# Calculus for physics

[redacted]



## Ex - 1
## Section - I

1 (a) $\frac{d}{dx}\left[(x^2-3x+3)(x^2+2x-1)\right]$

$$= (2x-3)(x^2+2x-1) + (x^2-3x+3)(2x+2)$$

$$= 2x^3 + 4x^2 - 2x - 3x^2 - 6x + 3$$
$$+ 2x^3 - 6x^2 + 6x + 2x^2 - 6x + 6.$$

$$= 4x^3 - 3x^2 - 8x + 9$$

$\cancel{(b) \quad (2x^2-3)(x^4+x^2-1) + (x^3-3x+1)(4x^3+2x)}$

$\cancel{=}$

(2) $y = \frac{x+1}{x-1}$.

$$\frac{dy}{dx} = \frac{\frac{d}{dx}(x+1)\cdot(x-1) - \frac{d}{dx}(x-1)\cdot(x+1)}{(x-1)^2}$$

$$= \frac{1(x-1) - 1(x+1)}{(x-1)^2}$$

$$= \frac{\cancel{x}-1-\cancel{x}-1}{(x-1)^2} = \frac{-2}{(x-1)^2}.$$

(3) $y = \dfrac{x}{x^2+1}$.

$$\frac{dy}{dx} = \frac{(x^2+1)\frac{d}{dx}x - x\frac{d}{dx}(x^2+1)}{(x^2+1)^2}.$$

$$= \frac{(x^2+1) - x(2x)}{(x^2+1)^2}$$

$$= \frac{1-x^2}{(x^2+1)^2}.$$

(4) $y = ax+b / cx+d$. ,

$$\frac{dy}{dx} = \frac{(cx+d)a - c(ax+b)}{(cx+d)^2}$$

$$= \frac{\cancel{ax} + da - \cancel{cax} - cb}{(cx+d)^2}$$

$$= (da-cb)/(cx+d)^2.$$

(5) $z = \dfrac{x^2+1}{3(x^2-1)} + (x^2-1).(1-x)$

$$\frac{dz}{dx} = \frac{[(2x)\,3(x^2-1) - 3(2x)(x^2+1)]}{(3(x^2-1))^2} + (2x)(1-x) + (x^2-1)(-1)$$

$$= \frac{-4x}{3(x^2-1)^2} + 1+2x+3x^2.$$

6. $y = \dfrac{1-x^3}{1+x^3}$.

$$\frac{dy}{dx} = \frac{(-3x^2)(1+x^3) - (1-x^3)(3x^2)}{(1+x^3)^2}$$

$$= \frac{-3x^2 - 3x^5 - 3x^2 + 3x^5}{(1+x^3)^2}$$

$$= \frac{-6x^2}{(x^3+1)^2}.$$

7. $y = \dfrac{2}{x^3-1}$.

$$\frac{dy}{dx} = \frac{0 - 2(3x^2)}{(x^3-1)^2} = -\frac{6x^2}{(x^3-1)^2}$$

8. $y = x^2 - x + 1 / a^2 - 3$ $\quad$ ($(a^3-3)$ is a constant)

$$\frac{dy}{dx} = \frac{1}{a^3-3}(2x-1)$$

9. $y = \dfrac{1-x^3}{\sqrt{\pi}}$

$$\frac{dy}{dx} = \frac{-3x^2}{\sqrt{\pi}}.$$

10. $\dfrac{dy}{dx} = \cos x - \sin x$

(11) $y = x / 1-\cos x$

$$\frac{dy}{dx} = \frac{1.(1-\cos x) - (\sin x).x}{(1-\cos x)^2}$$

$$= \frac{1-\cos x - x\sin x}{(1-\cos x)^2}$$

(12) $y = \tan x / x$

$$\frac{dy}{dx} = \frac{\sec^2 x \cdot x - \tan x \cdot 1}{x^2}$$

$$= \frac{x - \sin x \cdot \cos x}{x^2 \cos^2 x}$$

(13) $y = x\sin x + \cos x$

$$\frac{dy}{dx} = x\cos x + 1.\sin x + \cancel{\cos x} - \sin x$$

$$= x\cos x$$

(14) $y = \frac{\sin x}{x} + \frac{x}{\sin x}$

$$\frac{dy}{dx} = \frac{\cos x . x - 1.\sin x}{x^2} + \frac{1.\sin x - x\cos x}{\sin^2 x}$$

$$= (x\cos x - \sin x)\left(\frac{1}{x^2} - \frac{1}{\sin^2 x}\right)$$

(15) $y = \frac{\sin x}{1+\cos x}$

$$\frac{dy}{dx} = \frac{\cos x(1+\cos x) - (-\sin x)\sin x}{(1+\cos x)^2} = \frac{1+\cos x}{(1+\cos x)^2}$$

(16) $y = \dfrac{x}{\sin x + \cos x}$

$$\frac{dy}{dx} = \frac{1.(\sin x + \cos x) - x(\cos x - \sin x)}{(\sin x + \cos x)^2}$$

(17) $y = \dfrac{x \sin x}{1 + \tan x}$

$$\frac{dy}{dx} = \frac{(1+\tan x)(1.\sin x + x.\cos x) - x\sin x.\sec^2 x}{(1+\tan x)^2}$$

(18) $y = \cos^2 x$

$$\frac{dy}{dx} = 2\cos x.(-\sin x) = -\sin 2x.$$

(19) $y = \frac{1}{4}\tan^4 x$

$$\frac{dy}{dx} = \frac{1}{\cancel{4}}(\cancel{4}\tan^3 x . \sec^2 x).$$

$$= \tan^3 x . \sec^2 x$$

(20) $y = \cos x - \frac{1}{3}\cos^3 x$

$$\frac{dy}{dx} = -\sin x - \frac{1}{\cancel{3}}.\cancel{3}\cos^2 x(-\sin x)$$

$$= -\sin x(1 - \cos^2 x) = -\sin^3 x.$$

(21) $y = 3\sin^2 x - \sin^3 x$

$$\frac{dy}{dx} = 3.2\sin x\cos x - 3\sin^2 x \cos x.$$

$$= 6\sin x\cos x\left(1 - \frac{1}{2}\sin x\right)$$

$$= \frac{3}{2}\sin 2x\,(2 - \sin x)$$

(22) $y = \frac{1}{3}\tan^3 x - \tan x + x$

$\frac{dy}{dx} = \frac{1}{\cancel{3}}\cancel{3}\tan^2 x \cdot \sec^2 x - \sec^2 x + 1$

$= \tan^2 x \sec^2 x - \cancel{\sec^2 x} + \cancel{\sec^2 x} - \tan^2 x$

$= \tan^2 x (\sec^2 x - 1)$

$= \tan^4 x.$

(23) $y = x \sec^2 x - \tan x$

$\frac{dy}{dx} = 1 \cdot \cancel{\sec^2 x} + x \cdot 2 \sec x \cdot \sec x \tan x - \cancel{\sec^2 x}$

$= 2x \cdot \frac{\sin x}{\cos^3 x}$

~~(24)~~ $\cancel{y = \sec^2 x + \csc^2 x}$

$\cancel{\frac{dy}{dx} = 2 \sec x \sec x \tan x + 2 \csc x (-\csc x \cot x)}$

$\cancel{= 2 \frac{\sin x}{\cos^3 x} - \frac{2 \cos x}{\sin^3 x}}$

(24) $\cancel{y = x^2 \log_3 x}$

$\cancel{= x^2 \frac{\log_e x}{\log_e 3}} = \frac{1}{\log_e 3} x^2 \log_e x$

$\cancel{\frac{dy}{dx} = \frac{1}{\log_e 3} \cdot \left[2x \ln x + x^2 \frac{1}{x}\right].}$

Refer back for (24)

(25) $y = \ln^2 x$

$$\frac{dy}{dx} = 2 \ln x \cdot \frac{1}{x}.$$

(26) $y = x \log_{10} x = \dfrac{x \log_e x}{\log_e 10}$

Refere left

$$= \frac{x \ln x}{\log_e 10}$$

$$\frac{dy}{dx} = \frac{1}{\log_e 10} \left[ x \frac{1}{x} + 1 . \ln x \right].$$

$$= \frac{1}{\ln 10} (1 + \ln x)$$

(27) $y = (\ln x)^{1/2}$

$$\frac{dy}{dx} = \frac{1}{2} . \ln x \cdot \frac{1}{x}.$$

~~(28)~~ $\cancel{y = \dfrac{x-1}{\log_2}}$

(29) $\displaystyle\int \sqrt{x}\, dx = \int x^{1/2}\, dx = \frac{x^{1/2+1}}{\frac{1}{2}+1} + c$

$$= \frac{x^{3/2}}{3/2} + c$$

(29) $$\int x^{n/m}\,dx = \frac{x^{\frac{n}{m}+1}}{\frac{n}{m}+1} + C$$

(30) $$\int \frac{dx}{x^2} = \int x^{-2}\,dx = \frac{x^{-2+1}}{-2+1} + C$$

$$= \frac{x^{-1}}{-1} + C$$

(31) $$\int \frac{dx}{2\sqrt{x}} = \int \frac{1}{2} x^{-1/2}\,dx = \frac{1}{2}\,\frac{x^{-1/2+1}}{-\frac{1}{2}+1} + C$$

$$= x^{1/2} + C$$

(32) $$\int (1-2u)\,du = \int du - \int 2u\,du$$

$$= u - \frac{u^2}{2} + C.$$

# Calculus for physics

(5)

## Ex- II

(1) $\cancel{x^{-3}}$ $\frac{d}{dx}\left(x^{-1/2}\right)$

$= -\frac{1}{2} \cdot x^{-3/2}$.

(2) $\frac{d}{dx}\left(\frac{1}{(ax+b)^2}\right) = \frac{d}{dx}\left(ax+b\right)^{-2}$

$= -2 \cdot (ax+b)^{-3} \cdot a$

(3) $\frac{d}{dx}\left(x^3 + \frac{1}{x^3} + 8\right)$

$= 3x^2 + \frac{d}{dx}\left(x^{-3}\right) + \frac{d}{dx} 8$

$= 3x^2 - 3x^{-4} + 0$

(4) $\frac{d}{dx} \sin\left(x^3\right)$

$= \cos x^3 \cdot 3x^2$

(5) $\frac{d}{dx}\left(4x^3 - 5\right)^{1/2}$

$= \frac{1}{2}\left(4x^3-5\right)^{-1/2} \cdot 12x^2$

(6) $\frac{d}{dx} \sin(\ln x)$

$= \cos(\ln x) \cdot \frac{1}{x}$.

(7) $\frac{d}{dx} (2x^2+1)^{1/2}$

$= \frac{1}{2} (2x^2+1)^{-1/2} \cdot 4x$.

(8) $\frac{d}{dx} e^{\sqrt{2x}}$

$= e^{\sqrt{2x}} \cdot \frac{d}{dx} (\sqrt{2x})$

$= e^{\sqrt{2x}} \cdot \sqrt{2} \cdot \frac{1}{2} \cdot x^{-1/2}$.

(9) $\frac{d}{dx} (x^4 - 2\sin x + 3\cos x)$

$= 4x^3 - 2\cos x - 3\sin x$

(10) $\frac{d}{dx} (x^2 \sin x \ln x)$

$= \left(\frac{d}{dx}(x^2)\right) \sin x \ln x + x^2 \frac{d}{dx} (\sin x) \ln x$

$+ x^2 \sin x \frac{d}{dx} (\ln x)$

$= 2x \sin x \ln x + x^2 \cos x \ln x + x^2 \sin x \frac{1}{x}$

11 $\frac{d}{dx}\left(\frac{x^2+1}{x+1}\right)$

$$= \frac{2x(x+1) - (x^2+1)(1)}{(x+1)^2}$$

$$= \frac{2x^2 + 2x - x^2 - 1}{(x+1)^2}$$

~~12~~ $\cancel{\frac{d}{dx}\left(\frac{\sin x - x\cos x}{x\sin x + \cos x}\right)}$

$$= \cancel{\frac{\cos x - x(-\sin x)}{(x\sin x + \cos x)^2}}$$

12 $V = \frac{4}{3}\pi r^3.$

$$\frac{dV}{dr} = \frac{4}{3}\pi \frac{d}{dr}(r^3)$$

$$= \frac{4}{3}\pi \cdot 3r^2 = 4\pi r^2.$$

13

$xy = c^2$

Differentiating both sides w.r.t $x$, we have

$\cancel{x\frac{dy}{dx}}$ $\frac{d}{dx}(xy) = \frac{d}{dx}(c^2)$

$\therefore \left(\frac{d}{dx}x\right)y + x\cdot\left(\frac{d}{dx}y\right) = 0$

$\therefore y + x\cdot\frac{dy}{dx} = 0 \quad \therefore \frac{dy}{dx} = -\frac{y}{x}$

(14) $\cancel{x^y = y^x}$

~~Differentiating both sides.~~

$$\cancel{\frac{d}{dx}(x^y) = \frac{d}{dx}(y^x)}$$

$\therefore \cancel{x^y}$

(14) $x^y = y^x$

$$\ln(x^y) = \ln(y^x)$$

$$\therefore\ y \ln x = x \ln y$$

Differentiate both sides.

$$\frac{d}{dx}(y \ln x) = \frac{d}{dx}(x \ln y)$$

$$\Rightarrow \frac{dy}{dx} \cdot \ln x + y \cdot \frac{1}{x} = \frac{d}{dx} x \ln y + x \frac{1}{y} \frac{dy}{dx}$$

$$\Rightarrow \frac{dy}{dx}\left(\ln x - \frac{x}{y}\right) = \left(\ln y - \frac{y}{x}\right)$$

$$\frac{dy}{dx} = \frac{\ln y - y/x}{\ln x - x/y}$$

(15) $x = at^2, \quad y = 2at$

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{2a}{2at} = \frac{1}{t}$$

(16) $$\int x^{1/5}\, dx = \frac{x^{\frac{1}{5}+1}}{\frac{1}{5}+1} = \frac{x^{6/5}}{\frac{6}{5}} + C$$

(17) $\int \frac{1}{(ax+b)^2}\,dx$

$\int (ax+b)^{-2}\,dx$

Let $ax+b = t$

$a\,dx = dt$

$dx = \frac{dt}{a}$

$\int t^{-2} \cdot \frac{dt}{a} = \frac{t^{-2+1}}{-2+1} \cdot \frac{1}{a}$

$= \frac{t^{-1}}{-1}\,\frac{1}{a}$

$= \frac{(ax+b)^{-1}}{-1} \cdot \frac{1}{a} + c$

(18) $\int \sin x \cos x\,dx$

Let $\sin x = t$

$\therefore \cos x\,dx = dt$

$= \int t \cdot dt = \frac{t^2}{2}$

$= \frac{(\sin x)^2}{2} + c$

(19) $\int \frac{x}{x^2+a^2}\,dx$

Let $x^2+a^2 = t$

$2x\,dx = dt$

$= \int \frac{1}{2}\,\frac{dt}{t} = \frac{1}{2}\ln t = \frac{1}{2}\ln|x^2+a^2| + c$

(20) $\int_{-\pi/2}^{\pi/2} \cos x \, dx$

$= [\sin x]_{-\pi/2}^{\pi/2}$

$= [\sin \frac{\pi}{2} - \sin -\frac{\pi}{2}]$

$= 1-(-1) = 2.$

(21) $\int_0^{\pi/2} \sqrt{1+\cos x} \, dx$

$= \int_0^{\pi/2} \sqrt{2\cos^2 \frac{x}{2}} \, dx$

$= \sqrt{2} \int_0^{\pi/2} \cos \frac{x}{2} \, dx$

Let $\frac{x}{2} = t$

$\frac{dx}{2} = dt.$

$= \sqrt{2} \int_0^{\pi/4} \cos t \cdot (dt) 2$

$= 2\sqrt{2} \, [+\sin t]_0^{\pi/4}$

when $x = 0, \ t = 0$

$x = \frac{\pi}{2}, \ t = \frac{\pi}{4}$

$= 2\sqrt{2} \, [\sin \frac{\pi}{4} - \sin 0]$

$= 2\sqrt{2} \cdot \frac{1}{\sqrt{2}} = \underline{\underline{2}}.$

(22) $\int (1-x)\sqrt{x} \, dx = \int \left(x^{1/2} - x^{3/2}\right) dx$

$\frac{x^{3/2}}{3/2} - \frac{x^{5/2}}{5/2} + c$

(23) $\int \frac{\sin^2 x + \cos^2 x}{\sin^2 x \cdot \cos^2 x}\, dx$

$= \int \left(\frac{1}{\cos^2 x} + \frac{1}{\sin^2 x}\right) dx$

$= \int (\sec^2 x + \operatorname{cosec}^2 x)\, dx$

$= \tan x - \cot x + C$

(24) $\int \frac{1}{1+e^{-x}}\, dx$

$= \int \frac{e^x}{\underbrace{e^x+1}_{t}}\, dx$

then $e^x\, dx = dt$

$= \int \frac{dt}{t} = \ln t = \ln(1+e^x) + c$

(25) $\int \frac{\operatorname{cosec}^2 x}{1+\cot x}\, dx$

Let $1+\cot x = t$

then $-\operatorname{cosec}^2 x\, dx = dt$

$= \cancel{\int} \int \frac{-dt}{t}$

$= -\ln t = -\ln|1+\cot x| + c$

(26) $\int \frac{\ln x}{x}\, dx = \int (\ln x)(\ln x)'\, dx$

$= \frac{(\ln x)^2}{2} + c$

(27) $\int_0^{\pi/2} (\sin x + \cos x)\, dx$

$= \left[-\cos x + \sin x\right]_0^{\pi/2}$

$= -\left[\cos\frac{\pi}{2} - \cos 0\right] + \left[\sin\left(\frac{\pi}{2}\right) - \sin 0\right]$

$= -\left[0 - 1\right] + \left[1 - 0\right]$

$= +1 + 1 = 2$

(28) $\int_0^{\infty} e^{-x}\, dx$

Let $-x = t$ , $-dx = dt$

then when $x = 0$, $t = 0$

when $x = \infty$, $t = -\infty$.

$= \int_0^{-\infty} e^{t} \cdot (-dt)$

$= -\int_0^{-\infty} e^{t}\, dt =$

$= -\left[e^{t}\right]_0^{-\infty} + c$

$= -\left[e^{-\infty} - e^{0}\right] + c$

$= -\left[0 - 1\right] + c = 1$

## Ex-III

9

① $\dfrac{dR}{dt} = .05\,cm/s.$

$A = \pi R^2$

$\dfrac{dA}{dt} = \dfrac{d}{dt}(\pi R^2) = 2\pi R \dfrac{dR}{dt}$

$= \cancel{2\pi(3cm)\left(\frac{.05cm}{s}\right)}$

$= \cancel{3\pi(.1)\frac{cm^2}{s}}$

$= 2\pi\,(3.2cm)\left(.05\frac{cm}{s}\right)$

$= \cancel{2}\pi\,(3.2)(.1)\dfrac{cm^2}{s}$

$= .32\,\pi\,\dfrac{cm^2}{s}.$

② Let edge $= a$

$V =$ Volume $= a^3.$

$\dfrac{dV}{dt} = \dfrac{dV}{da}\cdot\dfrac{da}{dt}$

$= 3a^2\cdot\dfrac{da}{dt}$

$= 3\,(10)^2\cdot 3\,\dfrac{cm^3}{s}$

$= 900\,\dfrac{cm^3}{s}$

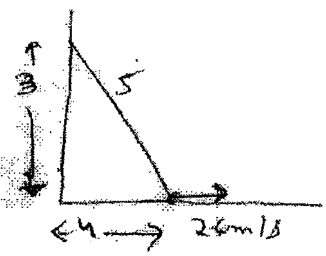
③ $\dfrac{dV}{dt} = 900\,cm^3/s.$ $\quad V = \frac{4}{3}\pi r^3.$

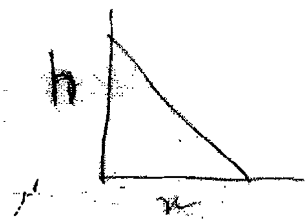
But $\dfrac{dV}{dt} = 4\pi r^2\dfrac{dr}{dt} \Rightarrow 900 = 4\pi(15)^2\dfrac{dr}{dt}$

$$900 = 4\pi \cdot \cancel{225} \, \frac{dr}{dt}$$

$$\frac{1}{\pi} = \frac{dr}{dt}$$

(4)




$$x^2 + h^2 = 5^2$$

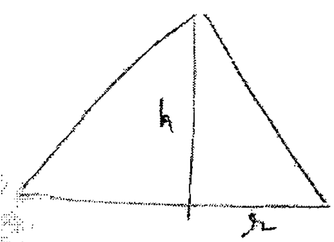
Differentiating, $2x \frac{dx}{dt} + 2h \frac{dh}{dt} = \frac{d}{dt}(5)^2$

$$\Rightarrow x \frac{dx}{dt} + h \frac{dh}{dt} = 0$$

$$\Rightarrow 4(2) + 3 \frac{dh}{dt} = 0$$

$$\therefore \boxed{\frac{dh}{dt} = \frac{-8}{3} \text{ cm/s}}$$

(5)




$$V = \frac{1}{3} \pi r^2 h.$$

$$\frac{dV}{dt} = \frac{d}{dt} \left( \frac{\pi}{3} r^2 h \right)$$

$$= \frac{\pi}{3} \frac{d}{dt} (r^2 h)$$

$$= \frac{\pi}{3} \left[ r^2 \frac{d}{dt} h + h \cdot \frac{dr^2}{dt} \right]$$

$= \frac{\pi}{3}\left[r^2 \frac{dh}{dt} + h \cdot 2r\frac{dr}{dt}\right]$.

Also $h = \frac{1}{6}r \Rightarrow$

$\Rightarrow 6h = r$ and $6\frac{dh}{dt} = \frac{dr}{dt}$.

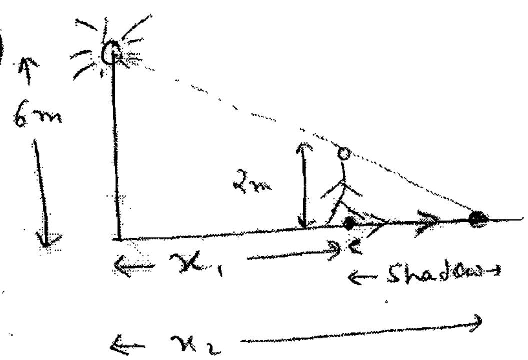
$\therefore \frac{dV}{dt} = \frac{\pi}{3}\left[36h^2\frac{dh}{dt} + h \cdot 2(6h) \cdot 6\frac{dh}{dt}\right]$

$\Rightarrow 12 = \frac{\pi}{3}\left[36(4)^2 \cdot \frac{dh}{dt} + 72 \cdot (4)^2 \cdot \frac{dh}{dt}\right]$

$\Rightarrow \cancel{12} = \frac{\pi}{\cancel{3}}\frac{dh}{dt} \quad 4^2 \cdot \cancel{\frac{36}{3}}\ \cancel{[1+2]}$

$\boxed{\frac{1}{48\pi} = \frac{dh}{dt}}$.

(6)




$\frac{dx_1}{dt} = 5\frac{km}{h}$.

Shadow length $= x_2 - x_1$

Also from similiar triangles,

$\frac{x_2 - x_1}{2} = \frac{x_2}{6}$

$3(x_2 - x_1) = x_2$

$3x_2 - 3x_1 = x_2$

$\boxed{x_2 = \frac{3}{2}x_1}$

To find.

$$\frac{d\,(\text{shadow length})}{dt}$$

$$= \frac{d}{dt}(x_2 - x_1)$$

$$= \frac{d}{dt}\left(\frac{3}{2}x_1 - x_1\right)$$

$$= \frac{d}{dt}\left(\frac{x_1}{2}\right) \qquad = \frac{1}{2}\left(\frac{d}{dt}x_1\right)$$

$$= \frac{1}{2} \cdot 5 \text{ km/h}$$

$$= \frac{5}{2} \text{ km/h}.$$

7) Let the numbers be $x_1$ and $x_2$ ..

Then, $x_1 + x_2 = 15$.

To minimise, $x_1^2 + x_2^2$.

or $x_1^2 + (15 - x_1)^2$.

$x_1^2 + (15 - x_1)^2$ will be minimum if

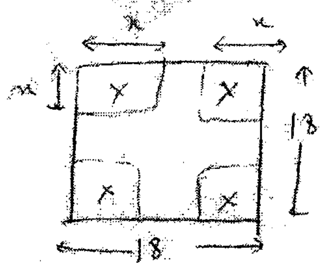
$$\frac{d}{dx_1}\left[(x_1^2) + (15 - x_1)^2\right] = 0$$

$$\therefore\ 2x_1 + 2(15 - x_1)(-1) = 0$$

$$x_1 - 15 + x_1 = 0$$

$$\therefore\ x_1 = \frac{15}{2}, \quad x_2 = \frac{15}{2}$$

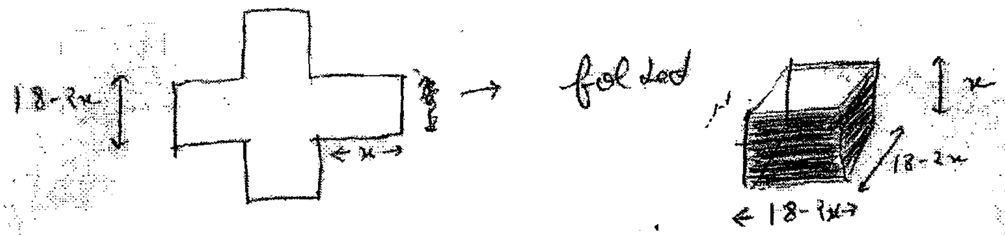
(8)




Let the edge of square be $x$.

then,,

Remaining figure.




Volume $= (18-2x)^2 \cdot x$..

$\frac{dV}{dx} = 0$

$2(18-2x).(-2)x + (18-2x)^2 . 1 = 0$

$\Rightarrow (-4x + 18 - 2x)(18-2x) = 0$

$\Rightarrow (18-6x)(18-2x) = 0$

either $x = 3$

or $x = 9$.

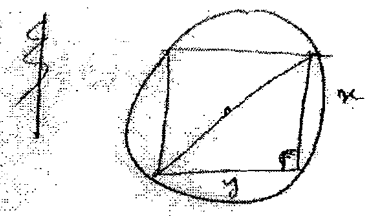
when $x = 9 \rightarrow$ Volume 0.

maximum volume will be

when $x = 3$.

$\underline{\underline{x = 3\text{cm}}}$.

(9)




Let the sides of rectangle be $x$ and $y$.

$$x^2+y^2 = (2r)^2$$

Differentiating w.r.t $x$,

$$2x + 2y\frac{dy}{dx} = 0$$

~~Differentiat~~

$$\therefore \boxed{x + y\frac{dy}{dx} = 0} \quad \text{(1)}$$

Area $= x.y.$

For area to be maximum,

$$\frac{dA}{dx} = 0$$

$$\therefore \boxed{1.y + x.\frac{dy}{dx} = 0} \quad \text{(2)}$$

From (1) and (2), we get $x = y$.

$\therefore$ Rectangle is a square

(10)

$$V = 29.4 - 9.8t$$

when height is maximum,

$$\frac{dh}{dt} = 0.$$

But $\frac{dh}{dt} = V = 0$ when $29.4 - 9.8t = 0$

$$\therefore \boxed{t = 3}$$

$$h = \int_0^3 v\,dt = \int_0^3 (29.4 - 9.8t)\,dt$$

$$= 29.4\,[t]_0^3 - 9.8\left[\frac{t^2}{2}\right]_0^3$$

$$= 29.4\times 3 - \frac{9.8\times 9}{2}$$

$$= 88.2 - 44.1 = 44.1\text{ m}$$

(1) $s = \frac{t^4}{4} - 4t^3 + 16t^2$.

(12)

(a) starting position $\Rightarrow s = 0$.

$$\frac{t^4}{4} - 4t^3 + 16t^2 = 0$$

$$t^2 ( t^2 - 16t + 64) = 0$$

$$t^2 ( t - 8)^2 = 0$$

$$\therefore t = 0, \quad t = 8$$

(b) $v = \frac{ds}{dt}$

$$= t^3 - 12t^2 + 32t$$

If $v = 0$, then $t^3 - 12t^2 + 32t = 0$

$$\therefore t ( t^2 - 12t + 32) = 0$$

$$\therefore t ( t - 8) ( t - 4) = 0$$

$$t = 0, \; t = 4, \; t = 8$$

(2) $m = 3\,kg.$

$s = 1 + t + t^2$

$K.E. = \frac{1}{2} m v^2$

$v = \frac{ds}{dt} = 0 + 1 + 2t$

At $t = 5s$, $v = 1 + 2 \times 5 = 11\,cm/s.$

$K.E. = \frac{1}{2} \times 3 \times (11)^2 \times 1000$ ergs

$= 1.18 \times 10^5$ ergs

(3) $s = t^3 - 4t^2 - 3t$

$v = \frac{ds}{ds} = 3t^2 - 8t - 3.$

$\Rightarrow 3t^2 - 9t + t - 3$

$\Rightarrow 3t(t-3) + 1(t-3) = 0$

$\Rightarrow (3t+1)(t-3) = 0$

$\therefore \boxed{t = 3}$ when $v = 0$.

$a = \frac{dv}{dt} = 6t - 8$

At $t = 3$,

$a = 6 \times 3 - 8 = 10$

~~(4)~~ $\cancel{V = t^3 - t^2 - 2t} + 2$

$\cancel{x = 4 \text{ after } t} = 2.$

(4) $t = \sqrt{x} + 3$

$v = 0 \Rightarrow \frac{dx}{dt} = 0.$

Differentiating both sides w.r.t. t.

$$\frac{dt}{dt} = \frac{d}{dt}(\sqrt{x} + 3)$$

$$\therefore 1 = \frac{1}{2\sqrt{x}} \cdot \frac{dx}{dt}$$

$$2\sqrt{x} = \frac{dx}{dt}$$

$\therefore$ when $\frac{dx}{dt} = 0$, then $x = 0$.

(5) $V = 3t^2 + 2t + 1.$

$$\frac{ds}{dt} = v$$

$$ds = v\,dt.$$

$$\int ds = \int v\,dt = \int_0^{10} (3t^2 + 2t + 1)\,dt$$

$$= [t^3 + t^2 + t]_0^{10}$$

$$= 1000 + 100 + 10$$

$$= 1110$$

(6) $V = 9t^2 - 8t$

Fourth second means from $t = 3\,s$ to $t = 4\,s$.

$$\frac{ds}{dt} = v$$

$$\therefore ds = v\,dt$$

$$\therefore s = \int ds = \int v\,dt$$

$$= \int_3^4 (9t^2 - 8t)\,dt$$

$$= \left[9\frac{t^3}{3} - 8\frac{t^2}{2}\right]_3^4$$

$$\Rightarrow \left[3t^3 - 4t^2\right]_3^4$$

$$= 3[64 - 27] - 4[16 - 9]$$

$$= 192 - 81 - 64 + 36$$

$$= 83\,m.$$

(7) $v = 6t^2 + 4$

$$s = \int ds = \int v\,dt$$

$$= \int (6t^2 + 4)\,dt = \left[2t^3 + 4t\right]_0^5$$

$$= 2[125] + 4[5]$$

$$= 270\,m$$

(8) $s = s_0 + v_0 t + \frac{1}{2} g t^2$

$$\frac{ds}{dt} = 0 + v_0 + (2gt)\frac{1}{2}$$

$$= v_0 + gt$$

(9) $x = t^4 - 12t^2 - 40$

(i) At $t = 2$, $x = (2)^4 - 12(2)^2 - 40$

$$= 16 - 48 - 40$$

$$= -72 \text{ m}$$

(ii) $v = \frac{dx}{dt}$

$$= 4t^3 - 24t$$

At $t = 2$, $v = 4(2)^3 - 24(2)$

$$= 32 - 48$$

$$= -16 \text{ m/s}$$

(iii) $a = \frac{dv}{dt} = 12t^2 - 24$

At $t = 2$, $a = 12(2)^2 - 24$

$$= 48 - 24 = 24.$$