IIT – JEE: 2023 **AITS – 9** **DATE: 11/05/23**

ADVANCED (ANSWER KEY)

PAPER - I

Que.	1	2	3	4	5	6	7	8	9	10
Ans.	A	D	C	A	B	A	C	B	A	C
Que.	11	12	13	14	15	16	17	18	19	20
Ans.	B	C	A	B	D	A	1	6	4	A-Q, B-S C-R, D-R
Que.	21	22	23	24	25	26	27	28	29	30
Ans.	C	D	A	B	C	C	B	C	B	C
Que.	31	32	33	34	35	36	37	38	39	40
Ans.	B	A	C	C	C	A	9	5	8	A-P, S B-P, S C-Q, R, S D-Q, R
Que.	41	42	43	44	45	46	47	48	49	50
Ans.	A	A	A	B	C	D	A	D	A	B
Que.	51	52	53	54	55	56	57	58	59	60
Ans.	C	B	D	C	D	D	1	8	6	A-P, B-Q C-Q, D-P

PAPER - II

Que.	1	2	3	4	5	6	7	8	9	10
Ans.	B	A	C	C	D	B	A	D	B	B
Que.	11	12	13	14	15	16	17	18	19	20
Ans.	9	1	2	7	3	3	3	8	1	4
Que.	21	22	23	24	25	26	27	28	29	30
Ans.	C	A	B	B	C	B	C	B	C	B
Que.	31	32	33	34	35	36	37	38	39	40
Ans.	2	3	5	4	8	4	3	4	3	8
Que.	41	42	43	44	45	46	47	48	49	50
Ans.	C	B	D	A	C	C	A	C	B	A
Que.	51	52	53	54	55	56	57	58	59	60
Ans.	2	2	3	6	6	3	2	6	6	7

Note : **Detailed solution to this test is available on Tuesday after 02.00 pm on our website.:www.iitianspace.com**

ANSWER KEY

1.	[B]	2.	[A]	3.	[C]	4.	[C]	5.	[D]
6.	[B]	7.	[A]	8.	[D]	9.	[B]	10.	[B]
11.	[9]	12.	[1]	13.	[2]	14.	[7]	15.	[3]
16.	[3]	17.	[3]	18.	[8]	19.	[1]	20.	[4]

SOLUTION

1. **(B)**

True reading $= R_0 + C = (M.S.R + V.S.R) + C$

$= N + n \times (V.C) + C = 3.1 + 4 \times 0.01 - 0.07$

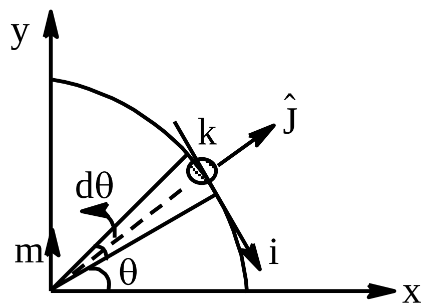
$= 3.07\,\text{cm}$

2. **(A)**

$$B = \mu_0 J = \mu_0 \sigma v = \frac{\mu_0 \varepsilon_0 vV}{d}$$

3. **(C)**

$$e = \int (\vec{V} \times \vec{B}).\vec{d\ell} = \int_0^{\pi/2} \omega r \cos\theta k \times \left(\frac{km\cos\theta}{r^3}\hat{i} + \frac{2km\sin\theta}{r^3}\hat{j} \right).rd\theta(-\hat{i}) = \frac{km\omega}{r}$$

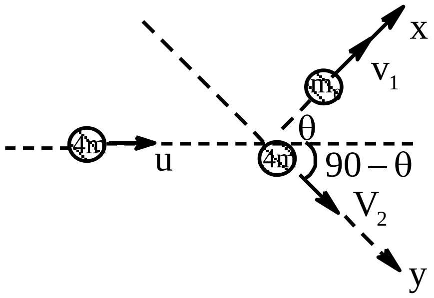


$$= \frac{\mu_0 m\omega}{4\pi r}$$

4. **(C)**

Laser gun (assume monochromatic) follow law of refraction

5. **(D)**



CLM along $y : V_2 = u\sin\theta$

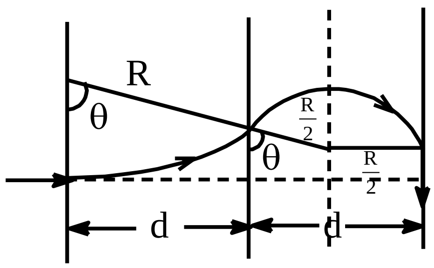
CLM along $x : V_1 = \dfrac{4mu\cos\theta}{m_0}$

$\text{loss} = \frac{1}{2}4mu^2 - \left(\frac{1}{2}m_0 v_1^2 + \frac{1}{2}4mv_2^2\right) > 0$

$\therefore m_0 > 4m$

6. **(B)**
Motion will unsymmetrical of same radius

7. **(A)**



$\frac{R}{2} + \frac{R}{2}\sin\theta + R\sin\theta = 2d \Rightarrow R = d \Rightarrow \frac{mv}{qB} = d$

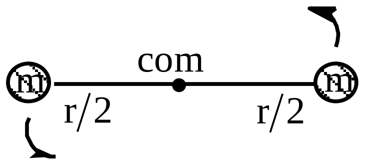
$$\therefore V = \frac{8}{9}\times 10^7 \, m/s$$

8. **(D)**
Body shape will change slightly hence

9. **(B)**
$r + a + t = 1 \quad \therefore a = 0.2 = e, \quad E = \sigma T^4 e = 100\times 0.2 = 20w/m^2, P = \sigma e A T^4 = 200w$

10. **(B)**
$\frac{1}{2}mv^2 = \frac{1}{2}m(v+\Delta v)^2 + E_1, \frac{1}{2}mv^2 = \frac{1}{2}m(v-\Delta v)^2 + E_2$, on comparing $E_2 > E_1$

11.



$$\frac{Ke^2}{r^2} = \frac{2mv^2}{r} .....(1)$$

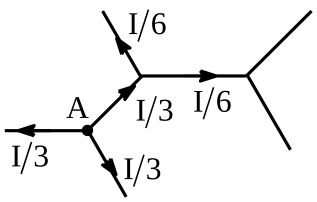
$mvr = nh/2\pi$ ….. (2)

Solving (1) and (2)

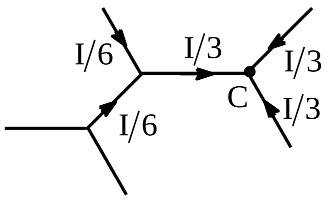
$$\text{Total eng} = -\frac{Ke^2}{2r} = -\frac{12.5}{n^2}\text{keV}$$

$$\text{Photon eng.} = 12.5\left(1-\frac{1}{4}\right) = 9.37\,\text{keV}$$

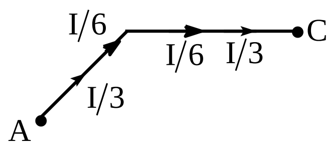
12. Considering only A



Considering only C:



Superimpose both the diagram



$$R_{AC} = \frac{V_{AC}}{I} = R$$

13. Use $\vec{V}_{P,Q} = \vec{\omega} \times \overrightarrow{QP}$, for ICOR $P(x_0, y_0)$ and $O(0,0)$

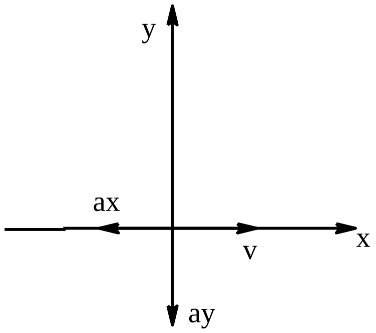
For ICOR $P(x_0, y_0)$ and $A(1,2)$

For ICOR $P(x_0, y_0)$ and $B(2,1)$

We will get $(x_0, y_0) = (0,-2)$

$\therefore (x_0 - y_0) = 0 + 2 = 2$

14.



$$\left(V - a_x \Delta t\right)^2 + \left(a_y \Delta t\right)^2 = \left(\frac{V}{2}\right)^2 \quad \text{......... (1)}$$

$$\left(V - 2a_x \Delta t\right)^2 + \left(2a_y \Delta t\right)^2 = \left(\frac{V}{4}\right)^2 \quad \text{....... (2)}$$

$$\left(V - 3a_x \Delta t\right)^2 + \left(3a_y \Delta t\right)^2 = x^2 \quad \text{........ (3)}$$

Solving (1), (2), (3) $x = \frac{\sqrt{7}}{4} V$

15. $N_1 = m_1 g \cos\alpha - m_1 A \sin\alpha$

$N_2 = m_2 g \cos\alpha + m_2 A \sin\alpha$

Where $N_1 = N_2$ So, $A = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) g \cot\alpha > g \tan\alpha$

$\therefore m_2 < m_1 \cos 2\alpha$

$k + k_1 = 2 + 1 = 3$

16. By CLM $P_1 = P_1' + P_2' \Rightarrow P_1^2 + P_1'^2 - 2P_1P_1'\cos\theta = P_2'^2$ .... (1)

By COE $\frac{P_1^2}{2m_1} = \frac{P_1'^2}{2m_1} + \frac{P_2'^2}{2m_2}$ .... (2)

Solving (1), (2) eliminate P_2', we will get

$$P_1'^2\left(1 + \frac{m_2}{m_1}\right) - 2P_1P_1'\cos\theta + P_1^2\left(1 - \frac{m_2}{m_1}\right) = 0$$

$\Delta \geq 0$ so, $\theta_{max} = \sin^{-1}\left(\frac{m_2}{m_1}\right) = 30^0$

17. By CAM $mv_0 \frac{r_0}{2} = m\left(v_y\right)_{rel} \frac{r}{2}$ ..... (1)

By COE $-\frac{Gm^2}{r_0} + \frac{1}{2}\left(\frac{m}{2}\right)v_0^2 = \frac{-Gm^2}{r} + \frac{1}{2}\left(\frac{m}{2}\right)\left(v_y\right)_{rel}^2$ .... (2)

Solving (1), (2), $\frac{r_{max}}{r_{min}} = 3$

18. $Mg - T - 6\pi\eta r_1 v = 0$ ..... (1)

$mg + T - 6\pi\eta r_2 v = 0$ ...... (2)

Solving (1), (2) $T = \frac{4}{3}\pi\rho g\left(r_1^2 r_2 - r_2^2 r_1\right)$ $\therefore \frac{dT}{dr_2} = 0$ $\therefore r_1 = 2r_2$

$$\frac{M}{m} = 8$$

19. for $w_1, \varepsilon = \frac{\ell}{2}\left[\left(\frac{\varepsilon_P}{1+2}\right)\frac{2}{\ell}\right]$..... (1)

for $w_2, \varepsilon = \frac{2\ell}{3}\left[\left(\frac{\varepsilon_p}{1+R}\right)\frac{R}{\ell}\right]$ .... (2)

Solving (1) and (2)

$R = 1\Omega$

20. $\frac{h_A}{h_B} = \frac{\rho_A S_A}{\rho_B S_B} = (4)$

ANSWER KEY

21.	**(C)**	22.	**(A)**	23.	**(B)**	24.	**(B)**	25.	**(C)**
26.	**(B)**	27.	**(C)**	28.	**(B)**	29.	**(C)**	30.	**(B)**
31.	**(2)**	32.	**(3)**	33.	**(5)**	34.	**(4)**	35.	**(8)**
36.	**(4)**	37.	**(3)**	38.	**(4)**	39.	**(3)**	40.	**(8)**

SOLUTION

21. **(C)**

22. **(A)**

23. **(B)**

24. **(B)**

25. **(C)**

$$CH_3COOH \rightleftharpoons CH_3COO^- + H^+$$

$$\frac{1}{21}-x \qquad x \qquad \frac{0.1}{21}+x$$

$$10^{-5} = \frac{\left(\frac{0.1}{21}+x\right)(x)}{\left(\frac{1}{21}-x\right)} \simeq \left(\frac{0.1}{21}\right)\left(\frac{21}{1}\right).\, x \qquad \text{(ignoring x)}$$

$$x \simeq 10^{-4}\ M$$

26. **(B)**

27. **(C)**

28. **(B)**

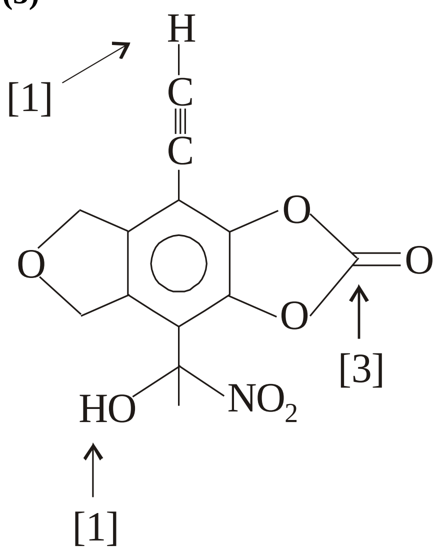
29. **(C)**

30. **(B)**

31. **(2)**

32. **(3)**

33. **(5)**



34. **(4)**

35. **(8)**

36. **(4)**

37. **(3)**

38. **(4)**

39. **(3)**

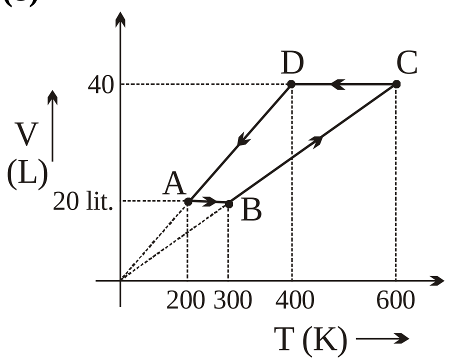
7- 4=3

→ Axial plane may contains two C – F bonds, one C – H bond and P - atom
Hence maximum number of atoms that may lie in one plane = 7

→ Similarly equatorial plane may also contain three C – H bonds and one P - atoms.

→ No. of planes that may contain maximum number of seven atoms = 4

40. **(8)**



$W_{AB} = W_{CD} = 0$

$W_{BC} = -nR\Delta T = -1 \times R \times 300 = 300\ R$

$W_{DA} = -nR\Delta T = -1 \times R \times -200 = 200\ R$

$W_{total} = -100\ R$

$\Rightarrow$ $|W| = 8$ atm. lit.

ANSWER KEY

41.	(C)	42.	(B)	43.	(D)	44.	(A)	45.	(C)
46.	(C)	47.	(A)	48.	(C)	49.	(B)	50.	(A)
51.	(2)	52.	(2)	53.	(3)	54.	(6)	55.	(6)
56.	(3)	57.	(2)	58.	(6)	59.	(6)	60.	(7)

SOLUTION

41. **(C)**

$$\tan^{-1}\left(\frac{1}{2}\tan 2A\right)+\tan^{-1}(\cot A)+\tan^{-1}(\cot^3 A)$$

$$=\tan^{-1}\left(\frac{1}{2}\tan 2A\right)+\tan^{-1}\left(\frac{\cot A+\cot^3 A}{1-\cot^4 A}\right)+\pi \qquad (\because \cot A>1)$$

$$=\tan^{-1}\left(\frac{\tan A}{1-\tan^2 A}\right)+\tan^{-1}\left(\frac{\cot A}{1-\cot^2 A}\right)+\pi$$

$$=\tan^{-1}\left(\frac{\tan A}{1-\tan^2 A}\right)-\tan^{-1}\left(\frac{\tan A}{1-\tan^2 A}\right)+\pi=\pi=4\tan^{-1}1$$

42. **(B)**

$$2<x^2<3\Rightarrow \sqrt{2}<x<\sqrt{3}\Rightarrow \frac{1}{\sqrt{2}}>\frac{1}{x}>\frac{1}{\sqrt{3}}\Rightarrow \left\{\frac{1}{x}\right\}=\frac{1}{x}$$

$(\because x>0)$

$\{x^2\}=x^2-2$

$$\Rightarrow \frac{1}{x}=x^2-2\Rightarrow x^3-2x-1=0\Rightarrow x=-1,\frac{1\pm\sqrt{5}}{2}$$

$$\therefore x=\frac{1+\sqrt{5}}{2}\quad x>0$$

43. **(D)**

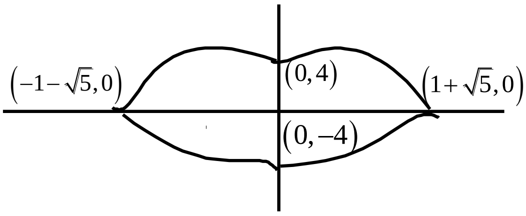
$$\int_0^1 \lim_{n\to\infty}\sum_{k=0}^{n}\frac{x^{k+2}.2^k}{k!}dx=\int_0^1 x^2.\lim_{n\to\infty}\sum_{k=0}^{n}\frac{(2x)^k}{k!}.dx$$

$$=\int_0^1 x^2.e^{2x}dx=\frac{e^2-1}{4}$$

44. **(A)**

Curve $|y|=-(1-|x|)^2+5$ is symmetric about both axes, required area $=4\int_0^{\sqrt{5}+1} y.dx$

$$=4\int_0^{\sqrt{5}+1}\left(5-(x-1)^2\right)dx$$



$$=\frac{8}{3}\left(7+5\sqrt{5}\right)$$

45. **(C)**

$$\frac{xdy}{x^2+y^2}=\left(\frac{y}{x^2+y^2}-1\right)dx \Rightarrow \frac{xdy-ydx}{x^2+y^2}=-dx$$

$$\Rightarrow \int d\left(\tan^{-1}\left(\frac{y}{x}\right)\right)=-\int dx$$

$$\tan^{-1}\left(\frac{y}{x}\right)=-x+c \Rightarrow \frac{y}{x}=\tan(c-x)$$

46. **(C)**

$$\Delta=\begin{vmatrix}[x]+1 & [y] & [z]\\ [x] & [y]+1 & [z]\\ [x] & [y] & [z]+1\end{vmatrix} \qquad R_1 \to R_1-R_2 \text{ then } R_2 \to R_2-R_3$$

$$\Delta=\begin{vmatrix}1 & -1 & 0\\ 0 & 1 & -1\\ [x] & [y] & [z]+1\end{vmatrix}=1([z]+1+[y])+[x]$$

$$\Delta=[x]+[y]+[z]+1 \qquad \Delta_{max}=2+0+1+1=4$$

47. **(A)**

$$\vec{n}_{OAB}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & 2 & 1\\ 2 & 1 & 3\end{vmatrix}=5\hat{i}-\hat{j}-3\hat{k}\ ;\ \vec{n}_{ABC}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & -1 & 2\\ -2 & -1 & 1\end{vmatrix}=\hat{i}-5\hat{j}-3\hat{k}$$

$$\theta=\cos^{-1}\left(\frac{\vec{n}_1.\vec{n}_2}{|\vec{n}_1|.|\vec{n}_2|}\right)=\cos^{-1}\left(\frac{19}{35}\right)$$

48. **(C)**

The man is one step away from starting point in 2 way …….(i)
6 step forward & 5 step back
5 step forward & 6 step back ……..(ii)

$$={}^{11}C_6\left(0(0.6).4^5\right)^6+{}^{11}C_5(0.4)^5(0.6)^6=0.37$$

49. **(B)**

Let $h(x)=(f(x)-f(a))(g(b)-g(x))$

Then $h(x)$ is continuous & differentiability & $h(a)=h(b)=0$. Using Rolle's theorem.
$(\exists c \in (a,b)$ such that $h'(c)=0$

$\Rightarrow f'(c)(g(b)-g(c))-g'(c)(f(c)-f(a))=0$

$\Rightarrow \frac{f(c)-f(a)}{g(b)-g(c)}=\frac{f'(c)}{g'(c)}$

50. **(A)**

$xf(x)=3f^2(x)+2 \Rightarrow f(x)+xf'(x)=6f(x).f^1(x)$

$\Rightarrow f^1(x)=\frac{f(x)}{6f(x)-x}$

$I=\int \frac{2x(x-6f(x))+f(x)}{(6f(x)-x)(x^2-f(x))^2}, dx=-\int \frac{2x-f^1(x)}{(x^2-f(x))^2}dx$

$I=\frac{1}{x^2-f(x)}+c$

51. **(2)**

$a\bar{a}=1, b\bar{b}=4, c\bar{c}=9$

$|9ab+4ac+bc|=|abc\bar{c}+abc\bar{b}+abc\bar{a}|=12$

$\Rightarrow |abc||\bar{a}+\bar{b}+\bar{c}|=12$

$\Rightarrow 6|\bar{a}+\bar{b}+\bar{c}|=12 \Rightarrow |a+b+c|=2$

52. **(2)**

Let $x=I+f$ $\qquad [x]=\frac{2x\{x\}}{x+\{x\}}$

$\Rightarrow I=\frac{2f(I+f)}{I+2f} \Rightarrow f=0, \frac{1}{\sqrt{2}}$ $\qquad f^2=0 \Rightarrow I=0 \Rightarrow x=0$, not possible

$\therefore f=\frac{1}{\sqrt{2}}$ $\qquad I=\pm\sqrt{2}\left(\frac{1}{\sqrt{2}}\right)=\pm 1$

$\therefore x=1+\frac{1}{\sqrt{2}}, -1+\frac{1}{\sqrt{2}}$

53. **(3)**

$A \equiv \left(ct, \frac{c}{t}\right)$ normal at A again cuts hyperbola at $B(ct')$

$t'=-\frac{1}{t^3}$

$M_{OA}=\frac{1}{t^2}, M_{OB}=t^6 \Rightarrow \tan\alpha=\frac{1/t^2-t^6}{1+t^4}$

$\tan A = \frac{p^2 - 1/p^2}{1+1} = \frac{p^4 - 1}{2p^2}$

$\frac{\tan\alpha}{\tan A} = -2 \Rightarrow$ convert in $\sin\theta \,\&\, \cos\theta$ & take compendo dividend

$\Rightarrow \frac{\sin(\alpha - A)}{\sin(\alpha + A)} = 3$

54. **(6)**

Equation of plane is $\frac{x}{6} + \frac{y}{4} + \frac{z}{2} = 1 \Rightarrow 2x + 3y + 6z - 12$ sphere centre should be (r, r, r) cause it touches all planes. $xy, zy, zx \,\&\, r < 2$ to be inside tetrahedron, distance from centre to any plane should 'r'.

Equation of plane $2x + 3y + 6z - 12 = 0$

$r = \left|\frac{2r + 3r + 6r - 12}{7}\right|$

$r = 3, 2/3 \qquad\qquad r \neq 3$

$r = 2/3 \Rightarrow 9r = 6$

55. **(6)**

$p_1 = \frac{2\Delta}{a}, P_2 = \frac{2\Delta}{b}, P_3 = \frac{2\Delta}{c}$

$\therefore P_1 + P_2 + P_3 \geq 3(P_1P_2P_3)^{1/3} \therefore P_1 + P_2 + P_3 \geq 3(P_1P_2P_3)^{1/3} = 3\left(\frac{(2\Delta)^3}{abc}\right)^{1/3} = 6\Delta\left(\frac{1}{abc}\right)^{1/3}$

$= 6rs\left(\frac{1}{abc}\right)^{1/3} = 6r\left(\frac{a+b+c}{2}\right)\left(\frac{1}{abc}\right)^{1/3} = 9r\left(\frac{a+b+c}{3}\right)\left(\frac{1}{abc}\right)^{1/3}, \frac{a+b+c}{3} = A.M., (abc)^{1/3} = G.M$

$A.M / G.M \geq 1$

$P_1 + P_2 + P_3 \geq 9r$

$P_1 + P_2 + P_3 \geq 9 \times 2/3 \Rightarrow \sum P_i \geq 6$

56. **(3)**

$(\tan\alpha)x + (\sin\alpha)y = \alpha \qquad : (\alpha \operatorname{cosec}\alpha)x + (\cos\alpha)y = 1$

$x = \frac{\alpha\cos\alpha - \sin\alpha}{\sin\alpha - \alpha} \Rightarrow \lim_{x\to 0} x - h \Rightarrow h = 2$

$y = \frac{\alpha - x\tan\alpha}{\sin\alpha} \Rightarrow k = \lim_{\alpha\to 0}\left(\frac{\alpha}{\sin\alpha} - \frac{x}{\cos\alpha}\right)$

$K = 1 - 2 = -1 \qquad \left[\because \lim_{x\to 0} x = 2\right]$

$h - k = 2 - (-1) = 3$

57. **(2)**

Let $A = B \qquad 2A + C = \pi \qquad \& \; 2\tan A + \tan C = 300$

$\tan 2A = -\tan C$

$2\tan A - \tan 2A = 300 \qquad\qquad$ if $\tan A = x$

$\Rightarrow 2x - \frac{2x}{1-x^2} = 300 \Rightarrow x^3 - 150x^2 + 150 = 0$

Let $f(x) = x^3 - 150x^2 + 150 \Rightarrow f'(x) = 3x^2 - 300x$

$\Rightarrow f'x$ has roots 0 & 100

$f(0).f(100) < 0$

Hence $f(x)$ has 3 roots which are distinct. Sum of roots of $f(x) = 150 > 0$. Product $= -150 < 0$

$\Rightarrow$ Exactly 2 roots are $+ve$ & are in negative, in isosceles Δ equal angles are always less than 90°

$\Rightarrow$ So only 2 non-similar isosceles Δ are possible

58. **(6)**

$I = \int \sin 4x.e^{\tan^2 x}.dx = 2\int \sin 2x.\cos 2x.e^{\tan^2 x}dx$

$I = 4\int \sin x \cos x \left(\frac{1-\tan^2 x}{1+\tan^2 x}\right).e^{\tan^2 x}.dx$

$\tan^2 x = t$

$2\tan x.\sec^2 xdx = dt$

$= -2\int \left\{\frac{1}{(1+t)^2} - \frac{2}{(1+t)^3}\right\} e + dt = -2\frac{dt}{(t+1)^2} + C$

$A = 2, C = 4$

$+C = 6$

59. **(6)**

$\int_1^{xy} f(t)dt = y\int_1^x f(t)dt + x\int_1^y f(t)dt$

Partial diff. w.r.t .x

$f(x,y).y = y.f(x) + f\int_1^y f(t)dt$

$x = 1 \quad yf(y) = y.f(1) + \int_1^y f(t)dt \qquad f(1) = 3$

$yf(y) = 3y + \int_1^y f(t)dt$

Diff. w.r.t'y $\quad yf'(y) + f(y) = 3 + f(y)$

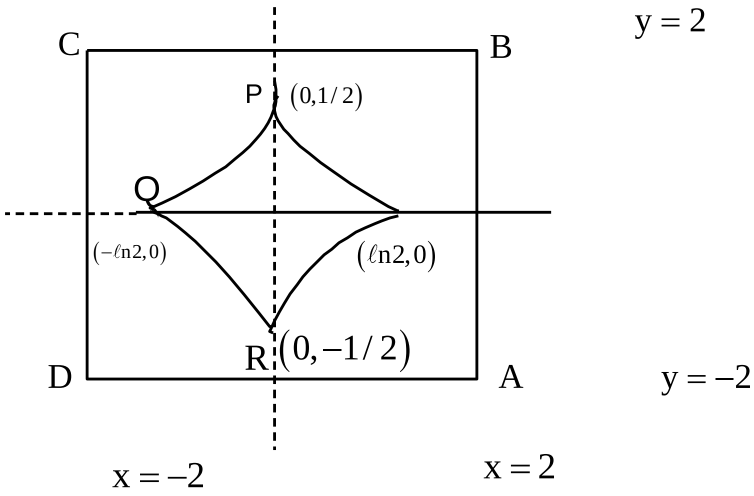
$\Rightarrow f(y) = 3\ell ny + c \qquad f(1) = 3 \Rightarrow c = 3$

$\Rightarrow f(y) = 3\ln y + 3 \Rightarrow f(e) = 3\ell ne + 3 = 6$

60. **(7)**

$\frac{|x|+|y|}{2} + \frac{|x|-|y|}{2} \le 2$

$\rightarrow$ required region $|x| \le 2, |y| \le 2, |x| + |y| \le 4$



$|y| = e^{-|x|} - \frac{1}{2}$ is symmetric about x & y

Required area is shaded part (A)

$A = \text{DABCD} - (\text{area curve PQRS})$

$$A = 16 - 4\int^{\ell n 2}\left(e^{-x} - \frac{1}{2}\right)dx$$

$A = 14 + \ell n 4$

$$\frac{A - \ell n 4}{2} = 7$$

ANSWER KEY

1.	[A]	2.	[D]	3.	[C]	4.	[A]	5.	[B]
6.	[A]	7.	[C]	8.	[B]	9.	[A]	10.	[C]
11.	[B]	12.	[C]	13.	[A]	14.	[B]	15.	[D]
16.	[A]	17.	[1]	18.	[6]	19.	[4]		

20. $[A-q; B-s; C-r; D-r.]$

SOLUTION

1. **(A)**

By conservation of energy

$$\frac{1}{2}m_{red}v_{rel}^2 = \frac{1}{2}kx^2 \therefore x = v\sqrt{\frac{m}{k}}$$

2. **(D)**

No dispersion for single wavelength.

3. **(C)**

Power delivered by battery $= E_{rms}(I_{rms})_1 \cos\theta_1 + E_{rms}(I_{rms})_2 \cos\theta_2$

$$= 130 \times 10 \times \frac{5}{13} + 130 \times 13 \times \frac{6}{10}$$

$$= 1514 \text{ watt}$$

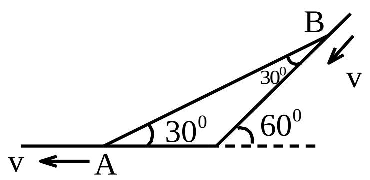
4. **(A)**

$$d\sin\theta = \Delta x = \text{path difference} = (2n+1)\frac{\lambda}{2}$$

$$\sin\theta = \left(\frac{2n+1}{6}\right) \le 1 \therefore n \le 2.5$$

$n = 0, 1, 2$ (three minima on one side)

5. **(B)**



$$\omega = \frac{(v_1)_{rel}}{L} = \frac{v}{L}$$

6. **(A)**

Use concept of LC oscillation

$$\frac{1}{2}CV_0^2 - \frac{\left(CV_0\cos\left(\frac{t}{\sqrt{LC}}\right)\right)^2}{2C} = \frac{Q_{max}^2}{4C}$$

$$\therefore Q_{max} = \left(\frac{CV_0}{\sqrt{2}}\right)$$

7. **(C)**

Rate of heat generation $= |2fv\cos 135|$

$$= \sqrt{2}\ \text{J/sec}$$

8. **(B)**

$$mvr = \frac{nh}{2\pi}, \frac{GMm}{r^2} = \frac{mv^2}{r} \qquad En = -\frac{GMm}{r} + \frac{1}{2}mv^2 = -\frac{GMm}{2r}$$

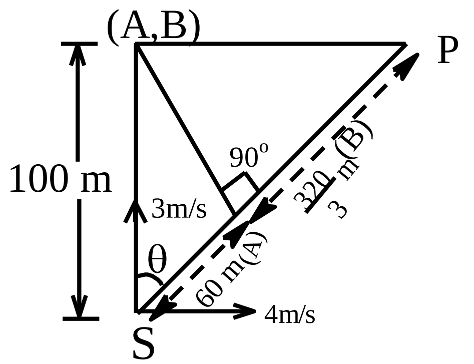
9. **(A)**

$$KE = \frac{1}{2}\mu\ell(mv)^2 = \frac{Tm^2a}{2}$$

10. **(C)**

$$\Delta x = 100\lambda = d\sin\theta \therefore \theta = 30^0 \quad \tan\theta = \frac{y}{D} \therefore y = \frac{1}{\sqrt{3}}m$$

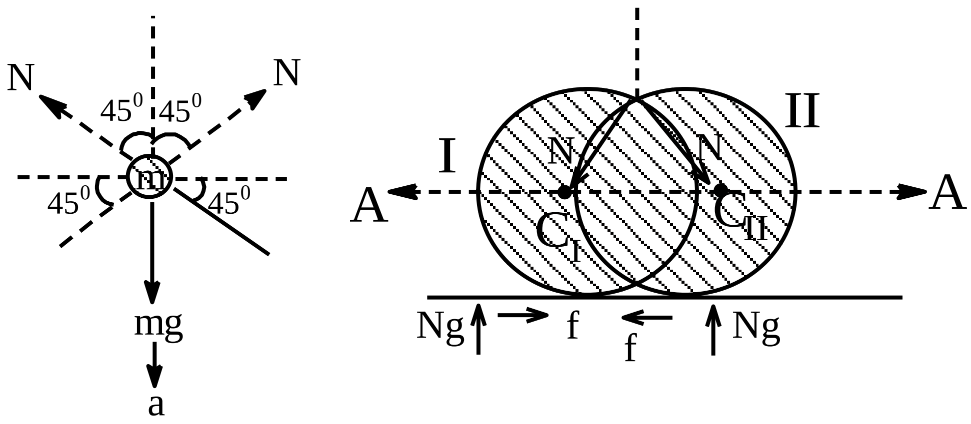
11 to 13.



$$t_A = \frac{60}{5} = 12\,\text{sec}$$

$$t_B = \frac{320}{3\times 5} = \frac{64}{3}\text{sec}$$

14 to 16



$a = A$ ….. (1)

$mg - 2N\cos 45^0 = ma$. …… (2)

$N\sin 45^0 - f = MA$ ….. ….(3)

$fR = MR^2\alpha$ …….. (4)

Solving (1), (2), (3) and (4)

$a = \frac{10}{9} m/s^2, f = \frac{2}{9} N, N_g = \frac{22N}{9}$

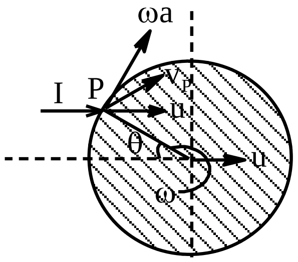
17. $\lambda_{eva} = \frac{\ln 2}{8}, \lambda_{suc} = \frac{\ln 2}{24}$

$\lambda_{eq} = \lambda_{eva} + \lambda_{suc} = \frac{\ln 2}{6} = \frac{\ln 2}{t_{1/2}} \therefore t_{1/2} = 6h$

$N = \frac{N_0}{2^4} = \frac{16}{16} = 1kg$

18. $\frac{\lambda}{2\pi\varepsilon_0 r} = E \therefore r = 6m$

19.



$V_P = \left(u^2 + (\omega a)^2 + 2u(\omega a)\cos(90-\theta)\right)^{1/2}$ where $\tan\theta = \frac{\sqrt{3}}{2}$ (By equating velocities)

$= \left(7v^2 + 7v^2\sin^2\theta + 14v^2\sin^2\theta\right)^{1/2}$

$= 4v$

20. (A) $C=\left(\frac{C}{C+V_S}\right)f\lambda',\qquad \lambda'=\left(\frac{C+V_S}{f}\right)$

(B) $C=\left(\frac{C}{C-V_S}\right)f\,\lambda',\qquad \lambda'=\left(\frac{C-V_S}{f}\right)$

(C) $\left(\frac{C+V_S}{C+V_\omega}\right)\left[\left(\frac{C-V_\omega}{C-V_s}\right)f\right]\lambda'=(C+V_S),\lambda'=\left(\frac{C+V_\omega}{C-V\omega}\right)\left(\frac{C-V_S}{f}\right)$

(D) $\left(\frac{C}{C+V_\omega}\right)\left[\left(\frac{C-V_\omega}{C-V_S}\right)f\right]\lambda'=C,\lambda'=\left(\frac{C+V_\omega}{C-V_\omega}\right)\left(\frac{C-V_S}{f}\right)$

ANSWER KEY

21.	**(C)**	22.	**(D)**	23.	**(A)**	24.	**(B)**	25.	**(C)**
26.	**(C)**	27.	**(B)**	28.	**(C)**	29.	**(B)**	30.	**(C)**
31.	**(B)**	32.	**(A)**	33.	**(C)**	34.	**(C)**	35.	**(C)**
36.	**(A)**	37.	**(9)**	38.	**(5)**	39.	**(8)**		

40. **(A) : P, S** **(B) : P, S** **(C) : Q, R, S** **(D) : Q, R**

SOLUTION

21. **(C)**

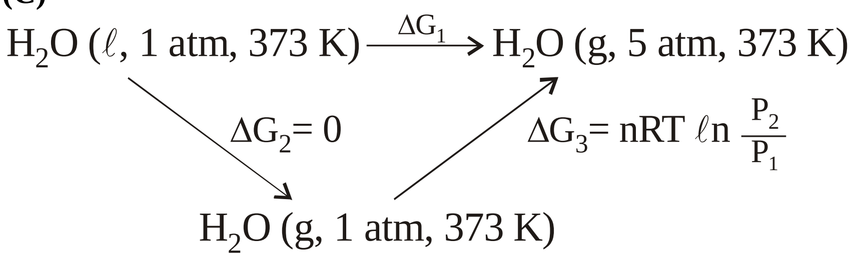
22. **(D)**

23. **(A)**

24. **(B)**

25. **(C)**

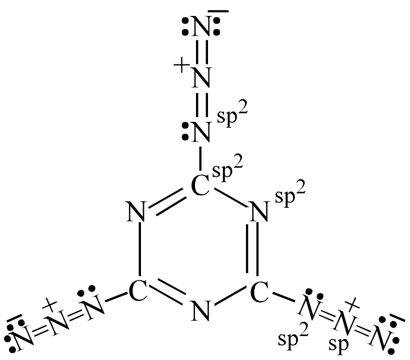
26. **(C)**



$$\Delta G_1 = \Delta G_3 = 5 \times 2 \times 373\ ln\frac{5}{1}$$

$$= 3730\ ln5$$

27. **(B)**



28. **(C)**

29. **(B)**

30. **(C)**

31. **(B)**

$$P = \frac{10}{30}\times 0.6 + \frac{20}{30}\times 0.9 = 0.2 + 0.6 = 0.8$$

32. **(A)**

$y_A = \frac{n_A}{10}$ $\quad$ $y_B = \frac{n_B}{10}$

$x_A = \frac{10-n_A}{20}$ $\quad$ $x_B = \frac{20-n_B}{20}$

$\frac{n_A}{10}P_T = \frac{10-n_A}{20}\times 0.6$ $\quad$ $\frac{n_B}{10}P_T = \frac{20-n_B}{20}\times 0.9$

$\frac{n_A}{n_B} = \frac{10-n_A}{20-n_B}\times\frac{2}{3}$

$60n_A - 3n_An_B = 20n_B - 2n_An_B$

$n_An_B - 60n_A + 20n_B = 0$

$n_A(10 - n_A) - 60n_A + 20(10 - n_A) = 0$

$-n_A^2 + 10n_A - 60n_A - 20n_A + 200$ $\quad$ $n_A^2 + 70n_A - 200 = 0$

$n_A = \frac{-70 \pm \sqrt{4900+800}}{2}$

$n_A = \frac{-70\pm 75.5}{2} = \frac{5.5}{2} = 2.75$

$n_A = 2.75$ $\quad$ $n_B = 7.75$

$P_T = \left(\frac{7.25\times 0.6 + 12.75\times 0.9}{20}\right) = 0.791$

33. **(C)**

$$\frac{1}{P} = \frac{1/3}{0.6} + \frac{2/3}{0.9} = \frac{1}{1.8} + \frac{2}{2.7} \Rightarrow P = 0.77$$

34. **(C)**

Ore is Ag_2S

then, $Ag_2S + 4NaCN \xrightleftharpoons{Air} 2Na[Ag(CN)_2] + Na_2SO_4$
(complex)

Complex $[Ag(CN)_2]^-$ is linear and diamagnetic

35. **(C)**

A $\rightarrow$ Sodium ethylxanthate acts as collector

B $\rightarrow$ Levigation can be used for sulphide ore to remove majority of gangue when density difference is high.

C → Froth Floatation can also be used for nonsulphide ores having sulphide impurities, and the ore is recovered by using suitable activator.
D → During roasting impurities like S, As are removed as their volatile oxides SO_2, As_2O_3

36. **(A)**

37. **(9)**

38. **(5)**

39. **(8)**

40. **(A) : P, S** **(B) : P, S** **(C) : Q, R, S** **(D) : Q, R**

ANSWER KEY

41.	(A)	42.	(A)	43.	(A)	44.	(B)	45.	(C)
46.	(D)	47.	(A)	48.	(D)	49.	(A)	50.	(B)
51.	(C)	52.	(B)	53.	(D)	54.	(C)	55.	(D)
56.	(D)	57.	(1)	58.	(8)	59.	(6)		

60. (A–P, B–Q, C–Q, D–P)

SOLUTION

41. **(A)**

$$\tan\alpha = \frac{\tan(k+1)\alpha - \tan k\alpha}{1+\tan(k+1)\alpha\cdot\tan\alpha}$$

$\Rightarrow$ $\tan(k+1)\alpha\cdot\tan\alpha = -1+\cot\alpha\{\tan(k+1)\alpha-\tan\alpha\}$

$$S=\sum_{k=1}^{n}\tan(k+1)\alpha\cdot\tan\alpha = \text{'n' times}(-1)+\cot\alpha\{\tan(n+1)\alpha-\tan\alpha\}$$

$$=-n+\cot\alpha\cdot\tan(n+1)\alpha-1$$

$$=\cot\alpha\cdot\tan(n+1)\alpha-n-1$$

42. **(A)**

$$\frac{1}{a+\omega}+\frac{1}{b+\omega}+\frac{1}{c+\omega}+\frac{1}{d+\omega}=\frac{1}{\omega} \qquad .....1$$

Taking conjugate $\Rightarrow \dfrac{1}{a+\omega^2}+\dfrac{1}{b+\omega^2}+\dfrac{1}{c+\omega^2}+\dfrac{1}{d+\omega^2}=\dfrac{1}{\omega}$ ....(2)

Taking difference (1) & (2)

$$\sum\left(\frac{1}{a+\omega}-\frac{1}{a+\omega^2}\right)=\frac{1}{\omega}-\frac{1}{\omega^2}$$

$$\sum\frac{\omega^2-\omega}{(a+\omega)(a+\omega^2)}=\frac{\omega^2-\omega}{\omega^3}=\frac{\omega^2-\omega}{1}$$

$$\sum\frac{1}{a^2-a+1}=1$$

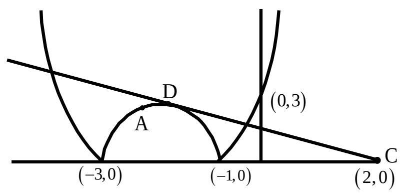
43. **(A)**

If $|x^2+4x+3|=mx-2m$ has exactly 3 solutions then curve $y_1=|x^2+4x+3|$ & $y_2=m(x-2)$ inters at exactly 3 points $\Rightarrow$ that implies y_2 is tangent to y_1 at one point & cut at 2 points

$-3<x<-2 \Rightarrow y_1=-(x^2+4x+3)$

$\Rightarrow$ $m(x-2)=-(x^2+4x+3)$ has equal roots

$(m+4)^2-4(3-2m)=0$



$m^2+16m+4=0$

$\Rightarrow \quad m=-8\pm 2\sqrt{15}$

Required $m=-8+2\sqrt{15}$

A is $\equiv(-1,1)$ $m_{AC}=-1/3$

$-8-2\sqrt{15}<-1/3<-8+2$

$m_{AC}<m_{DC}$

44. **(B)**

Number of sequences can be formed $=4^5$

Number of sequences including $BAD=3.4^2$

$\underline{B}\ \underline{A}\ \underline{D}\ \_\ \_\ ,\ \_\ \underline{B}\ \underline{A}\ \underline{D}\ \_\ ,\ \_\ \_\ \underline{B}\ \underline{A}\ \underline{D}$

Total number of sequence which does not include

$BAD=4^5-3.4^2=976$

45. **(C)**

Middle term of $(1+x)^{2n}=2n_C\cdot x^n=t_{n+1}$

t_{n+1} is greatest term too then

$t_{n+1}\ge t_n$ ...(1) $\qquad t_{n+1}\ge t_{n+2}$ ...(2)

.........(1) $\quad 2n_{C_n}x^n\ge 2n_{C_{n-1}}x^{n-1}$

$\Rightarrow \quad (n+1)x\ge n\Rightarrow x\ge\dfrac{n}{n+1}$

.........(2) $\quad 2n_{C_n}x^n\ge 2n_{C_{n+1}}x^{n+1}$

$\Rightarrow \quad nx\le n+1 \quad\Rightarrow\quad x\le\dfrac{n+1}{n}$

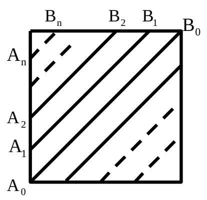
$\Rightarrow \quad x\in\left[\dfrac{n}{n+1},\dfrac{n+1}{n}\right]$

46. **(D)**

Length of diagonal is $n\sqrt{2}$

Hence side of square is $=n$

Lines $A_0B_0,\ A_1B_1,\ A_2B_2$ are parallel and each pair of most closest lines is at distance $\dfrac{1}{\sqrt{2}}$.



$\Rightarrow \quad A_0A_1=1,\ A_1A_2=1,\ ....A_{n-1}A_n=1$

$\Rightarrow \quad A_nB_n=\sqrt{2},\ A_{n-1}B_{n-1}=2\sqrt{2},\ A_{n-2}B_{n-2}=3\sqrt{2},\ A_1B_1=(n-1)\sqrt{2}$

And these type of lines are on both side of diagonal

Hence required sum is

$\Rightarrow \quad 2\left\{\sqrt{2}+2\sqrt{2}+3\sqrt{2}+.......(n-1)\sqrt{2}\right\}+\underset{\text{diagonal}}{2\sqrt{2}}$

$\Rightarrow \quad 2\sqrt{2}\dfrac{n(n-1)}{2}+n\sqrt{2} \quad\Rightarrow\quad n^2\sqrt{2}$

47. **(A)**

Lets take co-ordinate of A, B, C are $\left(\frac{r_1}{2},\frac{\sqrt{3}r_1}{2}\right)\left(\frac{r_2}{2},\frac{\sqrt{3}r_2}{2}\right)$ & $\left(\frac{r_3}{2},\frac{\sqrt{3}r_3}{2}\right)$

$r \to$ distance of point from origin

$x^3+y^3+3xy+5x^2+3y^2+4x+5y-1=0$

$$\left(\frac{r}{2}\right)^2+\left(\frac{\sqrt{3}r}{2}\right)^3+3\left(\frac{r}{2}\right)\left(\frac{\sqrt{3}r}{2}\right)+5\left(\frac{r}{2}\right)^2+3\left(\frac{\sqrt{3}r}{2}\right)+4\left(\frac{R}{2}\right)+\left(\frac{\sqrt{3}R}{2}\right)-1=0$$

$$R^3\left(\frac{1+3\sqrt{3}}{8}\right)+R^2(..........)+R(..........)-1=0$$

We need $r_1\cdot r_2\cdot r_3$

$$r_1\cdot r_2\cdot r_3=\frac{1}{\left(1+3\sqrt{3}\right)/8}=\frac{8}{3\sqrt{3}+1}=\frac{8\left(3\sqrt{3}-1\right)}{26}=\frac{4\left(3\sqrt{3}-1\right)}{13}$$

48. **(D)**

Both parabolas are symmetric about line $y=x$.

Hence minimum distance is distance between their parallel lines with slope '1', incase those parallel lines do not intersect

$P_1:(y-4)^2=4(x-6),\ P_2:(x-4)^2=4(y-6)$

Tangent on P_1 $\qquad y-4=1(x-6)+\frac{1}{1}$

$y=x-1$

Required distance $=2\left|\frac{1}{\sqrt{2}}\right|=\sqrt{2}$

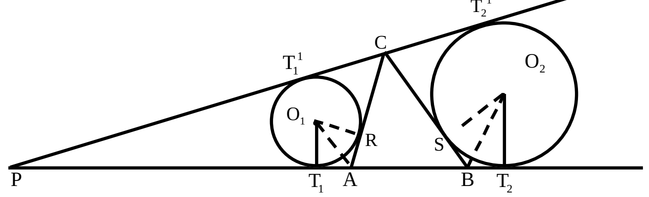
49. **(A)**

Normal at P is bisector of angle between focal distance SP & $S'P \Rightarrow \frac{SQ}{S'Q}=\frac{SP}{S'P}$

$SP=a-ex, S'P=a+ex,\ a=4,\ e=\sqrt{7}/4$

$$\frac{SQ}{S'Q}=\frac{4-\sqrt{7}/4\times 2}{4+\sqrt{7}/4\times 2}=\frac{8-\sqrt{7}}{8+\sqrt{7}}$$

50. **(B)**



$O_1T_1=O_1R=r_1,\ O_2S=O_2T_2=r_2$

$\angle RO_1A=30^o,\ \angle T_2O_2B=30^o$

$T_1T_2=T_1^1T_2^1 \Leftarrow$ length of D.C.T.

$$T_1T_2 = T_1A + AB + BT_2 = RA + AB + BS = \frac{r_1}{\sqrt{3}} + a + \frac{r_2}{\sqrt{3}}$$

$$T_1^1T_2^1 = CT_1^1 + CY_2^1 = CR + CS = CA - RA + CB - RB$$

$$= 2a - \left(\frac{r_1 + r_2}{\sqrt{3}}\right)$$

$$\Rightarrow \quad \frac{r_1 + r_2}{\sqrt{3}} + a = 2a - \left(\frac{r_1 + r_2}{\sqrt{3}}\right) \Rightarrow r_1 + r_2 = \frac{\sqrt{3}a}{2}$$

51. **(C)** 52. **(B)** 53. **(D)**

Equation of incident beam is $\lambda(x+2)+(2-y)=0$

$\Rightarrow$ rays are coming from $(-2,2)$

Similarly rays converge at $(2,2)$

Hence foci of ellipse are $S_1(2,2)$ & $S_2(-2,2)$

Centre is $(0,2)$ further given circle is auxiliary circle so its radius $= 3 =$ semi major axis (a)

$S_1S_2 = 2ae = 4 \Rightarrow e = 2/3$

The triangle have largest area it the point of incidence coincides with an end of minor axis.

$$\Delta = \frac{1}{2} \cdot b \cdot 2ae = a\sqrt{1-e^2} \cdot ae = 2\sqrt{5}$$

Since for every position of required point an ellipse. Total distance travelled is $\Rightarrow PS_1 + PS_2 = 2a = 6$

So no such point exist for which it is least.

54. **(C)** 55. **(D)** 56. **(D)**

$$m = \lim_{x\to\pm\infty} \frac{y}{x} = \lim_{x\to\pm\infty} \frac{x^2+2x-1}{x} = 1$$

$$c = \lim_{x\to 1\pm\infty} (y - mx) = \lim_{x\to\pm\infty} (f(x) - x) = \lim_{x\to\pm\infty} \left|\frac{x^2+2x-1}{x} - x\right.$$

$c = 2$

$y = x + 2$

Similarly 55. $y = x$ 56. $y = x + 1$

57. **(1)**

Let A_t, $t = 1,2,3,4,5,6$ be the set of days on which the friend is present at dinner & B_t be the set of days on which friend is absent at dinner

Then $|A_t| = |B_t| = 7$

Also $|A_i \cap A_j| = 5$, $|A_i \cap A_j \cap A_k| = 4$

$|A_i \cap A_j \cap A_k \cap A_\ell| = 3$, $|A_i \cap A_j \cap A_k \cap A_\ell \cap A_m| = 2$

& $|A_1 \cap A_2 \cap A_3 \cap A_4 \cap A_5 \cap A_6| = 1$

Where i, j, k, ℓ, m vary from 1 to 6 & distinct number of days/dinners at which at least one friend was present $= |A_1 \cup A_2 \cup A_3 \cup A_4 \cup A_5 \cup A_6|$

$$\sum|A_i|-\sum|A_i\cap A_j|+\sum|A_i\cap A_j\cap A_k|-\sum|A_i\cap A_j\cap A_k\cap A_\ell|+\sum|A_i\cap A_j\cap A_k\cap A_m|-|A_1\cap A_2\cap........A_6|$$

$$=6_{C_1}\times 7-6_{C_2}\times 5+6_{C_3}\times 4-6_{C_4}\times 3+6_{C_5}\times 2-6_{C_6}\times 1=13$$

Total number of dinners $=|At|+|Bt|=14$

Number of dinners person had alone $=14-13=1$

58. **(8)**

For given condition A should be scalar matrix if each element of diagonal is ‘a’

$$\text{tr}(A)=3a=12\Rightarrow a=4$$

$$\det(A)=4^3\Rightarrow\sqrt{\det(A)}=8$$

59. **(6)**

Number of blue socks x, number of red socks y

$(x>y)\ x+y\le 17$

$$\frac{{}^xC_2+{}^yC_2}{{}^{x+y}C_2}=\frac{1}{2}\Rightarrow x^2+y^2-2xy-x-y=0\Rightarrow(x-y)^2=(x+y)$$

$(x+y)$ should be a square of number

For largest ‘y’ $\quad x+y=16\Rightarrow x-y=4$

$\Rightarrow\quad y=6$

60. **(A–P, B–Q, C–Q, D–P)**

(A) $f(-x)=\dfrac{-x}{e^{-x}-1}-\dfrac{x}{2}+1=\dfrac{xe^x}{e^x-1}-x+\dfrac{x}{2}+1$

$$=\frac{x}{e^x-1}+\frac{x}{2}+1=f(x)\Rightarrow f(x)\text{ is even}$$

(B) $f(x)$ is even connot be one-one

(C) $\lim\limits_{x\to 0}f(x)=\lim\limits_{x\to 0}\left(\dfrac{x}{e^x-1}+\dfrac{x}{2}+1\right)=1+0+1=2$

$\forall x>0\ f(x)>0\ \&\ f(x)$ is even hence $f(-x)>0$

$\Rightarrow\quad f(x)>0\qquad\forall x\in R$ hence not onto

(D) True.