IIT – JEE: 2024 **TW TEST (Flagship)** **DATE: 10/07/22**

TOPIC: VECTORS

Answers & Solutions

1. (B)

$\left|\vec{a}-\vec{b}\right| = \sqrt{a^2+b^2-2ab\cos\theta}$ ($\theta = \angle$ between $\vec{a}$ & $\vec{b}$)

$30 = \sqrt{11^2+23^2-2\times 11\times 23\cos\theta} \Rightarrow \cos\theta = -\dfrac{250}{506}$

$\therefore \left|\vec{a}+\vec{b}\right| = \sqrt{a^2+b^2+2ab\cos\theta}$

$= \sqrt{11^2+23^2-250} = 20$

2. (B)

Let forces are F and 2F and angle between them is θ and resultant makes an angle α with the force F.

$\tan\alpha = \dfrac{2F\sin\theta}{F+2F\cos\theta} = \tan 90 = \infty$

$\Rightarrow F+2F\cos\theta = 0 \quad \therefore \cos\theta = -1/2 \text{ or } \theta = 120^0$

3. (B)

$A\times B$ is a vector $\perp$ to both $\overline{A}$ and $\overline{B}$

Now $\overline{A}\times\overline{B} = (\bar{i}-2\bar{j}+\bar{k})\times(3\bar{i}+\bar{j}-2\bar{k}) = 3\bar{i}+5\bar{j}+7\bar{k}$

Now, $B = \dfrac{\overline{A}\times\overline{B}}{|\overline{A}\times\overline{B}|}$

$= \dfrac{3\bar{i}+5\bar{j}+7\bar{k}}{\sqrt{3^2+5^2+7^2}} = \dfrac{3\bar{i}+5\bar{j}+7\bar{k}}{\sqrt{83}}$

Hence, (B) is correct.

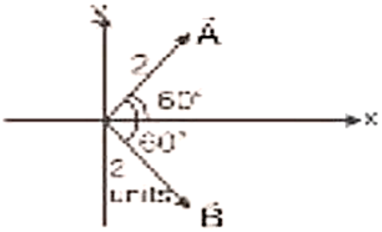
4. (A)

$\vec{r}_F - \vec{r}_1 =$ Displacment $\vec{r}_F =$ Final position vector

$\bar{r} =$ Initial position vector

$\therefore \vec{r}_F = \vec{r}_1 +$ Displacement

5. (B)



$\overline{A} = 2\cos 60^0 \hat{i} + 2\sin 60^0 \hat{j} = \hat{i} + \sqrt{3}\hat{j}$

$\vec{B} = 2\cos 60^0 \hat{i} - 2\sin 60^0 \hat{j} = i - \sqrt{3}j$

$\therefore \vec{A} + \vec{B} = 2\hat{i}$

6. (D)

$|\vec{A} + \vec{B}| = |\vec{A}| = |\vec{B}| = K$ Let

$|\vec{A} + \vec{B}| = \sqrt{A^2 + B^2 + 2AB\cos\theta}$

$K = \sqrt{K^2 + K^2 + 2KK\cos\theta}$

7. (C)

If a vector $\vec{A} = a\hat{i} + b\hat{j} + c\hat{k}$ makes angles α, β and γ with x,y and z axes respectively then

$\cos\alpha = \frac{a}{\sqrt{a^2 + b^2 + c^2}}, \cos\beta = \frac{b}{\sqrt{a^2 + b^2 + c^2}}$ and $\cos\gamma = \frac{c}{\sqrt{a^2 + b^2 + c^2}} = \frac{1}{\sqrt{2}} \Rightarrow \gamma = 45^o$

$\therefore \cos\alpha = \frac{1}{\sqrt{1^2 + 1^2 \left(\sqrt{2}\right)^2}} = \frac{1}{2} \Rightarrow \alpha = 60^0$

$\cos\beta = \frac{1}{\sqrt{1^2 + 1^2 + \left(\sqrt{2}\right)^2}} = \frac{1}{2} \Rightarrow \beta = 60^0$

8. (A)

$\vec{F}_{net} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 = (P + 1)\hat{i} + 4\hat{j}$

$\vec{F}_{net} = 5 \Rightarrow P + 1 = 3$ or $P + 1 = -3$

$\Rightarrow P = 2$ or $P = -4$

$\therefore$ Product of possible values of P is -8.

9. (D)

$\frac{\pi}{3}$ radians

$\vec{a}+\vec{b}+\vec{c}=0$

$-\vec{c}=\vec{a}+\vec{b}$

$|-\vec{c}|-=|\vec{a}+\vec{b}|$

$|-\vec{c}|=\sqrt{a^2+b^2+2ab\cos\theta}$

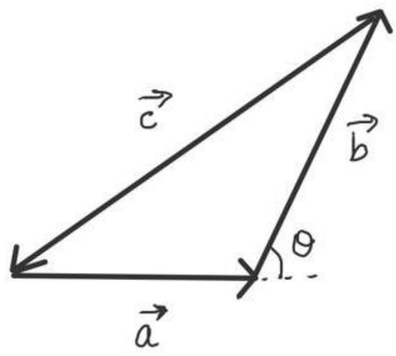
$7=\sqrt{3^2+5^2+2(3)(5)\cos\theta}$

$49=34+30\cos\theta$

$15=30\cos\theta$

$\cos\theta=\frac{1}{2}$

$\theta=\frac{\pi}{3}$



10. (D)

Vector joining A:(4, -4, 0) and B: (-2, -2, 0) is given by

$\overline{AB}=(-2-4)\hat{i}+(-2-(-4))\hat{j}+(0-0)\hat{k}=-6\hat{i}+2\hat{j}$

$|\overline{AB}|=\sqrt{(-6)^2+(2)^2}=\sqrt{40}=2\sqrt{10}$

11. (B)

Let the required vector be $\vec{A}$

$\vec{A}+(\hat{i}-2\hat{j}+2\hat{k})+(2\hat{i}+\hat{j}-\hat{k})=\hat{i}$

$\Rightarrow \vec{A}=-2\hat{i}+\hat{j}-\hat{k}$

12. (D)

$\vec{a}.\hat{i}=3$

13. (A)

$\vec{B}=\vec{C}+\vec{D}$

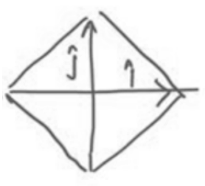
$\quad=5\hat{i}+5\hat{j}-5\hat{k}$

$\vec{B}=5(\hat{i}+\hat{j}-\hat{k})$

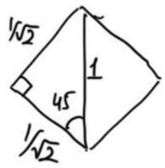
$\vec{B}=5\vec{A}$

Hence $\vec{B}$ and $\vec{A}$ are like vectors

14. (A)



Diagonals are $\perp^r$ and equal in length hence the 11gm is a square.



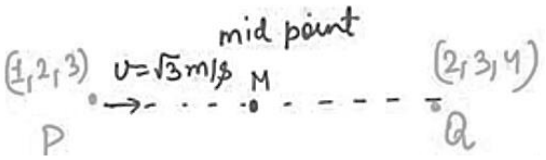
Area of square = (side)^2

$$=\left(\frac{1}{\sqrt{2}}\right)^2=\frac{1}{2}$$

15. (A)

16. (A)

$\mu=\sqrt{3}\,\text{m/s}$



$|\overrightarrow{PQ}|=\sqrt{3}\,\text{m}$

$|\overrightarrow{PM}|=\frac{\sqrt{3}}{2}\,\text{m}$

$t=\frac{|\overrightarrow{PM}|}{\text{speed}}=\frac{\sqrt{3}}{2\sqrt{3}}\sec=\frac{1}{2}$ seconds

17. (C)

$(\vec{a}-\vec{b})\times(\vec{a}+\vec{b})$

$=\left(\vec{a}\times\vec{a}\right)+\vec{a}\times\vec{b}-\vec{b}\times\vec{a}-\left(\vec{b}\times\vec{b}\right)$

$=2\left(\vec{a}\times\vec{b}\right)$ $\left[\because \vec{a}\times\vec{a}=\vec{b}\times\vec{b}=\vec{0} \text{ and } \vec{a}\times\vec{b}=-(\vec{b}\times\vec{a})\right]$

18. (D)

$\because \cos^2\alpha+\cos^2\beta+\cos^2\gamma=1$

$\sin^2\alpha+\sin^2\beta+\sin^2\gamma=3-(\cos^2\alpha+\cos^2\beta+\cos^2\gamma)$

=2

19. (A)

$|\hat{p}-\hat{q}|=\sqrt{|\hat{p}|^2+|\hat{q}|^2+2|\hat{p}||\hat{q}|\cos(180-\theta)}$

$=\sqrt{1+1-2\cos\theta}=\sqrt{2(1-\cos\theta)}$

$=\sqrt{4\sin^2(\theta/2)}=2\sin(\theta/2)$

20. (B)

$\hat{a}.\hat{b}=\cos 60\cos 45+\sin 60\sin 45$

$=\cos(60-45)$

$=\cos 15^0$

IIT – JEE: 2024 **TW TEST (Flagship)** **DATE: 10/07/22**

TOPIC: QUDRTIC EQUATIONS

Answers & Solutions

21. (D)

Since a, b, c $\in$ integers & one root is $3+\sqrt{5}$

other root has to be $3-\sqrt{5} \Rightarrow \boxed{\text{sum}=6}$

22. (A)

Product of roots $=\dfrac{\alpha^2+1}{\alpha}=-2$

$\Rightarrow \alpha^2+2\alpha+1=0 \qquad \Rightarrow (\alpha+1)^2=0$

$\Rightarrow \alpha=-1$

23. (C)

Given $\alpha+\beta=-b$

$\alpha\beta=-c$

$\therefore$ quadratic whose roots b, c is $x^2-(b+c)x+bc=0$

Or $x^2+(\alpha+\beta+\alpha\beta)x+\alpha\beta(\alpha+\beta)=0$

24. (D)

$x^2+ax+10=0$

$x^2+bx-10=0$

Common root condition

$400=(b-a)(-10)(a+b)$

$\therefore a^2-b^2=40$

25. (D)

For roots imaginary $D<0 \Rightarrow 25-4k<0 \Rightarrow \boxed{k>\dfrac{25}{4}}$

Hence least integral k is 7

26. (A)

If p, q roots of $x^2+px+q=0 \qquad \Rightarrow p+q=-p$ &

$pq=q \Rightarrow q=0$ or $p=1$

If $q=0 \Rightarrow p=0$ or if $p=1, q=-2$

27. (B)

Clearly $x^3+2x^2+2x+1=0$

$\Rightarrow$ $\alpha+\beta+\gamma=-2$ we know

$\alpha\beta+\beta\gamma+\gamma\alpha=2$ $\alpha^2+\beta^2+\gamma^2=(\alpha+\beta+\gamma)^2-2(\alpha\beta+\beta\gamma+\gamma\alpha)$

$=4-4=0$

28. (C)

For equal roots $D=0$

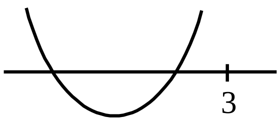
$\Rightarrow (m+2)^2-4(m^2-4m+4)=0$

$\Rightarrow m^2+4m+4-4m^2+16m-16=0$

$\Rightarrow 3m^2-20m+12=0$ $\Rightarrow (3m-2)(m-6)=0$

$\Rightarrow m=2/3, m=6$

29. (B)



For both roots to be less than 3

(i) $D \geq 0$ $\Rightarrow$ $a \in (-\infty, 3]$

(ii) $\frac{-b}{2a} < 3$ $\Rightarrow$ $a \in (-\infty, 3)$

(iii) $f(3) > 0$ $\Rightarrow$ $a \in (-\infty, 2) \cup (3, \infty)$

Upon taking intersection $a \in (-\infty, 2)$

30. (D)

Let $y=\frac{x^2-2x+5}{x^2-2x-8}$

$\Rightarrow$ $(y-1)x^2-2(y-1)x-(8y+5)=0$

For x real $D \geq 0$ $4(y-1)^2+4(y-1)(8y+5) \geq 0$

$\Rightarrow (y-1)[y-1+8y+5] \geq 0$

$\Rightarrow (y-1)(9y+4) \geq 0$

$\Rightarrow y \in \left(-\infty, \frac{-4}{9}\right] \cup [1, \infty)$

But for no x, y =1 $\Rightarrow$ Range $\left(-\infty, -\frac{4}{9}\right] \cup (1, \infty)$

31. (D)

Let quadratic be $ax^2+bx+c=0$ ..(i)

$\therefore$ 1^{st} student $ax^2+bx+p=0$ roots 3, 2 $\Rightarrow 5=-b/a$

2^{nd} student $ax^2+qx+c=0$ roots $-6,1 \Rightarrow -6=c/a$

Hence equation (i) becomes $x^2-5x-6=0$

$$\Rightarrow (x-6)(x+1)=0 \quad \text{roots } -1, 6$$

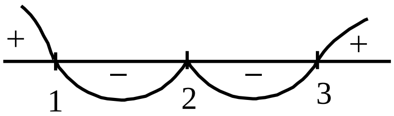
32. (C)

Given $(x-\alpha)(x-\beta)=(x-a)(x-b)-c$

$\Rightarrow$ $(x-a)(x-b)=(x-\alpha)(x-\beta)+c$

The roots of $(x-\alpha)(x-\beta)+c=0$ are a, b

33. (B)



Given $(x-1)^7(3-x)^5(x-2)^4>0$

$\Rightarrow$ $(x-1)^7(x-3)^5(x-2)^4<0$

Hence $x \in (1,2)\cup(2,3)$

34. (A)

$$x-\frac{1}{x^2-4}=2-\frac{1}{x^2-4}$$

$\Rightarrow x=2$ but 2 is not indomain $\Rightarrow$ No solution

35. (A)

$$x=\sqrt{1+\sqrt{1+\sqrt{1+.....}}}$$

$\Rightarrow x=\sqrt{1+x}$ $\Rightarrow x^2-x-1=0$

$$\Rightarrow x=\frac{1+\sqrt{5}}{2}$$

Negative solution rejected

36. (C)

Given $C=2,\ \frac{-b}{2a}=1$ & $\frac{-d}{4a}=3$

$\Rightarrow b=-2a$ & $2-\frac{b^2}{4a}=3$

$\Rightarrow 2-a=3 \Rightarrow a=-1, b=2$

So quadratic is $-x^2+2x+2=0$

Or $x^2 - 2x - 2 = 0$

$\Rightarrow$ both roots irrational

37. (D)

Let roots be $\alpha, \alpha/3$

$\Rightarrow \frac{4\alpha}{3} = \frac{1+4\lambda}{3}$ & $\frac{\alpha^2}{3} = \frac{\lambda^2+2}{3}$

$\Rightarrow \alpha = \frac{1+4\lambda}{4}$ $\Rightarrow$ $\frac{(4\lambda+1)^2}{16} = \lambda^2 + 2$

$\Rightarrow$ $\cancel{16\lambda^2} + 32\lambda + 1 = \cancel{16\lambda^2} + 32$

$\lambda = \frac{31}{32}$

38. (B)

If $a+b+c=0$ $\Rightarrow$ one root is 1

Clearly (B) option isn't feasible

As if $\sec\theta = 1$ $\Rightarrow \operatorname{cosec}\theta \to \infty$ & vice-versa

39. (A)

$\sqrt{x+1} - \sqrt{x-1} = \sqrt{4x+1}$

Square $x+1+x-1-2\sqrt{x^2-1} = 4x+1$

$-2\sqrt{x^2-1} = 2x+1$

Square $4\left(\cancel{x^2} - 1\right) = \cancel{4x^2} + 4x + 1$

$\Rightarrow$ $x = -\frac{5}{4} \to$ Which doesn't satisfy original equation

40. (C)

If $-3+5i$ is a root

$\Rightarrow -3-5i$ is another root

Sum = $-6 = -P$ $\therefore p+q = 40$

Product = $34 = q$