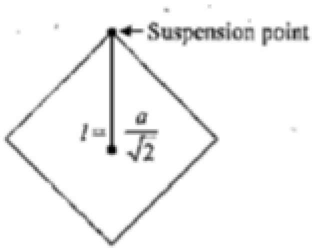
IIT – JEE: 2023 **TW TEST (ADV)** **DATE: 10/07/22**

TOPIC: SIMPLE HARMONIC MOTION

Answers & Solutions

1. (D)



Time period of a compound pendulum is given as

$$T = 2\pi\sqrt{\frac{I}{mgl}} = 2\pi\sqrt{\frac{m\frac{(a^2+a^2)}{12} + m\left(\frac{a}{\sqrt{2}}\right)^2}{mg.\frac{a}{\sqrt{2}}}}$$

$$= 2\pi\sqrt{\frac{\left(\frac{a}{6}+\frac{a}{2}\right).\sqrt{2}}{g}} = 2\pi\sqrt{\frac{2\sqrt{2}a}{3g}}$$

2. (D)

Centre of mass of system is

$$x_{cm} = \frac{(\lambda x)\frac{x}{2} + M\left(\frac{h}{2}\right)}{M + \lambda x}$$

Time period is maximum when

$$\frac{dx_{cm}}{dx} = 0$$

$$\Rightarrow (M+\lambda x)\lambda x - \left(\frac{\lambda x^2}{2} + \frac{Mh}{2}\right)\lambda = 0$$

$$(Mx + \lambda x^2) - \frac{\lambda x^2}{2} - \frac{Mh}{2} = 0$$

$$\lambda x^2 + 2Mx - Mh = 0$$

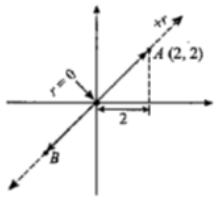
3. (B)

Let the line joining AB represent axis ‘r’. By the conditions given ‘r’ coordinate of the particle at time t is

$r = 2\sqrt{2}\cos\omega t$

$\omega = \frac{2\pi}{T} = \frac{2\pi}{2} = \pi$

$\Rightarrow r = 2\sqrt{2}\cos\pi t$



$x = r\cos 45^0 = \frac{r}{\sqrt{2}} = 2\cos\pi t$

$\Rightarrow a_x = -\omega^2 x = -\pi^2 2\cos\pi t$

$\Rightarrow F_x = ma_x = -4\pi^2\cos\pi t$

4. (B)

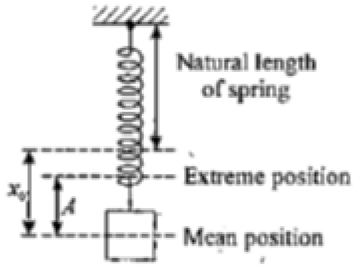
Torque about hinge

$2.5g.4\cos\theta - 1g.100\cos\theta = 0$

So rod remains stationary after the release.

5. (A)

The spring is never compressed. Hence spring shall exert least force on the block when the block is at topmost position.



$F_{least} = kx_0 - kA = mg - m\omega^2 A = mg - 4\frac{\pi^2}{T^2}mA$

6. (B, C)

When speed of block is maximum, net force on block is zero. Hence at that instant spring exerts a force of magnitude ‘mg’ on block.

At the instant block is in equilibrium position, its speed is maximum and compression in spring is x given by

$kx = mg$ …..(1)

From conservation of energy

$mg(L + x) = \frac{1}{2}kx^2 + \frac{1}{2}mv_{max}^2$ …..(2)

From (1) and (2) we get $v_{max} = \frac{3}{2}\sqrt{gL}$

$V_{max} = \frac{3}{2}\sqrt{gL}$ and $\omega = \sqrt{\frac{k}{m}} = 2\sqrt{\frac{g}{L}}$

$$\Rightarrow A = \frac{V_{max}}{\omega} = \frac{3}{4}L$$

Hence time taken t, from start of compression till block reaches mean position is given by

$$x = A\sin\omega t_0 \text{ where } x = \frac{L}{4}$$

$$\Rightarrow t_0 = \sqrt{\frac{L}{4g}}\sin^{-1}\frac{1}{3}$$

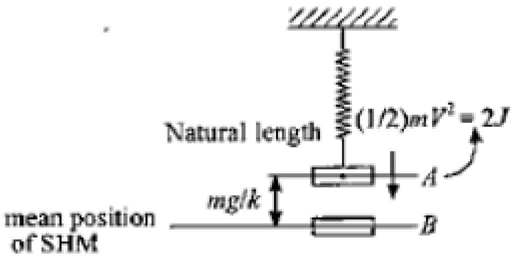
Time taken by block to reach from mean position to bottom

most position is $\frac{2\pi}{4\omega} = \frac{\pi}{4}\sqrt{\frac{L}{g}}$

Hence the required time = $\frac{\pi}{4}\sqrt{\frac{L}{g}} + \sqrt{\frac{L}{4g}}\sin^{-1}\frac{1}{3}$

7. (A, D)

Maximum KE is acquired by the block when it passes the mean position of SHM where $\Sigma F = 0$ or $mg = kx$.



$$x = mg/k = \frac{10}{200} = \frac{1}{20}\text{m}$$

Applying work energy theorem from position A and B on the block

$$K_f - K_t = W_{gravity} + W_{spring}$$

$$\Rightarrow K_f - 2J = 10N\left(\frac{1}{20}m\right) + \left[-\frac{1}{2}\times 200\times\left(\frac{1}{20}\right)^2\right]$$

$$\Rightarrow k_f = 2.25J$$

$$KE_{max} = \frac{1}{2}kA^2$$

$$A = \sqrt{\frac{2(KE_{max})}{k}} = \sqrt{\frac{2\times(2.25)}{200}} = \sqrt{\frac{9}{400}} = \frac{3}{20}m = 15cm$$

8. (A, B, D)

Just before collision, both P and Q arrive at their equilibrium position

$$\Rightarrow V_P = \omega\frac{A}{2} = \sqrt{\frac{k}{m}}\frac{A}{2}$$

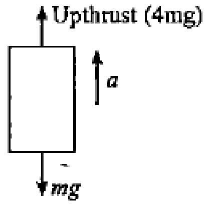
Speed of Q just before collision is

$$V_Q = \omega A = \sqrt{\frac{k}{m}}A$$

The block shall meet after time $t = \frac{T}{4}$, where T is time period of either isolated spring block system.

$$t = \frac{T}{4} = \frac{1}{4} 2\pi\sqrt{\frac{m}{k}} = \frac{\pi}{2}\sqrt{\frac{m}{k}}$$

9. (A, C)



$\Rightarrow a = 3g$

The density of liquid is four times that of cylinder, hence in equilibrium position one fourth of the cylinder will remain submerged. So as the cylinder is released from initial position, it moves distance $\frac{3l}{4}$ to reach its equilibrium position. The upward motion in this duration is SHM.

Therefore attained velocity is $v_{max} = \omega A$

Where $\omega = \sqrt{\frac{4g}{l}}$ and $A = \frac{3l}{4}$

Thus $v_{max} = \frac{3}{2}\sqrt{gl}$

The require time is one fourth of time period of SHM. Therefore $t = \frac{\pi}{2\omega} = \frac{\pi}{4}\sqrt{\frac{l}{g}}$

10. (A, C)

When 3 kg mass is released the amplitude of its oscillations is 2m and at a distance 1 m from the equilibrium position we can find the speed of it using the relation $v = [(k/m)(A^2 - x^2)]^{1/2}$ then by conservation of momentum we can find the resulting speed of the combined mass and the new amplitude using the above relation which gives options (A) and (C) are correct.

11. (A, B, C)

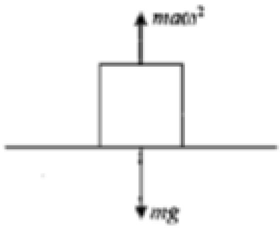
Comparing the graph with the relation in velocity and displacement $v = [(k/m)(A^2 - x^2)]^{1/2}$ we get option (A) and (C) are correct and maximum acceleration of the particle is given by $a_{max} = kA/m$ which gives option (B) is also correct.

12. (B, C, D)

Due to the Pseudo force on block (considered external) its mean position will shift to a distance mg/K above natural length of spring as net force now is mg in upward direction so total distance of block from new mean position is 2mg/K which will be the amplitude of oscillations hence option (C) is correct. During oscillations spring will pass through the natural length hence option (D) is correct. As block is oscillating under spring force and other constant forces which do not affect the SHM frequency. Hence option B is correct.

13. (B, C, D)

Block loses contact at the highest point. Then



$mg = ma\omega^2$

$\Rightarrow \omega = \sqrt{\frac{g}{a}} = \sqrt{\frac{10}{0.4}} = 5 rads^{-1}$

$\Rightarrow T = \frac{2\pi}{5} s$

At lowest point

$N = mg + ma\omega^2$

$N = 2mg$ (form(1)

Halfway down from mean position

$N = mg + k\left(\frac{a}{2}\right)$

$= mg + \frac{m\omega^2 a}{2}$

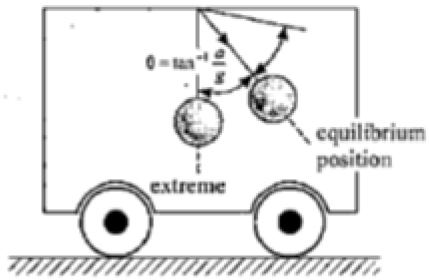
=1.5 mg

Block has maximum velocity when displacement (and thus acceleration) is zero, and thus has N=mg

14. (B, D)

Bob will oscillate about equilibrium position with amplitude $\theta = \tan^{-1}\left(\frac{a}{g}\right)$ for any value of a.

If a < < g, motion will be SHM and then



Time period will be $2\pi\sqrt{\frac{l}{\sqrt{a^2+g^2}}}$

15. (A, B, C)

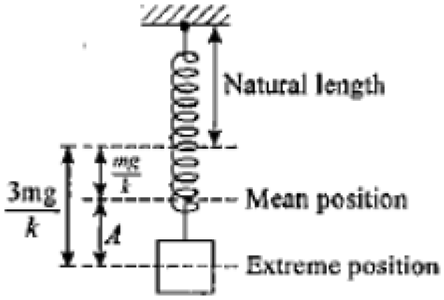
As no external force is acting on the system both blocks will oscillate about their center of mass. If compression is 6cm then by mass moment property of center of mass we use $m_1\Delta x_1 = m_2\Delta x_2$ so the amplitude of 3 kg mas sis 4 cm and that of 6kg mass is 2 cm hence option (B) 9s correct. The time period can be obtained either by using concept of reduced mass or by splitting the spring in two parts about center of mass of the two blocks in series combination which gives the 3kg block oscillates with a spring constant 1200 N/m for which time period is $T = 2\pi(m/k)^{1/2} = \pi/10 \sec$ hence option (A) is correct. The maximum momentum of 6 kg block will be when it will pass through mean positon and given as $mA\omega = 2.4 kgm/s$ hence option (C) is correct.

16. (8)

The period of small oscillation of a hinged/pivoted rigid body is given by the expression of time period of a compound pendulum given as $T = 2\pi(I/mgd)^{1/2}$ when I is the moment of inertia and d is the distance of center of mass from the hinge. Here we use $I = 2\left(\frac{m}{2}\right)\frac{l^2}{3} = \frac{ml^2}{3}$ and $d = \frac{l}{2\sqrt{2}}$

17. (2)

Just after cutting the string extension in spring $= \frac{3mg}{k}$ and extension in the spring when block is in mean position $= \frac{mg}{k}$



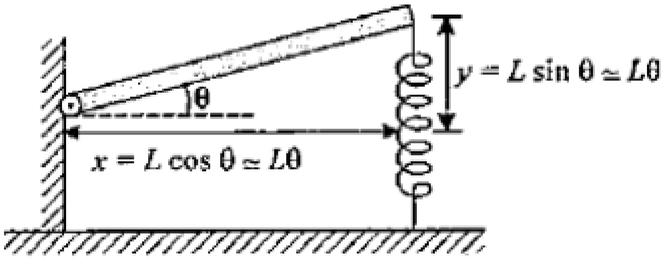
$\Rightarrow$ Amplitude of oscillation is given as

$A = \frac{3mg}{k} - \frac{mg}{k} = \frac{2mg}{k}$

18. (3)

Restoring torque on rod after small angle tilt is

$\tau_R = -kyL = -KL^2\theta$ (Since $y \cong L\theta$ from figure)



$\Rightarrow kL^2\theta = -\frac{mL^2}{3}.\alpha$

$\Rightarrow \alpha = -\frac{3k}{m}.\theta$

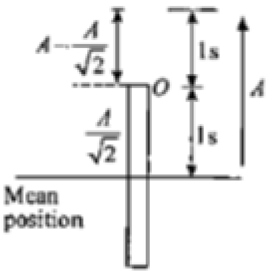
Comparing with $\alpha = -\omega^2\theta$

We get $\omega = \sqrt{\frac{3k}{m}}$

Note: Torque due to mg was already balanced so it is not taken in calculation.

19. (1)

Time period of motion =6+2=8s from mean position to the highest point of the wall, it takes is an covers distance $\frac{A}{\sqrt{2}}$



Thus, $A-\frac{A}{\sqrt{2}}=0.3\text{m}$

$\Rightarrow A=1.0\text{m}$

20. (1.5)

Given that from graph

$\frac{1}{2}mV_m^2=15\times10^{-3}$

$V_m=\sqrt{0.150}\text{m/s}$

$A\omega=\sqrt{0.150}\text{m/s}$

$L\theta_{m.}\cdot\sqrt{\frac{g}{L}}=\sqrt{0.150}\text{m/s}$

$\Rightarrow\sqrt{gL}=\frac{\sqrt{0.150}}{100\times10^{-3}}$

$\Rightarrow L=\frac{0.150}{0.1}=1.5\text{m}$

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TOPIC: ISOMERISM

Answers & Solutions

21. (A)
In B, C, D there is plane of symmetry so optically inactive.

22. (C)
Here the whole structure is planar.

23. (A)
Aromatic enol is more stable here than non-aromatic enol.

24. (B)
1st and 2nd are optically active.

25. (D)
Axial-axial

26. (A, C)
Non superimposable on mirror image.

27. (C, D)
di-substituted cyclic compounds.

28. (A, B, C, D)
All show tautomerism.

29. (A, C)
Racemic mixture is optically inactive.

30. (A, C, D)
Non superimposable mirror images.

31. (A, B)
Geometrical isomers are diastereomers.

32. (A, C, D)
In B no alpha H.

33. (A, B, D)
D is meso compound.

34. (A, C, D)
Enantiomers are non-superimposable mirror images.

35. (B, D)
In A and C no alpha H present.

36. (4)
Total number of stereo sites is 2 and molecule is unsymmetrical so stereoisomers is 4.

37. (3)
Z-Z, E-Z, E-E

38. (8)
Total stereo sites are 3 and molecule is unsymmetrical so 8.

39. (4)
II, IV, V, VII

40. (2)
Cis and trans and both are optically inactive.

Answers & Solutions

41. (B)

$$\frac{x\cos\theta}{3\sqrt{3}}+y\sin\theta=1.$$

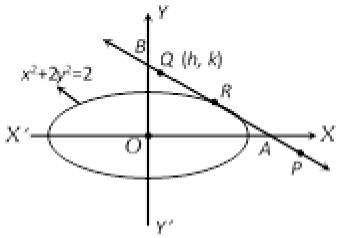
Sum of in intercepts $=3\sqrt{3}\sec\theta+\operatorname{cosec}\theta=f(\theta)$, (say)

$$f'(\theta)=\frac{3\sqrt{3}\sin^3\theta-\cos^3\theta}{\sin^2\theta\cos^2\theta}.$$

At $\theta=\frac{\pi}{6}$, $f(\theta)$ is minimum.

42. (C)

Let the point of contact be



$R\equiv\left(\sqrt{2}\cos\theta,\sin\theta\right)$

Equation of tangent AB is

$$\frac{x}{\sqrt{2}}\cos\theta+y\sin\theta=1 \Rightarrow A\equiv\left(\sqrt{2}\sec\theta,0\right);\ B\equiv\left(0,\operatorname{cosec}\theta\right)$$

Let the middle point Q of AB be (h, k)

$$\Rightarrow h=\frac{\sec\theta}{\sqrt{2}},k=\frac{\operatorname{cosec}\theta}{2} \Rightarrow \cos\theta=\frac{1}{h\sqrt{2}},\sin\theta=\frac{1}{2k}$$

$$\Rightarrow \frac{1}{2h^2}+\frac{1}{4k^2}=1$$

Required locus is $\frac{1}{2x^2}+\frac{1}{4y^2}=1$

Trick: The locus of mid-points of the portion of tangents to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ intercepted between axes is $a^2y^2+b^2x^2=4x^2y^2$

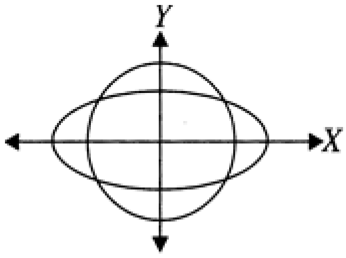
i.e. $\frac{a^2}{4x^2}+\frac{b^2}{4y^2}=1$ or $\frac{1}{2x^2}+\frac{1}{4y^2}=1$

43. (B)

The radius of circle having *SS*' as diameter is $r = ae$.

If it cuts an ellipse, then

$r > b$ or $ae > b$ or $e^2 > \frac{b^2}{a^2}$ or $e^2 > 1-e^2$ or $e^2 > \frac{1}{2}$ or $e > \frac{1}{\sqrt{2}}$ or $e \in \left(\frac{1}{\sqrt{2}}, 1\right)$



44. (B)

Here, $a^2+2 > a^2+1$ or $a^2+1 = (a^2+2)(1-e^2)$ or $a^2+1 = (a^2+2)\frac{5}{6}$ or $6a^2+6 = 5a^2+10$

or $a^2 = 10-6 = 4$ or $a = \pm 2$

Latus rectum $\frac{2(a^2+1)}{\sqrt{a^2+2}} = \frac{2\times 5}{\sqrt{6}} = \frac{10}{\sqrt{6}}$

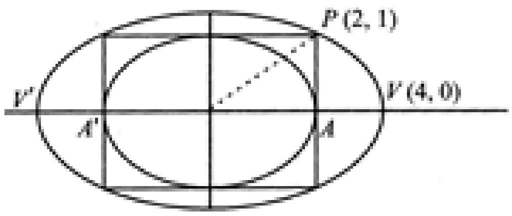
45. (B)

$x^2+4y^2 = 4 \Rightarrow \frac{x^2}{4}+\frac{y^2}{1} = 1$

So $a = 2, b = 1$

Thus *P* is (2, 1).

The required ellipse is $\frac{x^2}{a^2}+\frac{y^2}{b^2} = 1 \Rightarrow \frac{x^2}{4^2}+\frac{y^2}{b^2} = 1$



The point (2, 1) lies on it.

So, $\frac{4}{16}+\frac{1}{b^2} = 1 \Rightarrow \frac{1}{b^2} = 1-\frac{1}{4} = \frac{3}{4} \Rightarrow b^2 = \frac{4}{3}$

$\therefore \frac{x^2}{16}+\frac{y^2}{\left(\frac{4}{3}\right)} = 1 \Rightarrow \frac{x^2}{16}+\frac{3y^2}{4} = 1$

$\Rightarrow x^2+12y^2 = 16$

46. (AB)

Let $E_1 : \frac{x^2}{a^2}+\frac{y^2}{b^2} = 1$ where $a > b$ and $E_2 : \frac{x^2}{c^2}+\frac{y^2}{d^2} = 1$ where $c < d$

Also, $S : x^2+(y-1)^2 = 2$

Tangent at $P(x_1, y_1)$ to *S* is $x+y = 3$

To find point of contact put $x = 3 - y$ is *S*.

We get *P* (1, 2) Writing equation of tangent in parametric form

$$\frac{x-1}{\frac{-1}{\sqrt{2}}} = \frac{y-2}{\frac{1}{\sqrt{2}}} = \pm\frac{2\sqrt{2}}{3}$$

$$x = \frac{-2}{3}+1 \text{ or } \frac{2}{3}+1 \text{ and } y = \frac{2}{3}+2 \text{ or } \frac{-2}{3}+2$$

$$\Rightarrow x = \frac{1}{3} \text{ or } \frac{5}{3} \text{ and } y = \frac{8}{3} \text{ or } \frac{4}{3}$$

$$\therefore \quad Q\left(\frac{5}{3},\frac{4}{3}\right) \text{ and } R\left(\frac{1}{3},\frac{8}{3}\right)$$

Equation of tangent to E_1 at Q is

$$\frac{5x}{3a^2}+\frac{4y}{3b^2} = 1 \text{ which is identical to } \frac{x}{3}+\frac{y}{3} = 1$$

$$\Rightarrow a^2 = 5 \text{ and } b^2 = 4 \Rightarrow e_1^2 = 1-\frac{4}{5} = \frac{1}{5}$$

Equation of tangent to E^2 at R is

$$\frac{x}{3c^2}+\frac{8y}{3d^2} = 1 \text{ identical to } \frac{x}{3}+\frac{y}{3} = 1$$

$$\Rightarrow c^2 = 1, d^2 = 8 \Rightarrow e_1^2 = 1-\frac{1}{8} = \frac{7}{8}$$

$$\therefore \quad e_1^2+e_2^2 = \frac{43}{40},\ e_1e_2 = \frac{\sqrt{7}}{2\sqrt{10}},\ \left|e_1^2-e_2^2\right| = \frac{27}{40}$$

47. (A, D)

Equation of tangent

$$\frac{x.4\cos\theta}{16}+\frac{y.\frac{16}{\sqrt{11}}\sin\theta}{\frac{256}{11}} = 1$$

$$\frac{x\cos\theta}{4}+\frac{\sqrt{11}}{16}\sin\theta y = 1$$

$\because$ above equation is tangent to $x^2+y^2-2x = 15$ centre (2, 0) radius 4.

48. (AB)

$$\Rightarrow 4\left(x-2y+1\right)^2+9\left(2x+y+2\right)^2 = 25$$

$$\Rightarrow 4\left(\frac{x-2y+1}{\sqrt{5}}\right)^2+9\left(\frac{2x+y+2}{\sqrt{5}}\right)^2 = 5$$

$$4X^2+9Y^2 = 5$$

$$\Rightarrow \frac{X^2}{5/4}+\frac{Y^2}{5/9} = 1 \text{ where } \begin{cases} X = \frac{x-2y+1}{\sqrt{5}} \\ Y = \frac{2x+y+2}{\sqrt{5}} \end{cases}$$

Which is standard form of equation of ellipse.

$\therefore \quad a^2 = \frac{5}{4},\ b^2 = \frac{5}{9} \qquad \Theta b^2 = a^2\left(1-e^2\right)$

$\therefore \quad \frac{5}{9} = \frac{5}{4}.\left(1-e^2\right)$

$\Rightarrow \quad e^2 = \frac{5}{9} \ \Rightarrow \ e = \frac{\sqrt{5}}{3}$

49. (ABCD)

Equation of tangent $y = \frac{x}{t} + at$

$x - yt + at^2 = 0 \qquad ...(1)$

Normal at θ to $\frac{x^2}{5} + \frac{y^2}{4} = 1$

$\frac{x\sqrt{5}}{\cos\theta} - \frac{2y}{\sin\theta} = 1$

$2y\cos\theta = x\sqrt{5}\sin\theta - \sin\theta\cos\theta \qquad ...(2)$

From (1) and (2)

$t = \frac{2\cos\theta}{\sqrt{5}\sin\theta} = \frac{2}{\sqrt{5}}\cot\theta, t^2 = \frac{-\cos\theta}{\sqrt{5}} \Rightarrow -\frac{\cos\theta}{\sqrt{5}} = \frac{4\cos^2\theta}{5\sin^2\theta}$

$\cos\theta = 0$ or $\sqrt{5}\sin^2\theta + 4\cos\theta = 0$

$\cos\theta = -\frac{1}{\sqrt{5}}, \cos\theta = 0 \ \Rightarrow \ t = 0$

$t^2 = \frac{1}{5}$

50. (AC)

$x^2\tan^2\alpha + y^2\sec^2\alpha = 1 \ \Rightarrow \ \frac{x^2}{\cot^2\alpha} + \frac{y^2}{\cos^2\alpha} = 1$

$\because \ \cos^2\alpha = \cot^2\alpha\left(1-e^2\right) \ \Rightarrow \ \sin^2\alpha = \left(1-e^2\right)$

$\therefore \ \ e^2 = \cos^2\alpha\left(\alpha / \neq 90^o\right)$

$e = \cos\alpha$

$\because$ Latustrectum $= \frac{1}{2} = \frac{2b^2}{a}$

$\Rightarrow \ a = 4b^2 \ \Rightarrow \ \cot\alpha = 4\cos^2\alpha$

$\Rightarrow \ \frac{1}{\sin\alpha} = 4\cos\alpha \ \Rightarrow \ \sin 2\alpha = \frac{1}{2}$

$\sin 2\alpha = \frac{1}{2}$

$2\alpha = n\pi + (-1)^n \frac{\pi}{6}$

$\alpha = \frac{n\pi}{2} + (-1)^n \frac{\pi}{12}$

For $n = 0$

$\alpha = \frac{\pi}{12}$ and for $n = 1$

$\alpha = \frac{\pi}{2} - \frac{\pi}{12} = \frac{5\pi}{12}$

51. (AD)

$\frac{x^2}{9} + \frac{y^2}{4} = 1$

$4 = 9\left(1 - e^2\right) \Rightarrow 1 - e^2 = \frac{4}{9}$

$e = \frac{\sqrt{5}}{3}$

F_1, F_2 are $\left(\pm\sqrt{5}, 0\right)$

Area $= \sqrt{10}$

$\Rightarrow \frac{1}{2} \times \left(2\sqrt{5}\right) \times |y| = \sqrt{10} \Rightarrow |y| = \sqrt{2} \Rightarrow y = \sqrt{2}$ or $-\sqrt{2}$

52. (AC)

$\sqrt{(x-5)^2 + (y-7)^2} + \sqrt{(x+1)^2 + (y+1)^2} = 12$

$SP + S'P = 2a \Rightarrow$ conic is ellipse

$2ae = \sqrt{36 + 64}$

$2a = 12 \Rightarrow e = \frac{5}{6}$

Major axis $y - 7 = \frac{7+1}{5+1}(x - 5)$

$\Rightarrow 4x - 3y + 1 = 0$

53. (AC)

Focal property of ellipse

$PS + PS' = 2a; a > b$

$PS + PS' = b;\ a < b$

$PS \cos\alpha + PS' \cos\beta = 2ae$ ...(i)

$PS \sin\alpha - PS' \sin\beta = 0$ ...(ii)

$PS + PS' = 2a$ ...(iii)

From (i) and (ii), we get

$PS = \frac{2ae \sin\beta}{\sin(\alpha+\beta)}, PS' = \frac{2ae \sin\alpha}{\sin(\alpha+\beta)}$ ...(iv)

From (iii) and (iv)

$e(\sin\alpha + \sin\beta) = \sin(\alpha+\beta)$

$\therefore \quad e.2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2} = 2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha+\beta}{2}$

$\therefore \quad e\left(\cos\frac{\alpha}{2}\cos\frac{\beta}{2}+\sin\frac{\alpha}{2}\sin\frac{\beta}{2}\right)=\cos\frac{\alpha}{2}\cos\frac{\beta}{2}-\sin\frac{\alpha}{2}\sin\frac{\beta}{2}$

$\therefore \quad \tan\frac{\alpha}{2}\tan\frac{\beta}{2}=\frac{1-e}{1+e}=\frac{2a\left(1-a\sqrt{a^2-b^2}\right)}{b^2}\cdot\frac{2a\left(a-\sqrt{a^2-b^2}\right)-b^2}{b^2}$

$\therefore$ (C) is correct option and (D) is incorrect.

54. (AB)

The tangent at any point

$P(3\cos\theta, 2\sin\theta)$ is $\frac{x}{3}\cos\theta+\frac{y}{2}\sin\theta=1$

The tangent at the extremities of major axis are x = 3 and x = –3 where the points T and T' are $\left(3, \frac{2(1-\cos\theta)}{\sin\theta}\right)$ and $\left(-3, \frac{2(1+\cos\theta)}{\sin\theta}\right)$

Diameter form equation of circle

$x^2+y^2-4(\operatorname{cosec}\theta)y-5=0$

This passes through points given in (A) and (B).

55. (B)

$x-y-5=0 \qquad ...(1)$

$\frac{x^2}{4}+\frac{y^2}{1}=1 \qquad ...(2)$

$\because$ any point on (1) is (h, h – 5)

$\therefore$ Equation of chord of contact is :

$\Rightarrow xh+4hy-20y=4$

$\Rightarrow h(x+4y)-4(5y+1)=0 \qquad ...(3)$

$\because$ (3) always passes through point of intersection of $5y+1=0$ and $x+4y=0$

$\Rightarrow$ required fixed point $\left(\frac{4}{5},-\frac{1}{5}\right)$.

56. (0.50)

57. (3.00)

58. (10)

59. (15)

Here, 2a = 10m and $2ae=8m$;

$\therefore \quad e=\frac{4}{5}, a=5m$

$\therefore \quad b^2=a^2\left(1-e^2\right)=9 \Rightarrow b=3$

Thus, required area $=\pi ab=15\pi$ sq. metre.

60 (27)

By symmetry the quadrilateral is a rhmoubus.

So area is four times the area of the right angled triangle formed by the tangent and axes in the Ist quadrant. Now, $ae = \sqrt{a^2 - b^2} \Rightarrow ae = 2$

$\Rightarrow$ Tangent (in first quadrant) at end of latus rectum $\left(2, \frac{5}{3}\right)$ is

$\frac{2}{9}x + \frac{5}{3}\frac{y}{3} = 1$ i.e. $\frac{x}{9/2} + \frac{y}{3} = 1$

Area $= 4 \cdot \frac{1}{2} \cdot \frac{9}{2} \cdot 3 = 27$ sq. unit