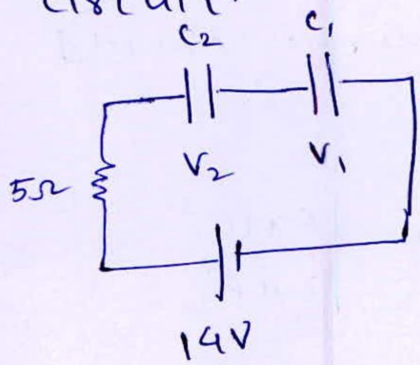
1] ②

In steady state, no current will flow through the circuit.



$\therefore V_1 + V_2 = 14$

$\therefore \frac{Q}{C_1} + \frac{Q}{C_2} = 14$

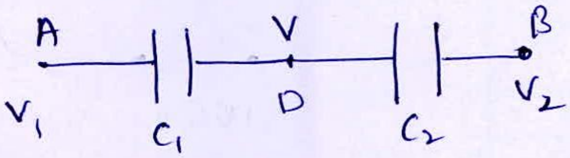
$\therefore Q\left(\frac{1}{5} + \frac{1}{2}\right) = 14$

$\therefore Q = \frac{14 \times 10}{7}$

$\therefore Q = 20\ \mu C$

$\therefore C_2 = \frac{Q}{V_2} \Rightarrow V_2 = \frac{Q}{C_2} = \frac{20}{2} = 10V$

2] ③



$V_1 - V = \frac{Q}{C_1}$ — ①

$V - V_2 = \frac{Q}{C_2}$ — ②

charge on both capacitors will be same as they are in series.

$\therefore Q = C_2(V - V_2)$ ... (from ②)

Putting this in ①,

$V_1 - V = \frac{C_2(V - V_2)}{C_1}$

$\therefore C_1V_1 - C_1V = C_2V - C_2V_2$

$\therefore C_1V_1 + C_2V_2 = (C_1 + C_2)V \Rightarrow V = \frac{C_1V_1 + C_2V_2}{C_1 + C_2}$

3] (4)

$Q_1 = C_1 V_1 = 1 \times 6 = 6 \times 10^{-3}\,C$

$Q_2 = C_2 V_2 = 2 \times 4 = 8 \times 10^{-3}\,C$

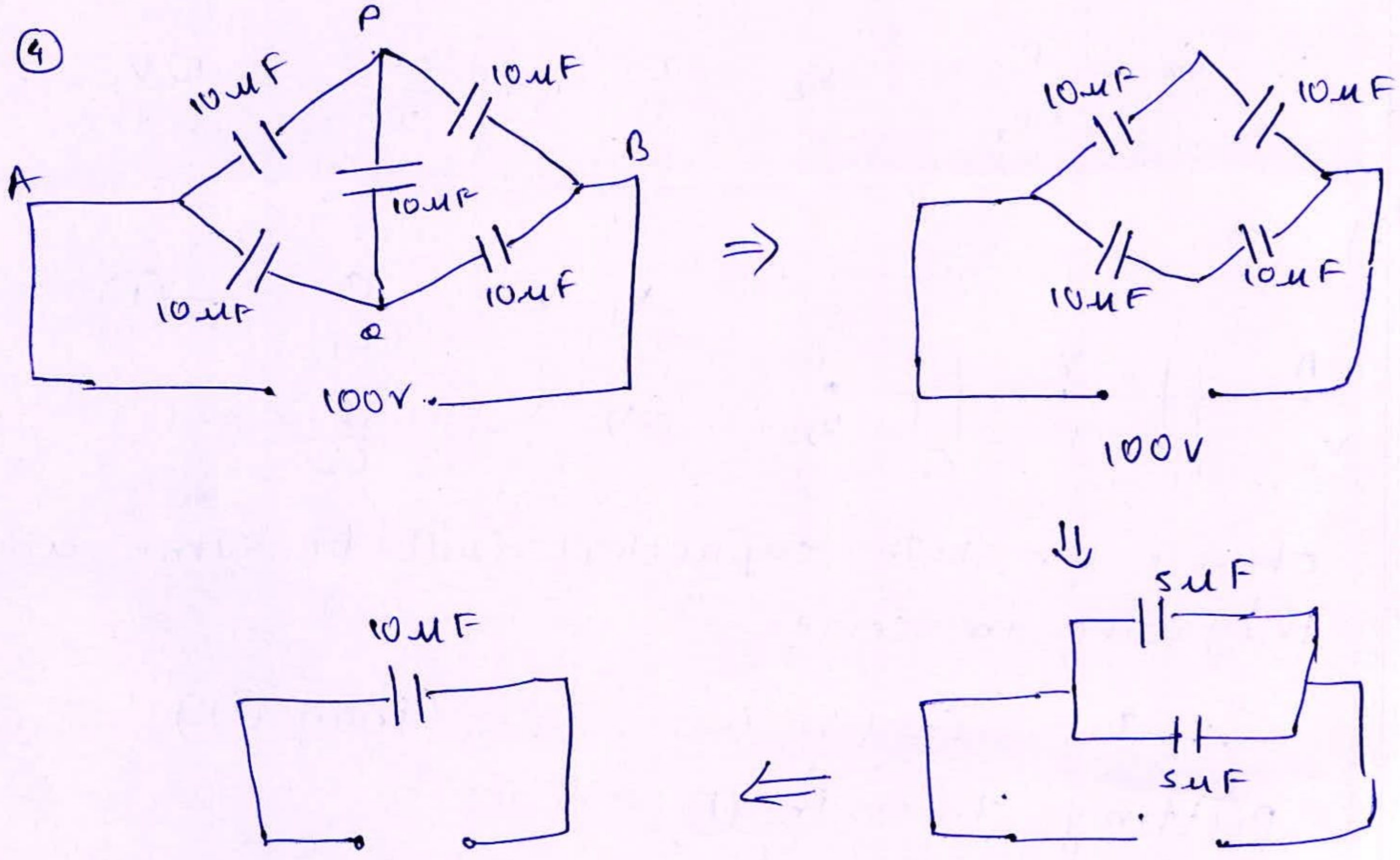
$\therefore$ When connected in series charge on both capacitors will be same, so the charge should not exceed $6 \times 10^{-3}\,C$.

$$\therefore V_1 + V_2 = \frac{Q_1}{C_1} + \frac{Q}{C_2} = \frac{6 \times 10^{-3}}{1\,\mu F} + \frac{6 \times 10^{-3}}{2\,\mu F}$$

$$= (6 + 3)\,kV$$

$$= 9\,kV$$

4] (4)



5] 3

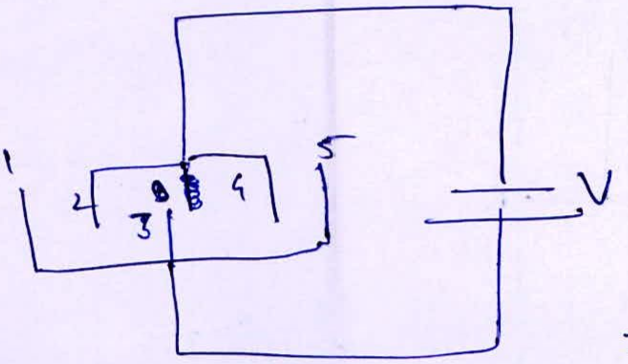
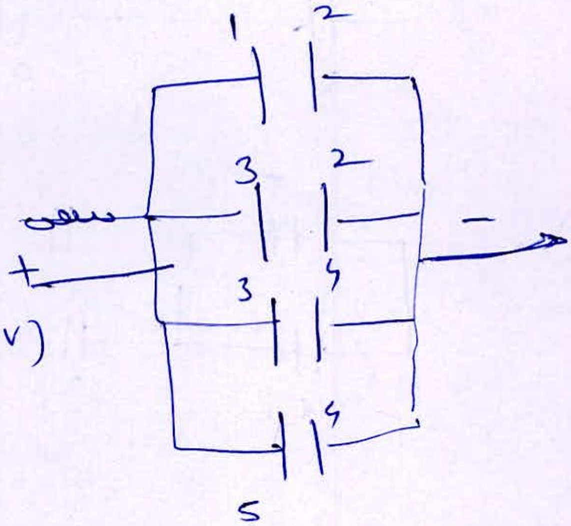


Plate 3, 1 and 5 will be at potential V and 2 and 4 will be at the potential of 0.



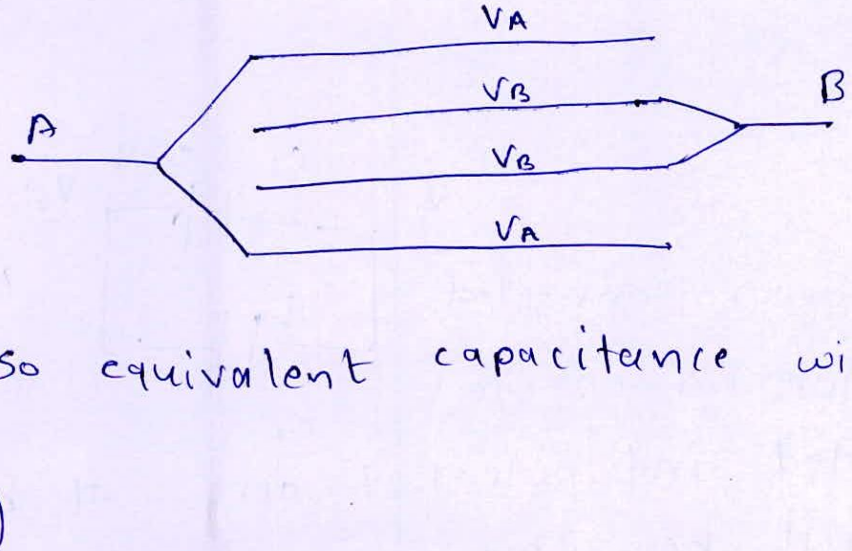
charge on 1 = $+CV$

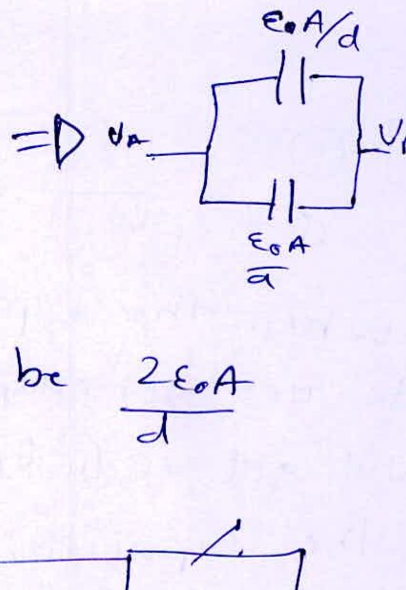
charge on plate 4 = $-CV + (-CV)$

$= -2CV$

6

6] 2

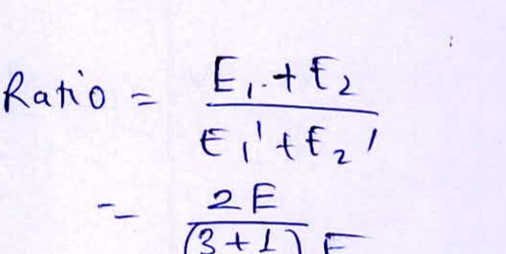




So equivalent capacitance will be $\frac{2\varepsilon_0 A}{d}$

7] 3

$E_1 = E_2 = \frac{Q^2}{2C} = \frac{1}{2}CV^2 = E$



when dielectric is inserted

$E_1' = \frac{1}{2}CKV^2 = KE_1$

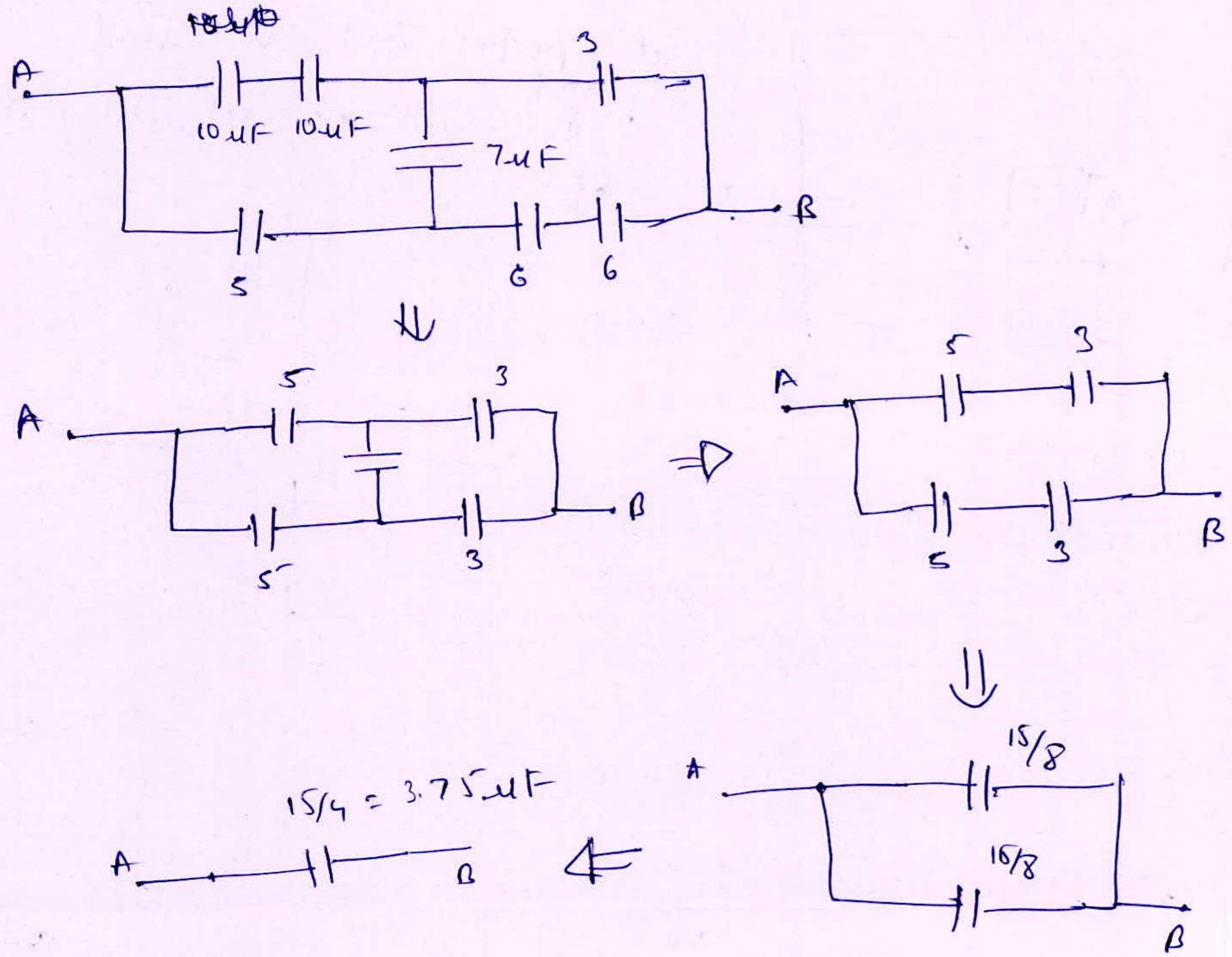
$E_2' = \frac{Q^2}{2KC} = \frac{1}{K}E_2$

$\therefore$ Ratio $= \frac{E_1 + E_2}{E_1' + E_2'}$

$= \frac{2E}{(3+\frac{1}{3})E}$

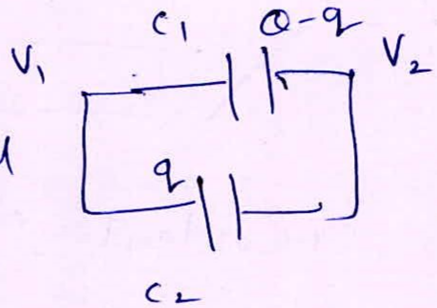
$\therefore$ Ratio $= \frac{6}{10} = \frac{3}{5}$

8] (1)



10] (1)

$Q = C_1 V$



When the capacitor is connected to another capacitor, the charge will get redistributed and potential across both the capacitors will be same.

$$\therefore \frac{q}{C_2} = \frac{Q-q}{C_1} \Rightarrow qC_1 = QC_2 - qC_2$$

$$\therefore q(C_1+C_2) = QC_2 \Rightarrow q = \frac{QC_2}{C_1+C_2}$$

$$\therefore V_2 - V_1 = \frac{q}{C_2} = \frac{\frac{QC_2}{C_1+C_2}}{C_2} = \frac{Q}{C_1+C_2} = \frac{C_1 V}{C_1+C_2}$$

11] (1)

$$U = \frac{1}{2} C_1 V^2$$

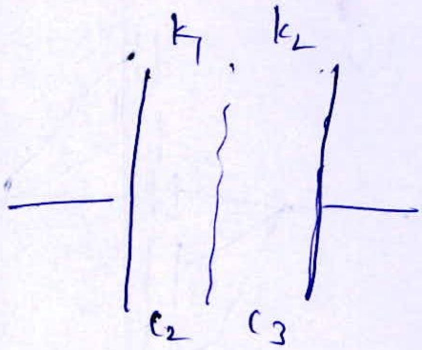
$$V_2 = \frac{1}{2} C_1 (V_2 - V_1)^2 + \frac{1}{2} C_2 (V_2 - V_1)^2$$

$$= \frac{1}{2} (C_1 + C_2)(V_2 - V_1)^2$$

$$= \frac{1}{2} (C_1 + C_2) \frac{C_1^2 V^2}{(C_1 + C_2)^2} = \frac{1}{2} \frac{C_1^2 V^2}{C_1 + C_2} = \frac{1}{2} C_1 V^2 \times \frac{C_1}{C_1 + C_2}$$

$$\therefore V_2 = \frac{U C_1}{C_1 + C_2}$$

12] (2)



$$C_2 = k_1 \frac{\varepsilon_0 A}{d/2} = \frac{2 k_1 \varepsilon_0 A}{d}$$

$$C_3 = k_2 \frac{\varepsilon_0 A}{d/2} = \frac{2 k_2 \varepsilon_0 A}{d}$$

$$\therefore C_{eq} = \frac{C_2 C_3}{C_2 + C_3} = \frac{\left(\frac{\varepsilon_0 A}{d}\right)^2 4 k_1 k_2}{\left(\frac{\varepsilon_0 A}{d}\right)(2k_1 + 2k_2)}$$

$$\therefore C_{eq} = \frac{2 k_1 k_2}{k_1 + k_2} \frac{\varepsilon_0 A}{d}$$

$$C = \frac{\varepsilon_0 A}{d}$$

$$\therefore C_{eq} = \left(\frac{2 k_1 k_2}{k_1 + k_2}\right) C$$

14] (2)

As the capacitors are in series, the charges on both capacitors are same

$$q = CV \Rightarrow V = \frac{q}{C_1}$$

$\therefore \Delta V \propto \frac{1}{C_1}$ As the change in potential across C_1 is greater, $C_1 < C_2$

16] (4)

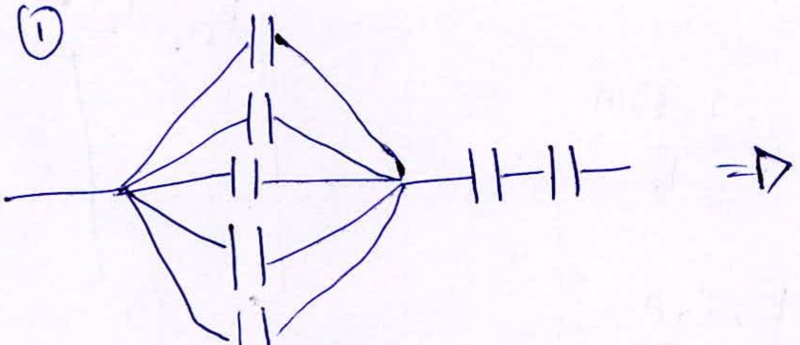
As the capacitors are in series, the charges on the capacitors will be same.

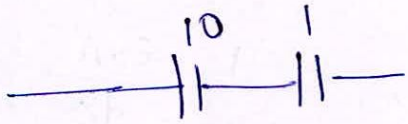
17] (1)

The charge on +2C point will become zero.

∴ The force will be zero.

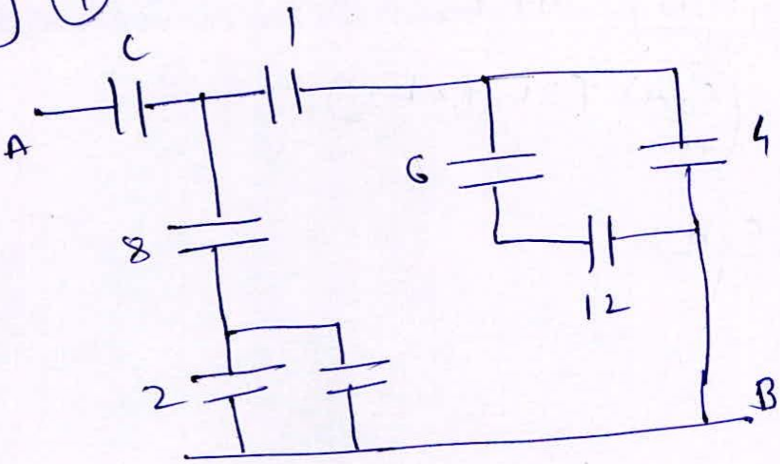
18] (1)

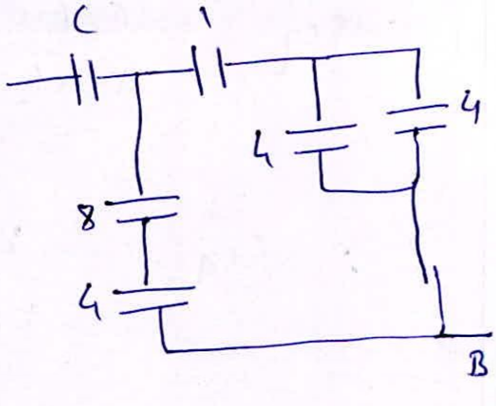


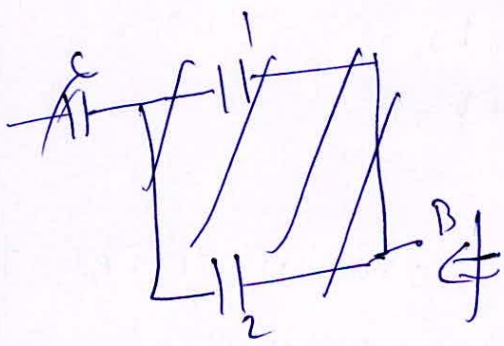


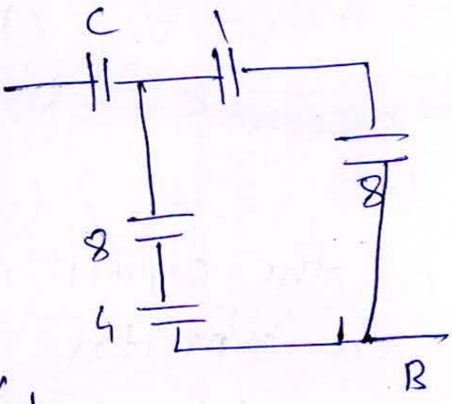
$\therefore C_{eq} = \frac{10 \times 1}{11} = \frac{10}{11}\ \mu F$

19] (1)



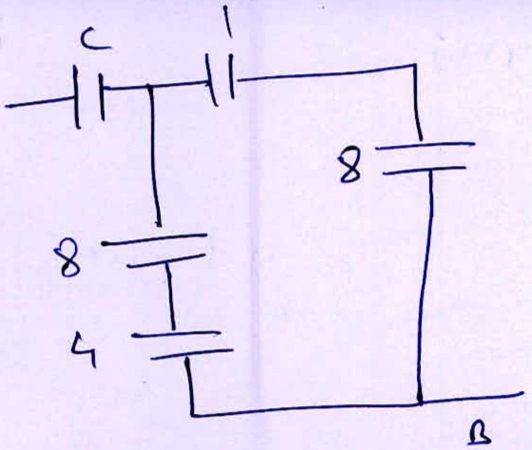




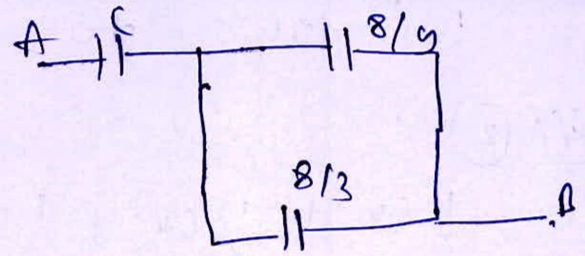


Now, $\frac{C \times \frac{2}{3}}{C + \frac{2}{3}} = 1 \Rightarrow \frac{2C}{3C+2} = 1$

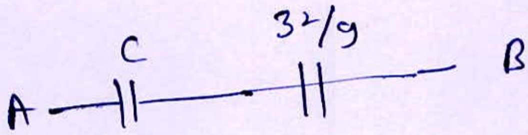
$\therefore 2C = 3C + 2$



$\Rightarrow$



$\Downarrow$



$$\therefore \frac{C \times \frac{32}{9}}{C + \frac{32}{9}} = 1$$

$$\therefore \frac{32C}{32 + 9C} = 1 \Rightarrow 32C = 32 + 9C$$

$$\therefore 23C = 32$$

$$\boxed{\therefore C = \frac{32}{23} \mu F}$$

20] 1

$$C_1 = \frac{\varepsilon_0 A}{d}$$

$$C_2 = \frac{K_1 \varepsilon_0 A/2}{d} + \frac{K_2 \varepsilon_0 A/2}{d}$$

($\because$ the capacitors will be in series)

$$\therefore C_2 = \frac{K_1 + K_2}{2} \left(\frac{\varepsilon_0 A}{d} \right)$$

$$\therefore C_2 = \frac{K_1 + K_2}{2} C_1$$

Windows to JEE main

1) (2)

$E = \frac{1}{2}CV^2 + \frac{1}{2}CV^2 + \frac{1}{2}CV^2 + \ldots$ n times

$\therefore E = \frac{1}{2}nCV^2$

2) (1)

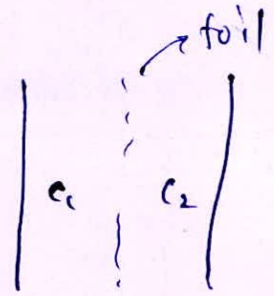
$E = 4\pi\varepsilon_0 R$

$\therefore E = \frac{R}{K} = \frac{1}{9\times10^9} = 1.1\times10^{-10}$

3) (1)

when aluminium foil is inserted, the two capacitances will be in series

$\therefore$ The net capacitance will decrease



4) (4)

work done will be equal to the energy stored

$\therefore W = \frac{1}{2}CV^2$

$= \frac{1}{2}\frac{Q^2}{C}$

$= \frac{64\times10^{-36}}{2\times10^{-4}} = 32\times10^{-32}\,J$

5) (1)

For n plates, number of capacitors formed will be n-1

$\therefore C_{eq} = (n-1)C$

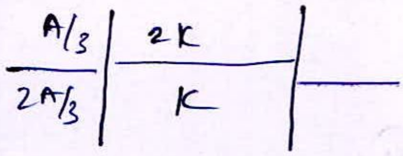
10] 3

$$q = CV$$

$$\frac{dq}{dt} = \frac{C\,dV}{dt} \Rightarrow 2 = 10^{-6} \times \frac{dV}{dt}$$

$$\therefore \frac{dV}{dt} = 2 \times 10^{6} \text{ V/s.}$$

11] 4



The capacitors will be in parallel

$$C_0 = \frac{K\varepsilon_0 A}{d}$$

$$C = \frac{2K\varepsilon_0 A/3}{d} + \frac{K\varepsilon_0\, 2A/3}{d}$$

$$= \frac{2}{3}\frac{K\varepsilon_0 A}{d} + \frac{2}{3}\frac{K\varepsilon_0 A}{d}$$

$$= \frac{4}{3}\frac{K\varepsilon_0 A}{d}$$

12]

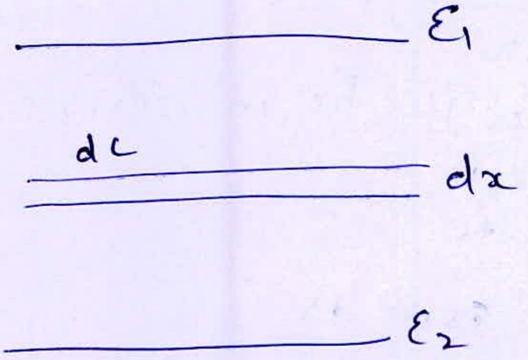
Potential on both can be zero, therefore charge on both capacitors should be equal.

So, $C_1 V_1 = C_2 V_2$

$\therefore C_1 \times 120 = C_2 \times 200$

$\therefore 3C_1 = 5C_2$

13]



ε at any x will be given by

$$\varepsilon_x = \varepsilon_1 + \left(\frac{\varepsilon_2 - \varepsilon_1}{d}\right)x$$

Capacitance for a small element will be given by

$$dC = \frac{\varepsilon_x A}{d} = \frac{\left(\varepsilon_1 + \left(\frac{\varepsilon_2 - \varepsilon_1}{d}\right)x\right)A}{dx}$$

All the capacitances will be in series.

$$\therefore \frac{1}{C_{eq}} = \int \frac{1}{dC}$$

$$= \int_0^d \frac{dx}{\left(\varepsilon_1 + \left(\frac{\varepsilon_2 - \varepsilon_1}{d}\right)x\right)A}$$

$$= \frac{d}{(\varepsilon_2 - \varepsilon_1)A} \ln\left(\varepsilon_1 + \left(\frac{\varepsilon_2 - \varepsilon_1}{d}\right)x\right)\Big|_0^d$$

$$= \frac{d}{(\varepsilon_2 - \varepsilon_1)A}\left[\ln(\varepsilon_2) - \ln(\varepsilon_1)\right]$$

$$\frac{1}{C_{eq}} = \frac{d}{(\varepsilon_2 - \varepsilon_1)A} \ln(\varepsilon_2/\varepsilon_1)$$

$$\therefore C_{eq} = \frac{(\varepsilon_2 - \varepsilon_1)A}{d \ln(\varepsilon_2/\varepsilon_1)}$$

14] (3) $\quad dC = \dfrac{(k_0 + \lambda x)\,\varepsilon_0 A}{dx}$; $\quad C_0 = \dfrac{\varepsilon_0 A}{d}$ (12)

All these will be in series.

dC — dx

$$\therefore \frac{1}{C_{eq}} = \int \frac{1}{dC}$$

$$= \int \frac{dx}{\varepsilon_0 A (k_0 + \lambda x)}$$

$$\therefore = \frac{1}{\varepsilon_0 A} \int \frac{dx}{k_0 + \lambda x}$$

$$= \frac{1}{\varepsilon_0 A}\frac{1}{\lambda} \ln (k_0 + \lambda x)\Big|_0^d$$

$$= \frac{1}{\lambda \varepsilon_0 A}\left(\ln (k_0 + \lambda d) - \ln k_0\right)$$

$$= \frac{1}{\lambda \varepsilon_0 A} \ln\left(1 + \frac{\lambda d}{k_0}\right)$$

$$\therefore C_{eq} = \frac{\lambda \varepsilon_0 A}{\ln\left(1 + \frac{\lambda d}{k_0}\right)}$$

$$\therefore C_{eq} = \frac{\lambda d}{\ln\left(1 + \frac{\lambda d}{k_0}\right)} C_0.$$

15]

$$\cancel{C_{eq} = \frac{k \varepsilon_0 A x}{d} + \frac{\varepsilon_0 A (L - x)}{d}}$$

$$\cancel{U = \frac{Q^2 d}{2(k\varepsilon_0 A x + \varepsilon_0 A (L-x))}}$$

$\cancel{A = wL}$

15] (3)

$$C_{eq} = \frac{K\varepsilon_0 \bar{w} x}{d} + \frac{\varepsilon_0 \bar{w}(l-x)}{d}$$

$$\therefore U = \frac{Q^2}{2C_{eq}}$$

$$\therefore U = \frac{Q^2 d}{2(K\varepsilon_0 \bar{w} x + \varepsilon_0 \bar{w}(l-x))}$$

$$F = -\frac{dU}{dx}$$

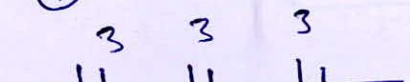
$$\therefore F = \frac{Q^2 d}{2(K\bar{w}\varepsilon_0 x + \varepsilon_0 \bar{w}(l-x))^2} \times (K\varepsilon_0 \bar{w} - \varepsilon_0 \bar{w})$$

$\therefore$ At $x = 0$

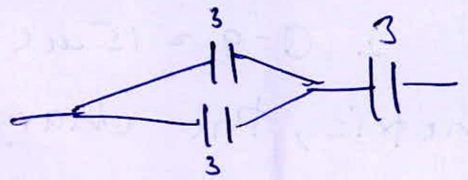
$$F = \frac{Q^2 d\, \varepsilon_0 \bar{w}(K-1)}{2(\varepsilon_0 \bar{w} l)^2}$$

$$\therefore F = \frac{Q^2 d (K-1)}{2\varepsilon_0 \bar{w} l^2}$$

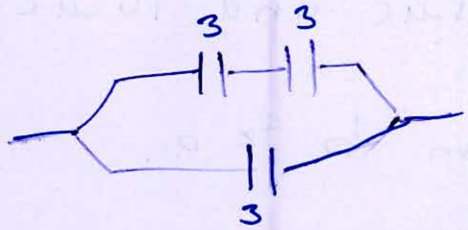
16] (4)



$C_{eq} = \frac{3}{3} = 1\,\mu F$



$\frac{1}{C_{eq}} = \frac{1}{6} + \frac{1}{3} \Rightarrow C_{eq} = 2\,\mu F$



$C_{eq} = \frac{3}{2} + 3 = 4.5\,\mu F$

We cannot get $6\,\mu F$ as C_{eq}

17] 4

$$E \times d = V = \frac{Q}{KC}$$

$$\therefore E \times d = \frac{Qd}{K \varepsilon_0 A}$$

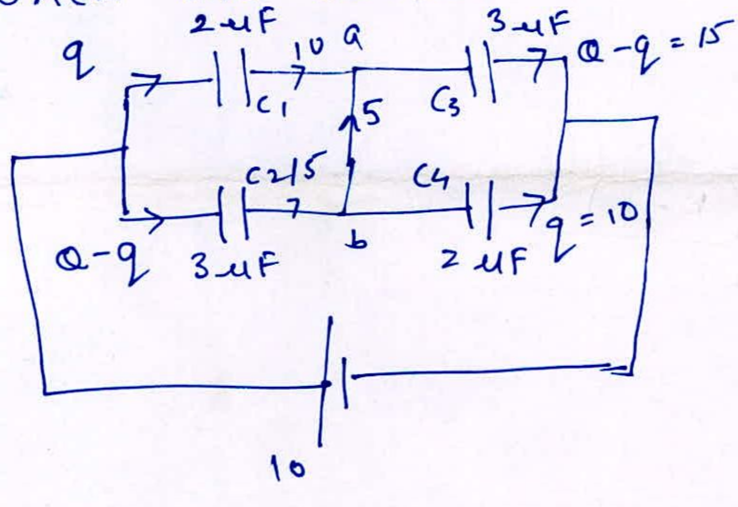
$$\therefore \frac{Q}{A} = E \times K \times \varepsilon_0$$

$$= 3 \times 10^4 \times 2.2 \times 8.85 \times 10^{-12}$$

$$\approx 6 \times 10^{-7} \, C/m^2$$

18] 1

when the switch is closed,



$$\frac{1}{C_{eq}} = \frac{1}{5} + \frac{1}{5}$$

$$\therefore C_{eq} = \frac{5}{2}$$

$$\therefore Q = C_{eq} V$$

$$= \frac{5}{2} \times 10$$

$$Q = 25 \mu C$$

Now, the 2μF and 3μF are in parallel

$$\therefore \frac{q}{2} = \frac{Q-q}{3} \Rightarrow \frac{q}{2} = \frac{25-q}{3}$$

$$\therefore 3q = 50 - 2q$$

$$\therefore 5q = 50 \Rightarrow q = 10 \mu F \qquad \therefore Q - q = 15 \mu C$$

Now as the structure is symmetric, the charges through C_3 and C_4 will be 15μC and 10μC respectively

∴ 5μC charge will flow from b to a.