# HOME ASSIGNMENT

**Q.1 (B)**

$$S = \sum_{m=0}^{100} 100C_m (x-3)^{100-m} 2^m$$

$$\Rightarrow S = (x-3+2)^{100} = (x-1)^{100}$$

Now, coefficient of $x^{53}$ in S, $(x-1)^{100}$ is $(-1)^{53}\ {}^{100}C_{53}$

**Q.2 (B)**

Let $E = \left(\sqrt{5}+1\right)^5 - \left(\sqrt{5-1}\right)^5$

$$\Rightarrow E = 2\left\{{}^5C_1\left(\sqrt{5}\right)^4 + {}^5C_3\left(\sqrt{5}\right)^2 + {}^5C_5\left(\sqrt{5}\right)^0\right\}$$

$$\Rightarrow E = 2\{125 + 50 + 1\} = 352$$

**Q.3 (A)**

Since, coefficient of $(3r)^{th}$ term in $(1+x)^{2n}$ equals coefficient of $(r + 2)^{th}$ term in $(1+x)^{2n}$

$$\therefore {}^{2n}C_{3r-1} = {}^{2n}C_{r+1}$$

$$\Rightarrow 3r - 1 + r + 1 = 2n$$

$$\therefore \boxed{r = \frac{n}{2}}$$

**Q.4 (C)**

$$t_{r+1} = {}^9C_r \left(\frac{4}{3}x^2\right)^{9-r} \left(\frac{-3}{2x}\right)^r$$

$$\Rightarrow t_{r+1} = {}^9C_r \frac{2^{18-3r}}{3^{9-2r}} (-1)^r \ x^{18-3r}$$

For term independent of x, we get

18 – 3r = 0

$\boxed{r = 6}$

**Q.5 (B)**

$$\frac{n(n-1)}{2!}\,x^2 = \frac{-1}{8}x^2$$

$$\Rightarrow n(n-1) = -\frac{1}{4}$$

$$\Rightarrow 4n^2 - 4n + 1 = 0$$

$$\therefore \boxed{n = \frac{1}{2}}$$

**Q.6 (D)**

$$t_{n+1} = {}^nC_n\left(2^{\frac{1}{3}}\right)^{n-n}\left(\frac{-1}{\sqrt{2}}\right)^n$$

$$\Rightarrow\left(\frac{1}{33^{\frac{2}{3}}}\right)^{\log_3 8} = (-1)^n\,2^{\left(\frac{-n}{2}\right)}$$

$$\Rightarrow 3^{\frac{-5}{3}\log_3 8} = (-1)^n\,2^{\left(\frac{-n}{2}\right)}$$

$$\Rightarrow \frac{1}{32} = \frac{(-1)^n}{2^{\frac{n}{2}}}$$

$$\therefore \boxed{n = 10}$$

$$\text{Now, } t_5 = {}^{10}C_4\left(2^{\frac{1}{3}}\right)^{10-4}\frac{(-1)^4}{\left(2^{\frac{1}{2}}\right)^4} \Rightarrow t_5 = {}^{10}C_4$$

**Q.7 (A)**

$$t_5 + t_6 = 0 \Rightarrow {}^nC_4(a)^{n-4}b^4 - {}^nC_5(a)^{n-5}b^5 = 0$$

$$\Rightarrow {}^nC_4 a = {}^nC_5 b$$

$\Rightarrow \frac{a}{b} = \frac{{}^{n}C_5}{{}^{n}C_4} = \frac{n-4}{5}$

**Q.8 (A)**

General term, $t_{r+1} = {}^{4n-2}C_r (i)^r x^r$

$r = 2, 6, \ldots\ldots\ldots\ldots, 4n-2$

$\therefore$ n terms

**Q.9**

Coefficient of $t^{32}$ in $\left(1+t^{12}\right)\left(1+t^{24}\right)\left(1+t^{2}\right)^{12}$

$\Rightarrow$ Coefficient of $t^{32}$ in $\left(1+t^{12}+t^{24}+t^{36}\right)\left(\sum_{r=0}^{12} {}^{12}C_r t^{2r}\right)$

$\Rightarrow$ Coefficient of $t^{32}$ is ${}^{12}C_{10} + {}^{12}C_4 = 561.$

**Q.10 (C)**

$E = \left(1-2x^3+3x^5\right)\left(\sum {}^{8}C_r x^{-r}\right)$

Coefficient of x in e is $(-2)\ {}^{8}C_2 + 3\ {}^{8}C_4 = 154$

**Q.11 (A)**

General term, $t_{r+1} = {}^{15}C_r\ 2^{\frac{1}{2}(15-r)}\ 3^{\frac{1}{2}r}$

Now, $r \in w$ and $0 \le r \le 15$

Also, 15 – r is EVEN and r is EVEN

$\Rightarrow$ r is ODD and r is EVEN

$\Rightarrow \therefore r \in \phi$

Hence, no radical term exist in the given expansion $\Rightarrow$ all terms are irrational.

$\therefore$ number of irrational terms = 16

**Q.12 (A)**

General form, $t_{r+1} = {}^{55}C_r\ x^{\frac{1}{5}(55-r)} y^{\frac{1}{10}r}$

$r \in w\ ;\ 0 \le r \le 55$

$\therefore$ 55 – r is a multiple of 5 and r is a multiple of 10

$\Rightarrow$ r = 0, 10, 20, 30, 40, 50

$\therefore$ 6 terms are rational.

**Q.13 (C)**

$$\Rightarrow\ 2\,{}^nC_5 = {}^nC_4 + {}^nC_6$$

$$\Rightarrow \frac{{}^nC_4}{{}^nC_5} + \frac{{}^nC_6}{{}^nC_5} = 2$$

$$\Rightarrow \frac{5}{n-4} + \frac{n-5}{6} = 2$$

$$\Rightarrow\ n^2 - 21n + 108 = 0$$

$$\Rightarrow\ (n-9)\ (n-12) = 0$$

$\therefore$ n = 9, 12

**Q.14 (A)**

$${}^5C_2\ xh3x^{2\log_{10} x} = 10^6 \Rightarrow\ x^{3+2\log_{10} x} = 10^5$$

$$\therefore\ 3\log_{10} x + 2(\log_{10} x)^2 - 5 = 0$$

$$\Rightarrow\ 2\left(\log_{10} x\right)^2 + 5\lg o_{10}x - 2\left(\log_{10} x\right) - 5 = 0$$

$$\Rightarrow\ \log_{10} x = \frac{-5}{2}, 1$$

$$\therefore\ x = 10^{\frac{-5}{2}}, 10$$

**Q.15 (D)**

$$^8C_5 \frac{1}{x^8} x^{10}\left(\log_{10} x\right)^5 = 5600 \Rightarrow x^2\left(\log_{10} x\right)^5 = 100$$

$$\therefore \boxed{x = 10}$$

**Q.16 (A)**

$$t_3 = {}^nC_2\ 2^{x(n-2)}\left(\frac{1}{4^x}\right)\ ;\ t_2 = {}^nC_1\ 2^{x(n-1)}\left(\frac{1}{4^x}\right)^1$$

$$\frac{t_3}{t_2} = 7 \Rightarrow \frac{{}^nC_2}{{}^nC_1}\ \frac{2^{x(n-2)}}{2^{x(n-1)}}\ \frac{\left(\frac{1}{4^x}\right)^2}{\left(\frac{1}{4^x}\right)^1} = 7$$

$$\Rightarrow \frac{n-1}{2}\ \frac{1}{2^x}\left(\frac{1}{4}\right)^x = 7$$

Also

$${}^nC_1 + {}^nC_2 = 36 \Rightarrow n^2 + n - 72 = 0$$
$$\Rightarrow n = 8$$

$$\therefore\ 2^{3x+1} = 2^0$$

$$\Rightarrow \boxed{x = \frac{-1}{3}}$$

**Q.17 (C)**

$${}^9C_4(2a)^5\left(\frac{a^2}{4}\right)^4\ ;\ -{}^9C_5(2a)^4\left(\frac{a^2}{4}\right)^5$$

**Q.18 (C)**

$$\sum_{r=0}^{n} r^2\ {}^nC_r\ x^r\,(1-x)^{n-r}$$

$$\Rightarrow \sum_{r=0}^{n} nr\ {}^{n-1}C_{r-1}\ x^r\,(1-x)^{n-r}$$

$$\Rightarrow \sum_{r=0}^{n} n(r-1+1)^{n-1}C_{r-1}\, x^r (1-x)^{n-r}$$

$$\Rightarrow \sum_{r=0}^{n} n(n-1)^{n-2}C_{r-2}\, x^r\ (1-x)^{n-r} + n^{n-1}C_{r-1}\, x^r (1-x)^{n-r}$$

$$\Rightarrow n(n-1)x^2 \sum_{r=2}^{n} {}^{n-2}C_{r-2}\ x^{r-2}\ (1-x)^{n-2-(r-2)} + nx\sum_{r=1}^{n} {}^{n-1}C_{r-1}\, x^{r-1}(1-x)^{(n-1)(r-1)}$$

$$\Rightarrow n(n-1)x^2 (x+1-x)^{n-2} + nx(1+1-x)^{n-1}$$

$$\Rightarrow n(n-1)x^2 + nx = nx(nx - x + 1)$$

$= nx\ (nx + y)$

**Q.19 (A)**

$$t_3 < t_4 > t_5 \Rightarrow {}^{10}C_2\,(2)^8\left(\frac{3}{8}|x|\right)^2 < {}^{10}C_3\,(2)^7\left(\frac{3}{8}|x|\right)^3 > {}^{10}C_4\,(2)^6\left(\frac{3}{8}|x|\right)^4$$

$$\Rightarrow {}^{10}C_2\ 2\left(\frac{3}{8}|x|\right)^{-1} < {}^{10}C_3 > {}^{10}C_4\ \frac{1}{2}\left(\frac{3}{8}|x|\right)$$

$$\Rightarrow \frac{{}^{10}C_2}{{}^{10}C_3}\ 2\cdot\frac{8}{3} < |x| \qquad\Bigg|\qquad \frac{{}^{10}C_3}{{}^{10}C_4}\ 2\cdot\frac{8}{3} > |x|$$

$$\Rightarrow \frac{3}{10-2}\cdot 2\cdot\frac{8}{3} < |x| \qquad\Bigg|\qquad \frac{4}{10-3}\cdot 2\cdot\frac{8}{3} > |x|$$

$$\therefore \boxed{|x| > 2} \qquad\Bigg|\qquad |x| < \frac{64}{21}$$

$$\Rightarrow 2 < |x| < \frac{64}{21}$$

**Q.20 (C)**

Term independent of x = ${}^{10}C_5\,(x\sin\alpha)^5\left(\frac{1}{x}\cos\alpha\right)^5$

or $\frac{{}^{10}C_5}{2^5}(2\sin\alpha\cos\alpha)^5 = \frac{{}^{10}C_5}{2^5}$ {as greatest value of 2 sin x cos x is 1}

**Q.21 (B)**

Let $t_{r+1}$ is the greatest term, then $t_{r+1} > t_r$

$$\Rightarrow {}^nC_r\ (2x)^{n-r}\ 7^r\ >\ {}^nC_{r-1}(2x)^{n-r+1}\ 7^{r-1}$$

$$\Rightarrow \frac{{}^nC_r}{{}^nC_{r-1}}\ 7\ >\ 2x$$

$$\Rightarrow \frac{n-r+1}{r}\ 7\ >\ 6$$

$$\Rightarrow 7(n+1) > 13r$$

$$\Rightarrow r < \frac{77}{13}$$

$\therefore$ $6^{th}$ terms the greatest term.

**Q.22 (A)**

$(n_1 - n_2)\ (n_1 - n_2 - 1) = 30$ .......( 1 )

$(n_1 + n_2)\ (n_1 + n_2 - 1) = 90$ ........( 2 )

$\therefore n_1 = 8$ and $n_2 = 2$

**Q.23 (D)**

$${}^{n-1}C_r = (k^2 - 3)\frac{n}{r+1}\ {}^{n-1}C_r$$

$$k^2 - 3 = \frac{r+1}{n}$$

I : $k^2 - 3 > 0$

II : $k^2 - 3 < 1$

**Q.24 (C)**

$${}^nC_r \cdot r! = 5040\left({}^{n-1}C_{r-1} + {}^{n-1}C_r\right) \Rightarrow {}^nC_r.r! = 5040.\,{}^nC_r$$

$$\Rightarrow r = 7$$

**Q.25 (A)**

$$^{35}C_8 + \sum_{r=1}^{7} {}^{42-r}C_7 + \sum_{r=1}^{5} {}^{47-r}C_{40-r}$$

$$\Rightarrow {}^{35}C_8 + \left({}^{41}C_7 + {}^{42}C_7 + {}^{43}C_7 + .......+ {}^{35}C_7\right) + \left({}^{46}C_7 + {}^{45}C_7 + {}^{44}C_7 + .....+ {}^{42}C_7\right)$$

By ${}^{n-1}C_{r-1} + {}^{n-1}C_r = {}^{n}C_r$,

$$^{35}C_8 + {}^{35}C_7 + {}^{36}C_7 + ... + {}^{41}C_7 = {}^{42}C_8 \;\&$$

$$^{42}C_8 + {}^{42}C_7 + {}^{43}C_7 + ... + {}^{46}C_7 = {}^{47}C_8$$

**Q.26 (D)**

$$(1+x)^{131}\left(1-x+x^2\right)^{130} = (1+x)\left(1+x^3\right)^{130}$$

Now each term in the expansion of $(1+x^3)^{130}$ will be of type $x^{3r}$.

Hence in $(1+x)\left(1+x^3\right)^{130}$ terms will be of type $x^{3r}$ & $x^{3r+1}$.

But no number of type 3r & 3r + 1 can be 65.

Hence coefficient of $x^{65} = 0$.

**Q.27 (A)**

Number of ways to distribute 8 identical objects in 3 distinct groups = ${}^{8+2}C_2 = \dfrac{10 \cdot 9}{2!} = 45$.

**Q.28 (B)(D)**

Number of ways to distribute n identical objects in 5 distinct groups = ${}^{n+4}C_4$

**Q.29 (B)**

$(1+2)^n = 6561$

$\Rightarrow n = 8 \;\therefore$ Greatest term is $5^{th}$ term

**Q.30 (D)**

$$\frac{n^2+n-14}{2} = \frac{n(n+1)}{2} - 7$$

$$\Rightarrow x^{\frac{n^2+n-14}{2}} = \frac{x.x^2.x^3...x^n}{x^7}$$

Hence we need to find those terms which are product of all the $x^r$ terms in each bracket except those in which the sum of powers is 7.

Such terms are $(x^7),(x.x^6),(x^2.x^5),(x^3x^4),(x.x^2.x^4),(x.x^3.x^3)$

Hence required coefficient is $-(7)+(1.6)+(2.5)+(3.4)-(1.2.4)=13$

**Q.31 (B)**

$$\frac{1}{n!}\left[\frac{n!}{1!\,(n-1)!}+\frac{n!}{3!\,(n-3)!}+.....\right]=\frac{1}{n!}\left({}^nC_1+{}^nC_3+.....\right)$$

$$=\frac{1}{n!}\,2^{n-1}$$

**Q.32 (B)**

$$S=1+\left(-\frac{2}{3}\right)\left(-\frac{1}{2}\right)+\frac{\left(-\frac{2}{3}\right)\left(-\frac{2}{3}-1\right)}{2!}\left(\frac{1}{2}\right)^2+\frac{\left(-\frac{2}{3}\right)\left(-\frac{2}{3}-1\right)\left(-\frac{2}{3}-2\right)}{3!}\left(-\frac{1}{2}\right)^3+........\infty$$

$$\Rightarrow S=\left(1-\frac{1}{2}\right)^{-\frac{2}{3}}=2^{2/3}.$$

**Q.33 (B)**

$$S=2+\frac{5}{2!3}+\frac{5.7}{3!3^2}+\frac{5.7.9}{4!3^3}+...$$

$$S=1+\frac{\left(\frac{3}{2}\right)}{1!\left(\frac{3}{2}\right)}+\frac{\left(\frac{3}{2}\right)\left(\frac{5}{2}\right)}{2!\left(\frac{3}{2}\right)^2}+\frac{\left(\frac{3}{2}\right)\left(\frac{5}{2}\right)\left(\frac{7}{2}\right)}{3!\left(\frac{3}{2}\right)^3}+\frac{\left(\frac{3}{2}\right)\left(\frac{5}{2}\right)\left(\frac{7}{2}\right)\left(\frac{9}{2}\right)}{4!\left(\frac{3}{2}\right)^4}+...$$

$$S=1+\frac{\left(\frac{3}{2}\right)}{1!}\left(\frac{2}{3}\right)+\frac{\left(\frac{3}{2}\right)\left(\frac{5}{2}\right)}{2!}\left(\frac{2}{3}\right)^2+\frac{\left(\frac{3}{2}\right)\left(\frac{5}{2}\right)\left(\frac{7}{2}\right)}{3!}\left(\frac{2}{3}\right)^3+\frac{\left(\frac{3}{2}\right)\left(\frac{5}{2}\right)\left(\frac{7}{2}\right)\left(\frac{9}{2}\right)}{4!}\left(\frac{2}{3}\right)^4+...$$

$$S=\left(1-\frac{2}{3}\right)^{-\frac{3}{2}}=3\sqrt{3}$$

**Q.34 (A)**

$$(1-ax)^{-1}(1-bx)^{-1}(1-cx)^{-1}$$

= (1 + ax + …) (1 + bx + …) (1 + cx + …)

Hence coefficient of x = (a + b + c).

**Q.35 (A)**

$$\frac{\left(\frac{1}{2}\right)\left(\frac{1}{2}-1\right)\left(\frac{1}{2}-2\right)}{3!}\left(\frac{3}{2}x\right)^3 = \frac{27}{128}x^3$$

**Q.36 (A)**

$$1+(1+x)+(1+x^2)+......+(1+x)^n = \frac{(1+x)^{n+1}-1}{x}$$

Coefficient of $x^k$ is ${}^{n+1}C_{k+1}$

**Q.37 (D)**

$$S=\sum_{r=0}^{n}(2r+1)\ {}^nC_r$$

$$S=2\sum_{r=0}^{n} n^{n-1}\ C_{r+1} + \sum_{r=0}^{n} {}^nC_r = 2\cdot 2^{n-1} + 2^n$$

$$= (n+1)\,2^n$$

**Q.38 (C)**

Coefficient of $x^0$ in $(1+x)^n\left(1+\frac{1}{x}\right)^n = {}^{2n}C_n$

Coefficient of $x^0$ in $(1+x)^n\left(1-\frac{1}{x}\right)^n = 0$

$$\therefore C_1^{\,2} + C_3^{\,2} + .........+ C_n^{\,2} = \frac{{}^{2n}C_n}{2} = \frac{(2n)!}{(n!)^2\,2}$$

**Q.39 (A)**

Coefficient of $x^{n-1}$ in $(1+x)^{n-1} + (1+x)^n + .....+(1+x)^{n+r-1}$

$$x^{n-1} \to (1+x)^{n-1}\left\{\frac{(1+x)^{r+1}-1}{(1+x)-1}\right\}$$

$$x^n \to (1+x)^{n+r} - (1+x)^{n-1} \text{ i.e. } {}^{n+r}C_n$$

**Q.40 (A)**

Coefficient of $x^r$ in $(1+x)^n\left(1+\frac{1}{x}\right)^n$

$\Rightarrow$ coefficient of $x^r$ in $\frac{(1+x)^{2n}}{x^n}$

$\Rightarrow$ coefficient of $x^{n+r}$ in $(1+x)^{2n}$ i.e. ${}^{2n}C_{r+n}$

**Q.41 (A)**

$$C_0 + C_1x + C_2x^2 + .........+C_nx^n = (1+x)^n$$

Difference w.r.t. x, we get

$$1C_1 + 2C_2x + 3C_3x^2 + ......+nC_nx^{n-1} = n(1+x)^{n-1}$$

$$1C_1x + 2C_2x^2 + 3C_3x^3 + ......+nC_nx^n = nx(1+x)^{n-1}$$

Differentiating again and then put x = 1.

$$1^2C_1 + 2^2C_2 + 3^2C_3 + .....+n^2C_n = n\ 2^{n-1} + n(n-1)2^{n-2}$$

$$= n(2+n-1)2^{n-2}$$

$$= n(n+1)\ 2^{n-2}$$

**Q.42 (B)**

$$(1+x)^n x = \sum_{r=0}^{n}\left({}^nC_r x^{r+1}\right) \Rightarrow \int_0^1 (1+x)^n x\,dx = \sum_{r=0}^{n}\left({}^nC_r\int_0^1 x^{r+1}\right)$$

$$\Rightarrow\left[\frac{x(1+x)^{n+1}}{n+1}-\frac{(1+x)^{n+2}}{(n+1)(n+2)}\right]_0^1=\left[\sum_{r=0}^{n}\frac{{}^nC_r}{r+2}x^{r+2}\right]_0^1$$

$$\Rightarrow\frac{2^{n+1}}{n+1}-\frac{2^{n+2}-1}{(n+1)(n+2)}=\sum_{r=0}^{n}\frac{{}^nC_r}{r+2}.$$

**Q.43 (C)**

$$\int_0^1\int_0^x(1+x)^n\,dx\,dx=\int_0^1\frac{(1+x)^{n+1}-1}{n+1}dx$$

$$=\frac{1}{n+1}\left[\frac{(1+x)^{n+2}}{n+2}-x\right]_0^1$$

$$=\frac{1}{n+1}\left\{\frac{2^{n+2}-1}{n+2}-2\right\}$$

$$=\frac{2^{n+2}-2n-5}{(n+1)(n+2)}$$

**Q.44 (D)**

$$(1+x)^n=\sum_{r=0}^{n}{}^nC_r\,x^r\Rightarrow x^3(1-x^2)^n=\sum_{r=0}^{n}(-1)^r\,{}^nC_r\,x^{2r+3}$$

$$\Rightarrow\frac{d}{dx}\left\{x^3(1-x^2)^n\right\}=\sum_{r=0}^{n}(-1)^r(2r+3)\,{}^nC_r\,x^{2r+2}$$

$$\Rightarrow 3x^2(1-x^2)^n-2nx^4(1-x^2)^{n-1}=\sum_{r=0}^{n}(-1)^r(2r+3)\,{}^nC_r\,x^{2r+2}$$

For $x=1,\quad \sum_{r=0}^{n}(-1)^r(2r+3)\,{}^nC_r=0$

**Q.45 (D)**

$$\frac{{}^nC_r}{(r+1)(r+2)}=\frac{{}^{n+2}C_{r+2}}{(n+1)(n+2)}\Rightarrow\sum_{r=0}^{n}\frac{2^{r+2}.\,{}^nC_r}{(r+1)(r+2)}=\frac{1}{(n+1)(n+2)}\sum_{r=0}^{n}2^{r+2}\,{}^{n+2}C_{r+2}$$

Now $(1+x)^{n+2}=\sum_{r=0}^{n+2}{}^{n+2}C_r x^r$

For $x=2$, we get $\sum_{r=0}^{n+2}{}^{n+2}C_r\,2^r=3^{n+2}$

$$\Rightarrow {}^{n+2}C_0\, 2^0 + {}^{n+2}C_1\, 2^1 + \sum_{r=0}^{n} 2^{r+2}\ {}^{n+2}C_{r+2} = 3^{n+2}$$

$$\Rightarrow \frac{1}{(n+1)(n+2)} \sum_{r=0}^{n} 2^{r+2}\ {}^{n+2}C_{r+2} = \frac{3^{n+2} - 1 - 2(n+2)}{(n+1)(n+2)}$$

**Q.46 (B)**

$$\sum_{r=0}^{n} r^3 \left(\frac{n-r+1}{r}\right)^2 = \sum_{r=0}^{n} r\,(n-r+1)^2$$

$$= \sum_{r=0}^{n} (n+1)^3 r - \sum_{r=0}^{n} 2\,(n+1)\, r^2 + \sum_{r=0}^{n} r^3$$

$$= \frac{1}{12}\, n\,(n+1)^2\,(n+2)$$

**Q.47 (B)**

$$t_r = (r+2-2)\frac{{}^{2n}C_r}{r+2} = {}^{2n}C_r - \frac{2\ {}^{2n}C_r}{r+2}$$

$$\Rightarrow S = \sum_{r=0}^{n} t_r = 2^{2n} - 2\int_0^1 (1+x)^{2n}\, x\ dx$$

$$= 2^{2n} - 2\left[\frac{x(1+x)^{2n+1}}{2n+1} - \frac{(1+x)^{2n+2}}{2n+2}\right]_0^1$$

$$= 2^{2n} - 2\left[\frac{2^{2n+2}-1}{2n+2} - \frac{2^{2n+1}-1}{2n+1}\right]$$

**Q.48 (D)**

$$S = \left({}^{n}C_0 + {}^{n}C_1\right) + \left(C_0 + C_2\right) + (C_0 + C_3) + \ldots\ldots\ldots + (C_0 + C_n)$$

$$+ (C_1 + C_2) + (C_1 + C_3) + \ldots\ldots + (C_1 + C_n) +$$

$$(C_2 + C_3) + \ldots\ldots + (C_2 + C_n) + (C_{n-1} + C_n)$$

$$\therefore S = \frac{n\,2^n}{2} - 2\cdot 2^n$$

$= \frac{n-4}{2}\, 2^n$

$= (n-4)\, 2^{n-1}$

**Q.49 (B)**

Required sum $= \frac{1}{2}\left\{\left(\sum_{r=0}^{n} C_r\right)^2 - \left(\sum_{r=0}^{n} C_r^{\,2}\right)\right\}$

$= \frac{1}{2}\left\{ (2^n)^2 - {}^{2n}C_n \right\}$

**Q.50 (A)**

$S = \frac{1}{2}\left\{ \left(n\ 2^{n-1}\right)^2 - \left\{\sum_{r=0}^{n-1} r^2\ {}^{n}C_r^{\,2}\right\}\right\}$

$\Rightarrow\ S = \frac{n^2}{2}\left\{2^{2n-2} - {}^{2n-2}C_{n-1}\right\}$

**Q.51 (B)**

$S = n\left(C_0^2 + C_1^2 + \ldots\ldots + C_n^2\right) + 2 \sum\sum_{0 \le i < j \le n} C_i C_j$

$S = n\left({}^{2n}C_n\right) + \left\{(2^n)^2 - {}^{2n}C_n\right\}$

$S = n\ {}^{2n}C_n + 2^{2n} - {}^{2n}C_n = (n-1)\, {}^{2n}C_n + 2^{2n}$

**Q.52 (D)**

$S = \sum\sum_{0 \le i < j \le n} (i+j)(C_i + C_j)$ …..( 1 )

Writing the series from last term to first term gives

$S = \sum\sum_{0 \le i < j \le n} (n-i+n-j)\ \left({}^{n}C_{n-i} + {}^{n}C_{n-j}\right)$

$= \sum\sum_{0 \le i < j \le n} (2n-i-j)\ \left({}^{n}C_i + {}^{n}C_j\right)$ ……( 2 )

Adding ( 1 ) and ( 2 ), we get

$$S = n \times \sum_{0 \le i < j \le n}\sum \left(C_i + C_j\right) = n^2.2^n$$

**Q.53 (A)**

$$S = \sum_{r=0}^{n} (-1)^r \ {}^nC_r \left(\frac{1}{2}\right)^r + \sum_{r=0}^{n} (-1)^r \ {}^nC_r \left(\frac{3}{4}\right)^r + \sum_{r=0}^{n} (-1)^r \ {}^nC_r \left(\frac{7}{8}\right)^r + \ldots\ldots\ldots\infty$$

$$\Rightarrow S = \left(1 - \frac{1}{2}\right)^n + \left(1 - \frac{3}{4}\right)^n + \left(1 - \frac{7}{8}\right)^n + \ldots\ldots\ldots\infty$$

$$\Rightarrow S = \frac{1}{2^n} + \left(\frac{1}{2^n}\right)^2 + \left(\frac{1}{2^n}\right)^3 + \ldots\ldots\infty$$

$$\Rightarrow S = \frac{\frac{1}{2^n}}{1 - \left(\frac{1}{2^n}\right)}$$

$$\therefore \boxed{S = \frac{1}{2^n - 1}}$$

**Q.54 (C)**

$$S = \sum_{i=0}^{r} {}^{n_1}C_{r-i} \ {}^{n_2}C_i$$

Coefficient of $x^r$ in $(1+x)^{n_1}(1+x)^{n_2} = (1+x)^{n_1+n_2}$

$$= {}^{n_1+n_2}C_r$$

**Q.55 (C)**

$$C_0 + C_1x + C_2x^2 + C_3x^3 + \ldots\ldots\ldots + C_nx^2 = (1+x)^n$$

$$C_0 + \frac{C_1}{2} + \frac{C_2}{3} + \frac{C_3}{4} + \ldots\ldots + \frac{C_n}{x^{n+1}} = \int_0^1 (1+x)^n dx$$

$$= \frac{2^{n+1} - 1}{n+1}$$

$$-C_0+\frac{C_1}{2}-\frac{C_2}{3}+\frac{C_3}{4}+.......+(-1)^{n+1}\frac{C_n}{n+1}=-\int_0^1(1-x)^n dx=\frac{-1}{n+1}$$

Adding we get $2\left(\frac{C_1}{2}+\frac{C_2}{4}+\frac{C_5}{6}+.....\right)=\frac{2^{n+1}-2}{n+1}$.

**Q.56 (D)**

$$S=1-\frac{n(1+x)}{1+nx}+\frac{n(n-1)}{2!}\frac{1+2x}{(1+nx)^2}-\frac{n(n-1)(n-2)}{3!}\frac{1+3x}{(1+nx)^3}+......\infty$$

$$\Rightarrow S=\left\{1-\frac{n}{1!}\left(\frac{1}{1+nx}\right)+\frac{n(n-1)}{2!}\left(\frac{1}{1+nx}\right)^2-\frac{n(n-1)(n-2)}{3!}\left(\frac{1}{1+nx}\right)^3+...\right\}$$
$$+x\left\{-\frac{n}{1!}\left(\frac{1}{1+nx}\right)+\frac{2n(n-1)}{2!}\left(\frac{1}{1+nx}\right)^2-\frac{3n(n-1)(n-2)}{3!}\left(\frac{1}{1+nx}\right)^3+...\right\}$$

$$\Rightarrow S=\left\{1-\frac{n}{1!}\left(\frac{1}{1+nx}\right)+\frac{n(n-1)}{2!}\left(\frac{1}{1+nx}\right)^2-\frac{n(n-1)(n-2)}{3!}\left(\frac{1}{1+nx}\right)^3+...\right\}$$
$$-\frac{nx}{1+nx}\left\{1-\frac{(n-1)}{1!}\left(\frac{1}{1+nx}\right)+\frac{(n-1)(n-2)}{2!}\left(\frac{1}{1+nx}\right)^2+...\right\}$$

$$\Rightarrow S=\left(1-\frac{1}{1+nx}\right)^n-\frac{nx}{1+nx}\left(1-\frac{1}{1+nx}\right)^{n-1}$$
$$\Rightarrow S=\left(\frac{nx}{1+nx}\right)^n-\left(\frac{nx}{1+nx}\right)^n=0.$$

**Q.57 (A)**

Coefficient of $x^n$ in $(1+x+2x^2+3x^3+.....+nx^n)^2$ equals coefficient of $x^n$ in $(1+x+2x^2+3x^3+......\infty)^2$

Let $S=1+x+2x^2+3x^3+......\infty$

$xS=\quad x+x^2+2x^3+.......\infty$

$(1-x)S=1\quad +x^2+x^3+.......\infty$

$$(1-x)S=1+\frac{x^2}{1-x}$$

$$\Rightarrow S=\frac{1}{1-x}+\frac{x^2}{(1-x)^2}=(1-x)^{-1}+x^2(1-x)^{-2}$$

Now, $S^2=(1-x)^{-2}+2x^2(1-x)^{-3}+x^4(1-x)^4$

Coefficient of $x^2$ in $S^2$ $=\ {}^{n+1}C_1+2\ {}^{n-2+2}C_2+\ {}^{n-4+3}C_3$

$$=(n+1)+2\ \frac{n(n-1)}{2}+\frac{(n-1)(n-2)(n-3)}{6}\ =\ \frac{n(n^2+11)}{6}$$

**Q.58 (A)**

$$S=\sum_{r=0}^{n}{}^{n}C_r\sin rx\cdot\cos(n-r)x \qquad .....(1)$$

$$S=\sum_{r=0}^{n}{}^{n}C_{n-r}\sin(n-r)x\cos rx$$

$$\Rightarrow S=\sum_{r=0}^{n}{}^{n}C_r\sin(n-r)x\cos rx \qquad ......(2)$$

Adding ( 1 ) and ( 2 )

$$S=\frac{1}{2}\sum_{r=0}^{n}{}^{n}C_r\sin nx=2^{n-1}\sin nx$$

**Q.59 (A)**

$$S=\sum_{r=0}^{n}{}^{n}C_r\,a^r\,b^{n-r}\cos\left(rB-\left(n-r\right)A\right)$$

Let $Z_1=\cos A+i\sin A,\ Z_2=\cos B+i\sin B$

Then $\mathrm{Re}\left(Z_2{}^{r}\,\overline{Z_1}^{\,n-r}\right)=\cos\left(rB-\left(n-r\right)A\right)$

$$\Rightarrow S=\mathrm{Re}\left(\sum_{r=0}^{n}{}^{n}C_r\,a^r\,b^{n-r}\,Z_2{}^{r}\,\overline{Z_1}^{\,n-r}\right)$$

$$\Rightarrow S=\mathrm{Re}\left(aZ_2+b\overline{Z_1}\right)^n$$

$$\Rightarrow S=\mathrm{Re}\left(a\left(\cos B+i\sin B\right)+b\left(\cos A-i\sin A\right)\right)^n$$

$\Rightarrow S = \text{Re}\left(a\cos B + ia\sin B + b\cos A - ib\sin A\right)^n$

Now by sine rule $a\sin B = b\sin A$ & by projection formula $a\cos B + b\cos A = c$

$\Rightarrow S = c^n$

**Q.60 (B)**

$3\cdot(80+1)^9 = 3\{80k+1\}$

**Q.61 (C)**

$^{50}C_{25} = \dfrac{50!}{25!\,25!}$

$18 = 2\times 3^2$

Index of 2 in 50! = 47

Index of 9 in 50! = 5

$\therefore$ Index of 18 in 50! = 5

Similarly index of 18 in 25! = 2

$\therefore$ Net index of 18 in $^{50}C_{25} = 1$

**Q.62 (C)**

$(100+1)^{100} - 1 = {}^{100}C_0(100)^{100} + {}^{100}C_{99}(100)^{99} + ......... + {}^{100}C_1 100 + 1 - 1$

**Q.63 (B)**

$$\frac{n^n\left(\dfrac{n+1}{2}\right)^{2n}}{\left(\dfrac{n+1}{2}\right)^3} = n^n\left(\frac{n+1}{2}\right)^{2n-3} \geq 1,\ n \in N$$

Hence $n^n\left(\dfrac{n+1}{2}\right)^{2n} \geq \left(\dfrac{n+1}{2}\right)^3$, $n \in N$.

**Q.64 (C)**

$99^n + 1 = (100-1)^n + 1$

$\Rightarrow 99^n + 1 = {}^nC_0 100^n - {}^nC_1 100^{n-1} + {}^nC_2 100^{n-2} - {}^nC_3 100^{n-3} + ... + {}^nC_{n-1} 100$

Hence there will be two zeros at the end.

**Q.65 (C)**

$$\left\{\frac{3^{2003}}{28}\right\}=\left\{\frac{9\times(28-1)^{667}}{28}\right\}=\left\{\frac{9\times\left({}^{667}C_0 28^{667}-{}^{667}C_1 28^{n-1}+{}^{667}C_2 28^{n-2}-{}^{667}C_3 28^{n-3}+\ldots+{}^{667}C_{666} 28-1\right)}{28}\right\}$$

$$\Rightarrow\left\{\frac{3^{2003}}{28}\right\}=\left\{\frac{28k+19}{28}\right\}=\frac{19}{28}$$

**Q.66 (A)**

Let $\left(8+3\sqrt{7}\right)^n=[p]+f$ & $\left(8-3\sqrt{7}\right)^n=f'$, where $f=p-[p]$ & $0\le f'<1$

Adding the two expansions gives $[p]+f+f'=2n$

Hence $f+f'\in I$

$\Rightarrow f+f'=1$

$\Rightarrow 1-f=f'$

$\Rightarrow (1-f)p=\left(8-3\sqrt{7}\right)^n\left(8+3\sqrt{7}\right)^n=1.$

**Q.67**

Let $\left(2+\sqrt{3}\right)^n=[x]+f$ & $\left(2-\sqrt{3}\right)^n=f'$, where $f=x-[x]$ & $0\le f'<1$

Adding the two expansions gives $[x]+f+f'=2n$

Hence $f+f'\in I$

$\Rightarrow f+f'=1$

$\Rightarrow 1-x+[x]=f'$

Now $x-x^2+x[x]=x(1-x+[x])=xf'$

$\Rightarrow x-x^2+x[x]=\left(2+\sqrt{3}\right)^n\left(2-\sqrt{3}\right)^n=1$

**Q.68 (D)**

$$(1+0.0001)^{10000}=\left(1+\frac{1}{10^4}\right)^{10^4}=1+{}^{10^4}C_1\frac{1}{10^4}+{}^{10^4}C_2\frac{1}{10^8}+{}^{10^4}C_3\frac{1}{10^{12}}+\ldots+{}^{10^4}C_{10^4}\frac{1}{10^{10^4}}$$

$$(1+0.0001)^{10000}=1+1+\frac{9999}{10^4\times 2!}+\frac{9999\times 9998}{10^8\times 3!}+\ldots+\frac{1}{10^{10^4}}$$

Clearly except the first two terms all the rest are non integers

So the The positive integer just greater than $(1+0.0001)^{10000}$ is 3.

**Q.69**

$\left(\sqrt{3}+1\right)^{2m} = \left(4+2\sqrt{3}\right)^{m} = 2^{m}\left(\sqrt{3}+2\right)^{m}$

Further let $\left(2+\sqrt{3}\right)^{m} = I+f$ & $\left(2-\sqrt{3}\right)^{m} = f'$, where I is integer & $0 < f, f' < 1$

Adding the two expansions gives $I + f + f' = 2n$, n being an integer.

Hence $f + f' \in I \Rightarrow f + f' = 1$

$\Rightarrow I = 2n - 1$ i.e. integer just less than $\left(\sqrt{3}+2\right)^{m}$ is odd

Hence integer just greater than $\left(\sqrt{3}+2\right)^{m}$ must be even

Therefore integer just greater than $\left(\sqrt{3}+1\right)^{2m}$ has $2^{m+1}$ as a factor.

**Q.70 (C)**

$\forall\ k \in \{1, 2, 3, ......, p-1\}$

k is one factor of p! i.e. p! = kI

$\therefore\ p! + k = kI + k$

$= k(I+1)$

Which is always divisible by k.

Hence, there is no prime in the given sequence.

## MORE THAN ONE OPTION CORRECT TYPE

**Q.1 (A , D)**

$f(x) = \left(\sqrt{x^2+1} + \sqrt{x^2-1}\right)^6 + \left(\sqrt{x^2+1} - \sqrt{x^2-1}\right)^6$

$= 2\left[{}^6C_0 (x^2+1)^3 + {}^6C_2 (x^2+1)^2 (x^2-1) + {}^6C_4 (x^2+1)(x^2-1)^4 + {}^6C_6 (x^2-1)^3\right]$

$= [62x^6 - 120x^4 + .......]$

**Q.2 (B , C)**

$$\sum_{r=0}^{2n} a_r (1+(x-3))^r = \sum_{r=0}^{2n} b_r (x-3)^r$$

Now, $b_n$ is coefficient of $(x-3)^n$ on RHS

$\therefore$ Coefficient of $(x-3)^n$ on LHS is ${}^{x}C_n + {}^{n+1}C_n + {}^{n+2}C_n + ........ + {}^{2n}C_n$

$= {}^{2n+1}C_{n+1}$ or ${}^{2n+1}C_{n-1}$

**Q.3 (C , D)**

$$\left\{ \frac{\left(x^{\frac{1}{3}}\right)^3 + 1}{x^{\frac{1}{3}} - x^{\frac{1}{3}} + 1} - \frac{\left(x^{\frac{1}{2}}\right)^2}{x^{\frac{1}{2}}\left(x^{\frac{1}{2}} - 1\right)} \right\}^{10}$$

$$\Rightarrow \left\{ \left(x^{\frac{1}{3}} + 1\right) - \left(\frac{x^{\frac{1}{2}} + 1}{x^{\frac{1}{2}}}\right) \right\}^{10}$$

$$\Rightarrow \left\{ x^{\frac{1}{3}} - x^{-\frac{1}{2}} \right\}^{10}$$

$$t_{r+1} = {}^{10}C_r \left(x^{\frac{1}{3}}\right)^{10-r} (-1)^r \left(x^{-\frac{1}{2}}\right)^r$$

$$= {}^{10}C_r (-1)^r \, x^{\frac{10-r}{3} - \frac{r}{2}}$$

$$= {}^{10}C_r (-1)^r \, x^{\frac{20-5r}{6}}$$

$\therefore$ r = 4

**Q.4 ( A , D )**

$${}^{2n}C_n = \frac{1 \cdot 3 \cdot 5 \cdot 7 \cdot ....... \cdot (2n-1)}{n!} \, 2^n$$

**Q.5 (A , B , C )**

Put x = 1 , x = w and $x = w^2$

Where, w and $w^2$ are complex cube roots of unity and then add

**Q.6 (A , B , C )**

$^nC_1 a^{n-1} b = 135$ ……..( 1 )

$^nC_2 a^{n-2} b^2 = 30$ ……..( 2 )

$^nC_3 a^{n-3} b^3 = \frac{10}{3}$ ……..( 3 )

$\frac{(2)}{(1)} : \frac{^nC_2}{^nC_1} \frac{b}{a} = \frac{30}{135} = \frac{2}{9}$

$\left(\frac{n-1}{2}\right)\frac{b}{a} = \frac{2}{9}$ ……...( 4 )

$\frac{(3)}{(2)} : \left(\frac{n-2}{3}\right)\frac{b}{a} = \frac{1}{9}$ ……..( 5 )

$\frac{(5)}{(4)} : \frac{2}{3}\,\frac{n-2}{n-1} = \frac{1}{2}$

$\Rightarrow 4n - 8 = 3n - 3$

$\therefore \boxed{n = 5}$

Substituting in ( 4 ), we get

$\boxed{9b = a}$

Solving, we get

a = 3 and $b = \frac{1}{3}$

**Q.7 (A, D)**

$^nC_{r-1}, {^nC_r}, {^nC_{r+1}}$ are in AP

$$\Rightarrow \frac{{}^nC_{r-1}}{{}^nC_r} + \frac{{}^nC_{r+1}}{{}^nC_r} = 2$$

$$\Rightarrow \frac{r}{n-r+1} + \frac{n-r}{r+1} = 2$$

$$\Rightarrow r^2 + r + (n-r)(r_1 - r + 1) = 2(n-r+1)(r+1)$$

$$\Rightarrow r^2 + r + (14-r)(15-r) = 2(15-r)(r+1)$$

$$\Rightarrow r^2 + r + r^2 - 29r + 210 = 2(-r^2 + 14r + 15)$$

$$\Rightarrow 4r^2 - 56r + 180 = 0$$

$$\Rightarrow r^2 - 14r + 45 = 0$$

$$\therefore r = 9, 5$$

**Q.8 (A , B )**

$$\frac{P_{n+1}}{P_n} = \frac{{}^{n+1}C_a\ {}^{n+1}C_1\ {}^{n+1}C_2 \ldots\ldots\ldots\ {}^{n+1}C_n\ {}^{n+1}C_{n+1}}{{}^nC_0\ {}^nC_1 \ldots\ldots\ldots {}^nC_{n+1}\ {}^nC_n}$$

$$= \frac{(n+1)^{n+1}}{(n+1)!} \text{ or } \frac{(n+1)^n}{n!}$$

**Q.9 (C , D )**

$$N^r = \sum_{r=0}^{n} {}^nC_r \sin 2rx \qquad \ldots..(1)$$

$$= \sum_{r=0}^{n} {}^nC_r \sin 2(n-r)x \qquad \ldots..(2)$$

$$(1)+(2): N^r = \sum_{r=0}^{n} {}^nC_r \sin 2nx\, ocs(n-2r)x$$

$$N^r = \sin nx \sum_{r=0}^{n} {}^nC_r \cos(n-2r)x$$

$$D^r = \sum_{r=0}^{n} {}^nC_r \cos 2rx \qquad \ldots..(3)$$

$$= \sum_{r=0}^{n} {}^{n}C_r \cos(2(n-r)x) \qquad .....(4)$$

$$(3)+(4) : D^r = \sum_{r=0}^{n} {}^{n}C_r \cos nx \cos(n-2r)x$$

$$= \cos nx \sum_{r=0}^{n} {}^{n}C_r \cos(n-2r)x$$

$$\therefore \frac{N^r}{D^r} = \tan nx$$

**Q.10 (C)**

AM $\geq$ GM

$$(p)^{\frac{1}{n}} \leq \frac{nC_0 + (n-1)C_1 + (n-2)C_2 + ......... + C_n}{n}$$

$$(p)^{\frac{1}{n}} \leq \frac{n2^{n-1}}{n} = 2^{n-1}$$

$$\therefore (p) \leq 2^{n(n-1)}$$

**Q.11 (B)**

$$k = \frac{\left({}^{n+1}C_1\right)\left({}^{n+1}C_2\right)...........\left({}^{n+1}C_n\right)}{C_1C_2...........C_n} \qquad \because \left\{ \left({}^{n}C_r + {}^{n}C_{r+1}\right) = {}^{n+1}C_{r+1} \right\}$$

Refer solution to problem 8.

**Q.12 (A, D)**

Coefficient of $x^r$ in $(1+x)^n (1-x)^{-1}$

$$\left(\sum_{p=0}^{n+1} {}^{n}C_p x^p\right)\left(\sum_{q=1}^{\infty} x^q\right)$$

Coefficient of $x^r$ in $\sum_{p=0}^{n+1}\sum_{q=1}^{\infty} {}^{n}C_p x^{p+q}$ is $2^n$

$p + q = r$ where $p \leq n$

$\Rightarrow r - q \le n$

$\Rightarrow r \le n+q$

**COMPREHENSION TYPE**

**Q.13 (C)**

Coefficient of $x^0$ in $(1+x)^{2n}\left(1-\frac{1}{x}\right)^{2n}$

$x^0 \rightarrow \frac{(1-x^2)^{2n}}{x^{2n}}$

$x^{2n} \rightarrow (1-x^2)^{2n}\ (-1)^n\ {}^{2n}C_n$

**Q.14 (C)**

Coefficient of $x^{-1}$ in $(1+x)^n \left\{\left(1+\frac{1}{x}\right)^{n+1} - 1\right\} \frac{1}{n+1}$

$x^{-1} \rightarrow \frac{(1+x)^n}{n+1}\left\{\frac{(1+x)^{n+1} - x^{n+1}}{x^{n+1}}\right\}$

$x^n \rightarrow \frac{(1+x)^{2n+1} - x^{n+1}(1+x)^n}{n+1}$

$\frac{1}{n+1}\left\{{}^{2n+1}C_n\right\} = \frac{(2n+1)}{\{(n+1)!\}^2}$

**Q.15 (C)**

$\frac{C_1}{C_0} + 2^2\frac{C_2}{C_1} + 3^2\frac{C_3}{C_2} + \text{..........} + 50^2\frac{C_{50}}{C_{49}}$

$\left(\frac{n-0}{1}\right) + 2^2\left(\frac{n-1}{2}\right) + 3^2\left(\frac{n-3}{2}\right) + \text{.........} + 50^2\left(\frac{n-49}{2}\right)$

$1n + 2(n-1) + 3(n-3) + \text{........} + 50(n-49)$

$$\sum_{r=1}^{50}(n-r+1)r$$

$$\sum_{r=1}^{50}(50-r+1)r \qquad \{\because n = 50\}$$

$$51\sum_{r=1}^{50} r - \sum_{r=1}^{50} r^2$$

**Q.16 (A)**

$$S = \sum_{r=0}^{2n}\frac{r}{a_r} \qquad .........(1)$$

$$S = \sum_{r=2n}\frac{n-r}{a_{n-r}} = \sum_{r=0}^{2n}\frac{2n-r}{a_r} \qquad ..........(2)$$

$$(1) + (2) \quad S = \frac{2n}{2}\sum_{r=0}^{2n}\frac{1}{a_r}$$

**Q.17 (A)**

$$(1+x+x^2)^n = a_0 + a_1x + a_2x^2 + ............ + a_{2n}x^{2n}$$

$$S = a_0 - a_1^2 - a_2^2 - a_3^2 + ......... + - a_{2n}^2$$

S is coefficient of $x^0$ in $(1+x+x^2)^n\left(1-\frac{1}{x}+\frac{1}{x^2}\right)^n$

$$x^{2n} \rightarrow (1+x^2+x^4) = a_n$$

$$\therefore \boxed{k = 1}$$

**<u>MATRIX MATCH TYPE</u>**

Q.18

(A) Data inconsistent.

(B) $\rightarrow$ (q)

$33^{4n}+1$

Since $3^1=3$ , $3^2=9$ , $3^3=27$ , $3^4=81$ , $3^5=243$

$\therefore$ 3 has a cycle of 4 at its unit's place

$\therefore$ we need to obtain the remainder when $3^{4n}$ is divided by 4.

$3^{4n}=(4-1)^{4n}=4k+{}^{4k}C_{4k}(-1)^{4k}=4k+1$

Hence last digit in $3^{3^{4n}}+1$ is 4.

(C) $\rightarrow$ (p)

General term $= {}^mC_r\ {}^nC_s\,(-1)^8\,x^{r+8}$

r + s = 1 $\Rightarrow$ (r , s) $\equiv$ (0 , 1) , (1 , 0)

$\therefore {}^mC_0\,{}^nC_1+{}^mC_1\,{}^nC_0=3$

$\boxed{-n+m=3}$

r + s = 2 $\Rightarrow$ (r , s) $\equiv$ (0 , 2) , (1 , 1) , (2 , 0)

${}^mC_0{}^nC_2-{}^mC_1{}^nC_1+{}^mC_2{}^nC_0=-6$

$$\Rightarrow \frac{n(n-1)}{2}-mn+\frac{m(m-1)}{2}=-6$$

$\Rightarrow n(n-1)-2(n+3)n+(n+3)(n+3-1)=-12$

$\Rightarrow n^2-n+2n^2-6n+(n+3)(n+2)=-12$

$\Rightarrow -2n=-18$

$\therefore \boxed{n=9} \quad \Rightarrow \boxed{m=12}$

(D) $\rightarrow$ (q , r , s)

$$\left(1+2x^2+x^4\right)\left(\sum {}^nC_r\,x^r\right)=\sum_{k=0}^{n+4}a_kx^k$$

$a_1 = {}^nC_1 = n$

$a_2 = {}^nC_2 + 2\ {}^nC_0 = \frac{n(n-1)}{2} + 2 = \frac{n^2 - n + 4}{2}$

$a_3 = {}^nC_3 + 2\ {}^nC_1 = \frac{n(n-1)(n-2)}{6} + 2n$

$= \frac{n}{6}\left\{n^2 - 3n + 2 + 12\right\} = \frac{n(n^2 - 3n + 14)}{6}$

$(n^2 - n + 4) = n + \frac{n}{6}(n^2 - 3n + 14)$

$6n^2 - 6n + 24 = n^3 - 3n^2 + 20n$

$\Rightarrow n^3 - 9n^2 + 26n - 24 = 0$

$\therefore n = 2, 3, 4$

## FILL IN THE BLANKS

**Q.19**

By definition $t_4 > t_5$ and $t_4 > t_3$

$\Rightarrow {}^{10}C_3 (2)^7 \left(\frac{3}{8}|x|\right)^3 > {}^{10}C_4 (2)^6 \left(\frac{3}{8}|x|\right)^4$

$\Rightarrow 2 \cdot \frac{8}{3} > \frac{10-3}{4}|x|$

$\Rightarrow \frac{64}{21} > |x|$

$\therefore -\frac{64}{21} < x < \frac{64}{21}$

Also, $t_1 > t_3$

$$^{10}C_3(2)^7\left(\frac{3}{8}|x|\right)^3 > {}^{10}C_2(2)^8\left(\frac{3}{8}|x|\right)^2$$

$$\frac{10-2}{3}\ \frac{1}{2}\ \frac{3}{8}|x| > 1$$

$$|x| > 2$$

$$\therefore x < -2 \text{ or } x > 2$$

$$\text{Hence, } x \in \left(-\frac{64}{21}, -2\right) \cup \left(2, \frac{64}{21}\right)$$

**Q.20**

$$(59)^{69} = (60-1)^{59}$$

$$= \left\{ {}^{59}C_0\,(60)^{59} - {}^{59}c_1(60)^{58} + \ldots\ldots\ldots + {}^{59}C_{58}(60) - {}^{59}C_{59} \right\}$$

$$= (x00 + 59 \times 60 - 1)$$

Where, x is any sequence of digits

= x00 + 3539

$\therefore$ Last 2 digits are 39

# EXERCISE – 1

**Q.1**

$na = 8$ ……..( 1 )

$\frac{n(n-1)}{2!} a^2 = 24$ ……..( 2 )

$\Rightarrow \frac{n(n-1)}{2} \times \frac{64}{n^2} = 24$

$\Rightarrow 1 - \frac{1}{n} = 1 - \frac{1}{4}$

$\therefore$ n = 4 and a = 2

**Q.2**

$t_{r+1} = {}^{12}C_r \frac{1^{12-2r}}{3^r} (x)^{24-4r} y^{r-12}$

24 – 4r = 9 and r – 12 = – 3

$\Rightarrow r = 9$

$\therefore r \in \phi$

**Q.3**

$^{10}C_6 \ 3^{-2} (-1)^6 x^8 = \frac{70}{3} x^8$

**Q.4**

$t_{r+1} = {}^{10}C_r \left(\sqrt{\frac{x}{3}}\right)^{10-r} \left(\frac{\sqrt{3}}{2x^2}\right)^r$

$\Rightarrow t_{r+1} = {}^{10}C_r \ \frac{3^{r-5}}{2^r} \ x^{5-\frac{5}{2}r}$

For term independent of x , r = 2

$\therefore t_3 = {}^{10}C_2 \frac{3^{-3}}{2^2} = \frac{5}{12}$

**Q.5**

$2^n = 4096$

$\Rightarrow n = 12$

$\therefore$ Greatest coefficient $= {}^{12}C_6$

**Q.6**

$${}^nC_3\ p^{n-3}\ x^{n-4} = \frac{5}{2}$$

$\therefore n = 4$

$$\Rightarrow 4p = \frac{5}{2}$$

$$\Rightarrow p = \frac{5}{8}$$

**Q.7**

If $t_{r+1}$ is the greatest term, then $t_r < t_{r+1} > t_{r+2}$

Now, $t_{r+1} > t_{r+2} \Rightarrow\ {}^8C_r\left(\frac{4}{3}\right)^{8-r} > {}^8C_r\left(\frac{4}{3}\right)^{8-(r+1)}$

$$\Rightarrow \frac{8-r}{r+1} < \frac{4}{3}$$

$\Rightarrow 24 < 7r + 1$

$$\Rightarrow r > \frac{23}{7} = 3 + \frac{2}{7}$$

$\therefore$ Greatest term is obtained for r = 4 Or $t_5 = {}^8C_5\left(\frac{4}{3}\right)^3 = \frac{3584}{27}$

**Q.8**

$$\frac{-6-r}{r+1} < \left(\frac{4}{13}\right);\ r \in I$$

$\Rightarrow -78 - 130 < 4r + 4$

$\Rightarrow 17r < -82$

**Q.9**

General term in the expansion of $\left(ax^2 + \frac{1}{bx}\right)^{11}$ is ${}^{11}C_r \frac{a^{11-r}}{b^r} x^{22-3r}$

For coefficient of $x^7$, we have

$\Rightarrow \boxed{r = 5}$

$\therefore$ coefficient of $x^7$ is ${}^{11}C_5 \frac{a^6}{b^5}$

Also, coefficient of $x^7$ in $\left(ax - \frac{b}{x^2}\right)^{11}$ is ${}^{11}C_6 \frac{a^5}{b^6}$

By given situation, we get $\boxed{ab = 1}$

**Q.10**

$$S = \sum_{r=1}^{n} nr \ {}^{n-1}C_{r-1}$$

$$\Rightarrow S = n\left[\sum_{r=1}^{n}(r-1+1)\ {}^{n-1}C_{r-1}\right]$$

$$\Rightarrow S = n\left[(n-1)\sum_{r=1}^{n} {}^{n-2}C_{r-2} + \sum_{r=1}^{n} {}^{n-1}C_{r-1}\right]$$

$$\Rightarrow S = n\left\{(n-1)2^{n-2} + 2^{n-1}\right\}$$

$$\Rightarrow \boxed{S = n\ (n+1)2^{n-2}}$$

**Q.11**

$$(2 - x + 3x^2)^5 = \sum \frac{5!}{p!\,q!\,r!}\, 2^p (-x)^q (3x^2)^r$$

Where, $p + q + r = 5$ ; $p, q, r \in w$

$$= \sum \frac{5!}{p!\, q!\, r!} 2^p (-1)^q 3^r \ x^{q+2r}$$

$q + 2r = 5$

| | | | | |
|---|---|---|---|---|
| p | 0 | 1 | 2 | NA |
| q | 5 | 3 | 1 | NA |
| r | 0 | 1 | 2 | 3 |

$$\therefore \text{ coefficient of } x^5 = \frac{5!}{5!}(-1) + \frac{5!}{3!}(6) + \frac{5!}{(2!)h2} - (36) - 961$$

**Q.12**

$E = 9^{17} = (8+1)^{17} = 8k + 1$

$\therefore$ Remainder when divided by 3 is 1.

**Q.13**

$$\left(\sqrt{2}+1\right)^6 + \left(\sqrt{2}-1\right)^6 = 2\left\{ {}^6C_0 \sqrt{2}^6 + {}^6C_2 \sqrt{2}^4 + \ldots \right\}$$

= Even Integer

Now, $\left(\sqrt{2}+1\right)^6 = I + f$ ; $I \in$ Integer & $f \in [0, 1)$

And $\left(\sqrt{2}-1\right)^6 = f'$; $0 \le f' < 1$

$\therefore I + f + f'$ = Even Integer

$\Rightarrow f + f'$ = Integer always and is equal to 1

**Q.14**

$t_{r+1} = {}^{15}C_r \, x^{60-7r}$

Foe coefficient of $x^{32}$ , r = 4

$\therefore t_5 = {}^{15}C_4 \, x^{32}$

**Q.15**

( i ) $t_{6+1} = {}^{12}C_6 \dfrac{y^6x^3}{5^6}(-1)^6 \dfrac{5^6}{x^6y^3} = {}^{12}C_6 x^3 y^3$

( ii ) There will be two middle terms i.e. $8^{th}$ and $9^{th}$ terms.

$t_8 = {}^{15}C_7 \; 2^8(-1)3^7 x = -\,{}^{15}C_7 2^8 3^7 x$

And $t_9 = {}^{15}C_8 \, 2^7 \, 3^8 \, \dfrac{1}{x}$

**Q.16**

$t_{r+1} = {}^{20}C_r \dfrac{2^{20-r}}{3^r}(-1)^r x^{10-\frac{r}{2}-\frac{r}{3}}$

For term independent of x

$10 - \dfrac{5r}{6} = 0$

$\Rightarrow \boxed{r = 12}$

$\therefore$ Term independent of x is ${}^{20}C_{12} \dfrac{2^8}{3^{12}}$

**Q.17**

$2^{2n} - 3n - 1 = (3+1)^n - 3n - 1$

$= {}^nC_2 3^2 + {}^n C_3 3^3 + ...... + {}^nC_n 3^n$

$= 9\left({}^nC_n + {}^nC_3 3 + ....... + {}^nC_n 3^{n-2}\right)$

Which is divided by 9.

**Q.18**

Assume $E = \left(4+\sqrt{10}\right)^n + \left(4-\sqrt{10}\right)^n$

Such that $\left(4+\sqrt{10}\right)^n = I + f \quad$ ……..( 1 )

And $\left(4-\sqrt{10}\right)^{n} = f'$ .......( 2 )

Where, $0 \le f\ , f' < 1$

Also, $E = 2\left\{ {}^{n}C_{0}4^{n} + {}^{n}C_{2}4^{n-2}(10) + ......\right\}$

$\Rightarrow$ I + f + f' = Even integer { using ( 1 ) and ( 2 )}

Since, I is an integer $\Rightarrow$ f + f' must be an integer

$\Rightarrow$ f + f' = 1

$\therefore$ I + 1 = Even Integer

**Q.19**

$a_{r} = {}^{n}C_{r}$

$$\text{LHS} = \frac{{}^{n}C_{1}}{{}^{n}C_{0}} + 2\,\frac{{}^{n}C_{2}}{{}^{n}C_{1}} + 3\,\frac{{}^{n}C_{3}}{{}^{n}C_{2}} + \ldots\ldots\ldots\ldots + n\,\frac{{}^{n}C_{1}}{{}^{n}C_{n-1}}$$

$$\Rightarrow \text{LHS} = n + 2\,\frac{(n-1)}{2} + 3\,\frac{(n-2)}{3} + \ldots\ldots. + n = \frac{n(n+1)}{2}$$

**Q.20**

$$E = (1-x-x^{2}(1-x))^{6} = (1-x)^{6}(1-x^{2})^{6}$$

$$\Rightarrow E = \left\{\sum_{r=1}^{6}(-1)^{r}\ {}^{6}C_{r}\,x^{r}\right\}\left\{\sum_{s=1}^{6}(-1)^{s}\ x^{2s}\right\}$$

$$\Rightarrow E = \sum_{r=1}^{6}\sum_{s=1}^{6}(-1)^{r+s}\ {}^{6}C_{r}\ {}^{6}C_{s}\,x^{r+2s}$$

For coefficient of $x^{7}$ ; r + 2s = 7 ; $r, s \in w$ and $r, s \le 6$

| | | | |
|---|---|---|---|
| r | 5 | 3 | 1 |
| s | 1 | 2 | 3 |

$\therefore$ Coefficient of $x^{7}$ in E is

$$(-1)^{6}\ {}^{6}C_{5}\,{}^{6}C_{1} + (-1)^{5}\ {}^{6}C_{3}{}^{6}C_{2} + (-1)^{4}\ {}^{6}C_{1}\ {}^{6}C_{3}$$

$= -144$

**Q.21**

$$S = {}^nC_0 + \frac{1}{2}\,{}^nC_1 + \frac{1}{3}\,{}^nC_2 + \ldots\ldots\ldots + \frac{1}{n+1}\,{}^nC_n$$

$$S = \frac{1}{n+1}\left\{\frac{n+1}{1}\,{}^nC_0 + \frac{n+1}{2}\,{}^nC_1 + \frac{n+1}{3}\,{}^nC_2 + \ldots\ldots + \frac{n+1}{n+1}\,{}^nC_n\right\}$$

$$S = \frac{1}{n+1}\left\{{}^{n+1}C_1 + {}^{n+1}C_2 + {}^{n+1}C_3 + \ldots\ldots\ldots + {}^{n+1}C_{n+1}\right\}$$

$$\therefore\ S = \frac{(2^{n+1}-1)}{(n+1)}$$

**Q.22**

Let $E = (3x-2)^n (1-x)^{-2}$

$$\Rightarrow E = (3x-2)^n \sum_{k=0}^{\infty}({}^{k+1}C_1\ x^k)$$

$$\Rightarrow E = \sum_{r=0}^{n} {}^nC_r\, 3^{n-r}(-1)^r\, 2^r\, x^{n-r} \sum_{k=0}^{\infty}(k+1)\,x^k$$

$$\Rightarrow E \sum_{r=0}^{n}\sum_{k=0}^{\infty}\ {}^nC_r(-1)^r\ 3^{n-r}2^r(k+1)\,x^{n+k-r}$$

For coefficient of $x^n$, we have

$$E = 3^n \sum_{r=k=0}^{n}\sum\ {}^nC_r\,(-1)^r\left(\frac{2}{3}\right)^r(k+1)\,x^n$$

$$\Rightarrow E = 3^n \sum_{r=0}^{n}(r+1)\ {}^nC_r\left(\frac{-2}{3}\right)^r$$

$$\Rightarrow E = 3^n\left[\sum_{r=0}^{n} n\ {}^{n-1}C_{r-1}\left(\frac{-2}{3}\right)^r + \sum_{r=0}^{n}\ {}^nC_r\left(\frac{-2}{3}\right)^r\right]$$

$$\Rightarrow E = 3^n\left\{\frac{-2}{3}\left(1-\frac{2}{3}\right)^{n-1} + \left(1-\frac{2}{3}\right)^n\right\}$$

$$\Rightarrow E = 3^n\left\{\frac{-2}{3} + 1\,\frac{-2}{3}\right\}\left(1 - \frac{2}{3}\right)^{n-1}$$

$$\Rightarrow \boxed{E = -1}$$

**Q.23**

$$S = 1 + \left(\frac{-2}{3}\right)\left(\frac{-1}{2}\right) + \frac{\left(\frac{-2}{3}\right)\left(\frac{-2}{3} - 1\right)}{2!}\left(\frac{-1}{2}\right)^2 + \frac{\left(\frac{-2}{3}\right)\left(\frac{-2}{3} - 1\right)\left(\frac{-2}{3} - 2\right)}{3}\left(\frac{-1}{2}\right)^3 + \ldots\ldots\infty$$

$$\therefore S = \left(1 - \frac{1}{2}\right)^{\frac{-2}{3}} = \left(\frac{1}{2}\right)^{\frac{-2}{3}} = (2)^{\frac{2}{3}} = \sqrt[3]{4}$$

**Q.24**

$$S = \sum_{k=0}^{r}(-1)^k\ {}^nC_{k+1}\ \ {}^{n+k}C_k$$

Coefficient of $x^{-1}$ in the expansion of

$${}^nC_1\,\frac{1}{x}(1+x)^n - {}^nC_2\,\frac{1}{x^2}(1+x)^{n+1} + {}^nC_3\,\frac{1}{x^3}(1+x)^{n+2}$$

$$- {}^nC_4\,\frac{1}{x^4}(1+x)^{n+3} + \ldots\ldots + (-1)^r\ {}^nC_{r+1}\,\frac{1}{x^{r+1}}(1+x)^{n+r}$$

$$+ \ldots\ldots\ldots + (-1)^{n-1}\ {}^nC_n\,\frac{1}{x^n}(1+x)^{n+(n-1)}$$

$$-(1+x)^{n-1}\left\{-\ {}^nC_1\left(\frac{1+x}{x}\right) + {}^nC_2\left(\frac{1+x}{x}\right)^2 - {}^nC_3\left(\frac{1+x}{x}\right)^3 + \ldots\ldots + (-1)^n\ {}^nC_n\left(\frac{1+x}{x}\right)^n\right\}$$

$$-(1+x)^n\left\{\left(1 - \left(\frac{1+x}{x}\right)\right)^n - 1\right\}$$

$$-(1+x)^n\left\{\left(\frac{-1}{x}\right)^n - 1\right\}$$

**Q.25**

$$(1+x^2)^2\,(1+x)^2 = \sum_{k=0}^{n+4} a_k x^k$$

$$= (a_0 + a_1 x + a_2 x^2 + a_2 x^3 + .......)$$

$$a_1 = {}^nC_1$$

$$a_2 = {}^nC_2 + 2\ {}^nC_0$$

$$a_3 = {}^nC_3 + 2^n C_1$$

Since, $a_1, a_2$ and $a_3$ are in AP ( given )

$$\Rightarrow 2\left({}^nC_2 + 2\right) = n + {}^nC_3 + 2n$$

$$\Rightarrow n(n-1) + 2 = 3n + {}^nC_3$$

$$\Rightarrow n^2 - 4n + 2 = {}^nC_3$$

$$\Rightarrow n^2 - 9n^2 + 26n - 24 = 0$$

$$\Rightarrow n = 2,\ 3,\ 4$$

$\therefore$ n = 3 or 4 only are possible solutions

**Q.26**

$$9^{n+1} - 8n - 9 = (8+1)^{n+1} - 8n - 9$$

$$= 1 + (n+1)8 + 8^2\{k\} - 8n - 9$$

= $64k$ ; $k \in I$

**Q.27**

$$E = 1 + (1+x) + (1+x)^2 + (1+x)^3 + ..... + (1+x)^n$$

$$\Rightarrow E = (1+x)^{n+1} - 1$$

$\therefore$ Coefficient of $x^3$ in $E = {}^{n+1}C_3$

**Q.28**

$$S=\sum_{r=0}^{n} {}^{2n+1}C_r=\sum_{r=0}^{n} {}^{2n+1}C_{2n+1-r}$$

$$\text{But } \sum_{r=0}^{n} {}^{2n+1}C_{2n+1-r}=\sum_{r=n+1}^{2n+1} {}^{2n+1}C_r$$

$$\Rightarrow 2S=\sum_{r=0}^{2n+1} {}^{2n+1}C_r=2^{2n+1}$$

$$\Rightarrow S=2^{2n}$$

**Q.29**

$$S=\sum_{r=0}^{n}(-1)^r\ {}^nC_r\left(a+rd\right)=a\sum_{r=0}^{n}(-1)^r\ {}^nC_r+d\sum_{r=0}^{n}(-1)^r\ r\ {}^nC_r=0$$

**Q.30**

$$(1+x+x^2)^n=a_0+a_1x+a_2x^2+a_3x^3+.....+a_{2n}x^{2n}$$

Replace x by $\frac{1}{x}$

$$\left(1+\frac{1}{x}+\frac{1}{x^2}\right)^n=a_0+\frac{a_1}{x}+\frac{a_2}{x^2}+..........+\frac{a_{2n}}{x^{2n}}$$

$$\Rightarrow (1+x+x^2)^n=a_0x^{2n}+a_1x^{2n-1}+a_2x^{2n-2}+.......+a_{2n}$$

On comparing the 2 series, we get $a_r=a_{2n-r}$

Now, sum of all coefficients in the series

$$a_0+a_1+a_2+.......+a_{n-1}+a_n+a_{n+1}+........+a_{2n}=3^n$$

Here $a_0=a_{2n}$ , $a_1=a_{2n-1}$ & $a_{n-1}=a_{n+1}$

$$\therefore 2(a_0+a_1+a_2+a_3+.........+a_{n-1})+a_n=3^n$$

$$\Rightarrow a_0+a_1+a_2+......+a_n=\frac{1}{2}\left(3^n+a_n\right)$$

**Q.31**

$$S=\sum_{r=0}^{n} r(n-r)\left({}^{n}C_r\right)^2$$

$$\Rightarrow S=\sum_{r=0}^{n} nr\ {}^{n}C_r\ {}^{n}C_r-\sum_{r=0}^{n} r^2\ {}^{n}C_r{}^{n}C_r$$

$$\Rightarrow S=\sum_{r=0}^{n} n^2\ {}^{n-1}C_{r-1}\ {}^{n}C_r-n^2\sum_{r=0}^{n}\left({}^{n-1}C_{r-1}\right)^2$$

$$\Rightarrow S=n^2\left({}^{2n-1}C_{n-1}-{}^{2n-2}C_{n-1}\right)$$

$$\therefore S=n^2\left({}^{2n-1}C_{n}-{}^{2n-2}C_{n-1}\right)$$

**Q.32**

$\left(\sqrt{3}+1\right)^{2n}=I+f$ ; where , I is an integer and $f\in[0,1)$

Assuring $\left(\sqrt{3}-1\right)^{2n}=f'$ ; $0\le f'<1$

Now, $\left(\sqrt{3}+1\right)^{2n}+\left(\sqrt{3}-1\right)^{2n}=2\ \{\text{integer}\}$

I + f + f’= Even

$\Rightarrow$ f + f’ = Integer $\Rightarrow$ f + f’ =1 or f’ = 1 – f

$\therefore (I+f)\ (I-f)=\left(\left(\sqrt{3}-1\right)\left(\sqrt{3}+1\right)\right)^{2n}=2^{2n}$

**Q.33**

$$\text{LHS}={}^{n}C_{n-1}+{}^{n+1}C_{n-1}+{}^{n+2}C_{n-1}+........+{}^{n+m-1}C_{n-1}$$

$$\Rightarrow \text{LHS = coefficient of } x^{n-1}\text{ in }(1+x)^n+(1+x)^{n+1}+(1+x)^{n+2}+.....+(1+x)^{m+n-1}$$

$$\Rightarrow \text{LHS = coefficient of } x^{n-1}\text{ in }(1+x)^n\left\{\frac{(1+x)^m-1}{x}\right\}$$

$$\Rightarrow \text{LHS = coefficient of } x^{n}\text{ in }(1+x)^{m+n}-(1+x)^n$$

$\Rightarrow$ LHS $= {}^{m+n}C_n - 1$

Similarly RHS = $= {}^{m+n}C_m - 1 = {}^{m+n}C_n - 1$

$\therefore$ LHS = RHS

**Q.34**

**(a)**

$$\frac{(1-x)^n}{x} = \sum_{r=0}^{n} (-1)^r \, {}^nC_r x^{r-1} \Rightarrow \frac{(1-x)^n - 1}{x} = \sum_{r=1}^{n} (-1)^r \, {}^nC_r x^{r-1}$$

$$\Rightarrow \int_0^1 \frac{(1-x)^n - 1}{x} dx = \sum_{r=1}^{n} (-1)^r \frac{{}^nC_r}{r}$$

$$\text{Now} \int_0^1 \frac{(1-x)^n - 1}{x} dx = -\int_0^1 \frac{1-(1-x)^n}{1-(1-x)} dx = -\int_0^1 \left\{1 + (1-x) + (1-x)^2 + \ldots + (1-x)^{n-1}\right\} dx$$

$$= -\left\{1 + \frac{1}{2} + \frac{1}{3} + \ldots + \frac{1}{n}\right\}$$

$$\text{Hence} \sum_{r=1}^{n} (-1)^r \frac{{}^nC_r}{r} = -\left\{1 + \frac{1}{2} + \frac{1}{3} + \ldots + \frac{1}{n}\right\}$$

$$\text{Or} \sum_{r=1}^{n} (-1)^{r+1} \frac{{}^nC_r}{r} = 1 + \frac{1}{2} + \frac{1}{3} + \ldots + \frac{1}{n}$$

**(b)**

$$(1-x)^n = \sum_{r=0}^{n} (-1)^r \, {}^nC_r x^r$$

$$\Rightarrow \int_0^x (1-x)^n dx = \sum_{r=1}^{n} (-1)^r \, {}^nC_r \int_0^x x^r dx$$

$$\Rightarrow \frac{1-(1-x)^{n+1}}{n+1} = \sum_{r=0}^{n} (-1)^r \, {}^nC_r \frac{x^{r+1}}{r+1}$$

$$\Rightarrow \frac{1-(1-x)^{n+1}}{1-(1-x)} = (n+1) \sum_{r=0}^{n} (-1)^r \, {}^nC_r \frac{x^r}{r+1}$$

$$\text{Now} \int_0^1 \frac{1-(1-x)^{n+1}}{1-(1-x)} dx = (n+1) \sum_{r=0}^{n} (-1)^r \, {}^nC_r \int_0^1 \frac{x^r}{r+1} dx$$

$$\int_0^1 \left\{1+(1-x)+(1-x)^2+...+(1-x)^n\right\}dx=(n+1)\sum_{r=0}^{n}(-1)^r\frac{{}^nC_r}{(r+1)^2}$$

Hence $\displaystyle\sum_{r=0}^{n}(-1)^r\frac{{}^nC_r}{(r+1)^2}=\frac{1}{n+1}\left\{1+\frac{1}{2}+\frac{1}{3}+...+\frac{1}{n+1}\right\}$

**Q.35**

$$S=\sum_{r=0}^{n}(r+1)^2\ {}^nC_r(-1)^r$$

$$\Rightarrow S=\sum_{r=0}^{n}(r^2+2r+1)\ {}^nC_r(-1)^r$$

$$\Rightarrow S=\sum_{r=0}^{n}r^2\ {}^nC_r(-1)^r+2\Rightarrow S=\sum_{r=0}^{n}{}^nC_r\ r(-1)^r+\Rightarrow S=\sum_{r=0}^{n}{}^nC_r(-1)^r$$

$$A=\sum_{r=0}^{n}(-1)^r\ r^2\ \frac{n}{r}\ {}^{n-1}C_{r-1}$$

$$=\sum_{r=0}^{n}(-1)^r\ r\,n\ {}^{n-1}C_{r-1}=\sum_{r=0}^{n}(-1)^r\,(r-1+1)_n\ {}^{n-1}C_{r-1}$$

$$=\sum_{r=0}^{n}(-1)^r n\left\{(n-1)\ {}^{n-2}C_{r-2}+{}^{n-1}C_{r-1}\right\}$$

$$=n(n-1)\sum_{r=2}^{n}(-1)^r\ {}^{n-2}C_{r-2}+n\sum_{r=1}^{n}(-1)^r\ {}^{n-1}C_{r-1}$$

$$=n(n-1)\ (1-1)^{n-2}+n(1-1)^{n-1}=0$$

Similarly, B = 0 & C = 0

$\therefore \boxed{S=0}$

**Q.36**

$$\sum_{r=0}^{n}(-1)^r\ {}^nC_r\frac{1+\frac{r}{n}\log 10}{(1+\log 10)^r}=\sum_{r=0}^{n}(-1)^r\ {}^nC_r\frac{1}{(1+\log 10)^r}+\log 10\times\sum_{r=0}^{n}(-1)^r\frac{r\ {}^nC_r}{n}\frac{1}{(1+\log 10)^r}$$

$$= \sum_{r=0}^{n} (-1)^r \, {}^{n}C_r \frac{1}{(1+\log 10)^r} - \frac{\log 10}{1+\log 10} \times \sum_{r=0}^{n} (-1)^{r-1} \, {}^{n-1}C_{r-1} \frac{1}{(1+\log 10)^{r-1}}$$

$$= \left(1 - \frac{1}{1+\log 10}\right)^n - \frac{\log 10}{1+\log 10}\left(1 - \frac{1}{1+\log 10}\right)^{n-1} = 0 .$$

## Binomial Theorem

**1 (A)**

40. (A)

Clearly it is van denounced 2

Rewrite the caprenian as ${}^{n}C_{n}.{}^{n}C_{r}+{}^{n}C_{n-1}{}^{n}C_{r+1}+.........$ By vandermont 2

${}^{2n}C_{n+r}$

41. (C)

$(abcd)^{10}\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}+\frac{1}{d}\right)^{10}$

So insted find cell of $\frac{1}{a^2}\frac{1}{b^6}.\frac{1}{c}\frac{1}{d}$ in $\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}+\frac{1}{d}\right)^{1}$

$\frac{10!}{2!6!}=2520$

42. Let term be $\frac{5!}{n_1!n_2!n_3!}\left(x^2\right)^{n_1}(-x)^{n_2}(-2)^{n_3}$

Now $n_1+n_2+n_3=5$ & $2n_1+n_2=5$

Put $n_{1q}=0\Rightarrow n_2=5\,\&\,n_3=0$

Coefficient is $-1$

If $n_1=1\Rightarrow n_2=3, n_3=1$

Coefficient is $\frac{5!}{3!}\times 2=40$

If $n_1=2\Rightarrow n_2=1, n_3=2$

Coefficient is $-\frac{5!}{2!2!}\times 4\Rightarrow -120$

Add all coefficient $-81$

43. Let term be $\frac{20!}{n_1!n_2!n_3!}(1)^{n_1}(-x)^{n_2}(y)^{n_3}$

$n_1+n_2+n_3=20$

$n_2=2$ Coefficient $\frac{20!}{15!3!2!}$

$n_3=3$

$n_1=15$

44. (C)

We can write it as

$(1+x)\left((1+x)\left(1-x+x^2\right)\right)^{100}$

$(1+x)\left(1+x^3\right)^{100}$

$(1+x)\left({}^{100}C_0 + {}^{100}C_1x^3 + ........{}^{100}C_{100}x^{300}\right)$

Clearly:- This multiplication we can't get power of form $3r+2$

45. Let term be $\frac{30!}{n_1!n_2!n_3!}\left(x^3\right)^{n_2}\left(-x^6\right)^{n_3}$

$n_1 + n_2 + n_3 = 30$

$3n_2 + 6n_3 = 28$

LHS is multiple of 3 but RHS is not cell 0

**1(B)**

37. $\left(1+2x+3x^2\right)^{10} = a_0 + a_1x + .......a_{20}a^{20}$ …………(1)

Diff $10\left(1+2x+3x^2\right)^9\left(2+6m\right) = a_1 + 2a_2x + ........$

Put $x = 0$

$20 = a_1$ In equation (1)

Put $x = 1$

$a_0 + a_1 + a_2 + ..........a_{20} = 6^{10}$

Clearly coefficient of $x^{20}$ is $3^{10}$

Use PNC to find coefficient of $x^2$

$3.{}^{10}C_1 + 4.\frac{10}{2}$

$30 + 180 = 210$

38. ${}^{min}C_m = A \,\&\, B = {}^{m+n}C_n$

Clearly $A = B$

39. General tan ${}^{100}\left(r(i)\right)^r$

$A = 1,3,5,7,.........99 \Rightarrow 50$ terms

40. $(abc)^{12}\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)^{12}$

$\Rightarrow$ Find coefficient of $\frac{1}{a^4}\frac{1}{b^2}\frac{1}{c^6}$ in $\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)^{12}$

Then $\frac{12!}{4!2!6!}$

41. (B)

Put $x = 1$

$a_0 + a_1 + a_2 + a_3..........a_{12} = 0$

Put $x = -1$

$1 - a_1 + a_2 - a_3......... + a_{12} = 2^6$

Add $2\left(1+a_2+a_4+.....a_12\right)=2^5$

$a_2+a_4+.......a_{12}=2^5$

42. Expression becomes

$$1+\frac{n}{1}+\frac{n\left(n-1\right)}{2!}+......\frac{n!}{n!}$$

$$^nC_0+^nC_1+\frac{n}{2}+.........^nC_n=2^n$$

43. $$\sum_{k=1}^{\infty}\sum_{r=0}^{k}\frac{1}{3}k.^kC_r$$

$$\sum_{k=1}^{\infty}\frac{1}{3^k}\left(2^k\right)$$

Which is a G.P.

$$\frac{\frac{2}{3}}{1-\frac{2}{3}}=2$$

44. $$\sum_{r=1}^{40}r.^{40}C_r.^{30}C_r$$

$$40\sum_{r=1}^{40}{}^{39}C_{r-1}.^{30}C_r$$

Or $40\sum_{r=1}^{40}{}^{39}C_{r-1}{}^{30}C_{30-r}$ vandermand 2

$$40\left({}^{69}C_{29}\right)$$

45. (A)

$$\sum_{r=1}^{15}\frac{\left(r+2-2\right).2^r}{\left(r+2\right)!}$$

$$\sum\frac{\left(r+2\right).2^r-2^{r+1}}{\left(r+2\right)!}$$

$$\sum_{r=1}^{15}\frac{2^r}{\left(r+1\right)!}-\frac{2^{r+1}}{\left(r+2\right)!}$$

$$=\frac{2}{21}-\frac{2^{16}}{17!}$$

**2(A)**

26. $^{69}C_{3r-1} + {}^{69}C_{3r} = {}^{69}C_{r^2} + {}^{69}C_{r^2} + {}^{69}C_{r^2-1}$

By pascal is

$^{70}C_{3r} = {}^{70}C_{r^2}$

$\Rightarrow$ Either $r^2 = 3r$ or $r^2 = 70 - 3r$

$r = 0$ or $r = 3$ $\quad\quad$ $r^2 - 3r - 70 = 0$

Discard $\quad\quad$ $(r+7)(r-10) = 0$

$r = 10$

27. Put $x = 1$ gives sum of coefficient

$S = (a-1)^n$ clearly A,C,D

28. (D)

$f(m) = \sum_{r=0}^{m} {}^{30}C_{30-r}\,{}^{20}C_{m.r}$

$= \sum {}^{30}C_{30-r}\,{}^{20}C_{20-m+r}$

$f(m) = {}^{80}C_{50-m}$ or $f(m) = {}^{80}C_m$

Now clearly $^{80}C_m$ max $^{50}C_{25}$

A,B, C not correct

By vandermond $\sum_{r=0}^{n} \left({}^{n}C_r\right)^2 = {}^{2n}C_n$

29. $\left({}^{m}C_0 - {}^{m}C_1 y + \ldots.. {}^{m}C_m\right)\left({}^{n}C_0 + {}^{n}C_1 y + \ldots\ldots {}^{n}C_n y\right)$

$a_1 = {}^{n}C_0\,{}^{n}C_1 - \frac{n}{0} m_4 = 10$

or $n - m = 10$

$a_2 = {}^{m}C_0\,{}^{n}C_2 - {}^{m}C_1\,{}^{n}C_1 + {}^{m}C_2\,{}^{n}C_0 = 10$

$\frac{n(n-1)}{2} - mn + \frac{m(m-1)}{2} = 20$

$n^2 - n - 2mn + m^2 - m = 40$

$(m.n)^2 - (m+n) = 40$

$n > m$ $\quad\quad$ $m + n = 60$

$m + n = 60$

$n - m = 10$

30. (A,C,D)

Clearly it is a,g,p

$N = \frac{51^{50} - 1}{50}$

$= \frac{1}{5}\left((1+50)^{50} - 1\right)$

$$= \frac{1}{50}\left( {}^{50}C_1.50 + {}^{50}C_2 50^2 + ......{}^{50}C_{50} 50 \right)$$

$$= {}^{50}C_1 + {}^{50}C_2 50 + .......{}^{50}C_{50}.50^{49}$$

Clearly a multiple of 50

# EXERCISE – ( 2 )

**Q.1**

Coefficient of $t_{(2r+3)+1} = {}^{18}C_{2r+3}$

and Coefficient of $t_{(r-3)+1} = {}^{18}C_{r-3}$

Now, ${}^{18}C_{2r+3} = {}^{18}C_{r-3}$

Either 2r + 3 = r – 3 or 3r = 18

$\Rightarrow$ r = 6 or r = 6

$\therefore \boxed{r = 6}$

**Q.2**

Since, $\alpha + \beta = \left(8 + 3\sqrt{7}\right)^n$ ; $0 \le \beta < 1$ ; $\alpha \in I^+$

Let $f = \left(8 - 3\sqrt{7}\right)^n$ ; $0 \le f < 1$

Now, $\alpha + \beta + f = \left(8 + 3\sqrt{7}\right)^n - \left(8 - 3\sqrt{7}\right)^n$

$= 2\left\{{}^nC_0 8^n \left(3\sqrt{7}\right)^0 + {}^nC_2 8^{n-2}\left(3\sqrt{7}\right)^2 + ......\right\}$

= 2 (Integer)

Since $\alpha \in I^+$

$\therefore$ $\beta +$ f = Integer

Also, $0 \le \beta < 1$ & $0 \le F < 1$

$\Rightarrow 0 \le \beta + f < 2$

$\therefore \beta + f = 0$ or 1

But $f \ne 0$

$\therefore \beta + f = 1$

$\Rightarrow f = 1-\beta$

Now, $(\alpha+\beta)\ (1-\beta) = (\alpha+\beta)f = 1$

**Q.3**

$$S = (x+3)\left\{\frac{\left(\frac{x+2}{x+3}\right)^n - 1}{\left(\frac{x+2}{x+3}\right) - 1}\right\} = \frac{(x+3)^{n-1}\left\{\left(\frac{x+2}{x+3}\right)^n - 1\right\}}{-1}$$

$S = (x+3)^n - (x+2)^n$

Coefficient of $x^r$ in $S = {}^nc_r\left(3^{n-r} - 2^{n-r}\right)$

**Q.4**

$$S = (1+x)^{100}\left\{\frac{\left(\frac{x}{1+x}\right)^{101} - 1}{\left(\frac{x}{1+x}\right) - 1}\right\}$$

$$\Rightarrow S = -(1+x)^{101}\left\{\left(\frac{x}{1+x}\right)^{101} - 1\right\}$$

$\Rightarrow S = (1+x)^{101} - x^{101}$

$\therefore$ Coefficient of $x^{50}$ is in $S = {}^{101}C_{50}$

**Q.5**

$S_n$ = Coefficient of $x^1$ in $(1+x)^n\left(1+\frac{1}{x}\right)^n$

= Coefficient of $x^{n+1}$ in $(1+x)^{2n} = {}^{2n}C_{n+1}$

Also, $S_{n+1} = {}^{2(n+1)}C_{n+2}$

Now, $\frac{S_{n+1}}{S_n} = \frac{{}^{2n+2}C_{n+2}}{{}^{2n}C_{n+1}} = \frac{15}{4}$

$$\Rightarrow \frac{(2n+2)!}{(n+2)!\,n!} \times \frac{(n+1)!\,(n-1)!}{(2n)!} = \frac{15}{4}$$

$$\Rightarrow \frac{(2n+2)\,(2n+1)}{(n+2)(n)} = \frac{15}{4}$$

$$\Rightarrow 4(4n^2+6n+2) = 15n^2+30n$$

$$\Rightarrow n^2-4n+8=0$$

**Q.6**

$$x=1 \Rightarrow a_0+a_1+a_2+.....+a_{2n}+......+a_{4n-1}+a_{4n}=3^{2n}$$

$$x=-1 \Rightarrow a_0-a_1+a_2-....+a_{2n}+.....-a_{4n-1}+a_{4n}=1$$

ADD : $2(a_0+a_2+a_4+....+a_{2n}+....+a_{4n}) = 3^{2n}+1$

Now. For the given expansion $a_{4n-r} = a_r$

$$\therefore\ 2(a_0+a_2+a_4+.......+a_{2n}+.....+a_4+a_2+a_0) = 3^{2n}+1$$

$$\Rightarrow 2(2(a_0+a_2+a_4+......)+a_{2n}) = 9^n+1$$

$$\Rightarrow 2\left\{2(a_0+a_2+a_4+......)+2a_{2n}-a_{2n}\right\} = 9^n+1$$

$$\therefore a_0+a_2+a_4+......+a_{2n} = \frac{9^n+1+2a_{2n}}{4}$$

**Q.7**

$$S=(n-1)^2\ {}^nC_{n-1}+(n-3)^2\ {}^NC_{n-3}+(n-5)^2\ {}^nC_{n-5}+.......+1^2\ {}^nC_1$$

$$\therefore S = \sum_{r=1}^{n} r^2\ {}^nC_{2r-1}$$

Now, ${}^nC_0+{}^nC_1x+{}^nC_2x^2+{}^nC_3x^3+.......+{}^nC_nx^\nu = (1+x)^n$

Differentiating w.r.t. x, we get

$$1\,{}^nC_1+2\,{}^nC_2x+3\ {}^nc_3x^2+\ 4\ {}^nC_4x^3+......=n(1+x)^{n-1}$$

Multiply by x and differentiating again w.r.t. x

$$1^2\ {}^nC_1 + 2^2\ {}^nC_2 x + 3^2\ {}^nc_3 x^2 + 4^2\ {}^nC_4 x^3 + \ldots\ldots = n(1+x)^{n-1} + xn(n-1)(1+x)^{n-2}$$

$$= n(1+x)^{n-2}\{(1+x) + x(n-1)\}$$

Put x = 1 and then x = – 1, then add, we have

$$2\{1^2\ {}^nC_1 + 3^2\ {}^nC_2 + \ldots..\} = n\ 2^{n-2}\{2+n-1\}$$

$$\therefore 1^2 C_1 + 3^2 C_2 + \ldots\ldots = n(n+1)2^{n-3}$$

**Q.8**

$$\sum_{0\le i}\sum_{j\le n}\left({}^nC_i - {}^nC_j\right)^2$$

$$\sum_{0\le i}\sum_{j\le n}\left\{(C_i)^2 + \left({}^nC_j\right)^2 - 2\left({}^nC_i\ {}^nC_j\right)\right\}$$

$$(n+1)\sum\left({}^nC_i\right)^2 - 2\cdot 2^n\cdot 2$$

$$(n+1)\ {}^{2n}C_n - 2\cdot 2^{2n}$$

**Q.9**

$$\int_0^1 x(1+x)^n\,dx = \int_0^1 (1+x)^{n+1} - (1+x)^n\,dx$$

$$= \left\{\frac{(1+x)^{n+2}}{n+2} - \frac{(1+x)^{n+1}}{n+1}\right\}\int_0^1$$

$$= \frac{2^{n+2}}{n+2} - \frac{2^{n+1}}{n+1} + \frac{1}{(n+1)(n+2)}$$

$$= 2^{n+1}\left\{\frac{n}{(n+1)(n+2)}\right\} + \frac{1}{(n+1)(n+2)}$$

$$= \frac{n\,2^{n+1}+1}{(n+1)(n+2)}$$

**Q.10**

$$\sum_{r=0}^{n}(-1)^r\ {}^nC_r\left(\left(\frac{1}{2}\right)^r+\left(\frac{3}{4}\right)^r+\left(\frac{7}{8}\right)^r+\ldots\right)=\sum_{r=0}^{n}(-1)^r\ {}^nC_r\left(\left(1-\frac{1}{2}\right)^r+\left(1-\frac{1}{4}\right)^r+\left(1-\frac{1}{8}\right)^r+\ldots\right)$$

$$=\sum_{r=0}^{n}(-1)^r\ {}^nC_r\left(1-\frac{1}{2}\right)^r+\sum_{r=0}^{n}(-1)^r\ {}^nC_r\left(1-\frac{1}{4}\right)^r+\sum_{r=0}^{n}(-1)^r\ {}^nC_r\left(1-\frac{1}{8}\right)^r+\ldots$$

$$=\left(1-\left(1-\frac{1}{2}\right)\right)^n+\left(1-\left(1-\frac{1}{4}\right)\right)^n+\left(1-\left(1-\frac{1}{8}\right)\right)^n+\ldots$$

$$=\frac{1}{2^n}+\frac{1}{4^n}+\frac{1}{8^n}+\ldots=\frac{\frac{1}{2^n}}{1-\frac{1}{2^n}}=\frac{1}{2^n-1}$$

**Q.11**

Let $\frac{1}{{}^nC_r}$ , $\frac{1}{{}^nC_{r+1}}$ , $\frac{1}{{}^nC_{r+2}}$ are in AP

$$\Rightarrow \frac{2}{{}^nC_{r+1}}=\frac{1}{{}^nC_r}+\frac{1}{{}^nC_{r+2}}$$

$$\Rightarrow 2=\frac{{}^nC_{r+1}}{{}^nC_r}+\frac{{}^nC_{r+1}}{{}^nC_{r+2}}=\frac{n-r}{r+1}+\frac{r+2}{n-(r+1)}$$

$$\Rightarrow 2\left(n-(r+1)\right)(r+1)=n^2-(r+1+r)n+(r+1)r+(r+1)(r+2)$$

$$\Rightarrow n^2-(2r+1+2r+2)n+(r+1)(2r+2)+2(r+1)^2=0$$

$$\Rightarrow n^2-(4r+3)n+4(r+1)^2=0$$

$$\therefore\ \Delta=(4r+3)^2-4\cdot4\cdot(r+1)^2$$

$$=(4r+3)^2-(4r+4)^2<0$$

$\Rightarrow$ n is an imaginary quantity the assumption is incorrect

**Q.12**

We know ${}^{2n}C_n=\sum_{r=0}^{n}\left({}^nC_r\right)^2$

But $\sqrt{\frac{\sum_{r=0}^{n}\left({}^{n}C_{r}\right)^{2}}{n+1}} > \frac{\sum_{r=0}^{n}{}^{n}C_{r}}{n+1}$ $\{\text{by R.M.S.} \geq \text{A.M.}\}$

$$\Rightarrow \sum_{r=0}^{n}\left({}^{n}C_{r}\right)^{2} > \frac{2^{2n}}{n+1}$$

$$\Rightarrow \frac{(2n)!}{(n)!(n)!} > \frac{2^{2n}}{n+1} > \frac{2^{2n}}{2n+1}$$

**Q.13**

$x = 1 : 4^{20} = a_0 + a_1 + a_2 + a_3 + ......... + a_{40}$

$x = -1 : 2^{20} = a_0 - a_1 + a_2 - a_3 + ..... + a_{40}$

Adding, we get

$$a_0 + a_2 + a_4 + ..... + a_{38} + a_{40} = \frac{2^{40} + 2^{20}}{2}$$

Now, Replace x by $\frac{1}{x}$, we have

$$(x^2 + x + 2)^{20} = x^{40}\left(a_0 + \frac{a_1}{x} + \frac{a_2}{x^2} + ...... + \frac{a_{40}}{x^{40}}\right)$$

$$= a_0x^{40} + a_1x^{39} + a_2x^{38} + ...... + a_{40}$$

Now the constant term on LHS is $2^{20}$

$\therefore a_{40} = 2^{20}$

Now, $a_0 + a_2 + a_4 + ....... + a_{38} = 2^{39} + 2^{19} - a_{40}$

$= 2^{39} + 2^{19} + 2^{20}$

$= 2^{39} - 2^{19}$

$\therefore$ n = 39 m = 19

**Q.14**

$$\int_0^1 (1-x^2)^n\,dx = \int_0^1 (1-x^2)^n\, x^0 dx$$

$$I_n = (1-x^2\ \ x\int_0^1 - \int_0^1 n\,(1-x^2)^{n-1}(-2x)x\,dx$$

$$I_n = 0 - 2n\int_0^1 (1-x^2)^{n-1}(1-x^2-1)dx$$

$$I_n = -2n\int_0^1 (1-x^2)^n\,dx + 2n\int_0^1 (1-x^2)^{n-1}\,dx$$

$$(1+2n)\,I_n = 2n\ I_{n-1}$$

$$\Rightarrow \frac{I_n}{I_{n-1}} = \frac{2n}{2n+1}$$

$$\therefore \frac{I_n}{I_0} = \frac{I_n}{I_{n-1}} \cdot \frac{I_{n-1}}{I_{n-3}}\ \frac{I_{n-2}}{I_{n-3}} \ldots\ldots\ldots \frac{I_1}{I_0}$$

$$\therefore I_n = \frac{2n}{2n+1}\ \frac{2(n-1)}{2n-1} \ldots\ldots\ldots \frac{6}{7}\ \frac{4}{5}\ \frac{2}{3}\ I_0$$

$$\therefore I_n = \frac{2\cdot 4\cdot 6\cdot \ldots\ldots(2n)}{3\cdot 5\cdot 7\cdot \ldots\ldots\ldots(2n+1)}1$$

**Q.15**

Let $\left(7+4\sqrt{3}\right)^n = I+f$ & $\left(7-4\sqrt{3}\right)^n = f'$, where $0 \le f' < 1$

Adding the two expansions gives $I+f+f' = 2n$

Hence $f+f' \in I$

$\Rightarrow f+f' = 1$

$\Rightarrow 1-f = f'$

$\Rightarrow (1-f)(I+f) = \left(7-4\sqrt{3}\right)^n \left(7+4\sqrt{3}\right)^n = 1.$

**Q.16**

$$E = (50-1)^{50} - (10-1)^{50}$$

$$= \left\{ {}^{50}C_0\ 50^{50} - {}^{50}C_1\, 50^{44} + ......+ - {}^{50}C_{47}(50)^3 + {}^{50}C_{48}(50)^2 - {}^{50}C_{49}50 + {}^{50}C_{50} \right\}$$

$$-\left\{ {}^{50}C_0\, 10^{50} - {}^{50}C_1 10^{48} + .......+ - {}^{50}C_{47}(10)^3 + {}^{50}C_{48}(10)^2 - {}^{50}C_{49}(10) + {}^{50}C_{50} \right\}$$

For last 3 digits

$${}^{50}C_{48}(2500-100) - {}^{50}C_{49}(50-10) + (1-1)$$

0000 – 000 = 000

**Q.17**

$13^{99} - 19^{93}$ will be an even number so it is sufficient to prove that it is divisible by 81.

$$13^{99} - 19^{93} = 2197^{33} - 6859^{31}$$

$$= \left(10 + 2187\right)^{33} - \left(55 + 6804\right)^{31}$$

$$= 10^{33} - 55^{31} + 81k_1$$

Now $10^{33} - 55^{31} = \left(1+9\right)^{33} - \left(1+54\right)^{31}$

$$= 1 + 33\times 9 - 1 - 31\times 54 + 81k_2$$

$$= 17\times 81 + 81k_2$$

**Q.18**

$$E = \frac{1}{{}^{2n+1}C_r} + \frac{1}{{}^{2n+1}C_{r+1}}$$

$$E = \frac{{}^{2n+1}C_{r+1} + {}^{2n+1}C_r}{{}^{2n+1}C_{r+1}\ {}^{2n+1}C_r} = \frac{{}^{2n+1}C_{r+1}}{\frac{2n+1}{r+1}\ {}^{2n}C_r\ \ {}^{2n+1}C_r}$$

$$= \frac{\frac{2n+2}{r+1}\ {}^{2n+1}C_r}{\frac{2n+1}{r+1}\ {}^{2n}C_r\ \ {}^{2n+1}C_r} = \frac{2n+2}{(2n+1)\ {}^{2n}C_r}$$

**Q.19**

$$S = \sum_{r=1}^{2n-1} \frac{(-1)^{r-1} r}{^{2n}C_r} = \left( \sum_{r=0}^{2n} \frac{(-1)^r r}{^{2n}C_r} \right) + 2n$$

Replace r by 2n – r, we have

$$S = \sum_{r=0}^{2n} \frac{(-1)^{2n-r}\, 2n - r}{^{2n}C_{2n-r}} + 2n$$

$$2S = \sum_{r=0}^{2n} \frac{(-1)^r\, 2n}{^{2n}C_r} + 4n$$

$$S = n \sum_{r=0}^{2n} \frac{(-1)^r}{^{2n}C_r} + 2n$$

**Q.20**

$$\frac{\frac{1}{3}\left(1+\frac{1}{3}\right)}{1\cdot 2} \cdot \left(\frac{3}{5}\right)^2 - \frac{\frac{1}{2}\left(1+\frac{1}{3}\right)\left(2+\frac{1}{3}\right)}{1\cdot 2\cdot 3}\left(\frac{3}{5}\right)^3 + \frac{\frac{1}{2}\left(1+\frac{1}{3}\right)\left(2+\frac{1}{3}\right)\left(3+\frac{1}{3}\right)}{1\cdot 2\cdot 3\cdot 4}\left(\frac{3}{5}\right)^4 - \text{.........}\infty$$

$$\Rightarrow 1 + \left(\frac{-1}{3}\right)\left(\frac{3}{5}\right) + \frac{\frac{1}{3}\left(1+\frac{1}{3}\right)}{2!}\left(\frac{3}{5}\right)^2 + \text{........}\infty - 1 - \left(\frac{-1}{3}\right)\left(\frac{3}{5}\right)$$

$$\Rightarrow \left(1+\frac{3}{5}\right)^{\frac{-1}{3}} - 1 + \frac{1}{5}$$

$$\Rightarrow \left(\frac{8}{5}\right)^{\frac{-1}{3}} - \frac{4}{5} = \left(\frac{5}{8}\right)^{\frac{1}{3}} - \frac{4}{5}$$

**Q.21**

$$\sum_{r=0}^{n} \frac{r+1}{r+2}\ {}^nC_r\, x^r$$

$$\sum_{r=0}^{n} {}^nC_r\, x^r - \sum_{r=0}^{n} \frac{1}{r+2}\ {}^nC_r\, x^r$$

$$(1+x)^n - \int_0^x (1+x)^n x\,dx$$

$$(1+x)^n - \int_0^x \left\{(1+x)^{n+1} - (1+x)^n\right\} dx$$

$$(1+x)^n - \frac{(1+x)^{n+2}}{n+2}\Big|_0^x + \frac{(1+x)^{n+1}}{n+1}\Big|_0^x$$

$$(1+x)^n \left\{1 - \frac{(1+x)^2}{n+2} + \frac{(1+x)}{n+1}\right\}$$

**Q.22**

$$p_r = \frac{1\cdot 3\cdot 5.....(2r-1)}{2\cdot 4\cdot 6..(2r)}$$

$$p_r = \frac{1\cdot 2\cdot 3\cdot 4\cdot 5.....(2r-1)(2r)}{\left\{2\cdot 4\cdot 6.....(2r)\right\}^2}$$

$$= \frac{(2r)!}{2^{2r}(r!)^2} = {}^{2r}C_r\left(\frac{1}{4}\right)^r$$

$\therefore p_r$ is coefficient of $x^r$ in $(1+x)^{2r}$; $x = \dfrac{x}{4}$

Now, the required expansion is the coefficient of $x^{2r+1}$ in the expansion of $(1+x)^{2r}\left(1+\dfrac{1}{x}\right)^{2r}$

Coefficient of $x^{4r+1}$ in $(1+x)^{4r}$

**Q.23**

**(a)**

${}^nH_r =$ Coefficient of $x^r$ in the expansion $\left(1+x+x^2+...+x^r\right)^n$

$\Rightarrow {}^nH_r =$ Coefficient of $x^r$ in the expansion $\left(1-x^{r+1}\right)^n\left(1-x\right)^{-n} = {}^{n+r-1}C_r$

**Q.25**

$$\left\{1+2x+3x^2+.........+(n+1)x^n\right\} \approx \left\{(1-x)^{-2}\right\}^2 (1-x)^{-4}$$

$\therefore$ coefficient of $x^r$ in $(1-x)^{-4}$ is ${}^{r+3}C_3$

**Q.26**

$E = {}^{72}C_{36} - 1$

$$\Rightarrow E = \frac{72\cdot 71\cdot 70......37}{(36)!} - 1$$

$$\Rightarrow E = \frac{(73-1)\ (73-3).......(73-1)}{(36)!} - 1$$

$$\Rightarrow E = \frac{73k + 36!}{36!} - 1$$

$$\Rightarrow E = 73\left(\frac{k}{36}\right) + 1 - 1 = 73\lambda$$

**Q.27**

$$(1+x^2-x^3)^9 = \sum \frac{9!}{p!\,q!\,r!}(x^{2q})(-1)^r x^{3r}$$

$2q+3r=8$ and p + q + r = 9

| | | |
|---|---|---|
| p | 5 | 6 |
| q | 4 | 1 |
| r | 0 | 2 |

$\therefore$

$$\text{Coefficient of } x^8 = \frac{9!}{5!\,4!} + \frac{9!}{6!\,1!\,2!}$$

$$= \frac{9\cdot 8\cdot 7\cdot 6}{24} + \frac{9\cdot 8\cdot 7}{2}$$

= 378

**Q.28**

Coefficient of $x^1$ in $(1+x+x^2)^n\left(1-\frac{1}{x}+\frac{1}{x^2}\right)^n$

$\Rightarrow$ coefficient of $x^{2n+1}$ $\left(1+x^2+x^4\right)^n$ i.e. <u>ZERO</u>

**Q.29**

$$\sum_{r=0}^{n}(-1)^r\frac{{}^nC_r}{{}^{r+3}C_r}=6\sum_{r=0}^{n}(-1)^r\frac{{}^nC_r}{(r+1)(r+2)(r+3)}$$

$$=\frac{6}{(n+1)(n+2)(n+3)}\sum_{r=0}^{n}(-1)^r\ {}^{n+3}C_{r+3}$$

$$=\frac{6}{(n+1)(n+2)(n+3)}\left\{0-\left({}^{n+3}C_0-{}^{n+3}C_1+{}^{n+3}C_2\right)\right\}=\frac{3}{n+3}$$

**Q.30**

$t_r=(r+1-1)r!=(r+1)!-r!$

$\therefore S=\sum_{r=1}^{n}t_r=(n+1)!-1$

## Binomial Theorem

**Ex-3**

1. $P_k(x) = \dfrac{x^k - 1}{x - 1}$

$\therefore \sum_{k=1}^{n} {}^nC_k\, p_k(n) = \sum_{k=1}^{n} \dfrac{{}^nC_k\left(x^k - 1\right)}{x - 1} = \dfrac{(1+n)^n - 2^n - 1 + 1}{n - 2}$

$= \dfrac{(1+x)^n - 2^n}{x - 1}$

$2^{n-1} P_n\left(\dfrac{1+n}{2}\right) = 2^{n-1}\left(\dfrac{\left(\dfrac{1+x}{2}\right)^n - 1}{\left(\dfrac{1+x}{2}\right) - 1}\right) = \dfrac{(1+x)^n - 2^n}{x - 1}$

2. $\left[\left(2+\sqrt{5}\right)^r\right] - 2^{p+1} = \left(2+\sqrt{5}\right)^p - \left(\sqrt{5}-2\right)^p - 2^{p+1}$

$= 2\left({}^PC_1\left(\sqrt{5}\right)^{p-1}.2 + {}^PC_3\left(\sqrt{5}\right)^{p-3}2^3 + ......{}^PC_P 2^P\right) - 2^{P+1}$

$= 2\left({}^PC_1\left(\sqrt{5}\right)^{P-1}.2 + {}^PC_3\left(\sqrt{5}\right)^{P-3}2^3 + .......2^P - 2^P\right)$

$= 2\left({}^PC_1\left(\sqrt{5}\right)^{p-1}.2 + {}^PC_3\left(\sqrt{5}\right)^{P-3}2^3 + .......{}^PC_{P-2}\left(\sqrt{5}\right)^2 2^{P-2}\right)$

${}^PC_1, {}^PC_3, .......{}^PC_{P-2}$ are all divisible by P

$\therefore \left[2+\left(\sqrt{5}\right)^P\right] - 2^{P+1}$ is divisible by P

3. $\dfrac{{}^{2n}C_n}{n+1} = \dfrac{((n+1)-n)}{n+1}\,{}^{2n}C_n$

$= {}^{2n}C_n - \dfrac{n}{n+1}\dfrac{(2_n)!}{n!n!}$

${}^{2n}C_n - \dfrac{(2n)!}{(n+1)!(n-1)!}$

$= {}^{2n}C_n - {}^{2n}C_{n-1} = \text{integer}$

4. Cauchy Schwartz $\left(\sum a_i b_i\right) \le \sqrt{\sum a_i^{\,2}}\sqrt{\sum b_i^{\,2}}$

$a_i = 1 \qquad b_i = \sqrt{{}^nC_i}$

$\sum\sqrt{{}^nC_i} < \sqrt{\sum 1}.\sqrt{\sum {}^nC_i}$

$\sum\sqrt{{}^nC_i} < \sqrt{n}\sqrt{2^n - 1}$

By AM GM $\sqrt{n\left(2^n - 1\right)} < \dfrac{n + 2^n - 1}{2}$

$\therefore \sum\sqrt{{}^nC_i} < 2^{n-1} + \dfrac{n-1}{2}$

5. For sum of coefficient put $x = 1$

$\therefore 2^n = 4096 \Rightarrow n = 12$

$\therefore$ greatest coefficient $= {}^{12}C_6 = 924$

6. Same as (4)

7. $\sum r^2\, {}^nC_r = \sum\left(r\left(r^1\right)^n C_r + r\,{}^nC_r\right)$

$= n(n-1)\sum {}^{n-2}C_{r-2} + n\sum {}^{n-1}C_{r-1}$

$= n(n-1)2^{n-2} + n\,2^{n-1}$

$= n^2 2^{n-2} - n\,2^{n-2} + 2n\,2^{n-2}$

$= n(n+1)2^{n-2}$

8. $a_r = {}^nC_r$

$T_r = r\times\frac{{}^nC_r}{{}^nC_{r-1}} = n-r+1$

$\therefore \sum(n-r+1) = (n+1)\sum 1 - \sum r$

$n(n+1) - \frac{n(n+1)}{2} = \frac{n(n+1)}{2}$

9. $(3x-2)^n : T_r\,(-1)^r\,{}^nC_r\,(2)^r\,(3x)^{n-r}$

$(1-x)^{-2} : T_r = (r+1)x^r$

$\therefore$ coefficient of $x^n = \sum(-1)^r\,{}^nC_r\,2^r 3^{n-r},(r+7)$

$= 3^n\sum(-1)^r\left({}^nC_r.r + {}^nC_r\right)\left(\frac{2}{3}\right)^r$

$= 3^n\sum(-1)^r\left(n.{}^{n-1}C_{r-1}\left(\frac{2}{3}\right)^2 + {}^nC_r\left(\frac{2}{3}\right)^r\right)$

$= 3^n\left(-\frac{2}{3}\sum(-1)^{r-1}\,{}^{n-1}C_{r-1}\left(\frac{2}{3}\right)^{r-1} + \sum(-1)^r\,{}^nC_r\left(\frac{2}{3}\right)^r\right)$

$= 3^n\left(\frac{-2n}{3^n} + \frac{1}{3^n}\right) = -2n+1$

10. $(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \ldots\ldots\ldots\ldots$

$= 1 + \frac{2}{6} + \frac{2.5}{6.12} + \ldots\ldots\ldots\ldots\ldots$

$nx = \frac{2}{6} = \frac{1}{3}$ and $\frac{n(n-1)}{2!}x^2 = \frac{2\times 5}{6\times 12} = \frac{5}{36}$

$\therefore (nx)^2\left(1-\frac{1}{n}\right) = \frac{5}{18} \Rightarrow \left(1-\frac{1}{n}\right)\times\frac{1}{9} = \frac{5}{18}$

$\frac{1}{n} = 1 - \frac{5}{2} = -\frac{3}{2} \Rightarrow n = \frac{-2}{3}$

$x = \frac{1}{3n} = \frac{1}{3}\times\frac{-3}{2} = -\frac{1}{2}$

$$\therefore (1+x)^n = \left(1-\frac{1}{2}\right)^{-2/3} = (4)^{1/3}$$

11. $$-1+(n+1)-\frac{n(n^2-1^2)}{2!}$$

$$= -\left(1-(n+1)+\frac{n(n^2-1^2)}{2!}..........\right)$$

$$(1-x)^n = 1-nx+\frac{n(n-1)}{2!}x^2...............$$

$$\therefore x = \frac{n+1}{n}$$

$$= -\left(1-\frac{n+1}{n}\right)^n$$

12. $$1+(1+x)+(1+x^2)+........(1+x)^n$$

$$= \frac{1.\left((1+x)^{n+1}-1\right)}{1+x-1}$$

$$= \frac{(1+x)^{n+1}-1}{x}$$

$\therefore$ Coefficient of $x^3$ will be coefficient of $x^4$ in $(1+x)^{n+1}$

$\therefore {}^{n+1}C_4$

13. $${}^{2n+1}C_0 + {}^{2n+1}C_1 + {}^{2n+1}C_2 + ......{}^{2n+1}C_n + {}^{2n+1}C_{n+1} + ..........{}^{2n+1}C_{2n+1} = 2^{2n+1}$$

$${}^{2n+1}C_0 + {}^{2n+1}C_1 + {}^{2n+1}C_2 + ......{}^{2n+1}C_n + {}^{2n+1}C_n + {}^{2n+1}C_{n+1}.....{}^{2n+1}C_0 = 2^{2n+1}$$

$$\Rightarrow 2\left({}^{2n+1}C_0 + {}^{2n+1}C_1 + {}^{2n+1}C_2 + ......{}^{2n+1}C_n\right) = 2^{2n+1}$$

$$\therefore {}^{2n+1}C_0 + {}^{2n+1}C_1 + {}^{2n+1}C_2 + ......{}^{2n+1}C_n = 2^{2n}$$

14. $${}^nC_0.a - {}^nC_1(a+d) + {}^nC_2(a+2d)..........+(-1)^n\, {}^nC_n(a+nd)$$

$$\sum_{r=0}^{n}(-1)^r\, {}^nC_r(a+rd) = a\sum_{r=0}^{n}(-1)^r\, {}^nC_r + d\sum_{r=0}^{n}(-1)^r\, r\,{}^nC_r$$

$$= d\sum_{r=1}^{n}(-1)^r .n.\, {}^{n-1}C_{r-1}$$

$$= dn\sum_{r=1}^{n}(-1)^r\, {}^{n-1}C_{r-1}$$

$$= 0$$

15. $$(1+x++x^2)^n = \sum_{k=0}^{2n} a_k x^k$$

$$x=1 \Rightarrow 3^n = a_0 + a_1 + a_2 + ......a_n + a_{n+1} + .......a_{2n}$$

$$\Rightarrow 3^n = 2(a_0 + a_1 + a_2 + .......a_n) - a_n$$

$$\Rightarrow a_0 + a_1 + a_2 + ........+ a_n = \frac{a_n + 3^n}{2}$$

16. $\sum\left(r^{n}C_{r}\right)n\,{}^{n}C_{r}-\left(r\,{}^{n}C_{r}\right)^{2}$

$=\sum n^{2}\left({}^{n-1}C_{r-1}\,{}^{n}C_{r}-\left({}^{n-1}C_{r-1}\right)^{2}\right)$

$=\sum n^{2}\,{}^{n-1}C_{r-1}\left({}^{n}C_{r}-{}^{n-1}C_{r-1}\right)$

${}^{n}C_{r}+{}^{n}C_{r-1}={}^{n+1}C_{r+1}\Rightarrow{}^{n}C_{r-1}={}^{n+1}C_{r+1}-{}^{n}C_{r}$

$n^{2}\sum\left({}^{n-1}C_{r-1}\right)\left({}^{n-1}C_{r-2}\right)$

$n^{2}\left({}^{n-1}C_{0}\,{}^{n-1}C_{1}+{}^{n-1}C_{1}{}^{n-1}C_{2}+............\right)$

Let us find ${}^{n}C_{0}{}^{n}C_{r}+{}^{n}C_{1}{}^{n}C_{r+1}+{}^{n}C_{2}{}^{n}C_{r+2}+............$

$(1+x)^{n}={}^{n}C_{0}+{}^{n}C_{1}x+{}^{n}C_{2}x^{2}+{}^{n}C_{3}x^{3}+.............+{}^{n}C_{n}x^{n}$

$\left(1+\frac{1}{x}\right)^{n}={}^{n}C_{0}+\frac{{}^{n}C_{1}}{x}+\frac{{}^{n}C_{2}}{x^{2}}+\frac{{}^{n}C_{3}}{x^{3}}+..........+\frac{{}^{n}C_{n}}{x^{n}}$

$\therefore{}^{n}C_{0}\,{}^{n}C_{r}+{}^{n}C_{r+1}+{}^{n}C_{r}{}^{n}C_{r+2}+............$ is coefficient of $x^{r}$ is the product $(1+x)^{n}\left(1+\frac{1}{x}\right)^{n}=\frac{(1+x)^{2n}}{x^{n}}$

$\therefore$ it is coefficient of $x^{n+r}$ in $(1+x)^{2n}$

$={}^{2n}C_{n+r}$

From (1) $LHS=n^{2}\times{}^{2n-2}C_{n}=n^{2}\times{}^{2n-2}C_{n-2}$

and (2) $=n^{2}\left({}^{2n-1}C_{n}-{}^{2n-2}C_{n-1}\right)$

22. $(x+3)^{n-1}+(n+3)^{n-2}(n+2)+(n+3)^{n-3}(n+2)^{2}+.......+(n+2)^{n-1}$

$$=\frac{(x+3)^{n-1}\left(\left(\frac{x+2}{x+3}\right)^{n}-1\right)}{\left(\frac{x+2}{x+3}\right)-1}=(x+3)^{n}-(x+2)^{n}$$

$\therefore$coefficient of $x^{r}={}^{n}C_{r}\left(3^{n-r}-2^{n-r}\right)$

23. $(1+x)^{100}+x(1+x)^{99}+x^{2}(1+x)^{98}+.....x^{100}$

$$=\frac{(1+x)^{100}\left(\left(\frac{x}{1+x}\right)^{101}-1\right)}{\frac{x}{1+x}-1}=(1+x)^{101}-x^{101}$$

$\therefore$coefficient of $x^{50}={}^{101}C_{50}$

24. $(1+x)^{n}={}^{n}C_{0}+{}^{n}C_{1}x+{}^{n}C_{2}n^{2}+{}^{n}C_{3}n^{3}+..........+{}^{n}C_{n}x^{n}$

$\left(1+\frac{1}{x}\right)^{n}={}^{n}C_{0}+{}^{n}C_{1}\left(\frac{1}{x}\right)+{}^{n}C_{2}\left(\frac{1}{x^{2}}\right)+.........+{}^{n}C_{n}\left(\frac{1}{x^{n}}\right)$

$\therefore{}^{n}C_{0}{}^{n}C_{1}+{}^{n}C_{1}{}^{n}C_{2}+....................+{}^{n}C_{n-1}.{}^{n}C_{n}$ is coefficient of $x$ in the product

$(1+x)^{n}\left(1+\frac{1}{x}\right)^{n}=\frac{(1+x)^{2n}}{x^{n}}$

$$={}^{2n}C_{n+1}$$

$$\therefore \frac{{}^{2n+2}C_{n+2}}{{}^{2n}C_{n+1}}=\frac{15}{4}$$

$$\Rightarrow \frac{(2n+2)!}{(n+2)!n!}\times\frac{(n+1)!(n-1)!}{(2n)!}=\frac{15}{4}$$

$$\Rightarrow \frac{(2n+2)(2n+1)}{(n+2)n}=\frac{15}{4}$$

$$\Rightarrow 16n^2+24n-18=15n^2+30n$$

$$\Rightarrow n^2-6n+8=0$$

$$\Rightarrow (n-1)(n-2)=0$$

$$\Rightarrow n=2 \text{ or } n=4$$

25. $(1+x+x^2)^{2n}=a_0+a_1x+a_2x^2+......+a_{4-n}x^{4n}$

$$x=1\Rightarrow 9^n=a_0+a_1+a_2+a_3+a_n+.........+a_{4n}$$

$$9^n=a_0+a_1+a_2+.......a_{2n-1}+a_{2n}+a_{2n+1}+.....a_{4n}$$

$$9^n+a_{2n}=2(a_0+a_1+a_2+a_3+......a_{2n})$$

$$\frac{1}{2}(9^n+a_{2n})=a_0+a_1+a_2+.......+a_{2n} \qquad \text{\_\_\_\_\_\_\_\_\_\_(1)}$$

$$x=-1\Rightarrow 1=a_0-a_1+a_2-a_3+............+a_{4n}$$

$$1+a_{2n}=2(a_0-a_1+a_2-a_3+......+a_{2n})$$

$$\frac{1}{2}(1+a_{2n})=a_0-a_1+a_2-a_3+......a_{2n} \qquad \text{\_\_\_\_\_\_\_\_\_\_(2)}$$

Add (1) and (2)

$$\frac{1}{2}(9^n+2a_{2n}+1)=2(a_0+a_2+a_4+......+a_{2n})$$

$$\therefore a_0+a_2+a_4+.............+a_{2n}=\frac{1}{4}(9^n+2a_{2n}+1)$$

28. $(1+x)^n={}^nC_0+{}^nC_1x+{}^nC_2x^2+{}^nC_3x^3+........{}^nC_nx^n$

$$\therefore x(1+x)^n={}^nC_0x+{}^nC_1x^2+{}^nC_2x^2+.........+{}^nC_nx^{n+1}$$

Integrating $\int x(1+x)^n\,dx=\frac{{}^nC_0x^2}{2}+\frac{{}^nC_1x^3}{3}+........\frac{{}^nC_nx^{n+2}}{n+2}$

We get given expression by putting $x=1$ in $\int n(1+x)^n.dx$

$$\int x(1+x)^n\,dx=\frac{x(1+x)^{n+1}}{n+1}-\int\frac{(1+x)^{n+1}}{n+1}$$

$$=\frac{2^{n+1}}{n+1}-\frac{(1+x)^{n+2}}{(n+1)(n+2)}+c \qquad c=\frac{1}{(n+1)(n+2)}$$

$$=\frac{2^{n+1}}{n+1}-\frac{2^{n+2}}{(n+1)(n+2)}+\frac{1}{(n+1)(n+2)}$$

$$=\frac{(n+2)(2^{n+1})-2^{n+2}+1}{(n+1)(n+2)}=\frac{n.2^{n+1}+1}{(n+1)(n+2)}$$

32. $\left(1+x+2n^{2}\right)^{20}=a_0+a_1x+a_2x^2+.....+a_{40}x^{40}$

$x=1\Rightarrow 2^{40}=a_0+a_1+a_2+.....+a_{40}x^{40}$

$x=-1\Rightarrow 2^{20}=a_0-a_1+a_2................+a_{40}$

$\therefore 2\left(a_0+a_2+a_4+......+a_{38}+a_{40}\right)=2^{40}+2^{20}$

$a_0+a_2+a_4+..........+a_{38}+a_{40}=2^{39}+2^{19}$

$x=\frac{1}{x}\Rightarrow\left(x^2+x+2\right)^{20}=a_0x^{40}+a_1x^{39}+a_2x^{38}+......+a_{40}$

$x=0\Rightarrow a_{40}=2^{20}$

$\therefore a_0+a_2+a_4+........+a_{38}=2^{39}+2^{19}-2^{20}\qquad =2^{39}-2^{19}$

$\therefore n=39, m=19$

33. $\left(1-x^2\right)^n=1-{}^nC_1+{}^nC_2x^4+.............+\left(-1\right)^n C_nx^{2n}$

$\int\left(1-x^2\right)^n_{dx}=x-\frac{{}^nC_1x^3}{3}+\frac{{}^nC_2x^5}{5}+......\frac{\left(-1\right)^n C_n}{2n+1}x^{2n}$

$\therefore$ we get given expression by putting $x=1$ in $\int\left(1-x\right)^n\ dx$

$x=\sin\theta\Rightarrow dn=\cos\theta d\theta$

$\therefore\int\cos^{2n+1}\theta d\theta$

34. $\frac{1}{{}^{2n+1}C_r}+\frac{1}{{}^{2n+1}C_{r+1}}=\frac{r!\left(2n-r+1\right)!+\left(r+1\right)!\left(2n-r\right)!}{\left(2n+1\right)!}$

$\frac{r!\left(2n-r\right)!\left(2n-r+1+r+1\right)}{\left(2n+1\right)!}$

$=\frac{\left(2n+2\right)}{2n+1}\frac{\left(r!\right)\left(2n-r\right)!}{\left(2n\right)!}$

$=\frac{2n+2}{2n+1}\times\frac{1}{{}^{2n}C_r}$

$S=\frac{1}{{}^{2n}C_1}-\frac{2}{{}^{2n}C_2}+\frac{3}{{}^{2n}C_3}+................+\frac{2n-1}{{}^{2n}C_{2n+1}}$

$S=\frac{2n-1}{{}^{2n}C_1}-\frac{2n-2}{{}^{2n}C_2}+\frac{2n-3}{{}^{2n}C_3}+.......+\frac{1}{{}^{2n}C_{2n-1}}$

Adding, $2S=2n\left(\frac{1}{{}^{2n}C_1}-\frac{1}{{}^{2n}C_2}+\frac{1}{{}^{2n}C_3}+.........+\frac{1}{{}^{2n}C_{2n+1}}\right)$

$S=\frac{n\left(2n+1\right)}{2n+2}\left(\frac{1}{{}^{2n+1}C_1}+\frac{1}{{}^{2n+1}C_2}-\frac{1}{{}^{2n+1}C_2}-\frac{1}{{}^{2n+1}C_3}........+\frac{1}{{}^{2n+1}C_{2n-1}}+\frac{1}{{}^{2n+1}C_{2n}}\right)$

$=\frac{n}{2}\frac{\left(2n+1\right)}{\left(n+1\right)}\left(\frac{1}{2n+1}+\frac{1}{2n+1}\right)$

$=\frac{n}{n+1}$

36. $\sum_{r=0}^{n}\frac{r+1}{r+2}{}^nC_rx^n=\sum_{r=0}^{n}{}^nC_rx^r-\sum_{r=0}^{n}\frac{{}^nC_rx^r}{r+2}$

$$\sum_{r=0}^{n} {}^nC_r x^r = (1+x)^n$$

$$\Rightarrow \sum_{r=0}^{n} {}^nC_r\, x^{r+1} = x(1+x)^n$$

Integrating , $\sum_{r=0}^{n} \frac{{}^nC_r x^{r+2}}{r+2} = \int x(1+x)^n = \frac{x(1+x)^{n+1}}{n+1} - \frac{(1+x)^{n+2}}{(n+1)(n+2)}$

$$= \frac{(1+x)^{n+1}}{n+1}\left(x - \frac{(1+x)}{n+2}\right)$$

$$= \frac{(1+x)^{n+1}}{(n+1)(n+2)}(nx+x-1)$$

$$\therefore \sum_{r=0}^{n} \frac{{}^nC_r x^r}{r+2} = \frac{(1+x)^{n+1}}{(n+1)(n+2)}(nx+x-1)$$

$$\therefore \sum_{r=0}^{n} \frac{r+1}{r+2}\, {}^nC_r x^r = (1+x)^n - \frac{(1+x)^{n+1}}{(n+1)(n+2)}(nx+x-1)$$

38. There are r identical things to be distributed among
(a) n different objects
$\therefore {}^nH_r = {}^{n+r-1}C_{n-1} = {}^{n+r-1}C_r$

(b) $\sum_{k=0}^{n} a^{n-k} \sum_{r=0}^{k} b^r c^{k-r} = \sum_{k=0}^{n} a^{n-k} \left( \frac{\left(\frac{b}{c}\right)^{k+1} - 1}{\frac{b}{c} - 1} \right) c^k$

$$= \sum_{k=0}^{n} a^{n-k} \frac{b^{k+1} - c^{k+1}}{b-c}$$

$$= \frac{1}{b-c}\left(a^n b + a^{n-1}b^2 + \ldots\ldots + a^0 b^{n+1} - \left(a^n c + a^{n-1}c^2 + \ldots\ldots + a^0 c^{n+1}\right)\right)$$

$$= \frac{1}{b-c}\left( \frac{a^n b\left(\left(\frac{b}{a}\right)^{n+1} - 1\right)}{\frac{b}{a} - 1} - \frac{a^n\left(\left(\frac{c}{a}\right)^{n+1} - 1\right)}{\frac{c}{a} - 1} \right)$$

$$= \frac{(a-c)\left(b^{n+2} - a^{n+1}b\right) + \left(c^{n+2} - a^{n+1}c\right)(b-a)}{(b-c)(b-a)(a-c)}$$

$$= \frac{(a-c)b^{n+2} + (b-a)c^{n+2} + (c-b)a^{n+2}}{(a-b)(b-c)(c-a)}$$

40. $\left(1+2n+3x^2+\ldots\ldots+(n+1)x^n\right)\left(1+2n+3n^2+\ldots..+(n+1)x^n\right)$

Coefficient of $x^r = \sum_{t=0}^{r} \text{coefficient of } x^t \times \text{coefficient of } x^r = t$

$$= \sum_{t=0}^{r} (t+1)(r-t+1)$$

$$=(r+2)\sum_{t=0}^{r}(t+1)-\sum_{t=0}^{r}(t+1)^2$$
$$=\frac{(r+2)(r+1)(r+2)}{2}-\frac{(r+1)(r+2)(2r+3)}{6}$$
$$=\frac{1}{6}(r+1)(r+2)(3r+6-2r-3)=\frac{1}{6}(r+1)(r+2)(r+3)$$

41. $72!=36!(83-36)(73-35).........(73-1)$

$=36!(73k+36!)$ ( k is some integer)

$\therefore 72!-(36!)^2=73k^{'}(K'=36!k)$

$\therefore (36!)^2\left({}^{72}C_{36}-1\right)$ is divisible by 73

Since 73 is prime m $(36!)^2$ is not divisible by 73

$\therefore {}^{72}C_{36}-1$ is divisible by 73.

42. $\left(1+x+x^2\right)^n=\sum_{k=0}^{2n}a_kx^k$

$$\left(1+\left(-\frac{1}{x}\right)+\left(-\frac{1}{x}\right)^2\right)^n=\sum_{k=0}^{2n}(-1)^k a_k\left(\frac{1}{x}\right)^k$$
$$\therefore \frac{\left(x^2-x+1\right)}{x^{2n}}=\sum_{k=0}^{2n}(-1)^k a_k\left(\frac{1}{x}\right)^k$$

$\therefore \sum_{k=0}^{2n-1}a_ka_{k+1}(-1)^k$ = coefficient of $x$ in

$$\frac{\left(x^2-x+1\right)^n\left(x^2+x+1\right)^n}{x^{2n}}$$

$\therefore \sum_{k=0}^{2n-1}a_k\,a_{k+1}(-1)^k=$ coefficient of $x^{2n+1}$in $\left(x^4+x^2+1\right)^n$

$\therefore$Coefficient of $x^{2n+1}$ in $\left(1-x^6\right)^n\left(1-x^2\right)^{-n}$

All the terms in $\left(1-x^6\right)^n\left(1-x^2\right)^{-n}$ are even $\therefore$coefficient of $x^{2n+1}=0$

43. $\sum_{r=0}^{n}(-1)^r\frac{{}^nC_r}{{}^{r+3}C_r}=\sum_{r=0}^{n}(-1)^r\frac{n!}{r!(n-r)!}\times\frac{r!3!}{(r+3)!}$

$$=\frac{3!}{(n+1)(n+2)(n+3)}\sum_{r=0}^{n}{}^{n+3}C_{r+3}(-1)^r$$
$$=-\frac{3!}{(n+1)(n+2)(n+3)}\sum_{r=0}^{n}(-1)^{r+3}\,{}^{n+3}C_{r+3}$$
$$=\frac{-3!}{(n+1)(n+2)(n+3)}\left(-(1-x)^{n+3}-1+{}^{n+3}C_1x-{}^{n+3}C_2x^2\right)$$

Put $x=1=\frac{-3!}{(n+1)(n+2)(n+3)}\left(-1+n+3-\frac{(n+3)(n+2)}{2}\right)$

$$=\frac{-3!}{(n+1)(n+2)(n+3)}\left((n+2)-\frac{(n+3)(n+2)}{2}\right)$$

$$= -\frac{3!}{(n+1)(n+3)}\left(\frac{2-n-3}{2}\right) = \frac{3!}{(2n+6)}$$

44. $U_n = \left(\sqrt{3}+1\right)^{2n} + \left(\sqrt{3}-1\right)^{2n}$

$$8U_n = 4U_{n-1} = 8\left(\sqrt{3}+1\right)^{2n} + 8\left(\sqrt{3}-1\right)^{2n} - 4\left(\sqrt{3}+1\right)^{2n-2} - 4\left(\sqrt{3}-1\right)^{2n}$$

$$= 4\left(\sqrt{3}+1\right)^{2n-2}\left(2\left(\sqrt{3}+1\right)^2 - 1\right) + 4\left(\sqrt{3}-1\right)^{2n-2}\left(2\left(\sqrt{3}-1\right)^2 - 1\right)$$

$$= 4\left(\sqrt{3}+1\right)^{2n-2}\left(7+4\sqrt{3}\right) + 4\left(\sqrt{3}-1\right)^{2n-2}\left(7-4\sqrt{3}\right)$$

$$= 4\left[\left(\sqrt{3}+1\right)^{2n-2}\left(2+\sqrt{3}\right)^2 + \left(\sqrt{3}-1\right)^{2n-2}\left(2-\sqrt{3}\right)^2\right]$$

$$= \left(\sqrt{3}+1\right)^{2n-2}\left(4+2\sqrt{3}\right)^2 + \left(\sqrt{3}-1\right)^{2n-2}\left(4-2\sqrt{3}\right)^2$$

$$= \left(\sqrt{3}+1\right)^{2n-2}\left(\sqrt{3}+1\right)^4 + \left(\sqrt{3}-1\right)^{2n-2}\left(\sqrt{3}-1\right)^4$$

$$= \left(\sqrt{3}+1\right)^{2n+2} + \left(\sqrt{3}-1\right)^{2n+2}$$

$$= U_{n+1}$$

48. $(1+x)^n = \sum {}^nC_r x^r$

$$\left(1-\frac{1}{x}\right)^{2n} = \sum (-1)^r \, {}^{2n}C_r \left(\frac{1}{x}\right)^r$$

$\therefore$ give expression is coefficient of $\frac{1}{x^n}$ in $(1+x)^n\left(1-\frac{1}{x}\right)^{2n}$

Coefficient of $\frac{1}{x^n}$ in $\frac{\left(1-x^2\right)^n(1-x)^n}{x^{2n}}$

Coefficient of $x^n$ in $\left(1-x^2\right)^n(1-x)^n$

49. We need coefficient of $x^n$ in

$${}^nC_0\left[(1+x)^2\right]^n - {}^nC_1\left[(1+x)^2\right]^{n-1} + {}^nC_2\left[(1+x)^2\right]^{n-2} \ldots.$$

$\therefore$ Coefficient of $x^n$ in $\left[(1+x)^2 - 1\right]^n$

$= x^n(x+2)^n$

$\therefore$ Coefficient of $x^n$ is constant term in $(x+2)^n = 2^n$

50. $(1-x)^n = \sum (-1)^r C_r x^r$

$$\left(1+x+x^2\right)^n = \sum a_i x^i$$

$$\frac{\left(x^2+x+1\right)^n}{x^{2n}} = \sum a_i\left(\frac{1}{x}\right)^i$$

$\therefore \sum (-1)^r (a_i)(c_i)$ is constant term in $\frac{(1-x)^n\left(x^2+x+1\right)^n}{x^{2n}}$

$\therefore$ Coefficient of $x^{2n}$ in $\left(1-x^3\right)^n$

$$\left(1-x^3\right)^n=\sum(-1)^r\,{}^nC_r\left(x^3\right)^r$$

$$\therefore 3r=2n\Rightarrow r=\frac{2n}{3}$$

$$\therefore \sum(-1)^r(a_i)(c_i)=(-1)^{2n/3}\,{}^nC_{2n/3}=(-1)^{2n/3}\,{}^nC_{n/3}$$

19. (a) $(1-x)^n=\sum(-1)^r\,{}^nC_rx^r\Rightarrow\frac{(1-x)^n}{x}=\sum(-1)^r\,{}^nC_rx^{r-1}$

$$\therefore\int_0^1\frac{(1-x)^n}{x}dx=\int_0^1\frac{1}{x}+\sum_{i=1}^{n}(-1)^r\frac{{}^nC_r}{r}$$

$$\Rightarrow\frac{{}^nC_1}{1}-\frac{{}^nC_2}{2}+\frac{{}^nC_3}{3}-.............-(-1)^n\frac{{}^nC_n}{n}=\int_0^1\frac{1-(1-x)^n}{x}xdx$$

$$=\int_0^1\left\{1+(1-x)+(1-x)^2+......(1-x)^{n-1}\right\}dx$$

$$=x+\frac{(1-x)^2}{-2}+\frac{(1-x)^3}{-3}+........\frac{(1-x)^n}{n}\Bigg|_{x=0}^{1}$$

$$=1+\frac{1}{2}+\frac{1}{3}+......+\frac{1}{n}$$

(b) $(1-x)^n={}^nC_0-{}^nC_1x+{}^nC_2x^2.............+(-1)^n\,{}^nC_nx^n$

Integrating, $\frac{(1-x)^{n+1}}{-(n+1)}={}^nC_0x-\frac{{}^nC_1x^2}{2}+\frac{{}^nC_2x^3}{3}+......+\frac{(-1)^n\,{}^nC_nx^{n+1}}{n+1}+k$

Put $x=0$ $k=-\frac{1}{n+1}$

Dividing by x, $\frac{(1-x)^{n+1}}{-(n+1)x}={}^nC_0-{}^nC_1\frac{x}{2}+\frac{{}^nC_2}{3}x^3+......+\frac{(-1)^n\,{}^nC_nx^{n+1}}{n+1}-\frac{1}{(n+1)}$

$$\Rightarrow\frac{1}{n+1}\left(\frac{1-(1-x)^{n+1}}{x}\right)=\sum(-1)^r\,{}^nC_r\frac{x^r}{r+1}$$

$$\Rightarrow\frac{1}{n+1}\left(1+(1-x)+(1-x)^2+.......+(1-x)^n\right)=\sum(-1)^2\,{}^nC_1\frac{x^r}{r+1}$$

$$\Rightarrow\frac{1}{n+1}\int_0^1\left[1+(1-x)+(1-x)^2+.....+(1-x)^n\right]=\sum(-1)^r\frac{{}^nC_r}{(r+1)^2}$$

$$=\frac{1}{n+1}\left(x+\frac{(1-x)^2}{-2}+\frac{(1-x)^3}{-3}+........\frac{(1-x)^{n+1}}{-(n+1)}\right)_{n=0}^{1}$$

$$=\frac{1}{n+1}\left(1+\frac{1}{2}+\frac{1}{3}+.....+\frac{1}{n+1}\right)$$

20. $(1-x)^n={}^nC_0-{}^nC_1x+{}^nC_2x^2+...........+(-1)^n\,{}^nC_nx^n$

$x(1-x)^n={}^nC_0x-{}^nC_1x^2+{}^nC_2x^3+.........+(-1)^n\,{}^nC_nx^{n+1}$

Differentiating, $(1-x)^n-nx(1-x)^{n-1}={}^nC_0-2^nC_1x+{}^{3n}C_2x^2+.........+(-1)^n(n+1)^nC_nx^n$

$x(1-x)^n-nx^2(1-x)^{n-1}={}^nC_0x-{}^{2n}C_1x^2+3^nC_2x^3+...........+(-1)^n(n+1)^nC_nx^n$

Differentiating, $(1-x)^n - nx(1-x)^{n-1} - 2nx(1-x)^{n-1} + nx^2(n-1)(1-x)^{n-2}$

$= {}^nC_0 - 2^2\,{}^nC_1x + 3^2\,{}^nC_2x^2 + ..... + (-1)^n(n+1)^2\,{}^nC_nx^n$

Put $x = 1, {}^nC_0 - 2^2\,{}^nC_1 + 3^2\,{}^nC_2 + ..... + (-1)^n(n+1)^2\,{}^nC_n = 0$

21. $\sum_{r=0}^{n}(-1)^r\,{}^nC_r\left(\frac{1}{1+\log 10}\right)^r + \sum_{r=0}^{n}(-1)^r \times \frac{r}{n}\log 10\,{}^nC_r\left(\frac{1}{1+\log 10}\right)^r$

$= \left(1-\frac{1}{1+\log 10}\right)^n + \sum_{r=0}^{n}(-1)^r \log 10.\,{}^{n+1}C_{r-1}\left(\frac{1}{1+\log 10}\right)^r$

$= \left(\frac{\log 10}{1+\log 10}\right)^n - \frac{\log 10}{1+\log 10}\sum_{r=0}^{n}(-1)^{r-1}\,{}^{n-1}C_{r-1}\left(\frac{1}{1+\log 10}\right)^{r-1}$

$= \left(\frac{\log 10}{1+\log 10}\right)^n - \frac{\log 10}{1+\log 10}\left(1-\frac{1}{1+\log 10}\right)$

$= 0$

26. $\left(1+\frac{1}{x}\right)^n - \left(1-\frac{1}{x}\right)^n = 2\left(\frac{{}^nC_1}{x} + \frac{{}^nC_3}{x^3} + \frac{{}^nC_5}{x^5} + .....\right)$

$(x+1)^n - (x-1)^n = 2\left({}^nC_1x^{n-1} + {}^nC_3x^{n-3} + {}^nC_5x^{n-5} + ......\right)$

Differentiating $n\left[(x+1)^{n-1} - (x-1)^{n-1}\right] = 2\left((n-1)^n C_1x^{n-2} + (n-3)^n C_3x^{n-4} + ........\right)$

$nx\left[(x+1)^{n-1} - (x-1)^{n-1}\right] = 2\left((n-1)\,{}^nC_1x^{n-1} + (n-3)\,{}^nC_3x^{n-3} + .......\right)$

Differentiating $n\left[\left\{(x+1)^{n-1} - (x-1)^{n-1}\right\} + x(n-1)\left\{(x+1)^{n-2} - (x-1)^{n-2}\right\}\right.$

$= 2\left((n-1)^2\,{}^nC_1x^{n-2} + (n-3)^2\,{}^nC_3x^{n-4} + ........\right)$

Put $x = 1, n\left[2^{n-1} + (n-1)2^{n-2}\right] = 2\left((n-1)^2\,{}^nC_1 + (n-3)^2\,{}^nC_3 + .......\right)$

$n\left[2^{n-1} - 2^{n-2} + n.2^{n-2}\right] = 2\left((n-1)^2\,{}^nC_1 + (n-3)^2\,{}^nC_3 + ........\right)$

$n\,2^{n-2}(n+1) = 2\left((n-1)^2\,{}^nC_1 + (n-3)^2\,{}^nC_3 + .......\right)$

$\therefore (n-1)^2\,{}^nC_1 + (n-3)^2\,{}^nC_3 + ...... = 2^{n-3}n(n+1)$

29. $\sum_{r=0}^{n}(-1)^r.\,{}^nC_r\left(\frac{1}{2}\right)^r + \sum_{r=0}^{n}(-1)^r\,{}^nC_r\left(\frac{3}{4}\right)^r + \sum_{r=0}^{n}(-1)^r\,{}^nC_r\left(\frac{7}{8}\right)^r + .......$

$= \left(1-\frac{1}{2}\right)^n + \left(1-\frac{3}{4}\right)^n + \left(1-\frac{7}{8}\right)^n + ........$

$= \left(\frac{1}{2}\right)^n + \left(\frac{1}{4}\right)^n + \left(\frac{1}{8}\right)^n + ..........$

$= \frac{\left(\frac{1}{2}\right)^n}{1-\left(\frac{1}{2}\right)^n}$

$= \frac{1}{2^n - 1}$

30. Let 3 consecutive coefficients be ${}^nC_{r-2}, {}^nC_r, {}^nC_{r+1}$

If they are in HP, ${}^nC_r = \dfrac{2\times {}^nC_{r-1}\times {}^nC_{r+1}}{{}^nC_{r-1}+{}^nC_{r+1}}$

$$\therefore \frac{n!}{r!(n-r)!} = \frac{2}{\dfrac{1}{{}^nC_{r+1}+\dfrac{1}{{}^nC_{r-1}}}} = \frac{2\times n!}{(r+1)!(n-r-1)!+(r-1)!(n-r+1)!}$$

$$\frac{1}{r!(n-r)!} = \frac{2}{r!(n-r)!}\left(\frac{1}{\dfrac{r+1}{n-r}+\dfrac{n-r+1}{r}}\right)$$

$$\frac{r+1}{n-r}+\frac{n-r+1}{r} = 2$$

$$r^2+r+n^2-nr+n-nr+r^2-r = 2nr-2r^2$$

$$4r^2-4nr+n^2+n=0$$

$(2r-n)^2+n=0$, which is not possible

31. $2^{2n} = {}^{2n}C_0 + {}^{2n}C_1 + {}^{2n}C_2 + \ldots\ldots\ldots\ldots + {}^{2n}C_{2n}$

Now ${}^{2n}C_i < {}^{2n}C_n \forall n \in N$ and ${}^{2n}C_i = {}^{2n}C_n$ for $i = n$

$$\therefore 2^{2n} < {}^{2n}C_n + {}^{2n}C_n + {}^{2n}C_n + \ldots\ldots\ldots\ldots + {}^{2n}C_n$$

$$2^{2n} < (2n+1)\,{}^{2n}C_n$$

$$\frac{4^n}{2n+1} < \frac{(2n)!}{n!n!}$$

27. $\displaystyle\sum_{0\le i}\sum_{<j\le n}\left({}^nC_i - {}^nC_j\right)^2 = (C_0-C_1)^2+(C_0-C_2)^2+(C_0-C_3)^2+\ldots\ldots.+(C_0-C_n)^2+(C_1-C_2)^2+\ldots\ldots..$

$$+(C_1-C_n)^2+\ldots\ldots.+(C_{n-1}C_n)^2$$

$$= n\left(C_0^2+C_1^2+C_2^2+\ldots\ldots C_n\right)^2 - 2p \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(1)$$

Now, $(C_0+C_1+C_2+\ldots\ldots+C_n)^2 = \sum C_r{}^2 + 2p$

$$\Rightarrow 2p = 2^{2n} - {}^{2n}C_n$$

$\therefore$ from (1)

$$\sum_{0\le i}\sum_{<j\le n}\left({}^nC_i - {}^nC_j\right)^2 = n\left({}^{2n}C_n\right) - \left(2^{2n} - {}^{2n}C_n\right)$$

$$= (n+1)\,{}^{2n}C_n - 2^{2n}$$

35. $(1+x)^n = 1 - \dfrac{1}{5} + \dfrac{1}{5}.\dfrac{4}{10} - \dfrac{1.4.7}{5.10.15}$..................as

$nx = -\dfrac{1}{5}$ and $\dfrac{n(n-1)}{2}x^2 = \dfrac{4}{50}$

$$\frac{\left(-\dfrac{1}{5}\right)\left(-\dfrac{1}{5}-x\right)}{2} = \frac{2}{25}$$

$$1+5x = 4 \Rightarrow x = \frac{3}{5}$$

$$n\times\frac{3}{5}=-\frac{1}{5}\Rightarrow n=\frac{-1}{3}$$

$$\therefore\left(1+\frac{3}{5}\right)^{-1/3}=1-\frac{1}{5}+5$$

$$\Rightarrow S=\sqrt[3]{\frac{5}{8}}-\frac{4}{5}$$

$$\frac{1}{{}^{2n}C_1}-\frac{2}{{}^{2n}C_2}+\frac{3}{{}^{2n}C_3}............+\frac{(2n-1)}{{}^{2n}C_{2n-1}}$$

$$\frac{1}{{}^{2n}C_{2n-1}}-\frac{2}{{}^{2n}C_{2n-2}}.......................+\frac{2n-1}{{}^{2n}C_1}$$

$$2S=2n\left[\frac{1}{{}^{2n}C_1}-\frac{1}{{}^{2n}C_2}+\frac{1}{{}^{2n}C_3}.....+\frac{1}{{}^{2n}C_{2n-1}}\right]$$

$$S=n^k\left[\frac{1}{2^{n+1}C_1}+\frac{1}{{}^{2n+1}C_2}-\frac{1}{{}^{2n+1}C_2}-\frac{1}{{}^{2n+1}C_3}\right]$$

$$S=n \qquad \frac{(n(n-1)(n-2))}{3\times25}\times x^3$$

$$\frac{1}{5}.\frac{4}{10}.\frac{(n-2).x}{3}$$

$$\frac{1}{5}.\frac{4}{10}\times\frac{1}{3}\times\left[-1-2x-\frac{21}{25}\right]$$

$$\frac{1}{5}.\frac{4}{10}\times\frac{1}{3}\times\frac{17}{25}$$

$$1-1+\frac{1}{5}.\frac{4}{10}$$

$$nx=1 \qquad \frac{n(n-1)}{2}\times x^2=\frac{1}{5}.\frac{4}{10}$$

$$x=\frac{-21}{25}$$

$$\frac{n-1}{2n}=\frac{1}{5}.\frac{4}{10}$$

$$x=\frac{25}{21} \qquad 25n-25=4n$$

$$21n=25$$

$$\frac{(n-2)(n-1)!(n+1)(n+2)}{2!3!}$$

$${}^{n}C_3$$

$$(n+2)C_3 \qquad \frac{(n+2)(n+1)n}{3!}\times\frac{(n-r)(n-r+1)......n(n+1)(n+2).....(n+r)}{r!(r+1)!}$$

$${}^{n+r}C_r$$

$$\frac{n!\times(n+r)!}{r!(r)!(r+1)!}$$

$${}^{n}C_{n-(r+1)},{}^{n+r}C_r.n!$$

$${}^{n}C_{r+1}.{}^{n+r}C_r.n!$$