IIT – JEE: 2024 **TW TEST (ADV)** **DATE: 21/05/23**

TOPIC: GRAVITATION

Answer Key & Solution

1. (D)

By law of conservation of energy, we get

$$\left(U+K\right)_{surface}=\left(U+K\right)_{centre}$$

Now, for a solid sphere, we have

$$U_{surface}=-\frac{GMm}{R} \text{ and } U_{centre}=-\frac{3}{2}\frac{GMm}{R}$$

$$\Rightarrow \quad -\frac{GMm}{R}+\frac{1}{2}m\left(0\right)^2=-\frac{3}{2}\frac{GMm}{R}+\frac{1}{2}mv^2$$

$$\Rightarrow \quad \frac{1}{2}mv^2=-\frac{GMm}{R}-\left(-\frac{3}{2}\frac{GMm}{R}\right)$$

$$\Rightarrow \quad \frac{1}{2}mv^2=\frac{1}{2}\frac{GMm}{R}$$

$$\Rightarrow \quad v=\sqrt{\frac{GM}{R}}=\frac{v_c}{\sqrt{2}}$$

2. (C)

$$F=\int_h^{h+L}\frac{G\left(\frac{M}{L}dx\right)m}{x^2}=\frac{GMm}{L}\int_h^{h+L}x^{-2}\,dx$$

$$\Rightarrow \quad F=\frac{GMm}{L}\left(\left.\frac{x^{-2+1}}{-2+1}\right|_h^{h+L}\right)$$

$$\Rightarrow \quad F=-\frac{GMm}{L}\left(\frac{1}{h+L}-\frac{1}{h}\right)$$

3. (C)

By Law of Conservation of Energy, we have

$$\left(U+K\right)_{axis}=\left(U+K\right)_C$$

If m_0 be the mass of the particle, then

$$-\frac{Gmm_0}{\sqrt{r^2+r^2}}+\frac{1}{2}m_0\left(0\right)^2=-\frac{Gmm_0}{r}+\frac{1}{2}m_0v^2$$

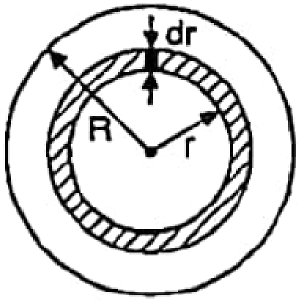
$$\Rightarrow \quad \frac{1}{2}m_0v^2=\frac{Gmm_0}{r}\left(1-\frac{1}{\sqrt{2}}\right)$$

$$\Rightarrow \quad v = \sqrt{\frac{2Gm}{r}\left(1 - \frac{1}{\sqrt{2}}\right)}$$

4. (C)

$$dU = -\frac{Gmdm}{r}$$

$$\Rightarrow \quad dU = -\frac{G\left(\frac{4}{3}\pi r^3 \rho\right)\left(4\pi r^2 dr \rho\right)}{r}$$



$$\Rightarrow \quad dU = -\frac{16\pi^2 G\rho^2}{3} r^4 dr$$

$$\Rightarrow \quad U = -\frac{16}{3}\pi^3 G\left(\frac{M}{\frac{4}{3}\pi R^3}\right)^2 \int_0^R r^4\, dr$$

$$\Rightarrow \quad U = \left(-\frac{16}{3}\pi^2 G\right)\left(\frac{M^2}{\frac{16}{9}\pi^2 R^6}\right)\left(\frac{R^5}{5}\right)$$

$$U = -\frac{3}{5}\frac{GM^2}{R}$$

5. (D)

$L = mvr$

Also, $\frac{mv^2}{r} = \frac{GMm}{r^2}$

From equations (1) and (2), we get

$L = m\sqrt{GMr}$

$\Rightarrow \quad L \propto r^{1/2}$

6. (AC)

$$E_g = -\frac{dV}{dx}$$

If $E_0 = 0$, then V = constant and this constant may also be zero.

7. (AD)

At two positions, when the planet is closest to the sun (perigee) and when it is farthest from the sun (apogee), velocity vector is perpendicular to force vector i.e., work done is zero. In one, complete revolution work done is zero.

8. (AD)

The field inside the shell is zero and so potential inside the shell is constant equal to the value that exists at the surface i.e. $-\frac{GM}{a}$.

9. (ABCD)

By Law of Conservation of Mechanical Energy, we get

$$(U+K)_\infty=(U+K)_r$$

$$\Rightarrow\quad 0+0=\frac{-Gm(4m)}{r}+\frac{1}{2}\mu v_r^2$$

$$\Rightarrow\quad \frac{G(m)(4m)}{r}=\frac{1}{2}\mu v_r^2 \qquad ...(1)$$

Where, $\mu=$ reduced mass $=\frac{(m)(4m)}{m+4m}=\frac{4m}{5}$ and

$v_r=$ relative velocity of aaproach

Substituting (1), the total kinetic energy is

$$K=\frac{G(m)(4m)}{r}$$

$$\Rightarrow\quad K=\frac{Gm^2}{r}$$

Net torque of two equal and opposite forces acting on two objets is zero. Therfore, angular momentum will remain conserved. Intially both the objects were stationary i.e., angular momentum about any point was zero. Hence, angular momentum of both the particles about any point will be zero at all instants.

10. (ABD)

Kinetic energy, $KE=\frac{GMm}{2r}$

Potential energy, $PE=-\frac{GMm}{r}$ and

The total energy, $E=-\frac{GMm}{2r}$

Kientic energy is always positive and $KE\propto\frac{1}{r}$

Potential energy is negative and $|PE|\propto\frac{1}{r}$

Similarly total energy is also negative and $|E|\propto\frac{1}{r}$

Also, $|E|<|PE|$, so from the graph we observe that A is kinetic energy, B is potential energy and C is total energy of the satellite.

11. (ABC)

Both the stars will revolve about their centre of mass.

So, if the centre of mass be at a distance x from $2m$, then

$$x=\frac{2m(0)+mr}{3m}=\frac{r}{3}$$

So, $r_1 = \frac{2r}{3}$ and $r_2 = \frac{r}{3}$

ω and T will be same for both the stars, so

$$K_1 = \frac{1}{2} I_1 \omega^2 \text{ and } K_2 = \frac{1}{2} I_2 \omega^2$$

$$\Rightarrow \quad \frac{K_1}{K_2} = \frac{I_1}{I_2} = \frac{m\left(\frac{2r}{3}\right)^2}{2m\left(\frac{r}{3}\right)^2} = 2$$

$$L_1 = I_1 \omega \text{ and } L_2 = I_2 \omega$$

$$\Rightarrow \quad \frac{L_1}{L_2} = \frac{I_1}{I_2} = 2$$

12. (A)

Gravitational field is the acceleration due to gravity.

$$\text{So, } g = \begin{cases} \dfrac{GM}{r^2} & \underset{(\text{outside})}{r \ge R} \\ \dfrac{4}{3}\pi G \rho r & \underset{(\text{Inside})}{r < R} \end{cases}$$

$$\Rightarrow \quad \frac{F_1}{F_2} = \frac{r_2^2}{r_1^2} (\text{Outside})$$

Where $r_1 > R$ and $r_2 > R$ and $\frac{F_1}{F_2} = \frac{r_1}{r_2} (\text{Inside})$

Where $r_1 < R$ and $r_2 < R$

13. (AD)

$$U_i = \frac{-GMm}{R} = U\,(given)$$

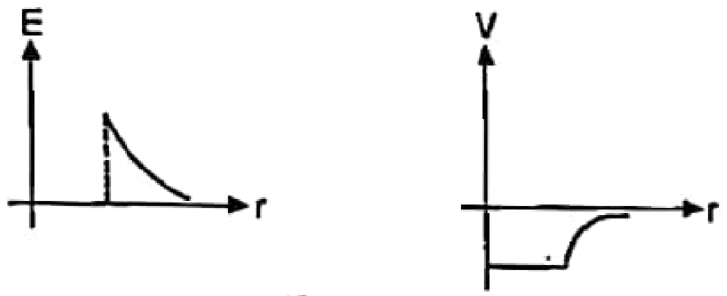
$$\Delta U = U_f - U_i = \frac{-GMm}{(R+R)} + \frac{GMm}{R}$$

$$\Rightarrow \quad \Delta U = \frac{GMm}{2R} = -\frac{U}{2}$$

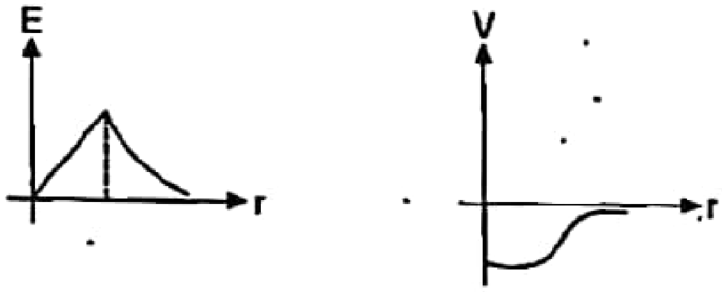
Same is the case with potential.

14. (ABC)

E-r and V-r graphs for a spherical shell and a solid sphere are shown here.



For a shell



For a solid sphere

15. (BCD)

$$T^2 = \frac{4\pi^2}{GM}\left(\frac{r_A + r_P}{2}\right)^3 \qquad \left\{\because r = \frac{r_A + r_P}{2}\right\}$$

$$\Rightarrow \; T^2 = \frac{\pi^2}{2GM}\left(r_A + r_P\right)^2 \text{k}$$

By Law of Conservation of Angular Momentum

$$mv_A r_A = mv_P r_P$$

$$\Rightarrow \; v_A r_A = v_P r_P$$

16. (3.00)

Since, $g_P = \frac{GM_\rho}{R_P^2} = \frac{4}{3}G\pi R_P \rho_P$

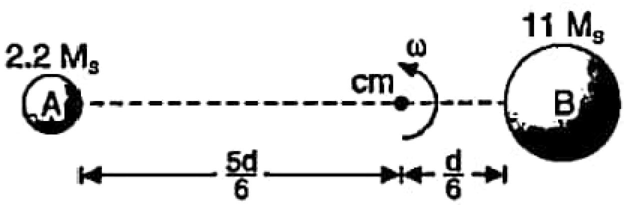
$$\Rightarrow \; \frac{g_P}{g_e} = \frac{R_\rho \rho_P}{R_e \rho_e}$$

Also, $v_e = \sqrt{2gR}$

$$\Rightarrow \; \frac{v_P}{v_e} = \sqrt{\frac{g_\rho R_P}{g_0 R_0}} = \left(\frac{g_P}{g_e}\right)\sqrt{\frac{\rho_e}{\rho_P}} = \frac{\sqrt{6}}{11} \times \sqrt{\frac{3}{2}}$$

$$\Rightarrow \; v_P = 3\,\text{kms}^{-1}$$

17. (6.00)



$$\frac{\text{Total angular momentum about cm}}{\text{Angular momentum of } B \text{ about cm}} = \frac{L}{L_B}$$

$$\Rightarrow \frac{L}{L_B} = \frac{(2.2\,M_S)\left(\omega\frac{5d}{6}\right)\left(\frac{5d}{6}\right) + (11M_S)\left(\omega\frac{d}{6}\right)\left(\frac{d}{6}\right)}{(11M_S)\left(\omega\frac{d}{6}\right)\left(\frac{d}{6}\right)} = 6$$

18. (7.00)

Let E be the gravitational field at x due to the complete sphere. If E_1 be the field due to hole and E_2 be the field due to the remaining portion, then we have

$$E = E_1 + E_2$$

$$\Rightarrow E_2 = E - E_1$$

$$\Rightarrow E_2 = \frac{GM}{x^2} - \frac{Gm}{\left(x - \frac{R}{2}\right)^2} \qquad \text{...(1)}$$

Where, $M = \frac{4}{3}\pi R^3 \rho_0$ and $m = \frac{4}{3}\pi\left(\frac{R}{2}\right)^3 \rho_0$

Substituting the values in equation (1), we get

$$E_2 = -\left(\frac{\pi G \rho_0 R^2}{6}\right)\left[\frac{1}{\left(x - \frac{R}{2}\right)^2} - \frac{8}{x^2}\right]$$

$$E_2 = -\left(\frac{\pi G \rho_0 R^3}{6}\right)\left[\frac{1}{\left(2R - \frac{R}{2}\right)^2} - \frac{8}{(2R)^2}\right]$$

$$E_2 = -\frac{\pi G \rho_0 R^3}{6}\left(\frac{4}{9R^2} - \frac{2}{R^2}\right)$$

$$E_2 = -\frac{\pi G \rho_0 R}{6}\left(\frac{4-18}{9}\right)$$

$$\Rightarrow E_2 = \frac{14}{54}\pi G \rho_0 R$$

$$\Rightarrow E_2 = \left(\frac{7}{27}\right)\pi G \rho_0 R$$

Since, $E = \left(\frac{a}{a+20}\right)\pi G \rho_0 R$

$$\Rightarrow \frac{a}{a+20} = \frac{7}{27}$$

$$\Rightarrow a = 7$$

19. (1.03)

$$\frac{v_{\max}}{v_{\min}} = \frac{1+e}{1+e} = \left[\frac{1+0.0167}{1-0.0167}\right] = 1.033$$

20. (10.00)

By Law of Conservation of Enerngy, we have

$$(U+K)_{\text{surface}} = (U+K)_{\text{at }\infty}$$

$$\Rightarrow \quad -\frac{GmM}{R} + \frac{1}{2}mu^2 = 0 + \frac{1}{2}mv^2$$

$$\Rightarrow \quad -\frac{2GM}{R} + u^2 = v^2$$

Since, $v_e = \sqrt{\frac{2GM}{R}}$

$$\Rightarrow \quad -v_e^2 + u^2 = v^2$$

$$\Rightarrow \quad v^2 = -(11.2)^2 + (15)^2$$

$$\Rightarrow \quad v^2 = -125 + 225$$

$$\Rightarrow \quad v = 10\,\text{km}\,\text{s}^{-1}$$

IIT – JEE: 2024 **TW TEST (ADV)** **DATE: 21/05/23**

TOPIC: CHEMICAL KINETICS

Answer Key & Solution

21. (A)
$k_{obs} = k.k_c = 1.2 \times 10^{-1} \times 1.4 \times 10^{-2} = 1.68 \times 10^{-6}$ mole^{-1} L min^{-1}
Rate = k_{obs} $[NO]^2$ $[H_2]$ = $1.68 \times 10^{-6} \times 0.5^2 \times 0.5$
= 2.1×10^{-7} mole L^{-1} min^{-1}

22. (A)
Catalyst affect only activation energy. It brings down the activation energy of reaction.

23. (A)
$A \rightarrow 2B + C$
P 0 0
P – x 2x x
At equilibrium
180 = P – x + 2x + x
180 = 90 + 2x
2x = 90, x = 45
$$K = \frac{2.303}{t}\log\frac{P}{P-x} = \frac{2.303}{10}\log\frac{90}{90-45} = \frac{2.303}{10}\log 2 = \frac{0.6932}{10}$$
$$= 0.6932 = \frac{0.06932}{60} = 1.1555 \times 10^{-3}\ \text{sec}^{-1}$$

24. (B)
Conceptual

25. (B)
$r_0 = K[A]^n [B]^m$
$r_1 = K[2A]^n [B/2]^m$
$r_2 = K\ 2^{n-m} [A]^n [B]^m$ or $r_1 = r \times 2^{n-m}$

26. (C)
Radioactivity follows first order kinetics

27. (ABCD)
Conceptual

28. (A)
Conceptual

29. (C)

$$K = \frac{2.303}{t}\log\frac{C_0}{C}$$

$$= \frac{2.303}{2\times10^4}\log\frac{800}{50}$$

$$= 1.38\times10^{-4}\ \sec^{-1}$$

30. (A)

Two reactants leads to bimolecular reaction may be of I or II order.

31. (A)

Rate constant 'K' is characteristic constant for a given reaction.

32. (ABD)

To increase the rate of reaction catalyst does

(i) decrease E_a
(ii) decreases E_a/RT
(iii) increases $-E_a/RT$
(iv) increases $e^{-E_a/RT}$
(v) increases k

33. (ACD)

$$-\frac{d[NH_3]}{dt} = \frac{k_1[NH_3]}{1+k_2[NH_3]} = \frac{k_1}{\frac{1}{[NH_3]}+k_2}$$

If $[NH_3]$ is very high $\frac{1}{[NH_3]}$ is very small than k_2

$$\therefore\ \frac{-d[NH_3]}{dt} = \frac{k_1}{k_2}\ \text{constant}$$

i.e. order is zero if $[NH_3]$ is very low $\frac{1}{[NH_3]}$ is very high than k_2

$$\therefore\ \frac{-d[NH_3]}{dt} = \frac{k_1}{1/[NH_3]}$$ $\therefore$ order is one.

34. (BC)

(A) It is $-k/2.303$
(B) It is of zero order

35. (ABC)

(A) $t_{x\%} \propto \frac{1}{a}$ for second order

(B) Rate = $kC_t = kC_0e^{-kt}$

$$\ell n\left(\frac{dc}{dt}\right) = \ell n\,(\text{rate}) = \ell n\,kC - kt$$ $\Rightarrow$ So, straight line

(C) k = $Ae^{-E_a/RT}$

Rate = k(conc.)^n = $Ae^{-E_a/RT}$ (conc.)^n

$$\ell n\,(\text{rate}) = -\frac{E_a}{RT} + \text{constant}$$

Slope = $\frac{-E_a}{R}$

(D) $\frac{t_{0.75}}{t_{0.5}}$ = ratio of time is constant with initial conc.

36. (60)

First order reaction

$$K = \frac{2.303}{t}\log\frac{a_0}{a_0 - x}$$

$$K = \frac{2.303}{90}\log\frac{a_0}{0.25a_0} \qquad ...(1)$$

$$= 0.0154$$

$$t = 60\% = \frac{2.303}{K}\log\frac{a_0}{a_0} \qquad ...(2)$$

$$= \frac{2.303}{0.0154}\times(1-0.602) = 59.51\,\text{min} \approx 60$$

37. (3)

After 2 seconds surface area becomes 1/4 th. Hence radius becomes 1/2 of initial therefore vol will become 1/8 th dissolved vol = 7/8

mass dissolved = 7/8 × 1/7 = 1/8 gm

molarity = $\frac{1}{8\times125} = \frac{1}{1000} = 10^{-3}$

38. (2)

$$t_{1/2} = 20\,\text{min},\ t'_{1/2} = 10\,\text{min}\ \&\ [A]'_0 = 2[A]_0$$

$$\because t_{1/2}\,\alpha\,\frac{1}{[A]_0^{n-1}} \Rightarrow \frac{t'_{1/2}}{t_{1/2}} = \left[\frac{[A]'_0}{[A]_0}\right]^{n-1}$$

$$\text{or}\ \frac{20}{10} = \left[\frac{2\times[A]_0^{n-1}}{[A_0]}\right]^{n-1} \rightarrow 2 = 2^{n-1} \rightarrow n-1 = 1 \Rightarrow n = 2$$

39. (3)

$r = k[A]^x[B]^y$, write r_1, r_2, r_3. Divide one over another, get x & y

$$x + y = 3$$

40. (4)

$$\frac{2.303}{k}\log\frac{100}{100-99.99} = \frac{2.303}{k}\log\frac{100}{100-90}\ \text{ or, }\ \log 10^4 = x\log 10 \Rightarrow x = 4$$

IIT – JEE: 2024 TW TEST (ADV) DATE: 21/05/23

TOPIC: DIFFERENTIABILITY

SOLUTIONS

41. (D)

(d) $f(x) = \begin{cases} -x, & \text{if } x \le 0 \\ x^2, & \text{if } 0 < x \le 1 \\ x^2 - x + 1, & \text{if } x > 1 \end{cases}$

For differentiability at $x = 0$,

$$Lf'(0^-) = \lim_{h \to 0} \frac{f(0-h) - f(0)}{-h}$$

$$= \lim_{h \to 0} \frac{-(0-h) - 0}{-h}$$

$$= -1$$

$$Rf'(0^+) = \lim_{h \to 0} \frac{f(0+h) - f(0)}{h}$$

$$= \lim_{h \to 0} \frac{(0+h^2) - 0}{h}$$

$$= 0$$

$$Lf'(0^-) \ne Rf'(0^+)$$

$\therefore f(x)$ is differentiable at $x = 0$.

For differentiability at $x = 1$,

$$Lf'(1^-) = \lim_{h \to 0} \frac{f(1-h) - f(1)}{-h}$$

$$= \lim_{h \to 0} \frac{(1-h)^2 - 1}{-h}$$

$$= \lim_{h \to 0} \frac{1 + h^2 - 2h - 1}{-h} = 2$$

$$Rf'(1^+) = \lim_{h \to 0} \frac{f(1+h) - f(1)}{h}$$

$$= \lim_{h \to 0} \frac{(1+h)^2 - (1+h) + 1 - 1}{h}$$

$$= \lim_{h \to 0} \frac{(1+h)(1+h-1)}{h} = 1$$

$\because \quad Lf'(1^-) \neq Rf'(1^+)$

$\therefore f(x)$ is not differentiable at $x = 1$.

42. (B)

(b) We have,

$$f(x) = \begin{cases} x\left(\dfrac{e^{1/x} - e^{-1/x}}{e^{1/x} + e^{-1/x}}\right), & \text{if } x \neq 0 \\ 0, & \text{if } x = 0 \end{cases}$$

Now, $\quad Lf'(0^-) = \lim_{h \to 0} \dfrac{f(0-h) - f(0)}{-h}$

$$= \lim_{h \to 0} \frac{-h\left(\dfrac{e^{-1/h} - e^{1/h}}{e^{-1/h} + e^{1/h}}\right)}{-h}$$

$$= \lim_{h \to 0} \left(\frac{e^{-2/h} - 1}{e^{-2/h} + 1}\right)$$

$$= -1$$

$$Rf'(0^+) = \lim_{h \to 0} \frac{f(0+h) - f(0)}{h}$$

$$= \lim_{h \to 0} \frac{h\left(\dfrac{e^{1/h} - e^{-1/h}}{e^{1/h} + e^{-1/h}}\right)}{h}$$

$$= \lim_{h \to 0} \frac{1 - e^{-2/h}}{1 + e^{-2/h}} = 1$$

$\because \quad Lf'(0^-) \neq Rf'(0^+)$.

$\therefore f(x)$ is not differentiable at $x = 0$.

43. (A)

(a) Given $f(x) = x^3 \operatorname{Sgn}(x)$

or $$f(x) = \begin{cases} x^3, \text{ if } x > 0 \\ 0, \text{ if } x = 0 \\ -x^3, \text{ if } x < 0 \end{cases}$$

To check differentiability at $x = 0$,

$$Lf'(0^-) = \lim_{h \to 0} \frac{f(0-h) - f(0)}{-h}$$

$$= \lim_{h \to 0} \frac{-(0-h)^3 - 0}{-h}$$

$$= \lim_{h - 0} \frac{h^3}{-h}$$

$$= 0$$

$$Rf'(0) = \lim_{h \to 0} \frac{f(0+h) - f(0)}{h}$$

$$= \lim_{h \to 0} \frac{(0+h)^3 - 0}{h}$$

$$= \lim_{h \to 0} \frac{h^3}{h} = 0$$

$\therefore f(x)$ is differentiable at $x = 0$.

44. (D)

(d) $f(x) = (x^2 - 1)|x^2 - 3x + 2| + \cos(|x|)$

$$= (x-1)(x+1)\cdot|(x-1)(x-2)| + \cos x$$

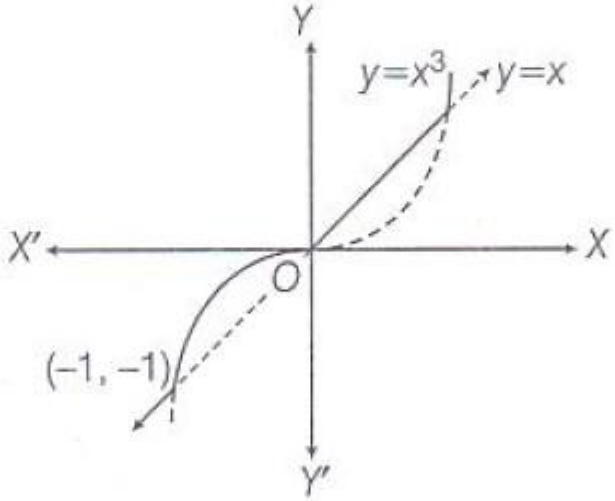
$(x-1)|x-1|$ and $\cos x$ are differentiable for all x, but $|x-2|$ is not differentiable at $x = 2$.

Hence, $f(x)$ is non-differentiable at $x = 2$.

45. (D)

(d) We have, $f(x) = \max.\{x, x^3\}$

$$\Rightarrow \quad f(x) = \begin{cases} x, & \text{if } x \le -1 \\ x^3, & \text{if } -1 \le x < 0 \\ x, & \text{if } 0 \le x \le 1 \\ x^3, & \text{if } 1 \le x \end{cases}$$



The continuous line shown in the above figure represent the graph of $f(x)$.

Clearly, $f(x)$ is not differentiable at $x = -1, 0, 1$.

46. (A, C)

(a,c) We have, $\lim_{x \to 0} \frac{f(x) - f(0)}{x - 0}$

$$= \lim_{x \to 0} \frac{\log \cos x}{\log(1 + x^2)}$$

$$= \lim_{x \to 0} \frac{\log(1 - 1 + \cos x)}{\log(1 + x^2)} \cdot \left(\frac{1 - \cos x}{1 - \cos x}\right)$$

$$= \lim_{x \to 0} \frac{\log\{1 - (1 - \cos x)\}}{1 - \cos x} \cdot \frac{1 - \cos x}{\log(1 + x^2)}$$

$$= - \lim_{x \to 0} \frac{\log\{1 - (1 - \cos x)\}}{-(1 - \cos x)} \cdot \frac{2\sin^2 x/2}{4(x/2)^2} \cdot \frac{x^2}{\log(1 + x^2)}$$

$$= -1/2$$

Hence, $f(x)$ is differentiable at $x = 0$, and $f(x)$ is continuous at $x = 0$.

47. (B, D)

(b,d) Given, $f(x) = \begin{cases} (x - e)2^{-2^{\frac{1}{e - x}}}, & x \neq e \\ 0, & x = e \end{cases}$

To check continuity at $x = e$,

$$\lim_{x \to e} f(x) = \lim_{x \to e} (x - e)\, 2^{-2^{\frac{1}{e - x}}} = 0$$

Also, $f(e) = 0$

$\because$ $\lim_{x \to e} f(x) = f(e) = 0$

$\therefore f(x)$ is continuous at $x = e$.

To check differentiability at $x = e$,

$$Lf'(e) = \lim_{h \to 0} \frac{f(e-h) - f(e)}{-h}$$

$$= \lim_{h \to 0} \frac{(-h)\,2^{(-2)^{1/h}} - 0}{-h} = \lim_{h \to 0} 2^{-2^{1/h}} = 0$$

$$Rf'(e) = \lim_{h \to 0} \frac{f(e+h) - f(e)}{h}$$

$$= \lim_{h \to 0} \frac{h \times 2^{-2^{1/h}} - 0}{h} = 1$$

$\therefore f(x)$ is non-differentiable at $x = e$.

Since, $f(x)$ is not differentiable at $x = e$

$\therefore$ It must gave sharp edge at $x = e$.

48. (A, B, C)

Given, $f(x) = 3(2x+3)^{2/3} + 2x + 3$

To check contunity at $x = 0$,

$$\text{LHL} = \lim_{h \to 0} 3\{2(0-h) + 3\}^{2/3} + 2(0-h) + 3$$

$$= \lim_{h \to 0} 3\{-2h + 3\}^{2/3} - 2h + 3$$

$$= 3 \cdot (3)^{2/3} + 3 = 3^{5/3} + 3$$

$$\text{RHL} = \lim_{h \to 0} 3\{2(0+h) + 3\}^{2/3} + 2(0+h) + 3$$

$$= \lim_{h \to 0} 3\{2h + 3\}^{2/3} + 2h + 3$$

$$= 3 \cdot (3)^{2/3} + 3 = 3^{5/3} + 3$$

$$f(0) = 3^{5/3} + 3$$

$\therefore f(x)$ is continuous at $x = 0$.

To check differentiability at $x = 0$,

$\therefore f(x)$ is not differentiable at $x = 0$.

To check continuity at $x = -3/2$,

$$\text{LHL} = \lim_{h \to 0} 3\{2(-3/2 - h) + 3\}^{2/3} + 2(-3/2 - h) + 3 = 0$$

$$\text{RHL} = \lim_{h \to 0} 3\{2(-3/2 + h) + 3\}^{2/3} + 2(-3/2 + h) + 3 = 0$$

$\therefore f(x)$ is continuous at $x = -3/2$.

To check differentiability at $x = -3/2$,

$$Lf'\left(-\frac{3}{2}\right) = \lim_{h \to 0} \frac{3\{2(-3/2 - h) + 3\}^{2/3} + 2(-3/2 - h) + 3}{-h}$$

$$= \lim_{h \to 0} \frac{3\{(-2h)^{2/3} - 3 - 2h + 3}{-h}$$

$$= \lim_{h \to 0} \frac{3\{(-2h)^{2/3} - 2h\}}{-h}$$

$$= \lim_{h \to 0} 3\{(-2)^{2/3} h^{-1/3} - 2\} = -6$$

$$Rf' = \left(-\frac{3}{2}\right) \lim_{h \to 0} \frac{3\left\{2\left(\frac{-3}{2} + h\right) + 3\right\}^{2/3} + 2\left(\frac{-3}{2} + h\right) + 3}{h}$$

$$= \lim_{h \to 0} \frac{3\{(2h)^{2/3}\} + 2h}{h} = \lim_{h \to 0} 3\{(2)^{2/3} h^{1/3} + 2\}$$

$$= 6$$

$Lf'(-3/2) \neq Rf'(-3/2)$

$\therefore f(x)$ is not differentiable at $x = -3/2$.

49. (A, B)

(a,b) We have,

$$f(x) = \begin{cases} 2x + 3, & \text{for} -3 \leq x < -2 \\ x + 1, & \text{for} -2 \leq x < 0 \\ x + 2, & \text{for } 0 \leq x \leq 1 \end{cases}$$

We check differentiability at $x = -2$

LHD = 2 and RHD = + 1

$\therefore f(x)$ is not differentiable not continuous at $x = 0$.

50. (A, D)

(a,d) We have, $f(x) = \cos \pi (|x| + [x])$

Check continuity at $x = \frac{1}{2}$,

$$\lim_{x \to 1/2} \cos \pi \left(|x| + \left[\frac{1}{2}\right]\right) = \lim_{x \to 1/2} \cos \pi x$$

$$= \cos \frac{\pi}{2} = 0, \text{ which is finite.}$$

So, it is continuous

$\therefore$ Continuity at $x = 0$,

$$\text{LHL} = \lim_{x \to 0^-} f(x) = \lim_{h \to 0} f(0 - h)$$

$$= \lim_{h \to 0} \cos \pi(|0 - h| + [-h])$$

$$= \lim_{h \to 0} \cos \pi (h - 1)$$

$$= \cos(-\pi) = -1$$

$$\text{RHL} = \lim_{h \to 0} f(0 + h)$$

$$= \lim_{h \to 0} \cos \pi (h + 0)$$

$$= \cos 0 = 1$$

$\therefore$ It is discontinuous at $x = 0$.

We know that $[x]$ is not diffrentiable at x = Integer

$\therefore$ Option (c) is incorrect.

$\therefore \quad f(x) = \cos \pi (|x| + 0) = \cos \pi |x| = \cos \pi x$

Hence, $f(x)$ is differentiable in $(0, 1)$.

51. (A, B, C)

(a,b,c) We have,

$$f(x) = |x+1|(|x| + |x-1|)$$

$$f(x) = \begin{cases} -(x+1)(-x-x+1); & -2 \le x < -1 \\ (x+1)(-x-x+1); & -1 \le x < 0 \\ (x+1)(x-x+1); & 0 \le x < 1 \\ (x+1)(x+x-1); & 1 \le x < 2 \end{cases}$$

$$= \begin{cases} -(x+1)(1-2x); & -2 \le x < -1 \\ (x+1)(1-2x); & -1 \le x < 0 \\ x+1; & 0 \le x < 1 \\ (x+1)(2x-1); & 1 \le x \le 2 \end{cases}$$

$$= \begin{cases} 2x^2+x-1; & -2 \le x < -1 \\ -2x^2-x+1; & -1 \le x < 0 \\ x+1; & 0 \le x < 1 \\ 2x^2+x-1; & 1 \le x \le 2 \end{cases}$$

Now, $$f'(x) = \begin{cases} 4x+1; & -2 \le x < -1 \\ -4x-1 & -1 \le x < 0 \\ 1; & 0 \le x < 1 \\ 4x+1; & 1 \le x \le 2 \end{cases}$$

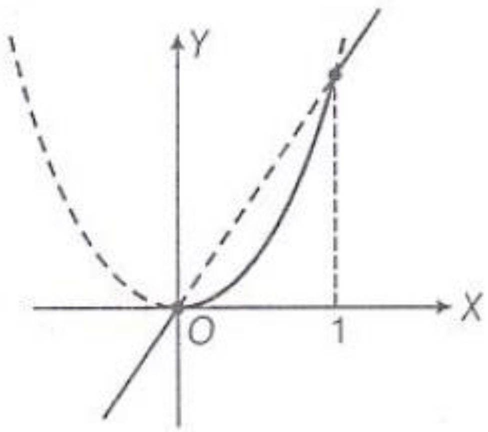
Clearly, $f(x)$ is not differentiable at $x = -1, 0$ and 1.

52. (A, C, D)

(a,c,d) We have,

$$h(x) = \min\{x, x^2\}$$

$$\therefore \quad h(x) = \begin{cases} x, & \text{if } x < 0 \\ x^2, & \text{if } 0 \le x < 1 \\ x, & \text{if } x \ge 1 \end{cases}$$



Clearly, $h(x)$ is continuous for all $x \in R$.

$$h'(x)=\begin{cases} 1, \text{ if } x<0 \\ 2x, \text{if } 0\le x<1 \\ 1, \text{ if } x\ge 1 \end{cases}$$

$\therefore \quad h'(x)=1$ for all $x>1$

and $h(x)$ is not differentiable at $x=0$ and 1.

53. (A, B)

(a,b) We have, $f(x)=1+[\cos x]x$

We know that in $0<x\le\frac{\pi}{2}$, $[\cos x]=0$

$\therefore f(x)=1$, which is a constant function.

$\therefore f(x)$ is continuous and differentiable everywhere.

54. (D)

(d) We have, for $-1<x<1$

$\Rightarrow \quad 0\le x\sin\pi x\le\frac{1}{2}$

Also, $x\sin\pi x$ becomes negative and numerically less than 1, when x is slightly greater than 1 and so by definition of [x].

$f(x)=[x\sin\pi x]=-1$, when $1<x<1+h$.

Thus, $f(x)$ is constant and equal to 0 in the closed internal [–1, 1] and so $f(x)$ is continuous and differentiable in the open interval (–1, 1).

At $x=1$, $f(x)$ is discontinuous, since

$$\lim_{h\to 0}(1-h)=0$$

and

$$\lim_{h\to 0}(1+h)=-1$$

$\therefore f(x)$ is not differentiable at $x=1$.

55. (B, C)

(b,c) We have,

$$f(x+y)=f(x)+f(y)$$

Clearly, $f(x)=Kx$ satisfy the above condition and which is differentiable at $x=0$.

We see that $f(x)$ continuous everywhere $f'(x)=K$, which is also continuous everywhere.

56. (3)

(3) We have,

$$\lim_{h\to 0}\frac{f(h)-f(-2h)}{h} \quad \left[\frac{0}{0}\text{ form}\right]$$

$$=\lim_{h\to 0}\frac{f'(h)+2f'(-2h)}{1} \quad \text{[by L' Hospital's rule]}$$

$$=f'(0)+2f'(0)$$

$$=3f'(0)=3$$

57. (7)

(7) We have,

$$f(x)=[x]\tan x(\pi x)$$

Now, $$Rf'(7)=\lim_{h\to 0}\frac{[7+h]\tan\pi(7+h)-7\tan 7\pi}{h}$$

$$=\lim_{h\to 0}\frac{7\cdot\tan\pi(7+h)-7\tan 7\pi}{h}$$

$$=7\cdot\lim_{h\to 0}\frac{\tan\pi(7+h)-\tan 7\pi}{h}=7\cdot\pi$$

$\therefore$ $k=7$

58. (5)

$$\because\ F(x)=\lim_{n\to\infty}\frac{f(x)+x^n g(x)}{1+x^n}=\begin{cases} g(x); & x>1\\ f(x); & 0\le x<1\\ \dfrac{f(1)+g(1)}{2}; & x=1\end{cases}$$

$$F(x) = \begin{cases} x^2 + px + 3; & 0 \le x < 1 \\ x + q; & x > 1 \\ \frac{p+q+5}{2}; & x = 1 \end{cases}$$

F(x) is derivable, $\therefore$ continuous too

$\therefore$ $1 + p + 3 = 1 + q \Rightarrow p - q = -3$ …(i)

L.H.D. at $x = 1$ = R.H.D. at $x = 1$

$\Rightarrow$ $2 + p = 1 \Rightarrow p = -1$

$\therefore$ q = 2

$\therefore$ $p^2 + q^2 = 5$ **Ans.**]

59. (5)

$f(g(x)) = \left| \left| |x| - 1 \right| - 2 \right|$

$$\Rightarrow f(g(x)) = \begin{cases} -x-3 & x < -3 \\ 3+x & -3 \le x < -1 \\ 1-x & -1 \le x < 0 \\ x+1 & 0 \le x < 1 \\ 3-x & 1 \le x < 3 \\ x-3 & 3 \le x \end{cases}$$

$f(g(x))$ is not differentiable at $x = -3, -1, 0, 1, 3$]

60. (0)

$\because$ $f(x)$ is always continuous

$\therefore$ $ax^2 + bx + c = 0 \quad \forall x \in (0, 6)$

$\therefore$ $a = b = c = 0$

$\therefore$ $f(x) = 0 \quad \forall x \in (0, 6)$

$\therefore$ $f(x)$ is always derivable.