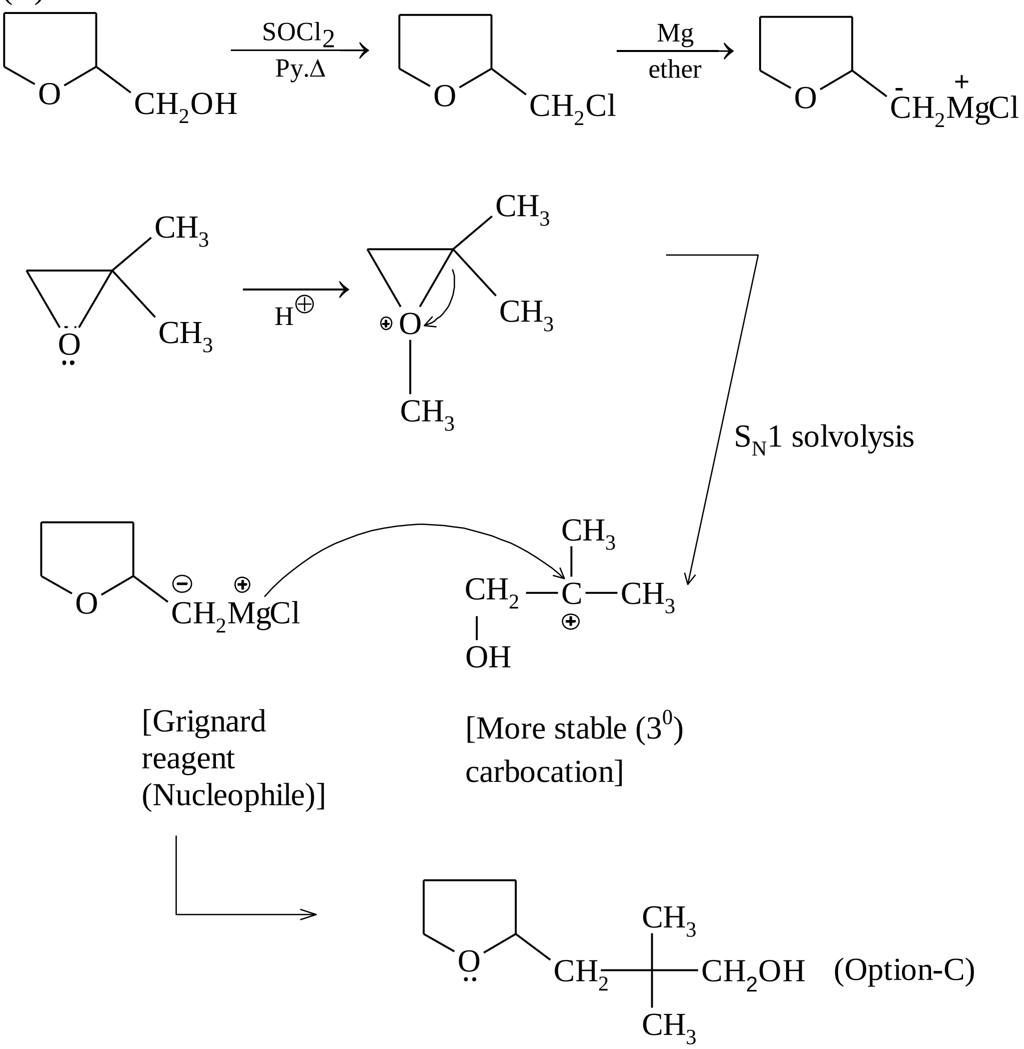

OH
CH_3
$CH=CH_2$
H^+
OH
CH_3
CH_3
Ring
Expansion
OH
CH_3
$CH-CH_3$
O
H
CH_3
CH_3
$-H^{\oplus}$
O
CH_3
CH_3

49. (B)

Complete reaction is as follows



50. (C)



51. (21)

Let us find the empirical/molecular formula of the crystal as

W	:	X	:	Y
(fcc)		(OV)		[TV]
[corner+face centre]		[bc+ec]		[alternate]

[After removal of all atoms

From two body diagonals] $\left[(8-4)\times\frac{1}{8}+(6-0)\times\frac{1}{2}\right]:\left[(1-1)+(12-0)\times\frac{1}{4}\right]:[4-2]$

$\Rightarrow\left(\frac{1}{2}+3\right):(3+0)\ :2$

$\Rightarrow\ 7:\ 6:4$

$\therefore$ The formula becomes, $W_7X_6Y_4\equiv W_aX_bY_c$

$\Rightarrow$ a= 7, b = 6, c = 4

$\Rightarrow\frac{2ab}{c}=\frac{84}{4}=21$

52. (3)

$$\underset{2\,\text{mmol}}{2KI}+\underset{1\text{mmol}}{HgI_2}\rightarrow K_2[HgI_4]\rightleftharpoons 2K^{\oplus}+[HgI_4]^{2-}$$ [No. of ions formed, n = 3]

$\Rightarrow$van't Hoff factor $i=1+\alpha(n-1)=1+1\times(3-1)=3$ [$\because$ for 100% dissociation $\alpha=1$]

53. (4)

The lightest known gas is H_2(M = 2g/mol)

Number of H-atoms $=4\times6\times10^{23}=24\times10^{23}$ atoms

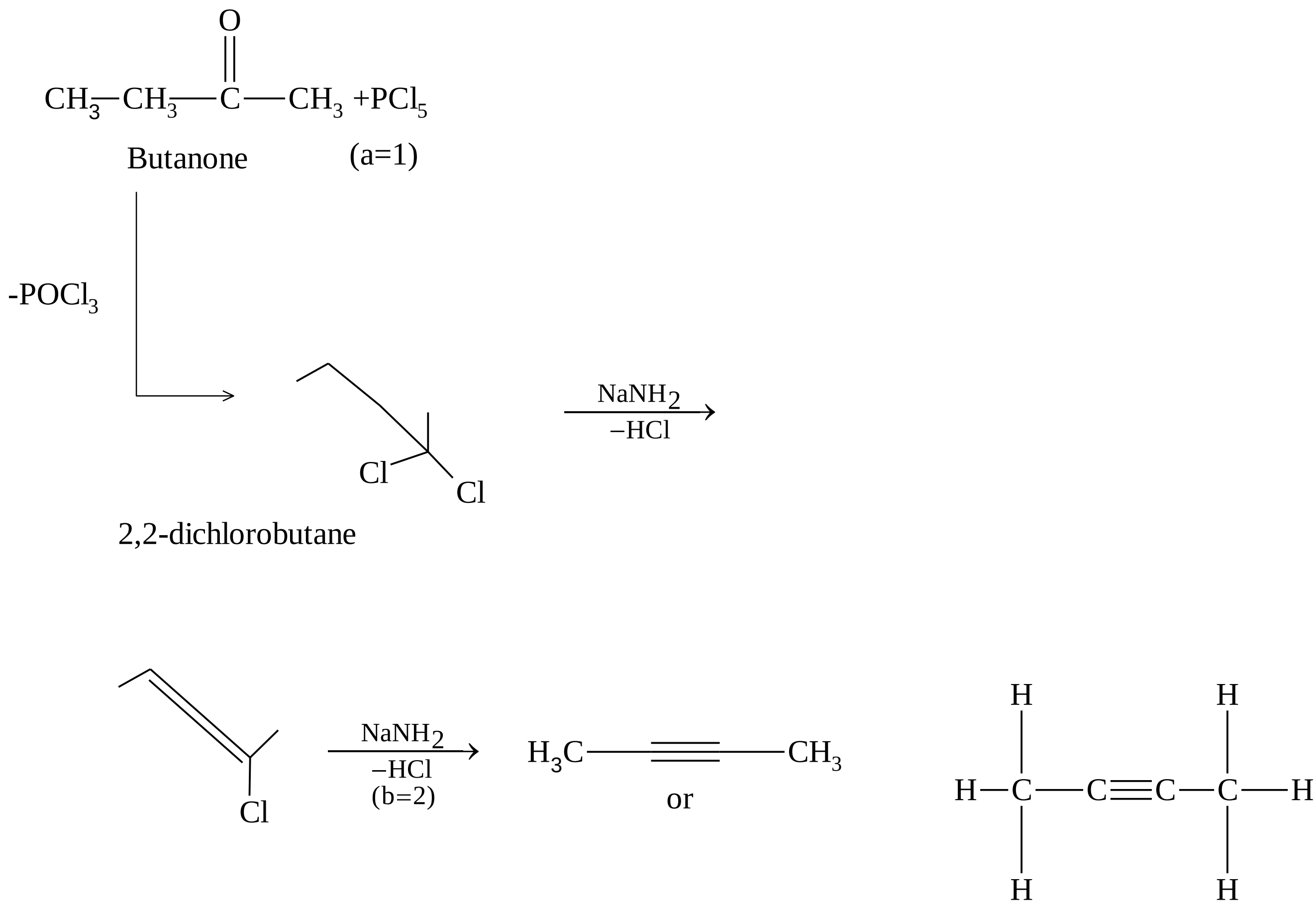
After removal of 24×10^{23} protons from 24×10^{23} atoms

$=24\times10^{23}$electrons are left

$\because$ Amount of charge left

$=\frac{24\times10^{23}}{6\times10^{23}}$=4 Faraday (X)

54. (3)



But-2-yne

Molar mass of but-2-yne (X)

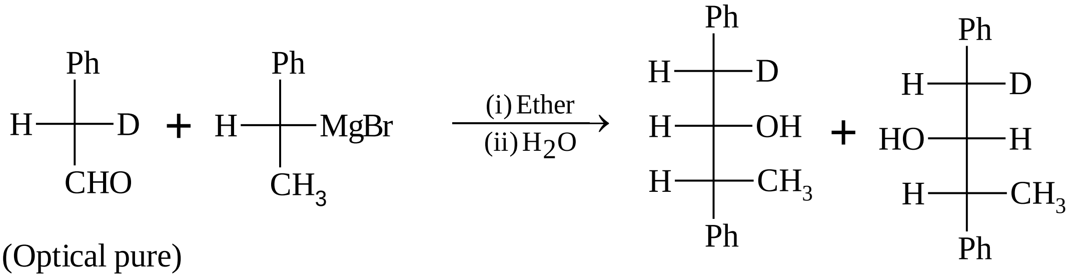
$= (12\times4)+(1\times6) = 54\ \text{amu} = x$

$$\Rightarrow \frac{X}{a+b} \div w_H = \left(\frac{54}{1+2}\right) \div 6 = \frac{18}{6} = 3.00$$

55. (2)

In the reaction, two stereoisomeric product will be formed. Total three chiral carbon atoms are present. The configuration at two chiral carbon atoms is same in the two isomers

However, the configuration at one chiral carbon atom is different in the two isomers.



(Optical pure)

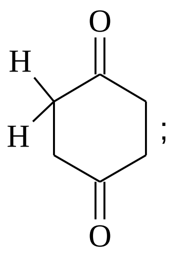
56. (5)

Tautomerisam will not be shown by those compounds which

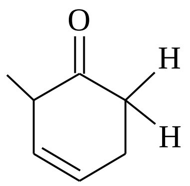
(i) do not have $\alpha-$hydrogen atom(s)

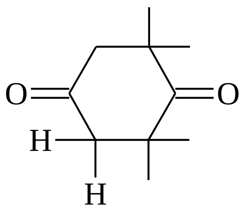
(ii) forms very unstable enol-structures

It will be shown by



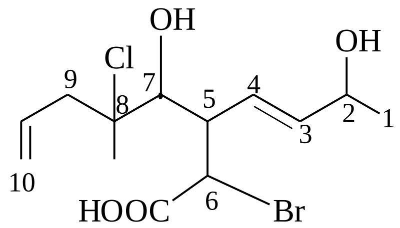
; $\overset{\alpha-H}{\downarrow}$ $C_6H_5CH_2CHO$





$\overset{\alpha-H}{\downarrow}$ $H_3CCOC(CH_3)_2COOEt$

57. (6)



Number of stereogenic centres

= Number of chiral carbon atom + Number of asymmetric – C = C –

=5(C_2,C_5,C_6,C_7 and C_8) + 1(C_3 – C_4 double bond)

=6

58. (3)

According to first order reaction, $-d[N_2O_5]/dt = k[N_2O_5]$. Then half-life period for first-order reaction is

$t_{1/2} = \frac{\ln 2}{k} = \frac{0.693}{k}$ So, $t_{1/2}$ is independent of initial concentration of the reactant.

With increase in temperature, k increases (according to Arrihenius equation), so $t_{1/2}$ decreases.

If the reaction completes to 99.61% completion, then

$$t = \left(\frac{2.303}{k}\right)\log\frac{a}{a-x}$$

Where a = 100, and (a – x) = (100 – 99.61) = 0.39

$$t_{99.61} = \left(\frac{2.303}{k}\right)\log\frac{100}{0.39}$$

$$= \left(\frac{2.303}{k}\right)2.41$$

$= 8 \times \frac{0.693}{k} = 8 \times t_{1/2}$

$\therefore$ Statements (a), (b) and (d) are correct

59. (4)

All are correct

60. (10)

BO of N_2^+ will be $\frac{10-5}{2} = 2.5 = X$

BO of O_2^- will be $\frac{10-7}{2} = 1.5 = Y$

BO of He_2^+ will be $\frac{4-3}{2} = 0.5 = Z$

BO of Be_2^+ will be $\frac{8-7}{2} = 0.5 = P$

$\therefore \left[\frac{X+Y+Z+P}{0.5}\right] = \frac{2.5+1.5+0.5+0.5}{0.5}$

$= \frac{5}{0.5} = 10$

PART (C) : MATHEMATICS

ANSWER KEY

61.	(C)	62.	(B)	63.	(A)	64.	(A)	65.	(A)
66.	(D)	67.	(D)	68.	(D)	69.	(B)	70.	(C)
71.	(C)	72.	(B)	73.	(A)	74.	(A)	75.	(A)
76.	(C)	77.	(B)	78.	(B)	79.	(C)	80.	(B)
81.	(1)	82.	(6)	83.	(18)	84.	(3)	85.	(11)
86.	(2)	87.	(1)	88.	(4)	89.	(5)	90.	(5)

SOLUTIONS

61. (C)

Since, the relation R is defined as $R=\{(x, y) \mid x, y$ are real numbers and $x = wy$ for some rational number $w\}$.

(A) **Reflexive** xRx as $x = 1x$

Here, $w = 1 \in$ Rational number

So, the relation R is reflexive.

(B) **Symmetric** $xRy \not\Rightarrow yRx$ as $0R1$ but $1 \not R 0$

So, the relation R is not symmetric.

Thus, R is not equivalence relation.

Now, for the relation S, defined as, $S = \left\{\left(\frac{m}{n}, \frac{p}{q}\right) \middle| m, n, p \text{ and } q \in \text{integers such that } n, q \neq 0 \text{ and } qm = pn\right\}$.

(A) **Reflexive** $\frac{m}{n} S \frac{m}{n} \Rightarrow mn = mn$ [true]

Hence, the relation S is reflexive.

(B) **Symmetric** $\frac{m}{n} S \frac{p}{q} \Rightarrow mq = np$

$\Rightarrow np = mq \Rightarrow \frac{p}{q} S \frac{m}{n}$

Hence, the relation S is symmetric.

(C) **Transitive** $\frac{m}{n} S \frac{p}{q}$ and $\frac{p}{q} S \frac{r}{s}$

$\Rightarrow mq = np$ and $ps = rq$

$\Rightarrow mq \cdot ps = np \cdot rq \Rightarrow ms = nr$

$\Rightarrow \frac{m}{n} = \frac{r}{s}$

$\Rightarrow \frac{m}{n} S \frac{r}{s}$

So, the relation S is transitive.

Hence, the relation S is equivalence relation.

62. (B)

$\because$ reflection of (5, 8) in BC will lie on circumcircle.

$\therefore$ (8, 5) will lie on circumcircle

$\therefore$ equation of circumcircle is

$$(x-2)^2+(y-3)^2=(8-2)^2+(3-5)^2$$

$$\Rightarrow \quad x^2+y^2-4x-6y-27=0$$

63. (A)

$$\sqrt{(x-0)^2+(y-1)^2}=\frac{|3x+4y+1|}{\sqrt{3^2+4^2}}$$

$e=1, S(0,1)$ & directrix $\equiv 3x+4y+1=0$

64. (A)

Equation of normal at $\left(ae, \frac{b^2}{a}\right)$ will be $\frac{a^2x}{ae}-\frac{b^2y}{\left(b^2/a\right)}=a^2-b^2$ it passes through (0, –b)

$$ab=a^2-b^2$$

$$\Rightarrow \left(\frac{b}{a}\right)^2+\left(\frac{b}{a}\right)-1=0$$

$$\frac{b}{a}=-\left(\frac{1\pm\sqrt{5}}{2}\right)$$

So, $\frac{b}{a}=\frac{\sqrt{5}-1}{2}$

$$e=\sqrt{1-\left(\frac{b}{a}\right)^2}=\sqrt{\frac{\sqrt{5}-1}{2}}$$

65. (A)

Homogenising the equation of hyperbola with the help of line

We have $\frac{x^2}{a^2}-\frac{y^2}{2a^2}=\left(\frac{x\cos\alpha+y\sin\alpha}{p}\right)^2$

Now this subtends an angle of 90^o at origin

So coefficient of x^2+ coefficient of $y^2=0$

i.e. $\frac{1}{a^2}-\frac{\cos^2\alpha}{p^2}-\frac{1}{2a^2}-\frac{\sin^2\alpha}{p^2}=0$

So, $\frac{1}{2a^2}=\frac{1}{p^2}$ $\qquad p=\sqrt{2}\,a$

66. (D)

$$\lim_{x\to 0}\frac{(1+x)^{1/x}-e+\frac{ex}{2}}{11x^2} \quad \left(\frac{0}{0}\text{form}\right)$$

By expansion

$$=\lim_{x\to 0}\frac{e\left(1-\frac{x}{2}+\lim_{x\to 0}\frac{11}{24}x^2+.....\right)-e+\frac{ex}{2}}{11x^2}=\frac{e}{24}$$

67. (D)

$$f'(x)=2x\left(a_1+2a_2x^2+......+na_n x^{2n-2}\right)$$

$$f'(x)=0 \Rightarrow x=0$$

$$f''(x)=2\left(a_1+6a_2 x^2+....+n(2n-1)a_n x^{2n-n}\right)$$

$$\left(f''(x)\right)_{x=0}=2a_1>0$$

$P(x)$ has only minimum at $x=0$

68. (D)

$$\therefore \quad I_n=\int \tan^n x\,dx=\frac{\tan^{n-1}x}{n-1}-I_{n-2}$$

$$I_7=\int \tan^7 x\,dx=\frac{\tan^6 x}{6}-\frac{\tan^4 x}{4}+\frac{\tan^2 x}{2}+\ln|\cos x|+c$$

69. (B)

$$\int\left(\frac{x^2+1}{(x+1)^2}\right)e^x\,dx$$

$$=\int\left(\frac{x^2-1+2}{(x+1)^2}\right)e^x\,dx$$

$$=\int\left(\frac{x-1}{x+1}+\frac{2}{(x+1)^2}\right)e^x\,dx$$

$$=\int\left(f(x)+f'(x)\right)e^x\,dx$$

$$=f(x)e^x+c$$

Where $f(x)=\dfrac{x-1}{x+1}$

$$f'(x)=\frac{2}{(x+1)^2}$$

$$f''(x)=\frac{-4}{(x+1)^3}$$

$$=\frac{12}{(x+1)^4}$$

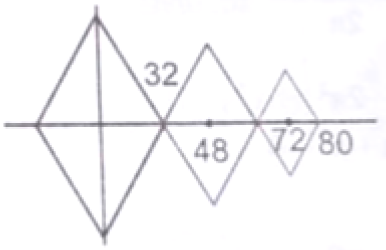
$$f''(1)=\frac{12}{16}=\frac{3}{4}$$

70. (C)

$$a_1=0,\ b_1=32,\ a_2=a_1+\frac{3}{2}b_1=48$$

$$b_2=\frac{b_1}{2}=16,\ a_3=48+\frac{3}{2}\cdot 16=72,\ b_3=8$$

So, the three loops from $i=1$ to $i=3$ are alike now area of i^{th} loop (square) $=\frac{1}{2}$(diagonal)^2



$$A_i=\frac{1}{2}(2b_i)^2=2(b_i)^2$$

$$\frac{A_{i+1}}{A_i}=\frac{2(b_{i+1})^2}{2(b_i)^2}=\frac{1}{4}$$

So the area from A GP series

So the sum of the GP upto infinite terms

$$\frac{A_1}{1-r}=2(32)^2\cdot\frac{1}{1-\frac{1}{4}}=\frac{8}{3}(32)^2 \text{ sq. units}$$

71. (C)

$$(1+y^2)+\left(x-e^{\tan^{-1}}\right)\frac{dy}{dx}=0$$

$$\Rightarrow\ \frac{dx}{dy}+\frac{x}{1+y^2}=\frac{e^{\tan^{-1}y}}{1+y^2}$$

$$\text{I.F.}=e^{\int\frac{1}{1+y^2}dy}=e^{\tan^{-1}y}$$

$$\Rightarrow\ x.e^{\tan^{-1}y}=\int\frac{e^{2\tan^{-1}y}}{1+y^2}dy$$

$$=\frac{1}{2}e^{2\tan^{-1}y}+c$$

$$\Rightarrow\ 2xe^{\tan^{-1}y}=e^{2\tan^{-1}y}+c'$$

72. (B)

$$x^2-2x=t\ge -1$$

$$t^2-3t+(k+2)=0$$

For four real solution t must lies in $(-1,\infty)$

$D = 9 - 4(k+2) > 0$

$9 - 4k - 8 > 0$

$4k < 1 \Rightarrow k < \frac{1}{4}$

$f(-1) > 0$

$1 + 3 + k + 2 > 0$

$k > -6$

So, $k \in \left(-6, \frac{1}{4}\right)$

73. (A)

Given, $S_n = \sum_{r=1}^{n} I(r) = n(2n^2 + 9n + 13)$

$S_r = r(2r^2 + 9r + 13)$

$S_{r-1} = (r-1)\left[2(r-1)^2 + 9(r-1) + 13\right]$

$I(r) = S_r - S_{r-1}$

$I(r) = 6r^2 + 12r + 6 = 6(r+1)^2$

$= \sqrt{6}\sum_{r=1}^{n} r + \sum_{r=1}^{n} \cdot 1 \Rightarrow \sqrt{6}\left[\frac{n(n+1)}{2} + n\right] \Rightarrow \frac{\sqrt{6}}{2}\left[n^2 + 3n\right] \Rightarrow \sqrt{\frac{3}{2}}\left(n^2 + 3n\right)$

74. (A)

Total no. of ways in which 6 persons can have their birth days $= 12 \times 12 \times 12 \times 12 \times 12 \times 12 = 12^6$

Out of 12 months, 2 month can be chosen in ${}^{12}C_2$ ways. Now birth days of six persons can fall in these two months in 2^6 ways. Out of these 2^6 ways, there are two ways when all six birth days fall in one month. So there are $(2^6 - 2)$ ways in which six birth days fall in chosen 2 months.

$\therefore$ required probability $= \frac{{}^{12}C_2\left(2^6 - 2\right)}{12^6} = \frac{341}{12^5}$

75. (A)

$$\Delta = \begin{vmatrix} \lambda & 1 & 1 \\ 1 & \lambda & 1 \\ 1 & 1 & \lambda \end{vmatrix} = 0$$

$C_1 \to C_1 + C_2 + C_3$

$$= (\lambda + 2)\begin{vmatrix} 1 & 1 & 1 \\ 1 & \lambda & 1 \\ 1 & 1 & \lambda \end{vmatrix} = 0$$

$C_2 \to C_2 - C_1, C_3 \to C_3 - C_1$

$= (\lambda+2)(\lambda-1)^2 = 0$

So, $\lambda = -2$ or 1

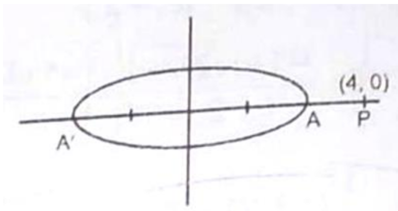
So, quadratic equation is $x^2-(-2+1)x+(-2\times1)=0$

$x^2+x-2=0$

76. (C)

$|z-1|+|z+3|\le 8$

$A'(-5,0)\ A(3,0)$



So, minimum $|z-4| = AP = 1$

Maximum $|z-4| = A'P = 9$

So, $|z-4|\in[1,9]$

77. (B)

$(\vec{r}-\vec{q})\times\vec{p}=0$

$\Rightarrow \vec{r}-\vec{q}=\lambda\vec{p},\ \vec{r}=\lambda\vec{p}+\vec{q}$ ..... (1) Now $\vec{r}\cdot\vec{s}=0$

$\Rightarrow (\lambda\vec{p}+\vec{q})\cdot\vec{s}=0 \qquad \Rightarrow \lambda \equiv \dfrac{\vec{q}\cdot\vec{s}}{\vec{p}\cdot\vec{s}}$ ..... (2) Put λ from (2) in (1) to get the results

78. (B)

Point of intersection of the line and the plane is (–2, –2, –5) ..... (i)

Image of (1, 2, –2) in the plane $2x+5y-4z-6=0$ is

$\left(-\dfrac{11}{45},-\dfrac{10}{9},\dfrac{22}{45}\right)$

$\therefore$ equation of the image line is $\dfrac{x+2}{79}=\dfrac{y+2}{40}=\dfrac{z+5}{247}$

79. (C)

Using $\tan\theta=\cot\theta-2\cot2\theta$

We get

$$E=(\cot\theta-2\cot2\theta)+2(\cot2\theta-2\cot4\theta)+4(\cot4\theta-2\cot8\theta)+....+2^{14}\left(\cot2^{14}\theta-2\cot2^{15}\theta\right)+2^{15}\cot2^{15}\theta$$

$=\cot\theta$

80. (B)

$p\Rightarrow q$ is logically equivalent to $\sim q\Rightarrow\sim p$

$\therefore (p\Rightarrow q)\Leftrightarrow(\sim q\Rightarrow\sim p)$ is a tatutology but not a contradiction

$\therefore (p\Rightarrow q)\Leftrightarrow(\sim q\Rightarrow\sim p)$ is a tatutology

81. (1)

The point B is (2, 1) (B is image of A about line $y = x$)

Image of A (1, 2) in the line $x-2y+1=0$ is given by $\frac{x-1}{1}=\frac{y-2}{-2}=\frac{4}{5}$

$\therefore$ coordinates of the point are $\left(\frac{9}{5},\frac{2}{5}\right)$.

Since this point lies on BC.

$\therefore$ equation of BC is $3x-y-5=0$

$\therefore$ $a+b=2$

82. (6)

$$\sum_{m=0}^{n} {}^{2n}C_{2m}\cdot\sum_{p=0}^{m} {}^{2m}C_{2p}$$

$$\sum_{m=0}^{n} {}^{2n}C_{2m}\cdot 2^{2m-1}=\frac{1}{2}\sum_{m=0}^{n} {}^{2n}C_{2m}\cdot 2^{2m}=\frac{1}{2}\cdot\left[\frac{1}{2}\left\{(1+2)^{2n}+(1-2)^{2n}\right\}\right]=\frac{1}{4}\cdot\left(3^{2n}+1\right)$$

83. (18)

Case – I If B is right on A

Subcase – I C is right on B then no. of ways = (4 – 1)! = 6

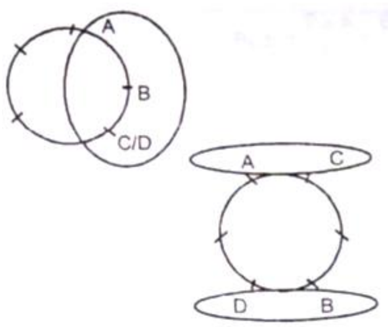
Subcase – II If D is right on B then no. of ways = (4 – 1)! = 6

Case – II If C is right on A

$\Rightarrow$ D must be right on B

$=(4-1)!=3!=6$

Hence total no. of ways is 6 + 6 + 6 = 18



84. (3)

$$\sin^{-1}\left(\frac{3\sin 2\theta}{5+4\cos 2\theta}\right)=\frac{\pi}{2}$$

Taking sin on both side

$$\frac{3\sin 2\theta}{5+4\cos 2\theta}=1$$

$$3\sin 2\theta=5+4\cos 2\theta$$

$$\frac{6\tan\theta}{1+\tan^2\theta}=5+4\left(\frac{1-\tan^2\theta}{1+\tan^2\theta}\right)$$

$$\tan^2\theta-6\tan\theta+9=0$$

$$\tan\theta=3$$

85. (11)

As given $\bar{x}=4, n=5$ and $\sigma^2=5.2$

If the remaining observations are x_1, x_2 then

$$\sigma^2 = \frac{\Sigma\left(x_i - \bar{x}\right)^2}{n} = 5.2$$

$$\Rightarrow \quad \frac{\left(x_1-4\right)^2+\left(x_2-4\right)^2+\left(1-4\right)^2+\left(2-4\right)^2+\left(6-4\right)^2}{5} = 5.2$$

$$\Rightarrow \quad \left(x_1-4\right)^2+\left(x_2-4\right)^2 = 9 \qquad \ldots\ldots \text{(i)}$$

Also $\bar{x} = 4 \Rightarrow \dfrac{x_1+x_2+1+2+6}{5} = 4$

$$\Rightarrow \quad x_1+x_2 = 11 \qquad \ldots.. \text{(ii)}$$

(i), (ii) $\Rightarrow x_1, x_2 = 4, 7$

86. (2)

$\because \quad \theta_1 - \theta = \theta - \theta_2$

$\Rightarrow \quad 2\theta = \theta_1 + \theta_2$

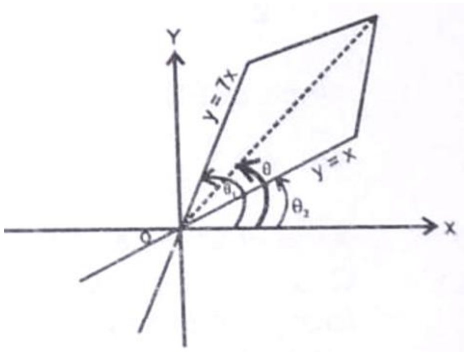
$\therefore \quad \tan 2\theta = \dfrac{\tan\theta_1 + \tan\theta_2}{1-\tan\theta_1 \tan\theta_2}$

$\Rightarrow \quad \tan 2\theta = \dfrac{7+1}{1-7} \quad \Rightarrow \quad \tan 2\theta = \dfrac{-4}{3}$

$\Rightarrow \quad \dfrac{2\tan\theta}{1-\tan^2\theta} = -\dfrac{4}{3}$

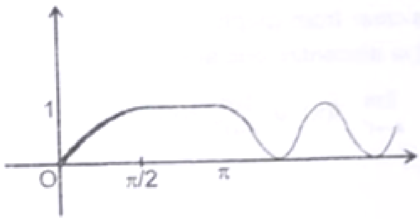
$\Rightarrow \quad \tan\theta = 2 \text{ or } -\dfrac{1}{2}$

$\therefore$ Slope of longer diagonal is = 2



87. (1)

$$g(x) = \begin{cases} f(x) & ; \quad 0 \le x \le \dfrac{\pi}{2} \\ 1 & ; \quad \dfrac{\pi}{2} < x \le \pi \\ \sin^2\dfrac{x}{2} & ; \quad x > \pi \end{cases}$$



88. (4)

Since $g(x)$ is inverse of $f(x)$

$g(f(x)) = x$

Differentiating w.r.t. x

$g'(f(x)) \cdot f'(x) = 1$

$x = 0,\ f(0) = 1$

$\therefore \quad g'(f(0)) \cdot f'(0) = 1$

$$g'(1)=\frac{1}{f'(0)}$$

$$\therefore \quad f'(0)=3$$

$$\therefore \quad g'(1)=\frac{1}{3}$$

89. (5)

For injective function differentiate

$$f'(x)=x^2+x+9$$

$$a>\frac{1}{4}$$

90. (5)

$$\arg\left(\frac{z-(10+6i)}{z-(4+6i)}\right)=\frac{\pi}{4}$$

$$\Rightarrow \tan^{-1}\frac{y-6}{x-10}-\tan^{-1}\frac{y-6}{x-4}=\frac{\pi}{4} \quad [\text{take } z=x+iy]$$

$$\Rightarrow \frac{\frac{y-6}{x-10}-\frac{y-6}{x-4}}{1+\frac{(y-6)(y-6)}{(x-10)(x-4)}}=1$$

$$\Rightarrow x^2+y^2-14x-18y+112=0$$

$$\Rightarrow (x-7)^2+(y-9)^2=18=\left(3\sqrt{2}\right)^2$$

$$\Rightarrow |z-(7+9i)|=3\sqrt{2}$$