SOLUTIONS

1. (B)

$\vec{A}.\vec{B}=0,$ then $\vec{A}\perp\vec{B}$

$\left(\hat{i}+\hat{j}+\hat{k}\right).\left(3\hat{i}+2\hat{j}-5\hat{k}\right)=3+2-5=0$

2. (A)

$(\vec{A}_1+2\vec{A}_2).(3\vec{A}_1-4\vec{A}_2)$

$=3A_1^2-4\vec{A}_1.\vec{A}_2+6\vec{A}_1.\vec{A}_2-8\vec{A}_2$

$=3A_1^2+2A_1.A_2-8A_2^2$ ….(1)

And $|\vec{A}_1+\vec{A}_2|=3$

$\Rightarrow A_1^2+A_2^2+2\vec{A}_1.\vec{A}_2=9$

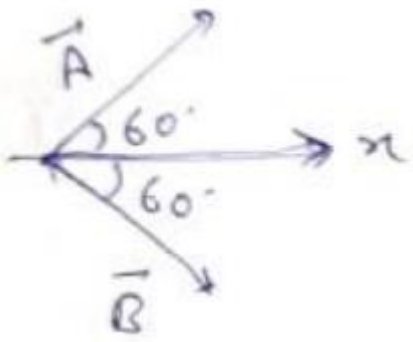
$4+9+2\vec{A}_1.\vec{A}_2=9$

$2\vec{A}_1.\vec{A}_2=-4$ …..(2)

From (1) and (2)

$(\vec{A}_1+2\vec{A}_2).(3\vec{A}_1-4\vec{A}_1)=-64$

3. (B)



$R=\sqrt{A^2+B^2+2AB\cos120^0}$

$=2cm\ along+x-axis$

4. (C)

$\left|\vec{A}\right|=\sqrt{21},\ \left|\vec{B}\right|=5$

$A+B\geq C$

5. (B)

Area $= \left|\vec{A} \times \vec{B}\right|$

$\frac{AB}{2} = AB\sin\theta$

$\sin\theta = \frac{1}{2}$

$\theta = 30^{o}$

6. (B)

$\frac{2}{5} = \frac{p}{7} = \frac{q}{3} \Rightarrow P = \frac{14}{5}$ & $q = \frac{6}{5}$

7. (C)

$\vec{F}_1 = 45\hat{i}, \vec{F}_2 = 20\,\hat{j}$

$\vec{F}_3 = -25\sin 37^{o}\,\hat{i} + 25\cos 37^{o} = -15\hat{i} + 20\,\hat{j}$

$F_{net} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 = 30\hat{i} + 40\,\hat{j}$

$\left|\vec{F}\right| = \sqrt{30^{20} + 40^{2}} = 50\,\text{N}$

8. (D)

9. (C)

10. (A)

11. (D)

12. (A)

13. (A)

14. (B)

Here, $x = 3\text{m}$, $y = 4\text{m}$ and $z = 5\text{m}$

$\therefore \quad |s| = \sqrt{x^2 + y^2 + z^2} = \sqrt{3^2 + 4^2 + 5^2}$

$= \sqrt{9 + 16 + 25} = \sqrt{50} = 5\sqrt{2}\,\text{m}$

15. (C)

AB = Position vector of B – Position vector of A.

$= \left(6\hat{i} + 7\,\hat{j} - 3\hat{k}\right) - \left(3\hat{i} + 4\,\hat{j} + 5\hat{k}\right)$

$= 3\hat{i} + 3\,\hat{j} - 8\hat{k}$

$\therefore \quad |AB| = \sqrt{3^2 + 3^2 + (-8)} = \sqrt{9 + 9 + 64} = \sqrt{82}\,\text{m}$

16. (B)

The resultant is $R = \hat{j}$

$$R = \hat{i} - 2\hat{j} + 2\hat{k} + 2\hat{i} + \hat{j} - \hat{k} + c$$

Or $\hat{j} = 3\hat{i} - \hat{j} + \hat{k} + c$

$\therefore \quad c = -3\hat{i} + 2\hat{j} - \hat{k}$

17. (C)

Let $a = 2\hat{i} + \hat{j} + 2\hat{k}$ and $b = \hat{i} - \hat{j} + \hat{k}$

$\because \; a \cdot b = |a||b| \cos\theta$

$$\therefore \; \cos\theta = \frac{a \cdot b}{|a||b|} = \frac{2 - 1 + 2}{\sqrt{4+1+4}\sqrt{1+1+1}} = \frac{1}{\sqrt{3}}$$

$$\therefore \; \theta = \cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$$

18. (B)

If a and b are perpendicular, $a \cdot b = 0$

Or $2x - 3 + 1 = 0$

$\therefore \quad x = 1$

19. (D)

$$|a + b| = \sqrt{a^2 + b^2 + 2ab\cos\theta}$$

$$= \sqrt{a^2 + b^2 + 2ab\cos 45^\circ}$$

$$= \sqrt{a^2 + b^2 + \sqrt{2}ab}$$

20. (D)

The displacement of the block is

$$s = r_B - r_i = r_B - r_A$$

$$= \left(4\hat{i} - 5\hat{j} + 6\hat{k}\right) - \left(\hat{i} + 2\hat{j} + 3\hat{k}\right) = \left(3\hat{i} - 4\hat{j} + 3\hat{k}\right) \text{m}$$

$\because \quad W = F \cdot s = \left(3\hat{i} + 4\hat{j} + 5\hat{k}\right) \cdot \left(3\hat{i} - 7\hat{j} + 3\hat{k}\right)$

$$= 9 - 28 + 15 = -4 \text{ N-m} = -4 \text{J}$$

IIT – JEE: 2025 **TW TEST** **DATE: 21/05/23**

TOPIC: MOLE CONCEPT

ANSWER KEY

21.	(B)	22.	(B)	23.	(C)	24.	(A)	25.	(A)
26.	(C)	27.	(A)	28.	(A)	29.	(B)	30.	(D)
31.	(A)	32.	(A)	33.	(D)	34.	(A)	35.	(A)
36.	(C)	37.	(C)	38.	(C)	39.	(D)	40.	(C)

SOLUTIONS

41. (B)

$\sec\theta+\tan\theta=3$

$\sec\theta-\tan\theta=\frac{1}{3}$

$2\sec\theta=3+\frac{1}{3}=\frac{10}{3}$

$\sec\theta=\frac{5}{3}$

$\therefore \quad \cos\theta=\frac{3}{5}$

42. (B)

$\sin\theta=\frac{4}{5}$ but θ lies in 2^{nd} or 4^{th}

Quadrant Hence, $\sin\theta=\frac{4}{5}$ or $-\frac{4}{5}$

43. (C)

$\Rightarrow \quad \sec^4\theta\left(\cos^2\theta\right)\left(1+\sin^2\theta\right)-2\tan^2\theta$

$\Rightarrow \quad \sec^2\theta\left(1+\sin^2\theta\right)-2\tan^2\theta \Rightarrow \sec^2\theta+\tan^2\theta-2\tan^2\theta$

$\Rightarrow \sec^2\theta-\tan^2\theta=1$

44. (D)

$d=10\theta$ where $\theta=\frac{360^o}{60^o}\times 20=\frac{2\pi}{3}$

$d=10\times\frac{2\pi}{3} \Rightarrow \frac{3d}{10\pi}=2$

45. (D)

$\sin 4<0$.

46. (C)

$\cos\left(A+B\right)=1 \Rightarrow A=-B$

$\therefore \ 2+\tan\left(-B\right)\cot B=2-1=1$

47. (C)

$\sin^2 A - \sin^2 B = \sin(A+B)\cdot\sin(A-B)$

$\therefore$ using this $K = \frac{1}{\sqrt{2}}$

$\therefore \quad \sqrt{2}K = 1$

48. (D)

If $A + B = 45^\circ$ then $(1+\tan A)(1+\tan B) = 2$

49. (B)

$\text{cosec}^2 x + 25\sec^2 x$

$1 + \cot^2 x + 25(1+\tan^2 x)$

$1 + \cot^2 x + 25 + 25\tan^2 x$

$26 + \underbrace{25\tan^2 x + \cot^2 x}$

$\geq 2\sqrt{25}$

≥ 10

$\therefore$ Least value of $\text{cosec}^2 x + 25\sec^2 x$ is 36.

50. (C)

$5\cos\theta + 3\left[\cos\theta\cdot\cos\left(\frac{\pi}{3}\right) - \sin\theta\cdot\sin\left(\frac{\pi}{3}\right)\right] + 3$

$5\cos\theta + \frac{3}{2}\cos\theta - \frac{3\sqrt{3}}{2}\sin\theta + 3$

$\frac{13}{2}\cos\theta - \frac{3\sqrt{3}}{2}\sin\theta + 3$

Now, $\lambda = \sqrt{\frac{169}{4} + \frac{27}{4}} + 3$

$= \sqrt{\frac{196}{4}} + 3$

$= 7 + 3 = 10$

$\mu = -7 + 3 = -4$

$\therefore \quad \lambda - \mu = 14$

51. (A)

$\cos^4\left(\frac{\pi}{8}\right) + \cos^4\left(\frac{3\pi}{8}\right) + \cos^4\left(\frac{5\pi}{8}\right) + \cos^4\left(\frac{7\pi}{8}\right)$

$\Rightarrow \quad \cos^4\left(\frac{\pi}{8}\right) + \cos^4\left(\frac{3\pi}{8}\right) + \cos^4\left(\frac{3\pi}{8}\right) + \cos^4\left(\frac{\pi}{8}\right)$

$\Rightarrow \quad 2\left[\cos^4\left(\frac{\pi}{8}\right) + \cos^4\left(\frac{3\pi}{8}\right)\right]$

$$\Rightarrow \quad 2\left[\cos^4\left(\frac{\pi}{8}\right)+\sin^4\left(\frac{\pi}{8}\right)\right]$$

$$\Rightarrow \quad 2\left[1-2\sin^2\left(\frac{\pi}{8}\right)\cos^2\left(\frac{\pi}{8}\right)\right]$$

$$\Rightarrow \quad 2\left[1-\frac{1}{2}\left(\sin^2\frac{\pi}{4}\right)\right]$$

$$\Rightarrow \quad 2\left[1-\frac{1}{2}\times\frac{1}{2}\right]=2\left[\frac{3}{4}\right]=\frac{3}{2}$$

52. (A)

$$(\alpha+\beta)-(\alpha-\beta)=2\beta$$

$$\tan\left[(\alpha+\beta)-(\alpha-\beta)\right]=\tan 2\beta$$

$$\frac{\tan(\alpha+\beta)-\tan(\alpha-\beta)}{1+\tan(\alpha+\beta)\cdot\tan(\alpha-\beta)}=\tan 2\beta$$

$$\therefore \quad \tan(\alpha+\beta)=\frac{3}{4} \text{ and } \tan(\alpha-\beta)=\frac{5}{12}$$

$$\Rightarrow \quad \tan 2\beta=\frac{\frac{3}{4}-\frac{5}{12}}{1+\frac{3}{4}\times\frac{5}{12}}=\frac{36-20}{48+15}$$

$$\tan 2\beta=\frac{16}{63}$$

53. (C)

$$\sin\theta\sin\left(60^\circ-\theta\right)\sin\left(60^\circ+\theta\right)=\frac{\sin 3\theta}{4}$$

$$\therefore \quad \sin 30^\circ\times\frac{\sin 30^\circ}{4}=\frac{1}{2}\times\frac{1}{8}=\frac{1}{16}.$$

54. (D)

$$\frac{1+\cos 20^\circ}{2}-\cos 50^\circ\left(\cos 10^\circ-\cos 50^\circ\right)$$

$$\frac{1+\cos 20^\circ}{2}-2\cos 50^\circ\sin 30^\circ\sin\left(20^\circ\right)$$

$$\frac{1+\cos 20^\circ}{2}-\cos 50^\circ\sin 20^\circ$$

$$\frac{1+\cos 20^\circ}{2}-\frac{1}{2}\left[\sin\left(70^\circ\right)-\sin\left(30^\circ\right)\right]$$

$$\frac{1+\cos 20^\circ-\sin 70^\circ+\sin 30^\circ}{2}$$

$$\frac{\frac{3}{2}+\cos 20^\circ-\cos 20^\circ}{2}=\frac{3}{4}$$

55. (C)

$$\frac{\sin\left(2^9\times\frac{\pi}{2^{10}}\right)}{2^9\times\sin\left(\frac{\pi}{2^{10}}\right)}\times\sin\left(\frac{\pi}{2^{10}}\right)=\frac{1}{512}$$

56. (A)

$$\sin\alpha+\sin\beta=\frac{21}{65}\ \Rightarrow\ 2\sin\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)=\frac{21}{65}$$

$$\cos\alpha+\cos\beta=\frac{-27}{65}\ \Rightarrow\ 2\cos\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)=\frac{-27}{65}$$

$\therefore\quad \tan\left(\frac{\alpha+\beta}{2}\right)=\frac{-21}{27}$

57. (D)

$\tan A\cdot\tan B+\tan B\cdot\tan C+\tan C\cdot\tan A=1$

$\frac{1}{2}+\frac{\tan C}{2}+\tan C=1$

$\frac{3\tan C}{2}=\frac{1}{2}\ \Rightarrow\ \tan C=\frac{1}{3}$

58. (A)

$$\frac{\cos(A-B)}{\cos(A+B)}=\frac{\cos A\cos B+\sin A\sin B}{\cos A\cos B-\sin A\sin B}$$

$$=\frac{1+\tan A\tan B}{1-\tan A\tan B}$$

$$=\frac{1+\frac{1}{3}}{1-\frac{1}{3}}=2$$

59. (D)

$\tan\left[2(A+B)\right]=\frac{2\tan(A+B)}{1-\tan^2(A+B)}$ $\qquad\left[\tan(A+B)=\frac{2+1}{1-2}=-3\right]$

$\therefore\quad \tan 2(A+B)=\frac{2\times(-3)}{1-(-3)^2}=\frac{-6}{-8}=\frac{3}{4}$

60. (D)

$$\frac{(\sin\theta+\cos\theta)^2}{(\cos\theta-\sin\theta)^2}=\frac{1+\sin 2\theta}{1-\sin 2\theta}=\frac{1+P}{1-P}$$