PART (A) : PHYSICS

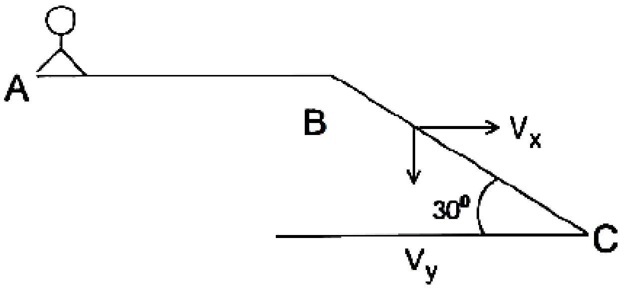
ANSWER KEY

1.	(B)	2.	(A)	3.	(A)	4.	(B)	5.	(B)
6.	(A)	7.	(A)	8.	(A)	9.	(B)	10.	(C)
11.	(A)	12.	(C)	13.	(A)	14.	(B)	15.	(D)
16.	(A)	17.	(C)	18.	(A)	19.	(B)	20.	(A)
21.	(5)	22.	(25)	23.	(24)	24.	(512)	25.	(100)
26.	(0)	27.	(5)	28.	(7)	29.	(7)	30.	(4)

SOLUTIONS

1. (B)

$\overline{V_R} = V_x\hat{i} + V_y\hat{j}$



For path AB $V_x = 5\,ms^{-1}$

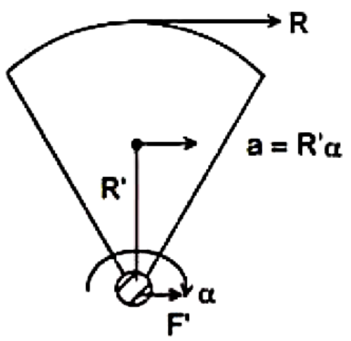
For path BC velocity of rain $\perp$ r to BC should be zero hence

$V_x \sin 30^\circ = V_y \cos 30^\circ\ V_y = \frac{5}{\sqrt{3}} ms^{-1}$

$V_R = \sqrt{V_x^2 + V_y^2} = \frac{10}{\sqrt{3}} ms^{-1}$

2. (A)

$\tau = I\alpha \Rightarrow \alpha = \frac{FR}{mR^2}.2 = \frac{2F}{mR}$

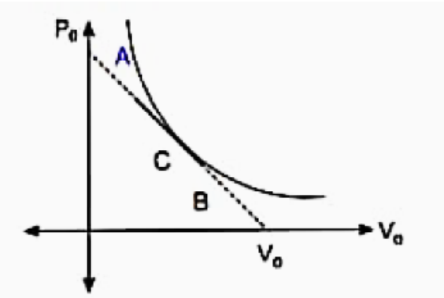


Position of C.O.M. $= \frac{2R}{3} \cdot \frac{\sin\left(\frac{\pi}{6}\right)}{\left(\frac{\pi}{6}\right)} = \frac{2R}{\pi} = R'$

Acceleration $F + F' = \frac{4R}{m\pi}.m$

$F' = \left(\frac{4-\pi}{\pi}\right)F$

3. (A)



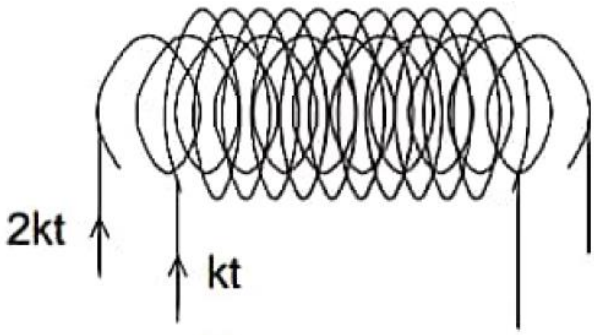
$-\frac{P_0}{V_0} = -\gamma \frac{P}{V}$

$-m = -\frac{\gamma(-mv + P_0)}{V}$ $\quad m = +\frac{P_0}{V_0}$

$mv(1+\gamma) = \gamma P_0$ $\quad V = \frac{\gamma P_0}{m(1+\gamma)} = \frac{5}{8}V_0$

4. (B)

$\phi = \mu_0 nk\pi r^2 + \mu_0 n2kt.\pi R^2$



$E.2\pi r = \frac{d\phi}{dt} = \mu_0 nk\pi\left(2R^2 + r^2\right)$

5. (B)

The point $x = a$ and b stable equilibrium point.

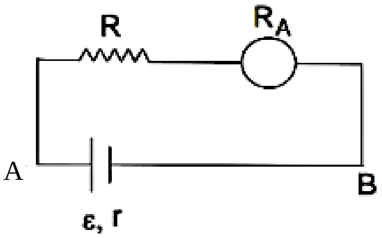
6. (A)

Since mass and specific heat of two different bodies are same

$$h_f = \frac{h_1 + h_2}{2} = h_1 = h_2$$

7. (A)

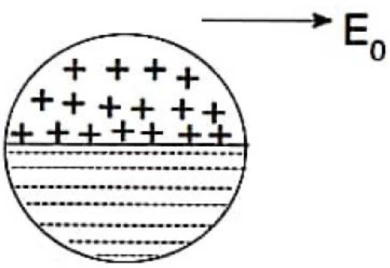
$$I = \frac{\varepsilon}{R + R_A + r}$$



$$V = \varepsilon - \frac{\varepsilon_r}{(R + R_A + r)} = \varepsilon - Ir = \frac{\varepsilon(R + R_A)}{(R + R_A + V)}$$

8. (A)

Dipole moment $\left(\frac{\sigma\pi R^2}{2}\right)\left(2.\frac{4R}{3\pi}\right)$



$$\frac{4R^3\sigma}{3}$$

$$f_r = Ma$$

$$\frac{4R^3\sigma}{3}E - f_r R = \frac{MR^2}{2}\alpha$$

$$\alpha = \frac{a}{R}$$

For pure rolling

$$a = \frac{8}{9}\frac{\sigma ER^2}{M}$$

9. (B)

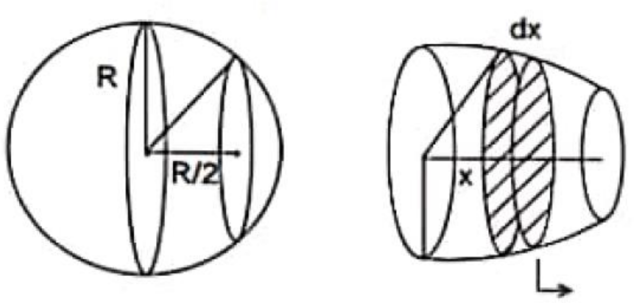
Current across inductor and capacitor are out of phase and same magnitude hence only resistance will be operated.

$2.5\sqrt{2}A$

10. (C)

$$\int \frac{1}{K}\cdot\frac{dx}{\pi\left(R^2-x^2\right)}$$

$$R_T=\frac{1}{K\pi}\int\frac{dx}{\left(R^2-x^2\right)}=\frac{1}{2K\pi R}\ln(3)$$



$$dR=\frac{1}{K}\frac{dx}{\pi\left(R^2-x^2\right)}$$

11. (A)

$$\overline{M}=10\left\{100\times10^{-4}\hat{i}+50\times10^{-4}\hat{j}+50\times10^{-4}\hat{k}\right\}$$

$$=\left(10\hat{i}+5\hat{j}+5\hat{k}\right)\times10^{-3}\,Am^2$$

$$\overline{B}=2\hat{i}-3\hat{j}+\hat{k}$$

$$\bar{\tau}=\overline{M}\times\overline{B}=10\times\sqrt{20}\times10^{-2}\,Nm=\sqrt{20}\times10^{-1}\,Nm$$

12. (C)

Distance between central bright and first minimum,

$$y=\frac{D\lambda}{d}$$

$$\frac{y_1}{y_2}=\frac{\lambda_1}{\lambda_2}$$

13. (A)

14. (B)

$$I_2=I_0\left(1-e^{-\mu x}\right)I=\frac{I_0}{2},x=1.5\,cm^{-1}$$

$$\mu=0.462\,cm^2/g$$

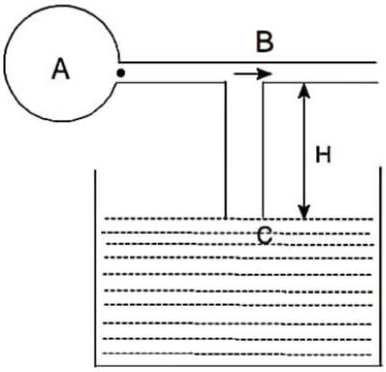
15. (D)

$$R=\sigma T^4\Rightarrow T^4=\frac{R}{\sigma}$$

$$\lambda T=b$$

16. (A)

$$P_B + \frac{1}{2}\sigma V^2 = P_A + P$$



$$P_B = \frac{P}{A} + P - \frac{1}{2}\sigma V^2$$

$$= \frac{P}{A} - \rho g H$$

$$V = \sqrt{\frac{2(P+\rho g H)}{\sigma}}$$

17. (C)

$$T^2 \propto (\text{semi} - \text{major axis})^3$$

18. (A)

$$V = V_1 + V_2 \text{ and } V_\gamma = V_1\gamma_1 + V_2\gamma_2$$

19. (B)

$$\vec{V} = \text{constant} \therefore \vec{a} = 0$$

$$\therefore \sum \vec{F} = 10 - f = 0 \Rightarrow f = 10N$$

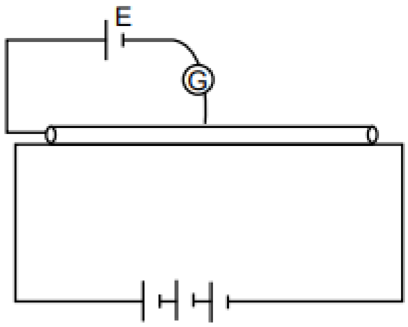
20. (A)

$$\tau = 0 \Rightarrow \alpha = 0 \Rightarrow \vec{\omega} = \text{constant } \& \ \vec{L} = I\vec{\omega} = \text{constant.}$$

21. (5)

Since $\overline{V_{AB}}$ and $\overline{R_{BA}}$ are antiparallel to each other initial separation is minimum at initial state.

22. (25)



$E_1 = K \times 75$

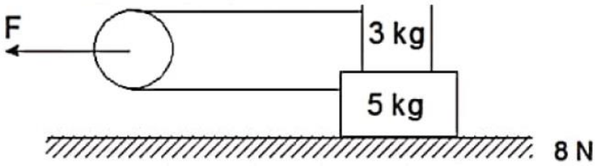
$E_1 = K \times x$

$\frac{E_1}{E_2} = \frac{3}{2} = \frac{75}{x} \Rightarrow x = 50\,cm$

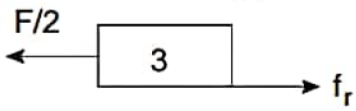
Difference = 75 – 50 cm

23. (24)

To start motion $F > 8M$



Acceleration $= \frac{F-8}{8}$

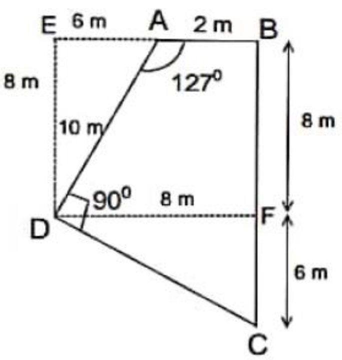


$\frac{F}{2} - f_r = 3\left(\frac{F-8}{8}\right) \Rightarrow \frac{F}{8} + 3 = f_r$

$\frac{F_{max}}{8} + 3 = f_{Lm} = 6N \Rightarrow F_{max} = 24\,Nx$

24. (512)

Portion DEA and DFC are identical hence moment of inertia of ABCD is same as ABFDEA.

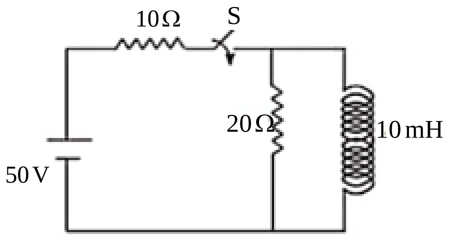


$$l = 2.\frac{M\times(BE)^2}{3}$$

$$= 512\,kgm^2$$

25. (100)

At steady state 5A current through indicator hence P.D. across indicator at $t = 0$ $20\times 5 = 100V$



26. (0)

No current will flow through the capacitor, so charge will be zero.

27. (5)

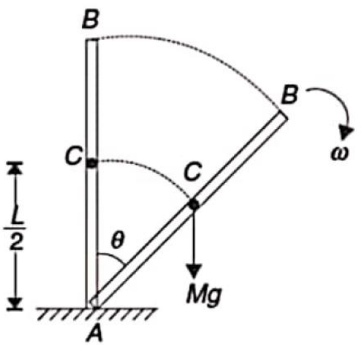
Elongation $\propto \frac{l}{Y}$. So, elongation in steel $= \frac{1}{4}$mm

Total elongation $= \frac{5}{4}$mm

28. (7)

$$\frac{1}{0.5} - \frac{3/2}{u} = \frac{1-\frac{3}{2}}{1,5}$$

29. (7)



Mechanical energy conservation

$$\frac{1}{2}I_A\omega^2 = Mg\frac{L}{2}(1-\cos\theta)$$

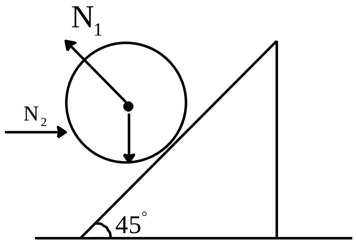
$$\frac{1}{2}\frac{ML^2}{3}.\omega^2 = \frac{MgL}{2}\left(1-\frac{4}{5}\right) \qquad \ldots\ldots(1)$$

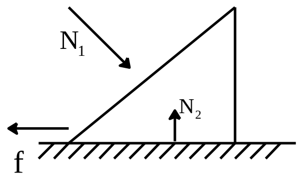
$$\omega = \sqrt{\frac{3}{5}\frac{g}{L}} = \sqrt{\frac{3}{5} \times \frac{10}{1.5}} = 2\,\text{rad/s}$$

$$v_{cm} = \frac{L}{2}.\omega = 1.5\,\text{m/s}$$

$\therefore$ momentum $= Mv_{cm} = 7.5\,\text{kg m/s}$

30. (4)





For Ball $\frac{N_1}{\sqrt{2}} = N_2$ and $\frac{N_1}{\sqrt{2}} = Mg \Rightarrow N_1 = \sqrt{2}Mg$

For wedge $N_3 = \frac{N_1}{\sqrt{2}} = Mg$

$\therefore$ Wedge will not move if

$$f_{max} \geq \frac{N_1}{\sqrt{2}}$$

$$\mu Mg \geq \frac{\sqrt{2}Mg}{\sqrt{2}} \Rightarrow \mu \geq 1$$

PART (B) : CHEMISTRY

ANSWER KEY

31.	(C)	32.	(C)	33.	(A)	34.	(C)	35.	(C)
36.	(C)	37.	(B)	38.	(B)	39.	(D)	40.	(A)
41.	(A)	42.	(A)	43.	(A)	44.	(A)	45.	(D)
46.	(B)	47.	(D)	48.	(C)	49.	(B)	50.	(D)
51.	(148)	52.	(43)	53.	(24)	54.	(3)	55.	(0)
56.	(4)	57.	(100)	58.	(1280)	59.	(0)	60.	(5)

SOLUTIONS

31. (C)

$$r = \frac{n^2h^2}{4\pi^2 kmZe^2}$$

$$\therefore \frac{r_2}{r_3} = \frac{2^2}{3^2}$$

$$\therefore r_3 = \frac{9}{4}r_2$$

So, (C) is the correct answer.

32. (C)

$$N_2(g) \quad + \quad 3H_2(g) \quad \rightleftharpoons \quad 2NH_3(g),$$

Partial pressures at equilibrium

0.8 0.4 [2.4 – (0.8 + 0.4) = 1.2]

Applying law of mass action,

$$K_P = \frac{[P_{NH_3}]^2}{[P_{N_2}][P_{H_2}]^3} = \frac{1.2\times1.2}{0.8\times0.4\times0.4\times0.4} \Rightarrow K_P = 28.125\ \text{atm}^{-2}$$

33. (A)

Each oxygen atom is tetrahedrally surrounded by four other oxygen atoms.

34. (C)

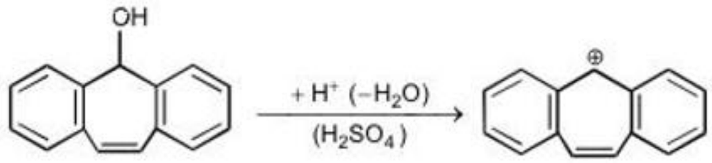
S_1 : Due to intermolecular H-bonding in HF it boils at higher temperature than HCl

S_2 : Mol. wt. of HBr < Mol. wt. of HI

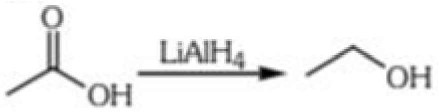
S_3 : Bond order of N_2 is more than N_2^+

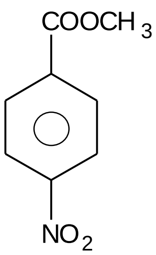
35. (C)

36. (C)
In this case negatively charged carbon is present between two electron attracting groups. As such it is a stable carbanion.

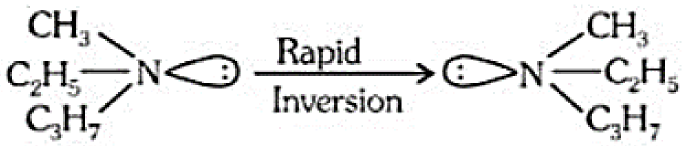
37. (B)

Stable carbocation (The central ring has also become aromatic and an aromatic tropylium cation is formed.) (Due to formation of more stable carbocation)

38. (B)
Ease of substitution of various types of H atoms is $3^o > 2^o > 1^o$.

39. (D)


40. (A)

There is more electron deficiency on carbonyl carbon.

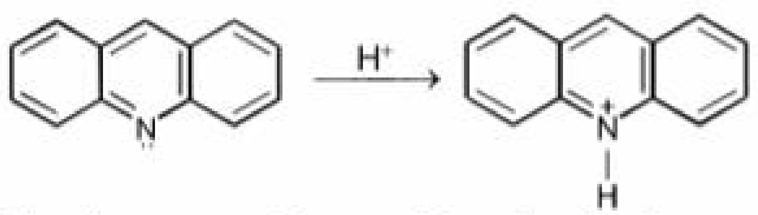
41. (A)
Since Y gives coupling reaction after diazotization, it suggest that Y can be aniline or benzene ring substituted aniline. Since Y has been obtained from Hofmann bromamide it means it has $-CONH_2$ group with benzene ring. Hence, it is $C_6H_5CONH_2$. Hence (A) is the correct answer.

42. (A)
The interconversion of d and ℓ-form are so fast that it is not possible to isolate these.
Hence (A) is the correct answer.


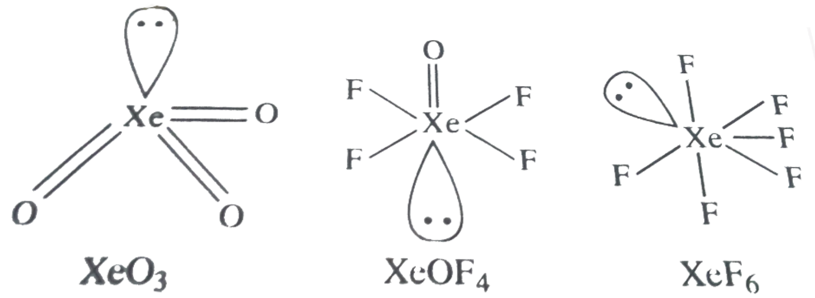
43. (A)
$CuFeS_2$ can be represented as $Cu_2S.Fe_2S_3$.

44. (A)

45. (D)
O_2^{2-} (18e⁻ species), Bond order = 1

46. (B)


The lone pair at N is not delocalized in the ring.
N atom does not show + m effect in other options. 1 ps. of N atom can show + m. Effect show they are least basic in nature.

47. (D)


48. (C)
$XeF_6 + 3H_2O \rightarrow XeO_3 + 6HF$

49. (B)
$$K_a = \frac{c\alpha^2}{1-\alpha} \Rightarrow 1.6\times10^{-5} = \frac{0.01\times\alpha^2}{1-\alpha}$$
$$\alpha = \sqrt{\frac{1.6\times10^{-5}}{0.01}} = \sqrt{1.6\times10^{-3}} = 0.04;\ \alpha = \frac{\wedge_m}{\wedge_m^\infty};\ \wedge_m = 0.04\times\left(380\times10^{-4}\ S\,m^2\,mol^{-1}\right)$$
$$\wedge_m = \frac{\kappa\times10^{-3}}{M};\ K = \frac{\wedge_m\times M}{10^{-3}}$$
$$\kappa = \frac{0.04\times380\times10^{-4}\times0.01}{10^{-3}} = 1.52\times10^{-2}\ S\,m^{-1}$$

50. (D)
$$Na_2[\overset{II}{Fe}(CN)_5NO^+] \xrightarrow{OH^-} Na_4[\overset{II}{Fe}(CN)_5NOS^-]$$
Purple solution

51. (148)

At initial condition,

Total pressure = 200 mm of Hg = P_{gas} + $P_{vapour\ water}$

$\Rightarrow \quad P_{gas} + 96 = 200$

$P_{gas} = 104$ mm of Hg (Initial pressure of air (or, gas)

When second container is connected

$P_1 = 104$ mm of Hg $\qquad P_2 = ?$

$V_1 = 1 \qquad V_2 = 2$ litre

$P_1V_1 = P_2V_2$

$104 \times 1 = P_2 \times 2$

$P_2 = 52$ mm of Hg (Final pressure of air)

After equilibrium is established, $P_{total} = P_2 + P_{vapour}$

$P_{total} = 52 + 96 = P_{gas} + P_{water} = 148$ mm of Hg at equilibrium.

52. (43)

According to Hanny-Smith equation

% Ionic character = 16 (4.0 – 2.1) + 3.5 $(4.0 – 2.1)^2$ = 43%

53. (24)

Let 2n be the number of moles of HI which is decomposed, the number of moles of H_2 and I_2 produced will be n mole each. Then molar concentrations of various species at equilibrium are

$[HI] = \frac{(5-2n)}{10}$ mol/L, $\qquad [H_2] = \frac{n}{10}$ mol/L and $[I_2] = \frac{n}{10}$ mol/L

Also, $K_C = \frac{[H_2][I_2]}{[HI]^2} = \frac{\frac{n}{10} \times \frac{n}{10}}{\left(\frac{5-2n}{10}\right)^2}$

$$0.0256 = \frac{n^2}{(5-2n)^2} \Rightarrow \frac{n}{(5-2n)} = 0.16$$

Solving for n, we get $n = 0.6$

$\therefore \quad [HI] = \frac{5 - 2 \times 0.6}{10} = \frac{3.8}{10} = 0.38$ mol/L

$[H_2] = \frac{0.6}{10} = 0.06$ mol/L

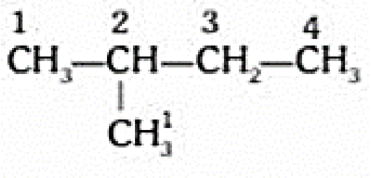
$[I_2] = \frac{0.6}{10} = 0.06$ mol/L

Percentage of HI decomposed $= \frac{2 \times 0.6}{5} = 0.24$ or 24%

54. (3)

Magnetic moment $= \sqrt{n(n+2)}$ B.M. , n = no. of unpaired electrons.

55. (0)
Isochoric process
$w = 0$

56. (4)


57. (100)
Given: $K_b = 2.16^oC$, $w = 0.15$ g, $\Delta T_b = 0.216^oC$, $W = 15$ g,

$\Delta T_b = \text{Molality} \times K_b$

$\Delta T_b = \frac{w}{M \times W} \times 1000 \times K_b \Rightarrow 0.216 = \frac{0.15}{M \times 15} \times 1000 \times 2.16$

$\Rightarrow M = \frac{0.15 \times 1000 \times 2.16}{0.216 \times 15} = 100$

58. (1280)
In $K_2SO_4 \cdot Al_2(SO_4)_3 \cdot 24H_2O$
No. of moles of 'O' = 40 × (moles of sample)

$\Rightarrow \frac{6400}{16} = 40 \times x$

$\Rightarrow x = 10.$

No. of moles of 'S' $= 4 \times$ (moles of sample)
$= 4 \times 10 = 40$

$\therefore$ Mass of 'S' = 40 × 32 g = 1280 g

59. (0)

60. (5)

Rate constant $= k = \frac{8.21 \times 10^{-2}}{0.0821 \times 300} \frac{\text{atm}}{\text{min}} = \frac{1}{300} \frac{\text{mole}}{\ell.\text{min}}$

$t_{1/2} = \frac{a}{2k} = \frac{2}{2k} = \frac{1}{k} = 300 \text{ min}$
$= 5 \text{ hr.}$

PART (C) : MATHEMATICS

ANSWER KEY

61.	(D)	62.	(C)	63.	(C)	64.	(B)	65.	(B)
66.	(A)	67.	(D)	68.	(D)	69.	(B)	70.	(B)
71.	(A)	72.	(D)	73.	(D)	74.	(C)	75.	(D)
76.	(D)	77.	(C)	78.	(B)	79.	(B)	80.	(D)
81.	(8)	82.	(4)	83.	(6)	84.	(2)	85.	(4)
86.	(19)	87.	(5)	88.	(1)	89.	(11)	90.	(12)

SOLUTIONS

61. (D)

We are given,

np = 4 ...(i)

And npq = 2 $\Rightarrow$ (4) q = 2

$\Rightarrow$ $q = \frac{1}{2}$

$\therefore$ $p = 1 - q = 1 - \frac{1}{2} \Rightarrow p = \frac{1}{2}$

Now, $n\left(\frac{1}{2}\right) = 4 \Rightarrow n = 8$ [From Eq. (i)]

$\therefore$ $p(X = 1) = {}^{n}C_1 pq^{n-1} = 8pq^7$

$= 8\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)^7$

$= 8 \times \frac{1}{2} \times \frac{1}{128} = \frac{1}{32}$

62. (C)

For acute angle, $\vec{a} \cdot \vec{b} > 0$

i.e. $2x^2 - 3x + 1 > 0$

$\Rightarrow$ $(2x-1)(x-1) > 0$

$\Rightarrow$ $2\left(x - \frac{1}{2}\right)(x-1) > 0$

$\Rightarrow$ $\left(x - \frac{1}{2}\right)(x-1) > 0$

$\Rightarrow$ $x \in \left(-\infty, \frac{1}{2}\right) \cup (1, \infty)$ ...(i)

$\therefore$ Axis of ordinate is Y-axis.

From question, $\vec{b} \cdot \hat{j} < 0 \Rightarrow x < 0$

i.e. $x \in (-\infty, 0)$ ...(ii)

From Eqs. (i) and (ii), we get

$x \in (-\infty, 0)$ i.e. for all x < 0

63. (C)

$\therefore$ General term in $(x + y + z)^{17}$ is

$$\frac{17!}{p!q!r!}x^p y^q z^r$$

Where p + q + r = 17

If p = 4, q = 10, r = 3

So, coefficient of $x^4y^{10}z^{13}$ is

$$\frac{17!}{4!10!3!}$$

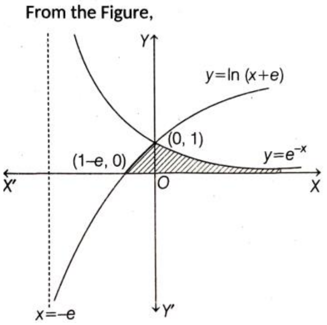
64. (B)

The given curves are $y = \text{In}(x+e)$ and $x = In\left(\frac{1}{y}\right)$

$\Rightarrow \frac{1}{y} = e^x$

$\Rightarrow y = e^{-x}$

From the figure.



$\therefore$ Required area

$$= \int_{1-e}^{0} \log_e(x+e)dx + \int_0^{\infty} e^{-x}dx$$

$$= \int_{1-e}^{0} \log t\, dt + \int_0^{\infty} e^{-x}dx$$

[$\therefore$ putting x + e = t]

$$= [t\,\text{in}t - t]_1^e - [e^{-x}]_0^{\infty}$$

= 1 + 1 = 2 sq units

65. (B)

$$\sum_{r=0}^{n}\sum_{s=0}^{n}(r^n \cdot C_r \cdot {}^nC_s + s \cdot {}^nC_r\, {}^nC_s)$$

$$\sum_{r=0}^{n}\sum_{s=0}^{n} r \cdot {}^n C_r \cdot {}^nC_s + \sum_{r=0}^{n}\sum_{s=0}^{n} s \cdot {}^n C_r \cdot {}^nC_s$$

$$= \sum_{r=0}^{n} r \cdot {}^n C_r \left(\sum_{r=0}^{n} {}^nC_s\right) + \sum_{r=0}^{n} s \cdot {}^n C_s \left(\sum_{r=0}^{n} {}^nC_r\right)$$

$$\sum_{r=0}^{n} r \cdot {}^n C_r \cdot 2^n + \sum_{s=0}^{n} s \cdot {}^n C_s \cdot 2^n$$

$$= 2^n\left[\sum_{r=0}^{n} r({}^nC_r)\right] + 2^n\left[\sum_{s=0}^{n} s({}^nC_s)\right]$$

$$= 2^n \left[\sum_{r=0}^{n} r \cdot \left(\frac{n}{r} \right)^{n-1} \cdot C_{r-1} \right] + 2^n \left[\sum_{s=0}^{n} s \cdot \left(\frac{n}{s} \right)^{n-1} \cdot C_{s-1} \right]$$

$$= 2^n \cdot (n \cdot 2^{n-1}) + 2^n (n \cdot 2^{n-1})$$

$$= n(2^{2n-1}) + n(2^{2n-1}) = n \cdot 2^{2n}$$

66. (A)

$|A| I + A.adj\ B = B$

$B + A.adjB.\ B = B^2$

$B + A.|B|\ I = B^2$

$B + A = b^2$

$\therefore\ |B + A| = |B^2| = 1$

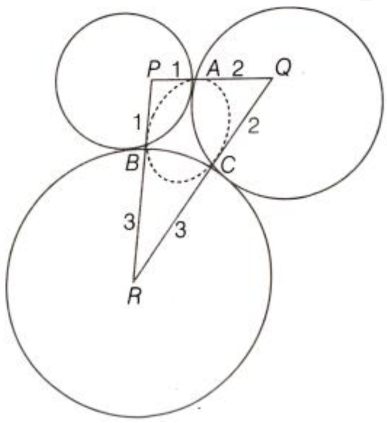
67. (D)

The circle ABC is incircle of ΔPQR,. $|PQ| = 3, |QR| = 5$ and $|PR| = 4$

$\Rightarrow\ \Delta PQR$ is a right-angled triangle

$\therefore$ Area of $\Delta PQR(\Delta) = \frac{1}{2} \times 3 \times 4 = 6$

And $s = \frac{1}{2}(3 + 4 + 5) = 6$



$\therefore$ Area of $\Delta PQR(\Delta) = r \cdot s$

$\Rightarrow\ r = \frac{\Delta}{s} = \frac{6}{6} = 1$

68. (D)

Number of ways,

$S = {}^{20}C_{10} + {}^{20}C_9 + {}^{20}C_8 + \ldots + {}^{20}C_0$

$\Rightarrow\ S = {}^{20}C_{10} + {}^{20}C_{11} + {}^{20}C_{12} + \ldots + {}^{20}C_{20}$

$2S = {}^{20}C_{10} + 2^{20}$

$S = \frac{1}{2}\,{}^{20}C_{10} + 2^{19}$

69. (B)

Given that,

$$\frac{dy}{dx}+\frac{1}{x}\tan y=\frac{1}{x^2}\tan y\cdot\sin y$$

$\Rightarrow$ $$\cot y\cdot \operatorname{cosec} y\frac{dy}{dx}+\frac{1}{x}\operatorname{cosec} y=\frac{1}{x^2}$$

Putting – cosec y = t, we get,

$$\frac{dt}{dx}-\frac{t}{x}=\frac{1}{x^2}$$

Which is linear differential equation.

$$IF=e^{\int-\frac{1}{x}dx}=e^{-\ln x}=\frac{1}{x}$$

Therefore, solution is

$$\frac{t}{x}=\int-\frac{1}{x^3}dx$$

$$\frac{t}{x}=\frac{1}{2x^2}+K$$

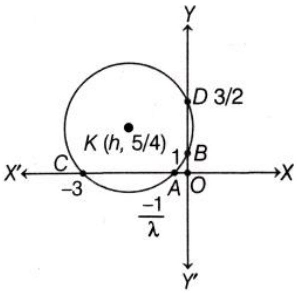
$\Rightarrow$ $2xt = 1 + 2kx^2$

$\Rightarrow$ $2x = \sin y\,(1 + cx^2)$, where c = 2K and K = constant of integration

70. (B)

If $\lambda\neq 0$, then $\lambda x-y+1=0$ and $x-2y+3=0$ meet the axes at

$$A\left(\frac{-1}{\lambda},0\right),B(0,1),C(-3,0),\text{ and } D\left(0,\frac{3}{2}\right).$$



The centre of circle lies on the perpendicular bisector of BD,

i.e. on $y=\frac{5}{4}$

let the centre of the circle be $k\left(h,\frac{5}{4}\right)$

then, $KC^2=KB^2$

$\Rightarrow$ $$(h+3)^2+\left(\frac{5}{4}\right)^2=h^2+\left(\frac{1}{4}\right)^2$$

$$h=\frac{-7}{4}$$

As, $KA^2=KB^2$, we get;

$$\left(\frac{-1}{\lambda}+\frac{7}{4}\right)^2+\left(\frac{5}{4}\right)^2=\left(\frac{-7}{4}\right)^2+\left(\frac{1}{4}\right)^2$$

$\Rightarrow \quad \left(\frac{1}{\lambda}-\frac{7}{4}\right)^2 = \frac{49}{16}-\frac{24}{16}=\frac{25}{16}$

$\Rightarrow \quad \frac{1}{\lambda}=\frac{7}{4}\pm\frac{5}{4}=3,\frac{1}{2}$

$\Rightarrow \quad \lambda=\frac{1}{3},2$

If $\lambda=0$, then lines become $-y+1=0$ and $x-2y+3=0$.

These meet the axes at (–3, 0), (0, 1) and $\left(0,\frac{3}{2}\right)$.

As, these three points are non-collinear, they lie on a circle.

Thus, $\lambda=0,2,\frac{1}{3}$

Hence, $\lambda\neq\frac{-1}{3}$

71. (A)

Given,

$f(x)=\sqrt[6]{4^x+8^{(2/3)(x-2)}-52-2^{2(x-1)}}$

f(x) is defined, if

$4^x+8^{(2/3)(x-2)}-52-2^{2(x-1)}\geq 0$

$\Rightarrow \quad 2^{2x}+2^{2(x-2)}-2^{2(x-1)}\geq 52$

$\Rightarrow \quad 2^{2x}[1+2^{-4}-2^{-2}]\geq 52$

$\Rightarrow \quad 2^{2x}\left(\frac{13}{16}\right)\geq 52$

$\Rightarrow \quad 2^{2x}\geq 64$

$\Rightarrow \quad 2x\geq 6$

$\Rightarrow \quad x\geq 3$

$\therefore \quad x\in[3,\infty)$

72. (D)

$\log_{1/2}\left(\frac{2a-5}{a-3}\right)\geq 0$

$\Rightarrow \quad \log_{1/2}\left(\frac{2a-5}{a-3}\right)\geq \log_{1/2}1$

$\Rightarrow \quad \frac{2a-5}{a-3}\leq 1 \Rightarrow \frac{2a-5}{a-3}-1\leq 0$

$\Rightarrow \quad \frac{a-2}{a-3}\leq 0, a\in[2,3)$

So, the only integral value of ‘a’ is 2.

Let $I=\int_0^{\sqrt{2}-1}\frac{1}{1+x}dx$

$=[\ln(1+x)]_0^{\sqrt{2}-1}=\ln\sqrt{2}$

$\Rightarrow \quad \frac{1}{r} = e^{\ln\sqrt{2}}$

$\Rightarrow \quad r = \frac{1}{\sqrt{2}}$

So, sum of GP

S = a + ar + ar^2 + ar^3 + …

$$\frac{a}{1-r} = \frac{2}{1-\frac{1}{\sqrt{2}}} = \frac{2\sqrt{2}}{\sqrt{2}-1}$$

73. (D)

Given,

$$z|z| - az + 1 = 0$$

Putting $z = a + 2i$, we have

$$(a+2i)\sqrt{a^2+4} = a(a+2i) - 1$$

$$= a^2 - 1 + 2ai$$

On comparing real and imaginary parts, we get

$$a\sqrt{a^2+4} = a^2 - 1$$

and $2\sqrt{a^2+4} = 2a$

$\Rightarrow \quad \sqrt{a^2+4} = \frac{a^2-1}{a} = a$

$\Rightarrow \quad a^2 = a^2 - 1$ Which is an absurd. Hence, no such complex number exists.

74. (C)

Since, coefficient of $x^2 + ax + 1$ are symmetrical.

So, roots will be $\alpha, \frac{1}{\alpha}$

($\therefore$ coefficient of x^2 = constant term)

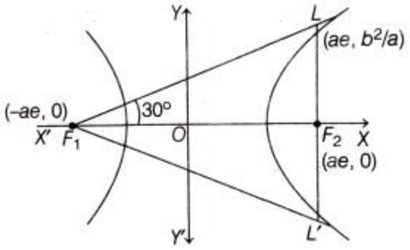
Now, $\lim_{x\to 1/\alpha} \frac{\sin(x^2+ax+1)}{(\alpha x - 1)}$

$$\lim_{x\to 1/\alpha} \frac{\sin\left[\left(x-\frac{1}{\alpha}\right)(x-\alpha)\right]}{\left(x-\frac{1}{\alpha}\right)(\alpha x-1)} \cdot \frac{\left(x-\frac{1}{\alpha}\right)(x-\alpha)}{(\alpha x-1)}$$

$$= \lim_{x\to 1/\alpha} \frac{(x-\alpha)}{\alpha} = \frac{\frac{1}{\alpha}-\alpha}{\alpha} = \frac{1-\alpha^2}{\alpha^2}$$

75. (D)

Slope of LF_1 is tan 30^o



So, $\tan 30^o = \dfrac{b^2/a}{2ae}$

$\Rightarrow \quad \dfrac{1}{\sqrt{3}} = \dfrac{a^2(e^2-1)}{2a^2e} = \dfrac{e^2-1}{2e}$

$\Rightarrow \quad e = \sqrt{3}$

76. (D)

$\therefore \quad x = 4t^2 - 1 \Rightarrow \dfrac{x+1}{4} = t^2$

And $y = 8t - 2 \Rightarrow \dfrac{y+2}{8} = t$

$\Rightarrow \quad \dfrac{x+1}{4} = \left(\dfrac{y+2}{8}\right)^2$

$\Rightarrow \quad (y+2)^2 = \dfrac{64}{4}(x+1)$

$\Rightarrow \quad (y+2)^2 = 16(x+1)$

Which is of the for $y^2 = 4AX$

Where y = y + 2,

4A = 16 ⇒ A = 4

X = x + 1

∴ Equation of latus rectum

X = A

X + 1 = 4 ⇒ x – 3 = 0

77. (C)

Given, $f(x) = \dfrac{1}{1-x}$

The point x = 1 is a discontinuous point of the function $f(x) = \dfrac{1}{1-x}$.

If $x \neq 1$, then $\lambda(x) = f(f(x)) = \dfrac{x-1}{x}$

Hence, x = 0 is a point of discontinuity of the function $\lambda(x)$.

If $x \neq 0$ and $x \neq 1$, then fofof(x) = x

Hence, $y = f^{3n}(x) = (f^3(x)^n) = x$ is continuous everywhere.

Therefore, 0 and 1 are the only points of discontinuity of y.

78. (B)

Since, 3^m ends in 3, 9, 7, 1 and $3^m + 3^n$ is divisible by 5. When either m is of the form (4k + 1) and n is of the form (4k + 3) or m is of the form 4k and n is of the form (4k + 2) and vice-versa.

There are 25 numbers of each types from 1 to 100.

∴ Required probability

$$= \frac{1}{4} \times \frac{1}{4} \times 2 + \frac{1}{4} \times \frac{1}{4} \times 2$$

$$= \frac{4}{16} = \frac{1}{4}$$

79. (B)

Given, curves are $y = \sqrt{4 - x^2}$ …(i)

$$y \geq \sqrt{2} \sin\left(\frac{\pi x}{2\sqrt{2}}\right) \quad \text{…(ii)}$$

Curves Eqs. (i) and (ii) intersect at $x = \sqrt{2}$.

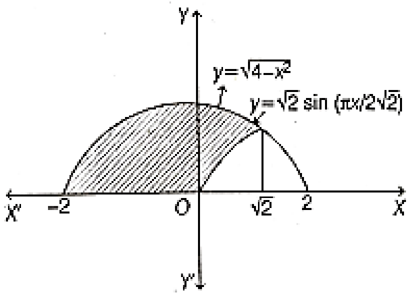
Area of the left of Y-axis π sq units and area of the right Y-axis

$$= \int_0^{\sqrt{2}} \sqrt{4 - x^2} - \sqrt{2} \sin\left(\frac{\pi x}{2\sqrt{2}}\right) dx$$

$$= \left[\frac{x\sqrt{4 - x^2}}{2} + \frac{4}{2}\sin^{-1}\frac{x}{2}\right]_0^{\sqrt{2}} + \frac{4}{\pi}\left[\cos\left(\frac{\pi x}{2\sqrt{2}}\right)\right]_0^{\sqrt{2}}$$

$$= \left(1 + 2 \cdot \frac{\pi}{4}\right) + \frac{4}{\pi}(0 - 1)$$

$$= 1 + \frac{\pi}{2} - \frac{4}{\pi} = \frac{2\pi + \pi^2 - 8}{2\pi}$$



∴ Required ratio $= \dfrac{2\pi^2}{2\pi + \pi^2 - 8}$

80. (D)

We have,

$$A_r = {}^{10}C_r, B_r = {}^{20}C_r \text{ and } C_r = {}^{30}C_r$$

$$\therefore \sum_{r=1}^{10} A_r(B_{10}B_r - C_{10}A_r)$$

$$= B_{10}\sum_{r=1}^{10} A_rB_r - C_{10}\sum_{r=1}^{10}(A_r)^2$$

$$= B_{10}\left(\sum_{r=1}^{10} {}^{10}C_r \cdot {}^{20}C_r\right) - C_{10}\sum_{r=1}^{10}({}^{10}C_r)^2$$

$$= B_{10}\left(\sum_{r=0}^{10} {}^{10}C_r \cdot {}^{20}C_r - 1\right) - C_{10} \times \left(\sum_{r=1}^{10} ({}^{10}C_r)^2 - 1\right)$$

$B_{10} \times \{\text{coefficient of } x^{20} \text{in} (1+x)^{10}$

$(1+x)^{20} - 1\} - C_{10} \times \{{}^{20}C_{10} - 1\}$

$$\left[\therefore (1+x)^n = \sum_{r=0}^{n} {}^nC_r \cdot x^r\right]$$

$$= B_{10} \times ({}^{30}C_{20} - 1) - C_{10} \times ({}^{20}C_{10} - 1)$$

$$= B_{10} \times ({}^{30}C_{10} - 1) - C_{10}({}^{20}C_{10} - 1)$$

$$= B_{10} \times (C_{10} - 1) - C_{10}(C_{10} - 1) = C_{10} - B_{10}$$

81. (8)

The given differential equation is

$$\frac{dy}{dx} = \frac{y}{x}\left(\log_e\left(\frac{y}{x}\right) + 1\right) \qquad \ldots(i)$$

Put $y = Vx \Rightarrow \frac{dy}{dx} = V + \frac{dV}{dx} \cdot x$

From Eq. (i),

$$V + x\frac{dV}{dx} = V(\ln V + 1)$$

$$\Rightarrow \quad \frac{dV}{V \cdot \ln V} = \frac{dx}{x}$$

On integrating,

ln(ln V) = ln x + ln c

$$\Rightarrow \quad \ln\left(\frac{y}{x}\right) = cx$$

$$\Rightarrow \quad y = x \cdot e^{cx} \qquad \ldots(ii)$$

When x = 1, y = 2

From Eq. (ii), $2 = 1 \cdot e^{c \cdot 1} \Rightarrow e^c = 2$

$\therefore \quad y = x \cdot (e^c)^x = x(2)^x$

$\therefore \quad y(2) = 2 \cdot (2)^2 = 8$

82. (4)

Centre of the circle lies on the line through (2, – 2) and perpendicular to x + y = 0, that is, on (x – 2) – (y + 2) = 0

or x – y = 4

as centre lies on the on the X-axis, the centre of circle is (4, 0) and its radius is

$\sqrt{(4-2)^2 + (-2-0)^2} = 2\sqrt{2}$

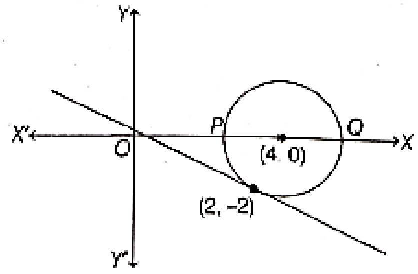
For maximum possible value of h. (h, k) will lie on Q.

$\therefore \quad Q = (4 + 2\sqrt{2}, 0)$,

Where $2\sqrt{2}$ is radius.

$\therefore h \le 4 + 2\sqrt{2}$

Thus, $h - 2\sqrt{2} \le 4$.



83. (6)

Two parabolas are equal. If the lengths of their latusrectum are equal.

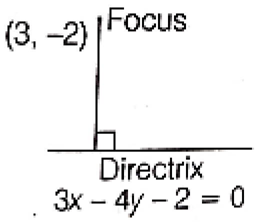
Now, the length of latusrectum of $y^2 = ax$ is a. the equation of second parabola can be written as

$$\sqrt{(x-3)^2+(y+2)^2} = \left(\frac{3x-4y-2}{5}\right)$$

So, focus is (3, –2) and the equation of the directrix is $3x-4y-2=0$

$\therefore$ length of latusrectum

$= 2\times$ (Distance between focus and directrix)



Thus, two parabolas are equal of a = 6.

84. (2)

Let $L_1 : \frac{x-1}{1} = \frac{y-(-3)}{-\lambda} = \frac{z-1}{\lambda} = S$

and $L_2 : \frac{x}{\left(\frac{1}{2}\right)} = \frac{y-1}{1} = \frac{z-2}{-1} = t$

i.e. $\frac{x-0}{1} = \frac{y-1}{2} = \frac{z-2}{-2} = t$

lines L_1 and L_2 will be coplanar, if

$$\begin{vmatrix} x_1-x_2 & y_1-y_2 & z_1-z_2 \\ l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \end{vmatrix} = 0$$

i.e. $\begin{vmatrix} 1 & -4 & -1 \\ 1 & -\lambda & \lambda \\ 1 & 2 & -2 \end{vmatrix} = 0$

$\Rightarrow$ $-5\lambda - 10 = 0$

$\Rightarrow$ $\lambda = -2$

$\therefore$ $\lambda + 4 = -2 + 4 = 2$

85. (4)

$\sin^{-1}(\sin 6x) = x$

$\Rightarrow \sin 6x = \sin x$

$\Rightarrow x = \frac{\pi}{7}, \frac{3\pi}{7}, \frac{5\pi}{7}, \pi, 0, \frac{2\pi}{5}, \frac{4\pi}{5}$

But $\frac{5\pi}{7}, \pi$ and $\frac{4\pi}{5}$ are rejected as any value of x

Which is solution of given equation can't be more than $\frac{\pi}{2}$.

Hence, number of solutions is 4

86. (19)

Using LMVT for some $c \in (1,6)$ such that

$f'(c) = \frac{f(6) - f(1)}{5}$

$= \frac{f(6)+2}{5} \geq 4.2$

$\Rightarrow f(6)+2 \geq 21$

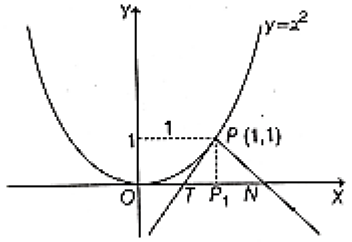
$\Rightarrow f(6) \geq 19$

Hence, smallest possible value of f(6) is 19.

87. (5)

Given, $y = x^2 \Rightarrow \frac{dy}{dx} = 2x$

$\therefore \left(\frac{dy}{dx}\right)_{(1,1)} = 2$



$\therefore$ Equation of tangent PT at (1,1) is

$(y-1) = 2(x-1)$ i.e. $y = 2x - 1$

So, $T \equiv \left(\frac{1}{2}, 0\right)$

Equation of normal PN at P(1,1) is

$y - 1 = -\frac{1}{2}(x-1)$ i.e. $2y = -x + 3$

So, $N \equiv (3,0)$

$\text{NT} = 3 - \frac{1}{2} = \frac{5}{2}$

Required area $= \frac{1}{2}\left(\frac{5}{2}\right)(1) = \frac{5}{4}$ sq units $= 1.25$ sq units

88. (1)

We have, $\cos y\left(\frac{dy}{dx}+e^{-x}\right)+\sin y\left(e^{-x}-\frac{dy}{dx}\right)=e^{e^{-x}}$

$\Rightarrow (\cos y-\sin y)\frac{dy}{dx}+(\cos y+\sin y)e^{-x}=e^{e^{-x}}$ ...(i)

Let cos y + sin y = z

$\Rightarrow (\cos y-\sin y)\frac{dy}{dx}=\frac{dz}{dx}$

$\therefore$ Eq. (i) reduces to $\frac{dz}{dx}+e^{-x}\cdot z=e^{e^{-x}}$,

which is linear differential equation whose integrating factor (IF)

$=e^{\int e^{-x}dx}=e^{-e^{-x}}$

and solution is

$z\times(IF)=\int e^{e^{-x}}\times(IF)dx$

$\Rightarrow z\cdot e^{-e^{-x}}=\int e^{e^{-x}}e^{-e^{-x}}dx$

$\Rightarrow z\cdot e^{-e^{-x}}=\int e^{(e^{-x}-e^{-x})}dx$

$\Rightarrow \int e^{o}dx=\int dx=x+c$

$\Rightarrow (\cos y+\sin y)e^{-e^{-x}}=x+c$ ...(ii)

But Eq. (ii) passes through $\left(0,\frac{-\pi}{4}\right)$.

So, c = 0

$\Rightarrow (\cos y+\sin y)e^{-e^{-x}}=x$ ...(iii)

Putting (t, 0) i.e. x = t and y = 0, in Eq. (iii), we get

$e^{-e^{-t}}=t$

$\Rightarrow \frac{t}{e^{-e^{-t}}}=1$

$\Rightarrow t\cdot e^{e^{-t}}=1$

$\Rightarrow \left|t\cdot e^{e^{-t}}\right|=|1|=1$

89. (11)

Given, curves are

y = ex ln x ...(i)

and $y=\frac{\ln x}{ex}$ ...(ii)

Points of intersection of Eqs. (i) and (ii) are $\left(\frac{1}{e},-1\right)$ and Q(1,0).

For curve (i),

y < 0 for 0 < x < 1

and $y\geq 0$ for $x\geq 1$

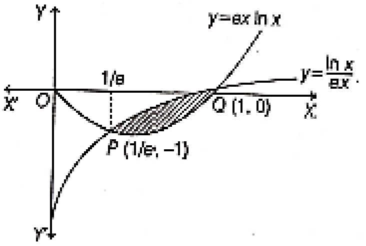
obviously $y\to 0$ when $x\to 0$

for curve (ii), $y<0$ for $0<x<1$

and $y\geq 0$ for $x\geq 1$

obviously $y \to 0$ when $x \to \infty$

Thus, the shape of the two curves are as shown in the figure below



$\therefore$ required area

$$= \left|\int_{1/e}^{1} ex\ln x dx\right| - \left|\int_{1/e}^{1} \frac{\ln x}{ex} dx\right|$$

$$= \left|\left[\frac{1}{2}ex^2(2\ln x - 1)\right]_{1/e}^{1}\right| - \left|\left[\frac{1}{2e}(\ln x)^2\right]_{1/e}^{1}\right|$$

$$= \left|\frac{1}{4e}(3 - e^2)\right| - \left|-\frac{1}{2e}\right|$$

$$= \frac{e^2 - 3}{4e} - \frac{1}{2e} = \frac{e^2 - 5}{4e} \quad \text{sq units}$$

Comparing, we get

$\alpha = 2, \beta = -5$ and $\gamma = 4$

Hence, $\alpha - \beta + \gamma = 2 + 5 + 4 = 11$

90. (12)

Let $F(x) = f^3(x)$

And F(x) is continuous and differentiable function is [1, 3] using Lagrange's mean value theorem, We get

$$\frac{F(3) - F(1)}{3 - 1} = F'(c)$$

$$\therefore \quad \frac{f^3(3) - f^3(1)}{2} = 3f^2(c) \cdot f'(c)$$

$$\Rightarrow \quad [f(3) - f(1)][f^2(3) + f^2(1) + f(3)f(1)] = 6f^2(c) \cdot f'(c)$$

(using the formula of $a^3 - b^3$)

On comparing, we get

k = 6

$\therefore$ 2k = 12