IIT – JEE: 2024 **TW TEST (MAIN)** **DATE: 20/05/23**

TOPIC: GRAVITATION

Answer Key & Solution

1. (B)

P.E. becomes less negative, and KE become less positive

2. (C)

Escape speed $\upsilon_{min} = \sqrt{2gR}$ Since we talking about another planet with different radius, gravitational force also changes.

$\upsilon_{min} = \sqrt{\frac{2GM}{R}} \Rightarrow \upsilon_{min} \propto \sqrt{\frac{1}{R}} \Rightarrow$

Given $R_2 = \frac{R_1}{4}$

$\therefore \frac{\upsilon_2}{\upsilon_1} = \sqrt{\frac{R_1}{\left(\frac{R_1}{4}\right)}} \Rightarrow \frac{\upsilon_2}{\upsilon_1} = \sqrt{4} \Rightarrow \upsilon_2 = 2\upsilon_1$

3. (A)

$m\upsilon'^2 = 2 \times \frac{1}{2} m\upsilon^2$

$= \sqrt{2}\,\upsilon_0$

$= \upsilon_e$

So escape

4. (D)

$P.E. = -\frac{Gm_1m_2}{r}$

$T.E. = -\frac{Gm_1m_2}{2r}$

$K.E. = +\frac{Gm_1m_2}{2r}$

5. (B)

$U_f - u_i = \frac{-GmM}{\left(R + \frac{R}{5}\right)} + \frac{GmM}{R}$

$$= \frac{-5GmM}{6R} + \frac{GmM}{R} = \frac{GmM}{6R} = \frac{mgR}{6}$$

6. (B)

$$V_P = -\frac{GM}{R}$$

As $R \downarrow \frac{GM}{R} \uparrow$ but dur to -ve it decreases.

7. (A)

Gravitational field strength at m_1 is

$$I_1 = \frac{Ggm_2}{d^2}$$

Gravitational field strength of m_2 is

$$I_2 = \frac{Ggm_1}{d^2}$$

$\Rightarrow \frac{Gg}{d^2} = \frac{l_1}{m_2} = \frac{-l_1}{m_1}$ (There is a minus sign since $\bar{l}_1$ and $\bar{l}_2$ are in opposite direction)

$$\Rightarrow \bar{l}_1 m_1 + \bar{l}_2 m_2 = 0$$

$$\bar{F}_{net} = 0$$

8. (C)

Let the point masses be

$p_1, p_2, p_3 ......p_a ....(a > 0)$

Now potential at x =0 due to p_1 be

$$P_1 = \frac{-Gm}{1}$$

Similarly $P_2 = \frac{-Gm}{2}$

$$P_3 = \frac{-Gm}{4}$$

So, total potential is given by

$$P = P_1 + P_2 + P_3 + ...$$

$$P = \frac{-Gm}{1} + \frac{-GM}{2} + \frac{-Gm}{4} + \frac{-Gm}{8} +$$

$$= -Gm\left(1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + ...\right)$$

This is a sum of infinite terms with common ratio $\frac{1}{2}$.

$$P = (-Gm)\,\frac{1}{\left(1 - \frac{1}{2}\right)} = -2GM$$

9. (B)

$$g = \frac{Gm}{R^2}$$

10. (D)

The value of gravity changes as we move away from or towards the centre of the Earth.

This is given by $9R = \frac{GM}{R^2}$, where M is the mass of a planet of radius R

So $M = \frac{4}{3}\pi R^3 \rho$; substituting in above equation

$$g_R = G\frac{4}{3}\pi\rho \times R$$

Since we want the value of g at depth (d) from the Earth's surface, we replace R by (R – d)

$$\Rightarrow g_d = G\frac{4}{3}\pi\rho \times (R - d)$$

$$\Rightarrow \frac{g_R}{g_d} = \frac{R}{(R - d)}$$

$$\Rightarrow \frac{g_d}{g_R} = \left(1 - \frac{d}{R}\right)$$

The given depth in the problem is d = 3200 km, substituting we get,

$$\frac{g_d}{g_R} = \left(1 - \frac{3200}{6400}\right)$$

$$g_d = 9.8 \times \left(1 - \frac{3200}{6400}\right)$$

$$g_d = 9.8/2$$

$$g_d = 4.9 ms^{-2}$$

11. (C)

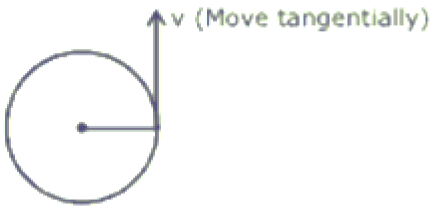
Escape speed is $\upsilon_e = \sqrt{2gR}$

$$\frac{\upsilon_1}{\upsilon_2} = \sqrt{\frac{g_1 R_1}{g_2 R_2}}$$

It is given $\frac{R_1}{R_2} = K_1$ and $\frac{g_1}{g_2} = K_2$

$$\frac{\upsilon_1}{\upsilon_2} = \sqrt{K_1 K_2}$$

12. (B)



13. (C)

$T \propto r^{3/2} \left[\omega = \frac{2\pi}{T}\right]$

$\omega \propto \frac{1}{r^{3/2}}$

$\left(\frac{\omega}{2\omega}\right) = \left(\frac{R_1}{r}\right)^{3/2}$

$\frac{R_1}{r} = \frac{1}{(2)^{2/3}}$

$R_1 = \frac{r}{(4)^{1/3}}$

14. (C)

$\upsilon = \sqrt{\frac{Gm}{r}} = \sqrt{\frac{Gm}{R + h}}$

$\upsilon_1 = \sqrt{\frac{Gm}{R + \frac{R}{2}}} = \sqrt{\frac{2}{3}}\,\upsilon$

15. (B)

$\frac{GMm}{r^2} = \frac{m\upsilon^2}{r}$

16. (C)

$\frac{1}{2}m\upsilon^2 = \frac{GM_e m}{R_e + h} = \frac{gR_e^2 m}{R_e + 4R_e}$

$\frac{1}{2}m\upsilon^2 = \frac{MgR_e}{5}$

17. (B)

$W = \frac{GmM}{R} - \frac{GmM}{nR + R}$

$= \frac{GmM}{R}\left[1 - \frac{1}{nH}\right] = \frac{GmM}{R}\left[\frac{n}{n+1}\right]$

$= mgR\left(\frac{n}{n+1}\right)$

18. (A)

When the 2 particles are at rest, at a separation d

Potential energy of the system = $\frac{-Gm}{d}$ and kinetic energy = 0

Let V be the speed at half the separation

Potential energy at $\frac{d}{2}$ is $=\frac{-2Gm^2}{d}$ and

Kinetic energy $=2\times\frac{1}{2}mV^2$

From conservation of energy principle,

$$\frac{-Gm^2}{d}=\frac{-2Gm^2}{d}+2\times\frac{1}{2}mV^2$$

$$\frac{-Gm^2}{d}=\frac{-2Gm^2}{d}+V^2$$

Hence $V=\sqrt{\frac{Gm}{d}}$

19. (C)

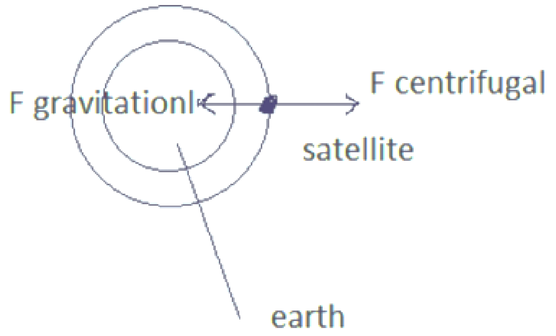
For a body revolving around the earth at any orbital radius, the gravitational force towards earth is countered by centrifugal force as shown in figure.

i.e. $F_g = F_\upsilon$

So, for a body hanging in the satellite

$W_1 + F_\upsilon = F_g \rightarrow W_1 = 0$

Similarly $W_2 = 0$. Thus $W_1 = W_2$



20. (A)

Here $G = 6.67\times10^{-11}$ N m^2/kg^2

Mass of the pulsar, $M = 1.98\times10^{30}$ kg

Radius of the pulsar, R = 12 km $=12\times10^3$ m

Acceleration due to gravity on the surface of the pulsar is

$$g=\frac{GM}{R^2}$$

Substituting the given numerical values, we get

$$g=\frac{(6.67\times10^{-11}Nm^2/kg^2)(1.98\times10^{30}kg)}{(12\times10^3m)^2}$$

$$=0.092\times10^{13}\ m/s^2 = 9.2\times10^{11}m/s^2$$

21. (1.5)

Time period $T^2=\frac{4\pi^2}{GM}R^3$

i.e. $T^2 \propto R^3$

$$\Rightarrow \left(\frac{T_n}{T_S}\right)^2 = \left(\frac{R_n}{R_S}\right)^3$$

Given $\frac{R_n}{R_S} = \frac{10^{13}}{10^{12}} = 10$

$$\left(\frac{T_n}{T_S}\right)^2 = 10^3$$

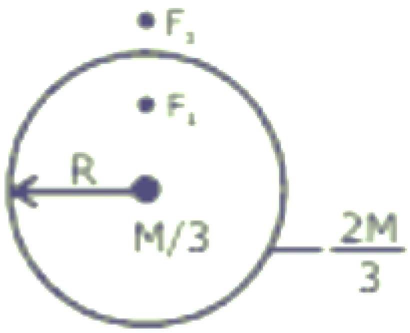
$$\Rightarrow \frac{T_n}{T_S} = 10^{3/2} = 10\sqrt{10}$$

22. (1.5)

$$g' = \frac{G(2M_e)}{(2R_e)^2} = \frac{g}{2}$$

$$T = 2\pi\sqrt{\frac{l}{g}} = 2$$

New time period $T' = 2\pi\sqrt{\frac{l}{g/2}} = 2\sqrt{2}$



23. (1)
As the stars will always be diametrically opposite each other they rotate with the same angular velocity.

24. (100)

$$\frac{1}{2}mv^2 = mgh = \frac{mGM}{R^2} \times 90 \qquad .....(1)$$

$$\frac{1}{2}mv^2 = \frac{mG\left(\frac{1}{10}M\right)}{\left(\frac{R}{3}\right)^2} \times G_1 \qquad .....(2)$$

From (1) and (2)

$$m\frac{GM}{R^2} \times 90 = \frac{9}{10}\frac{mGM}{R^2} \times h_1 \Rightarrow h_1 = 100 \text{ m}$$

25. (12)

$$T = \frac{2\pi}{\omega_{rel.}} = \frac{2\pi}{2\omega} = \frac{2\pi \times 24 \times hr.}{2 \times 2\pi}$$

$$T = 12\,hr.$$

26. (2)

$$v_0 = \sqrt{\frac{2GM_e}{R_e}} = \sqrt{\frac{2G\rho\frac{4}{3}\pi R_e^3}{R_e}}$$

$$= \sqrt{2G\rho\frac{4}{3}\pi R_e^2}$$

$$v' = \sqrt{2G\rho\frac{4}{3}\pi(2R_e)^2}$$

$$= 2v_0$$

27. (0.2)

$$g = \frac{GM_e}{R_e^2} = \frac{GM_e}{(5R_e)^2}$$

$$\frac{\frac{4}{3}\pi R_e^3\rho}{R_e^2} = \frac{\frac{4}{3}\pi(5R_e)^3\rho'}{(5R_e)^2}$$

$$\rho = 5\rho'$$

$$\rho' = \frac{\rho}{5}$$

28. (4)

Considering the origin of the coordinates system at 4*m*, we evaluate the positon of the centre of mass as

$$\frac{4m\times 0 + m\times r}{4m+m} = \frac{r}{5}$$

Thus the center of mass is $\frac{r}{5}$ from 4*m* and $\frac{4r}{5}$ from *m*.

The ratio of their kinetic energy is given as $\frac{\frac{1}{2}\left[I\omega^2\right]_{2m}}{\frac{1}{2}\left[I\omega^2\right]_m}$

As the angular velocity of the both the masses would be same we get the ratio of kinetic energy as

$$\frac{4m\left(\frac{r}{5}\right)^2}{m\left(\frac{4r}{5}\right)^2} = \frac{1}{4}$$

29. (1.4)

All point on the circumference of the ring are at distance $\sqrt{R^2+x^2}$ from the center *O* of the ring.

Force on the mass *m* at *P* :

$$F = \frac{GMm}{R^2+x^2}\times(2\cos\theta) \text{ where } \cos\theta = \frac{x}{\sqrt{R^2+x^2}}$$

Due to symmetry, vertical components of the forces from two symmetrical elements cancel out, the horizontal components add up, hence we get a factor of 2.

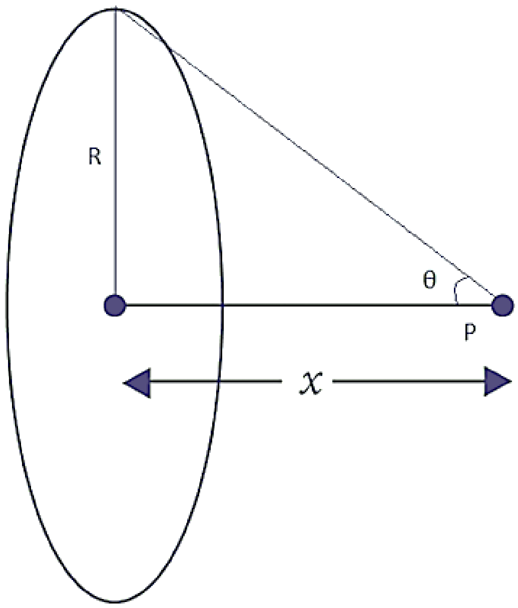
$$\therefore \quad F = \frac{GMm}{R^2+x^2}\times 2\frac{x}{\sqrt{R^2+x^2}}$$

When *F* is maximum, $\frac{dF}{dx} = 0$

$$\Rightarrow \left(R^2+x^2\right)^{-\frac{3}{2}}+x\left(-\frac{3}{2}\left(R^2+x^2\right)^{-\frac{3}{2}-1}\times 2x\right.$$

$$\Rightarrow \left(R^2+x^2\right)=3x^2$$

$$\Rightarrow x=\frac{R}{\sqrt{2}}$$



30. (0.5)

Gravitational force between 2 bodies is given by $F=\frac{Gm_1m_2}{R^2}$

Where, m_1 and m_2 are the masses of the bodes, G is the universal gravitational constant and R is the distance between them.

For a given distance, $F=\frac{Gm(1-X)m(X)}{R^2}$ is maximum when $X(1-X)$ is maximum.

By differentiation, we get,

$$\frac{dF}{dX}=\frac{Gm^2}{R^2}\frac{d}{dX}\left(X(1-X)\right)$$

$$\frac{dF}{dX}=\frac{Gm^2}{R^2}\frac{d}{dX}\left(X-X^2\right)$$

$$\frac{dF}{dX}=\frac{Gm^2}{R^2}(1-2X)=0$$

$$1-2X=0$$

$$X=\frac{1}{2}$$

The gravitational force of attraction has a maximum value at X = 1/2.

Answer Key & Solution

31. (D)

$$\frac{\Delta[NO_2]}{\Delta t}=\frac{2.4\times10^{-2}}{6}=4\times10^{-3}$$

$$-\frac{1}{2}\frac{\Delta[N_2O_5]}{\Delta t}=\frac{1}{4}\frac{\Delta[NO_2]}{\Delta t}$$

$$-\frac{\Delta[N_2O_5]}{\Delta t}=\frac{1}{2}\frac{\Delta[NO_2]}{\Delta t}=2\times10^{-3}$$

32. (C)

Rate constant depends upon temperature & catalyst.

33. (B)

$$r_1=K(a)^2(b)^{\frac{1}{2}}$$

$$r_2=K(2a)^2(4b)^{\frac{1}{2}}$$

$$\frac{r_2}{r_1}=8$$

34. (B)

$$K=10^{-2}\ \ell\ \text{mole}^{-1}\,\text{sec}^{-1}$$

$$=\frac{10^{-2}\times10^{3}}{6\times10^{23}}\times60\ \text{ml molecule}^{-1}\ \text{min}^{-1}$$

$$=10^{-21}$$

35. (B)

	$A_2(g)\rightarrow$	$B(g)+$	$\frac{1}{2}C(g)$
t = 0	100	0	0
t = 5 min	100 – p	p	$\frac{p}{2}$

$$100-p+p+\frac{p}{2}=120$$

$$100+\frac{p}{2}=120\ \Rightarrow\ p=40\ \text{min}$$

$$\text{Rate}=-\frac{\Delta P_{A_2}}{\Delta t}=\frac{40}{5}=8\ \text{mm/min}$$

36. (B)

$N_2(g)+3H_2(g)\rightarrow 2NH_3(g)$

$\frac{\Delta[NH_3]}{\Delta t}=10^{-3}\ kg/h=\frac{10^{-3}\times 10^{3}}{17}mol/h$

$-\frac{\Delta[H_2]}{\Delta t}=\frac{3}{2}\frac{\Delta[NH_3]}{\Delta t}=\frac{3}{2}\times\frac{10^{-3}\times 10^{3}}{17}\ mol/h$

$=\frac{3}{2}\times\frac{1}{17}\times 2\ gm/h$

$=\frac{3}{17}\times 10^{-3}\ kg/h=1.76\times 10^{-4}\ kg/h$

37. (C)

Rate = K

38. (B)

$1.837=(1.5)^n$

$n=1.5$

39. (A)

rate $=K[A]^m[B]^n$

$0.1=K(0.012)^m(0.035)^n$ ...(1)

$0.1=K(0.024)^m(0.035)^n$ ...(2)

$(2)\div(1)$

$1=2^m \Rightarrow m=0$

$0.8=K(0.024)^m(0.070)^n$ ...(3)

$(3)\div(2)$

$8=2^n \Rightarrow n=3$

rate = $K[B]^3$

40. (A)

$[A]=[A]_0-Kt$

$\frac{[A]_0}{4}=[A]_0-K(10) \Rightarrow K=\frac{3[A]_0}{4\times 10}$

$\frac{[A]_0}{10}=[A]_0-\frac{3[A]_0}{4\times 10}t$

$\frac{3[A]_0}{4\times 10}t=\frac{9[A]_0}{10}$

$t=12\ hr$

41. (C)

$$K = \frac{2.303}{t} \log \frac{a}{a - \frac{3a}{4}}$$

$$t = \frac{2.303}{K} \log 4$$

42. (B)

$$t_{\frac{1}{2}} \propto \frac{1}{[A]_0}$$

$$[A]_0 \xrightarrow{t_1} \frac{[A]_0}{2} \xrightarrow{2t_1} \frac{[A]_0}{4}$$

$$[A]_0 \xrightarrow{3t_1} \frac{[A]_0}{4}$$

43. (D)
[Reactant] decreases
[Product] increases
Rate of decreases in concentration of B is greater than A

44. (C)

$$t_{\frac{1}{2}} \propto \frac{1}{[A]_0^{n-1}}$$

$$\frac{60}{29} = \left(\frac{0.75}{1.55}\right)^{1-n}$$

$$1 - n = -1 \Rightarrow n = 2$$

45. (B)
Activation energy is different for different substance.

46. (C)

$$\log K = \log A - \frac{Ea}{2.303RT}$$

$\log K$ v/s $\frac{1}{T}$ graph is straight line.

47. (D)

$$\Delta H = y - x$$

$$\Delta H = E_{a,f} - E_{a,b}$$

$$y - x = E_{a,f} - z$$

$$E_{a,f} = y - x + z$$

48. (B)
$K = A$ if $T \to \infty$

49. (D)

$$r_1 = K(a)^n (b)^m$$

$$r_2 = K(2a)^n\left(\frac{b}{2}\right)^m$$
$$\frac{r_2}{r_1} = 2^n\left(\frac{1}{2}\right)^m = 2^{n-m}$$

50. (C)

$r \propto [NO]^2[O_2]$

If volume is tripled then concentration decreases to $\frac{1}{3}$rd .

51. (0.80)

$2.4\times10^{-5} = \left(3\times10^{-5}\right)[N_2O_5]$

$K = 0.8$

52. (0.00)

rate = K for zero order

53. (0.33)

$r = K(a)^m$

$2r = K(8)^m$

$2 = 8^m \Rightarrow 2 = 2^{3m}$

$3m = 1 \Rightarrow m = \frac{1}{3} = 0.33$

54. (2.00)

Unit of K for 2nd order reaction is $\ell\,mol^{-1}\,sec^{-1}$.

55. (0.00)

$\frac{[A]_0}{4} = [A]_0 - K \Rightarrow K = \frac{3[A]_0}{4}$

Time taken for completion $= \frac{[A]_0}{K}$

$= \frac{4}{3}hr = 1.33\,hr$

i.e. after 1.33 hr, [reactant] = 0

56. (2.00)

$t_{99} = \frac{2.303}{K}\log\frac{100}{1} = \frac{2.303}{K}\times2$

$t_{90} = \frac{2.303}{K}\log\frac{100}{10} = \frac{2.303}{K}\times1$

$t_{99} = 2\times t_{90}$

57. (30.00)

$K = \frac{0.693}{10}\min^{-1}$

Initial rate $= 6\times10^{21}$ molecules $ml^{-1}\ sec^{-1}$

Final rate $= 4.5\times10^{25}$ molecules $\ell^{-1}\ min^{-1}$

$$= \frac{4.5\times10^{25}}{10^3\times60} \text{ molecules } ml^{-1}\ sec^{-1}$$

$$= 0.75\times10^{21}$$

Ratio of rate is ratio of conc. for 1st order reaction.

$$t = \frac{2.303}{K}\log\frac{6\times10^{21}}{0.75\times10^{21}}$$

$$t = \frac{2.303}{0.693}\times10\times\log 8 = 30$$

58. (0.00)

For zero order, $t_{1/2} \propto [A]_0$

59. (3.00)

For 2nd order reaction, time taken for 75% reaction is 3 times of half life period.

60. (32.00)

2^5 times = 32 times

SOLUTIONS

61. **(B)**

62. **(B)**

63. **(B)**

64. **(C)**

65. **(C)**

66. **(D)**

67. **(C)**

68. **(C)**

69. **(B)**

70. **(D)**
$f(x) = x\,[x]$; $-1 \le x \le 3$.
Since, Greatest Integral function are not continuous as well as differentiable at Integers.
$\therefore$ Option (D) is correct answer.

71. **(B)**
at x = 0:
$f'(0-0) = \lim_{h\to 0} \frac{|0-h|-0}{-h} = -1$
$f'(0+0) = \lim_{h\to 0} \frac{|0+h|-0}{h} = 1$
Now, since $f'(0-0) \ne f'(0+0)$
$\Rightarrow$ f(x) is not differentiable at x = 0.

72. (B)
Differentiability at x = 0
$R[f'(0)] = \lim_{h\to 0} \frac{f(0+h)-f(0)}{h}$
$= \lim_{h\to 0} \frac{(0+h)^2-0}{h} = \lim_{h\to 0} h = 0$
$L[f'(0)] = \lim_{h\to 0} \frac{f(0-h)-f(0)}{-h}$

$= \lim_{h\to 0} \frac{-(0-h)-0}{-h} = -1$

$\because$ R [f ' (0)] $\neq$ L [f '(0)]

$\therefore$ f(x) is not differentiable at x = 0

Differentiability at x = 1

$R[f'(1)] = \lim_{h\to 0} \frac{f(1+h)^3 - f(1)}{h}$

$= \lim_{h\to 0} \frac{(1+h)^3 - (1+h) + 1 - 1}{h}$

$= \lim_{h\to 0} \frac{2h + 3h^2 + h^3}{h} = 2$

$L[f'(1)] = \lim_{h\to 0} \frac{f(1-h) - f(1)}{-h}$

$= \lim_{h\to 0} \frac{(1-h)-1}{-h}$

$= \lim_{h\to 0} \frac{-2h + h^2}{-h} = 2$

Thus R [f ' (1)] = L f '(1)]

$\therefore$ function f(x) is differentiable at x = 1

73. **(A)**

Since $f(1-0) = \lim_{x\to 1} 3^x = 3$

$f(1+0) = \lim_{x\to 1} (4-x) = 3$

and $f(1) = 3^1 = 3$

f(1–0) = f(1+0) = f(1)

$\therefore$ f(x) is continuous at x = 1

$\Rightarrow$ Again $f'(1+0) = \lim_{x\to 1^+} \frac{f(x) - f(1)}{x-1}$

$= \lim_{x\to 1} \frac{3^x - 3}{x-1}$

$= \lim_{h\to 0} \frac{3^{1+h} - 3}{h}$

$= 3 \lim_{h\to 0} \frac{3^h - 1}{h}$

= 3 log 3

and $f'(1+0) \quad \lim_{x\to 1^-} \frac{f(x) - f(1)}{x-1}$

$= \lim_{x\to 1} \frac{4 - x - 3}{x-1} = -1$

$\therefore$ f ' (1+0) $\neq$ f '(1–0) $\neq$ f(x) is not differentiable at x = 1.

74. **(A)**

When x < 0, $f(x) = \frac{x}{1-x}$

$f'(x) = \frac{1}{(1-x)^2}$ ...(1)

which exists finitely for all x < 0

Also when x > 0, $f(x) = \frac{1}{1+x}$

$\Rightarrow f'(x) = \frac{1}{(1+x)^2}$ ...(2)

which exists finitely for all x > 0. Also from (1) and (2) we have

$$\begin{cases} f'(0-0)=1 \\ f'(0+0)=1 \end{cases} \Rightarrow f'(0) = 1$$

Hence f(x) is differentiable $\forall\ x \in R$

75. **(B)**

When $x \neq 0$

$$f'(x) = 2x \sin\frac{1}{x} + x^2 \cos\frac{1}{x}.\left(-\frac{1}{x^2}\right)$$

$$= 2x \sin\frac{1}{x} - \cos\left(\frac{1}{x}\right)$$

Which exists finitely for all $x \neq 0$ and $f'(0) = \lim_{x\to 0}\frac{f(x)-f(0)}{x-0} = \lim_{x\to 0}\frac{x^2 \sin 1/x}{x} = 0$

$\therefore$ f is also derivable at x = 0. Thus

$$f'(x) = \begin{cases} 2x\sin\frac{1}{x} - \cos\frac{1}{x}, x \neq 0 \\ 0, \quad x = 0 \end{cases}$$

Also $\lim_{x\to 0} f'(x) = \lim_{x\to 0}\left(2x\sin\frac{1}{x} - \cos\frac{1}{x}\right)$

$$= 2 - \lim_{x\to 0}\cos\frac{1}{x}$$

But $\lim_{x\to 0}\cos\frac{1}{x}$ does not exist, so $\lim_{x\to 0} f'(x)$ does not exist.

Hence, f ' is not continuous (so not derivable) at x = 0.

76. (B)

(b) Given, $f(x) = \begin{cases} g(x)\cos\left(\frac{1}{x}\right), \text{ if } x \neq 0 \\ 0, \quad \text{if } x = 0 \end{cases}$

Since, $g(x)$ is an even function, then

$$g(x) = g(-x) \quad \Rightarrow \quad g'(x) = -g'(-x)$$

$$\Rightarrow \quad g'(0) = -g'(0) \quad \Rightarrow \quad g'(0) = 0$$

Now, $f'(0) = \lim_{h\to 0} y\frac{(0+h)\cos(1/h) - 0}{h}$

$$= \lim_{h\to 0}\frac{g(h)\cos(1/h)}{h}$$

$$= \lim_{h\to 0} g(0)\cos(1/h) = 0$$

77. (C)

(c) Given, $f(x) = \begin{cases} x\, e^{-\left(\frac{1}{|x|} + \frac{1}{x}\right)}, & x \neq 0 \\ 0, & x = 0 \end{cases}$

$$\Rightarrow \qquad f(x) = \begin{cases} x \cdot e^{-\left(-\frac{1}{x} + \frac{1}{x}\right)} = x, & x < 0 \\ x \cdot e^{-2/x}, & x > 0 \\ 0, & x = 0 \end{cases}$$

Clearly, $\lim\limits_{x \to 0^-} f(x) = \lim\limits_{x \to 0} x = 0$

$\lim\limits_{x \to 0^+} f(x) = \lim\limits_{x \to 0} x \cdot e^{-2/x} = 0$

Also, $f(0) = 0$

$\therefore \quad \lim\limits_{x \to 0^-} f(x) = \lim\limits_{x \to 0^+} f(x) = f(0)$

So, $f(x)$ is continuous at $x = 0$.

Now, $Lf'(0) = \lim\limits_{h \to 0} \dfrac{f(0-h) - f(0)}{-h}$

$= \lim\limits_{h \to 0} \dfrac{0 - h - 0}{-h} = 1$

$Rf'(0^+) = \lim\limits_{h \to 0} \dfrac{f(0+h) - f(0)}{h}$

$= \dfrac{(0+h) \cdot e^{-\frac{2}{0+h}} - 0}{h}$

$= 0$

$\because \quad Lf'(0) \neq Rf'(0^-)$

$\therefore f(x)$ is not differentiable at $x = 0$.

78. (D)

(d) We have, $\cos|x| = \begin{cases} \cos x, & \text{if } x \geq 0 \\ \cos(-x), & \text{if } x < 0 \end{cases}$

$\Rightarrow \quad \cos|x| = \cos x$ for all $x \in R$

Similarly, $\sin|x| = \begin{cases} \sin x, & \text{if } x \geq 0 \\ \sin(-x) = -\sin x, & \text{if } x < 0 \end{cases}$

Let $f(x) = \cos|x| + |x|$ and $g(x) = \cos|x| - |x|$

Since, $\cos x$ is everywhere differentiable and $|x|$ is not differentiable at $x = 0$.

Therefore, $f(x)$ and $g(x)$ are not differentiable at $x = 0$.

Let $$u(x) = \sin(|x|) + |x|, \text{ then}$$

$$u(x) = \begin{cases} \sin x + x, & x \geq 0 \\ -\sin x - x, & x < 0 \end{cases}$$

Now, $$Lf'(0^-) = \left\{\frac{d}{dx}(-\sin x - x)\right\}_{\text{at } x = 0}$$

$$= (-\cos x - 1)_{\text{at } x = 0} = -2$$

$$Rf'(0^+) = \left\{\frac{d}{dx}(\sin x + x)\right\}_{\text{at } x = 0}$$

$$= \{\cos x + 1\}_{\text{at } x = 0} = 2$$

Clearly, $$Lf'(0^-) \neq Rf'(0^+)$$

So, $\mu(x)$ is not differentiable at $x = 0$.

Let $$v(x) = \sin|x| - |x|, \text{ then}$$

$$v(x) = \begin{cases} -\sin x + x, & \text{if } x < 0 \\ \sin x - x, & \text{if } x \geq 0 \end{cases}$$

$$Lf'(0^-) = \left\{\frac{d}{dx}(-\sin x + x)\right\}_{\text{at } x = 0}$$

$$= \{-\cos x + 1\}_{\text{at } x = 0} = 0$$

$$Rf'(0^+) = \left\{\frac{d}{dx}(\sin x - x)\right\}_{\text{at } x = 0}$$

$$= (\cos x - 1)_{\text{at } x = 0} = \cos 0 - 1 = 0$$

$\therefore$ $$Lf'(0^-) = Rf'(0^+)$$

So, $v(x)$ is differentiable at $x = 0$.

79. (A)

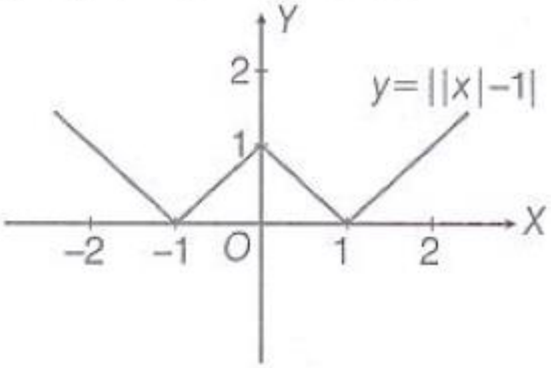
(a) Given, $f(x) = ||x| - 1|$

It is not differentiable, when $|x| = 0$ i.e. $x = 0$.

and when $|x| - 1 = 0$ i.e. $x = \pm 1$

Alternate Method

The graph of $y = ||x| - 1|$ is as follows



The graph of function has sharp turn at $x = -1, 0, 1$.

Hence, it is not differentiable at $x = -1, 0, 1$.

80. (D)

(d) $\lim_{x \to a} \frac{x^2 f(a) - a^2 f(x)}{x - a}$ $\left[\frac{0}{0} \text{ form}\right]$

$= \lim_{x \to a} \frac{2x f(a) - a^2 f'(x)}{1 - 0}$ [using L' Hospital's rule]

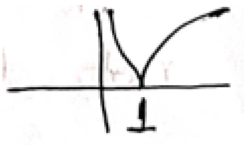
$= 2a f(a) - a^2 f'(a)$

81. (0)

$f(x)$ is not diff. at $x = 1$ and $x = 0$.

Hence in $[2, \infty)$ no point.

82. (1)

Graph  $f(x)$ is not differentiability at $x = 1$.

83. (1)

Here $f(x)$ in discount at $x = 1$ hence not differentiable.

84. (0)

Graph of $\text{sgn}(x^2 + 1)$ is which is always differentiable.

85. (2)

We know that differentiable $\pm$ not differentiable is not diff.

But at $x = 0$ $|\sin x| - |x|$ is differentiable

$|\sin x|$ is not differentiable at π and 2π

$|x|$ is differentiable at π and 2π

Hence 2 point of non differentiable.

86. (3)

On solving we get $f(x) = \ln x$

$f'(x) = \frac{1}{x} \Rightarrow f'\left(\frac{1}{3}\right) = 3$ Ans.

87. (3)

$f(x)$ differentiable $\Rightarrow$ continuity at $x = 1$

$\Rightarrow \quad a = b + 1 \qquad ...(1)$

Also $2a = b$ (Diff.) … (2)

Solving (1) and (2)

$a = -1$, $b = -2$

$|a + b| = 3$

88. (3)

$f(x)$ is discount at $x = 1, 2, 3$.

Hence not differentiable at 3 points.

89. (1)

Function is continuous but not differentiable for $K \in (0, 1]$.

Hence only integer is 1.

90. (2)

$\text{sgn}\left(x^2 - 1\right)$ is discount at $x = 1, -1$.

Hence, not differentiable.