COM & Conservation of P. Exercise # 1.

1.

$F_{cm} = M a_{cm}$

(D)

$\Rightarrow a_{cm} = \frac{F_{cm}}{M} = \frac{F}{M}$: independent of h (const)

2. If ball and box is a system then there's no external force. Hence V_{cm} remains const.

(B)

3. CM is at a distance r/2 from the centre of ring.

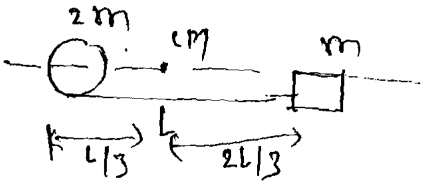
$F_{ext} = \frac{mv^2}{(r/2)} = \frac{2mv^2}{r}$

(C)

4. (E) ~~use~~ moment of mass about cm must be neutralised.

(5)

(c)



C.M should not shift along x-axis.

$$2m\,x = m \cdot \frac{2L}{3}(1-\cos\theta)$$

$$x = \frac{L}{3}(1-\cos\theta).$$

(6)

~~Same as ques. no.~~

(b) At max^m. compression both the blocks will have equal velocity

$$6 \times 2 - 3 \times 1 = (3+6)V$$

$$V = 1 \text{ m/s}$$

(7)

(d) Let x : Displacement of Boat (away from shore)

C.M should remain fixed.

$$5(4-x) = 20x$$

$$\boxed{x = 0.8 \text{ m}}$$

Distance of Dog from shore $= 6 + 0.8$
$= 6.8$ m

(8)

(c)

$$\vec{a}_{cm} = \frac{m_1\vec{a}_1 + m_2\vec{a}_2}{m_1+m_2} = \frac{m\vec{g}}{m+m} = \frac{\vec{g}}{2}$$

$$|\vec{a}_{cm}| = g/2,$$

(9) $\vec{V}_{cm} = \dfrac{m_1\vec{V_1} + m_2\vec{V_2}}{m_1+m_2} = \dfrac{10(2\hat{i}-7\hat{j}+3\hat{k}) + 2(-10\hat{i}+35\hat{j}-3\hat{k})}{10+2}$

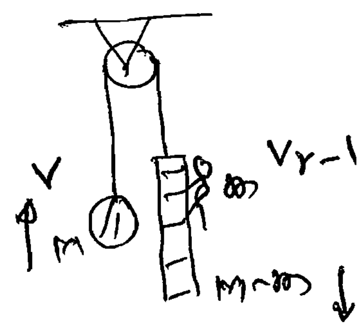
(b)

$= 2\hat{k}\ m/s$

(10) $\vec{V}_{cm} = \dfrac{MV + m(V_r - V) - (M-m)V}{2M}$

(b)

$= \dfrac{mV_r}{2M}$



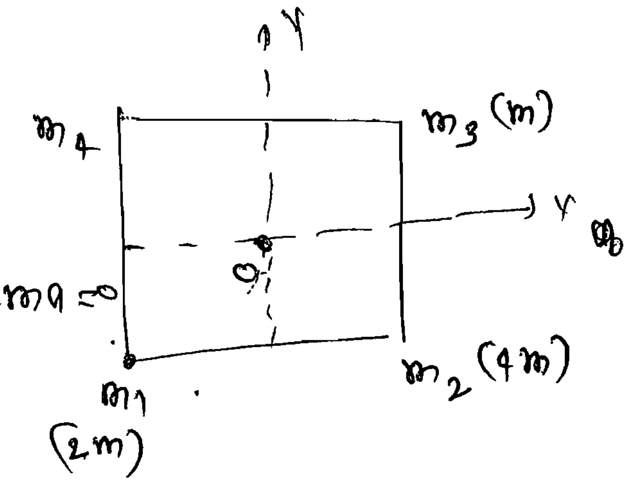
(11) motion of CM remains unaffected due to internal forces.

(a)

(12)

(b) c.m will follow the original Trajectory as if there is no explosion

(13) (d)



$x_{cm} = 0$

$\Rightarrow m_4 a + ma + 4ma - 2ma = 0$

$\Rightarrow m_4 + 5m = 2m$

$\boxed{m_4 = -3m}$

NOT possible

(14)

(b) cart and man will meet at cm of the system i.e $x = 5$

(15)

(c) There is no force in horizontal dir^n for the "gun shot" system. Hence

$\vec{V}_{cm} = 0$

E

(16)
(C) K.E of a system of particles

$= $ K.E of CM + K.E. of different particles in the frame of CM

K.E of CM $= \frac{1}{2}mv^2$

$\therefore$ K.E of system of particles $\geq \frac{1}{2}mv^2$

(17)
(a)

$$a = \frac{mg\sin 60^\circ - mg\sin 30^\circ}{2m} = \frac{(\sqrt{3}-1)g}{4}$$

$$\vec{a}_{cm} = \frac{m\vec{a}_1 + m\vec{a}_2}{m+m} = \frac{1}{2}(\vec{a}_1 + \vec{a}_2)$$

$$|\vec{a}_{cm}| = \frac{1}{2}|\vec{a}_1 + \vec{a}_2| = \sqrt{2}a = \frac{(\sqrt{3}-1)g}{4\sqrt{2}}$$

(18)
(c)

$$\vec{V}_{cm} = \frac{m_1\vec{v}_1 + m_2\vec{v}_2}{m_1+m_2} = \frac{\vec{v}_1 + \vec{v}_2}{2} = (\hat{i}+\hat{j})\ m/s$$

$$\vec{a}_{cm} = \frac{m_1\vec{a}_1 + m_2\vec{a}_2}{m_1+m_2} = \frac{\vec{a}_1+\vec{a}_2}{2} = \frac{3}{2}(\hat{i}+\hat{j})\ m/s^2$$

$\vec{V}_{cm} \parallel \vec{a}_{cm} \Rightarrow$ St. line path

(19)
(d)

$$\left.\begin{aligned} u_x &= 20\sqrt{2}\cos 45^\circ = 20\ m/s \\ u_y &= 20\sqrt{2}\sin 45^\circ = 20\ m/s \end{aligned}\right] t=0$$

$$t=1\text{ sec}, \left[\begin{aligned} V_x &= u_x = 20\ m/s \\ V_y &= u_y - gt = 10\ m/s \end{aligned}\right.$$

Due to explosion one part comes to rest. Hence from conservation linear momentum vertical component of second part will be $V_y' = 2V_y = 20\ m$

$$H_{max} = h_1 + h_2 = \left[20(1) - \frac{1}{2}g(1)^2\right] + \frac{V_y'^2}{2g}$$

$$= 20 + 15 = 35\ m$$

(20) (b) CM will move in a vertical line if $v_1 \cos\theta_1 = v_2 \cos\theta_2$. otherwise for any other values it will follow a parabolic path.

(21) (b) Initial x-coordinate of CM

$$x_i = \frac{4M(0) + M(5R)}{4M + M} = R \quad \text{(i)}$$

Let x_0 = x-coordinate of shell when the small sphere reaches the other extreme position

$$x_f = \frac{4M(x_0) + M(x_0 - 5R)}{4M + M} = x_0 - R \quad \text{(ii)}$$

surface is smooth $\Rightarrow x_{in} = x_f$

$$x_0 - R = R$$

$$\boxed{x_0 = 2R}$$

(22) (b) $a = \frac{2g - 1g}{2+1} = \frac{10}{3}\ m/s^2$

$$a_{cm} = \frac{2a - 1(a)}{2+1} = \frac{a}{3} = \frac{10}{9}\ m/s^2 \text{ (downward)}$$

$$\therefore S_{cm} = \frac{1}{2} a_{cm} t^2 = \frac{20}{9}\ m.$$

(23) (c) Range $= \frac{u^2 \sin 2\theta}{g} = 10\ m$

Net force in horizontal $dir^n = 0$

C.M remains stationary in horizontal dir^n

$$(60 + 40)x = 1(10)$$

$$\boxed{x = 0.1\ m}$$

(24) (d) CM cannot move towards left. It will always move towards right. Because wedge has a tendency to move left and only external force on the system is friction which will act towards right.

(25) (B) Example: Explosion

internal force can't change linear momentum

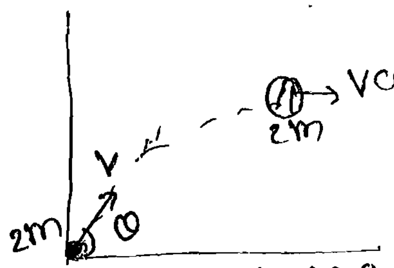
(26) (D)

Emission is due to internal force, hence linear momentum must be conserved,

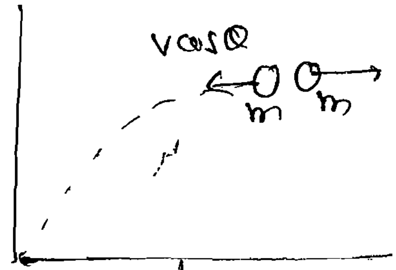
$$m\vec{v} = m_1\vec{v_1} + m_2\vec{v_2}$$

$\vec{v}$ must be parallel to $(m_1\vec{v_1} + m_2\vec{v_2})$

(27) (A)



Just before explosion



Just after explosion

Linear momentum conservation

$$2mv\cos\theta = mv' - mv\cos\theta$$

$$\boxed{v' = 3v\cos\theta.}$$

(28) (A)

Linear momentum conservation

$$mu = MV \quad \text{(I)}$$

Mechanical energy conservation

$$mg(R-r) = \frac{1}{2}mu^2 + \frac{1}{2}MV^2 \quad \text{(II)}$$

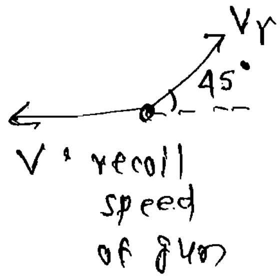
solving (I) & (II)

$$\boxed{V = \sqrt{10/3}}$$

m = 10 kg
M = 20 kg
R = 1.1 m
r = 0.1 m.

29

c



$$\vec{V}_{bullet} = \vec{V}_r + \vec{V}_{gun}$$

$$= (V_r \cos 45^\circ - V)\hat{i} + V_r \sin 45^\circ \hat{j}$$

$$\tan\theta = \frac{V_r \sin 45^\circ}{V_r \cos 45^\circ - V} > 1$$

$$\boxed{\theta > 45^\circ}$$

30

b

h << radius of Earth ⇒ g can be assumed to be constt.

Let v = velocity of block at height h/2. Then velocity of Earth will be v/3 (conservation of $\vec{p}$)

conservation of mechanical energy:

$$\frac{M}{3} g \frac{h}{2} = \frac{1}{2}\frac{M}{3}v^2 + \frac{1}{2}M(v/3)^2$$

$$v = \sqrt{\frac{3gh}{2}}$$

31

b

Linear momentum conservation

$$mu = (m+2m)V \Rightarrow \boxed{V = u/3} \quad (i)$$

Mechanical energy conservation

$$\frac{1}{2}mu^2 = \frac{1}{2}(m+2m)V^2 + mgh \quad (ii)$$

$$\boxed{u = \sqrt{3gh}}$$

32

a

$$\frac{1}{2}\left(\frac{m \cdot 2m}{m+2m}\right) V_{rel}^2 = \frac{1}{2}kx^2$$

$$V_{rel} = \left(\frac{3k}{2m}\right)^{1/2} x$$

(33) (C)

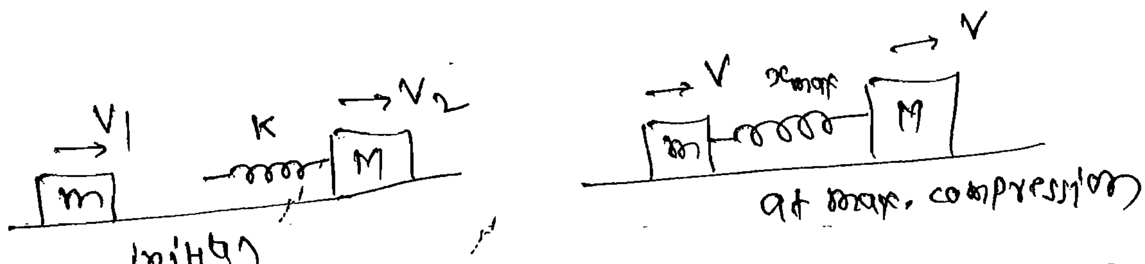
change in linear momentum, $\vec{\Delta P} = \vec{F}.\Delta t$

$= (m\vec{g})\Delta t$

$|\vec{\Delta P}| = mg(\Delta t)$

$= (1)(10)(1) = 10 \text{ kg m/s}.$

(34) (C)



$mV_1 + MV_2 = (m+M)V \Rightarrow \boxed{V = \frac{mV_1 + MV_2}{m+M}}$ — (I)

$\frac{1}{2}mV_1^2 + \frac{1}{2}MV_2^2 = \frac{1}{2}(m+M)V^2 + \frac{1}{2}Kx_{max}^2$

$mV_1^2 + MV_2^2 = (M+m)V^2 + \frac{1}{2}Kx_{max}^2$ — (II)

solving (I) & (II)

$x_{max} = (V_1 - V_2)\sqrt{\frac{Mm}{(M+m)K}}$

(35) (C)

$mV = (m+M)V'$

$V' = \frac{mV}{M+m}$,

36 (c)

Linear momentum is conserved.

$$|\vec{p}_1| = |\vec{p}_2| = p$$

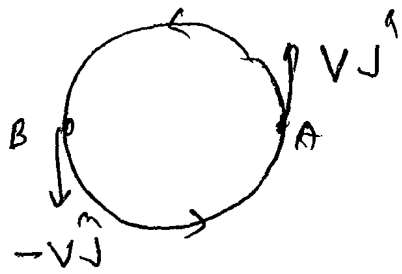
Energy released = Σ K.E of fragments

$$= \frac{p^2}{2m_1} + \frac{p^2}{2m_2}$$

37 (b)

$$\Delta \vec{P} = \vec{P}_B - \vec{P}_A$$
$$= -2mv\hat{j}$$

$$|\Delta \vec{P}| = 2mv$$



$\Delta KE = 0$

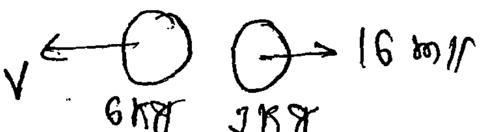
38 (a)

$$\Delta \vec{P} = \vec{P}_f - \vec{P}_{in}$$

$|\Delta \vec{P}|_{lead} = mv$

$|\Delta \vec{P}|_{Tennis} = m(v+v')$

39 (c)



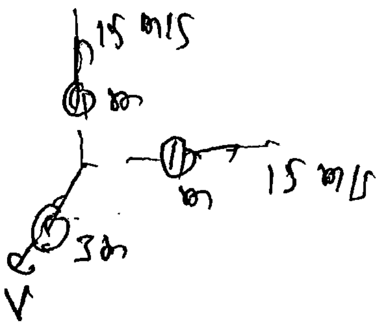
$P_{3kg} = P_{6kg} = 48$

$$KE = \frac{p^2}{2m} = \frac{(48)^2}{2\times 6} = 192\ J$$

40 (a)

Use linear momentum conservation

$$\vec{P}_{in} = \vec{P}_f$$
$$\Rightarrow \vec{P}_f = 0$$



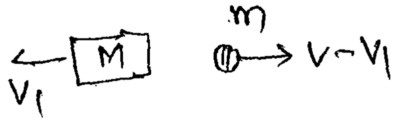
$$15\sqrt{2}\, m = 3mV$$

$$\boxed{V = 5\sqrt{2}\ m/s}$$

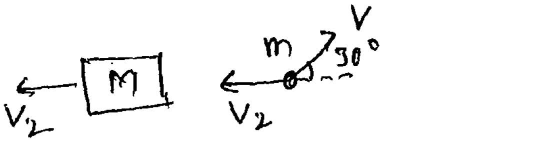
41 (c)

$$\vec{V}_{cm} = \frac{m_1\vec{V}_1 + m_2\vec{V}_2}{m_1+m_2} = \frac{28(10) + 4\cdot 0}{(10+4)} = 20\ m/s$$

(42)
(b)



case-I



case-2

$$m(v - V_1) = M V_1$$

$$\boxed{V_1 = \frac{mv}{M+m}}$$

$$m(v\cos 30^\circ - V_2) = M V_2$$

$$\boxed{V_2 = \frac{\sqrt{3}\, m v}{2(M+m)}}$$

$$V_1/V_2 = \frac{2}{\sqrt{3}}$$

(43)
(a)

$$\vec{F} = \frac{d\vec{P}}{dt} = 2\vec{B}t$$

When $\vec{a}$ and $\vec{v}$ are at 45°, $\vec{F}$ and $\vec{p}$ will also be at 45° and this will happen

at $t = \sqrt{A/B}$

$$\vec{F} = 2\vec{B}\sqrt{A/B}$$

(44)
(a)

At max^m extension velocity of both blocks will be equal.

$$(3+6)V = 6\times 2 - 3\times 1 = 9 \quad \left(\begin{array}{c}\text{momentum}\\ \text{conservation}\end{array}\right)$$

$$\boxed{V = 1\ m/s}$$

Mechanical energy conservation

$$\frac{1}{2}3.(1)^2 + \frac{1}{2}.6(2)^2 = \frac{1}{2}200.x_{max}^2 + \frac{1}{2}9.(1)^2$$

$$\boxed{x_{max} = 0.3\ m}$$

(45)
(c)

Impulse = change in linear momentum

$$\vec{F}.\Delta t = m(\vec{v_f} - \vec{v_{in}})$$

$$(2\hat{i} + \hat{j} + 3\hat{k})(2) = 1\left[\vec{v_f} - (2\hat{i} + \hat{j})\right]$$

$$\vec{v_f} = 6\hat{i} + 3\hat{j} + 6\hat{k}$$

$$|\vec{v_f}| = 9\ m/s$$

(46)
(b)

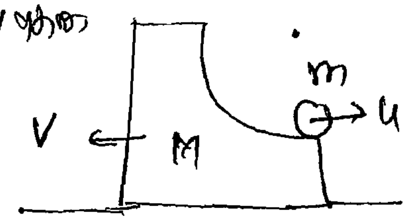
$F = 0 \Rightarrow t = 0.003\ sec$

$\text{Impulse} = \int_0^t F\,dt = 0.9\ NS$

(47)
(d)

Linear momentum conservation

$mu = MV$ — (i)



conservation of energy

$mgR = \frac{1}{2}MV^2 + \frac{1}{2}mu^2$ — (ii)

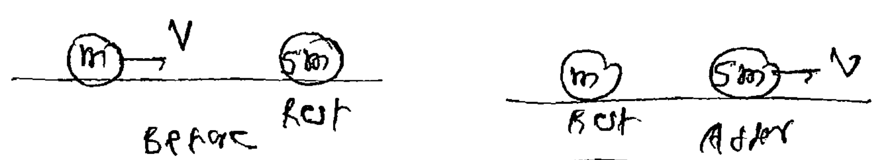
$V = \sqrt{\frac{2mgR}{M+m}}$

(48)
(d)

$\text{Force} = V\frac{dm}{dt} = 5(1) = 5N$

$a = \frac{F}{M} = \frac{5}{2} = 2.5\ m/s^2$

(49)
(a)



$mV = 5mV' \Rightarrow \boxed{V' = V/5}$

$e = \frac{V'-0}{V-0} = 1/5$

(50)
(c)

$V_1 + 2V_2 = 36$ — (i)

$\frac{V_1 - V_2}{36} = \frac{2}{3}$ — (ii)

$V_1 = 28\ m/s$
$V_2 = 4\ m/s$

$\text{loss} = \frac{1}{2}\cdot 1\cdot(12)^2 + \frac{1}{2}\cdot 2\cdot(24)^2 - \left[\frac{1}{2}\cdot 1\cdot 28^2 + \frac{1}{2}\cdot 2\cdot 4^2\right]$

$= 240\ J$

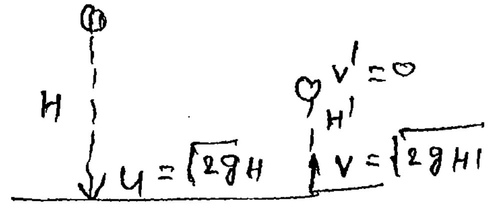
(51)
(a) After collision

$$V_2 = \left(\frac{1+e}{2}\right)u$$

$$V_1 = \left(\frac{1-e}{2}\right)u$$

$$V_2 = 2V_1 \Rightarrow e = 1/3.$$

(52)
(d)



$$v = eu \Rightarrow v^2 = e^2u^2$$

$$H' = e^2 H.$$

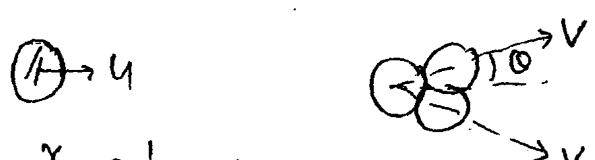
$$e^2 = H'/H = \frac{2.5}{10} = 1/4$$

$$\boxed{e = \frac{1}{2}}$$

$$I = F.\Delta t = \Delta P = mv + mu = m\left[\sqrt{2gH} + \sqrt{2gH'}\right]$$

$$F = \frac{m}{\Delta t}\sqrt{2g}\left[\sqrt{H} + \sqrt{H'}\right]$$

(53)
(c)



$$\sin\theta = \frac{r}{2r} = \frac{1}{2} \Rightarrow \theta = 30°$$

Linear momentum conservation

$$mu = 2mv\cos 30° \Rightarrow v = u/\sqrt{3}$$

$$e = \frac{v}{u\cos\theta} = \frac{u/\sqrt{3}}{u\sqrt{3}/2} = \frac{2}{3}$$

(54)
(b)

u = velocity before collision

v = velocity of ball after collision

$$= \sqrt{(u/\sqrt{2})^2 + (u/2\sqrt{2})^2} = \sqrt{5/8}\,.\,u.$$

Fractional loss in KE

$$= \frac{\frac{1}{2}mu^2 - \frac{1}{2}mv^2}{\frac{1}{2}mu^2} = 1 - (v/u)^2$$

$$= 3/8.$$

55) After collision balls exchange their velocities

(C)

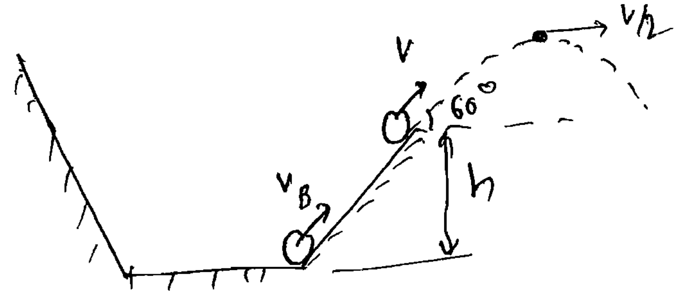
$$V_A = \sqrt{2gh}$$

$$V_B = \sqrt{2g(4h)} = 2\sqrt{2gh}$$

$$H_A = \frac{V_A^2}{2g} = h$$

$$H_B = \frac{13h}{4}$$

$$\frac{H_A}{H_B} = 4/13$$



(56) component of velocity parallel to wall remains unchanged whereas normal component will be

(b) revered and e times the initial component

$$\vec{V}_{in} = 2\hat{i} + 2\hat{j}$$

$$\vec{V}_f = -\frac{1}{2}(2\hat{i}) + 2\hat{j} = -\hat{i} + 2\hat{j}$$

(57) a = retardation due to friction $= \mu g = 2.5\ m/s^2$

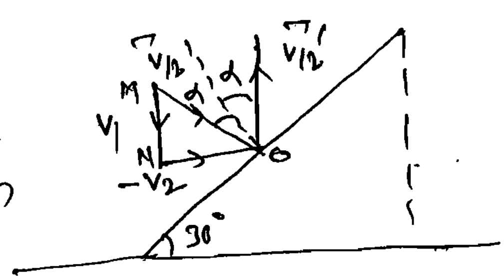
(a)

$$S = \frac{v^2}{2a} = \frac{(5)^2}{2(2.5)} = 5$$

$\therefore$ Final separation $= 5 - 2 = 3$ m.

(58) $\vec{V}_{12}$ = velocity of ball w.r.t wedge before collision

(b) $\vec{V}_{12}'$ = velocity of ball w.r.t wedge after collision



$\angle MON = 30°$

$$V_1/V_2 = \tan 30° = 1/\sqrt{3}$$

(59)

(b) $T = \frac{d}{(v/\sqrt{2})} + \frac{d}{(ev/\sqrt{2})} = \left(1+\frac{1}{e}\right)\frac{\sqrt{2}\,d}{v}$

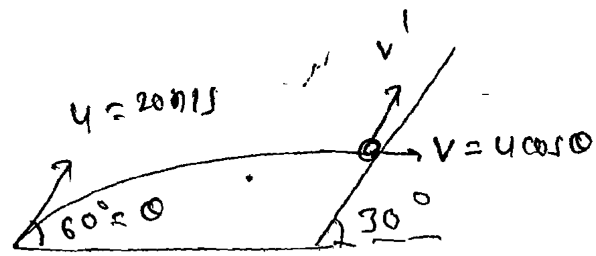
$$\frac{2v\cdot\sin 45^\circ}{g} = \left(1+\frac{1}{e}\right)\frac{\sqrt{2}\,d}{v}$$

$$\boxed{e = \frac{gd}{v^2 - gd}}$$

(60) $v = u\cos 60^\circ = 10\ m/s$

(a) since $e=0$, ball will not bounce, will move along the plane with velocity $v' = v\cos 30^\circ$

$= 5\sqrt{3}\ m/s$



H = max^n height attained

$$= \frac{u^2\sin^2 60^\circ}{2g} + \frac{v'^2}{2g} = 18.75\ m.$$

Exercise #2

(1) (c, d) ~~...~~

• Body may be along x-axis $\boxed{x \geq 0}$

(2) (A, B) Non-uniform mass distribution around mid-point of rod.

(3) $F_{ext} \neq 0$, $a_{cm} = \frac{F_{ext}}{m} \neq 0$

(B, D) But V_{cm} may be zero (motion under gravity)

(4) A, B

speed of man w.r.t ground

$= V - V_{car}$

$KE = \frac{1}{2} m (V - V_{car})^2$

$W = \Delta KE < \frac{1}{2} m V^2$

If he walks normal to rail

$V_c = 0$

$KE = \frac{1}{2} m V^2$

$W = \Delta KE = \frac{1}{2} m V^2$.

(5) B, C, D

$(K.E)_{system} = (KE)_{cm} + \Sigma (K.E)$ of particle relative to cm.

$(K.E)_{system} \geq (KE)_{cm}$.

(6) (b, c)

$$a_{cm} = \frac{m_1 a_1 + m_2 a_2}{m_1 + m_2} = \frac{mg + mg}{2m} = g \text{ (downward)}$$

$$(V_{cm})_x = (V_{cm})_y = 10 \text{ m/s}$$

$$H = \frac{(V_{cm})_y^2}{2g} = 5 \text{ m}$$

H_{max} of cm $= 20 + 5 = 25$ m

(7) (a, c)

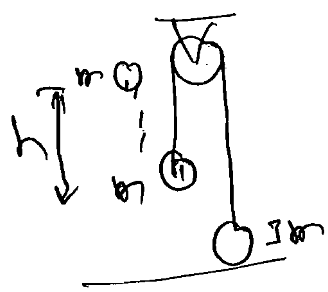
$$\vec{a}_1 = \vec{a}_2 \Rightarrow \vec{a}_{cm} = \vec{a}_1 = \vec{a}_2$$

(8) (a, c, d)

$$V_{cm} = \frac{mv_0 + M \cdot 0}{m + M} = \frac{mv_0}{M + m}$$

At max^m compression m & M will have equal velocity and will be rest in cm frame

(9) (a, b)



Just before striking : $u = \sqrt{2gh}$

After striking common speed

$$V = \frac{mu}{5m} = u/5$$

System : $|a| = \frac{3mg - 2mg}{5m} = \frac{g}{5}$

$$H = \frac{v^2}{2 \cdot |a|} = h/5$$

(10) (a, d) Till the block B returns to its mean position spring is compressed and hence there is a force on block A. once A leaves contact with wall, net force on system becomes zero.

$$V_B = \omega A = \sqrt{K/m} \cdot A = \sqrt{\frac{16}{1}} \cdot 1 = 4 \text{ m/s (right)}$$

$$V_{cm} = 2 \text{ m/s}$$

(11) (a, b, d) momentum conservation

$\frac{m}{2}u = \left(m + \frac{m}{2}\right)v$

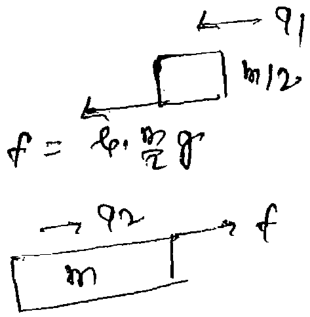
$\boxed{v = u/3}$

work done against friction

$= E_{in} - E_f$

$= \frac{1}{2}\cdot\frac{m}{2}\cdot u^2 - \frac{1}{2}\,\frac{3m}{2}\cdot\left(\frac{u}{3}\right)^2$

$= \frac{1}{6}mu^2 = \frac{2}{3}\left(\frac{1}{4}mu^2\right)$



$f = \mu\cdot\frac{m}{2}g$

$f = \mu\frac{m}{2}g$

$a_1 = \mu g\,,\quad a_2 = \mu g/2$

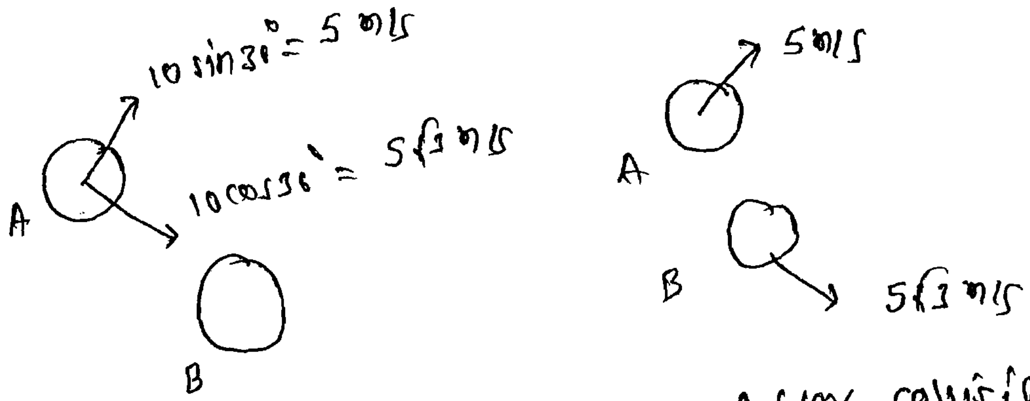
$a_{rel} = 3\mu g/2\,.$

(12) (b) (c)

B will give impulse $-J$ to A

$e = \frac{J-(P-J)}{P} = \frac{2J}{P} - 1\,.$

(13) (a, b, c)



Before collision

After collision

Normal component exchanged in elastic collision with equal mass.

(14) (a, b) Horizontal component of v remains unchanged while vertical component get multiplied by e.

$T_1 = \frac{2u_y}{g}\,,\quad T_2 = \frac{2\cdot eu_y}{g}$

$R_1 = \frac{2}{g}u_x\cdot u_y\,,\quad R_2 = \frac{2}{g}\cdot u_x(eu_y)$

$H_1 = \frac{u_y^2}{2g}\,,\quad H_2 = \frac{(eu_y)^2}{2g}$

$\frac{T_1}{T_2} = 1/e = \frac{R_1}{R_2}$

$\frac{H_1}{H_2} = 1/e^2$

(15) (a,b,c)

Before collision $V_A = \sqrt{2gH}, \quad V_B = 0$

After collision $V_A' = \left(\frac{m_A - m_B}{m_A + m_B}\right) V_A = -\frac{\sqrt{2gH}}{2}$

$$V_B' = \left(\frac{2m_A}{m_A+m_B}\right) V_A = \frac{\sqrt{2gH}}{2}$$

$$H_A = V_A'^2/2g = H/4$$

$$H_B = V_B'^2/2g = H/4$$

collision : perfectly inelastic

$$V = \frac{m_A V_A}{m_A + m_B} = \frac{\sqrt{2gH}}{4}$$

$$H = \frac{V^2}{2g} = H/16.$$

(16) (b,c)

m $\rightarrow V$ M

$V_1 = V$ $V_2 = 0$

Before

m $\rightarrow V_1'$ M $\rightarrow V_2'$

After

$$V_1' = \left(\frac{m-M}{m+M}\right) V = \left(\frac{1-x}{1+x}\right) V, \quad x = M/m$$

$\boxed{V_1' = \pm V/3} \Rightarrow x = \frac{M}{m} = \frac{1}{2}$ or 2

(17) (b,c)

Angle of incidence = Angle of reflection

$u = V$

(18) (b,c)

Momentum of the system is always conserved. Potential energy is max^m when both particles have equal velocity.

K.E = E (Before collision) $= \frac{1}{2}mV^2$

At max^m. P.E, $V = u/2$

$$KE = \frac{1}{2} \cdot 2m \cdot (u/2)^2 = \frac{1}{2}\left(\frac{1}{2}mu^2\right) = E/2$$

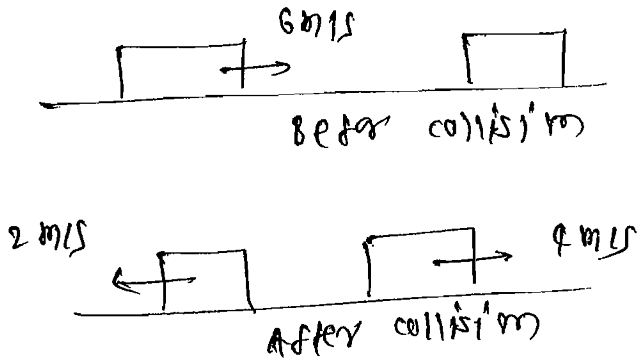
$(KE)_{min} = E - E/2 = E/2.$

(19) (a,b,c) use definition of oblique collision

(20)

(20)

(a, c)



$$e = \frac{4-(-2)}{6-0} = 1$$

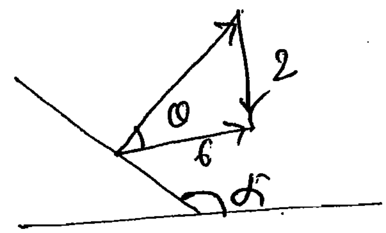
$$(V_{cm})_{in} = \frac{m_1 V_1 + m_2 V_2}{m_1 + m_2} = \frac{1 \times 6 + 2 \times 0}{1+2} = 2 \text{ m/s}$$

(21) (a, c, d)

Impulse = change in linear momentum

$= 2(\vec{V_2} - \vec{V_1})$

$= 2(3\hat{i} + \hat{j})$

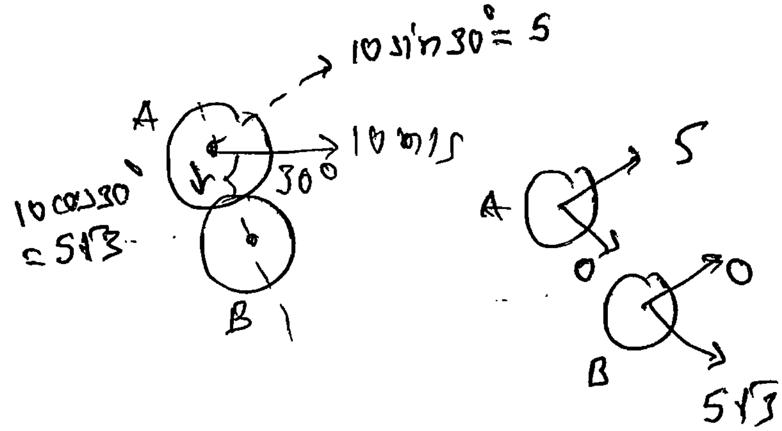


Impulse is in the normal to plane of collision

$\tan\theta = 2/6 = 1/3$

$\theta = \tan^{-1}(1/3) \Rightarrow \alpha = 90^\circ + \tan^{-1}(1/3)$

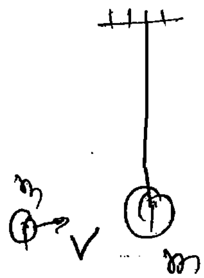
(22) (a, b, c)



(23) (23) (b, c, d)

K.E is not conserved during collision even in elastic collision.

(24) (a, b, c, d)



Inelastic: $v' = v/2$

$H_{max} = \frac{v'^2}{2g} = \frac{v^2}{8g}$

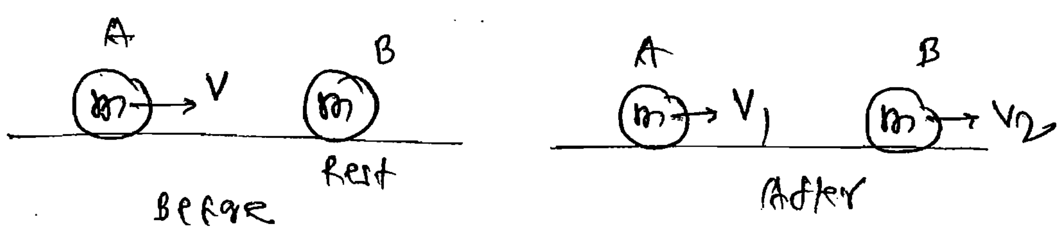
$(KE)_{after} = \frac{1}{2} \cdot 2m \cdot v'^2 = \frac{1}{4}m$

Elastic: $v' = v$

$H_{max} = v^2/2g$

(24)

(25) (b, c)



(25)

$mv = mv_1 + mv_2 \Rightarrow \boxed{v_1 + v_2 = v}$ (I)

$\frac{v_2 - v_1}{v} = e \Rightarrow \boxed{v_2 - v_1 = ev}$ (II)

$v_1 = (1-e)v/2$

$v_2 = (1+e)v/2$

(1). Exercise #3

use std. result,

(4) A → r

B → p

C → s

D → q

correction

(2) use $x_{cm} = \frac{\Sigma m_i x_i}{\Sigma m_i}$

$y_{cm} = \frac{\Sigma m_i y_i}{\Sigma m_i}$

A → q

B → s

C → p

D → R

(3) $\vec{v_1} = (2t)\hat{i}$, $\vec{a_1} = \frac{d\vec{v_1}}{dt} = 2\hat{i}$

$\vec{v_2} = (t^2)\hat{j}$, $\vec{a_2} = \frac{d\vec{v_2}}{dt} = 2t\hat{j}$

$\vec{F}_{cm} = m_1\vec{a_1} + m_2\vec{a_2} = 2\hat{i} + 2(2t)\hat{j} = 2\hat{i} + 4t\hat{j}$

$|\vec{F}_{cm}| = \sqrt{4+16t^2}$ $t = 2\text{sec}$, $|\vec{F}_{cm}| = \sqrt{68}$ units

$\vec{v}_{cm} = \frac{m_1\vec{v_1} + m_2\vec{v_2}}{m_1+m_2} = \frac{1(4\hat{i}) + 2(4\hat{j})}{1+2} = \frac{4\hat{i}+8\hat{j}}{3}$

$|\vec{v}_{cm}| = \frac{1}{3}\sqrt{16+64} = \sqrt{80}/3$ m/s

$\vec{s_1} = \int_0^2 \vec{v_1}\,dt = 4\hat{i}$

$\vec{s_2} = \int_0^2 \vec{v_2}\,dt = \frac{8}{3}\hat{j}$

$\vec{s}_{cm} = \frac{m_1\vec{s_1} + m_2\vec{s_2}}{m_1+m_2} = \frac{4}{3}\hat{i} + \frac{16}{9}\hat{j}$

$|\vec{s}_{cm}| = 20/9$

A → q

B → R

C → P

(4) $V_{cm} = \dfrac{m_1V_1 + m_2V_2}{m_1+m_2} = \dfrac{1(10) + (2)(-5)}{3} = 0$

$P_{cm} = 0$

Net force on system is zero, hence V_{cm} and P_{cm} will remain constant.

velocity of 1kg and 2kg blocks keep on decreasing initially and finally both of them stop as $V_{cm} = 0$.

A → R, S

B → R, S

C → φ

D → φ

(5) (A) $K = P^2/2m$, $K' = \dfrac{(3P)^2}{2m} = 9K$ (P)

% increase in K = 800%

(B) $P = \sqrt{2Km}$

$P' = \sqrt{2 \cdot 4K \cdot m} = 2P$ (T)

% increase in P = 100%

(C) $K = P^2/2m$

For small change

% increase in K = 2(% increase in P)

= 2% (S)

(D) $P = (2Km)^{1/2}$

% increase in P = $\frac{1}{2}$(% increase in K)

= 0.5% (R)

(6) 7, 8, 9

d, a, c

$(P_{in})_x = (P_f)_x \Rightarrow mu = mv_1\cos 30^\circ + mv_2\cos 30^\circ$

$\boxed{v_1 + v_2 = \frac{2}{\sqrt{3}}u}$ (i)

$(P_{in})_y = (P_f)_y \Rightarrow 0 = mv_1\sin 30^\circ - mv_2\sin 30^\circ$

$\Rightarrow \boxed{v_1 = v_2}$ (ii)

$\boxed{v_1 = v_2 = \frac{u}{\sqrt{3}}}$

$(K.E)_{Before} = \frac{1}{2}mu^2$, $(K.E)_{after} = \frac{1}{2}mv_1^2 + \frac{1}{2}mv_2^2$

$= mv^2 = mu^2/3$,

8 (10, 11, 12)

(c) • Height after n^{th} collision. $H_n = He^{2n}$.

$S = H_0 + 2H_1 + 2H_2 + \dots \infty$

$= H_0 + 2e^2H + 2e^4H + \dots = h\left(\frac{1+e^2}{1-e^2}\right)$

(a) • Time of ascent after n^{th} collision $= e^n t_0$

Where $t_0 = \sqrt{2h/g}$

$T = t_0 + 2t_1 + 2t_2 + \dots \infty = \sqrt{\frac{2h}{g}}\left(\frac{1+e}{1-e}\right)$

(b) • $P = (\Delta P)_1 + (\Delta P)_2 + \dots$

$= (mu + meu) + (meu + me^2u) + \dots$

$= mu(1+e) + mue(1+e) + \dots$

$= mu(1+e)[1 + e + \dots \infty] = mu\left(\frac{1+e}{1-e}\right)$

$= m\sqrt{2gh}\left(\frac{1+e}{1-e}\right)$

(8)

(a) At time t: $x_1 = 3+3t$, $y_1 = 0$

13, 14, 15

$x_2 = 0$, $y_2 = 9+6t$

(b)

$$x_{cm} = \frac{1\,x_1 + 2\,x_2}{1+2} = \frac{3+3t}{3} = 1+t$$

$$y_{cm} = \frac{1\cdot y_1 + 2\cdot y_2}{1+2} = \frac{18+12t}{3} = 6+4t$$

$$\boxed{y_{cm} = 4x_{cm} + 2}$$

(c) First particle will stop at $t_1 = \frac{v_1}{\mu g} = 1.5\,s$

second particle will stop at $t_2 = \frac{v_2}{\mu g} = 3\,sec$,

Hence $v_{cm} = 0$ when both v_1 & v_2 are zero

$$\boxed{t = 3\,sec}$$

(9)

(d) 1kg will stop at $x_1 = 3 + \frac{v_1^2}{2\mu g} = 5.25\,m$

$y_1 = 0$

2kg will stop at $y_2 = 9 + \frac{v_2^2}{2\mu g} = 18\,m$

$x_2 = 0$

$$x_{cm} = \frac{m_1x_1 + m_2x_2}{m_1+m_2} = \frac{1\cdot 5.25}{3} = 1.75\,m$$

$$y_{cm} = \frac{m_1y_1 + m_2y_2}{m_1+m_2} = \frac{36}{3} = 12\,m$$

$$\boxed{(1.75\,m,\ 12\,m)}$$

(10) (10) cm will follow the original parabolic trajectory

(16, 17, 18)

(a) $v^2 = u^2 - 2gh = (10)^2 - 2\cdot(10\cdot1) = 80$

$v = 4\sqrt{5}$ m/s

(b) $T = \frac{2u\sin\theta}{g} = \sqrt{2}$ sec $\qquad \frac{2\times 10\cdot\frac{1}{\sqrt{2}}}{10}$

(c) speed of Vcm will be min at its heighest position

$(V_{cm})_{min} = u\cos\theta = 10\cdot\frac{1}{\sqrt{2}}$

$= 5\sqrt{2}$

(19, 20, 21)

(11) • $P_1 = m_1v_1 = 1(15 - 8gt)$

$P_2 = m_2v_2 = 2\left(\frac{8m_1g}{m_2}\right)t = 8gt$

$P_1 + P_2 =$ initial momentum of 1kg block $= 15$ kg.m/s (a)

but $P_2 > P_1$ since $m_2 > m_1$

• slope $= \frac{dP}{dt} =$ force

forces are equal and opposite (c)

$|F_1| = |F_2| = 8m_1g = 4N$

• $15 - 8gt = 8gt$

$t = \frac{15}{2\cdot 8g} = 1.875$ sec (b)

(12) ~~Let~~ V = horizontal velocity of wedge at topmost point.

~~22, 23, 24~~ V_r = velocity of block relative to wedge.

From given condition $V_r = V$ (in magnitude)

Absolute velocity of block

$$V_b = 2V\cos 30^\circ = \sqrt{3}\,V$$

(22) momentum conservation:

$$1 \cdot V_0 = 2V + 1 \cdot V_b \cos 30^\circ$$

$$V_0 = 2V + \frac{3}{2}V$$

$$\boxed{V_0 = \frac{7V}{2}}$$

mechanical Energy conservation

$$\frac{1}{2} \cdot 1 \cdot V_0^2 = \frac{1}{2} \cdot 2 \cdot V^2 + \frac{1}{2} \cdot 1 \cdot (\sqrt{3}V)^2 + 1 \cdot 10 \cdot (1.45)$$

$$\boxed{V_0^2 - 5V^2 = 29}$$ (b)

$$\boxed{V_0 = 7 \text{ m/s}}$$

$$H = 1.45 + \frac{V_b^2 \sin^2 30^\circ}{2g} = 1.6 \text{ m}$$ (c)

$$J_H = 2V = 4V_0/7 = 4 \text{ N-s}$$

$$\frac{J_H}{J_V} = \tan 60^\circ \Rightarrow J_V = J_H \cot 60^\circ = 4/\sqrt{3} \text{ Ns}$$

$$J = \sqrt{J_H^2 + J_V^2}$$

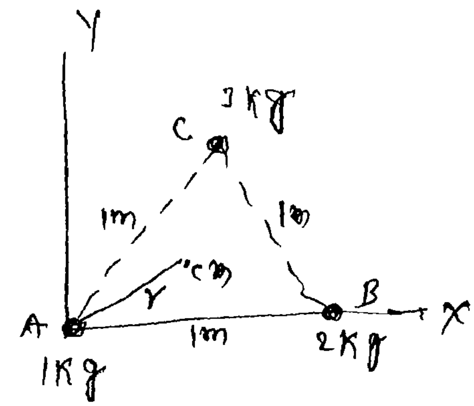
$$= 8/\sqrt{3} \text{ N-s}$$ (a)

1)

$$x_{cm} = \frac{1.0 + 2.1 + 3.\frac{1}{2}}{1+2+3} = \frac{7}{12}$$

$$y_{cm} = \frac{1.0 + 2.0 + 3.\frac{\sqrt{3}}{2}}{1+2+3} = \frac{3\sqrt{3}}{12}$$

$$r = \sqrt{x_{cm}^2 + y_{cm}^2} = \frac{\sqrt{19}}{6}$$

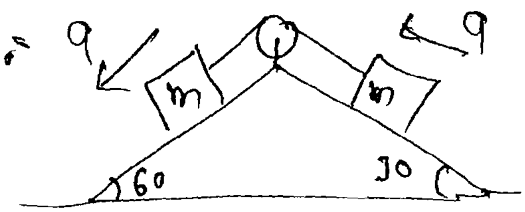


2) $\vec{F}_{ext} = 0$; CM remains at rest.

3) $$(\Delta x)_{cart} = \frac{m(\Delta x)_{\text{ball relative to cart}}}{m+M} = \frac{mL}{M+m}$$

4) $$(\Delta x)_{balloon} = \frac{m.(\Delta x)_{rel}}{m+M} = \frac{mL}{m+M}$$

5) $$a_A = a_B = \frac{g\sin 60 - g\sin 30}{2} = \frac{g}{4}(\sqrt{3}-1)$$



$$\vec{a}_{cm} = \frac{m_A\vec{a}_A + m_B\vec{a}_B}{m_A + m_B} = \frac{1}{2}(\vec{a}_A + \vec{a}_B)$$

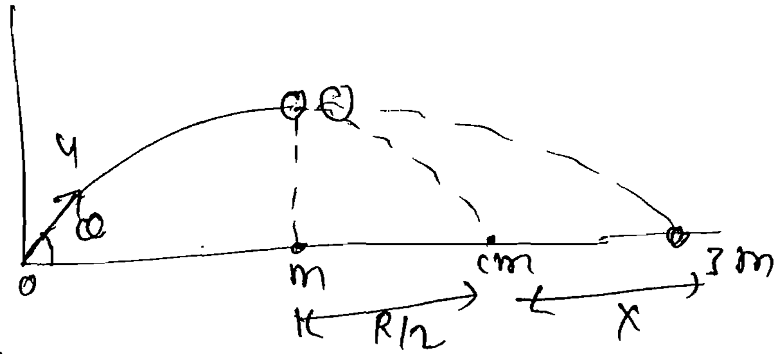
$$|\vec{a}_{cm}| = \frac{1}{2}|\vec{a}_A + \vec{a}_B| = \frac{1}{\sqrt{2}}a = \frac{g}{4\sqrt{2}}(\sqrt{3}-1)$$

6) ~~CM of man and bag will be falling vertically. Hence~~

$$\cancel{m x_{Bag} = M x}$$

$$\cancel{\boxed{x_{Bag} = \frac{Mx}{m}}}$$ ~~(left)~~

(7)



$m \cdot \frac{R}{2} = 3m \cdot x$

$x = \frac{R}{6} = 160\,m.$

$R = \frac{2u^2}{g}\sin\theta\cos\theta$

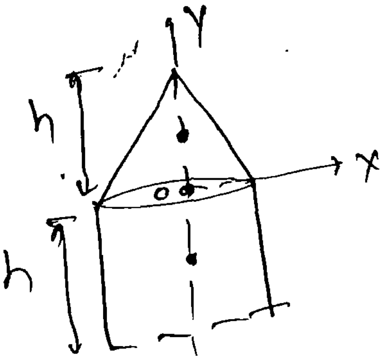
$= \frac{2}{10}(100)^2 \frac{3}{5}\cdot\frac{4}{5}$

$= 960\,m$

Distance of 3m from O $= 960 + 160$

$= 1120\,m.$

(8)



$Y_{cm} = \frac{m_1 Y_1 + m_2 Y_2}{m_1 + m_2}$

$= \frac{V_1 Y_1 + Y_2 \cdot V_2}{V_1 + V_2}$

$= \frac{\frac{1}{3}\pi R^2 h \cdot \frac{h}{4} + \left(-\pi R^2 h \cdot \frac{h}{2}\right)}{\frac{1}{3}\pi R^2 h + \pi R^2 h}$

$= -\frac{5h}{16}$

cm will be within cylinder.

(9)

$a_1 = g\sin 30^\circ = g/2$

$a_2 = g\sin 60 = \sqrt{3}g/2$

$\vec{a}_{cm} = \frac{m\vec{a}_1 + m\vec{a}_2}{2m} = \frac{\vec{a}_1 + \vec{a}_2}{2}$

$|\vec{a}_{cm}| = \frac{1}{2}|\vec{a}_1 + \vec{a}_2| = \frac{1}{2}\sqrt{a_1^2 + a_2^2} = g/2$

(10) $(x_{cm})_{in} = 44$ $\qquad (\Delta x)_{cm} = 44$

$(x_{cm})_f = 42$

since $\left(\vec{F}_{ext}\right)_x = 0$, cm should not shift

hence hinge will shift toward right by 44.

(11) $u_{cm} = \frac{m_1 v_1 + m_2 u_2}{m_1 + m_2} = \frac{u_1 + v_2}{2} = \frac{50 + 30}{2} = 40\, m/s\ (\uparrow)$

$a_{cm} = 10\, m/s^2\ (\downarrow)$

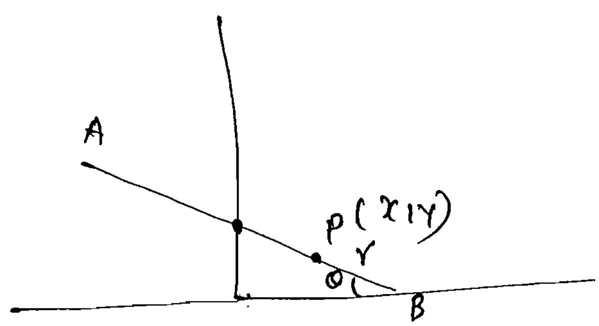
$(Y_{cm})_{in} = 20\, m.$

Addition height gained $= \frac{(u_{cm})^2}{2 a_{cm}} = \frac{(40)^2}{2(10)} = 80\, m$

$(H_{cm})_{max} = 20 + 80 = 100\, m.$

(12) (a) No force on rod along x-axis. Hence cm will fall vertically downward along y-axis

(b) co-ordinates of point P



$x = \left(\frac{L}{2} - r\right) \cos\theta$

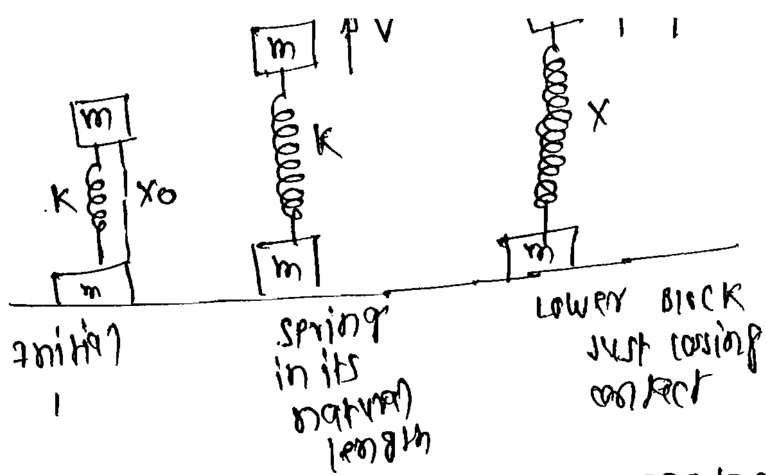
$y = r \sin\theta$

$$\boxed{\frac{x^2}{\left(\frac{L}{2} - r\right)^2} + \frac{y^2}{r^2} = 1}$$

Ellipse

Trajectory of point P.

(1)



when lower block just loses contact cm has already gained a height of $\frac{4mg}{K}$.

Now, using conservation of mechanical energy

$$\frac{1}{2}Kx_0^2 = \frac{1}{2}Kx^2 + \frac{1}{2}mv_1^2 + mg(x+x_0)$$

also $Kx = mg$

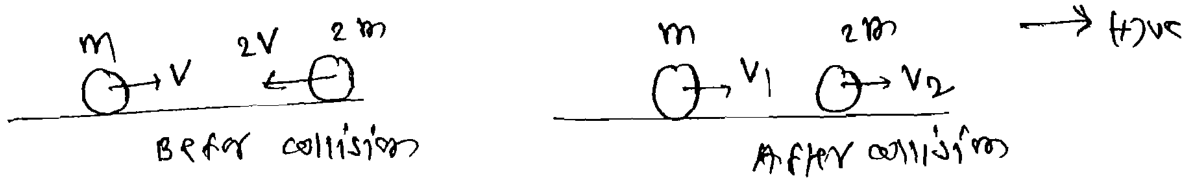
$$\boxed{v_1^2 = \frac{32(mg)^2}{Km}}$$

$$V_{cm} = \frac{mv_1 + m\cdot 0}{2m} = \frac{v_1}{2}$$

$$H = \frac{V_{cm}^2}{2g} = \frac{v_1^2}{8g} = \frac{4mg}{K}$$

$$(H_{cm})_{max} = \frac{4mg}{K} + \frac{4mg}{K} = \frac{8mg}{K}.$$

14



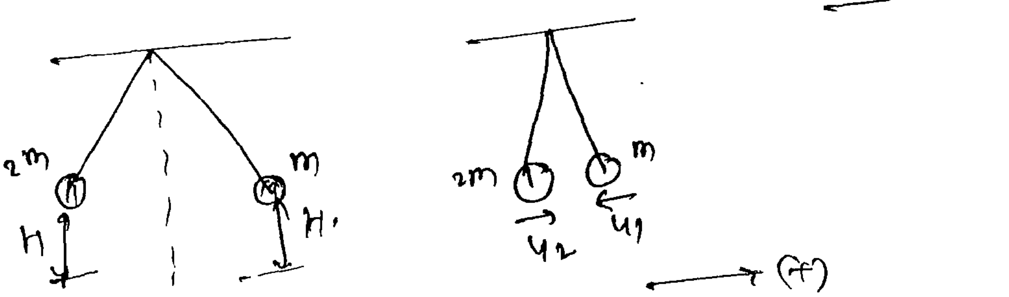
$$\vec{v}_1 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)\vec{u}_1 + \frac{2m_2}{m_1+m_2}\vec{u}_2$$

$$\vec{v}_2 = \left(\frac{2m_1}{m_1+m_2}\right)\vec{u}_1 + \left(\frac{m_2 - m_1}{m_1+m_2}\right)\vec{u}_2$$

$m_1 = m, \quad m_2 = 2m, \quad u_1 = V, \quad u_2 = -2V$

$$\boxed{\begin{aligned}\vec{v}_1 &= -3V\\ \vec{v}_2 &= 0\end{aligned}}$$

18



$u_1 = u_2 = \sqrt{2gH} = u$

$$\vec{v}_1 = \left(\frac{m_1 - m_2}{m_1+m_2}\right)\vec{u}_1 + \left(\frac{2m_2}{m_1+m_2}\right)\vec{u}_2$$

$$\vec{v}_2 = \left(\frac{2m_1}{m_1+m_2}\right)\vec{u}_1 + \left(\frac{m_2 - m_1}{m_1+m_2}\right)\vec{u}_2$$

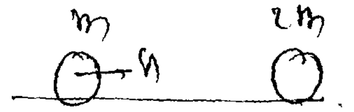
$u_1 = -u$ | $m_1 = m$

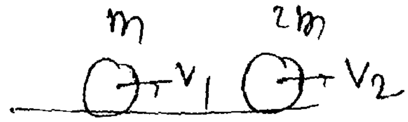
$u_2 = u$ | $m_2 = 2m$

$\vec{v}_1 = 5u/3 \quad \Rightarrow \quad H_1 = v_1^2/2g = \frac{25}{9}H$

$\vec{v}_2 = -u/3 \quad\quad H_2 = v_2^2/2g = H/9.$

(15)





$\rightarrow$ (4)

$$v_2 = \left(\frac{2m_1}{m_1+m_2}\right)u_1 = \frac{2 \cdot m}{m+2m}\,u = \frac{2u}{3}$$

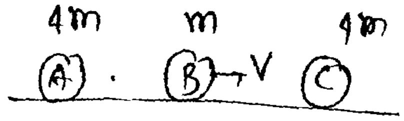
Loss in K.E of m = Gain in KE of 2m

$$= \frac{1}{2} \cdot 2m \cdot \left(\frac{2u}{3}\right)^2 = \frac{8}{9} \cdot \frac{1}{2} m u^2$$

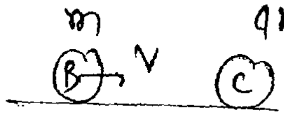
fractional loss in K.E of m

$$= \frac{\frac{8}{9} \cdot \frac{1}{2} m u^2}{\frac{1}{2} m u^2} = 8/9.$$

(19)



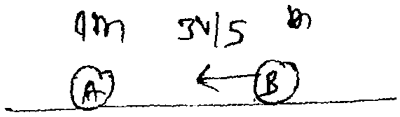
collision betn B and C.



Before

After

collision betn A and B

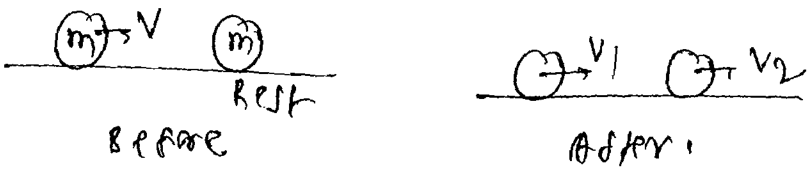


Before

After

velocity of B is less than C hence only 2 collisions will take place.

16



$$m v_1 + m v_2 = m v \Rightarrow \boxed{v_1 + v_2 = v} \quad (I)$$

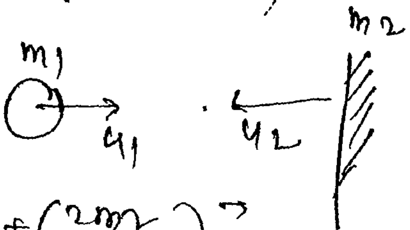
$$\frac{1}{2} m v_1^2 + \frac{1}{2} m v_2^2 = \frac{3}{4}\left(\frac{1}{2} m v^2\right)$$

$$v_1^2 + v_2^2 = \frac{3}{4} v^2 \quad (II)$$

solving (I) & (II) $\boxed{v_2 - v_1 = \frac{v}{\sqrt{2}}}$

$$e = \frac{v_2 - v_1}{v} = \frac{1}{\sqrt{2}}$$

20



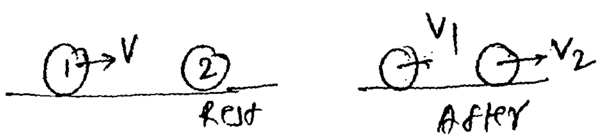
$m_2 >> m_1$

$$\vec{v_1} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)\vec{u_1} + \left(\frac{2m_2}{m_1 + m_2}\right)\vec{u_2} \longrightarrow (+)$$

$$= -\vec{u_1} + 2\vec{u_2}$$

$$= -2 + 2(-1) = -4$$

17



$$mV = m(V_1 + V_2) \Rightarrow \boxed{V_1 + V_2 = V} \quad (I)$$

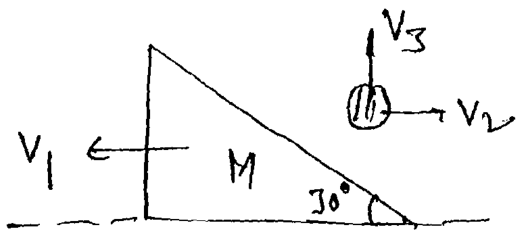
$$\boxed{V_2 - V_1 = eV} \quad (II)$$

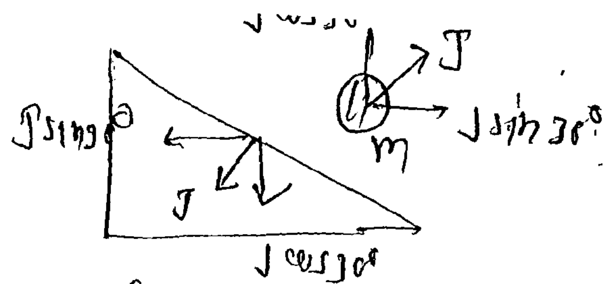
$$V_2 = \frac{(1+e)V}{2}$$

Generalise this

$$V_n = \frac{(1+e)^{n-1} \cdot V}{2^{n-1}}$$

21)





using impulse = change in linear momentum

$$J\sin 30° = MV_1 = mV_2 \quad \text{(I)}$$

$$J\cos 30° = m(V_3 + V_0) \quad \text{(II)}$$

using definition of e.

$$(V_1 + V_2)\sin 30° + V_3 \cos 30° = \frac{1}{2}(V_0 \cos 30°) \quad \text{(III)}$$

solving (I), (II) & (III)

$$V_1 = 1/\sqrt{3}, \quad V_2 = 2/\sqrt{3}, \quad V_3 = 0$$

22) component of velocity parallel to wall remains unchange i.e $2\hat{j}$
while component of velocity normal to wall gets reversed and multiplied by e.
i.e $-\frac{1}{2}(2\hat{i})$

$$\vec{V_f} = -\hat{i} + 2\hat{j}$$

23) $\vec{J} = \vec{P_f} - \vec{P_{in}} = m(\hat{i} + 3\hat{j}) - m(4\hat{i} - \hat{j})$
$= m(-3\hat{i} + 4\hat{j})$

Impulse direction is same as that of normal to wall

$$\hat{J} = \frac{\vec{J}}{|\vec{J}|} = \frac{1}{5}\left(-3\hat{i}+4\hat{j}\right)$$

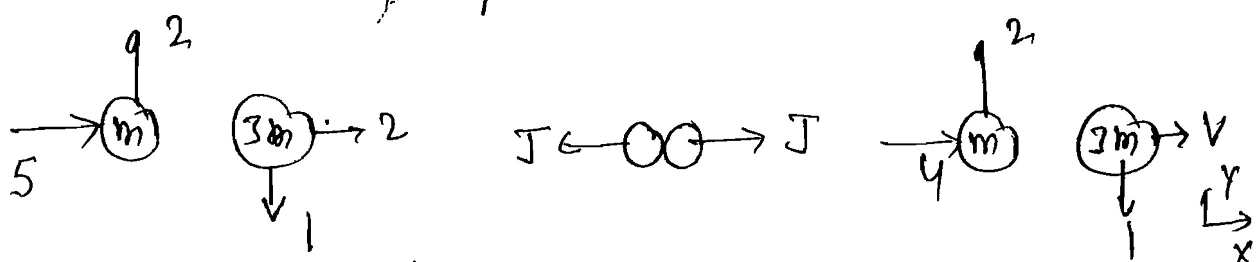
magnitude of velocity components in $\hat{J}$ dirn

$$= \left(4\hat{i}-\hat{j}\right)\cdot\frac{1}{5}\left(-3\hat{i}+4\hat{j}\right) = -16/5 \ (\text{Before})$$

$$\& \quad \left(\hat{i}+3\hat{j}\right)\cdot\frac{1}{5}\left(-3\hat{i}+4\hat{j}\right) = \frac{9}{5} \ (\text{After})$$

$$e = \frac{9/5}{|16/5|} \Rightarrow e = 9/16.$$

24)



conservation of linear momentum

$$5m + 3m(2) = mu + 3mV \quad \text{(i)}$$

Definition of e: $\quad \frac{5-2}{3} = V-u \quad$ (ii)

$$\boxed{u = 2,\ V = 3}$$

$$\vec{V}_A = 2\hat{i}+2\hat{j}, \quad \vec{V}_B = 3\hat{i}-\hat{j}.$$

Befor collision $\quad (KE)_A = \frac{1}{2}m\left(5^2+2^2\right) = 29m/2$

$\quad (KE)_B = \frac{1}{2}\cdot 3m\left(1^2+2^2\right) = 15m/2$

After collision $\quad (KE)_A = \frac{1}{2}m\left(2^2+2^2\right) = 4m$

$\quad (KE)_B = \frac{1}{2}\cdot 3m\left(3^2+1^2\right) = 15m$

Loss in KE $= 3m$.

For sphere B: $J = 3m(3-2) = 3m$.

(25) unit vector along line of impact

$$\hat{a} = \frac{1}{\sqrt{2}}(\hat{i} - \hat{j})$$

unit vector ⊥r to line of impact

$$\hat{b} = \frac{1}{\sqrt{2}}(\hat{i} + \hat{j}).$$

Sphere A

component of $\vec{V}_A$ along $\hat{a} = (\hat{i} + 2\hat{j}) \cdot \hat{a} = -\frac{1}{\sqrt{2}}$

component of $\vec{V}_A$ along $\hat{b} = (\hat{i} + 2\hat{j}) \cdot \hat{b} = 3/\sqrt{2}$

Sphere B

component of $\vec{V}_B$ along $\hat{a} = (-\hat{i} + 3\hat{j}) \cdot \hat{a} = -2\sqrt{2}$

component of $\vec{V}_B$ along $\hat{b} = (-\hat{i} + 3\hat{j}) \cdot \hat{b} = \sqrt{2}$

conservation of momentum & defn. of e along line of impact

$$m \cdot \frac{1}{\sqrt{2}} + 2m(2\sqrt{2}) = mu + 2mV$$

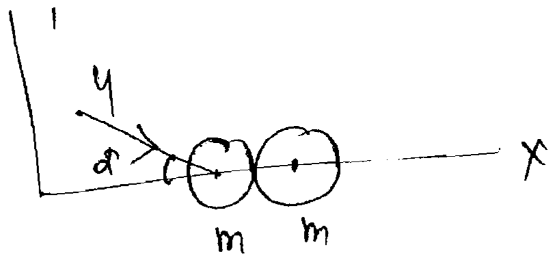
$$\frac{1}{2}\left(2\sqrt{2} - \frac{1}{\sqrt{2}}\right) = u - V$$

$$\Rightarrow \boxed{u = 2\sqrt{2},\ V = \frac{5}{4}\sqrt{2}}$$

$$(\vec{V}_A)_{\text{after collision}} = -u\hat{a} + \frac{3}{\sqrt{2}}\hat{b} = \frac{1}{2}(-\hat{i} + 7\hat{j})$$

$$(\vec{V}_B)_{\text{after collision}} = -V\hat{a} + \sqrt{2}\hat{b} = \frac{1}{4}(-\hat{i} + 9\hat{j})$$

(20)



momentum conservation along x-axis

$$m u \cos\alpha = m V_{1x} + m V_{2x}$$

$$\boxed{V_{1x} + V_{2x} = u\cos\alpha}$$

At the moment of max^m. deformation

$$V_{1x} = V_{2x}$$

$$\Rightarrow \boxed{V_{1x} = \frac{u\cos\alpha}{2}}$$

$$(KE)_{in} = \frac{1}{2} m u^2$$

$$(KE)_f = \frac{1}{2} m V_{1x}^2 + \frac{1}{2} m V_{1y}^2 + \frac{1}{2} m V_{2x}^2$$

$$= 2 \cdot \frac{1}{2} m V_{1x}^2 + \frac{1}{2} m V_{1y}^2$$

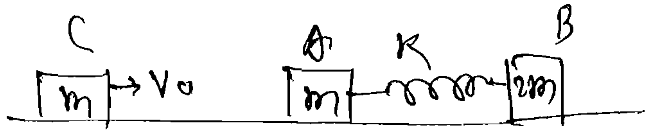
$$= m\frac{u^2\cos^2\alpha}{4} + \frac{m u^2 \sin^2\alpha}{2}$$

$$= \frac{3mu^2}{8} \quad (\alpha = 45^\circ)$$

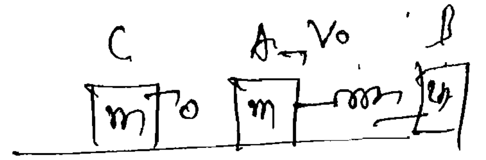
$$PE = (KE)_{in} - (KE)_f = \frac{mu^2}{2} - \frac{3mu^2}{8} = \frac{mu^2}{8}$$

$$\text{fraction} = \frac{mu^2/8}{mu^2/2} = 1/4 = 0.25$$

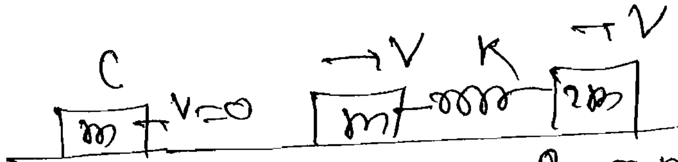
27)



Before collision



Just after collision



At max. compression

(i) conservation of linear momentum

$$mV_0 = (m+2m)V$$

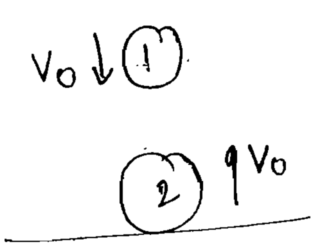
$$V = V_0/3$$

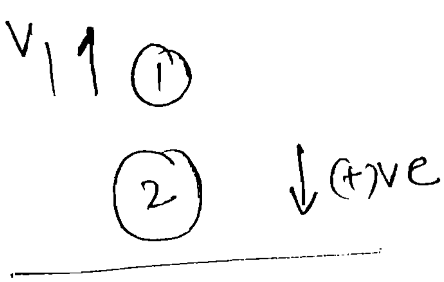
(ii) conservation of mechanical energy

$$\frac{1}{2}mV_0^2 = \frac{1}{2}(m+2m)V^2 + \frac{1}{2}Kx_0^2$$

$$\boxed{K = \frac{2}{3}\frac{mV_0^2}{x_0^2}}$$

28) Assuming balls are slightly separated when the superball hits the floor.





$V_0 = \sqrt{2gh}$

v_1 = velocity of ball (1) just after collision

$$v_1 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)u_1 + \left(\frac{2m_2}{m_1+m_2}\right)u_2$$

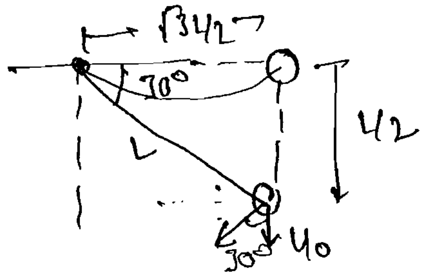
$$u_1 = v_0, \quad u_2 = -v_0, \quad \frac{m_1}{m_2} = \frac{m}{m} \approx 0$$

$$v_1 = -3v_0 = -3\sqrt{2gh}$$

H_1 = Ht. gained by ball 1

$$= \frac{v_1^2}{2g} = 9h.$$

(29)



$$u_0 = \sqrt{2 \cdot g \cdot \frac{L}{2}} = \sqrt{gL}$$

(i)

An impulsive force will act when string gets its natural length. component of u_0 along string will be zero, while component of u_0 $\perp^r$ to string will be same

$$(u_0)_\perp = u_0 \cos 30^\circ = u_0 \cdot \frac{\sqrt{3}}{2}$$

velocity of m_1 just before striking m_2 is

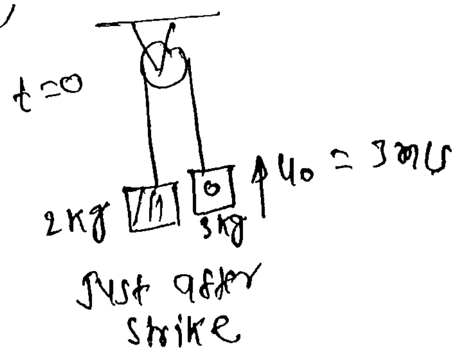
$$u_1 = \sqrt{\left(u_0 \frac{\sqrt{3}}{2}\right)^2 + 2 \cdot g \cdot \frac{L}{2}}$$

$$= \sqrt{\frac{7}{4} gL} = \sqrt{\frac{7}{4} \cdot 10 \cdot \frac{32}{5}} = 4\,m/s$$

(ii) $$V_{3m} = \frac{2m}{m+3m} \cdot 4 = 2\,m/s$$

$$(H_{3m})_{max} = \frac{(2)^2}{2g} = 0.20\,m.$$

30)



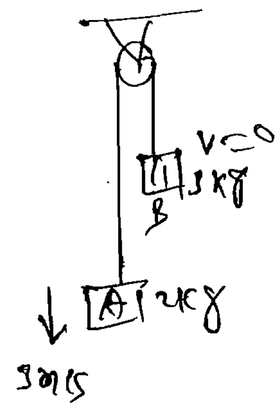
velocity of 3 kg will be zero

at $t = \frac{u_0}{g} = 0.3$ sec

Ht. gained by B $= \frac{3^2}{2g} = 0.45$ m

velocity of A at t = 0.3 sec

$= g(0.3) = 3$ m/s



An impulsive force (Tension) will act and after that both A & B will be moving together

$$3(2) = (3+2)V$$

$$\Rightarrow \boxed{V = 1.2 \text{ m/s}}$$

System will under retardation

$$a = \frac{3g - 2g}{3+2} = 2 \text{ m/s}^2$$

velocity of B will become zero

at $t = \frac{1.2}{2} = 0.6$ sec

Ht. gained further $= \frac{(1.2)^2}{2(2)} = 0.36$ m

(i) Time when B starts moving down

$= 0.3 + 0.6 = 0.9$ sec

(ii) max^m Ht. reached by B $= 0.45 + 0.36$

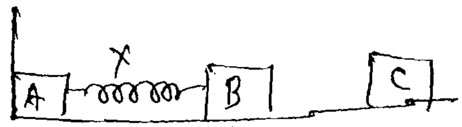
$= 0.81$ m

(iii) Loss $= \frac{1}{2} \cdot 1 \cdot (9)^2 - (3-2) \cdot 10 \cdot (0.81)$

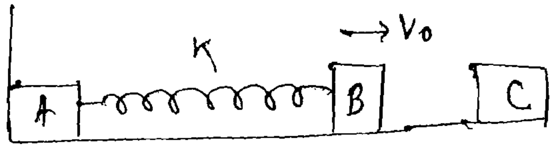
$= 40.5 - 8.1$

$= 32.4$ Joule

(vi)

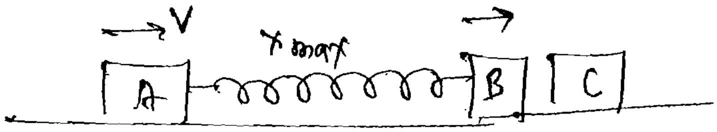


compressed



spring: natural length

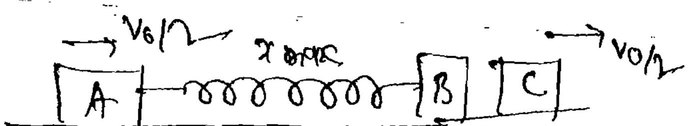
$$\frac{1}{2}kx^2 = \frac{1}{2}mv_0^2 \quad \text{(i)}$$



max^m extension in spring

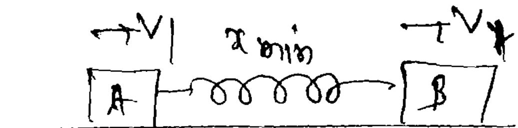
$$mv_0 = (m+m)V$$

$$\boxed{V = v_0/2}$$



just after collision

$$\frac{1}{2}mv_1^2 = \frac{1}{2}\cdot 2mV^2 + \frac{1}{2}k\,x_{max}^2 \quad \text{(ii)}$$



max^m compression in spring

$$m\frac{v_0}{2} = (m+m)V_1$$

$$\boxed{V_1 = v_0/4}$$

$$\frac{1}{2}m\left(\frac{v_0}{2}\right)^2 + \frac{1}{2}k\,x_{max}^2 = \frac{1}{2}\cdot 2m\cdot\left(\frac{v_0}{4}\right)^2 + \frac{1}{2}k\,(x_{min})^2 \quad \text{(iii)}$$

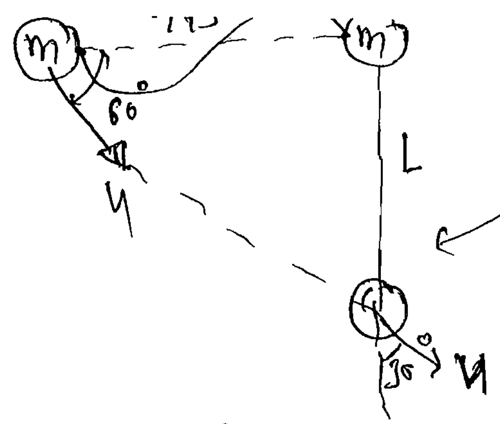
solving (i), (ii) & (iii)

$x_{min} = \sqrt{\frac{5}{8}}\,x$: max^m compression

and hence min^m separation bet^n A & B

$$= L - \sqrt{\frac{5}{8}}\,x.$$

32)



string will be in its natural length.

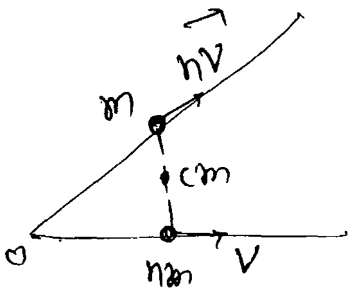
conservation of linear momentum along string

$mu \cos 30° = (m+m) V$

$V = \frac{u}{2} \cos 30° = \frac{\sqrt{3} u}{4}$

$J = mV = \frac{\sqrt{3} m u}{4}$ = impulse of the tension

33)



$V_1 = V_2 = V$

$\vec{V}_{cm} = \frac{nm\vec{V_1} + m(n\vec{V_2})}{m+nm} = \frac{n}{n+1}(\vec{V_1} + \vec{V_2})$

$|\vec{V}_{cm}| = \frac{n}{n+1}|\vec{V_1} + \vec{V_2}|$

$= \frac{n}{n+1}\sqrt{V_1^2 + V_2^2 + 2V_1V_2 \cos\varphi}$

$= \frac{n}{n+1} \cdot 2V \cos(\varphi/2)$

(34) For shell

$$H = \frac{u^2 \sin^2\theta}{2g} = 15 m$$

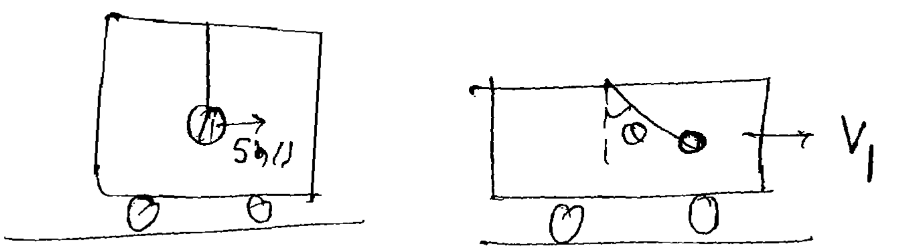
Hence it will strike the ball at its heighest position of its trajectory.

V_{shell} just before collision $= u\cos 60^\circ = 10$ m/s

collision betn shell & ball

conservation of linear momentum

$$1 \cdot 10 = (1+1)V \Rightarrow \boxed{V = 5 \text{ m/s}}$$



Just after collision

At highest point combined mass is at rest relative to trolley. Let V_1 be the their common speed then

$$2 \times 5 = (2+18) V_1 \Rightarrow \boxed{V_1 = \frac{1}{2} \text{ m/s}}$$

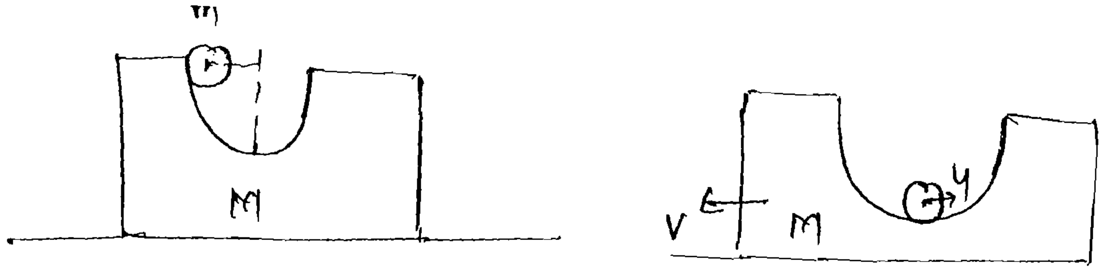
mechanical Energy conservation

$$\frac{1}{2} \cdot 2 \cdot (5)^2 - \frac{1}{2} \cdot (2+18)\left(\frac{1}{2}\right)^2 = 2 \cdot 10 \cdot 1 (1-\cos\theta)$$

$$\cos\theta = 0.125$$

$$\boxed{\theta \simeq 82.82^\circ}$$

(35)



(a) CM should not shift along horizontal direction

$m(R-r-x) = Mx$ x : movement of M towards left.

$\Rightarrow \boxed{x = \frac{m(R-r)}{M+m}}$

(b) conservation of mechanical energy

$mg(R-r) = \frac{1}{2}mu^2 + \frac{1}{2}MV^2$ — (I)

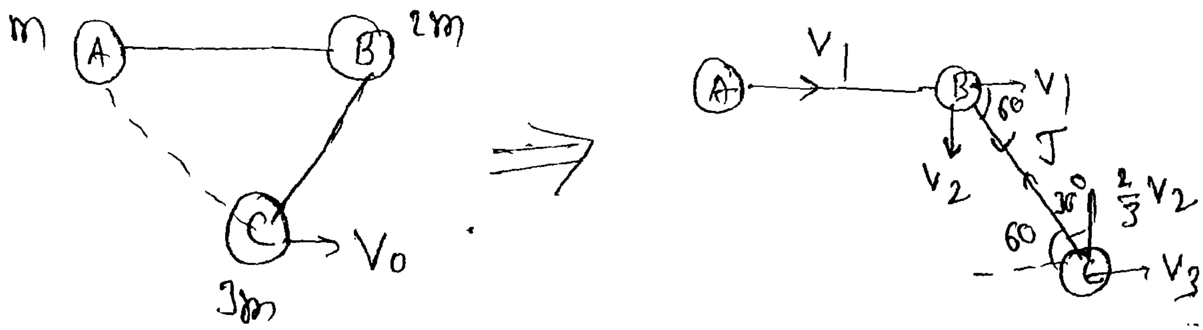
conservation of linear momentum

$mu = MV$ — (II)

solving (I) & (II)

$V = m\sqrt{\frac{2g(R-r)}{M(m+M)}}$.

(36)



momentum conservation along horizontal

$3mV_0 = mV_1 + 2mV_1 + 3mV_3$

$V_0 = V_1 + V_3$ — (I)

[illegible] of velocity along [illegible] of particles B
and C will be equal

$$v_1 \cos 60^\circ + v_2 \cos 30^\circ = v_3 \cos 60^\circ - \frac{2}{3} v_2 \cos 30^\circ$$

$$v_1 - v_3 + \frac{5}{\sqrt{3}} v_2 = 0 \quad \text{— (I)}$$

Impulse = change in linear momentum

$$J \cos 60^\circ = 3m(v_0 - v_3)$$

$$J/2 = 3m(v_0 - v_3) \quad \text{— (III)}$$

$$J \cos 30^\circ = 3m\left(\frac{2}{3} v_2\right)$$

$$\frac{\sqrt{3} J}{2} = 2mv_2 \quad \text{— (IV)}$$

solving (I), (II), (III) & (IV)

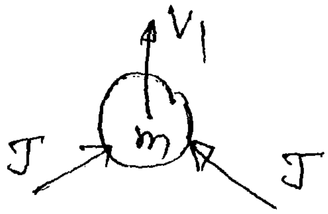
$$\boxed{v_1 = \frac{2v_0}{19}}$$

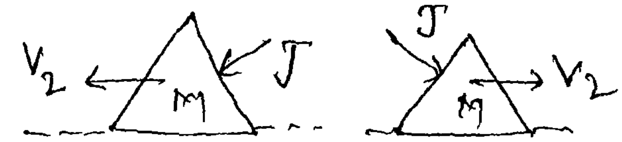
37)

$$2J\sin 30^\circ = mv_1 - (-mv_0)$$

$$J = m(v_1 + v_0) \quad \text{— (I)}$$

$$J\cos 30^\circ = Mv_2 \quad \text{— (II)}$$





From (I) & (II):

$$\boxed{\frac{2}{\sqrt{3}}Mv_2 = mv_1 + mv_0} \quad \text{— (III)}$$

$$e = \frac{v_1\cos 60^\circ + v_2\cos 30^\circ}{v_0\cos 60^\circ} = \frac{v_1 + \sqrt{3}v_2}{v_0} \quad \text{— (IV)}$$

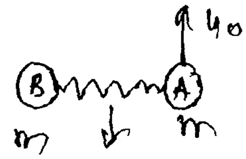
Solving (III) & (IV)

$$v_1 = \frac{(2eM - 3m)v_0}{2M + 3m}$$

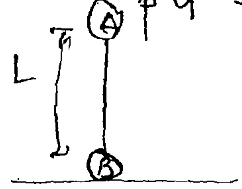
$$v_2 = \frac{\sqrt{3}(1+e)mv_0}{2M + 3m}.$$

38)

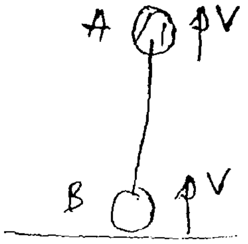
$$u = \sqrt{u_0^2 - 2gL} = \sqrt{8gL}$$



length of string = L



string just before tightening

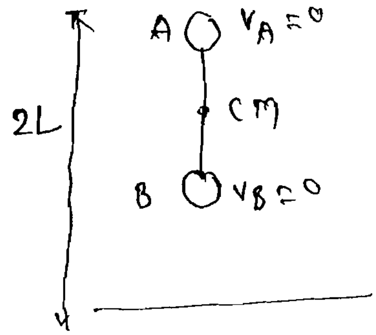


$$2mv = mu \Rightarrow v = \frac{u}{2} = \sqrt{2gL}$$

just after impulsive tension

$v_{cm} = \frac{mv + mv}{m+m} = v$

Maximum height attained by cm $= \frac{v_{cm}^2}{2 g_{m}}$



$= \frac{v^2}{2g} = L$

Height of A above ground surface $= 2L$

While returning velocity of A when it strikes the ground surface

$v_A = \sqrt{2g \cdot 2L} = \sqrt{4gL} = 2\sqrt{gL}$

39) a) Let v_1 = velocity of block after bullet just emerges from it

$\frac{1}{2} M v_1^2 = \frac{1}{2} K x^2 \Rightarrow v_1 = x\sqrt{\frac{K}{M}} = 1.5$ m/s

Now Let v_2 = speed of bullet when it comes out

$m u_0 = m v_2 + M v_1$

$5 \times 10^{-3} \times 400 = 5 \times 10^{-3} v_2 + 1 \times 1.5$

$\boxed{v_2 = 100 \text{ m/s}}$

(b) $E_{in} = \frac{1}{2} m u_1^2$

$E_f = \frac{1}{2} K x^2 + \frac{1}{2} m v_2^2$

Loss $= E_{in} - E_f$

≈ 379 J.

(40) Net force on table by the chain

= Thrust force + wt. of portion chain at that moment

$= v_r \frac{dm}{dt}$ + weight of x length of chain

$= v_r(\lambda v_r) + \frac{Mgx}{L}$

$= \frac{M}{L} v_r^2 + \frac{Mgx}{L}$

$= \frac{M}{L} . 2gx + \frac{Mgx}{L} = \frac{3Mgx}{L}$

The table will apply same force on chain in vertical upward direction.

(41) In variable mass system

$\vec{F}_{net} = \vec{F}_{ext} + \vec{F}_{Th}$.

$$\boxed{m\frac{d\vec{v}}{dt} = \vec{F}_{ext} + \vec{F}_{Th}}$$

where $\vec{F}_{Th}$ = Thrust force $= \vec{v}_{rel} . \frac{dm}{dt}$

m = mass at time "t"

$\vec{F}_{ext}$ = external force acting on main mass

$$\boxed{m\frac{dv}{dt} = \vec{F} - u\mu}$$

$\mu = \frac{dm}{dt}$

u = speed of jet relative to rocket.

(42) For rocket

$$\boxed{m\frac{dv}{dt} = v_r\left(-\frac{dm}{dt}\right) - mg}$$

$$dv = v_r\left(-\frac{dm}{m}\right) - g\,dt$$

$$\int_u^v dv = v_r\int_{m_0}^{m}\frac{-dm}{m} - g\int_0^t dt$$

$$\boxed{v = u - gt + v_r \ln\left(\frac{m_0}{m}\right)}$$

In this question $u = 0$, $g = 0$, $v_r = u$

$$\boxed{v(t) = u \ln(m_0/m).}$$

(43) (a) sand's rate of change of linear momentum

$= v_{rel}\cdot\frac{dm}{dt} = (0.75)5 = 3.75\ kg.m/s^2$

(b) since speed is const

$F_f = v_{rel}\cdot\frac{dm}{dt} = 3.75\ N$

(c) $F_{ext} = F_f = 3.75\ N$

(d) work done by F_{ext} in 1 sec

$= F_{ext}\cdot s = 3.75 \times 0.75 = 2.81\ J$

(e) K.E acquired by sand each sec

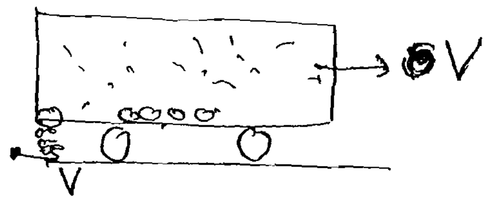
$= \frac{1}{2}\left(\frac{dm}{dt}\right)v^2$

$= \frac{1}{2}\cdot 5 \times (0.75)^2$

$\simeq 1.41$ Joule.

(f) Energy gets converted into internal energy due to friction

(44)



$V_r \simeq 0$

sand spills through a hole in the bottom of the cart hence relative velocity of sand, $V_r = 0 \Rightarrow F_{Th} = 0$

$$F_{net} = F$$

$$m\left(\frac{dv}{dt}\right) = F \quad , \text{where} \quad m = m_0 - bt.$$

$$dv = \frac{F\,dt}{m} = \frac{F\,dt}{m_0 - bt}$$

$$\int_0^v dv = F\int_0^t \frac{dt}{m_0 - bt}$$

$$\boxed{v = \frac{F}{b}\ln\left(\frac{m_0}{m_0 - bt}\right)}$$

$$\boxed{a = \frac{dv}{dt} = \frac{F}{m} = \frac{F}{m_0 - bt}}$$

(45.)

(a) chain has a constant speed.
Net force on it should be zero.

F = Wt. of length y of chain + Thrust force

$= \left(\frac{M}{L}\right) g y + \left(\frac{M}{L}\right) v_0^2$

$= \frac{M}{L}\left(g y + v_0^2\right)$

(b) Reaction of the floor

= Wt. of length (L−y) of chain

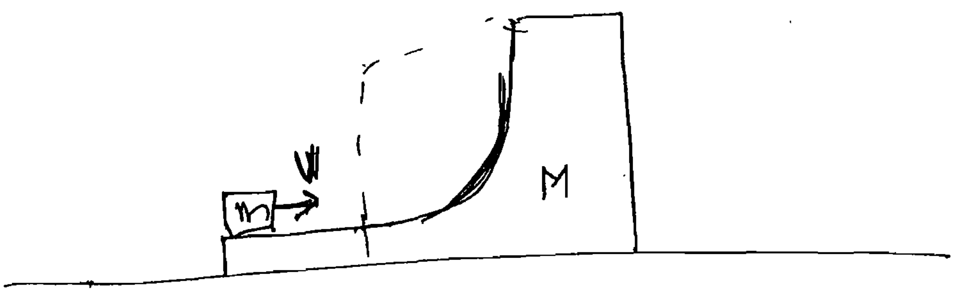
$= Mg\left(1 - \frac{y}{L}\right)$

(c) Energy lost during the lifting

= work done by applied force − increase in mechanical energy of chain

$= Fy - \left(\frac{M}{L} y\right) g \left(\frac{y}{2}\right) - \frac{1}{2}\left(\frac{M}{L} y\right) v_0^2$

$= \frac{My}{2L}\left(g y + v_0^2\right)$.


m
v
M