PART-1 : PHYSICS

SECTION–I : (Maximum Marks : 40)

- This section contains TEN questions.
- Each question has FOUR options (A), (B), (C) and (D). ONE OR MORE THAN ONE of these four option(s) is (are) correct.
- For each question, darken the bubble(s) corresponding to all the correct option(s) in the ORS
- For each question, marks will be awarded in one of the following categories :

Full Marks	: +4	If only the bubble(s) corresponding to all the correct option(s) is (are) darkened.
Zero Marks	: 0	If none of the bubbles is darkened.
Negative Marks	: –2	In all other cases.

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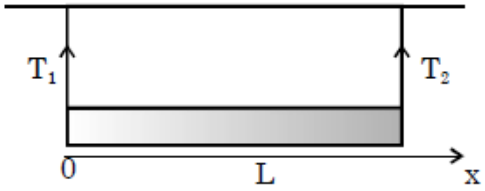
1. The linear mass density of a rod of length L varies from one end to the other as $\lambda_0\left(1+\frac{x^2}{L^2}\right)$, where x is the distance from one end, where λ_0 is a constant. The rod is suspended from a ceiling by two massless strings with tensions T_1 and T_2 in them (see figure). Then, which of the following statement(s) is/are **CORRECT** ?



(A) The mass of the rod is $\frac{2\lambda_0 L}{3}$.

(B) The center of gravity of the rod is located at $x=\frac{9L}{16}$.

(C) The tension T_1 in the left string is $\frac{7\lambda_0 Lg}{12}$.

(D) The tension T_2 in the right string is $\frac{3\lambda_0 Lg}{2}$.

Ans. (B, C)

(A) $M=\int_0^L \lambda dx=\lambda_0\int_0^L\left(1+\frac{x^2}{L^2}\right)dx$

$=\lambda_0\left[L+\frac{1}{L^2}\times\frac{L^3}{3}\right]=\frac{4\lambda_0 L}{3}$

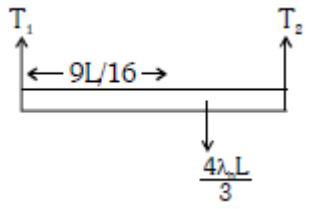
(B) $X_{CM}=\frac{\int dmx}{M}$

$=\frac{\int_0^L \lambda x dx}{M}=\frac{\lambda_0\int_0^L\left(x+\frac{x^3}{L^2}\right)dx}{M}$

$=\frac{\lambda_0\left[\frac{L^2}{2}+\frac{L^2}{4}\right]}{\frac{4\lambda_0 L}{3}}=\frac{3\lambda_0 L^2\times 3}{4\times 4\lambda_0 L}$

$X=\frac{9L}{16}$

(C)

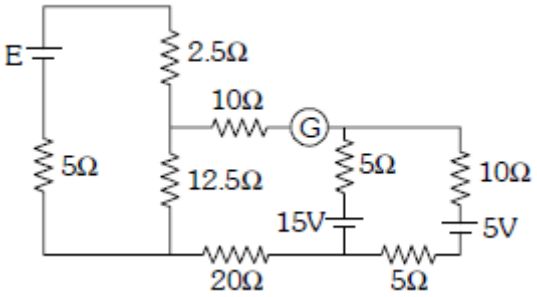


$$T_1 + T_2 = \frac{4\lambda_0 Lg}{3}$$

$$T_1 \times \frac{gL}{16} = T_2 \times \frac{7L}{16}$$

So, $T_1 = \frac{7\lambda_0 Lg}{12}$ $\quad$ $T_2 = \frac{9\lambda_0 Lg}{12}$

2. If galvanometer shows no deflection in the given circuit. The value of E is (all batteries are ideal)



(A) the value of E is 10 volt

(B) the value of E is 20 volt

(C) the value of current passing through 20 Ω resistance is zero

(D) the battery of emf 5 volt is charging at the rate of 2.5 W

Ans. (B,C,D)

The current in extreme right loop is $\frac{15-5}{20} = 0.5\,A$

The potential difference across the galvanometer is zero.

$15 - 0.5 \times 5 = i \times 12.5$

$12.5 = i \times 12.5 = 1A$

3. Which of the following statement(s) is/are correct?

(A) For hydrogen like atoms if quantum number is very large then the frequency of photon emitted when electron jumps from an orbit to next lower energy orbit is equal to frequency of revolution of electron in original orbit

(B) Electron debroglie wavelength depends on kinetic energy of electron

(C) Energy of electrons is quantized in Li^{++} atom

(D) Electrons cannot exists inside nucleus because they are attracted by protons inside nucleus

Ans. (A,B,C)

Frequency of revolution of electron is $f \propto \frac{1}{n^3}$

frequency of photon emitted $\nu \propto \left(\frac{1}{(n-1)^2} - \frac{1}{n^2}\right)$

$$\nu \propto \left[\frac{n^2-(n-1)^2}{[n(n-1)]^2}\right]$$

$$\nu \propto \frac{[(2n-1)]}{n^2(n-1)^2}$$

when $n >> 1$

$$\nu \propto \frac{2n}{n^4}$$

$$\nu \propto \frac{1}{n^3}$$

4. A circuit, containing an inductance and a resistance connected in series, has an AC source of 200 V, 50 Hz connected across it. An AC current of 10 A rms flows through the circuit and the power loss is measured to be 1kW.

(A) The inductance of the circuit is $\frac{\sqrt{3}}{10\pi}$ H.

(B) The frequency of the AC when the phase difference between the current and emf becomes $\pi/4$, with the above components is $\frac{50}{\sqrt{3}}$ Hz.

(C) The frequency of AC, at which the reactive power is half of the actual power loss is $\frac{25}{\sqrt{3}}$Hz

(D) The frequency of the AC when the phase difference between the current and emf becomes $\pi/4$, with the above components is $\frac{25}{\sqrt{3}}$Hz.

Ans. (A,B,C)

(A) The power loss in the circuit is

$$P_{loss} = V_{rms} \times I_{rms} \cos\phi$$

$$= 200\ V \times 10\ A \times \cos\phi = 1000\ W$$

$\cos\phi = \frac{1}{2}$; $\therefore \phi = \pi/3$ (since the circuit element is an inductor)

The inductance in the circuit is calculated as follows:

$$X_L = \frac{200}{10} \times \sin\frac{\pi}{3} = 10\sqrt{3}$$

$$\therefore \quad L = \frac{10\sqrt{3}}{2\pi \times 50} = \frac{\sqrt{3}}{10\pi} H$$

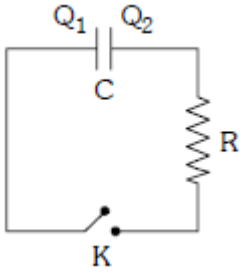
(B) $$\frac{\omega L}{R} = \tan\frac{\pi}{4} = 1$$

$$2\pi f \times \frac{\sqrt{3}}{10\pi} \times \frac{1}{10} = 1 \quad \text{or} \quad f = \frac{50}{\sqrt{3}} Hz$$

(C) $$\frac{P_{reactive}}{P_{loss}} = \frac{1}{2} = \tan\phi = \frac{\omega L}{R}$$

$$\therefore \ f = \frac{25}{\sqrt{3}} Hz, \text{ which is half of the previous value.}$$

5. The two plates of a capacitor of capacitance C are given charges Q_1 and Q_2. This capacitor is connected across a resistance R as shown key is closed at t = 0. Find the charges on the plates after time t.



(A) Total charge of the right plate $q_2 = \frac{Q_1 + Q_2}{2} - \left(\frac{Q_1 - Q_2}{2}\right) e^{-\frac{t}{RC}}$

(B) Total charge on the left plate $q = \frac{Q_1 + Q_2}{2} + \left(\frac{Q_1 - Q_2}{2}\right) e^{-\frac{t}{RC}}$

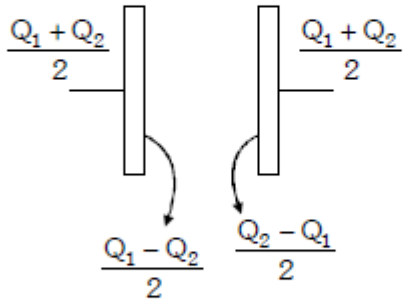
(C) Initial potential difference across the plates is given by $\frac{Q_1 - Q_2}{2C}$

(D) Initial potential difference across the plates is given by $\frac{Q_1 + Q_2}{2C}$

Ans. (A,B,C)

Initially the charges on the plates will be distributed as shown. P. d across the plates is give $\frac{Q_1 - Q_2}{2C}$ and is independent of the charge on the outer surfaces. Thus, when capacitor discharged, the charges on the outer surface do not change. Charge on the inner surface decreases according to the equation

$$q = \left(\frac{Q_1 - Q_2}{2}\right)e^{-\frac{1}{RC}}$$



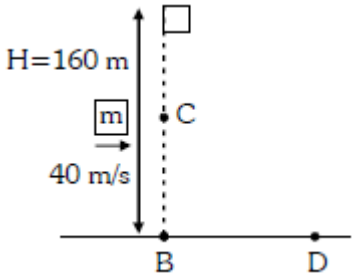
Total charge on the left plate

$$q = \frac{Q_1 + Q_2}{2} + \left(\frac{Q_1 - Q_2}{2}\right)e^{-\frac{t}{RC}}$$

And total charge of the right plate

$$q_2 = \frac{Q_1 + Q_2}{2} - \left(\frac{Q_1 - Q_2}{2}\right)e^{-\frac{t}{RC}}$$

6. A body of mass 150 gm is dropped from point A, takes time t_1 seconds to reach the point B at the ground. Once more, the same body is dropped from point A and when body traverses half the distance, a bullet of mass 50 gm moving horizontally with speed 40 m/s hits the falling body at point C and sticks to it. The total time of flight of the body in this case is t_2 seconds and body strikes the ground at point D. The air drag should be neglected -
(Take g = 10 m/s^2).



(A) distance between B and D is 20 m

(B) distance between B and D is 80 m

(C) speed of the body just before striking at point D is $10\sqrt{26}$ m/s

(D) speed of the body just before striking at point D is $15\sqrt{26}$ m/s

Ans. (A,C)

$h = 160\ m$

$$t_{1_0} = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2\times160}{10}} = 4\sqrt{2}\,sec.$$

Velocity of system after collision in vertical direction,

$v_{1_0} = 30$ m/s.

$$v_{2_0} = \frac{1}{4}\times40 = 10\,m/s$$

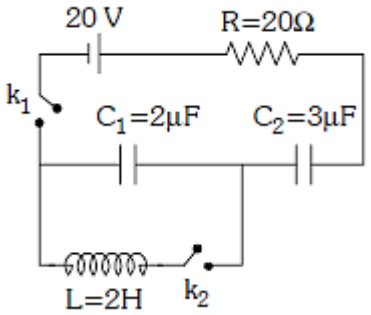
$$v_{10}^{'2} = 900 + 2\times10\times80$$

$$v_{10}^{'} = 50\,m/s$$

$$t_{20} = \frac{50-30}{10} = 2\,sec.$$

$t_{1_0} = 4$ sec. $\Rightarrow V_1 = 40$ m/s

7. A circuit shown in the figure in which k_1 is closed and k_2 is open. Inductor L can be connected in series to capacitor C_1 by closing switch k_2 and opening k_1.



(A) The switch k_1 is closed and k_2 is opened for long time. The charge on capacitor C_2 will be 24 μC

(B) At t = 0, when capacitors are fully charged, switch k_1 is opened and switch k_2 is closed, so that inductor is connected in series with capacitor C_1. The maximum charge will appear on capacitor C_1 at time t is $\frac{\pi}{500}$ sec.

(C) At t = 0, when capacitors are fully charged, switch k_1 is opened and switch k_2 is closed, so that inductor is connected in series with capacitor C_1. The maximum charge will appear on capacitor C_1 at time t is $\frac{\pi}{1000}$ sec.

(D) The maximum energy in inductor approximately will be 0.144 mJ

Ans. (A,B,D)

$$\frac{Q}{2} + \frac{Q}{3} = 20$$

$$Q = 24\,\mu C$$

$$\left[\omega = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{2\times2\times10^{-2}}} = \frac{10}{2} = 50\,rad/s\right]$$

$Q(t) = 24 \sin \omega t\ \mu C$

$Q(t) = (24 \sin \omega t)\ \mu C$

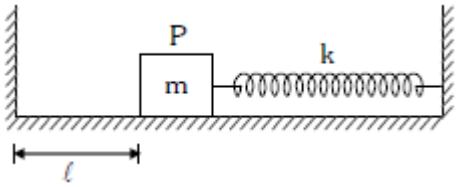
$$500\,t = \frac{\pi}{2}(-1)^n + \pi n$$

$$t = \frac{n\pi}{500} + (-1)^n \frac{\pi}{1000}$$

$$E_{max} = \frac{Q_0^2}{2C} = \frac{24\times10^{-3}\times24\times10^{-6}}{2\times2\times10^{-6}}$$

$$= 144\times10^{-6}\,J = 0.144\,mJ$$

8. Figure shows a block P of mass m resting on a smooth horizontal floor. The block is at a distance ℓ from a rigid wall on left side. Block is pushed toward right through $\frac{3\ell}{2}$ and released. When this block is passing through its mean position, a second block of mass m_1 is gently placed on it and stick to it. The combined system of two blocks move and finally just reaches the wall. Then



(A) $m_1 = \frac{5m}{8}$

(B) $m_1 = \frac{5m}{4}$

(C) Velocity of block m at mean position is $\frac{3\ell}{2}\sqrt{\frac{K}{m}}$ (before attaching block m_1)

(D) Velocity of block m at mean position is $\frac{3\ell}{4}\sqrt{\frac{K}{m}}$ (before attaching block m_1)

Ans. (B,C)

When block P is released from rest from a distance $\frac{3\ell}{2}$ towards right from mean position, thus this will be the amplitude of oscillation, so velocity of block when passing from its mean position is given as

$$v = A\omega = \frac{3\ell}{2}\sqrt{\frac{k}{m}} \qquad [\text{as } \omega = \sqrt{\frac{k}{m}}]$$

If mass m_1 is added to it and just after it velocity combined block becomes v_1, from momentum conservation we have

$$mv = (m + m_1)v_1 \quad \text{or} \quad v_1 = \frac{m}{(m+m_1)}\left(\frac{3\ell}{2}\sqrt{\frac{k}{m}}\right)$$

If this is the velocity of combined block at mean position, it must be given as

$$v_1 = Aw_1 \quad \left[\text{now } \omega_1 = \sqrt{\frac{k}{m+m_1}}\right]$$

where A_1 and ω_1 are the new amplitude and angular frequency of SHM of the block. It is given that combined block just reaches the left wall thus the new amplitude of oscillation must be ℓ so we have

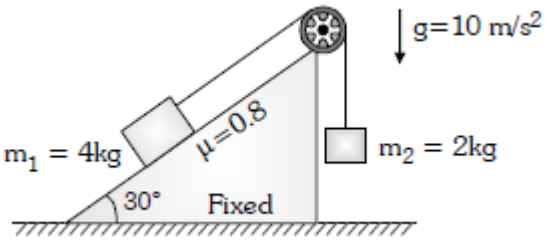
$$\frac{m}{(m+m_1)} \cdot \frac{3\ell}{2}\sqrt{\frac{k}{m}} = I_1\sqrt{\frac{k}{m+m_1}}$$

or $$\frac{3\sqrt{m}}{2\sqrt{m+m_1}} = 1$$

or $$9m = 4m + 4m_1$$

or $$m_1 = \frac{5}{4}m$$

9. Two blocks of masses m_1 and m_2 are connected through a massless inextensible string. Block of mass m_1 is placed at the fixed rigid inclined plane while the block of mass m_2 hanging at the other end of the string which is passing through a fixed massless frictionless pulley shown in figure. The coefficient of static friction between the block and the inclined plane is 0.8. The system of masses m_1 and m_2 is released from rest.



(A) the tension in the string is 20 N after releasing the system

(B) the contact force by the inclined surface on the block is along normal to the inclined surface

(C) the magnitude of contact force by the inclined surface on the block m_1 is $20\sqrt{3}$ N

(D) none of these

Ans. (A,B,C)

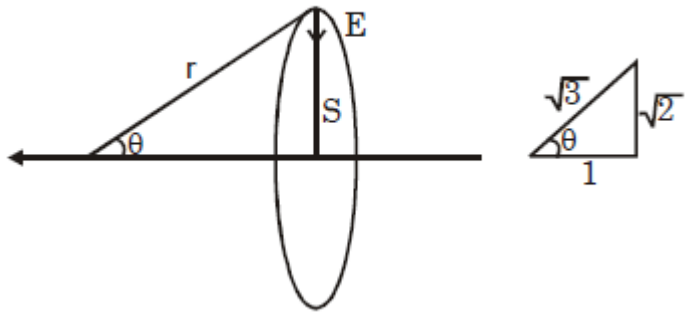
If the tendency of relative motion along the common tangent does not exist, then component of contact force along common tangent will be zero.

10 A stationary electric dipole $\vec{\phi} = p\hat{k}$ is situated at origin. A positive charge q, mass m executes circular motion of radius s at constant speed in the field of the dipole. Characterize the plane of orbit is :-

(A) The speed of charge particle is $v = \frac{1}{s}\sqrt{\frac{2qp}{\sqrt{3}\pi\epsilon_0 m}}$ (B) The speed of charge particle is $v = \frac{1}{s}\sqrt{\frac{qp}{3\sqrt{3}\pi\epsilon_0 m}}$

(C) Angular momentum of charge is $L = \sqrt{\frac{2qpm}{\sqrt{3}\pi\epsilon_0}}$ (D) Angular momentum of charge is $L = \sqrt{\frac{qpm}{3\sqrt{3}\pi\epsilon_0}}$

Ans. (B, D)



For dipole $\tan\alpha = \frac{1\tan\theta}{2}$ where α is angle of electric field with position vector

$\tan\theta = \sqrt{2}$

$\frac{S}{r} = \sin\theta$

$qE = \frac{mv^2}{R}$

$\frac{qkp}{mr^3}\sqrt{1+3\cos^2\theta} = \frac{v^2}{s}$

$r = \frac{s}{\sin\theta}$, $\cos\theta = \frac{1}{\sqrt{3}}$; since $= \frac{\sqrt{2}}{\sin\theta}$

$\frac{kpq}{ms^2}\frac{2\sqrt{2}}{3\sqrt{3}}\left(\sqrt{2}\right) = \frac{v^2}{s}$

$\frac{pq}{\pi\epsilon_0 m3\sqrt{3}} = v^2 \Rightarrow \frac{1}{s}\sqrt{\frac{pq}{\pi\epsilon_0 m3\sqrt{3}}}$

SECTION–II : (Maximum Marks : 24)

- This section contains EIGHT questions.
- The answer to each question is a SINGLE DIGIT INTEGER ranging from 0 to 9, both inclusive.
- For each question, darken the bubble corresponding to the correct integer in the ORS.
- For each question, marks will be awarded in one of the following categories :

Full Marks : +4 If only the bubble corresponding to the correct answer is darkened.
Zero Marks : 0 In all other cases.

1. A moving hydrogen atom makes a head on collision with another stationary H-atom. Before collision both the atoms are in ground state, and after collision both are getting excited and also move together. The minimum value of the kinetic energy of the incident hydrogen atom is x(13.6)eV. Fill the value of x in OMR sheet.

Ans. 3.00

After collision both atoms are moving together so collision between them must be completely inelastic and maximum loss in this collision is half of initial KE of atom. so to excite both after collision minimum must be $(10.2 + 10.2) = 20.4$eV

$$\frac{K}{2} \geq 20.4$$

$$K \geq 2 \times 20.4$$

$$K \geq 40.8$$

So $x = 3.00$

2. An inductor L = 50 mH carrying an initial current I_0 = 2.5 amp is connected across a non linear resistor in which voltage accross resistor varies, with current I according to $V = 10\ I^2$. How long after (in ms) current through inductor becomes 1.25 amp.

Ans. 2.00

At time 't'

$$-L\frac{di}{dt} - iR = 0$$

$$-L\frac{di}{dt} = iR = V = 10i^2$$

$$-\int_{2.5}^{1.25} \frac{di}{i^2} = \frac{10}{L}\int_0^t dt$$

$$\Rightarrow \left(\frac{1}{L}\right)_{2.5}^{1.25} = \frac{10}{50\times10^{-3}}[t-0]$$

$$\Rightarrow \frac{4}{5} - \frac{2}{5} = \frac{1000}{5} \times t$$

$$\therefore t = \frac{2}{1000} = 2ms$$

3. A wire is stretched between two fixed points separated by a distance of 2m such that tension in it is equal to 30.25 kgwt. The wire vibrates in its first overtone. A closed organ pipe of length 5/3 m is brought near the wire. The temperature of the gas in the pipe is 27°C. When the organ pipe is made to vibrate in second overtone, five beats are heard every second. If the tension in the string is reduced slightly then the number of beats heard per second is reduced to three. Find the linear (mass) density of the wire in gram/metre. Given C_p/C_v of gas in the organ pipe = 1.44 and its mean molar mass is 27.7 gm.

Ans. (1)

$$v = \sqrt{\frac{T}{m}};\ v_w = \frac{v}{\lambda} = \frac{v}{\ell} = \frac{1}{\ell}\sqrt{\frac{T}{m}}$$

$$\ell = 2m;\quad \lambda = \ell$$

For organ pipe:

$$v_{op} = \frac{v}{\lambda} = \frac{3v}{2\ell};\quad \ell_{pipe} = \frac{5\lambda}{4}(2^{nd}\text{ overtone})$$

where, $v_s = \sqrt{\frac{\gamma P}{\rho}} = \sqrt{\frac{\gamma \rho RT}{\rho M}} = \sqrt{\frac{\gamma RT}{M}} = 360\,m/s$

$$v_{op} = \frac{3\times 360}{4} = 270\,Hz$$

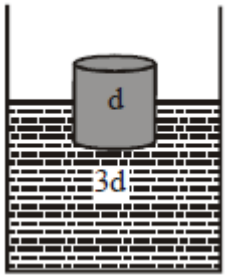
$\ell = 3\lambda/2;\quad \lambda = 2\ell/3$

$v_w - v_{op} = 5 \Rightarrow v_w = 275$ Hz

$$275 = \frac{1}{2}\times\sqrt{\frac{302.5}{m}}$$

$m = 10^{-3}$ kg/m

4. A cylindrical block of height 1 m is in equilibrium in a beaker as shown. Cross-sectional area of cylindrical block is one fourth of cross-sectional area of beaker. Density of cylindrical block is one third of liquid. Determine the time period of small oscillation (in seconds). (Given: $g = \pi^2$ m/s²)



Ans. 1

When block is displaced down by y, then level of liquid in beaker also rises.

$$\frac{A}{4}y = \left(A - \frac{A}{4}\right)y'$$

So, up thrust $= \left(y - \frac{y}{3}\right)\frac{A}{4}3dg$

$$\left(\frac{A}{4}\right)dha = -\left(\frac{4y}{3}\right)\frac{A}{4}(3d)g$$

$$a = -\left(\frac{4g}{h}\right)y$$

$$T = 2\pi\sqrt{\frac{h}{4g}} = \pi\sqrt{\frac{h}{g}}$$

5. An equi-convex lens of focal length 10 cm in air and refractive index ($\mu_g = 1.5$) is placed in a liquid whose refractive index varies with time as $\mu(t) = 1.0 + \frac{1}{10}t$. If the lens was placed in the liquid at t = 0 lens will act as concave lens of focal length 20 cm at t = 5n. Find n.

Ans. (2)

$$\frac{f_e}{f_a} = \frac{({}_a\mu_g - 1)\,{}_a\mu_t}{{}_a\mu_g - {}_a\mu_t}$$

$$\frac{-20}{10} = \frac{(1.5-1)_a\mu_t}{1.5}$$

$3 = 1.5\ {}_a\mu_t \Rightarrow t = 10$ sec.

6. A gas containing hydrogen like ions with atomic no Z, emits photons in transition n + 2 $\rightarrow$ n, where n = Z. These photons fall on a metallic plate and eject electrons having minimum de-Broglie wavelength λ of 5Å. Find the value of 'Z' if the work function of metal is 4.2 eV.

Ans. (2)

$$E = -(13.6\,eV)Z^2\left(\frac{1}{(Z+2)^2}-\frac{1}{Z^2}\right)$$

$$= -13.6\times\left(\frac{Z^2-(Z+2)^2}{(Z+2)^2}\right)=\frac{4(Z+1)\times 13.6}{(Z+2)^2}eV$$

Now energy of electron is $k=\frac{h^2}{2\lambda^2 m}$;

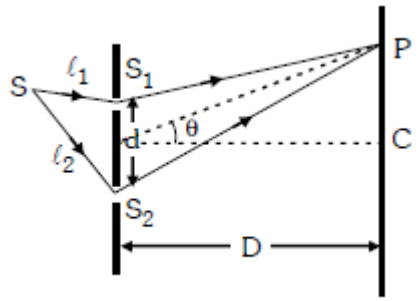
$$\left(\text{We have } \lambda=\frac{h}{\sqrt{2mk}}\right) \quad \text{or} \quad k = 6eV$$

So, $$\frac{4(Z+1)\times 13.6}{(Z+2)^2}=10.2\,eV$$

or $$\frac{(Z+1)}{(Z+2)^2}=\frac{3}{16}\Rightarrow (Z-2)(3Z+2)=0$$

So, the value of Z = 2 (neglecting the negative / fractional value)

7. In a Young's double slit experiment the light source is at distance $\ell_1 = 20\ \mu m$ and $\ell_2 = 40\ \mu m$ from the slits. The light of wavelength $\lambda = 500$ nm incident on slits separated at a distance d = 10 μm. A screen is placed at a distance D = 2m away from the slits as shown in the figure. If 10k maxima appear on the screen, then find the value of k. Round off your answer to the nearest integer, if required.



Ans. (4)

$$\sin\theta=\frac{1}{d}[\Delta x-(\ell_2-\ell_1)]=\frac{1}{d}(n\lambda-(\ell_2-\ell_1)]$$

$$=\frac{1}{10\times10^{-6}}[n\times500\times10^{-9}-20]$$

$$\sin\theta=2\left[\frac{n}{40}-1\right] \quad \text{Hence } \theta=\sin^{-1}\left[2\left(\frac{n}{40}-1\right)\right]$$

$$|\sin\theta|\le 1$$

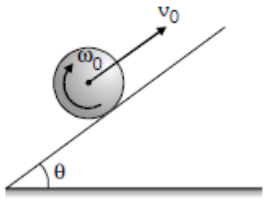
$$-1\le 2\left[\frac{n}{40}-1\right]\le 1$$

$$-20\le[n-40]\le 20$$

$$20\le n\le 60$$

Hence number of maxima = 60 – 20 = 40.

8. A sphere of radius r is projected up an inclined plane for which $\mu = \left(\frac{1}{7}\right)\tan\theta$ with a velocity v_0 and initial angular velocity $\omega_0 (v_0 > r\omega_0)$. The total time of rise is found to be $\frac{kv_0 + 4\omega_0 r}{18g\sin\theta}$. Find k.

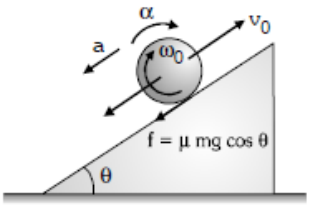


Ans. (7)

Initially $v_0 > r\omega_0$.

Therefore, there is forward slipping. Hence, friction will be downward.

Once, $v = r\omega$, force of friction becomes upward.



Initially

$$a = \frac{\mu mg\cos\theta + mg\sin\theta}{m} = \frac{8}{7}g\sin\theta$$

$\left(\mu = \frac{1}{7}\tan\theta\right)$

$$\alpha = \frac{\mu rmg\cos\theta}{\frac{2}{5}mr^2}$$

$\left(\mu = \frac{1}{7}\tan\theta\right)$

$$= \frac{5}{14}\frac{g\sin\theta}{r}$$

PART-2 : CHEMISTRY

SECTION–I : (Maximum Marks : 40)

- This section contains TEN questions.
- Each question has FOUR options (A), (B), (C) and (D). ONE OR MORE THAN ONE of these four option(s) is (are) correct.
- For each question, darken the bubble(s) corresponding to all the correct option(s) in the ORS
- For each question, marks will be awarded in one of the following categories :

Full Marks : +4 If only the bubble(s) corresponding to all the correct option(s) is (are) darkened.

Zero Marks : 0 If none of the bubbles is darkened.

Negative Marks : –2 In all other cases.

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1. $Cu^{2+}(aq.) + Zn(s) \longrightarrow Cu(s) + Zn^{2+}(aq.)$; $E^0 = 1.1$ V

$Zn^{2+}(aq.) + 2e^- \longrightarrow Zn(s)$; $E^0 = -0.76$V

$Cu^{2+}(aq.) + 4NH_3(aq.) \longrightarrow [Cu(NH_3)_4]^{2+}(aq.)$; $K_{eq} = 10^{20}$

If $\frac{2.303RT}{F} = 0.06$ then the E^0 of following reaction is -

$Cu(s) + 4NH_3(aq.) \longrightarrow [Cu(NH_3)_4]^{2+}(aq.) + 2e^-$

(A) 0.34 V (B) 0.76 V (C) 0.26 V (D) 0.14 V

Ans. (C)

$Cu(s) + 4NH_3(aq.) \longrightarrow [Cu(NH_3)_4]^{2+}(aq.) + 2e^-$ ..... (1)

$Cu^{2+}(aq.) + Zn(s) \longrightarrow Zn^{2+}(aq.) + Cu(s)$ ..... (2)

$Zn^{2+}(aq.) + 2e^- \longrightarrow Zn(s)$ ..... (3)

$Cu^{2+}(aq.) + 4NH_3(aq.) \rightarrow [Cu(NH_3)_4]^{2+}(aq.) + 2e^-$ ..... (4)

(1) = (4) — (2) — (3)

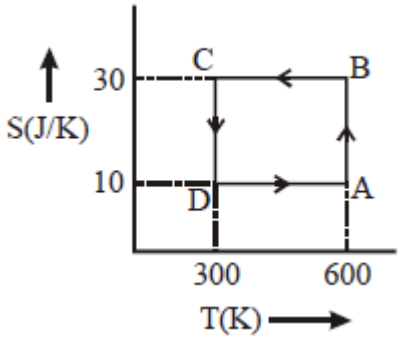
$-2FE^\circ = -RT \ln 10^{20} + 2F(1.1) - 2F(.76)$

$$E^0 = \frac{2.303RT}{2F}\log 10^{20} - 0.34$$

$$\Rightarrow \quad E^0 = \frac{0.06}{2} \times 20 - 0.34$$

$$= \quad 0.6 - 0.34 = 0.26 \text{ V}$$

2. For the reversible cyclic process, the correct information is/are -



(A) The efficiency of the cyclic process = 50 %

(B) Net workdone = – 6kJ

(C) Heat supplied to the system = + 12 kJ

(D) More work is done (magnitude wise) in BC than DA.

Ans. (A,B,C)

$$\eta = 1 - \frac{T_1}{T_2} = 1 - \frac{300}{600} = \frac{1}{2}$$

$$\Delta S_{AB} = 20\ J/K = \frac{q_{reversible}}{600}$$

$$\Rightarrow q_{AB} = 12000\ J$$

$$\therefore W_{AB} = -12000\ J$$

$$q_{BC} = q_{DA} = 0\ ;\ q_{CD} = -6000\ J$$

$$\therefore W_{CD} = +6000\ J$$

$$|W_{BC}| = |W_{DA}|$$

3. Which of the following compound(s) is/are oxidized by O_3.

(A) K_2MnO_4 (B) KI (C) $K_4[Fe(CN)_6]$ (D) BaO_2

Ans. (A,B,C,D)

F_2, atomic O & perxenate anion are powerful O.A. than O_3.

4. Which of the following statement(s) is/are CORRECT ?

(A) At 369K, both α and β forms of sulphur are stable to exist and this temperature is called transition temperature.

(B) BiF_3 is predominately covalent.

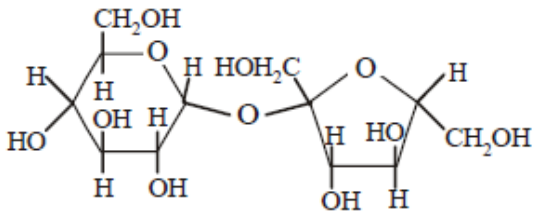
(C) Diborane catches fire spontaneously upon exposure to air.

(D) I_2O_5 is used in the estimation of carbon monoxide.

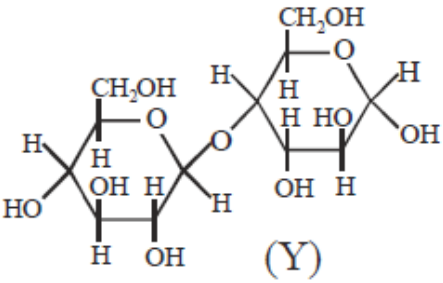
Ans. (A,C,D)

Theory based.

5. The correct statement(s) about the following sugars X and Y is(are)



(X)



(Y)

(A) X is a reducing sugar and Y is a non-reducing sugar.

(B) X is a non-reducing sugar and Y is a reducing sugar.

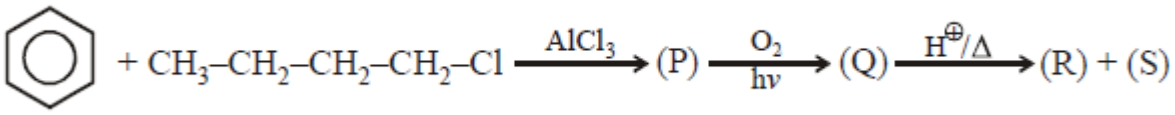
(C) The glucosidic linkages in X and Y are α and β , respectively.

(D) The glucosidic linkages in X and Y are β and α , respectively.

Ans. (B,C)

Hemiacetal linkage reduce fehling solution.

6. Choose the correct option(s) regarding following reaction sequence :



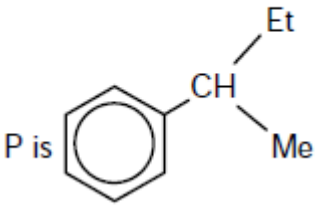
(A) Product (R) may be butanone

(B) Product (S) may be phenol

(C) Product (Q) is cumene hydroperoxide

(D) Product (P) is known as cumene

Ans. (A, B)

P is 

7. Which of the following reactions are correctly represented against their indicated colour ?

(A) $CuSO_4(aq) + NH_3(l)$ [excess] → Deep blue solution.

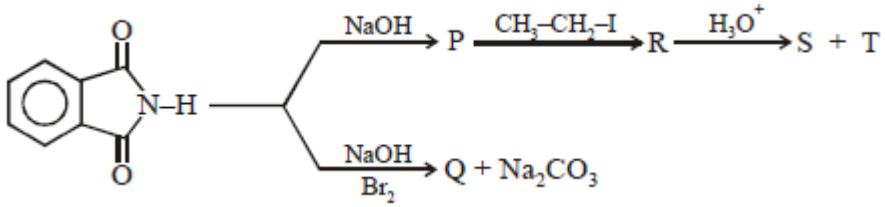
(B) $Na_3[Cu(CN)_4](aq) + H_2S \rightarrow$ Black ppt of Cu_2S.

(C) $BiCl_3(aq) + NaOH(aq) + Na_2[Sn(OH)_4](aq) \rightarrow$ Black ppt of Bi

(D) $Na_2[Fe(CN)_5(NO)](aq) + H_2S \xrightarrow[\text{medium}]{\text{alkaline}}$ Purple / Violet colour solution

Ans. (A, C, D)

Theory based.

8. 

If T can evolve effervescence of CO_2 with aq. $NaHCO_3$, then correct statement(s) is/are :

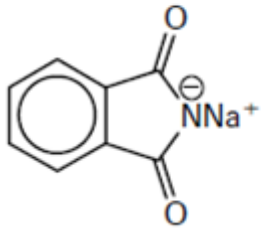
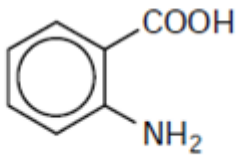
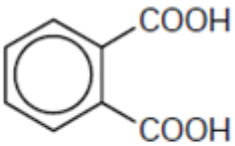
(A) S & Q can be distinguished by dye azo test

(B) T is most acidic among all isomeric benzenoid dicarboxylic acid

(C) Q & S can be distinguished by mustered oil test

(D) P, Q & T all are soluble in aq. $NaHCO_3$

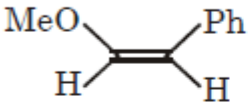
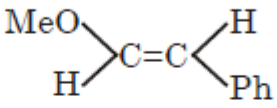
Ans. (A, B, D)

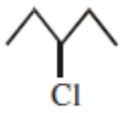
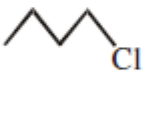
P ⇒  , Q ⇒  , T ⇒ 

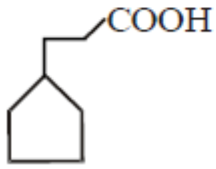
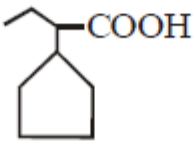
S ⇒ $CH_3CH_2NH_2$

9. Select **INCORRECT** statement (s) from the following :-

(A) $CH_3-\overset{\overset{O}{\|}}{C}-CH_3$ and $CH_2=CH-\overset{\overset{OH}{|}}{CH_2}$ are structural isomers.

(B)  and  are metamers

(C)  and  are positional isomers

(D)  and  are positional isomers

Ans. (B,C,D)

10 Select the **CORRECT** comparison(s) with respect to the indicated properties :

(A) $O^+ > O > O^-$ (electron affinity)

(B) F > O > Se (electronegativity)

(C) $OF_2 > O_2F_2 > H_2O_2$ (electronegativity of central atom)

(D) B > N > F (electron affinity)

Ans. (A,B,C)

$\overset{+2}{O}F_2 > \overset{+1}{O_2}F_2 > \overset{-1}{O_2}H_2$

← Inc. EN of oxygen

Fluorine has greater electron affinity than Be and N.

SECTION–II : (Maximum Marks : 24)

- This section contains EIGHT questions.
- The answer to each question is a SINGLE DIGIT INTEGER ranging from 0 to 9, both inclusive.
- For each question, darken the bubble corresponding to the correct integer in the ORS.
- For each question, marks will be awarded in one of the following categories :

Full Marks : +4 If only the bubble corresponding to the correct answer is darkened.
Zero Marks : 0 In all other cases.

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1. The reversible reaction :

$$Na_2SO_4.10H_2O \rightleftharpoons Na_2SO_4 + 10H_2O; \qquad \Delta H = +20 \text{ Kcal}$$

goes completely to the right at temperature above 32.4°C and remains completely on the left below this temperature. This system has been used in some solar houses for heating at night when temperature drops below 32.4°C. How many m^3 of fuel gas could be saved by the above process if a fixed charge of 161 kg $Na_2SO_4.10H_2O$ is taken ? Assume that the fuel value of the fuel gas is 5000 Kcal/m^3.

Ans. (2)

$$Na_2SO_4.10H_2O \longrightarrow Na_2SO_4 + 10H_2O; \qquad \Delta H = +20 \text{ Kcal}$$

500 mole

Heat atm = $20 \times 500 = 10000$ Kcal

$$\text{Gas saved} = \frac{10000}{5000} = 2m^3$$

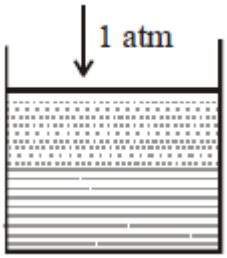
2. Packing fraction of two dimensional unit cell shown in figure is 'A' , then calculate value of $\left(A \times \frac{25}{3}\right)$ is.

[Take $\pi = 3.2$, $\sqrt{2} = 1.4$]

Ans. (8)

$$PF = \frac{1 \times \pi r^2 + 1 \times \pi(\sqrt{2}-1)}{(2r)^2} = 0.96$$

3. If an ideal binary solution of two volatile components A and B are in equilibrium with atmospheric pressure at T kelvin. Find $(P_A^\circ + P_B^\circ)$ in atmosphere, if the mole fraction of more volatile component in solution is 1/2.



Ans. (2)

$P_{Total} = 1$ atm

$$P_{total} = P_A^\circ\left(\frac{1}{2}\right) + P_B^\circ\left(\frac{1}{2}\right)$$

$$P_A^\circ + P_B^\circ = 2$$

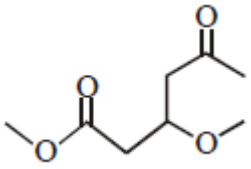
4. Find out the total number of species which can dimerize.

$BeCl_2$, AlF_3, $AlCl_3$, BeH_2, CaH_2, ClO_3, NO_2, CO_2

Ans. (5)

Molecule with incomplete octet of central atom or odd e^- on central atom will dimerize.

5. Total number of MeMgBr consumed when excess of MeMgBr treated with one molecule of the following compound:

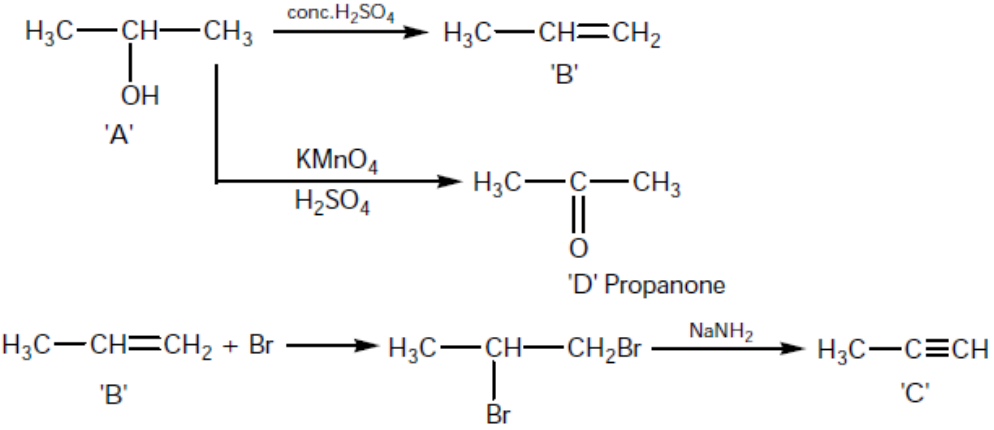


Ans. (3)

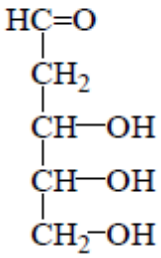
With ester G.R. will react twice & with ketone once.

6. An alcohol (A) on treament with conc. H_2SO_4 gave an alkene (B). The compound (B), on reacting with Br_2 and subsequent dehydrobromination with $NaNH_2$ produced a compound (C). The compound (C) with dil.H_2SO_4 in presence of $HgSO_4$ gave a compound 'D'. The compound D can also obtained by oxidation of A by acidicfied $KMnO_4$ or from dry heating of calcium acetate. Calculate the mass of 0.1 mol of 'C' in gms.

Ans. (4)



7. For the given aldopentose how many stereoisomers are possible in its D-furanose form.



Ans. 4

$2^2 = 4$.

8. Among the following, total number of possible metals present in anode mud, which is obtained by electrolytic refining of copper.

Fe, Ag, Au, Pt, Pb, Zn

Ans. 3

Ag, Au & Pt which are below the copper in the reactivity series.

PART-3 : MATHEMATICS

SECTION–I : (Maximum Marks : 40)

- This section contains TEN questions.
- Each question has FOUR options (A), (B), (C) and (D). ONE OR MORE THAN ONE of these four option(s) is (are) correct.
- For each question, darken the bubble(s) corresponding to all the correct option(s) in the ORS
- For each question, marks will be awarded in one of the following categories :

Full Marks	: +4	If only the bubble(s) corresponding to all the correct option(s) is (are) darkened.
Zero Marks	: 0	If none of the bubbles is darkened.
Negative Marks	: –2	In all other cases.

1. Consider two circles $S_1 \equiv x^2 + y^2 + 8x = 0$, $S_2 \equiv x^2 + y^2 - 2x = 0$. Let ΔPQR be formed by the common tangents to circles S_1 and S_2. Then which of the following holds good ?

(A) Incentre of ΔPQR is (1, 0)

(B) Equation of Radical axis of S_1 & S_2 is $y = 0$

(C) Product of slopes of direct common tangents is $\frac{-9}{16}$

(D) If tranverse common tangent intersect direct common tangents at point Q and R then QR equals 1

Ans. (A,C)

S_1 and S_2 touch each other externally

Equation of direct common tangents

$y = m\left(x - \frac{8}{3}\right)$ as point P is point of intersection of direct common tangents $\left(\frac{8}{3}, 0\right)$

condition of tangency $\frac{\left(m - \frac{8m}{3}\right)}{\sqrt{1+m^2}} = 1$

$\Rightarrow m^2 = \frac{9}{16}$

2. Which of the following limit(s) vanishes ?

(A) $\lim\limits_{x \to 0^+}\left(x^{x^n} - x^x\right)$

(B) $\lim\limits_{x \to 0^+} x^2 \ln\sqrt{\frac{1}{x}}$

(C) $\lim\limits_{x \to 0^+} x^{\ln(x+1)}$

(D) $\lim\limits_{x \to 0^+} \frac{10^x - 2^x - 5^x + 1^x}{x + \tan x}$

Ans. (B,D)

(A) $\lim\limits_{x\to 0^+} x^x = 1$

(B) $-\frac{1}{2}\lim\limits_{x\to 0^+} x^2 \ln x = -\frac{1}{2}\lim\limits_{x\to 0^+}\frac{1/x}{-2/x^3} = 0$

(C) $\ell = \lim x^{\ell n(x+1)}$

$\Rightarrow \ell n\, \ell = \lim\limits_{x\to 0^+} \ell n(x+1)\ell n\, x = 0$

$\Rightarrow \ell = 1$

(D) $\lim\limits_{x\to 0^+}\frac{\left(\frac{5^x-1}{x}\right)\left(\frac{2^x-1}{x}\right)x}{1+\frac{\tan x}{x}} = 0$

3. If $S_n = \frac{3}{4}+\frac{5}{36}+\frac{7}{144}+\frac{9}{400}+.....n$ terms and $P_n = \frac{1}{1-S_n}$, then

(A) Unit place digit of P_{40} is 1

(B) Ten's place digit of P_{40} is 8

(C) Ten's place digit of P_{50} is 0

(D) Ten's place digit of P_{50} is 6

Ans. (A,B,C)

$$S_n = \sum_{n=1}^{n} \frac{2n+1}{n^2(n+1)^2} = 1-\frac{1}{(n+1)^2}$$

$P_n = (n+1)^2$

$P_{40} = (41)^2 = 1681$

$P_{50} = (51)^2 = 2601$

4. Let $f : R \to R$ be a continuous function such that $f(f(f(x))) = x\ \forall\ x \in R$, then CORRECT option(s) is(are)

(A) $y = f(x)$ is a bijective function

(B) $y = f(x)$ is a strictly increasing function

(C) $y = f(x)$ is strictly decreasing function

(D) $y = f(x)$ intersects $y = x^2$ at exactly two distinct points.

Ans. (A,B,D)

Let $f(x_1) = f(x_2)$

$\Rightarrow f(f(f(x_1))) = f(f(f(x_2))) \Rightarrow x_1 = x_2$

So, f is one-one

as $f(x)$ is continuous & one-one then it will be either strictly increasing or strictly decreasing.

If $x_1 < x_2$ and f is decreasing

then $f(x_1) > f(x_2)$

$\Rightarrow f(f(x_1)) < f(f(x_2))$

$\Rightarrow f(f(f(x_1))) > f(f(f(x_2)))$

$\Rightarrow x_1 > x_2$ (which contradicts)

So, f is increasing

Now, say $f(x) \geq x$ $\Rightarrow f(f(x)) \geq f(x) \geq x$

$\Rightarrow f(f(f(x))) \geq f(x)$

$\Rightarrow x \geq f(x)$

So, $f(x) = x$

Similarly if $f(x) \leq x$ $\Rightarrow f(x) \geq x$

$\Rightarrow f(x) = x$

5. The bisector of $\angle B$ of an isosceles ΔABC with AB = AC meets the circumcircle of ΔABC at P and meets AC at R and BC produced meet AP at Q. If BC , CA, AB are a , b , c respectively then which of the following is(are) CORRECT ?

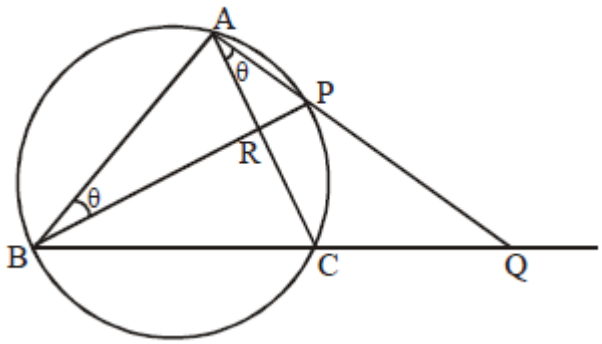
(A) ΔCQA is isosceles

(B) $BR = \dfrac{2ac}{a+c}\cos B/2$

(C) AQ = 2b cos B/2

(D) PQ = BC

Ans. (A,B,C)



$$\frac{AB}{SinB}=\frac{b}{\sin B/2}$$

$\Rightarrow AQ = 2\ b\cos B/2$

and $PQ.\ AQ = QC\ .QB$

$$\Rightarrow PQ=\frac{QC}{QA}.QB$$

In Δ AQC

$$\frac{QC}{\sin B/2}=\frac{QA}{\sin B}\Rightarrow\frac{QC}{QA}=\frac{1}{2\cos B/2}$$

In Δ ABQ $\qquad BQ=\dfrac{c\sin(A+B/2)}{\sin B/2}$

$$PQ=\frac{c\sin(A+B/2)}{\sin B}$$

6. Let $f : [-1, 1] \to [-1, 1]$, $f(x) = \sin(2\sin^{-1}x)$ and let $g(x) = \dfrac{f(x)}{\sqrt{1-x^2}}$, then -

(A) f(x) is one-one function

(B) f(x) is odd function

(C) g(x) is continuous in (–1,1)

(D) f(x) is continuous for all $x \in [-1, 1]$

Ans. (B,C,D)

(A) Let $\sin^{-1}x = \theta \Rightarrow \sin\theta = x$

$f(x) = \sin2\theta = 2\sin\theta\cos\theta = 2x\sqrt{1-x^2}$

$f(0) = 0 \Rightarrow$ at x = 0, 1, –1 (many-one)

(B) $f(x) + f(-x) = 0 \Rightarrow$ odd function

(C) g(x) = 2x, continuous in given interval

(D) f(x) is continuous function.

7. The value of $\lim\limits_{n\to\infty}\left[\lim\limits_{x\to0}\left(\left(1+\sin\frac{x}{7}\right)\left(1+\sin\frac{x}{7^2}\right).....\left(1+\sin\frac{x}{7^n}\right)\right)^{\frac{1}{\sin x\left[\prod\limits_{r=1}^{n}\left(1+\frac{x}{7^r}\right)\right]}}\right]$ is $e^{\frac{p}{q}}$ (where p & q are coprime), then which of the following is/are correct -

(A) p + q = 7 (B) p + q = 5 (C) q – p = 5 (D) q – p = 7

Ans. (A,C)

$$\lim_{n\to\infty}\lim_{x\to 0} e^{\frac{1}{\left(1+\frac{x}{7}\right)\left(1+\frac{x}{7^2}\right)\cdots\left(1+\frac{x}{7^n}\right)\sin x}\left[\left(1+\sin\frac{x}{7}\right)\left(1+\sin\frac{x}{7^2}\right)\cdots\left(1+\sin\frac{x}{7^n}\right)-1\right]}$$

$$= \lim_{x\to 0} e^{\frac{\left[1+\left(\sin\frac{x}{7}+\sin\frac{x}{7^2}+\ldots\right)+\left(\sin\frac{x}{7}\sin\frac{x}{7^2}+\ldots\right)\ldots-1\right]}{\left[1+\left(\frac{x}{7}+\frac{x}{7^2}+\ldots\right)+\left(\frac{x}{7}\cdot\frac{x}{7^2}+\ldots\right)\right]x}}$$

$$= e^{\frac{\frac{1}{7}+\frac{1}{7^2}+\ldots}{1}} = e^{\frac{\frac{1}{7}}{1-\frac{1}{7}}} = e^{\frac{1}{6}}$$

8. If $\sin^{-1}\left(\frac{\sqrt{x}}{2}\right)+\sin^{-1}\left(\sqrt{1-\frac{x}{4}}\right)+\tan^{-1}(y)=\frac{2\pi}{3}$, then

(A) Maximum value of x^2+y^2 is $\frac{49}{3}$

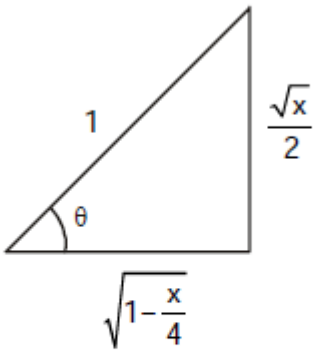
(B) Maximum value of x^2+y^2 is 4

(C) Minimum value of x^2+y^2 is $\frac{1}{3}$

(D) Minimum value of x^2+y^2 is 3

Ans. (A,C)

$$\sin^{-1}\left(\sqrt{1-\frac{x}{4}}\right)\Rightarrow \sin\theta=\sqrt{1-\frac{x}{4}}$$



$$\frac{\pi}{2}+\tan^{-1}y=\frac{2\pi}{3}\Rightarrow y=\frac{1}{\sqrt{3}} \quad\Rightarrow\quad 0\le x\le 4$$

Max. value of x^2+y^2 is $\frac{49}{3}$

Min value of x^2+y^2 is $\frac{1}{3}$

9. If the line $2y + 4x + 1 = 0$ intersects the parabola $y^2 = -x$ at A and B and O is the origin, then

(A) measure of $\angle AOB$ is $\frac{\pi}{2}+\tan^{-1}\left(\frac{3}{2\sqrt{5}}\right)$

(B) measure of $\angle AOB$ is $\tan^{-1}\frac{2\sqrt{5}}{3}$

(C) Circle described on AB as a diameter touches the line $4x-1=0$

(D) Tangents drawn at A and B to the parabola are not perpendicular

Ans. (A,C)

Clearly AB is a focla chord.

Circle described on AB on diameter touches the directrix.

Tangents at A and B must be perpendicular

Homogeneous $y^2 = -x$ from $2y + 4x + 1 = 0$

$y^2 = -x(-2y - 4x)$

$\Rightarrow \quad 4x^2 + 2xy - y^2 = 0$

$$\tan\theta=\frac{2\sqrt{1+4}}{4-1}=\frac{2\sqrt{5}}{3}$$

$\because$ $\angle AOB$ is obtuse.

$$\therefore \quad \angle AOB=\pi-\tan^{-1}\left(\frac{2\sqrt{5}}{3}\right)=\frac{\pi}{2}+\cot^{-1}\left(\frac{2\sqrt{5}}{3}\right)$$

10. If z_1 and z_2 are two complex number such that $z_1 + z_2 = z_1^2 + z_2^2 = \frac{2i}{\sqrt{3}}$ and $|Re(z_1)| = \sin\left(\frac{\pi}{k}\right)$ where $i=\sqrt{-1}$, then value of k is :

(A) 4 (B) 2.5 (C) 3.5 (D) None of these

Ans. (A)

$$z_1+z_2=z_1^{\,2}+z_2^{\,2}=\frac{2i}{\sqrt{3}} \bullet\bullet \quad .....(1)$$

$$\Rightarrow \quad (z_1+z_2)^2-2z_1z_2=\frac{2i}{\sqrt{3}}$$

$$\Rightarrow \quad z_1z_2=-\frac{2}{3}-\frac{i}{\sqrt{3}} \quad .....(2)$$

Let $z_1 = x+iy$ and $z_2 = -x+i\beta$ $\left\{\because z_1+z_2=\frac{2i}{\sqrt{3}}\right\}$

From eq. (1) $(-x^2-y\beta)=-\frac{2}{3},\ x(\beta-y)=-\frac{1}{\sqrt{3}}$

$\beta+y=\frac{2}{\sqrt{3}} \Rightarrow \beta=\frac{1}{\sqrt{3}}-\frac{1}{2\sqrt{3}x},\ y=\frac{1}{\sqrt{3}}+\frac{1}{2\sqrt{3}x}$

Putting the value of β and y

$12x^4 - 4x^2 - 1 = 0 \Rightarrow x \pm \frac{1}{\sqrt{2}}$

$\left|R_e(z_1)=\sin\left(\frac{\pi}{4}\right)\right| \Rightarrow k=4$

SECTION–II : (Maximum Marks : 24)

- This section contains EIGHT questions.
- The answer to each question is a SINGLE DIGIT INTEGER ranging from 0 to 9, both inclusive.
- For each question, darken the bubble corresponding to the correct integer in the ORS.
- For each question, marks will be awarded in one of the following categories :

Full Marks : +4 If only the bubble corresponding to the correct answer is darkened.
Zero Marks : 0 In all other cases.

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1. A vector $\vec{v}$ satisfying the equations $\vec{v}\times\left(\hat{a}\times\vec{b}\right)=\vec{b}+2\hat{a}$ and $\hat{a}\times\vec{v}+\hat{a}+\vec{b}=(\hat{a}\times\vec{b})\times\hat{a}$. If $\hat{a}$ is a unit vector and $\vec{b}=\hat{i}-\hat{j}+\hat{k}$ which is NOT collinear with $\hat{a}$, then $|\vec{v}|$ is

Ans. 1

$(\vec{v}.\vec{b})\hat{a}-(\vec{v}.\hat{a})\vec{b}=\vec{b}+2\hat{a}$

$\Rightarrow \quad \vec{v}.\vec{b}=2 \ \& \ \vec{v}.\hat{a}=-1$

Now $\hat{a}\times(\hat{a}\times\vec{v})+\hat{a}\times\vec{b}=\hat{a}\times\left(\vec{b}-(\hat{a}.\vec{b})\hat{a}\right)$

$\Rightarrow -\hat{a}-\vec{v}+\hat{a}\times\vec{b}=\hat{a}\times\vec{b}$

$\Rightarrow \quad \vec{v}=-\hat{a}$

2. Let matrix $P=\begin{bmatrix}\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}}\\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}}\end{bmatrix}$ and Q is another matrix such that $Q^TP = P^T$ and $\lambda Q^T = 2P^T - \sqrt{2}I$, then value of determinant of matrix $\text{adj}(I + Q^T).\text{adj}(\lambda P)$ is

(where I is unit matrix of order 2, adj(X) and X^T denote adjoint and transpose of matrix X respectively)

Ans. 4

$Q^TP = P^T$

$P^TQ = P$

$PP^TQ = P^2$

$Q = P^2$

$$\Rightarrow \quad Q=\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \Rightarrow \quad Q^T=\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

$$\therefore \quad \begin{bmatrix} 0 & -\lambda \\ \lambda & 0 \end{bmatrix}=\begin{bmatrix} 0 & -\sqrt{2} \\ \sqrt{2} & 0 \end{bmatrix}$$

$$\therefore \quad \lambda = \sqrt{2}$$

Now $\because \quad \sqrt{2}(I+Q^T) = 2P^T$

$\Rightarrow$ adj.$(I + Q^T)$.adj(λP)

$= \text{adj}(\sqrt{2}P^T).\text{adj}(\sqrt{2}P)$

$= 2\text{adj}(P^TP) = 2I$

3. The terms of an arithmetic sequence add to 465. The first term of sequence is increased by 1, the second term is increased by 3 and so on, in general, the k^{th} term is increased by the k^{th} odd positive integer. The terms of new sequence add to 690. If t_1 and t_n denotes first and last term of the original sequence respectively, then $\left(\frac{t_1+t_n}{31}\right)$ is

Ans. 2

Given $t_1 + t_2 + \ldots\ldots + t_n = 465 = \frac{n}{2}(t_1 + t_n)$ ...(i)

and $(t_1 + 1) + (t_2 + 3) + \ldots + (t_n + (2n-1)) = 690$ ...(ii)

on (ii) – (i) $\Rightarrow \sum_{n=1}^{n}(2n-1) = 225$

$\Rightarrow n^2 = 225 \quad \Rightarrow \quad n = 15$

Put value of n is equation (i), $(t_1 + t_n).\frac{15}{2} = 465$

$$\Rightarrow \quad \frac{t_1+t_n}{31} = 2$$

4. Let f(x) be an even function satisfying $f(x) + f(x+2) = f(x+1)\ \forall\ x \in R$, where $f(-13) = 4$, then $f(-5) + f(1)$ is equal to

Ans. 8

$f(x) + f(x+2) = f(x+1)$ ....(i)

$\Rightarrow$ $f(x+1) + f(x+3) = f(x+2)$ ....(ii)

Adding (i) and (ii) we get

$f(x) + f(x+3) = 0$

$\Rightarrow$ $f(x) = -f(x+3)$

$x \to x+3$

$\Rightarrow$ $f(x+3) = -f(x+6)$

$\Rightarrow$ $f(x) = f(x+6)$

$\Rightarrow$ Period of f(x) is 6

Now $f(-5) = f(5) = f(5-18) = f(-13) = 4$

and $f(1) = f(-1) = f(-1-12) = f(-13) = 4$

$\Rightarrow$ $f(-5) + f(1) = 8$

5. Let $\vec{a} = 2\hat{i} + \hat{j} + \hat{k}$, $\vec{b} = 3\hat{i} + \hat{j} - 5\hat{k}$ and if point of intersection of the lines $\vec{r} \times \vec{a} = \vec{b} \times \vec{a}$ and $\vec{r} \times \vec{b} = \vec{a} \times \vec{b}$ is $\vec{c}$, then $[\vec{a}\ \vec{b}\ \vec{c}]$ equals

Ans. (0.00)

$\vec{c} = \vec{a} + \vec{b} \Rightarrow \vec{a} + \vec{b} - \vec{c} = 0$

$\Rightarrow [\vec{a}\ \vec{b}\ \vec{c}] = 0$

6. Consider a function $f(x)$ which satisfies $f'(x) - \tan x . f(x) = \int_{-\pi/6}^{\pi/6} f(x)dx \ \forall\ x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ and $f(0) = 1$.

The value of $\left(f\left(\frac{\pi}{3}\right) - \sqrt{3}\ell n3\right)$ is

Ans. 2

$f'(x) - \tan x f(x) = k$

I.F. $= \cos x$

$\Rightarrow$ $f(x).\cos x = \int k \cos x dx$

$\Rightarrow f(x) = k\tan x + c\sec x$

$f(0) = 1 \Rightarrow C = 1$

$\Rightarrow$ $f(x) = k\tan x + \sec x$

Now $k = \int_{-\pi/6}^{\pi/6} f(x)dx = 2\int_{0}^{\pi/6} \sec x dx = \ell n3$

7. Volume of parallelopiped formed by planes whose equations are given by

$|6x - 2y + 3z| = 7$, $|2x - 3y - 6z| = 14$ and $|6x + 12y - 4z| = 7$, is (in cubic units)

Ans. 8

Parallelopiped formed by planes is a cuboid.

$\therefore$ Volume $= d_1 d_2 d_3 = 2.4.1$, where d_1, d_2 & d_3 are distances between parallel planes.

8. An ellipse with focii (1, 4) and (α, β) touches x-axis at (5, 0). Then value of (α –β) is

Ans. 5

(1, 4), (α, β) and reflection of (α,β) on x-axis and collinear

$$\begin{vmatrix} 1 & 4 & 1 \\ 5 & 0 & 1 \\ \alpha & -\beta & 1 \end{vmatrix} = 0 \Rightarrow \alpha - \beta = 5$$

PART-1 : PHYSICS

SECTION–I : (Maximum Marks : 32)

- This section contains **EIGHT** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is (are) correct.
- For each question, darken the bubble(s) corresponding to all the correct option(s) in the ORS
- For each question, marks will be awarded in one of the following categories :

Full Marks : +4 If only the bubble(s) corresponding to all the correct option(s) is (are) darkened.

Zero Marks : 0 If none of the bubbles is darkened.

Negative Marks : –2 In all other cases.

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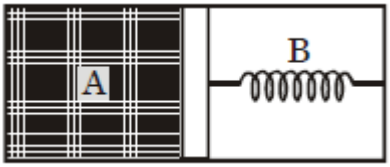
1. A thermally insulated chamber of volume $2V_0$ is divided by a frictionless and insulated piston of cross section area S into two equal parts A and B. Part A has an ideal diatomic gas at pressure P_0 and temperature T_0 and in part B is vacuum. A massless spring of force constant K is connected with the piston and the wall of the container as shown. Initially the spring is unstretched. The ideal gas in chamber A is allowed to expand slowly with the help of an external agent. After acheiving equilibrium state, external agent is removed. Let in equilibrium the spring be compressed by a length x_0. then



(A) Final pressure of the gas is Kx_0/S

(B) Work done by the gas is $\dfrac{5\left[P_0V_0 - kx_0^2 - \dfrac{kx_0V_0}{S}\right]}{2}$

(C) Magnitude of change in internal energy of the gas is $1/2\ kx_0^2$

(D) Work done by an external agent is $\dfrac{5V_0\left[\dfrac{kx_0}{S} - P_0\right]}{2} + 3kx_0^2$

Ans. (A,B,D)

$kx_0 = P \times S$

$P = \dfrac{kx_0}{S}$

Process is an adiabatic

$$W_{gas} = \frac{P_2V_2 - P_1V_1}{1-\gamma} = \frac{\dfrac{kx_0}{S}[V_0 + 5x_0] - P_0V_0}{1 - \dfrac{7}{5}}$$

$$= \frac{5\left[P_0V_0 - kx_0^2 - \frac{kx_0V_0}{S}\right]}{2}$$

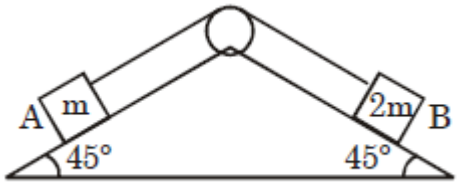
$$\Delta u = -W_{gas}$$

$$W_{Ext} + W_{gas} + W_{spring} = 0$$

$$W_{Ext} + \frac{5\left[P_0V_0 - kx_0^2 - \frac{kx_0V_0}{S}\right]}{2} + \frac{1}{2}kx^2 = 0$$

$$W_{Ext} = \frac{5V_0\left[\frac{kx_0}{S} - P_0\right]}{2} + 3kx_0^2$$

2. Two blocks A and B of mass m and 2m are placed on a fixed triangular wedge by massless inextensible string as shown. The pulley is massless and frictionless. The coefficient of friction between block A and the wedge is 2/3 and between B and the wedge is 1/3.
(Take : m = 1kg, g = 10 m/s^2)



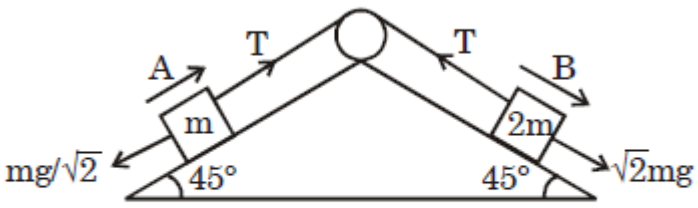
(A) Friction force between block A and wedge can be $\frac{10}{3\sqrt{2}}$ units

(B) Friction force between block B and wedge can be $\frac{10\sqrt{2}}{3}$ units

(C) Tension in the string can be $\frac{20\sqrt{2}}{3}$ units

(D) Maximum friction force between block B and wedge is $\frac{5}{\sqrt{2}}$ units

Ans. (A,B,C)



Maximum friction force on

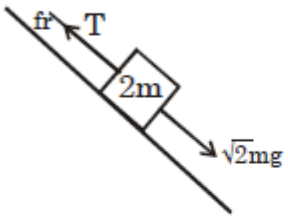
$$A = \frac{2}{3} \times \frac{mg}{\sqrt{2}} = \frac{\sqrt{2}mg}{3}$$

Maximum friction force on

$$B = \frac{1}{3} \times \sqrt{2}mg = \frac{\sqrt{2}mg}{3}$$

for checking equilibrium condition

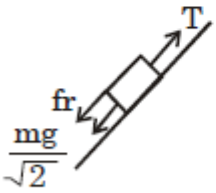
motion of B



$$\sqrt{2}mg - \frac{\sqrt{2}mg}{3} = T$$

$$\frac{2}{3}\sqrt{2}mg = T$$

Motion of A



$$fr = T - \frac{mg}{\sqrt{2}}$$

$$fr = \frac{2}{3}\sqrt{2}mg - \frac{mg}{\sqrt{2}}$$

$$fr = \frac{mg}{3\sqrt{2}} < \frac{\sqrt{2}mg}{3}$$

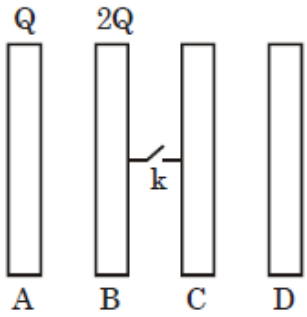
Both are at equilibrium

$$T = \frac{2}{3}\sqrt{2}mg$$

$$\text{friction force on A} = \frac{mg}{3\sqrt{2}}$$

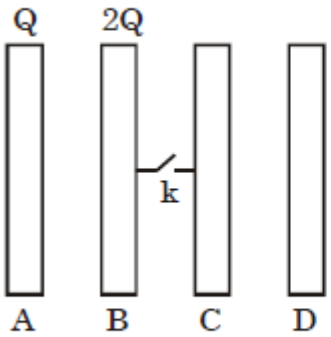
$$\text{friction force on B} = \frac{\sqrt{2}mg}{3}$$

3. Initially two plates (A and B) are given charges as shown. Find the final charges on the plates after the key k is closed. All plates are conducting, parallel and of infinite length and breadth.

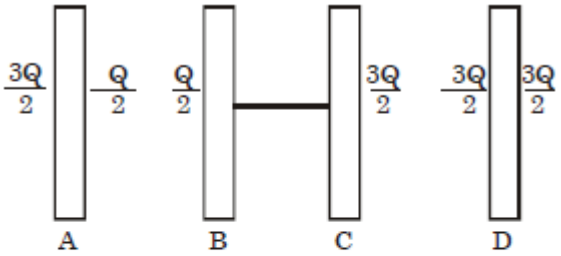


(A) Final charges on plate A is Q
(B) Final charge on plate B is 0
(C) Final charge on plate C is 3Q/2
(D) Final charge on plate D is 0

Ans. (A,C,D)



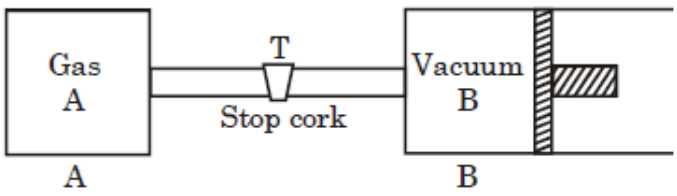
After switch on final charge distribution is like this



final charge on C is $\frac{3Q}{2}$

final charge on B is $\frac{Q}{2}$

4. An ideal diatomic gas is confined in a cylinder A of volume V_0. Then, cylinder is connected to another cylinder B with the help of tube of negligible volume. The cylinder B is fitted with movable piston, which can be adjusted from outside. Initially, piston is adjusted so that the volume of B is same as volume of A, i.e., V_0. B is evacuated and stop cork is opened so that gas expands and occupies the volume $2V_0$.



(A) During this free expansion, the internal energy of this system remains constant

(B) With open stop cork, now the piston is slowly moved to compress the gas back to cylinder A at constant temperature T. Then for n number of moles of gas, work done on the gas is nRT ℓn2

(C) The heat absorbed by the gas in the above process is –nRT ℓn2

(D) With open stop cork, now the piston is slowly moved to compress the gas back to cylinder A at constant temperature T. Then for n moles gas, work done on the gas is 1/2 (nRT ℓn2)

Ans. (A,B,C)

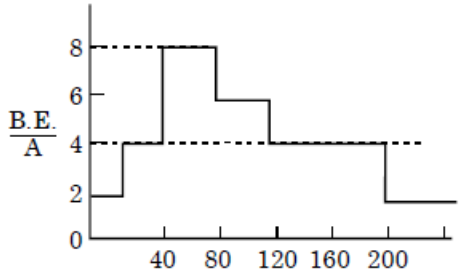
Temperature remain const. so internal energy remain same.

$$\text{W.D. by gas} = nRT\ell n\left(\frac{v_2}{v_1}\right) = nRT\ell n2$$

$\Delta Q = \Delta U + \Delta W$

$\Delta Q = 0 + nRT\ell n = nRT\ell n2$

5. The following histogram represents the binding energy per particle (B.E./A) in MeV as a function of the mass number A of a nucleus. A nucleus with mass number A = 180 fissions into two nuclei of equal masses. In the process



(A) 180 MeV of energy is released

(B) 180 MeV of energy is absorbed

(C) 360 MeV of energy is released

(D) 360 MeV of energy is absorbed

Ans. (C)

180° 90

$X \longrightarrow 2\,Y$

so energy released

$Q = 2 \times [90 \times 6] - [180 \times 4] = 360$ MeV

6. The orbiting period of Moon is T_M, orbiting at a height of $(r_M - R_E)$ above the Earth of radius R_E. A geostationary satellite is a satellite that always stays above the same location on the Equator. It stays at a height of $(r_S - R_E)$ above the Earth with an orbiting period of T_S. What is the ratio r_M/r_S in terms of T_M and T_S ?

(A) $(T_M/T_S)^{1/2}$ (B) T_M/T_S (C) T_S/T_M (D) $(T_M/T_S)^{2/3}$

Ans. (D)

Centripetal force = Gravitational force :

$$\frac{GMm}{r^2} = m\frac{4\pi^2}{T^2}r \Rightarrow r = \left(\frac{GMT^2}{4\pi^2}\right)^{1/3}$$

For the moon, we have

$$r_M = \left(\frac{GMT_M^2}{4\pi^2}\right)^{1/3}$$

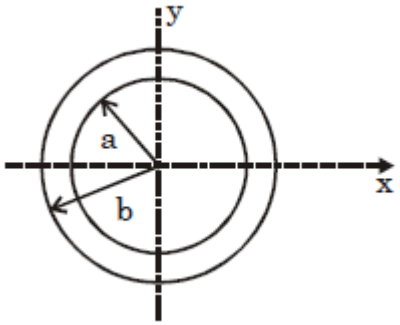
For the satellite, we have

$$r_S = \left(\frac{GMT_S^2}{4\pi^2}\right)^{1/3}$$

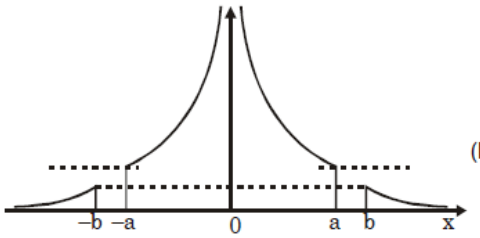
Therefore, we have

$$\frac{r_M}{r_S} = \left(\frac{GMT_M^2}{4\pi^2}\right)^{1/3} \Big/ \left(\frac{GMT_S^2}{4\pi^2}\right)^{1/3} = \left(\frac{T_M}{T_S}\right)^{2/3}$$

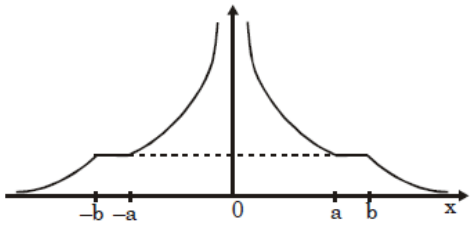
7. A positive point charge q is located inside a neutral hollow spherical conducting shell. The shell has inner radius a and outer radius b ; b – a is not negligible. The shell is centred on the origin. Which of the following is correct graph of electric field vs radial distance x or electric potential vs radial distance x. The point charge can be located anywhere inside shell on x-axis.



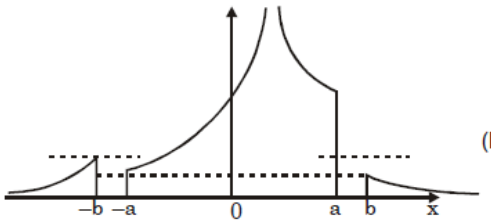
(A)



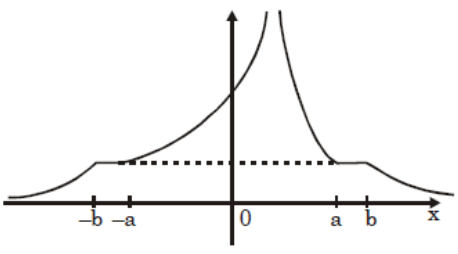
(B)



(C)



(D)



Ans. (A, B, D)

(A) Option is electric field v//s x graph and (B) option for potential when point charge is kept at centre.

(D) Option is for potential v/s x when point charge is not at centre.

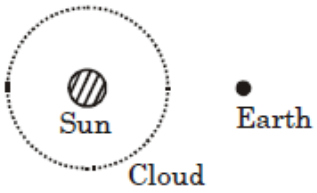
(C) Option is incorrect become magnitude of electric field at x = + b & x = –b must be same.

8. Assuming the Sun is a black body at a temperature of T_S and that the Earth is also a black body. Diameter of Sun subtends a very small angle θ radians at the centre of Earth. (Neglect sources of heat from the Earth itself). If an opaque cloud of dust particles in the form of a spherical shell with a radius equal to half the radius of the Earth's orbit were interposed between Earth and Sun centred on the Sun. Mark the CORRECT statement(s) : [Assume cloud of dust and earth to behave as black body]

(A) Equilibrium temperature of cloud (T_D) is $T_S\sqrt{\theta}$ (B) Equilibrium temperature of cloud (T_D) is $T_S\sqrt{\frac{\theta}{2}}$

(C) Equilibrium temperature of Earth (T_E) is $\frac{T_D}{2}$ (D) Equilibrium temperature of Earth (T_E) is $\frac{T_D}{4}$

Ans. (A,C)



R_S = Radius of sun

R = Radius of orbit of earth

Given,

$$\tan\theta = \frac{2R_S}{R} \Rightarrow \theta = \frac{2R_S}{R}$$ [θ is very small]

For equilibrium temperature of cloud,

$$\sigma 4\pi R_S^2 T_S^4 = \sigma 4\pi \left(\frac{R}{2}\right)^2 T_D^4 \Rightarrow T_D = T_S\sqrt{\theta}$$

For equilibrium temperature of earth,

$$\frac{\sigma 4\pi \frac{R^2}{4} T_D^4}{4\pi R^4} \pi R^2 = \sigma \cdot 4\pi R^2 T_E^4$$

$$\therefore T_E^4 = \frac{T_D^4}{16} \Rightarrow T_E = \frac{T_D}{2}$$

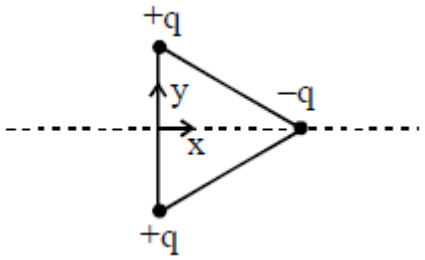
SECTION–II : (Maximum Marks : 24)

- This section contains **TWO** paragraphs.
- Based on each paragraph, there are **THREE** questions
- Each question has **FOUR** options (A), (B), (C) and (D) **ONE OR MORE THAN ONE** of these four option(s) is(are) correct.
- For each question, darken the bubble(s) corresponding to all the correct option(s) in the ORS.
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks	: +4	If only the bubble(s) corresponding to all the correct option(s) is (are) darkened.
Zero Marks	: 0	If none of the bubbles is darkened.
Negative Marks	: –2	In all other cases.

Paragraph for Questions 9 to 11

Two positive charges and a negative charge of same magnitude is fixed on vertices of an equilateral triangle. For this set up electric field is investigated on x-axis. Answer following questions for given case.



9. Select ***CORRECT*** statement(s) for electric field and electric potential at origin.

(A) $E = 0$ (B) $V > 0$ (C) $E \neq 0$ (D) $V < 0$

Ans. (B, C)

Field at origin is only due to −q and potential is more due to positive charges.

10. Select ***CORRECT*** statement(s) for direction of electric field on x-axis.

(A) Field at origin is towards right.

(B) Field just right of –q charge is left.

(C) Field far away on positive x-axis is towards right.

(D) Field far left on negative x-axis is towards left.

Ans. (A, B, C, D)

(A) At origin electric field is due to $-q$ charges.

(B) Just right of $-q$ charge the fieled is mainly due to $-q$ charge

(C,D) Far away from systen, the system is like a $+q$ charge ($+2q - q = +q$) so field will be away from charge.

11. Number of points of equilibrium on x-axis is

(A) 1 (B) 2 (C) 3 (D) 4

Ans. (B)

By continuity argument the points of zero electric field must be 2.

Paragraph for Questions 12 to 14

Two identical carts A and B each with mass m are connected via a spring with spring constant k. Two additional springs, identical to the first, connect the carts to two fixed points. The carts are free to oscillate under the effect of the springs in one dimensional frictionless motion. Initially all springs are at relax length.

12 If we remove the middle spring and cart A and B perform SHM (without colliding with each other) then ratio of their time period of A and B is

(A) 1 (B) $\frac{1}{2}$ (C) 0 (D) ∞

Ans. (A)

After removing middle spring both carts acts two spring block systems.

$$\frac{T_A}{T_B} = \frac{2\pi\sqrt{\frac{m}{k}}}{2\pi\sqrt{\frac{m}{k}}} = 1$$

13. Cart A and B (with all three springs) under suitable condition can oscillates in same phase. Lets say they are oscillating in same phase then for this motion select ***CORRECT*** option(s)

(A) Length of middle spring remains fixed.

(B) Length of middle spring varies with time .

(C) Angular frequency of oscillation of carts is $\sqrt{\frac{k}{m}}$.

(D) Angular frequency of oscillation of carts is $\sqrt{\frac{3k}{m}}$.

Ans. (A,C)

If both carts oscillates in same phase the middle spring length remains fiexd and effectively both blocks perform SHM with one spring so angular frequency $\omega = \sqrt{\frac{k}{m}}$.

14. Cart A and B (with all three springs) under suitable condition can oscillates out of phase. Lets say they are oscillating out of phase then for this motion select ***CORRECT*** option(s).

(A) Length of middle spring remains fixed.

(B) Length of middle spring varies with time.

(C) Angular frequency of oscillation of carts is $\sqrt{\frac{k}{m}}$.

(D) Angular frequency of oscillatlion of carts is $\sqrt{\frac{3k}{m}}$

Ans. (B, D)

For the out of phase situation, the separation between the objects is not constant, but there exists a point on the connecting spring that does not move. As such, either object can be though of moving under the influence of two springs in parallel, one with length L, the there with length L/2. That is equivalent to a spring with constant k in parallel with a spring of constant 2k. The net spring constant is then k + 2k = 3k.

As such, the frequency is given by

$$\omega = \sqrt{3k/m}$$

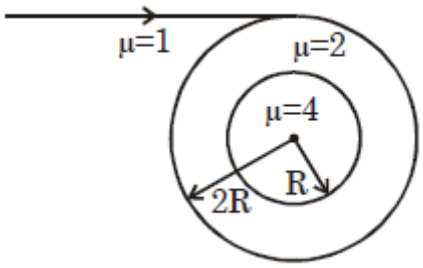
SECTION–III : (Maximum Marks : **8**)

- This section contains **FOUR** questions.
- The answer to each question is a SINGLE DIGIT INTEGER ranging from 0 to 9, both inclusive.
- For each question, darken the bubble corresponding to the correct integer in the ORS.
- For each question, marks will be awarded in one of the following categories :

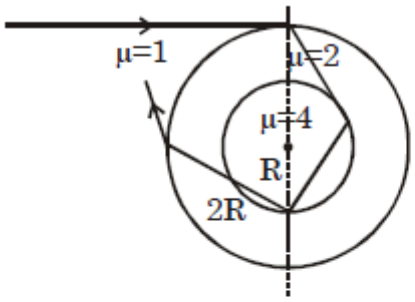
Full Marks : **+ 2** If only the bubble corresponding to the correct answer is darkened.

Zero Marks : **0** In all other cases.

15. Figure shows two spherical surface of radii R and 2R separating three transparent media of refractive index $\mu = 4$, $\mu = 2$ and $\mu = 1$. A ray of light travelling in medium $\mu = 1$ is incident on outer sphere tangentially. If the net deviation suffered by light ray in clockwise direction is β (in degree) then find value of $\beta/30$.



Ans. 8



By snell's law

$\mu_1 \sin i = \mu_2 \sin r$

$1 \sin 90° = 2 \sin r$

$\sin r = \frac{1}{2}$

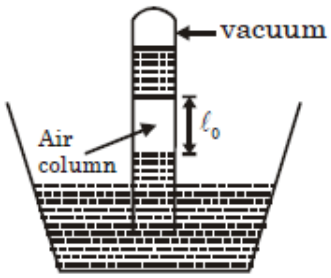
$r = 30°$

Deviation = $90 - r = 80°$

Total refraction = 4

Total deviation = $4 \times 60° = 240°$

16. At the middle of the mercury barometer tube there is a little column of air with the length ℓ_0 and there is vacuum at the top as shown. Under the normal atmospheric pressure and the temperature of 300 kelvin, ℓ_0 = 10 cm. Neglect expansion of the tube. The length of the air column if the temperature rises to 330 Kelvin will be α cm, then find $\alpha/11$:



Ans. 1

$T = 300$ k

$T = 330$ k

$V \propto T$

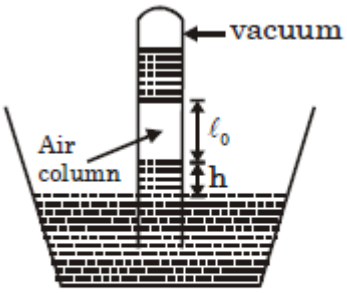
$$\frac{A\ell_0}{300} = \frac{A(\ell_0 + x)}{330}$$

$$(\ell_0 + x) = \frac{11\ell_0}{10}$$

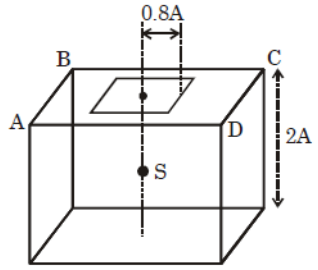
$\ell_0 = 10$ cm

$x = 1$ cm

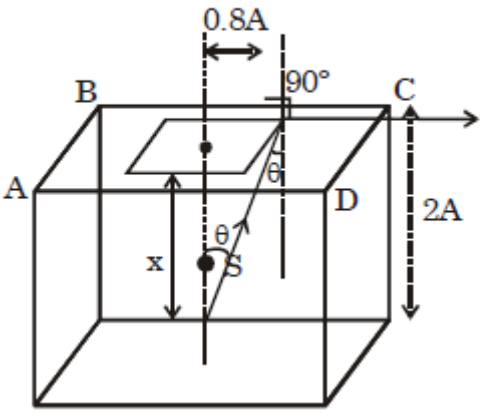
$(\ell_0 + x) = 11$ cm



17. A cube of side 2A has a point source S at its centre. All the faces of the cube are blackened except the top surface which has got a square sheet of black paper of side 1.6 A. If the source starts oscillating inside a negligibly thin tunnel pependicular to the direction of face ABCD, and having the mean position of oscillation at the centre of the cube, then the maximum amplitude of oscillation so that the source may not be seen from the top surface is found to be βA. Then find 10β. (Refractive index of material of cube $\mu = \sqrt{3.25}$. Assume black surface absorbs all light incident on it.)



Ans. 2



$$\tan\theta = \frac{0.8A}{x}$$

$$\sin\theta = \frac{0.8A}{\sqrt{x^2 + 0.64A^2}}$$

If particle is not visible at point 'A' then it will not be visible during its motion hence, by snell's law

$\mu \sin\theta = 1 \sin 90°$

$$\sqrt{3.25} = \frac{\sqrt{x^2 + (0.8A)}}{(0.8A)}$$

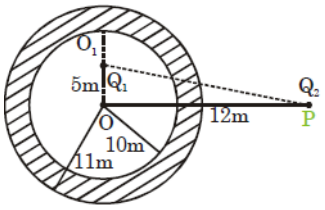
$x = 1.2\ A$

Amplitude $= 0.2A$

$\beta = 0.2$

$10\beta = 2$

18. A charge Q_1 is placed at O, inside a hollow conducting sphere having inner and outer radii as 10 m and 11 m as shown. The force experienced by Q_2 at P is F_1 and force experienced by Q_2 when Q_1 is placed at O_1 is F_2. Then F_1/F_2 is equal to



Ans. 1

Field due to charge placed inside conductor shell is independent of its position inside shell.

So, $F_1 = F_2$

$$\frac{F_1}{F_2} = 1$$

PART-2 : CHEMISTRY

SECTION–I : (Maximum Marks : 32)

- This section contains **EIGHT** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is (are) correct.
- For each question, darken the bubble(s) corresponding to all the correct option(s) in the ORS
- For each question, marks will be awarded in one of the following categories :

Full Marks : +4 If only the bubble(s) corresponding to all the correct option(s) is (are) darkened.

Zero Marks : 0 If none of the bubbles is darkened.

Negative Marks : –2 In all other cases.

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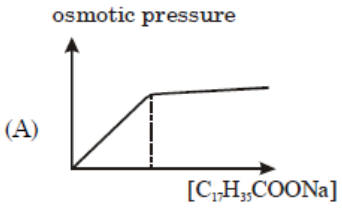
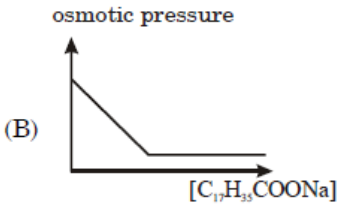
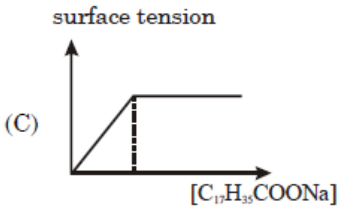
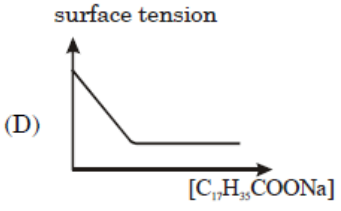
1. Select the correct option(s) about $C_{17}H_{35}COONa$ aq.solution.

(A) 

(B) 

(C) 

(D) 

Ans.(A,D)

2. In which of the following% s-character is higher in hybrid orbital of nitrogen used for bond formation as compared to NH_4^+ :-

(A) NH_3 (B) NF_4^+ (C) N_2H_2 (D) NO_3^-

Ans. (C, D)

3. **IA metal** $\xrightarrow[\text{Air}]{\text{Burn in}}$ mixture of product(s) **(X)**

(X) $\xrightarrow{\text{Water}}$ Alkaline solution + **gas Y**
(produce blue coloration with $CuSO_4$(aq.))

Select **CORRECT** option(s) :-

(A) Metal can be Li

(B) Metal can be Na

(C) Gas-Y is absorbed in H_2SO_4 but not in KOH

(D) Superoxide of metal is also present as major product in (X)

Ans. (A, C)

4. The **CORRECT** option(s) regarding extraction of pure aluminium from red bauxite is/are :

(A) Serpeck's process is used for ore concentration.

(B) Leaching of ore with aq. NaOH produces precipitate of $NaAlO_2$.

(C) Weakly acidic medium is used to precipitate $Al(OH)_3$ from solution containing $Na[Al(OH)_4]$.

(D) Only pure Al_2O_3 is used as electrolyte in Hall-Heroult's process of electrolytic reduction.

Ans. (C)

Serpeck's process is used for concentration of white bauxite.

Leaching of ore with NaOH produces $NaAlO_2$ in filtrate not as ppt.

5. In the metallurgy of gold from native ore, which step involves oxidation of mainly Au atoms.

(A) Leaching (B) Metal displacement in water by Zn

(C) Electrolytic refining (D) Cupellation

Ans. (A, D)

(A) Fact

(B) Probability of finding an electron is upto 90% in an orbital

(C) No of angular nodes are *l*

(D) For 1s, $|\Psi|^2$ is maximum at nucleus

6. For the reaction : A → product(s).

If plot between given parameter are straight lines then the correct match for order of reaction given with parameter is/are-

[A] reactant concentration at time 't' , k = rate constant

(A) $\ln\left[\frac{-d[A]}{dt}\right]$ v/s $\ln[A]$; order = 3

(B) [A] v/s t ; order = 1

(C) $\log t_{1/2}$ v/s $\log [A]_0$; order = –1

(D) $\frac{-d[A]}{dt}$ v/s $[A]^2$; order = 2

Ans. (A, C, D)

For this reaction

$$-\frac{d[A]}{dt} = k[A]^n \ \& \ t_{1/2} = \frac{1}{k(n-1)}\left[\frac{2^{n-1}-1}{A_0^{n-1}}\right]$$

7. Highly pure dilute solution of sodium in liquid ammonia :

(A) Shows blue colour.

(B) Exhibits electrical conductivity and paramagnetism

(C) The main species present in solution are solvated metal ion and solvated electron

(D) On long standing, it produce blue crystals of sodium amide and evolve N_2

Ans. (A,B,C)

$M + (x + y)\,NH_3 \longrightarrow [M(NH_3)_x]^+ + [e(NH_3)_y]^-$

The electrical conductivity is due to the solvated ions and blue colour is due to solvated electrons.

8. Which of the following option(s) represent the **CORRECT** IUPAC name of the given co-ordination compounds ?

(A) $Li[CuMe_2]$: Lithium dimethylcuprate(II).

(B) $[Os(Et)(NH_3)_5]Cl$: Pentaammineethylosmium(II) chloride.

(C) $K_3[Co(CN)_5H]$: Potassium pentacyanidoprotoniumcobaltate(I)

(D) $[Co(en)_3]Cl_3$: Tris-(ethane-1,2-diammine)cobalt(III) chloride.

Ans. (B)

$Li[CuMe_2]$: Ox.state of Cu = +1

$K_3^{+1}[Co(\overset{-1}{C}N)_5(\overset{-1}{H})]$: Ox. state of Co = +1

$In[Co(en)_3]Cl_3$, en is ethane-1,2-diamine
not diammine.

SECTION–II : (Maximum Marks : 24)

- This section contains **TWO** paragraphs.
- Based on each paragraph, there are **THREE** questions
- Each question has **FOUR** options (A), (B), (C) and (D) **ONE OR MORE THAN ONE** of these four option(s) is(are) correct.
- For each question, darken the bubble(s) corresponding to all the correct option(s) in the ORS.
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks	: +4	If only the bubble(s) corresponding to all the correct option(s) is (are) darkened.
Zero Marks	: 0	If none of the bubbles is darkened.
Negative Marks	: –2	In all other cases.

Paragraph for Questions 9 to 11

Metal-Insoluble Salt-Anion Half Cell:

In this half cell, a metal coated with its insoluble salt is in contact with a solution containing the anion of the insoluble salt. eg. Silver-Silver Chloride Half Cell:

This half cell is represented as $Cl^-/AgCl/Ag$. The equilibrium reaction that occurs at the electrode is $AgCl(s) + e^- \rightleftharpoons Ag(s) + Cl^-(aq)$

Gas-Ion Half Cell:

In such a half cell, an inert electrode, platinum or graphite is in contact with gas and a solution containing a specified ion. One of the most important gas-ion half cell is the hydrogen-gas-hydrogen ion half cell. In this half cell, purified H_2 gas at a constant pressure is passed over a platinum electrode which is in contact with an acid solution.

$H^+(aq) + e^- \rightleftharpoons 1/2\ H_2(g)$

9. E^0_{cell} for the following cell at 298 K is nearly :

$Ag(s)\ |AgCl(s)\ |\ Cl^-(aq.)\ ||\ Br^-(aq.)\ |\ AgBr(s)\ |\ Ag(s)$

(Given : K_{sp} of $AgCl = 2 \times 10^{-10}$, K_{sp} of $AgBr = 4 \times 10^{-13}$)

(A) 0.0 V (B) 0.16 V (C) –0.16 V (D) – 0.32 V

Ans. (C)

10. Calculate the emf of the cell at 25°C

$Pt\,|\,H_2(1.0\ atm)\,|\,CH_3COOH\ (0.1\ M)\,|\,|\,NH_3(aq,\ 0.01M)\,|\,H_2\ (1.0\ atm)\ |\ Pt$

(Given : $K_a(CH_3COOH) = 10^{-5}$, $K_b\ (NH_3) = 10^{-5}$)

(A) –0.44 V (B) –0.88 V (C) –0.22 V (D) –0.11 V

Ans. (A)

$CH_3COOH \rightleftharpoons CH_3COO^- + H^+$

$t = 0 \quad CM$

$t \quad C-C\alpha \qquad C\alpha \qquad C\alpha$

$$ka = \frac{(C\alpha)\ (C\alpha)}{(C - C\alpha)} = \frac{C\alpha^2}{1-\alpha}$$

if $\alpha << 1$

$k_a = C\alpha^2$

$$\alpha = \sqrt{\frac{k_a}{C}}$$

$$\Rightarrow [H^+] = C\alpha = \sqrt{k_a \times C} = \sqrt{10^{-5} \times 10^{-1}} = 10^{-3}$$

$NH_4OH \rightleftharpoons NH_4^+ + OH^-$

$$k_b = \frac{C\alpha^2}{1-\alpha}$$

if $\alpha << 1$

$$[OH^-] = C\alpha = \sqrt{k_b \times C} = \sqrt{10^{-5} \times 10^{-2}}$$

$$= \sqrt{10^{-7}} = 10^{-3.5}\ M$$

$$H_2 + \underset{10^{-10.5}M}{2H^+} \rightleftharpoons \underset{10^{-M}}{2H^+} + H_2$$

$$E = E^\circ - \frac{0.059}{2}\log\frac{(10^{-3})^2}{(10^{-10.5})^2}$$

$$E = \frac{-0.059}{2}\log\frac{10^{-6}}{10^{-21}}$$

$$= \frac{-0.059}{2} \times 15 = -0.44V$$

11. For the cell ,

Pt(s) | H_2(0.5 bar) | HCl (0.01M) | AgCl(s) | Ag(s)

$E^0_{cell} = 0.22V$ at 25°C. E_{cell} at 25°C is (log2 = 0.30)

(A) 0.0V (B) 0.45 V (C) 0.67 V (D) –0.23 V

Ans. (B)

Paragraph for Questions 12 to 14

Alkyl halides can undergo S_N1,S_N2,, E1 & E2 reactions. Outcome of a reaction is affected by factors like

(1) Nature of solvent (2) Temperature

(3) Structure of alkyl halide (4) Type of Base/Nucleophile

If reaction conditions are altered, outcome of a reaction can change.

Answer following questions.

12. Which of the following compound(s) can give same major product in S_N1 as well as in S_N2 reaction ?

(A) $Ph - CH_2 - Cl$

(B) $CH_2 = CH - CH_2 - Cl$

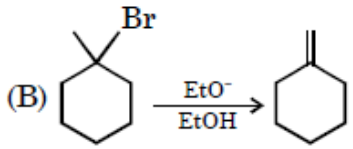
(C) $CH_3 - \underset{\displaystyle CH_3}{\underset{|}{CH}} - \underset{\displaystyle Cl}{\underset{|}{CH}} - CH_3$

(D) $CH_3 - \underset{\displaystyle CH_3}{\underset{|}{CH}} - Cl$

Ans. (A,B,D)

13 Which of the following reaction is **CORRECT** for given product as major product ?

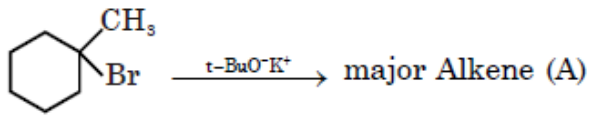
(A) $CH_3-\underset{}{CH_2}-\underset{F}{CH}-CH_3 \xrightarrow{\text{alc. KOH}/\Delta} CH_3-CH_2-CH=CH_2$

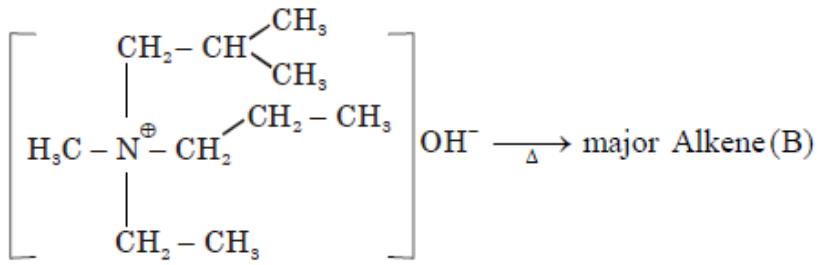
(B) 

(C) $CH_3-\underset{Br}{CH}-\underset{CH_3}{CH}-CH_3 \xrightarrow[t\ BuOH]{t\ BuO^-} CH_3-CH=\underset{CH_3}{C}-CH_3$

(D) $CH_3-\underset{CH_3}{\overset{CH_3}{C}}-Br \xrightarrow{\text{Aq KOH}} CH_3-\underset{CH_3}{\overset{CH_2OH}{C}}-H$

Ans. (A)

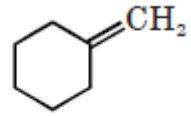
14.  major Alkene (A)



Total sum of α-H in Alkene (A) & Alkene (B) is/are

(A) 4 (B) 7 (C) 10 (D) 8

Ans. (A)

Compound A is  and

(4 α-H)

Compound B is $CH_2=CH_2$ (no α-H)

SECTION–III : (Maximum Marks : **8**)

- This section contains **FOUR** questions.
- The answer to each question is a SINGLE DIGIT INTEGER ranging from 0 to 9, both inclusive.
- For each question, darken the bubble corresponding to the correct integer in the ORS.
- For each question, marks will be awarded in one of the following categories :

Full Marks : **+ 2** If only the bubble corresponding to the correct answer is darkened.

Zero Marks : **0** In all other cases.

15. For the reaction : 3A (g) $\rightarrow$ 2B(g), the rate of formation of 'B' at 298 K, is represented as $\ln\left(\frac{d[B]}{dt}\right) = -0.04 + 2\times\ln[A]$. The order of reaction is-

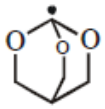
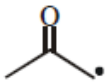
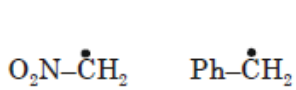
Ans. 2

$$-\frac{1}{3}\frac{d[A]}{dt} = +\frac{1}{2}\frac{d[B]}{dt} = K[A]^n$$

$$\text{or, } \ln\left(\frac{d[B]}{dt}\right) = \ln(2K) + n.\ln[A]$$

Hence, $n = 2$

16. Identify total number of free radicals which have sp^2 hybridised central atom (having free radical) :

Ans. 3

17. For reaction $A \rightarrow B$, the rate constant $k_1 = A_1(e^{-E_{a_1}/RT})$ and for the reaction $X \rightarrow Y$, the rate constant $k_2 = A_2 e^{-(E_{a_2}/RT)}$. If $A_1 = 10^9$, $A_2 = 10^{10}$, $E_{a_1} = 1200$ cal/mol and $E_{a_2} = 1800$ cal/mol, then the temperature at which $k_1 = k_2$ is $\frac{x}{2.303}$K. Find $Y = \frac{x}{100}$.

(Given; R = 2 cal/K-mol)

Ans. 3.00

$A \rightarrow B$

$$K_1 = A_1 \times e^{\frac{-Ea_1}{RT}}$$

$x \rightarrow y$

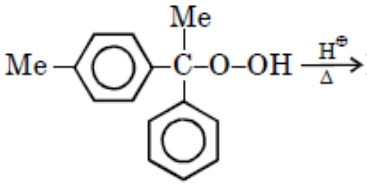
$$K_2 = A_2 \times e^{\frac{-Ea_2}{RT}}$$

if $k_1 = k_2$

$$10^9\, e^{\frac{-1200}{RT}} = 10^{10} e^{\frac{-1800}{RT}}$$

$$\frac{-1200}{RT} = \frac{-1800}{RT} + \ln 10$$

$$T = \frac{300}{2.303}K$$

18. 

$P_1 + NaOI \longrightarrow \underset{\text{yellow}}{P_3\downarrow} + P_4$

If x = molecular mass of gas obtained when P_3 is treated with Ag powder.
y = molecular mass of aromatic hydrocarbon obtained when P_4 is treated with sodalime/Δ.

Then value of $\left(\frac{y}{x}\right)$ is....

Ans. 3.00

y = 78 (benzene)

x = 26 (acetylene)

PART-3 : MATHEMATICS

SECTION-I : (Maximum Marks : 32)

- This section contains **EIGHT** questions.
- Each question has **FOUR** options (A). (B). (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is (are) correct.
- For each question, darken the bubble(s) corresponding to all the correct option(s) in the ORS
- For each question, marks will be awarded in one of the following categories :

Full Marks : +4 If only the bubble(s) corresponding to all the correct option(s) is (are) darkened.

Zero Marks : 0 If none of the bubbles is darkened.

Negative Marks : –2 In all other cases.

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1. The curve $C_1 : y = e^{-x}$ and $C_2 : y = e^{-x}\sin x$ $(x > 0)$ touch each other at infinitely many points. Let $0 < x_1 < x_2 < \ldots\ldots < x_n < \ldots\ldots$ be the abscissa of these points of contact. If A_n denotes the area bounded by two curves & ordinates $x = x_n$ & $x = x_{n+1}$, then

(A) $A_1 = \frac{1}{2}\frac{(e^{2\pi}-1)}{e^{5\pi/2}}$

(B) $A_2 = \frac{1}{2}\frac{(e^{2\pi}-1)}{e^{9\pi/2}}$

(C) A_1, A_2, A_3 are in A.P.

(D) A_1, A_2, A_3 are in G.P.

Ans. (A,B,D)

$$x_n = 2n\pi + \frac{\pi}{2}$$

$$\int e^{-x}(1-\sin x)\,dx = \frac{e^{-x}}{2}(\sin x + \cos x) - e^{-x}$$

{integration by parts}

$$\Rightarrow A_n = \int_{x_n}^{x_n+1} e^{-x}(1-\sin x)\,dx = \frac{1}{2}\left(e^{2\pi}-1\right)e^{\frac{-(4n+1)\pi}{2}}$$

2. If tangents are drawn to the curve $y = \frac{x}{1+x^2}$; such that these tangents are concurrent at $\left(1, \frac{1}{2}\right)$, then

(A) Number of such tangents are 3.

(B) Each of the tangents have exactly two points in common with the curve.

(C) Abscissa of point of contact for one of the tangents is $1+\sqrt{2}$.

(D) Abscissa of point of contact of one of the tangents is $1-\sqrt{2}$.

Ans. (A,C,D)

$$y = \frac{x}{1+x^2}$$

$$\frac{dy}{dx} = \frac{1-x^2}{\left(1+x^2\right)^2}$$

Let point of contact P(a,b), $A\left(1, \frac{1}{2}\right)$ then

slope of PA = tangent at P and $b = \frac{a}{1+a^2}$

$$\Rightarrow \frac{b-\frac{1}{2}}{a-1} = \frac{1-a^2}{\left(1+a^2\right)^2}$$

$$\Rightarrow \left(\frac{2a}{1+a^2}-1\right)\left(1+a^2\right)^2 = \left(1-a^2\right)(a-1)$$

$1 + a^2 = (1 + a)^2$

$\therefore a^2 - 2a - 1 = 0$

$\Rightarrow a = 1+\sqrt{2}$ or $a = 1-\sqrt{2}$

$\therefore$ There are three tangents possible at

$x = 1+\sqrt{2}, 1-\sqrt{2}, 1$

3. If $I_n = \int_0^1 \frac{(\ell nx)^n}{\sqrt{x}} dx, n \in N$, then

(A) I_n has finite value for all natural values of 'n'.

(B) $\frac{I_2}{I_1} + \frac{I_3}{I_2} + \frac{I_4}{I_3} = -4$

(C) $I_4 = -9$

(D) $I_8 = 40331$

Ans. (A)

$$I_n = \int_0^1 \frac{(\ell nx)^2}{\sqrt{x}} dx$$

$$\sqrt{x} = t$$

$$\Rightarrow I_n = 2^{n+1} \int_0^1 (\ell nt)^n \, dt$$

$$\text{Let } J_n = \int_0^1 (\ell nt)^n \, dt$$

$$\Rightarrow J_n = t(\ell nt^n)\Big|_0^1 - n\int_0^1 (\ell nt)^{n-1} \, dt$$

$$J_n = -nJ_{n-1}$$

$$\Rightarrow \frac{I_n}{2^{n+1}} = -n\frac{I_{n-1}}{2^n} \Rightarrow I_n = -2n\, I_{n-1}$$

$$\text{Now } \because \; I_1 = \int_0^1 \frac{\ell nx}{\sqrt{x}} = -4 \text{ is finite} \Rightarrow I_n \text{ is finite}$$

$\because$ I_n is always negative integer.

$\Rightarrow$ Only (A) is correct.

4. Consider a sequence of 10A's and 8 B's placed in a row. By a run we mean single or more letters of the same type placed side by side. For example following is an arrangement consisting of 4 runs of A & 4 runs of B : AAABBABBBAABAAAABB. The number of arrangements of 10 A's and 8 B's in a row so that there are -

(A) 4 runs of A and 4 runs of B is $2.{}^9C_3.{}^7C_3$

(B) 4 runs of A and 5 runs of B is $3.{}^9C_4.{}^7C_4$

(C) 3 runs of A and 3 runs of B is $2.{}^9C_2.{}^7C_2$

(D) 6 runs of A and 6 runs of B is $4.{}^9C_5.{}^7C_5$

Ans. (A,C)

$\uparrow_x B \uparrow_y B \uparrow_z B \quad B \quad B \uparrow_w B \quad B \quad B$

for 4 runs $\rightarrow$ select 3 places out of 7 between B's and one end. Let x,y,z,w be number of A's in this place $x + y + z + w = 10$,

$x,y,z,w \geq 1 \Rightarrow {}^7C_3$

$\Rightarrow$ Required arr. $= {}^2C_1.{}^7C_3.{}^9C_3$.

5. If the quadratic equations $x^2 + (\sin\alpha)x + \operatorname{cosec}\alpha = 0$, where $\alpha \in (0, \pi)$ and $2x^2 + x + 8\sin\frac{\beta}{2} = 0$, where $\beta \in (-\pi, \pi)$ have a common root, then $\alpha + \beta$ can be

(A) $\frac{\pi}{3}$ (B) $\frac{\pi}{2}$ (C) $\frac{7\pi}{6}$ (D) $-\frac{\pi}{6}$

Ans. (B,C)

$\alpha \in (0, \pi)$

$\Rightarrow$ $x^2 + (\sin\alpha)x + \operatorname{cosec}\alpha = 0$ has imaginary roots

$$\Rightarrow \quad \frac{1}{2} = \frac{\sin\alpha}{1} = \frac{\operatorname{cosec}\alpha}{8\sin\frac{\beta}{2}}$$

$$\Rightarrow \quad \sin\alpha = \frac{1}{2}, \operatorname{cosec}\alpha = 2 \text{ and } 8\sin\frac{\beta}{2} = 4$$

$$\Rightarrow \quad \alpha = \frac{\pi}{6} \text{ or } \frac{5\pi}{6} \text{ and } \beta = \frac{\pi}{3}$$

6. Let area of triangle formed by any line through point (1, 3) and co-ordinate axes in xy-plane is A. then

(A) If A = 8, then number of such lines is 4 (B) If A = 6, then number of such lines is 3

(C) If A = 4, then number of such lines is 2 (D) If A = 0, then number of such lines is 1

Ans. (A,B,C,D)

Let line is $(y - 3) = m(x - 1)$

or $mx - y + (3 - m) = 0$

$$A = \frac{(3-m)^2}{|2m|}$$

$\Rightarrow m^2 - 6x + 9 = \pm 2Am$

$\Rightarrow m^2 - 2(3 - A)m + 9 = 0$

or $m^2 - 2(3 + A)m + 9 = 0$

$D = 4A(A - 6)$ or $D = 4A(A + 6)$

7. $f(x) = (x - 1)\sec x - \ell n\left(\tan\left(\frac{\pi}{4} + \frac{x}{2}\right)\right), x \in (-\pi, \pi)$ then f(x) is –

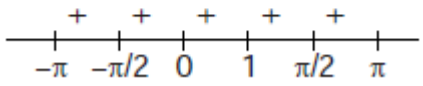
(A) Decreasing in (0, 1) (B) increasing in $(-\pi, -\pi/2)$

(C) Increasing in $(1, \pi/2)$ (D) Decreasing in $(-\pi/2, 0)$

Ans. (A, B, C)

$f'(x) = (x - 1)\sec x \tan x$

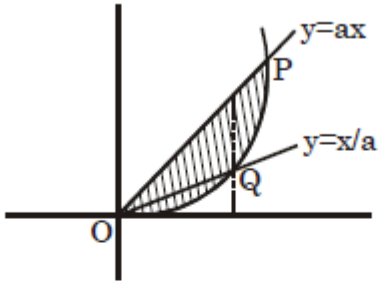
Now, sign scheme of f'(x) is



8. If the maximum area bounded by the curves $x^2 = 4ay$, $y = ax$, $y = \frac{\hat{}}{a}$, $1 \le a \le 2$ is α, then

(A) $a = \frac{3}{2}$ (B) $\alpha = 84$ (C) $a = 2$ (D) $\alpha = 48$

Ans. (B,C)



Points of intersection are $P(4a^2, 4a^3)$, $Q\left(4, \frac{4}{a}\right)$

required are $= \int_0^4 \left(ax - \frac{x}{a}\right)dx + \int_4^{4a^2} \left(ax - \frac{x^2}{4a}\right)dx$

$\alpha = \frac{16}{3a} - \frac{8}{a} + \frac{a^5}{3}$ which in increasing

$\Rightarrow$ maximum value of area occures of a = 2

$\Rightarrow$ α = 84

SECTION–II : (Maximum Marks : 24)

- This section contains **TWO** paragraphs.
- Based on each paragraph, there are **THREE** questions
- Each question has **FOUR** options (A), (B), (C) and (D) **ONE OR MORE THAN ONE** of these four option(s) is(are) correct.
- For each question, darken the bubble(s) corresponding to all the correct option(s) in the ORS.
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks : +4 If only the bubble(s) corresponding to all the correct option(s) is (are) darkened.

Zero Marks : 0 If none of the bubbles is darkened.

Negative Marks : –2 In all other cases.

Paragraph for Questions 9 to 11

A Pythagorean triplet (a, b , c) is such that $a^2 + b^2 = c^2$

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 2 \\ 2 & 2 & 3 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 2 & 2 \\ -2 & -1 & -2 \\ 2 & 2 & 3 \end{bmatrix} \qquad C = \begin{bmatrix} -1 & -2 & -2 \\ 2 & 1 & 2 \\ 2 & 2 & 3 \end{bmatrix}$$

9. If we write pythagorean triplet (a , b , c) in matrix form [a b c] then which of the following product is a pythagorean triplet ?

(A) [3 4 5] A (B) [3 4 5] B (C) [3 4 5] C (D) None of these

Ans. (B,C)

Option (A) $\Rightarrow [3, 4, 5] \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 2 \\ 2 & 2 & 3 \end{bmatrix} = [21\ 20\ 32]$

10. Which of the following(s) hold good ?

(A) $A^{-1} = \text{adj } A$

(B) $(AB)^{-1} = \text{adj } (AB)$

(C) $(BC)^{-1} = \text{adj } (BC)$

(D) $(ABC)^{-1} \neq \text{adj } (ABC)$

Ans. (A,B,C)

$|A| = 1 = |B| = |C|$

So $|AB| = 1 = |BC| = |CA| = |ABC|$

11. Tr $(A + B^T + 3C)$ equals (Where Tr (A) denotes trace of square matrix A)

(A) 17 (B) 15 (C) 19 (D) 18

Ans. (A)

Tr $(A+B^T + 3C) = 5 + 3 + 9 = 17$

Paragraph for Questions 12 to 14

A box contains n coins . Let $P(E_i)$ be the probability that exactly i out of n coins are biased. If $P(E_i)$ is directly proportional to $i\ (i + 1)$, $0 \le i \le n$

12. Proportionality constant is not equal to

(A) $\frac{3}{n(n^2+1)}$ (B) $\frac{1}{(n^2+1)(n+2)}$ (C) $\frac{3}{n(n+1)(n+2)}$ (D) $\frac{1}{(n+1)(n+2)(n+3)}$

Ans. (A,B,D)

13 What are the odd in favour of a coin selected at random is biased ? (where $n \to \infty$)

(A) 1 : 3 (B) 3 : 2 (C) 3 : 1 (D) 7 : 1

Ans. (C)

14. If a coin selected at random is found to be biased then probability that it is the only biased coin in the box

(A) $\frac{1}{(n+1)(n+2)(n+3)(n+4)}$ (B) $\frac{12}{n(n+1)(n+2)(3n+1)}$

(C) $\frac{24}{n(n+1)(n+2)(2n+1)}$ (D) $\frac{24}{n(n+1)(n+2)(3n+1)}$

Ans. (D)

$P(E_i) = Ki\ (i + 1)$

$$\sum P(E_i) = 1 \quad \Rightarrow \quad K = \frac{3}{n(n+1)(n+2)}$$

$$P(E) = \sum_{i=1}^{n} P(E_i) P(E/E_i)$$

$$= \sum Ki(i+1).\frac{i}{n} = \frac{(3n+1)(n+2)}{4n(n+2)}$$

When $n \to \infty$ $P(E) = 3/4$

$$\left(\frac{E_1}{E}\right) = \frac{K.\ 2.\frac{1}{n}}{\frac{(3n+1)(n+2)}{4n(n+2)}} = \frac{24}{n(n+1)(n+2)(3n+1)}$$

SECTION–III: (Maximum Marks : **8**)

- This section contains **FOUR** questions.
- The answer to each question is a SINGLE DIGIT INTEGER ranging from 0 to 9, both inclusive.
- For each question, darken the bubble corresponding to the correct integer in the ORS.
- For each question, marks will be awarded in one of the following categories :

Full Marks : **+ 2** If only the bubble corresponding to the correct answer is darkened.
Zero Marks : **0** In all other cases.

15. The shortest distance between the curves $f(x) = -6x^6 - 3x^4 - 4x^2 - 6$ & $g(x) = e^x + e^{-x} + 2$ is λ, then the value of $\frac{\lambda}{2}$ is

Ans. 5

$f(x) = -6x^6 - 3x^4 - 4x^2 - 6$

$g(x) = e^x + e^{-x} + 2$

$f(x)$ is maximum at $x = 0 \Rightarrow$ point $(0,-6)$

$g(x)$ is minimum at $x = 0 \Rightarrow$ point $(0,4)$

$\therefore$ minimum distance = 10.

16. If the value of $\sum_{n=1}^{\infty} \frac{\cos(2n)}{2^n} = \frac{a\cos(a) - 1}{5 - 2a\cos(a)}$, then the minimum positive integral value of 'a' is

Ans. 2

$$\sum_{n=1}^{\infty}\frac{\cos(2n)}{2^n}=\sum_{n=1}^{\infty}\frac{e^{i2n}+e^{-i2n}}{2}\cdot\frac{1}{2^n}=\frac{1}{2}\sum_{n=1}^{\infty}\left(\frac{e^{2i}}{2}\right)^n+\left(\frac{e^{-2i}}{2}\right)^n$$

$$=\frac{1}{2-e^{2i}}+\frac{1}{2-e^{-2i}}-1$$

$$=\frac{2\cos 2-1}{5-4\cos 2}\Rightarrow a=2$$

17. Tangent at P to $xy = c^2$ cuts x-axis & y-axis at M and N respectively. Rectangle OMVN is completed where O is origin. If MV cuts the curve at Q, such that MV = λ MQ, then λ equals

Ans. 4.00

$$\frac{x}{x_1}+\frac{y}{y_1}=2, \qquad MQ=\frac{C^2}{2x1}$$

$$MV=2y_1=\frac{2c^2}{x_1}, \text{ hence } \lambda=4$$

18. If three different polynomials $x^2 + ax + b$, $x^2 + x + ab$ and $ax^2 + x + b$ have exactly one common zero, where a, b are non-zero real numbers, then the value of $a + 2b$ is

Ans. 0

Let $X = \alpha$ be common zero then

$\alpha^2 + a\alpha + b = 0$ ....(1)

$\alpha^2 + \alpha + ab = 0$ ....(2)

$a\alpha^2 + \alpha + b = 0$ ....(3)

$(1) - (2) \Rightarrow (\alpha - 1)(\alpha - b) = 0$

(1) or (3) gives $b^2 + ab + b = 0$

and (3) gives $ab^2 + 2b = 0$

$b \neq 0 \Rightarrow ab = -2$

if $ab = -2$, $b^2 + b - 2 = 0$

$b = 1$ or -2

$a = -2$ or 1

$a = -2, b = 1$ and $a = 1, b = -2$ (does not satisfy the mentioned condition polynomial will be same)

$a + 2b = 0$