Ex. 1(A) ①

① equation can be written as $(x-2)^2+(y-3)^2=(3)^2$

(D) $\therefore r=3$, $2r=6$

② In equation of circle coefficient (x^2) = Coefficient (y^2) and cofficient(xy)

(D) $=0$

~~③ radius $= \sqrt{(3-2)^2+(6-(-1))^2} = \sqrt{50}$~~

~~$f=-3$, $g=-6$~~

~~$f^2+g^2-C=r^2$~~

~~$\Rightarrow 3^2+6^2-C=50$~~

~~$\Rightarrow C=-5$~~

~~$\therefore x^2+y^2+2(-3)x+2(-6)y-5=0$~~

③ radius $= \sqrt{(3-2)^2+(6-(-1))^2} = \sqrt{50}$

(A) $f=-2$, $g=1$

$f^2+g^2-C=r^2 \Rightarrow 4+1-C=50$

$\Rightarrow C=-45$

$\therefore x^2+y^2+2(-2)x+2(1)y-45=0$

④ diametric form: $(x-x_1)(x-x_2)+(y-y_1)(y-y_2)=0$

$(x-4)(x+12)+(y-3)(y+1)=0$

(B) $x^2+y^2+8x-2y-51=0$

⑤ ~~x-intercept $=1$,~~

~~$\Rightarrow 2\sqrt{g^2-C}=1 \Rightarrow g^2-C=\frac{1}{4} \Rightarrow g$~~

Let centre be (h,k)

$(h-1)^2+(k-0)^2=(h-1)^2+k^2$

$\Rightarrow h=\frac{1}{2}$.

$(h-0)^2+(k-0)^2=h^2+(k-1)^2 \Rightarrow k=\frac{1}{2}$.

Centre $\left(\frac{1}{2}, \frac{1}{2}\right)$ radius $=\sqrt{\left(\frac{1}{2}-0\right)^2+\left(\frac{1}{2}-0\right)^2}$

$=\sqrt{\frac{1}{4}+\frac{1}{4}}=\frac{1}{\sqrt{2}}$

②

⑥. $x^2+y^2-2gx+f^2=0$ → ~~Centa~~ Centre is $(g,0)$.

∴ radius = $\sqrt{(a-g)^2+b^2}$.

⑦ $(x,3)$ $(1,5)$ ends of diameter

∴ centre $=\left(\frac{x+1}{2}, \frac{3+5}{2}\right) = \left(\frac{x+1}{2}, 4\right)$

$\left(\frac{x+1}{2}, 4\right) = (2,4)$ ⇒ $\frac{x+3}{2}=2$ ⇒ $x=1$

$y=4$

⑧ ~~diametral~~ diametric form $(x-x_1)(x-x_2)+(y-y_1)(y-y_2)=0$.

$(x-0)(x-1)+(y-1)(y-1)=0$

⇒ $x^2-x+y^2-2y+1=0$.

⑨. parametric form

$x=r\cos\alpha$, $y=r\sin\alpha$.

radius $=2$ ⇒ $(2\cos\alpha, 2\sin\alpha)$.

⑩. $x^2+y^2-4x-4y=0$.

centre $(2,2)$ radius $=2\sqrt{2}$.

parametric form $=(2+2\sqrt{2}\cos\alpha, 2+2\sqrt{2}\sin\alpha)$.

(11). $x^2+y^2-2x+2y-2=0$.

centre $(1,-1)$ radius $= 2$.

parametric form $= (1+2\cos\alpha, -1+2\sin\alpha)$.

(12). $k(x^2+y^2)-x-y+k=0$.

$k\neq 0$ radius $= \sqrt{\frac{1}{4k^2}+\frac{1}{4k^2}-1}$

$= \sqrt{\frac{1-2k^2}{k^2}}$

~~$2-k^2>0$~~. $1-2k^2>0$ $\Rightarrow k\in\left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$.

$\Rightarrow 0<|k|<\frac{1}{\sqrt{2}}$.

(13). $px^2+(2-q)xy+3y^2-6qx+30y+6q=0$

$2-q=0$ for the equation to be a circle

$\Rightarrow$ $q=2$.

$px^2+3y^2-12x+30y+12=0$. x^2, y^2 coefficient should be equal

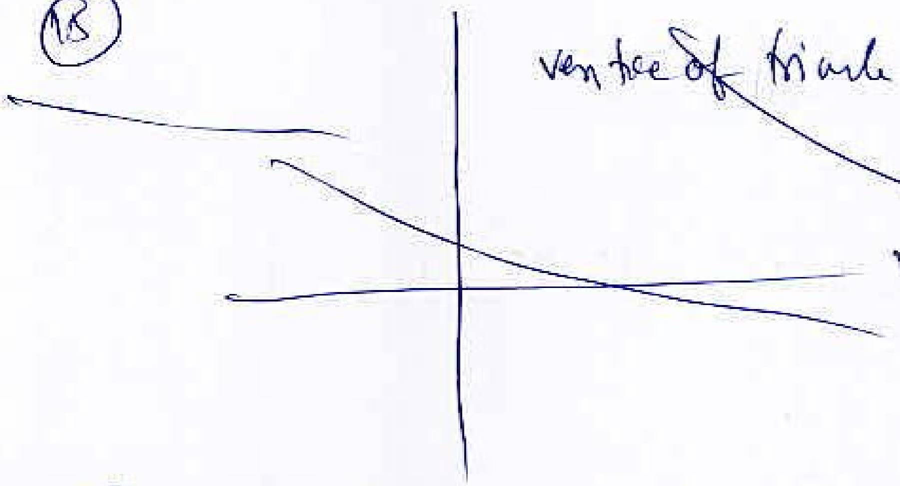
$\Rightarrow$ $p=3$

(14)

g^2+f^2 point circle radius $=0$

$\Rightarrow g^2+f^2-c=0 \Rightarrow g^2+f^2=c$

(15)



vertice of triangle will be $(0,0), (a,0)$ $(0,-b)$.

right angle at $(0,0)$.

(b) vertices of triangle are

$(0,0)$ $(a,0)$ $(0,-b)$.

right angle at $(0,0)$

circumcentre is mid point of hypotenuse

$= \left(\frac{a}{2}, -\frac{b}{2}\right)$

and radius $= \sqrt{\left(\frac{a}{2}\right)^2 + \left(-\frac{b}{2}\right)^2}$

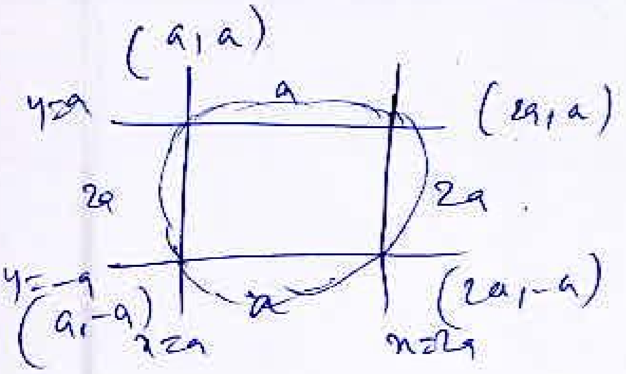
$\Rightarrow$ eqn of circle

$$\left(x - \frac{a}{2}\right)^2 + \left(y + \frac{b}{2}\right)^2 = \left(\sqrt{\left(\frac{a}{2}\right)^2 + \left(\frac{b}{2}\right)^2}\right)^2.$$

$$\Rightarrow x^2 - ax + \frac{a^2}{4} + y^2 + by + \frac{b^2}{4} = \frac{a^2}{4} + \frac{b^2}{4}$$

$$\Rightarrow x^2 + y^2 - ax + by = 0.$$

(16)



Centre of circle will be point of intersection of diagonals.

$\therefore$ Centre $= \left(\frac{a+2a}{2}, \frac{a-a}{2}\right) = \left(\frac{3a}{2}, 0\right)$

radius $= \sqrt{\left(\frac{3a}{2} - a\right)^2 + (0-a)^2}$

$\Rightarrow$ circle eqn : $\left(x - \frac{3a}{2}\right)^2 + (y-0)^2 = \left(\frac{a}{2}\right)^2 + a^2$.

$x^2 - 3ax + \frac{9a^2}{4} + y^2 = \frac{5a^2}{4}$.

$x^2 + y^2 - 3ax + a^2 = 0$.

(17). $(x-x_1)(x-x_2) + (y-y_1)(y-y_2) = 0$

$\Rightarrow x^2 - x(x_1+x_2) + x_1x_2 + y^2 - y(y_1+y_2) + y_1y_2 = 0$.

$x_1 + x_2 = -2 \quad x_1x_2 = -a^2 \quad y_1+y_2 = -4, \; y_1y_2 = -b^2$.

$\Rightarrow x^2 - x(-2) - a^2 + y^2 - y \; (-4) + -b^2 = 0$.

$\Rightarrow x^2 + y^2 + 2x + 4y - a^2 - b^2 = 0 \Rightarrow (x+1)^2 + (y+2)^2 = 5 + a^2 + b^2$

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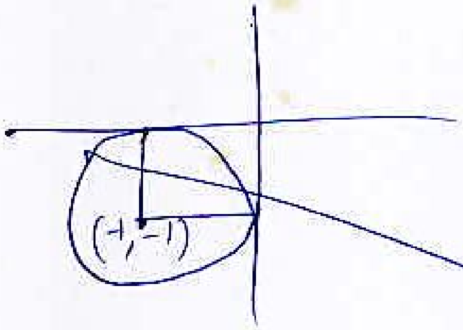
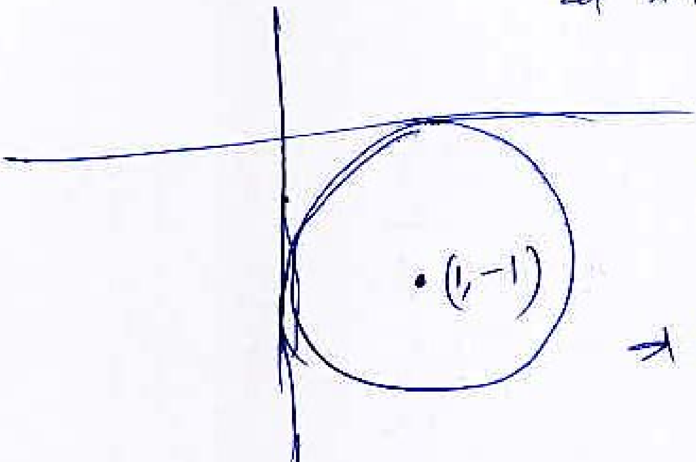


$(x+1)^2+(y+1)^2=1.$

$x^2+2x+1+y^2+2y+1=1$

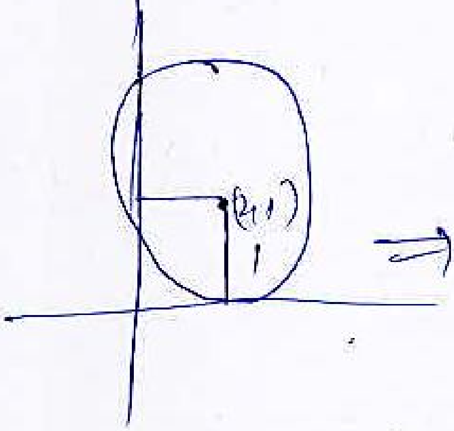
$\Rightarrow x^2+y^2+2x+2y+1=0.$



$(x-1)^2+(y+1)^2=1$

$\Rightarrow x^2+y^2-2x+2y+1=0.$

19.



$\Rightarrow$ radius = 1.

$\Rightarrow (x-2)^2+(y-1)^2=1.$

$\Rightarrow x^2-4x+y^2-2y+4=0.$

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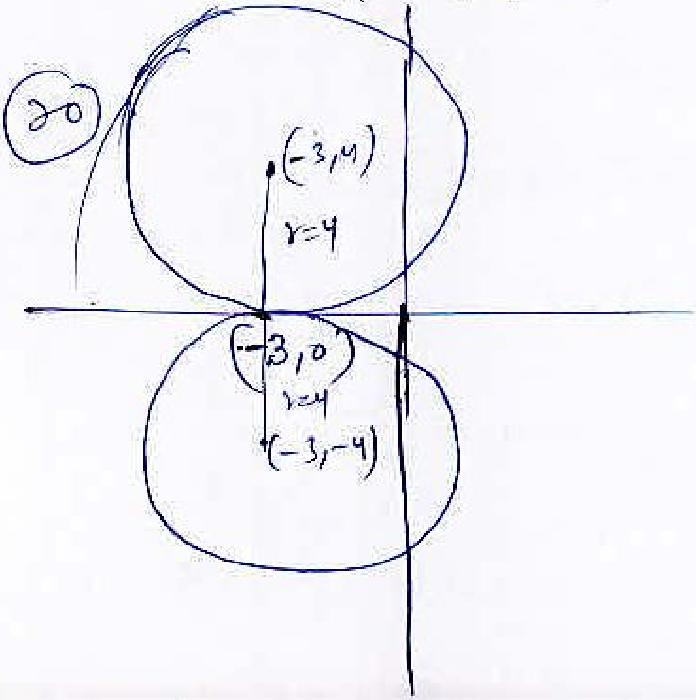
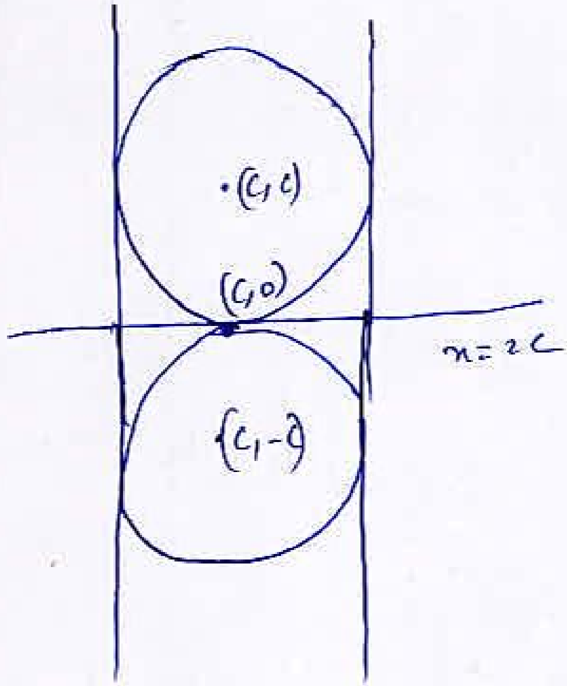


$\Rightarrow (x-3)^2+(y\pm4)^2=4^2$

$\Rightarrow x^2-6x+y^2\pm8y+9=0.$

(21)



$(x-c)^2+(y\pm c)^2=c^2$

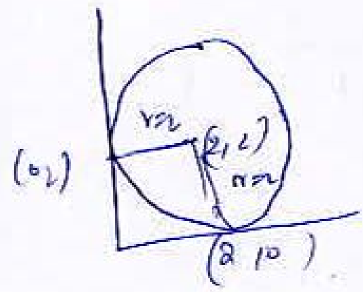
$x^2+y^2-2xc\pm 2yc+c^2=0.$

(22)

$x^2+y^2-4x-4y+4=0.$

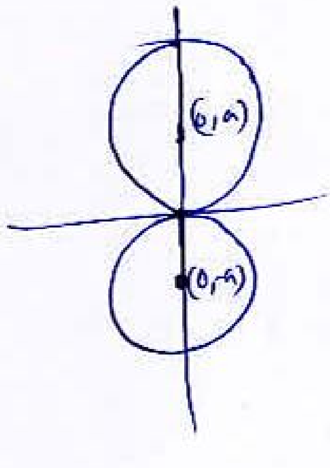
$(x-2)^2+(y-2)^2=4$

Touches both axis.



(23).

$x^2+y^2+2gx+2fy+c=0.$



$x^2+(y\pm a)^2=a^2$

$x^2+y^2\pm 2ay=0$

(24)

$x^2+y^2-2x-6y+9=0.$

$x=0 \Rightarrow y^2-6y+9=0 \Rightarrow (y-3)^2=0 \Rightarrow \quad (0,3)$

(25) $x^2+y^2+6y=0$.

passes through origin.

~~If $x=0 \Rightarrow y(y+6)=0 \Rightarrow y=0,-6$~~

If $y=0 \Rightarrow x=0$ ∴ mean touches x-axis at origin.

(26) ~~$x^2+y^2-x+y-1=0$~~

~~centre $\left(\frac{1}{2},-\frac{1}{2}\right)$ radii $=\sqrt{\frac{1}{4}+\frac{1}{4}+1}$~~

~~$=\sqrt{\frac{3}{2}}=\sqrt{1.5}$.~~

~~distance between centre & (1, 1)~~

~~$S_1=\sqrt{\frac{1}{4}+\frac{9}{4}}=\sqrt{2.5}$.~~

~~$S_1>r$ mean point is ~~inside~~ outside the circle.~~

(27). $S_{(0.1,3.1)} = 0.1^2+3.1^2-2\times0.1-4\times3.1$

(26). $(1,1)$ $S=x^2+y^2-x+y-1=0$.

$S(1,1) = 1+1-1+1-1=1>0$

∴ point lies outside the circle.

(27) $S(0.1, 3.1) = 0.1^2 + 3.1^2 - 2\times 0.1 - 4\times 3.1 + 3$

$= 0.02 > 0$

So $(0.1, 3.1)$ lies outside the circle.

(28) $(x-2)+(y+3)=0$, $(x-2)^2+(y-3)^2=11$.

centre of circle $=(2,3)$

$\perp^r$ distance from $(2,3)$ to line $(x-2)+(y+3)$

$$\frac{|(2-2)+(3+3)|}{\sqrt{2}} = \frac{|6|}{\sqrt{2}} = 3\sqrt{2}.$$

radius of circle $= \sqrt{11}$.

$\perp^r$ distance $= 3\sqrt{2} > \sqrt{11}$

$\therefore$ line ~~doesn't~~ doesn't touch the circle

(29) $3x+4y=m$, $x^2+y^2-10x=0$.

centre $=(5,0)$

radius $= 5$.

$\perp^r$ distance to $3x+4y=m$ from $(5,0)$

$$\frac{|15-m|}{5} = 5 \rightarrow \text{as it touches the circle}$$

$\Rightarrow 15-m=25$ (or) $15-m=-25$

$\Rightarrow m=-10,\ m=40$

(30) $x^2+y^2-4x-8y-5=0$.

$3x-4y=m$

centre $(2,4)$ radius $=\sqrt{25}=5$.

⊥r distance to line $3x-4y-m=0$.

$$d=\frac{|3\times 2-4\times 4-m|}{5}<5$$ to intersect the circle

$$\frac{|-m-10|}{5}<5.$$

$-25<m+10<25$.

$-35<m<15$.

(31).

$x+y=1$ meets circle $x^2+y^2=1$

at $(1,0)$ & $(0,1)$

∴ length of intercept $=\sqrt{2}$.

(32).

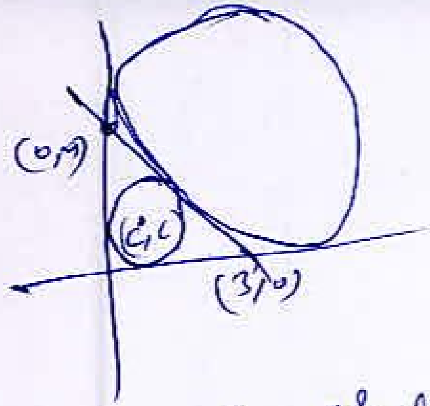
$x^2+y^2=r^2$ → eqn of circle with centre $(0,0)$ & radius r.

⊥r distance from $(0,0)$ to $5x+12y=1$ will be the radius of circle. $=\frac{|5\times 0+12\times 0-1|}{13}$

$=\frac{1}{13}=r$

⇒ $x^2+y^2=\frac{1}{169}$ ⇒ $169(x^2+y^2)=1$

(33)



radius $= \sqrt{c^2+c^2-c^2}$

$= c.$

$\therefore$ $\perp^r$ distance from (c,c) to line $\frac{x}{3}+\frac{y}{4}=1$ is c.

$\Rightarrow \frac{|4c+3c-12|}{5} = c$

$\Rightarrow |7c-12| = 5c$

$\Rightarrow 7c-12=5c \quad , 7c-12=-5c$

$\Rightarrow c=6, \quad c=1.$

~~but c=6 lies outside the $\triangle$ (0,0) (3,0), (0,4)~~

(34).

$x^2+y^2-2x+4y-4=0 \quad . \quad 2x-y+1=0.$

Centre $(1,-2)$ radius $= 3.$

$\perp^r$ distance from $(1,-2)$ to $2x-y+1$

$= \frac{|2\times1-(-2)+1|}{\sqrt5} = \frac{5}{\sqrt5} = \sqrt5.$

$\perp^r d(\sqrt5) < r(3)$ $\therefore$ the line is a chord.

3

(12)

(35). $y = x + c$, $x^2 + y^2 = 1$.

~~Centre (0, 0) r = 1~~

Substituting this in $x^2 + y^2 = 1$

$\Rightarrow x^2 + (x+c)^2 = 1 \Rightarrow 2x^2 + 2xc + c^2 - 1 = 0$.

for coincident points $D = 0$

$\Rightarrow 4c^2 - 4(c^2-1)\times 2 = 0$.

$\Rightarrow c = \pm\sqrt{2}$

(36). $x + y = 1$. Centre $(2, 3)$

mean ⊥^{r} distance from (2,3) to $x+y=1$ is radius

$$\Rightarrow r = \frac{|2+3-1|}{\sqrt{2}} = \frac{4}{\sqrt{2}} = 2\sqrt{2}.$$

∴ eqn of circle. $(x-2)^2 + (y-3)^2 = (2\sqrt{2})^2$.

$x^2 + y^2 - 4x - 6y + 5 = 0$.

(37). $12x - 5y - 17 = 0$, $24x - 10y + 44 = 0$.

both tangent are parallel

means distance between them will be diameter of circle. $\Rightarrow d = \frac{|22+17|}{13} = \frac{39}{13} = 3 \Rightarrow r = \frac{d}{2} = \frac{3}{2}$.

(38). $x^2+y^2=a^2$

Centre (0,0) ⊥r distance to $y=mx+c$

$$d=\frac{|c|}{\sqrt{m^2+1}}.$$

Chord length $=2\sqrt{r^2-d^2}$

$$=2\sqrt{a^2-\frac{c^2}{m^2+1}}=2b.$$

$\Rightarrow b^2(m^2+1)=a^2(m^2+1)-c^2.$

$\Rightarrow c^2=m^2+1(a^2-b^2).$

(39) $lx+my+n=0$. tangent to $x^2+y^2=r^2$

$(0,0)\rightarrow$ ⊥r to $lx+my+n=0$.

$$d=\frac{|n|}{\sqrt{l^2+m^2}}=r.$$

$\Rightarrow n^2=r^2(l^2+m^2)$

(40). $x^2+y^2=25$. $\theta=60^\circ$

$\Rightarrow$ slope of tangent $=m=\tan 60=\sqrt{3}$.

$\Rightarrow$ eqn of ~~tangent~~: $y=\sqrt{3}x+C$.

⊥r distance from ~~y=~~ (0,0) to $\sqrt{3}x+c=y$

$=5$.

$\Rightarrow \frac{|c|}{2}=5$. $\Rightarrow c=\pm 10$

$\Rightarrow$ Tangent Eqn: $y=\sqrt{3}x\pm 10$

(41). ~~tag~~ tangat slope at (x_1, y_1)

is $m = \frac{dy}{dx}(x_1, y_1)$

$2x\,dx + 2y\,dy = 0 \Rightarrow \frac{dy}{dx} = -\frac{x}{y}$

$\Rightarrow \left(\frac{dy}{dx}\right)_{(a\cos\alpha,\, a\sin\alpha)} = \frac{-a\cos\alpha}{a\sin\alpha} = -\cot\alpha.$

(42). $y = c$ tangent at $(1, 1)$

means ~~$y \neq 0$~~ $(1, 1)$ should lie on $y = c$

$\Rightarrow y = 1$

(43). $3x + 4y = 25. \rightarrow x^2 + y^2 = 25$

Centre $(0,0)$ $r = 5.$

foot of $\perp^r$ will be the point which lie on line and circle.

$\frac{x}{3} = \frac{y}{4} = \frac{\sqrt{125}}{5}.$

$\Rightarrow. \frac{x}{3} = \frac{y}{4} = 1$

$\Rightarrow x = 3, \quad y = 4$

$(3, 4)$

(44)

~~$S = T^2$~~

$y+f = m(x+g) \pm r\sqrt{1+m^2}$

~~$S T =$~~ $\Rightarrow y+2 = m(x-1) \pm \sqrt{5(1+m^2)}$

passes through (0, 1)

$\Rightarrow 3 = -m \pm \sqrt{5(1+m^2)}$.

$\Rightarrow 4m^2 + 9 + 6m = 5 + 5m^2$

$\Rightarrow 4m^2 - 6m - 4 = 0$.

$2m^2 - 3m - 2 = 0$

$\Rightarrow (2m+1)(m-2) = 0 \Rightarrow m = 2 \text{ (or) } -\frac{1}{2}$.

$\Rightarrow y+2 = 2(x-1) \pm \sqrt{5 \times 5}$

$\Rightarrow y+2 = 2x - 2 \pm 5$.

one ln. $2x - y + 1 = 0$.

another line. $y+2 = -\frac{1}{2}(x-1) \pm \sqrt{5 \times (1 + (-\frac{1}{2})^2)}$.

$y+2 = -\frac{x}{2} + \frac{1}{2} + \frac{5}{2}$.

$x + 2y - 2 = 0$.

(16)

(45).

slope of $4x+3y+5=0$ is $-\frac{4}{3}$

so slope to $\parallel$ to it is $-\frac{4}{3}$.

$\therefore$ slope of tangent :- $y = \frac{-4x}{3} + c$.

$\perp^r$ from centre $(3,-2)$ to $y = -\frac{4x}{3}+c$

equal to radius (5)

$$\Rightarrow \frac{|-4+2+c|}{\sqrt{\frac{4^2}{3^2}+1}} = 5$$

$\Rightarrow |c-2| = \frac{25}{3}$.

$\Rightarrow c-2 = \frac{25}{3}$, $c-2 = -\frac{25}{3}$

$\Rightarrow c = \frac{31}{3}$, $c = -\frac{19}{3}$.

line are $y = -\frac{4x}{3} + \frac{31}{3}$ $\quad y = -\frac{4x}{3} - \frac{19}{3}$

$\Rightarrow 4x+3y-31=0$. $\Rightarrow 4x+3y+19=0$.

(46) $5x+12y+8=0$.

$m = -\frac{5}{12}$.

slopes of line ⊥r to above $= \frac{12}{5}$.

$\left[y = \frac{12x}{5} + c \right]$

Centre of circle (11, 2).

radius = 10.

⊥r distance from (11, 2) to $y = \frac{12x}{5} + c$

$$\frac{\left| \frac{12 \times 11}{5} - 2 + c \right|}{\sqrt{\left(\frac{12}{5}\right)^2 + 1}} = 10.$$

$$\frac{|122 + 5c|}{13} = 10.$$

$\Rightarrow |122 + 5c| = 130$

$\Rightarrow 122 + 5c = 130$, $\quad 122 + 5c = -130$.

$\Rightarrow c = 8/5$, $\quad c = -\frac{252}{5}$.

line are. $\frac{12x}{5} - y + 8/5 = 0$, $\quad \frac{12x}{5} - y - \frac{252}{5} = 0$

$\Rightarrow 12x - 5y + 8 = 0$, $\quad 12x - 5y - 252 = 0$.

(18)

(47). $x^2+y^2=9$, $\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$

~~slope~~ $2x\,dx+2y\,dy=0$

$\Rightarrow \frac{dy}{dx} = -\frac{dx}{dy}$ slope at $\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right) = -1$

But normal will be $\perp^r$ to it so slope $= 1$.

$\therefore \left(y-\frac{1}{\sqrt{2}}\right) = \left(x-\frac{1}{\sqrt{2}}\right)1$

$\Rightarrow x-y=0$.

(48) $x^2+y^2-40x+10y=153$

~~2xdx~~

(42). Normal to a circle formula

$$\frac{y+f}{x+g} = \frac{y_1+f}{x_1+g}. \quad \left(x^2+y^2=9\right)$$

$x_1=\frac{1}{\sqrt{2}}, y_1=\frac{1}{\sqrt{2}}$. $g=0, f=0$.

$\Rightarrow \frac{y}{x} = \frac{\frac{1}{\sqrt{2}}}{\frac{1}{\sqrt{2}}}$ $\Rightarrow x-y=0$.

(48) Normal to a circle formula.

$x^2+y^2-40x+10y-153=0$

$$\frac{y+f}{x+g} = \frac{y_1+f}{x_1+g}$$

$g=-20, f=5$.

$x_1=4, y_1=-1$.

$$\frac{y+5}{x-20} = \frac{-1+5}{4-20}$$

$$\Rightarrow \frac{y+5}{x-20} = -\frac{1}{4}$$

$$\Rightarrow 4y+20 = -x+20.$$

$$\Rightarrow \boxed{x+4y=0.}$$

(49).

$$x^2+y^2-8x-2y+12=0.$$

$$(y_1=-1).$$

$$\Rightarrow x_1^2+1-8x_1+2+12=0 \qquad g=-4,\ f=-1.$$

$$\Rightarrow x_1^2-8x_1+15=0$$

$$\Rightarrow x_1 = 3, 5.$$

$$\therefore (3,-1)\quad (5,-1).$$

$$\frac{y+f}{x+g} = \frac{y_1+f}{x_1+g}$$

~~$\Rightarrow \frac{y+1}{x+4} = \frac{-1+1}{3+4}$, $\frac{y+1}{x+4} = \frac{-1+1}{5-4}$~~

~~$\Rightarrow (y+1)-1 = -2(x+4)$, $(y+1) = -2(x+4)$~~

~~$\Rightarrow 2x-y+7=0$, $2x+y+9=0.$~~

$$\frac{y-1}{x-4} = \frac{-1-1}{3-4}, \quad \frac{y-1}{x-4} = \frac{-1-1}{5-4}.$$

(20)

⇒. $2x-y-7=0$, $2x+y-9=0$.

50). ~~(a)~~ $ax+by+c=0$ is a normal

mean it should pass through center of circle

$(0,0)$ of $x^2+y^2=r^2$.

⇒ ~~$0+0+c=0$~~ ⇒ $c=0$ means it should be a diameter to the circle.

~~∴ $ax+by=0$ is the line.~~

∴ length of intercept $= 2\times r = 2r$.

51). Length of tangent $=\sqrt{S_1}$

$\Rightarrow \sqrt{5^2+3^2+2\times5+k\times3+D} = 7$

⇒ $61+3k=49$

⇒ $3k=-12$ ⇒ $k=-4$

(52) $\sqrt{S_1} = \sqrt{5^2+1+6\times5-4-3} = \sqrt{49}$

$=7.$

(53).

$$\sqrt{S_1} = \sqrt{2(2^2+3^2) - 7\times2 + 9\times3 - 11}$$

$$= \sqrt{2^2+3^2 - \frac{7\times2}{2} + \frac{9\times3}{2} - \frac{11}{2}}$$

$$= \sqrt{14}.$$

(54).

$$\frac{4}{3} = \frac{\sqrt{1^2+2^2+1+2-4}}{\sqrt{1^2+2^2-\frac{1}{3}-\frac{2}{3}+\frac{k}{3}}}$$

$$\Rightarrow \frac{16}{9} = \frac{4}{4+\frac{k}{3}}$$

~~$\Rightarrow 32 + \frac{8k}{3} = 36$~~

~~$\Rightarrow \frac{8k}{3} = 4 \Rightarrow k =$~~

$$\Rightarrow 64 + \frac{16k}{3} = 36 \Rightarrow \frac{16k}{3} = -28.$$

$$k = -\frac{21}{4}.$$

(55).

$S_1 S = T^2.$ $\quad S = x^2+y^2+10(x+y)+20-20.$

$S_{1(0,0)} = 20$ $\quad T = x\times0 + y\times0 + 10(x+0) + 10(y+0) + 20$

(22)

$\Rightarrow$ ~~$20(x^2+y^2+20(x+y)+20) = (10x^2+10y+20)^2$~~

$\Rightarrow$ ~~$5x^2+5y^2+100(x+y)-20 = 25x^2+25y^2+4-20x-20y+50xy$~~

$\Rightarrow$ ~~$20x^2+20y^2-120x-120y+50xy$~~

$\Rightarrow$ ~~$T = x+y+2=0$~~

$20(x^2+y^2+10(x+y)+20) = (10x+10y+20)^2$

$x^2+y^2+20x+20y+20 = 5x^2+5y^2+20+10xy+20x+20y+20$

$4x^2+4y^2+10xy = 0$

$2x^2+2y^2+5xy = 0$

(58) $\perp^r$ distance from $3x+y=0$ to $(2,-1)$

is $\frac{6-1}{\sqrt{10}} = \frac{5}{\sqrt{10}} = \sqrt{\frac{5}{2}}$ = radius

$S = (x-2)^2+(y+1)^2 = 5/2$.

as it pass through origin $y = mx$ will be the eqn.

$\Rightarrow$ $\perp^r$ from $(2,-1)$ to $y=mx$ will be equal to radius.

$\left|\frac{2m+1}{\sqrt{m^2+1}}\right| = \sqrt{\frac{5}{2}}$.

$$\frac{(2m+1)^2}{m^2+1} = \frac{5}{2}$$

$\Rightarrow 8m^2 + 2 + 8m = 5m^2 + 5$

$\Rightarrow 3m^2 + 8m - 3 = 0$

$3m(3m-1)(m+3) = 0$

$\Rightarrow m = \frac{1}{3},\ m = -3.$

$\Rightarrow$ line is $y = \frac{x}{3}$

$\Rightarrow x - 3y = 0.$

(57). $SS_1 = T^2.\quad S = x^2 + y^2 - 2x + 4y + 3$

$S_1 = 6^2 + (5)^2 - 2\times 6 + 4\times -5 + 3$

$= 32$

$T = 6x - 5y - (x+6) + 2(y-5) + 3$

$= 5x - 3y - 13.$

$SS_1 = T^2$

$\Rightarrow 32 \times (x^2 + y^2 - 2x + 4y + 3) = (5x - 3y - 13)^2$

$\Rightarrow 32x^2 + 32y^2 - 64x + 128y + 96 = 25x^2 + 9y^2 + 169 - 30xy - 130x + 78y.$

$\Rightarrow 7x^2 + 23y^2 + 66x + 50y + 30xy - 73 = 0.$

(58).

~~$\alpha = 2\tan^{-1}($~~

$$\tan\theta = \frac{2r\sqrt{S_1}}{S_1 - r^2}$$

$$\tan\theta = \frac{10\sqrt{25}}{25-25} = \frac{10\sqrt{25}}{0}$$

$$\theta = \pi/2$$

(59).

~~length of chord of contact~~

$S = x^2+y^2+4x+6y-12$

$(2,-3)$

~~$= \frac{2r\sqrt{S_1}}{\sqrt{r^2+S_1}}$~~

Equation of chord of contact

$$T = 0$$

$\Rightarrow$. $2x - 3y + 2(x+2) + 3(y-3) - 12 = 0$

$\Rightarrow$ $4x - 17 = 0$ $\Rightarrow 4x = 17.$

(60). $T = 0$

$\Rightarrow$ $5x - 3y = 10.$

(61) $x^2+y^2=8$.

$r=2\sqrt{2}$. director circle radius $=\sqrt{2}\times r$

$=\sqrt{2}\times 2\sqrt{2}$

$=4$.

$\Rightarrow x^2+y^2=(4)^2 \Rightarrow x^2+y^2=16$.

(62).

$\tan\theta = \dfrac{2r\sqrt{S_1}}{S_1-r^2}$.

$\theta=\frac{\pi}{2}$

$\Rightarrow$. $S_1-r^2=0$. $\Rightarrow x_1^2+y_1^2-a^2-a^2=0$

$\Rightarrow$ ~~dom~~ locus :- $x^2+y^2=2a^2$.

(63). centres are $(0,1)$ $(1,0)$

radius $r_1=3$, $r_2=5$.

distance between centre $=\sqrt{2}$.

~~$\sqrt{2} < \frac{r_1+r_2}{2} < 8$~~

~~mean circle intersect~~ in two part.

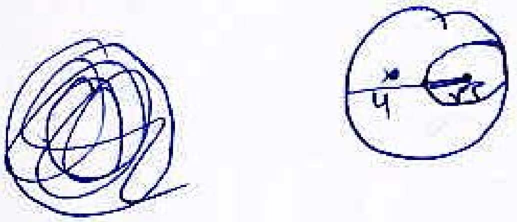
$\sqrt{2}<r_1, r_2$. mean one circle will lie inside the other.

(64) $(1, 2)$ $(0, 4)$

$r_1 = \sqrt{5}$, $r_2 = 4$.

distance between centre $= \sqrt{5}$.

$d = r_1$ means both touch internally.



(65). $r_1 = \sqrt{2}$, $C_1 (1, 0)$

$r_2 = 2\sqrt{2}$, $C_2 (0, 1)$.

$C_1C_2 = \sqrt{2}$.

$C_1C_2 = r_1$

means circles touch internally.

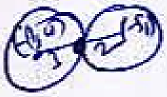
$\therefore$ common tangents $= 1$.

(66). $C_1(-1, 4)$ $C_2(-5, 1)$

$r_1 = 3$ $r_2 = 2$.

$C_1C_2 = 5$.

$r_1 + r_2 = 5$



~~cont~~

point of contact divides

(-1,4) & (-5,11)

in ratio 3 : 2

$\therefore$ point of contact $= \left(\frac{-15-2}{5}, \frac{3+8}{5}\right)$

(67).

$C_1 (3,1)$ $\quad$ $C_2 (-1,4)$

$r_1 = 3$ $\quad$ $r_2 = 2$.

$C_1C_2 = 5$.

$r_1 + r_2 = C_1C_2$

Touch each other externally.

(68).

~~$x^2 + y^2 = r_1^2$~~

$C_1 (0,10)$ $\quad$ $C_2 (10,0)$

$r_1 = r$ $\quad$ $r_2 = 8$.

To intersect.

$C_1C_2 < r_1 + r_2$, $C_1C_2 > r_1 - r_2$

$10 < r + 8$, $\quad$ $10 > r - 8$.

$r > 2$, $r < 18$ $\quad$ $2 < r < 18$

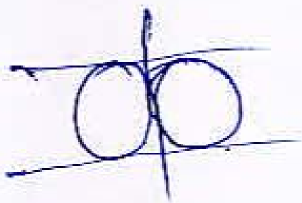
(69). $C_1 (2,3)$ $\quad C_2(-1,-1)$

$r_1 = 4$ $\quad r_2 = 1$

$C_1C_2 = 5$.

$C_1C_2 = r_1 + r_2$.

means circle with touch each other externally.



Total 3 common tangents

(70). let (h,k) be mid-point of chord. $x^2+y^2=25$.

⇒

Equation of chord with (h,k) mid-point given by

$T = S_1$

$\Rightarrow xh + yk = h^2+k^2$

$\Rightarrow 5\cos 45 = \sqrt{h^2+k^2}$

$\Rightarrow h^2+k^2 = \frac{25}{2}$.

∴ Locus. ~~h^2+k^2~~

$x^2+y^2=\frac{25}{2}$.

(71) centre $(0,0)$ $r=4$

$(2,-1)$

So chord will be $\perp^r$ to line going $(0,0)$ & $(2,-1)$

$\therefore$ slope of chord $= \frac{-1}{-\frac{1}{2}} = 2$.

$m=2$, $(2,-1)$

$y+1=(x-2)2$

$\Rightarrow 2x-y=5$.

(72) $x^2+y^2-6x+8y=0$, bisected at $(5,-3)$

$C(3,-4)$ $(5,-3)$.

So chord will be $\perp^r$ to the line ~~going~~ joining $(3,-4)$ & $(5,-3)$

slope of chord $= \frac{-1}{\frac{1}{2}} = -2$

$m=-2$, $(5,-3)$

$y+3=(x-5)-2$

$\Rightarrow 2x+y=7$

(73) $S_1 = x^2+y^2+13x-3y=0$

$S_2 = 2x^2+2y^2+4x-7y-25=0$

(30)

Let the equation of circle passing through point of intersection of S_1 & S_2 be:

$$S_1 + \mu S_2 = 0.$$

$\Rightarrow (x^2+y^2+13x-3y) + \mu(2x^2+2y^2+4x-7y-25)=0$

substitute $(1,1)$ in above eq^n.

$\Rightarrow \quad 12 - 24\mu = 0$

$\Rightarrow \quad \mu = \frac{1}{2}.$

$\therefore (x^2+y^2+13x-3y)+\frac{1}{2}(2x^2+2y^2+4x-7y-25)=0.$

$\Rightarrow 4x^2+4y^2+30x-13y-25=0.$

74

$$S_1 + \mu S_2 = 0.$$

$\Rightarrow (x^2+y^2-6) + \mu(x^2+y^2-6x+8)=0$

substitute $(1,1)$

$\Rightarrow \quad -4 + \mu 4 = 0 \quad \Rightarrow \mu = 1$

$\Rightarrow x^2+y^2-6+x^2+y^2-6x+8=0 \Rightarrow x^2+y^2-3x+1=0$

(75).

common chord → centre $\left(-\frac{3}{2}, -1\right)$ $r = \frac{3}{2}$

$S - S' = \left(x^2+y^2+3x+2y+1\right) - \left(x^2+y^2+3x+4y+2\right) = 0$

$\Rightarrow 2y+1=0$. or $y = -\frac{1}{2}$.

foot of ⊥r from any centre of the circle will be the centre of new circle with common chord as the diameter

∴ foot of ⊥r from $\left(-\frac{3}{2}, -1\right)$ to $y = -\frac{1}{2}$.

$$\frac{x+\frac{3}{2}}{0} = \frac{y+1}{1} = \left|-1+\frac{1}{2}\right|$$

$\Rightarrow x = -\frac{3}{2}$, $y = -\frac{1}{2}$.

radius $= r_1 = \sqrt{r^2 - d^2}$

$$= \sqrt{\frac{9}{4} - \left|-1+\frac{1}{2}\right|^2}$$

$$= \sqrt{\frac{9}{4} - \frac{1}{4}} = \sqrt{\frac{8}{4}}$$

$= \sqrt{2}$

$\left(x+\frac{3}{2}\right)^2 + \left(y+\frac{1}{2}\right)^2 = (\sqrt{2})^2 \Rightarrow 2(x^2+y^2) + 6x + 2y + 1 = 0$

(76) Common Chord: $S - S' = 0$

$\Rightarrow x^2+y^2-4x-4y-(x^2+y^2-16)=0$

$\Rightarrow -4x-4y+16=0$

$\Rightarrow x+y=4.$

(77). It subtends an angle 90° at origin.

77. Common chord of $x^2+y^2+5x-8y+1=0$, $x^2+y^2-3x+7y-25$

is $S-S'=0 \Rightarrow$ $8x-15y+26=0$

$\perp^r$ distance from centre of $x^2+y^2-2x=0$ $(1,0)$

to $8x-15y+26=0$

is $\frac{|8+26|}{\sqrt{8^2+15^2}} = \frac{34}{17} = 2.$

78. Length of Common chord.

$S-S'=0 \Rightarrow -2x+2y=0$

$x=y$ is the common chord.

length of comm chord

$= 2\sqrt{r^2 - d^2}$

d → ⊥ distance from centre of $x^2+y^2+4x+6y+4=0$
(−2, −3) to
Chord of Contact (x = y)

r = radius of $x^2+y^2+4x+6y+4=0$

= 3.

⇒ length of common chord $= 2\sqrt{9 - \left(\frac{+2+3}{\sqrt{2}}\right)^2}$

$= 2\sqrt{9 - \frac{1}{2}}$.

$= 2 \times \sqrt{\frac{17}{2}} = \sqrt{34}$

Q 292 ✓ length $S - S' = 0$ is common chord.

⇒ $-6x + 8y - 7 = 0$

length of common chord $= 2\sqrt{r^2 - d^2}$.

$x^2+y^2-6x-16=0$ → Centre (3, 0) $r = 5$.

⇒ $l = 2\sqrt{25 - \left(\frac{|(-25)|}{10}\right)^2}$

$= 2\sqrt{\frac{75}{4}} = 5\sqrt{3}$.

80) $S - S'$

$-2x + 2y = 0 \Rightarrow x = y.$

length of chord $= 2\sqrt{r^2 - d^2}$

Centre of $x^2 + y^2 + 5x + 7y + 9 = 0$

$\left(-\frac{5}{2}, -\frac{7}{2}\right)$. radius $= \sqrt{\frac{38}{4}}$

$$l = 2\sqrt{\frac{38}{4} - \left(\frac{\left(-\frac{5}{2} + \frac{7}{2}\right)}{\sqrt{2}}\right)^2}$$

$$= 2\sqrt{\frac{38}{4} - \frac{1}{2}} = 2\sqrt{\frac{36}{4}}$$

$$= 6.$$

81) $x^2 + y^2 + ax + by + c = 0.$

$x^2 + y^2 + dx + ey + f = 0.$

~~$2(c_1 + c_2)$~~ $2(g_1 g_2 + f_1 f_2) = c_1 + c_2.$

$\Rightarrow$ ~~$2(ad + be) = c + f$~~

$2 \times \left(\frac{a}{2} \times \frac{d}{2} + \frac{b}{2} \times \frac{d}{2}\right) = c_1 + c_2$

$\Rightarrow ad + bd = 2(c_1 + c_2)$

82) $g_1 = -2,\ g_2 = -5 \quad f_1 = -3,\ f_2 = -6.$

$a^2 = 13 - c_1,\ a^2 = 45 - c_2.$

$2(g_1g_2 + f_1f_2) = c_1 + c_2.$

$2(10 + 18) = 13 - a^2 + 64 - a^2.$

$\Rightarrow 2a^2 = 18 \Rightarrow a = 3.$

83)

$$\cos\theta = \frac{c_1 + c_2 - 2g_1g_2 - 2f_1f_2}{2\sqrt{g_1^2 + f_1^2 - c_1}\sqrt{g_2^2 + f_2^2 - c_2}}$$

$$= \frac{-9 - 7 - 2(-4) - 2(-4)}{2 \times \sqrt{26} \times \sqrt{24}}$$

$$= \frac{0}{2\sqrt{26}\sqrt{24}}$$

$\Rightarrow \theta = 90^\circ.$

(84)

$$\cos\theta = \frac{c_1 + c_2 - 2(g_1g_2 + f_1f_2)}{2 r_1 r_2}.$$

$2r_1r_2 = c_1 + c_2 - 2(g_1g_2 + f_1f_2).$

$\Rightarrow \sqrt{(g_1 - g_2)^2 + (f_1 - f_2)^2} = r_1 + r_2.$

Circle will touch each othe.

intersect at only one point.

85Q

Let cent (-g, -f)

~~cos 90 = C_1 ...~~

$\to$ g = +4

$x^2+y^2+2gx+2fy+4=0$.

~~Ab.~~ substitute (1,2).

~~$1^2+2^2+2\times5\times1+4$~~

$1^2+2^2+2g+4f+4=0$

$2g+4f=-9$.

$2(-g)+4(-f)=9$

$\to 2x+4y-9=0$.

86Q

~~as f~~ Let circle eqn.

$x^2+y^2+2gx+2fy+c=0$.

as it passes through origin c=0.

$\to x^2+y^2+2gx+2fy=0$.

$\cos 90 = \frac{4-2(-2g+f)}{2 r r_2}$ $\to 4+4g-2f=0$.

$\to f-2g=2$.

as centre $(-g,-f)$ lies on $x+y=4$

$\Rightarrow -g-f=4 \Rightarrow g+f=-4$.

$f-2g=2$.

$\Rightarrow 3g=-6 \Rightarrow g=-2$.

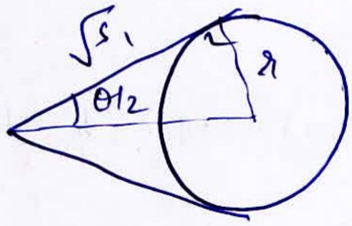
$f=-2$.

$\therefore$ eqn of circle

$$x^2+y^2-4x-4y=0$$

Circle 1(B) Pros Sir

1



$\tan\left(\frac{\theta}{2}\right) = \frac{r}{\sqrt{S_1}}$

Hence (B)

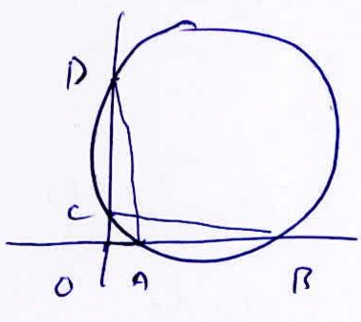
2

$d < |r_1 - r_2| \Rightarrow$ one circle inside the other

$d = \sqrt{2}$ & $r_1 - r_2 = b - a$

$\therefore \boxed{b - a > \sqrt{2}}$

(3)

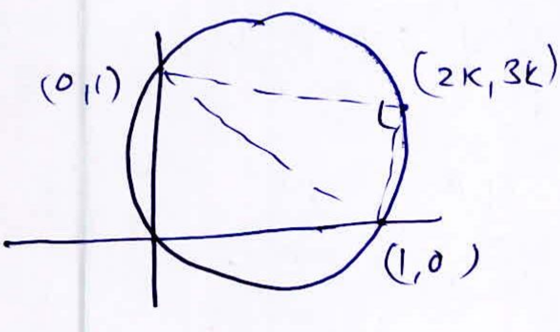


$OA \cdot OB = OC \cdot OD$

$\left(\frac{-c_1}{a_1}\right)\left(-\frac{c_2}{a_2}\right) = \left(\frac{-c_1}{b_1}\right)\left(\frac{-c_2}{b_2}\right)$

$\Rightarrow \boxed{a_1 a_2 = b_1 b_2}$

(4)



Peel $m_1 m_2 = -1$

$\frac{3K}{2K-1} \cdot \frac{3K-1}{2K} = -1$

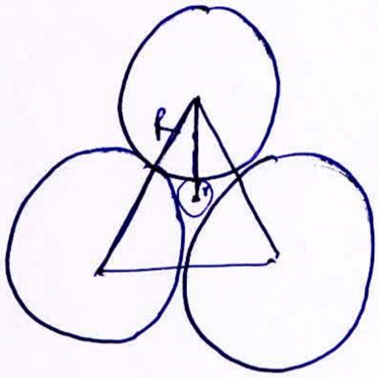
$\Rightarrow K = 5/13$

(5) That means centre $(-a,-b)$ will satisfy the radical Axis.

Radical Axis is $2(g-a)x + 2(f-b) + c - d = 0$

put $(-a,-b)$ Ans (A)

(6)



Let the Radii be r & R

clearly this is an Equilateral Δ with side length $2R$

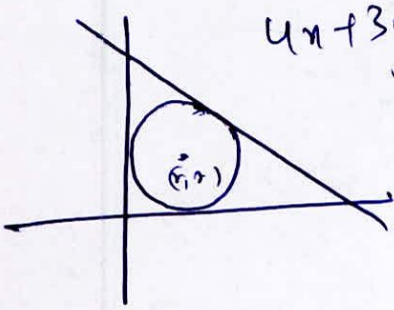
Also centre of smaller circle is Centroid $\therefore$ Altitude is divided in Ratio $2:1$

$\therefore \quad r+R = \frac{2}{3}(2R\sin 60)$

$r+R = \frac{2 \cdot 2R}{3} \cdot \frac{\sqrt{3}}{2}$

$r = \left(\frac{2}{\sqrt{3}} - 1\right) R$

(7)



Centre (r, r)
& radius r

$\therefore p = r$

$\therefore \left| \dfrac{4r+3r-6}{5} \right| = r$

$7r - 6 = 5r$ or $-5r$

$2r = 3$ or $r = 1/2$

For incircle $r = 1/2$

$\therefore$ Ans B

(8) Let the circle be

$(x^2+y^2-6x+8) + \lambda(x^2+y^2-6) = 0$

put $(1,1)$

$4 + \lambda(-4) = 0$

$\lambda = 1$

$\therefore$ Ans $2x^2+2y^2-6x+2=0$

(9) $|r_1 - r_2| < d < r_1 + r_2$

$C_1(1,3)$ $C_2(4,-1)$

$r_1 = r$ $r_2 = 3$

$\Rightarrow d = 5$

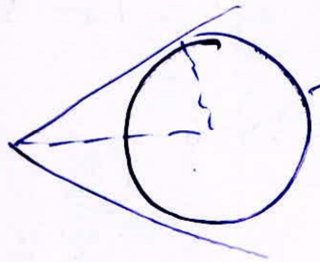
$|r-3| < 5 < r+3$

$\Rightarrow r < 8$ $\Rightarrow r > 2$

(10) let pt $P(x_1, y_1)$

$\therefore x_1^2 + y_1^2 + 2gx_1 + 2fy_1 + c = 0$

Now



(x_1, y_1)

$\to x^2+y^2+2gx+2fy + c\sin^2\alpha + (g^2+f^2)\cos^2\alpha = 0$

We know $\tan\frac{\theta}{2} = \frac{R}{\sqrt{S_1}}$

Here $R = \sqrt{g^2+f^2 - c\sin^2\alpha - g^2\cos^2\alpha - f^2\cos^2\alpha}$

$= \sqrt{g^2+f^2-c}\ \sin\alpha$

Also $\sqrt{S_1} = \sqrt{(x_1^2+y_1^2+2gx_1+2fy_1) + c\sin^2\alpha + (g^2+f^2)\cos^2\alpha}$

$= \sqrt{-c + c\sin^2\alpha + (g^2+f^2)\cos^2\alpha}$

$= \sqrt{g^2+f^2-c}\ \cos\alpha$

$\therefore \tan\left(\frac{\theta}{2}\right) = \tan\alpha$

$\therefore \theta = 2\alpha$

(11) External touch

$C_1(a,0)$ $C_2(0,b)$

$$d = r_1 + r_2$$

$$\sqrt{a^2+b^2} = \sqrt{a^2-c^2} + \sqrt{b^2-c^2}$$

Square

$$a^2+b^2 = a^2-c^2+b^2-c^2+2\sqrt{(a^2-c^2)(b^2-c^2)}$$

$$c^4 = a^2b^2-(a^2+b^2)c^2+c^4$$

$$\frac{1}{c^2} = \frac{1}{a^2}+\frac{1}{b^2}$$

(12) ∵ tangent $p = r$

$C(a\cos\alpha, a\sin\alpha)$ & $R = a$

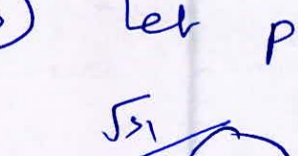
∴ $$|a\cos^2\alpha + a\sin^2\alpha - p| = a$$

$$|a-p| = a$$

∴ $a-p = a$ or $-a$

$p = 0$ or $2a$

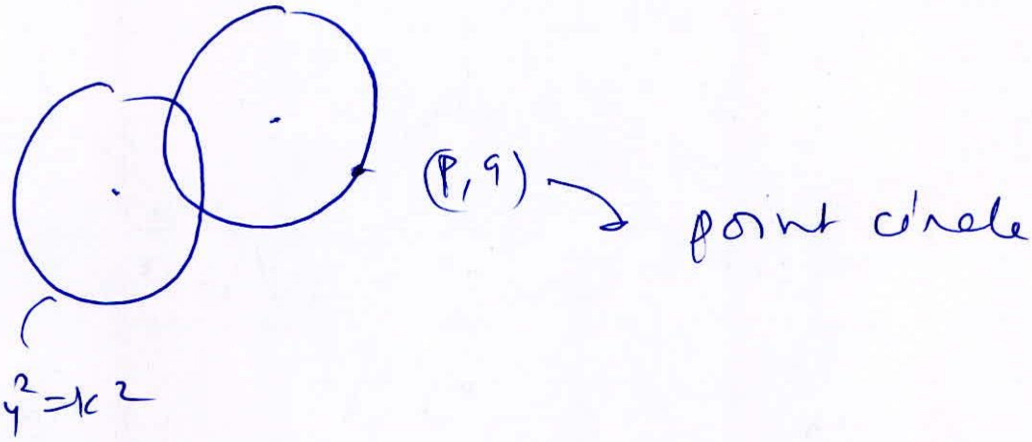
(13) Let pt be $(x_1,y_1) \Rightarrow x_1^2+y_1^2+2gx_1+2fy_1+\alpha=0$



∴ $L = \sqrt{S_1} = \sqrt{x_1^2+y_1^2+2gx_1+2fy_1+\beta}$

$= \sqrt{-\alpha+\beta}$

(14)



we can say the variable circle cuts 2 circles orthogonally

∴ its locus is the R·A of 2 circles

$$(x^2+y^2-k^2)-((x-p)^2+(y-q)^2)=0$$

Ans (A)

(13) $x\cos\theta+y\sin\theta+g\cos\theta+f\sin\theta+k=0$

touches $C(-g,-f)$ & $r=\sqrt{g^2+f^2-c}$

$$\Rightarrow \quad |-g\cos\theta-f\sin\theta+g\cos\theta+f\sin\theta+k|=\sqrt{g^2+f^2-c}$$

$$\Rightarrow \quad k^2=g^2+f^2-c$$

$$\boxed{k^2+c=g^2+f^2}$$

(16) let pt be $P(x_1, y_1)$

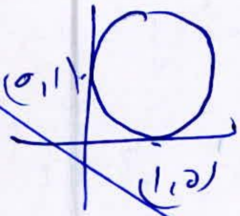
$$\therefore \frac{r_1}{r_2} = \frac{b}{a}$$

$$\frac{\sqrt{x_1^2 + y_1^2 - a^2}}{\sqrt{x_1^2 + y_1^2 - b^2}} = \frac{b}{a}$$

$$a^2 x_1^2 + a^2 y_1^2 - a^4 = b^2 x_1^2 + b^2 y_1^2 - b^4$$

$$(a^2 - b^2)(x_1^2 + y_1^2) - (a^2 - b^2)(a^2 + b^2) = 0$$

(17)

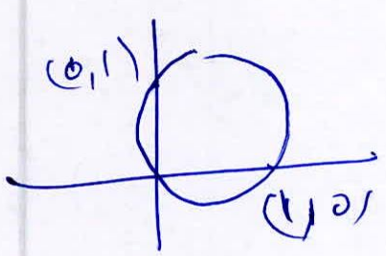


for smallest it can touch Axes @ (1,0) & (0,1)

centre (1,1) & radius 1

Ans c

(17)



the smallest circle is with (1,0) & (0,1) as diametrically opp end pts.

$$\therefore x(x-1) + y(y-1) = 0$$

Ans (D)

(18) chord of contact from (0, 0) is T = 0

$gx + fy + c = 0$

chord of contact from (g, f) is T = 0

$gx + fy + g(x+g) + f(y+f) + c = 0$

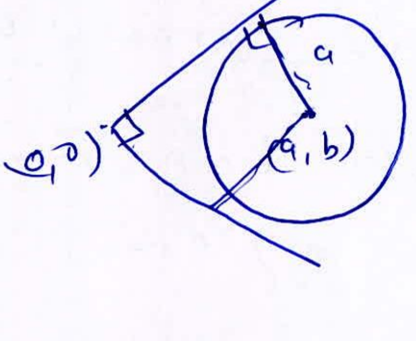
$2gx + 2fy + g^2 + f^2 + c = 0$

∴ distance between 2 lines is

$$\left| \frac{\frac{g^2+f^2+c}{2} - c}{\sqrt{g^2+f^2}} \right| = \left| \frac{g^2+f^2-c}{2\sqrt{g^2+f^2}} \right|$$

(19) standard $\frac{r(S_1)^{3/2}}{r^2+S_1}$ Hence option (A)

(20)

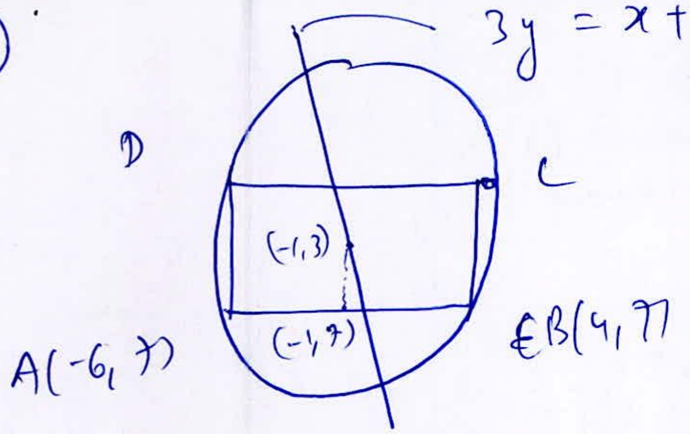


(0,0) & (a,b) at distance of $\sqrt{2}a$

$a^2 + b^2 = 2a^2$

$b^2 = a^2$

(21)



Now ⊥r bisector of AB also passes through centre

∴ its Eqn is $x = -1$

∴ centre is $(-1, 3)$

∴ $AB = 10$, $BC = 8$

∴ $A = 80$

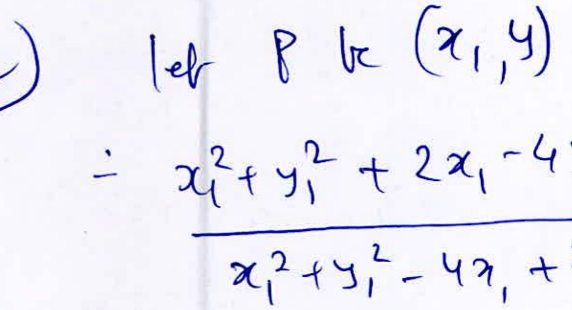
(22) let P be (x_1, y_1)

∴ $\dfrac{x_1^2 + y_1^2 + 2x_1 - 4y_1 - 20}{x_1^2 + y_1^2 - 4x_1 + 2y_1 - 44} = \dfrac{2}{3}$

$\Rightarrow x_1^2 + y_1^2 + 14x_1 - 16y_1 + 28 = 0$

Ans $(-7, 8)$

(23)



let pt of contact be (r, r)

∴ $\sqrt{(r-1)^2 + (r-1)^2} = 1$

$2(r^2 - 2r + 1) = 1$

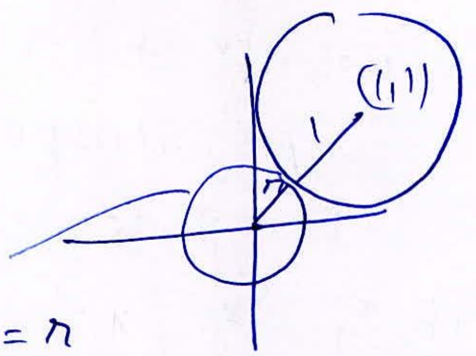
$2r^2 - 4r + 1 = 0$

$r = ~~2+\sqrt{2}~~$ or $2 - \sqrt{2}$

ignore

∴ A ~~$x^2 + y^2 = \ldots$~~ ~~$x^2 + y^2 = (2-\sqrt{2})^2$~~

(23)



let radius = r

$\sqrt{2} = r+1$

$r = \sqrt{2}-1$

∴ Eqⁿ $x^2+y^2 = (\sqrt{2}-1)^2$

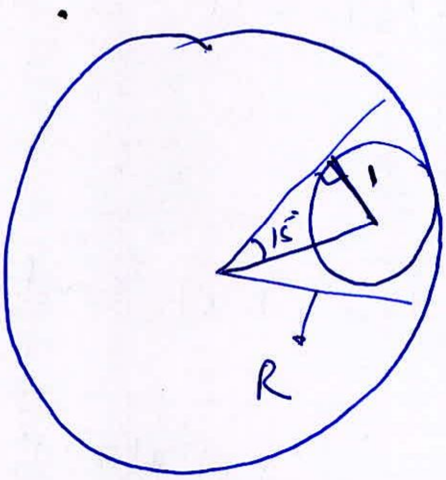
(24) clearly C_2 is D.C. of C_1

⇒ $C_2: x^2+y^2 = 8$

& $C_3: x^2+y^2 = 16$

$A = \pi(16-8) = 8\pi$ sq units

(25)



In the right angle △

$\frac{1}{R-1} = \sin 15°$

$R-1 = \frac{2\sqrt{2}}{\sqrt{3}-1}$

$= 1+\sqrt{6}+\sqrt{2}$

(26) centre will be

$$\frac{3x+4y-5+3x+4y-20}{2}=0$$

$$3x+4y-\frac{25}{2}=0$$

(27) let the circle by

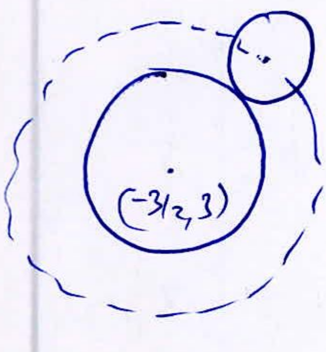
$$x^2+y^2+6x+8y+\lambda=0$$

$\because$ $(-3,2)$ lies $\therefore$

$$9+4-18+16+\lambda=0$$

$$\lambda=-11 \quad \therefore \text{Ans (B)}$$

(28)



let (x_1,y_1) be centre

$\therefore$ $d=R_1+R_2$

$$\left(x_1+\frac{3}{2}\right)^2+\left(y_1-3\right)^2=2+\frac{9}{2}$$

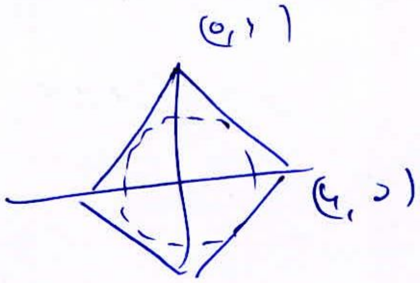
$\therefore$ option (B)

(29) distance between lines $=2$

$\therefore$ circle is $\quad x^2+y^2=4$

(31) $|x|+|y|=4$

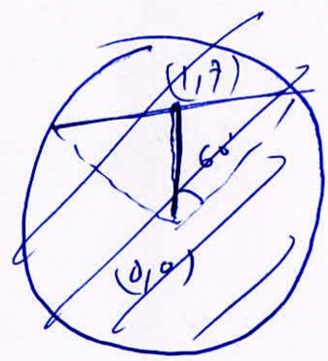
is a square

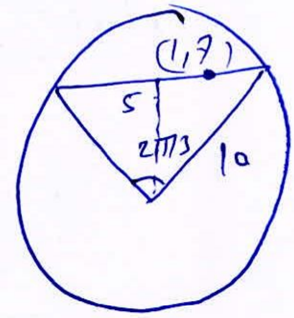


∴ circle centre $(0,0)$ $r = 2\sqrt{2}$

$$x^2+y^2=8$$

(32)





∴ ⊥r distance of line from $(0,0)$ is 5

let line be $y-7=m(x-1)$

or $mx-y+7-m=0$

∴ $p=5$ ∴ $\left|\frac{7-m}{\sqrt{1+m^2}}\right| = 5$

$(m-7)^2 = 25m^2+25$

$24m^2+14m-24=0$

$12m^2+7m-12=0$

$12m^2+16m-9m-12=0$

$(4m-3)(3m+4)=0$

$m = 3/4, \; -4/3$

∴ line is

$3y+4x-25=0$

(33)

$$R_1^2 = d^2 + 1^2$$

$$R_1^2 = 1 + 2(R_1 - 1)^2$$

$C_1(R_1\ R_1)$

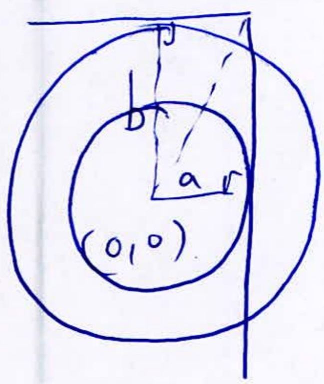
$C_2(1, 1)$

$$R_1^2 - 4R_1 + 3 = 0$$

$$R_1 = 3$$

(34)

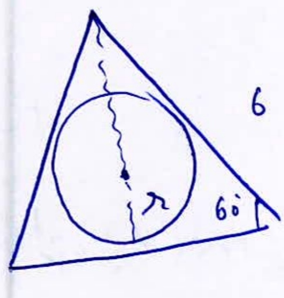
$P(h, k)$



$$OP^2 = a^2 + b^2$$

$$\boxed{h^2 + k^2 = a^2 + b^2}$$

(35)



now $r = \frac{1}{3}h$

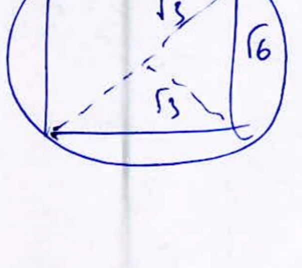
when $h = 6 \times \sin 60°$

$= 3\sqrt{3}$

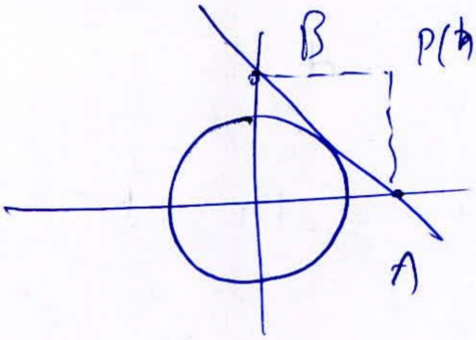
$\therefore \boxed{r = \sqrt{3}}$

Square inscribed

$A = 6$



(36)



let tangent be

$x\cos\theta + y\sin\theta = r$

$\therefore A\left(\frac{r}{\cos\theta}, 0\right)$

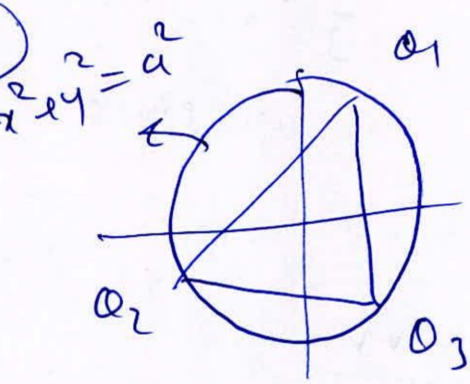
$B\left(0, \frac{r}{\sin\theta}\right)$

$\therefore$ ~~Mid pt~~

$\Rightarrow h = \frac{r}{\cos\theta}, \quad k = \frac{r}{\sin\theta}$

$$\therefore \boxed{\frac{1}{x^2} + \frac{1}{y^2} = \frac{1}{r^2}}$$

(37)



$\because \triangle$ is Equilateral

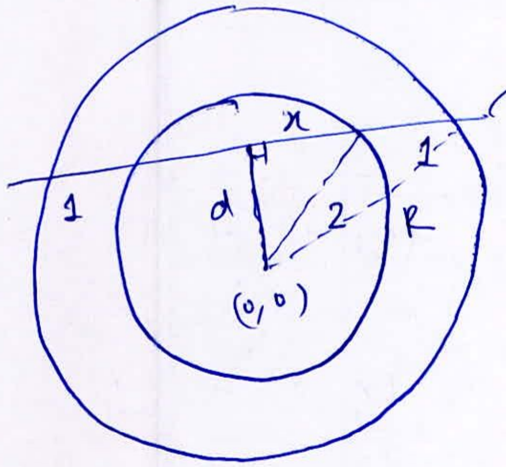
$\therefore$ Centroid also (0, 0)

i.e, $\left(\frac{a\sum\cos\theta_i}{3}, \frac{a\sum\sin\theta_i}{3}\right)$

is origin

$\Rightarrow \sum\cos\theta_i = 0$ & $\sum\sin\theta_i = 0$

(38)



Line: $x+y=2$

$d = \left|\frac{-2}{\sqrt{2}}\right| = \sqrt{2}$

Now $x = \sqrt{4-2}$

$x = \sqrt{2}$

$\therefore R^2 = d^2 + (x+1)^2$

$= 2 + (\sqrt{2}+1)^2$

$R^2 = 5+2\sqrt{2}$

$\therefore$ Circle is

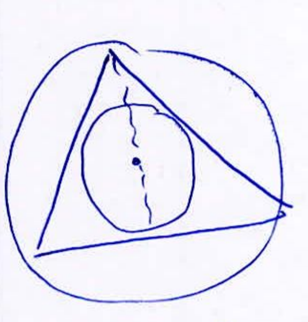
$x^2+y^2 = 5+2\sqrt{2}$

(39) $\therefore$ the pts will be intersecⁿ of $x=3$ with director circle $x^2+y^2=16$

$\Rightarrow 9+y^2=16 \quad \therefore y = \pm\sqrt{7}$

pts $(3, \sqrt{7})$, $(3-\sqrt{7})$ Ans (A)

(40)



Both are concentric $\therefore$ circumcentre is also $(-2,3)$

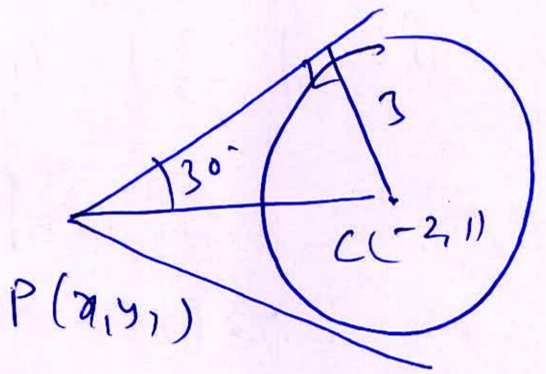
Also radii are in ratio 2:1

$\therefore R = 6$

$\therefore (x+2)^2 + (y-3)^2 = 36$

Ans (B)

(41)

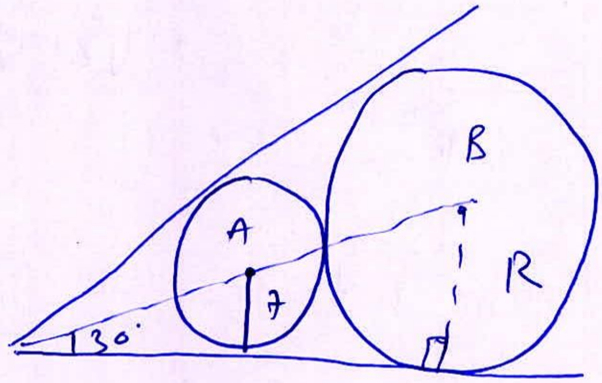


$$= \frac{3}{\sqrt{(x_1+2)^2+(y_1-1)^2}} = \frac{1}{2}$$

$$36 = (x_1+2)^2 + (y_1-1)^2$$

A A

(42)



$A(7\cot 30°, 7)$

or $A(7\sqrt{3}, 7)$

$B(R\cot 30°, R)$

or $B(\sqrt{3}R, R)$

Now $AB = R+7$

$$3(R-7)^2 + (R-7)^2 = (R+7)^2$$

Solve for R Ans $R = 21$

CIRCLES

EXERCISE – 2(A)

Q.1

$4x^2 + 4y^2 - 12x + 4y + 1 = 0$

$x^2 + y^2 - 3x + y + ¼ = 0$

Center = (3/2 , -1/2) ,

Radius = 3/2

$\angle ACB = 120^0$

⇨ $\angle ACP = 60^0$

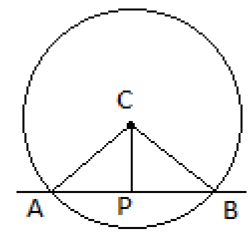
AC = 3/2

⇨ CP = $3/2 \cos 60^0$ = ¾

∴ locus of CP is : $(x-3/2)^2 + (y+1/2)^2 = 9/16$

⇨ $X^2 + y^2 - 3x + y + 31/16 = 0$

Ans: C



Q.2

$x^2 + y^2 - 2x - 6y = 0$ has Center : C_1 =(1 , 3) & Radius: $R_1 = 2$

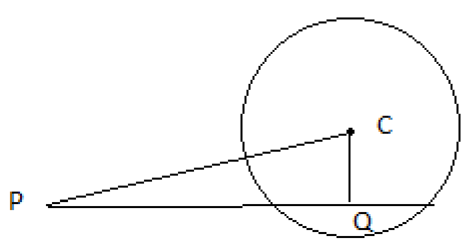
$C_2 = (2 , 1)$

Distance between centers : d= $\sqrt{5}$

$\therefore (R_2)^2 = (R_1)^2 + d^2 = 3$

Ans: (C)

Q.3



The locus of Q is circle with PC as diameter

P= (h , k) & C= (0 , 0)

Locus : (x-h)(x-0) + (y-k)(y-0) = 0

Ans :(B)

Q.4

Radical axis of two sides of triangle will pass through the common vertex and will perpendicular to third side.

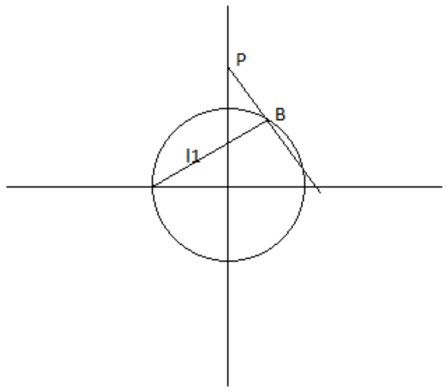
So, intersection of these axis will be orthocenter of triangle ABC

Ans: (D)

Q.5 The point of concurrence will be the pole of the line.

Ans: (D)

Q.6



B is the intersection point of l1 and circle

B = (6, 8), slope of line = ½

Therefore, slope of perpendicular line = -2

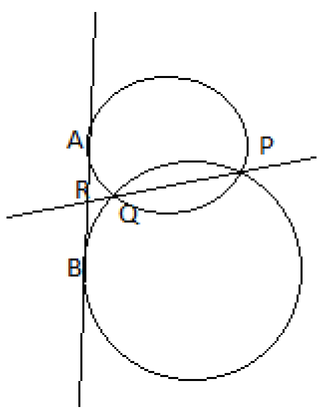
And equation of line will be : (y-8) = -2(x-6)

⇨ $2x + y = 20$

So, coordinates of P are (0,20) i.e. t = 20

Ans: (C)

Q.7



P =(1 ,1) & Q = (3,-1)

$AR^2 = (RQ)(RP)$ & $BR^2 = (RQ)(RP)$

⇨ R is the midpoint of AB

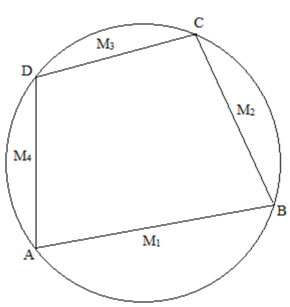
Equation of PQ : (y - 1) = -1(x - 1)

⇨ x + y =2, hence R is (0, 2)

Therefore, $AB^2 = 4(RP)(RQ)$ or $AB = 2\sqrt{6}$

Ans: (B)

Q.8



$M_1 = ½$, $M_2 = -3/4$ & $M_3 = ¼$

$\angle ABC = \Pi - \angle CDA$

⇨ $\frac{M1-M2}{1+M1M2} = -\frac{M3-M4}{1+M3M4}$

⇨ M4 = 9/2

Ans: (D)

Q.9

$d = R_1 + R_2$

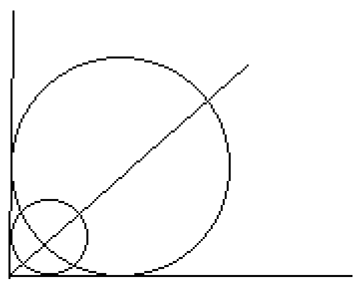
$d^2 - (R_1 - R_2)^2 = 4 * 35$

⇨ $R_1R_2 = 35$

⇨ $(R_1 , R_2) = 35*1$ or $7*5$

Ans: (C)

Q.10



Center lies on y = x

One end of the common chord =(a ,b)

Other end is the reflection in y=x i.e. = (b, a)

⇨ Radical axis : x + y = a + b

Q.11

Equation of the family of circle is $x^2 + y^2 - x + ky = 0$

Center $C_1 =(1/2 , -k/2)$ $R_1^2 = ¼ + k^2/4$

$X^2 + y^2 = 9$

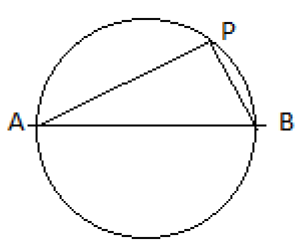
$C_2 = (0,0)$ $R_2= 3$

$\sqrt{(1} + k * k) = 3$

$k = + - 2\sqrt{2}$

Ans: (B)

Q.12

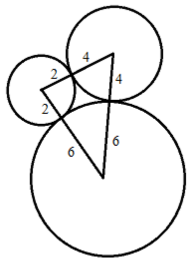


P lies on the circle $x^2 + y^2 = 50$

⇨ $50 = 1^2 + 7^2$ or $7^2 + 1^2$ or $5^2 + 5^2$

Ans: (C)

Q.13



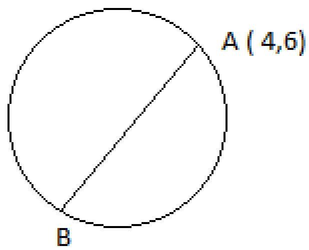
Radical center will be incenter of the triangle formed by the centers

PA = in radius of the triangle formed with sides 8,10,6

i.e = 2

Ans: (A)

Q.14



Let the other end of the chord be B (k, -6)

If B lies on the circle

$K^2 - 4k + 72 = 0$

K is non real

Ans: (A)

Q.15

Let A_k be (x_k, y_k) & let P be (x, y), then

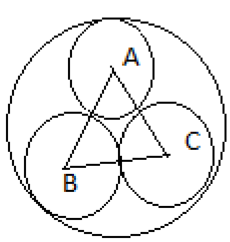
$PA_k^{\ 2} = (x - x_k)^2 + (y - y_k)^2$.

Now $nx^2 + ny^2 - 2(\sum_{k=1}^{n} x_k)x - 2(\sum_{k=1}^{n} y_k)y + \sum_{k=1}^{n} {x_k}^2 + \sum_{k=1}^{n} {y_k}^2 = \sum_{k=1}^{n} {d_k}^2$

Clearly this is equation of a circle.

Ans: (A)

Q.16



ABC is equilateral triangle

Radius of the required circle is $r + \frac{2r}{2\sin(\frac{\Pi}{3})}$

$= r\ (1 + 2/\sqrt{3})$

Ans: (B)

Q.17

Length of common chord, $\ell = \frac{2r_2r_1 \sin\theta}{PQ}$, where $\cos\theta = \frac{PQ^2 - r_1^2 - r_2^2}{2r_1r_2}$, P & Q are centers and r_1 & r_2 are the radii.

Ans:(B)

Q.18

OMPN is cyclic quadrilateral

⇨ Diameter of the circle is OP

⇨ Radius = OP/2

⇨ I.e radius = 5/2

Ans:(B)

Q.19

Let the center of w be c(x,y)

Then the radius is the length of the tangent

$R^2 = x_1^2 + y_1^2 - k^2$

⇨ $(x-x_1)^2 + (y-y_1)^2 = x_1^2 + y_1^2 - k^2$

⇨ $X^2+y^2 - 2xx_1 - 2yy_1 + k^2 = 0$

It passes through (a,b)

⇨ $2ax_1 + 2by_1 - (a^2+b^2+k^2) = 0$

Ans: (A)

Q.20

If 3x + 4y=c is a tangent then [c]/5 =5

⇨ C = + - 5

Ans: (C)

Q.21

Let the line be (y-2)=m(x-2)

Perpendicular distance from center = $|6m|/\sqrt{(1+m*m)}$

R=5 gives $R^2 - d^2 = 4$

$25 - 36m^2/(1+m^2) = 16$

$25 + 25m^2 - 36m^2 = 16 + 16m^2$

$27m^2 = 9$ gives $3m^2=1$

Ans: (D)

Q.22

$(x-3)^2 + (y-4)^2 = 10$

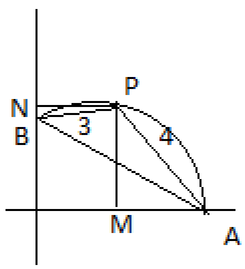
$x-3 = a$

$y-4 = b$

then $a^2 + b^2 < 10$

Ans: (D)

Q.23



$\angle APB = \Pi/2$

$PM = y$

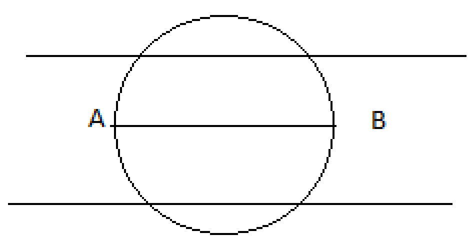
$PN = x$

Therefore, triangle PNB and Triangle PAM are similar

⇨ $x/3 = y/4$

Ans: (C)

Q.24



AB is the diameter of the circle ½ AB.h = 5

h=2

radius of the circle = 5/2

hence 4 points (C)

Q.25

The equation of the circle is

$(x-x_1)(x-x_2) + (y-y_1)(y-y_2) = 0$

$X^2 + 2ax - b^2 + y^2 + 2px - q^2 = 0$

Ans: (C)

Q.26

Both the circle pass through (0,0)

If they touch each other then tangent at (0,0) should coincide

⇨ $gx + fy = 0$ and $g_1x + f_1y = 0$ are same

⇨ $g/g_1 = f/f_1$

Ans: (A)

Q.27

Let the tangent be $x\cos\theta + y\sin\theta = r$

⇨ $A = (r/\cos\theta, 0)$

And $B= (0, r/\sin\theta)$

⇨ $C= (r/\cos\theta, r/\sin\theta)$

⇨ $1/x^2 + 1/y^2= 1/r^2$

Ans:(B)

Q.28

The common chord should be the diameter of $x^2 +y^2=16$

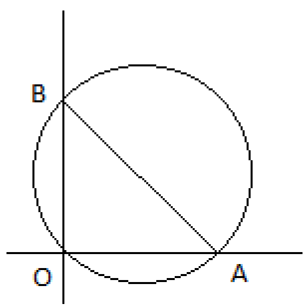
Chord is $3x-4y=0$

$X^2+y^2-16+k(3x-4y)=0$, hence Radius = 5

⇨ $K=\pm 6/5$

Ans:(A)

Q.29



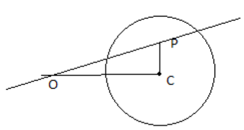
If OA = a , OB = b, hence centroid G(h,k) = (a/3,b/3)

$a^2 + b^2 = 36r^2$

$h^2+k^2 = 4r^2$

Ans: (C)

Q.30

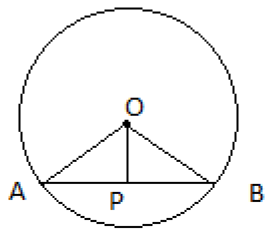


Locus of P is the circle on OC as diameter

⇨ $X^2 + y^2 – ax = 0$

Ans: (C)

Q.31



$\angle AOP = \Pi/4$

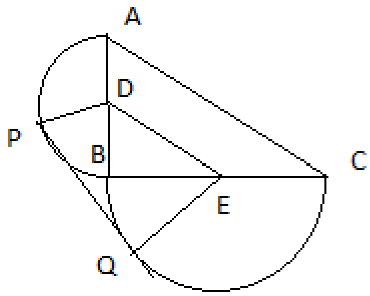
⇨ $OP = \sqrt{2}$

⇨ $OP^2 = 2$

⇨ $X^2 + y^2 = 2$

Ans: (D)

Q.32



D, E are mid points of AB , BC

⇨ $DE = 5/2$

⇨ $\angle DPQ = \angle PQE = \Pi/2$

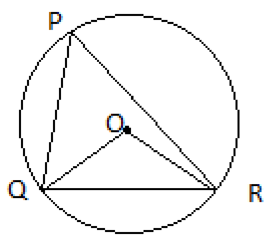
⇨ $PQ^2 = DE^2 - (QE - PD)^2$

$= 25/4 - 1/4 = 6$

⇨ $PQ = \sqrt{6}$

Ans: (B)

Q.33



$OQ = 5$

$OR = 5$

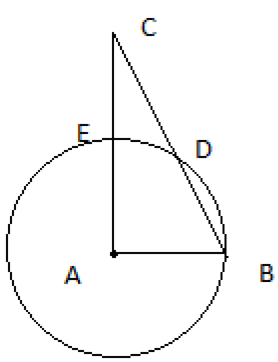
$RQ = 5\sqrt{2}$

$\therefore \angle QOP = \Pi/2$

And $\angle QPR = ½ \angle QOP$ or $\Pi - \angle QOP = \Pi/4$ or $3\Pi/4$.

Ans: (D)

Q.34



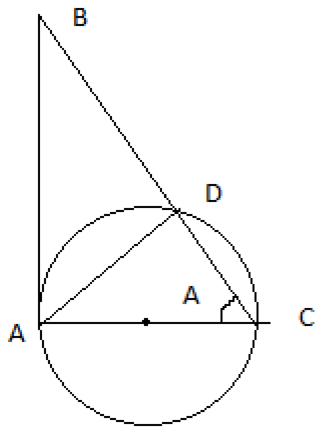
CD.CB = power of the point = $CA^2 - EA^2$

$AC^2 = 16*36 + (BC^2 - AC^2)$

$2AC^2 = 16*36 + 36^2$ gives $AC = 6\sqrt{26}$

Ans: (B)

Q.35



If AC = 2r, then DC = 2rcosθ

AD=2rsinθ, AB=2rtanθ & BC=2rsecθ

⇨ $AC^2 = AB^2 . AD^2/(AB^2 - AD^2)$

Ans: (D)

Q.36

A moves on the circle $x^2 + y^2 = 9$

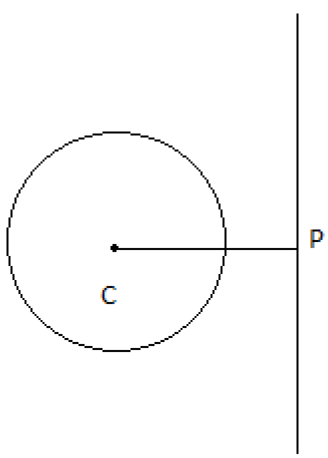
Let A = (3cosθ,3sinθ)

Then the centroid G = (cosθ,sinθ)

⇨ $X^2 + y^2 = 1$

Ans: (A)

Q.37



$x^2 + y^2 = 6x - 8y$

Center = (3,-4)

Perpendicular distance from center to line

d = CP = |-9 +16-25|/5

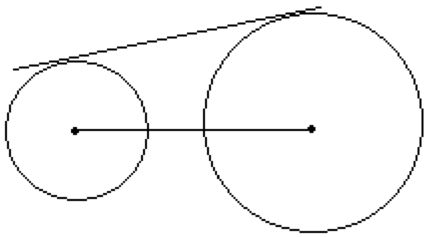
=18/5

Radius = 5

Shortest distance = |18/5 -5| =7/5

Ans: (A)

Q.38



Centers $C_1 = (10,0)$ & $C_2 = (-15,0)$

$R_1=6$ & $R_2=9$

$d = C_1C_2 = 25$

$d > R_1+R_2$

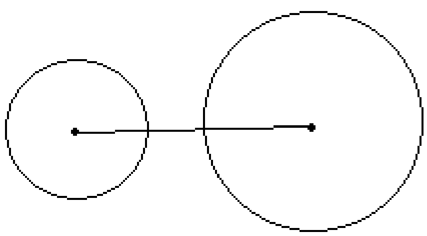
⇨ Circles are non intersecting

Length of the direct common tangent = $(d^2 - (R_1 - R_2)^2)^{1/2} = \sqrt{616}$

$PQ = (d^2 - (R_1 + R_2)^2)^{1/2} = 20$

Ans: (c)

Q.39



$C_1 = (0,1)$ & $C_2 = (4,9)$, $R_1=2$ & $R_2=2$

$C_1C_2 > R_1+R_2$

Center of the smallest circle is mid point of $C_1C_2=(2,5)$

Ans: (D)

Q.40 Equation of the family of circles is

$(x-2)^2 + (y-5)^2 + k(2x-y+1) = 0$

Center = $[-(k-2), (k+10)/2]$

Lies on $x-2y = 4$

$K=-6$

$\therefore$ radius = $3\sqrt{5}$

Ans : (A)

–

<u>**CIRCLES**</u>

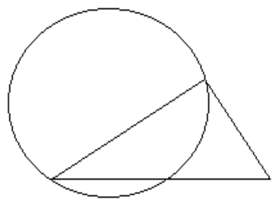
<u>**EXERCISE – 2 (B)**</u>

Q.1

⇒**Case : 1 [D]**

Three distinct lines from a triangle (1) in-circle and (3) Ex-circle are possible

Ans : 4

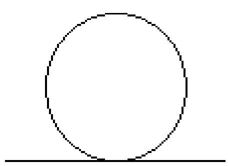


⇒**Case : 2 [A, D]**

Three line are concurrent lines

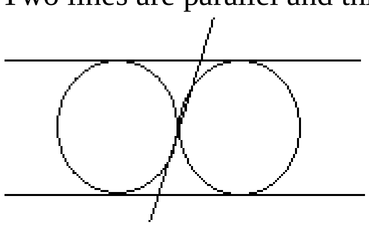
Ans : 0

Possbile Ans : 4



⇒**Case : 3 [C]**

Two lines are parallel and third is intersecting then 2 circle are possible.



⇒**Q.2 [B, D]**

(-1,1) (0,6) , (5,5) from a right angle isosceles triangle with (0,6)

Circumcenter = mid-point of hypotenuse.

$\Rightarrow$ (3,2)

Slope = $\frac{2}{3}$: slope of tangent

Slope of normal : $\frac{-3}{2}$

$\Rightarrow y - 2 = \frac{-3}{2}(x - 3)$ ……(1) : eq$^{\underline{n}}$ of normal.

Equation of circle $(x-3)^2 + (y-2)^2 = r^2$

(5 ,5) will satisfy this eq$^{\underline{n}}$

$\Rightarrow (5-3)^3 + (5-2)^2 = r^2$

$\Rightarrow r^2 = 13$

Equation of circle : $(x-3)^2 + (y-2)^2 = 13$

$\Rightarrow$From (1) $y = 2 - \frac{3}{2}(x-3)$

Substituting this in circle equation.

$$\Rightarrow (x-3)^2 + \left(2 - \frac{3}{2}(x-3) - 2\right)^2 = 13$$

$$\Rightarrow (x-3)^2 + \frac{9}{4}(x-3)^2 = 13$$

$$\Rightarrow (x-3)^2 \frac{13}{4} = 13$$

$$\Rightarrow (x-3)^2 = 4$$

$$\Rightarrow (x-3) = \pm 2$$

$\Rightarrow x - 3 = 2 \quad \text{or} \ -2$

$\Rightarrow \therefore \quad x = 1 \text{ or } x = 5$

$\Rightarrow y = 5 \text{ or } y = -1$

$\Rightarrow (1, 5); \qquad (5, -1)$

Q.3 [A, B, C, D]

$$\frac{x - x_1}{\cos\theta} = r \qquad ....(1)$$

$\Rightarrow x = x_1 + r\cos\theta$

$$\Rightarrow \frac{y - y_1}{\sin\theta} = r \qquad ....(2)$$

$\Rightarrow y = y_1 + r\sin\theta$

$$\Rightarrow r = \frac{x - x_1}{\cos\theta}$$

Substituting this in (2)

$$\Rightarrow y - y_1 = \frac{(x - x_1)}{\cos\theta}\sin\theta = \tan\theta(x - x_1) \qquad ......(C)$$

Adding (1) and (2)

$\Rightarrow x + y = x_1 + y_1 + r(\cos\theta + \sin\theta)$

If θ is constant ....(A)

$\Rightarrow x^2 = (x_1 + r\cos\theta)^2 = x_1^2 + r^2\cos^2\theta + x\,r\cos\theta$

$\Rightarrow y^2 = y_1^2 + r^2\sin^2\theta + y\,r\sin\theta$

$\Rightarrow x^2 + y^2 = x_1^2 + y_1^2 + r^2 + r(x\cos\theta + y\sin\theta)$

If r = constant and θ varies

It represents equation of circle ………..(B)

And centers will be $\frac{r}{2}\cos\theta$, $\frac{r}{2}\sin\theta$ …………(D)

Q.4 [B, C]

$L_2 : x + y = 1 \Rightarrow y = 1 - x$

$\Rightarrow S_1 : x^2 + y^2 - x + 3y = 0$

Substituting value of y in equation of circle.

$\Rightarrow x^2 + (1-x)^2 - x + 3(1-x) = 0$

$\Rightarrow x^2 - 3x + 2 = 0$

$\Rightarrow (x_2 - 2)(x_1 - 1) = 0$

Difference of roots $x_2 - x_1 = d = 1$

$\therefore$ intercept length $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$\Rightarrow \sqrt{2}\, d$ ……(1)

or $(x_2 - x_1) = (y_2 - y_1)$

$\Rightarrow L_1 : y = mx$. (passing through origin)

Substituting this in equation of circle

$\Rightarrow x^2(1 + m^2) + (3m - 1)x = 0$

$\Rightarrow x = \frac{1-3m}{1+m^2}$

Difference of roots $x_2 - x_1 = d^1 = \frac{1-3m}{1+m^2}$

$\therefore$ intercept length $= \sqrt{(x_2 - x_1)^1 + (y_2 - y_1)^2}$

$\Rightarrow d^1\sqrt{1 + m^2}$ ……(2)

Equating (1) and (2)

$\Rightarrow \sqrt{2}d = d^1\sqrt{1 + m^2}$

$\Rightarrow \sqrt{2} = \dfrac{1 - 3m}{\sqrt{1 + m^2}} \Rightarrow \sqrt{2}\left(1 + m^2\right) = 1 - 3m$

$\Rightarrow 2\left(1 + m^2\right) = (1 - 3m)^2$

$\Rightarrow 7m^2 - 7m + m - 1 = 0$

$\Rightarrow m = +1\ ;\ m = \dfrac{1}{7}$

$\therefore$ equation of L_1 : $x - y = 0$ $7y + x = 0$

Q.5 [A, B, D]

$S_1 = x^2 + y^2 = 4$

$\Rightarrow S_2 = x^2 + y^2 - 2x - 4y + 4 = 0$

$\Rightarrow R_1 = 2$

$\Rightarrow R_2 = \sqrt{1 + 4 - 4} = 1$

$\Rightarrow C_1 = (0, 0)$ $C_2 = (1, 2)$

$\Rightarrow C_1C_2 = \sqrt{5} < R_1 + R_2$

$\therefore$ True common tangents ……(A)

Let the variable point be P (h,k)

$$\Rightarrow h^2 + k^2 - 4 = h^2 + k^2 - 2h - 2k + 4$$

$$\Rightarrow 2h + 4k - 8 = 0$$

$$\Rightarrow h + 2k - 4 = 0$$

$$\Rightarrow x + 2y - 4 = 0 \qquad \ldots\ldots.(B)$$

For orthogonality

$$\Rightarrow 2\left(g_1 g_2 + f_1 f_2\right) = C_1 + C_2$$

$$\Rightarrow 2\left(0 + 0\right) = 4 - 4$$

$$\Rightarrow \text{True} \qquad \ldots\ldots\ldots.(D)$$

Q.6 [C, D]

Let equation of line passes through (1,0) with slope m_1

$$\text{Eq}^{\underline{n}} : \frac{y-0}{x-1} = m_1 \qquad \Rightarrow \frac{y}{x-1} = m_1$$

similarly,

$$\Rightarrow m_2 = \frac{y-0}{x+1} \quad \Rightarrow \frac{y}{x+1} = m_2$$

Angle between the two lines is 45°

$$\Rightarrow \therefore \tan 45^\circ = \left|\frac{m_1 - m_2}{1 - m_1 m_2}\right|$$

$$\Rightarrow \pm 1 = \frac{\left|\frac{y}{x-1} - \frac{y}{x+1}\right|}{1 + \frac{y}{x-1}\frac{y}{x+1}} \quad \Rightarrow \pm 1 = \frac{2y}{x^2 + y^2 - 1}$$

$$\Rightarrow x^2 + y^2 - 1 = 2y \qquad x^2 + y^2 - 1 = -2y$$

$\Rightarrow x^2 + y^2 - 2y = 1$ $\quad\quad$ $x^2 + y^2 + 2y - 1 = 0$

$\Rightarrow$ centers : (0 , - 1) $\quad\quad$ centers : (0 , 1)

$\Rightarrow radius = \sqrt{2}$ $\quad\quad$ $radius = \sqrt{2}$

Q.7 [A, B]

Equation of circle will be $x^2 + y^2 = 1$

y = 1 + c (x + 3)

Substituting this in eq$^{\underline{n}}$ of circle.

$$\Rightarrow x^2 + \left(1 + x\left(x+3\right)^2\right) = 1$$

$$\Rightarrow x^2 + 1 + c^2\left(x+3\right)^2 + 2c\left(x+3\right) = 0$$

$$\Rightarrow x^2 + c^2\left(x^2 + 9 + 6x\right) + 2c\left(x+3\right) = 0$$

$$\Rightarrow \left(1 + c^2\right)x^2 + \left(6c^2 + 2c\right)x + 9c^2 + 6c = 0$$

Coincident points. $\quad$ D = 0

$$\Rightarrow \left(6c^2 + 2c\right)^2 - 4\left(9c^2 + 6c\right)\left(1 + c^2\right) = 0$$

$$\Rightarrow -32c^2 - 24c = 0$$

$\Rightarrow$ C = 0 ; $\quad$ C = $-\dfrac{3}{4}$

Q.8 [A, C]

$$a + b = d^2$$

$$\Rightarrow b = d^2 - a$$

Substituting this in eq$^{\underline{n}}$ of circle.

$\Rightarrow al^2 - bm^2 + 2dl + 1 = 0$

$\Rightarrow al^2 - \left(d^2 - a\right)m^2 + 2dl = 0$

$\Rightarrow a = -d^2 + a \quad \Rightarrow \quad d = 0$

$\Rightarrow \therefore$ eq$^{\underline{n}}$ of circle.

$\Rightarrow al^2 + am^2 + 1 = 0$

Q.9 [B, C, D]

$S_1 : x^2 + y^2 + 2x + 4y + 1 = 0$

$\Rightarrow S_2 : x^2 + y^2 - 4x + 3 = 0$

$\Rightarrow S_3 : x^2 + y^2 + 6y + 5 = 0$

$\Rightarrow S_1 - S_2 \Rightarrow 6x + 4y - 2 = 0$

$\Rightarrow 3x + 2y - 1 = 0$ ……(1)

$\Rightarrow S_1 - S_3 \Rightarrow 2x - 2y - 4 = 0$

$\Rightarrow x - y - 2 = 0$ …….(2)

Solving (1) and (2)

3x + 2y = 1

+ 2x – 2y = – 2

5x = – 1

$\Rightarrow x = -\frac{1}{5}; \quad y = x - 2$

$\Rightarrow y = \frac{-1}{5} - 2 = \frac{-11}{5}$

$\therefore$ Radical center lies in 4th quadiant.

$\Rightarrow C_1 = \{-1, -2\}$ $r_1 = 2$

$\Rightarrow C_2 = \{2, 0\}$ $r_2 = 1$

$\Rightarrow C_3 = \{0, 3\}$ $r_3 = 2$

Let the co-ordinates of center of circle be {+ g , + f} and radius be

C , therefore.

SS_1 are orthogonal.

i.e. 2{ - g + (- 2f)} = c + 2

$\Rightarrow$2g + 2f + c + 2 = 0 ……….(1)

$\Rightarrow SS_2$: 2{2g} = c + 1

$\Rightarrow$4g – c – 1 = 0 ………..(2)

$\Rightarrow SS_3$: 2{+ 3f} = c + 2

$\Rightarrow$ 6f – c – 2 = 0 ………..(3)

(1) – (2)

$\Rightarrow$6g + 2f + 1= 0 ………..(4)

(1) – (3)

$\Rightarrow$2g + 8f= 0 ………..(5)

Solving (4) and (5)

$$\begin{array}{r} 6g + 2f + 1 = 0 \\ -6g + 24f \quad = 0 \\ \hline -22f \quad = 0 \end{array}$$

$$\Rightarrow f = +\frac{1}{22}$$

$$\Rightarrow g = -\frac{2}{11}$$

$\Rightarrow$ c = 1 ………(C)

Q.10. [A, B, C, D]

$$y = \frac{c^2}{x}$$

$$\Rightarrow x^2 + \frac{c^4}{x^2} = a^2$$

$$\Rightarrow x^4 - a^2x^2 + c^4 = 0$$

$\Rightarrow x_1 + x_2 + x_3 + x_4$ sum of roots = 0

$$\Rightarrow x_1 + x_2 + x_3 + x_4 = c^4$$

Similarly for y.

$$x = \frac{c^2}{y}$$

$$\Rightarrow \frac{c^4}{y^2} + y^2 = a^2$$

$$\Rightarrow y^4 - a^2y^2 + c^4 = 0$$

$$\Rightarrow y_1 + y_2 + y_3 + y_4 = 0$$

$$\Rightarrow y_1y_2y_3y_4 = c^4$$

Q.11

$$C_1 : x^2 + y^2 - x - 2y - 5 = 0$$

$$\Rightarrow C_2 : x^2 + y^2 + 2x - 3y - 7 = 0$$

$$\Rightarrow C_1 - C_2 : -3x + y - 12 = 4$$

$\Rightarrow 3x - y + 16 = 0$ .....(1)

$\Rightarrow m_1 = 3$

$\Rightarrow C_1\left(\frac{1}{2}, 1\right)$; $C_2\left(-1, \frac{3}{2}\right)$

Line joining C_1C_2 is

$$\Rightarrow \frac{y-1}{x-\frac{1}{2}} = \frac{1-\frac{3}{2}}{\frac{1}{2}+1}$$

$$\Rightarrow \frac{2(y-1)}{2x-1} = \frac{2-3}{1+2} = -\frac{1}{3}$$

$\Rightarrow 6y - 6 = -2x + 1$

$\Rightarrow 2x + 6y = 7$

$\Rightarrow m_2 = -\frac{1}{3}$

$\Rightarrow m_1m_2 = -1$ $\therefore$(A)

$\Rightarrow$Radical axis $C_1 - C_2 = x^2 + y^2 - x - 2y - 5 - x^2 - y^2 - 2x + 3y + 7 = 0$

$\Rightarrow 3x - y + 12 = 0$ ......(2) (B)

Distance between line (1) and (2)

$$\Rightarrow \frac{16-12}{\sqrt{9+1}} = \frac{4}{\sqrt{10}}$$

Paragraph – I

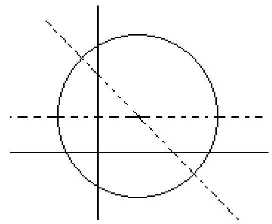
$A:\{(x\,,\,y):\geq 1\}$

$B:\{(x\,,\,y): x^2+y^2-4x-2y-4=0\}$

$C:\{(x\,,\,y): x+y=\sqrt{2}\}$

$x^2+y^2-4x-2y-4=0$

center (2 , 1) radius = 3



Q.12 1 point

Q.13

$(x+1)^2+(y-1)^2 \Rightarrow$ distance of point (x , y)from (-1 , 1)

$\Rightarrow (x-5)^2+(y-1)^2 \Rightarrow$ distance of point (x , y) from (5 , 1)

$\Rightarrow$ (– 1, 1) and (5 , 1) are the end points of diameter

$\Rightarrow \therefore (5+1)^2+(1-1)^2 = 36$ {from Pythagoras theorem }

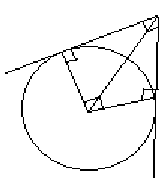
Q.14

Radius of the locus point is $r\sqrt{2}$

$\Rightarrow 3\sqrt{2}$

Area enclosed $\pi\left(2\sqrt{2}\right)^2-\pi r^2$

$\Rightarrow \pi r^2 = 9\pi$



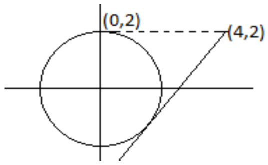
<u>Paragraph – 2</u>

Q.15

$x^2+y^2=4$

form f g use one tangent is y = 2

Then, y – 2 = m (x – 4) be the other tangent.



$\Rightarrow$y = 2 + m(x - 4)

substituting this in equation of circle.

$$\Rightarrow x^2 + c_2 + m(x-4)^2 = 4$$

$$\Rightarrow x^2(1+m^2) + (4m - m^2 8)x + 4m + m^2 16 - 16m$$

For D = 0

$$\Rightarrow (4m - m^2 8)^2 - 4(1+m^2)(4 + m^2 16 - 16m)$$

$$\Rightarrow -48m^2 + 69m = 0$$

$$\Rightarrow m = \frac{4}{3}$$

$\Rightarrow \therefore \theta$ will be varies between $\left(45^\circ, 60^\circ\right)$

Q.16 [B]

$$y - 2 = \frac{y}{3}(x-4)$$

For x intercept y = 0

$$\Rightarrow x = \frac{5}{2}$$

Q.17 [B]

$$x\cos\alpha + y\sin\alpha = 1$$

$\Rightarrow$ x intercept $= \dfrac{1}{\cos\alpha}$

$\Rightarrow$ y intercept $= \dfrac{1}{\sin\alpha}$

$\Rightarrow (h\ ,\ k) = \left(\dfrac{1}{2\cos\alpha}\ ,\dfrac{1}{2\sin\alpha}\right)$ co-ordinates of mid-points.

$$\Rightarrow \therefore \left(\frac{1}{2h}\right)^2 + \left(\frac{1}{2k}\right)^2 = 1$$

$$\Rightarrow h^{-2} + k^{-2} = 2^{-2}$$

$$\Rightarrow x^{-2} + y^{-2} = 2^{-2}$$

<u>Paragraph – 3</u>

Q.18

$$Ca := \frac{x^2}{4} - ax + a^2 + a - 2$$

$$\Rightarrow \ f(0) < 0$$

$$\Rightarrow a^2 + a - 2 < 0$$

$$\Rightarrow (a+2)(a-1) \ < \ 0$$

$$\Rightarrow a \in \ \in (-2, 1)$$

$\Rightarrow \therefore$ two integer values.

Q.19 [A]

vertex $\equiv -\dfrac{b}{2a} = \dfrac{a}{\frac{1}{2}} = 2a$

$$\Rightarrow -\frac{D}{4a} \quad = -\frac{\left(a^2 - (a^2 + a - 2)\right)}{4} \quad = \quad -\left(a^2 - a^2 - a + 2\right)$$

$\Rightarrow$a – 2.

$\Rightarrow \therefore$ vertex = (2a , a – 2)

$$\Rightarrow \text{locus} : \frac{x}{2} = y + 2$$

$\Rightarrow x = 2y + 4$

Q.20

For a = 3

$$\Rightarrow Ca : y = \frac{x^2}{4} - 3x + 10 \qquad c = y = 2 - \frac{x^2}{4}$$

$$\Rightarrow \frac{x^2}{4} - 3x + 10 = 2 - \frac{x^2}{4}$$

$$\Rightarrow \frac{x^2}{2} - 3x + 8 = 0$$

$\Rightarrow x^2 - 6x + 10 = 0$

Sum of the roots $m_1 + m_2 = 6$

<u>Paragraph – 4</u>

Q.21 [B]

$ax^2 + 3xy - 2y^2 - 5x + 5y + c = 0$

Represent pair of perpendicular lines

$\Rightarrow$a + b= 0$\Rightarrow$ a – 2 = 0 $\Rightarrow$ a = 2

$\Rightarrow \therefore 2x^2 + 3xy - 2y^2 - 5x + 5y + c = 0$

To find x intercept y = 0

$\Rightarrow 2x^2 - 5x + c = 0$

$\Rightarrow x_1$ *and* x_2 are two roots then

$\Rightarrow x_1 + x_2 = \frac{5}{2}$

To find y intercept x = 0

$\Rightarrow -2y^2 + 5y + c = 0$

$\Rightarrow 2y^2 - 5y - c = 0$

$\Rightarrow y_1$ *and* y_2 are two roots then

$\Rightarrow y_1 + y_2 = \frac{5}{2}$

$\Rightarrow \therefore x_1 + x_2 + y_1 + y_2 = \frac{5}{2} + \frac{5}{2} = 5$

Q.22 [C]

So line $2y^2 - 5y - c = 0$; $c = \frac{-23}{2}$

$\therefore$ and find the value of A , B

Find the orthocenter and circumter.

Q.23 [D]

$2\left(S_1 \times f_2 + g_1 g_2\right) = C_1 + C_2$

$\Rightarrow \therefore 2\left(5\right) = -\frac{23}{2} + k$

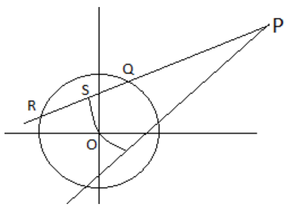
$\Rightarrow k = \frac{3}{2}$

<u>Paragraph – 5</u>

Q.24

When Q & R will be the diameter then S will be the origin.

So, the curve on which S lies is on area of circle OP as diameter



Q.25 [D]

(PQ) (PR) $\neq PS^2$

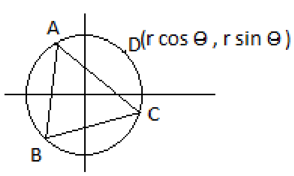
$\Rightarrow$As S is the mid-point of Q and R.

Q.26 [A]

Then AD + DC = BD

$$\Rightarrow (\beta+\gamma-\alpha)(\gamma+\alpha-\beta)(\alpha+\beta-\gamma)=0$$

Therefore



<u>Paragraph – 6</u>

Q.27 [C]

$$S_1 : x^2 + y^2 + 2x - 3 = 0$$

$$\Rightarrow S_2 : x^2 + y^2 - 1 = 0$$

$\Rightarrow S_3 : x^2 + y^2 + 2y - 3 = 0$

$\Rightarrow \ C_1 : (1, 0)$ $\qquad R_1 = 2$

$\Rightarrow \ C_2 : (0, 0)$ $\qquad R_2 = 1$

$\Rightarrow \ C_3 : (0, 1)$ $\qquad R_3 = 2$

$\Rightarrow \ R = \sqrt{R_1^2 + R_2^2 + R_3^2} = \sqrt{4 + 1 + 4} = 3$

Q.28 [D]

$x^2 + y^2 + 2gx + 2fy + c = 0 \ : \ S$

S and S_1 are orthogonal

$\Rightarrow$2(g) = – 3 + c

$\Rightarrow$2g = – 3 + c ……….(1)

S and S_2 are orthogonal

$\Rightarrow$2(0 + 0) = – 1 + c

$\Rightarrow$c = 1 ………..(2)

S and S_3 are orthogonal

$\Rightarrow$2(f + 0) = – 3 + c

$\Rightarrow$2f= – 3 + c …………(3)

Solving (1) and (2)

$\Rightarrow$2g= – 3 + 1 = – 2

$\Rightarrow$g = – 1

Solving (1) and (3)

$\Rightarrow$f = – 1

$\Rightarrow \therefore$ (a , b + r) = 3

Q.29 [C]

Let $x^2 + y^2 + 2gx + 2fy + c = 0$ be the circle touches S_1

$\Rightarrow$ 2x – 3 +2gx + 2fy + c = 0 ……………(1)

Touches S_2 at (1 , 0)

$\Rightarrow$ – 1 + 2gc + 2f + c = 0

$\Rightarrow$ – 1 + 2g + c = 0 …………….(2)

Passes through (3 , 2)

$\Rightarrow$ 9 + 4 + 6g + 4y + c = 0 ……………..(3)

Solving (1) , (2) and (3)

We get radius = 2

<u>Assertion and Reasoning Type</u>

Q.30. $C_1 : x^2 + y^2 – 6x – 4y + 4 = 0$

$\Rightarrow$ Center (3 , 2)

$\Rightarrow C_2 : x^2 + y^2 – 8x – 6y + 23 = 0$

$\Rightarrow (3)^2 + (2)^2 - 8(3) - 6(2) + 23 = 0$

$\Rightarrow 0 = 0$

$\Rightarrow$ S – 2 true

$\Rightarrow$ S – 1 true

$\Rightarrow$ but not correct explonation.

Q.31 [A]

$S : x^2 + y^2 - 6x + 8y - 75 = 0$

Center : $(+3 , -4)$

Radius : 10

Equation of director circle.

$\Rightarrow (x-3)^2 + (y+4)^2 = 2 \times 100$

$\Rightarrow x^2 + 9 - 6x + y^2 + 16 + 8y = 200$

$\Rightarrow x^2 + y^2 - 6x + 8y - 175 = 0$

$\Rightarrow$ (13, 6)

$\Rightarrow (13)^2 + (6)^2 - 6(13) + 8(6) - 175 = 0$

$\Rightarrow \therefore$ P lies on the director circle.

Q.32 [C]

$x^2 + y^2 - 4x + 8y - 16 = 0$

$\Rightarrow$Center : (2 , - 4)

$\Rightarrow$One line can pass through (2 , -3) and (2 , -4)

$\Rightarrow$as (2 , -3) lies inside the circle.

$\Rightarrow S_2$ – false

Q.34 [D]

Let P $\equiv$(h , k)

$\Rightarrow$hx + ky – 3(x + h) – 4(y + h) + 5 = 0

should passes through (6 , 8)

$\Rightarrow$6h + 8k – 18 – 34 – 4(1) – 4k + 5 = 0

$\Rightarrow$3h + 4k – 18 – 32 + 5 = 0

$\Rightarrow$3h + 4k – 45 = 0

$\Rightarrow$5 – 1 $\longrightarrow$ false.

$\Rightarrow$S (6 , 8) : $(6)^2 + (8)^2 - 6(6) - 8(8) + 5 = 0$

$\Rightarrow$36 + 64 – 36 – 64 + 5 > 0

$\Rightarrow \therefore$ P lies outside the circle

Q.35 [C]

L : k(x – y – 4) + (7x + y + 20) = 0

$\Rightarrow$ x – y = 4

$\Rightarrow$ x = 4 + y

$\Rightarrow$ 7x + y + 20 = 0

$\Rightarrow$ 7(4 + y) + y +20 = 0

$\Rightarrow$ 28 + 7y + y +20 = 0

$\Rightarrow$ 48 + 8y = 0

$\Rightarrow$ y = – 6

$\Rightarrow$ x = – 2

$\Rightarrow$ Center (– 2, – 6)

$\Rightarrow x^2 + y^2 + 4x + 12y - 60 = 0$

$\Rightarrow$ Center (- 2 , - 6)

$\Rightarrow \therefore S_1 \longrightarrow$ true.

$\Rightarrow S_2 \longrightarrow$ false.

Q.36 [A]

Radical axis $2\,(1)\,x + 2\left(0 + \frac{1}{2}\right)y + (-4 - 1) = 0$

$\Rightarrow$ 2a + y = 5

$\Rightarrow$ (0 , 5) lies on the radical axis

$\Rightarrow S_2 \longrightarrow$ True

$\Rightarrow S_1 \longrightarrow$ True

Q.37 [A]

C_1 and C_2 always intersect at two point and they are always orthogonal.

orthogonal

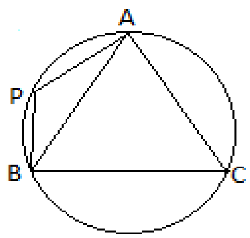
$\Rightarrow (x-a)^2 + y^2 = a^2 \quad x^2 + (y-a)^2 = a^2$

Q.38 [D]

(x + y) is always equal to z

$\Rightarrow$ S – 1 false

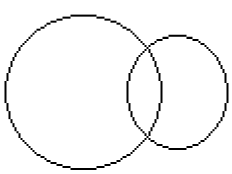
$\Rightarrow$ S – 2 true



<u>Matric Match</u>

Q.39

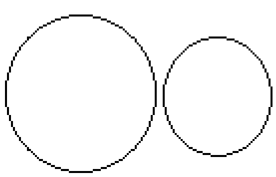
A)



two common tangent

A → P , Q

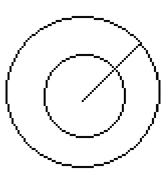
B)



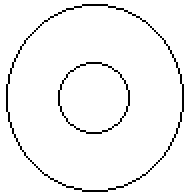
have common normal

B → Q , P

C)



C → Q

D) 

$d \to Q, S$

Q.40

$$\Rightarrow x^2 + y^2 - 20y + 90 = 0$$

(A) [P,Q, R]

$$\Rightarrow (kx)^2 + x^2 - 20(kx) + 90 = 0$$

$$\Rightarrow x^2 (1+k)^2 - 20kx + 90 = 0$$

$$\Rightarrow D \geq 0$$

$$\Rightarrow k^2 = \frac{360}{400} = \frac{9}{10}$$

$$\Rightarrow k = \frac{\sqrt{3}}{\sqrt{10}}$$

(B) [Q, R]

$$\Rightarrow x^2 + y^2 + px + py - 7 = 0$$

$$\Rightarrow x^2 + y^2 - 10x + 2py + 1 = 0$$

$$\Rightarrow 2\left(\frac{p}{2}(-5) + \frac{p}{2}(p)\right) = -7 + 1$$

$$\Rightarrow -5p + p^2 = -6$$

$$\Rightarrow p^2 - 5p + 6 = 0$$

$$\Rightarrow (p-3)(p-2) = 0$$

(C) [Q, R, S]

$\Rightarrow x^2 + y^2 + 2\lambda x + 4 = 0$

$\Rightarrow g^2 + f^2 - c > 0$

$\Rightarrow 0^2 + \lambda^2 - 4 > 0$

$\Rightarrow \lambda^2 > 4$

$\Rightarrow \lambda > \pm 2$

$\Rightarrow x^2 + y^2 - 4\lambda y + 8 = 0$

$\Rightarrow g^2 + f^2 - c > 0$

$\Rightarrow 0 + (-2\lambda)^2 + 8 > 0$

$\Rightarrow 4\lambda^2 + 8 > 0$

$\Rightarrow \lambda^2 > -2$

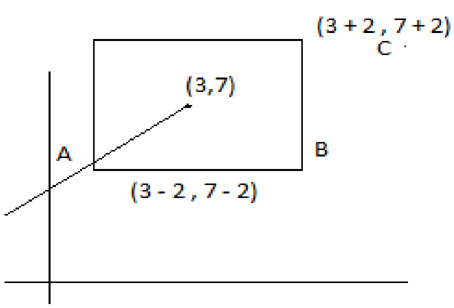
$\Rightarrow \lambda = Q, R, S$

(D) [P, S]

$\Rightarrow A \equiv (3 - 2, 7 - 2) \equiv (1, 3)$

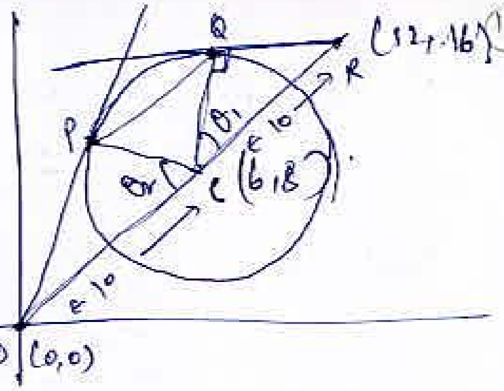
$\Rightarrow C \equiv (3 + 2, 7 + 2) \equiv (5, 9)$

$\Rightarrow \therefore$ Lies on (x = y)



Ex. 2(B)

41) shortest distance = $OP + \widehat{PQ} + QR$
where OP and QR are tangents to the given circle from O & R



$OP = \sqrt{(-6)^2 + (-8)^2 - 25} = \sqrt{75} = 5\sqrt{3}$

$QR = \sqrt{6^2 + 8^2 - 25} = \sqrt{75} = 5\sqrt{3}$

$\tan\theta_1 = \frac{QR}{CQ} = \frac{5\sqrt{3}}{5}$, $\tan\theta_2 = \frac{OP}{CP} = \frac{5\sqrt{3}}{5}$

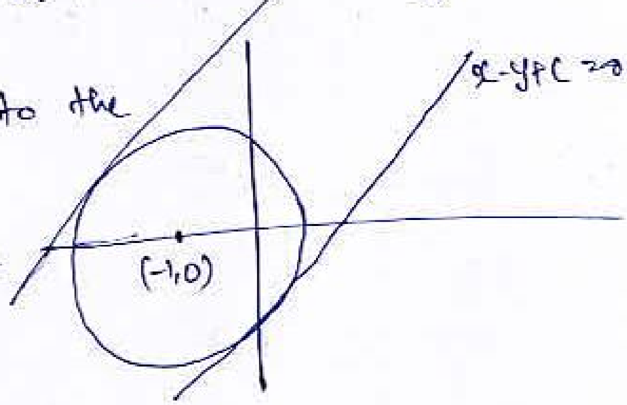
$\theta_1 = \pi/3$, $\theta_2 = \pi/3$

$\therefore \angle PCQ = \pi/3$

$\widehat{PQ} = 5\pi/3$

total distance $= OP + \widehat{PQ} + QR = 10\sqrt{3} + \frac{5\pi}{3}$

42) The line should be tangent to the given circle



$\therefore$ perpendicular distance from centre to the line = radius

$\left|\frac{c-1}{\sqrt{2}}\right| = \sqrt{2}$

$(c-1) = \pm 2 \Rightarrow c = 3$ or -1

$\therefore x - y + 3 = 0$ or $x - y - 1 = 0$

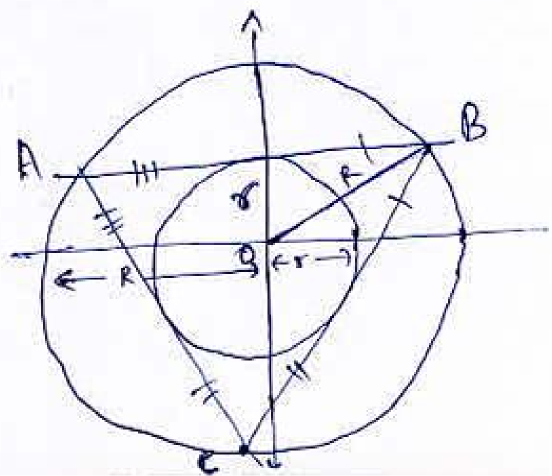
Now centre of circle should not satisfy $x - y + c \geq 0$

$\therefore x - y + c < 0$

$-1 - 0 + c < 0 \Rightarrow c < 1$

$\therefore c = -1$

43) $AB = BC = AC = 2\sqrt{R^2 - r^2}$



$\therefore \angle ABC = 60^\circ$

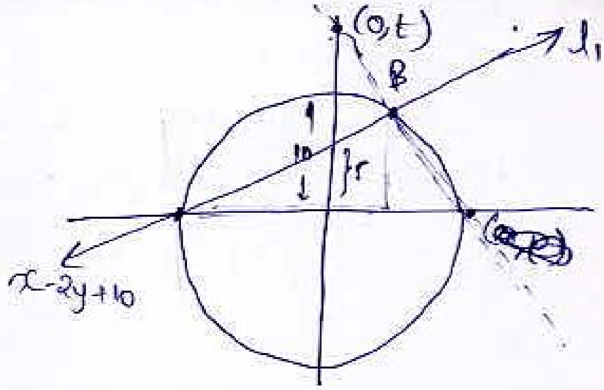
$\angle OBA = 30^\circ$

$\sin 30^\circ = \frac{r}{R} = \frac{1}{2}$

(2)

(44) equation of perpendicular line =

$2x + y = t$



Point B: $x - 2y + 10 = 0$

$4x + 2y - 2t = 0$

$5x = -10 + 2t \Rightarrow x = \frac{-10+2t}{5}$

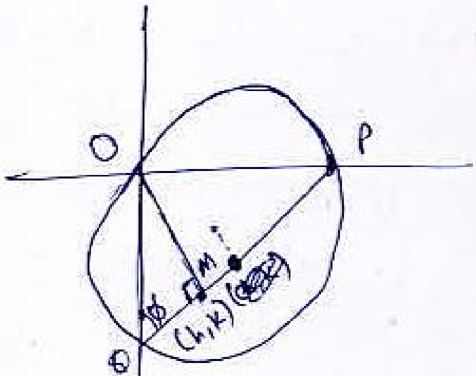
$2y = x + 10 = \frac{-10+2t}{5} + 10 = \frac{40+2t}{5}$

$\Rightarrow y = \frac{20+t}{5}$

$x^2 + y^2 = 100$

$\Rightarrow \left(\frac{-10+2t}{5}\right)^2 + \left(\frac{20+t}{5}\right)^2 = 100 \Rightarrow t = 20$

(45) $OM = \sqrt{h^2+k^2}$



$OQ = \frac{\sqrt{h^2+k^2}}{\sin\phi}$ $\quad OP = \frac{\sqrt{h^2+k^2}}{\cos\phi}$

∵ OPQ is Right angled triangle

∴ Circumcentre lies on hypotenous.

∴ $\frac{h^2+k^2}{\cos^2\phi} + \frac{h^2+k^2}{\sin^2\phi} = 4a^2$

Equation of QP : $\frac{x}{OP} + \frac{y}{OQ} = 1$

$\Rightarrow \frac{x\cos\phi}{\sqrt{h^2+k^2}} + \frac{y\sin\phi}{\sqrt{h^2+k^2}} = 1$

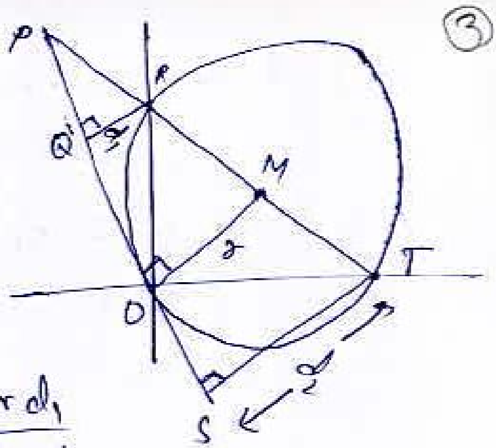
∵ (h,k) lies on QP $\Rightarrow \frac{h\cos\phi}{\sqrt{h^2+k^2}} + \frac{k}{\sqrt{h^2+k^2}}\sin\phi = 1$

∴ $\sin(\theta+\phi) = 1$ where $\sin\theta = \frac{h}{\sqrt{h^2+k^2}}$

$\Rightarrow \phi = \frac{\pi}{2} - \theta$

∴ $\frac{h^2+k^2}{\sin^2\theta} + \frac{h^2+k^2}{\cos^2\theta} = 4a^2 \Rightarrow (h^2+k^2)^2\left(\frac{1}{h^2} + \frac{1}{k^2}\right) = 4a^2$

(47) $\Delta PQR \sim \Delta POM$



$$\frac{PR}{PM} = \frac{PR}{PR+r} = \frac{d_1}{r} \Rightarrow PR = \frac{rd_1}{r-d_1}$$

$\Delta PQR \sim \Delta PST$

$$\frac{PR}{PT} = \frac{PR}{PR+2r} = \frac{d_1}{d_2} \Rightarrow PR = \frac{2rd_1}{d_2-d_1}$$

$$PR = \frac{rd_1}{r-d_1} = \frac{2rd_1}{d_2-d_1}$$

$$\Rightarrow d_2 - d_1 = 2r - 2d_1$$

$$\Rightarrow 2r = d_2 + d_1$$

(48) power of $(a,b) = a^2+b^2-2(t^2-3t+1)a - 2(t^2+2t)b + t$

Power of $(a,b) = k \Rightarrow \frac{d}{dt}(\text{Power of } (a,b)) = 0 \ \forall t$

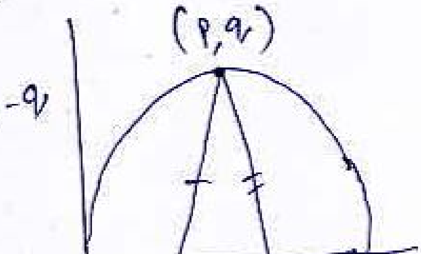
$\Rightarrow -4at + 6a - 4tb - 4b + 1 = 0$

$\Rightarrow -(4a+4b)t + (6a-4b+1) = 0 \quad \forall t$

$\therefore 4a+4b = 0$ and $6a - 4b + 1 = 0$

$\Rightarrow a = -b$ $\qquad b = \frac{1}{10}, \ a = -\frac{1}{10}$

(49) Since x axis is bisecting the chord
Hence its other point will have ordinate $= -q$



$x^2+y^2 = px + qy$

$y = -q \Rightarrow x^2 + q^2 = px + q(-q)$

$x^2 - px + 2q^2 = 0$

This has 2 real roots.

$\therefore p^2 - 8q^2 > 0$

(4)

50) Let equation of line be $y - mx = m$ (since it passes through $(-1, 0)$) $m \in Q$

P is point of intersection of this line and given circle and x coordinate of $P \neq -1$

$$\therefore x^2 + (mx + m)^2 = 1$$

$$m^2 (x+1)^2 = 1 - x^2$$

$$m^2 = \frac{1-x}{1+x} \quad (\because x \neq -1)$$

$$\therefore \frac{m^2 - 1}{m^2 + 1} = -x$$

$$\therefore \forall m \in Q \qquad x \in Q$$

(51) equation of line passing through $(x_1, -1)$ and $(x_2, 1)$ is

$$(y-1) = \frac{2}{x_2 - x_1}(x - x_2)$$

$$y(x_2 - x_1) - x_2 + x_1 = 2x - 2x_2$$

$$\Rightarrow y(x_2 - x_1) - 2x + x_1 + x_2 = 0$$

~~this~~ distance of this line from origin is 1

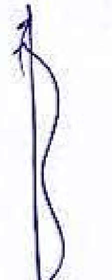
$$\Rightarrow \left| \frac{x_1 + x_2}{\sqrt{(x_2 - x_1)^2 + 4}} \right| = 1$$

$$\Rightarrow (x_1 + x_2)^2 = (x_2 - x_1)^2 + 4$$

$$x_1^2 + x_2^2 + 2x_1 x_2 = x_1^2 + x_2^2 - 2x_1 x_2 + 4$$

$$\Rightarrow x_1 x_2 = 1$$

(52)



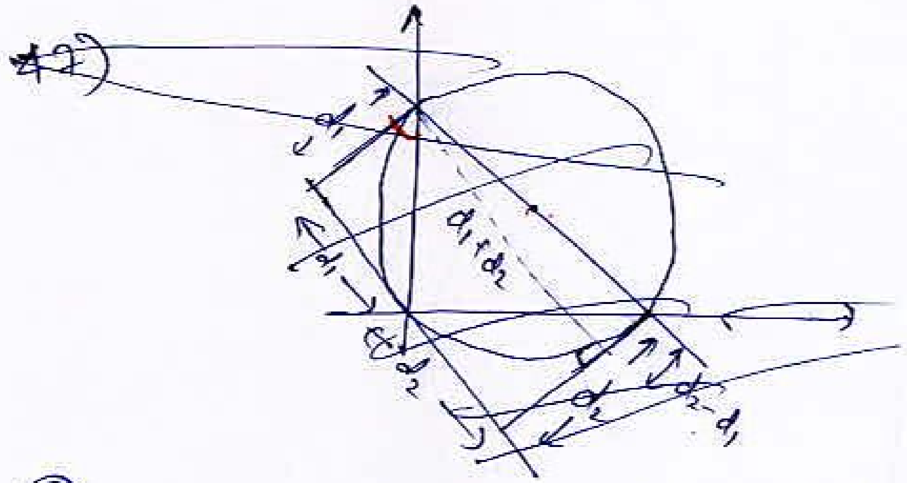
46) reflection of (a,b) in the line $y=x$ is (b,a) (5)

and raidius $= |a|$.

So, $f=-b$ $g=-a$ & $b^2+a^2-c=a^2$

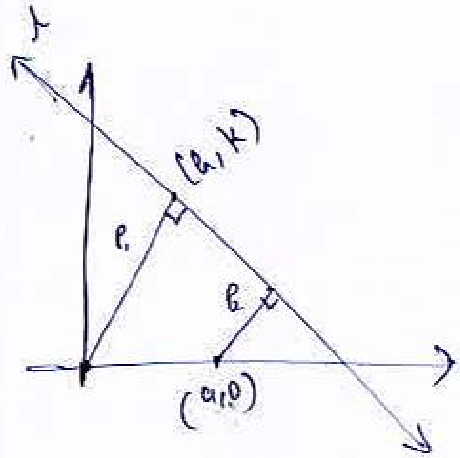
$\Rightarrow C = b^2$

$\therefore$ equation of circle :- $x^2+y^2-2bx-2ay+b^2=0$



52) equation of line l: $(y-k) = -\frac{h}{k}(x-h)$

$\Rightarrow yk + hx - h^2 - k^2 = 0$



$p_1 = \sqrt{h^2+k^2}$

$p_2 = \left| \frac{ha - h^2 - k^2}{\sqrt{h^2+k^2}} \right|$

$\therefore \sqrt{x^2+y^2} \times \left| \frac{xa - x^2 - y^2}{\sqrt{x^2+y^2}} \right| = k^2$

$\Rightarrow x^2+y^2-xa = k^2$ or $x^2+y^2-xa+k^2=0$

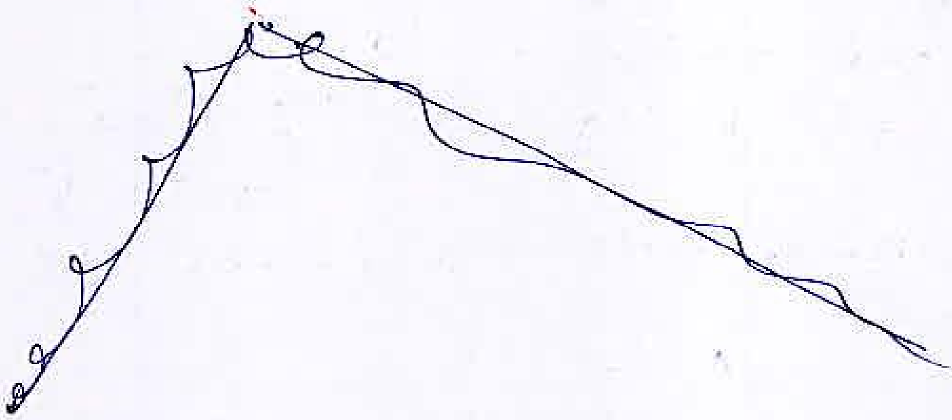
$r^2 = f^2+g^2-C$

$= \left(\frac{a}{2}\right)^2 + (0)^2 \pm k^2$

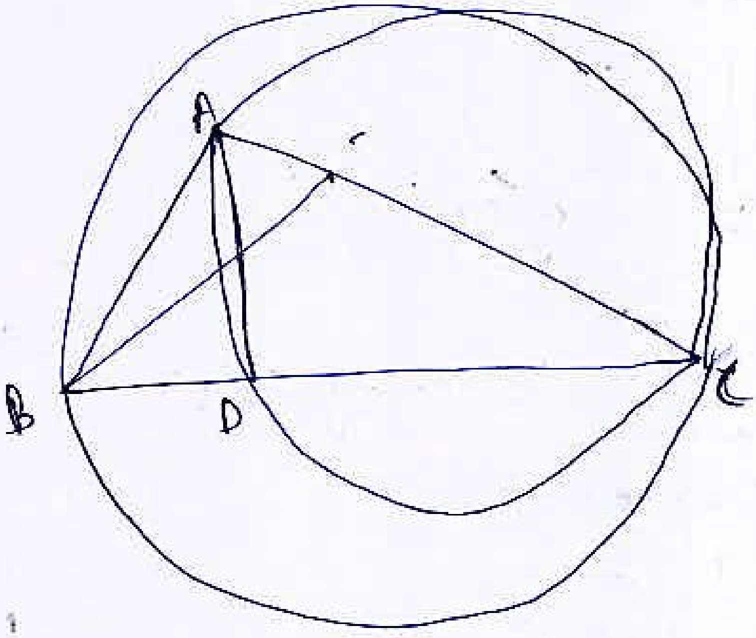
$\therefore r^2 = \frac{a^2}{4} + k^2$ $\left(\because a^2 < 4k^2\right)$

(6)

~~S3~~



(S3)



~~AD~~ since AC is diameter.

Hence $AD \perp BC$

∴ Radical axis are altitudes

Hence radical centre is orthocentre.

Passage 6

$m^2 = \frac{4l^2 + 6l + 1}{5}$

(16) Let the centre of the circle be (a, b)

Hence $r = \left|\frac{la + mb + 1}{\sqrt{l^2 + m^2}}\right| = \sqrt{5}\left|\frac{la + mb + 1}{\sqrt{5l^2 + 4l^2 + 6l + 1}}\right|$

$= \sqrt{5}\left|\frac{la + mb + 1}{3l + 1}\right|$

Since radius is constant $\forall\, l, m$

~~Hence $l = 3$ $m = 0$ and r~~

Hence $a = 3$, $b = 0$, $r = \sqrt{5}$

(17) Let P be $(h, 1 - h)$

Circle : $(x - 3)^2 + (y - 0)^2 = (\sqrt{5})^2$

$\Rightarrow x^2 - 6x + y^2 + 4 = 0$

Chord of contact from $(h, 1 - h)$

$\Rightarrow xh - 3(x + h) + (1 - h)y + 4 = 0$

$\Rightarrow h(x - y - 3) - 3x + y + 4 = 0$

Always passes through $x - y - 3 = 0$

$-3x + y + 4 = 0$

$-2x + 1 = 0 \Rightarrow x = 1/2$

$\frac{1}{2} - y - 3 = 0 \Rightarrow y = -5/2$

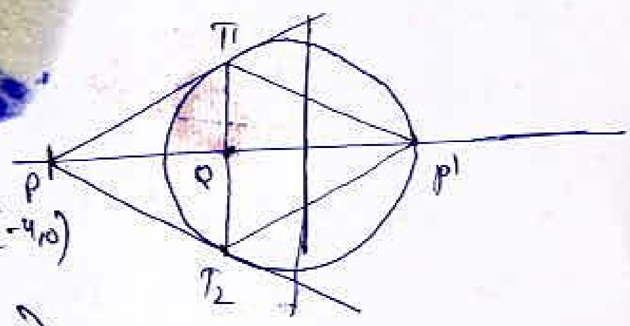
(18) Power of $(2, -3) = (2)^2 - 6(2) + (-3)^2 + 4$

$= 5$

Hence 2 tangents can be drawn.

(8) Passage - 7

(19) $x^2+y^2=4$



Chord of contact T_1T_2 : $x(-4)+y(0)=4$

$\Rightarrow x=-1$

$T_1 : (\sqrt{3},-1)$ $\quad T_2 : (-\sqrt{3},-1)$

In ΔT_1QP $\tan P = \frac{T_1Q}{PQ} = \frac{\sqrt{3}}{3} \Rightarrow \angle P = 30°$

Hence $\angle T_1PT_2 = 60°$ & $PT_1 = PT_2$

$\Rightarrow PT_1T_2$ is equilateral triangle

$\therefore$ dividing PQ in the ratio $2:1$

$\left(\frac{-4\times 1+(-1)\times 2}{1+2}, 0\right) = (-2,0)$

(20) area of $\Delta PT_1P' = \frac{1}{2}$ Rhombus $(PT_1P'T_2)$

are of $\Delta P'T_1T_2 = \frac{1}{2}$ Rhombus $(PT_1P'T_2)$

$\therefore ar(\Delta PT_1P') = ar(\Delta PT_1T_2)$

(21) Now T_1T_2 : $x(h)+y(0)=4 \Rightarrow x=\frac{4}{h}$

$P' = (x_1, 0)$

$\therefore \left(\frac{h+x_1}{2}\right) = \frac{4}{h}$

$x_1 = \left(\frac{8-h^2}{h}\right)$

$\therefore \left(\frac{8-h^2}{h}\right)^2 + 0^2 = 4 \Rightarrow \frac{8-h^2}{h} = \pm 2$

$h^2+2h-8=0$ or $h^2-2h-8=0$

$(h+4)(h-2)=0$ $\quad (h-4)(h+2)=0$

$\therefore h = \pm 4$ $\quad T_1T_2 : x = \pm 1$

$l(T_1T_2) = 2\sqrt{3}$ $\quad PP' = 6$

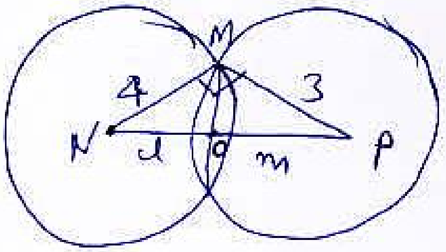
$ar(\text{Rhombus } PT_1P'T_2) = \frac{1}{2}\times 2\sqrt{3}\times 6 = 6\sqrt{3}$

(12) Match matrix

(5) (A) In ΔMNP

$$(l+m)^2 = 3^2+4^2$$

$$\Rightarrow l+m = 5$$



In ΔMON and MOP

$$OM^2 = 16 - l^2 = 9 - m^2$$

$$\Rightarrow l^2 - m^2 = 7$$

$$\Rightarrow l - m = 7/5$$

$$l + m = 5$$

$$l = \frac{16}{5}$$

$\therefore$ length of chord $= 2\sqrt{16 - l^2} = \frac{24}{5}$

(B) ~~The chord of contact is diameter to~~

Common chord is diameter to the first circle

~~A~~ Common chord :- $S_1 - S_2 = 0$

$$\Rightarrow 6x + 4y + (p+q) = 0$$

Since this is diameter to the first circle it passes through $(-2, -6)$

$$\therefore p + q = 36$$

(C) centre of circle $= (-\sqrt{2}, 1/2)$

Chord through $(\sqrt{2}, 1/2)$: $\left(y - \frac{1}{2}\right) = m\left(x - \sqrt{2}\right)$

pt. of intersection with x axis

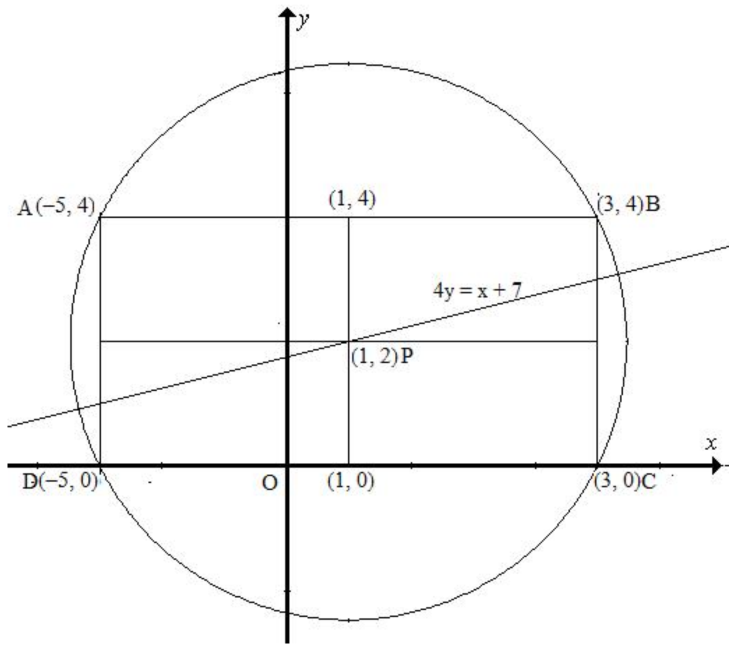
$$= \sqrt{2} - \frac{1}{2m}$$

Since it is bisected at this point

$$m\left(\frac{1/2 - 0}{\frac{1}{2m} - 2\sqrt{2}}\right) = -1$$

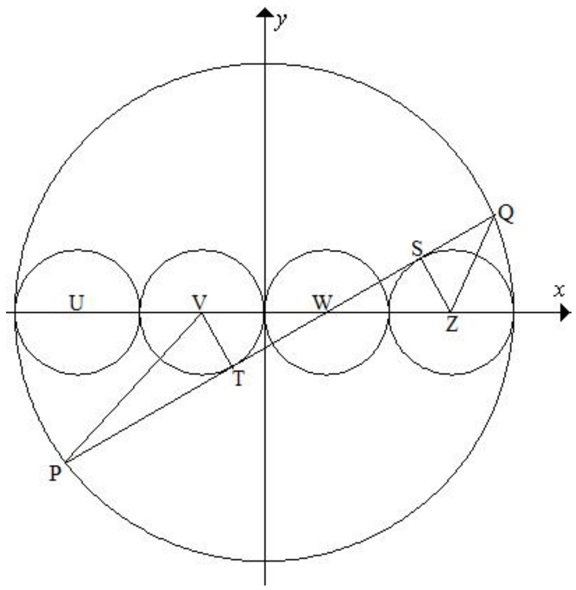
CIRCLES
EXERCISE – 2 (C)

Q.1



Refer the adjoining figure.

Q. 2



$W(1,0)$ gives equation of PQ as $mx - y - m = 0$.

As it touches B i.e. $x^2 + y^2 + 2x = 0$, hence

$$\left|\frac{-2m}{\sqrt{1+m^2}}\right| = 1 \Rightarrow m = \frac{1}{\sqrt{3}}.$$

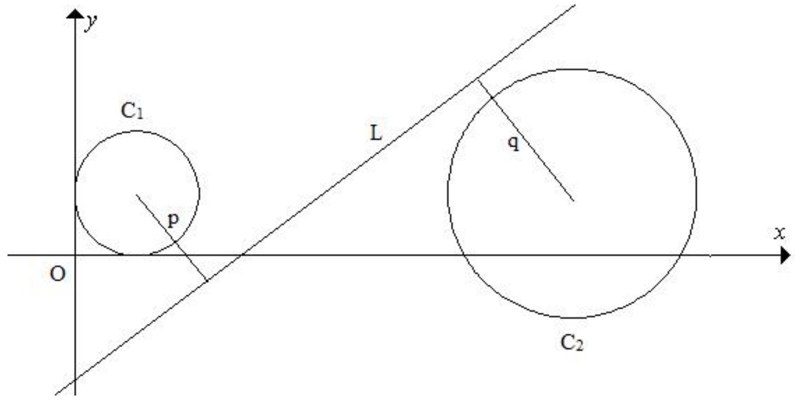
Hence parametric coordinates of any point on PQ at a distance r from $(1,0)$ will be

$\left(1 + \frac{r\sqrt{3}}{2}, \frac{r}{2}\right)$. Substituting these in $x^2 + y^2 = 16$

gives $r^2 + \sqrt{3}r - 15 = 0$.

Difference of roots of this will be $PQ = \sqrt{63}$.

Q.3



For L to lie between C_1 & C_2, r_1(radius of C_1) $< p$ & r_2(radius of C_2) $< q$.
Also centers of C_1 & C_2 must lie on either sides of L.

$r_1 < p \Rightarrow \left|\frac{3\times1-4\times1+k}{5}\right| > 1 \Rightarrow k < -4$ or $k > 6$.

$r_1 < q \Rightarrow \left|\frac{3\times8-4\times1+k}{5}\right| > 2 \Rightarrow k < -30$ or $k > -10$.

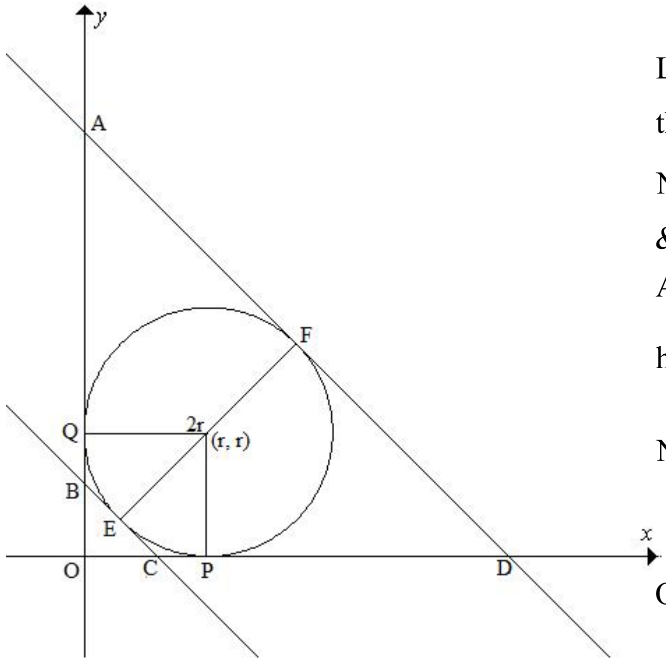
Also $(3\times1-4\times1+k)(3\times8-4\times1+k) < 0 \Rightarrow -20 < k < 1$.

Hence $-10 < k < -4$.

Q.4

The lines $y = x+10$ & $y = x-6$ are parallel so diameter of the required circle will be distance between these lines i.e. $8\sqrt{2}$.
Also the center of this circle will lie on mid line between these lines i.e. $(x-y+10)+(x-y-6)=0 \Rightarrow x-y+2=0$.
Let the center be (h,k), then $h-k+2=0$.
As the circle touches y−axis hence $h = 4\sqrt{2}$ & $k = 2+4\sqrt{2}$.
$h+k = 2+8\sqrt{2}$.

Q.5 incomplete



Let eq. of BC be $x+y=c$ &

that of AD be $x+y=c+2\sqrt{2}\,r$

Now $BC=\sqrt{OB^2+OC^2}=c\sqrt{2}$

& $AD=c\sqrt{2}+8r$

Also distance of $x+y-c=0$ from (r,r) is r,

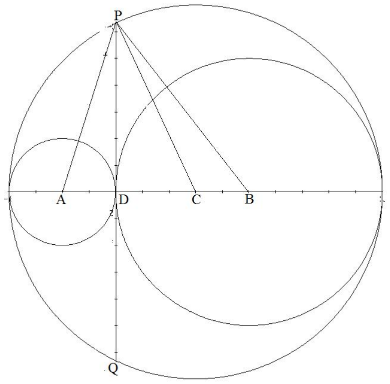
hence $\left|\frac{2r-c}{\sqrt{2}}\right|=r$ or $2r-c=\sqrt{2}\,r$

Now Area of $ABCD=\frac{1}{2}(AD+BC)\times 2r$

$$=\left(2\sqrt{2}\,c+8r\right)\times r$$

Or $\left(4\sqrt{2}-8+8\right)\times r^2=900\sqrt{2}$ hence $r=15$.

Q.6



Let A, B & C be the centers of C_1, C_2 & C_3 respectively.

Given $AD=4$ & $BD=10$

$\Rightarrow$ radius of $C_3=14$ & $CD=6$.

Now $DP^2+CD^2=CP^2$

or $DP=\sqrt{14^2-6^2}$

Hence $PQ=8\sqrt{10}$.

Q.7

(i) Common chord of the given circles is $2x+1=0$. Any circle belonging to the same family will be given by $x^2+y^2+2x+3y+1+\lambda(2x+1)=0$

or $x^2+y^2+2(\lambda+1)x+3y+\lambda+1=0$

Now as $2x+1=0$ is diameter hence $-(\lambda+1)=-\frac{1}{2}$ i.e. $\lambda=-\frac{1}{2}$.

Required circle is $x^2+y^2+x+3y+\frac{1}{2}=0$ & diameter $=2\sqrt{\frac{1}{4}+\frac{9}{4}-\frac{1}{2}}$. Hence $K=8$.

(ii) Let any chord of the given curve be $y = mx + c$. Homogenising $y^2 = 8x$ gives

$y^2 = 8x\left(\frac{y - mx}{c}\right)$ or $8mx^2 - 8xy + cy^2 = 0$.

This equation represents pair of straight lines joining end points of the chord to the origin hence this must be a pair of mutually perpendicular lines.

$\Rightarrow 8m + c = 0$.

Now equation of chord becomes $y = m(x - 8)$. This passes through $(8,0)$. Hence $W = 8$.

(iii) $H = S_1 = 3^2 + 0 + \frac{5}{2} \times 0 - 8 = 1$.

Hence $KWH = 64$.

Q.8

Centers of any circle touching x – axis & $y = mx$ will lie on angle bisector between these.

Let $m = \tan\theta$, then the above mentioned bisector will be $y = x\tan\frac{\theta}{2}$.

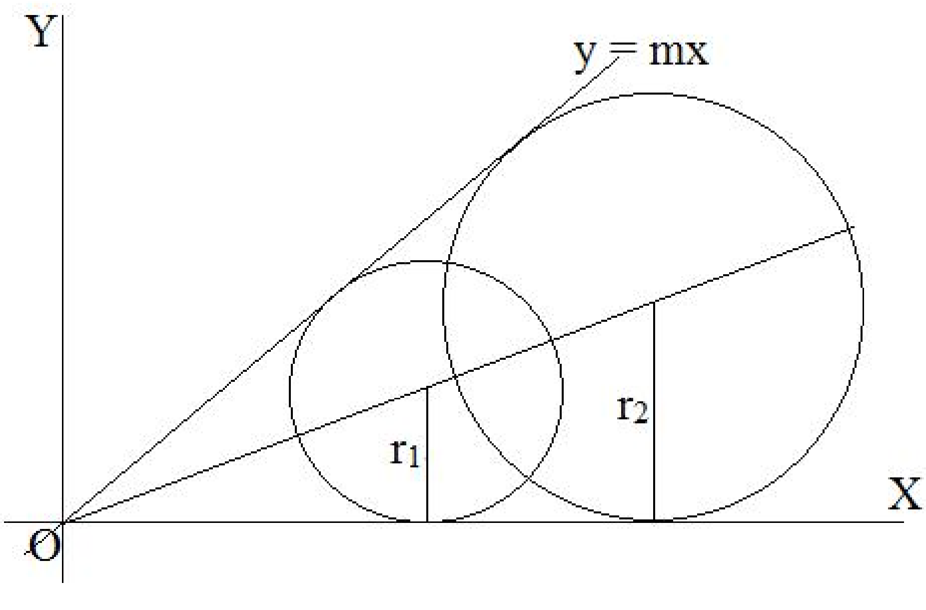
Let center of such a circle be at a distance r from origin, then

Center will be $\left(r\cos\frac{\theta}{2}, r\sin\frac{\theta}{2}\right)$, where radius $= r\sin\frac{\theta}{2}$.

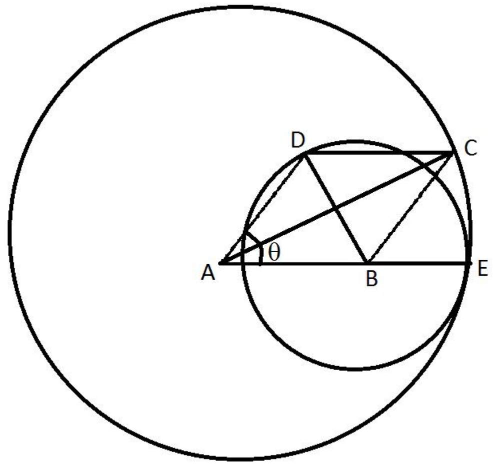
As this circle passes through $(6,4)$ hence $\left(r\cos\frac{\theta}{2} - 6\right)^2 + \left(r\sin\frac{\theta}{2} - 4\right)^2 = r^2\sin^2\frac{\theta}{2}$.

Or $\left(\cos^2\frac{\theta}{2}\right)r^2 - \left(12\cos\frac{\theta}{2} + 8\sin\frac{\theta}{2}\right)r + 52 = 0$.

Now as given $r_1r_2 = \frac{52}{3}$ hence



Q.9



$$AB = AD = 10,\ \angle BAD = \theta$$
$$AC^2 = 10^2 + 10^2 + 2\times 10\times 10\times \cos\theta$$
$$BD^2 = 10^2 + 10^2 - 2\times 10\times 10\times \cos\theta$$
$$AC^2 + BD^2 = 400$$
$$AE - BE = AB = 10$$
$$AE = AC\ \&\ BE = BD \Rightarrow AC - BD = 10$$
$$\text{Area} = \frac{AC\times BD}{2} \Rightarrow \text{Area} = \frac{\left(AC^2 + BD^2\right) - \left(AC - BD\right)^2}{4}$$
$$\Rightarrow \text{Area} = 75$$

Q.10

Given $\sin^2 A + \cos^2 B = 1$

Now $E = \left(\tan C - \sin A\right)^2 + \left(\cot C - \cos B\right)^2$

$$\Rightarrow E = \tan^2 C + \cot^2 C + \sin^2 A + \cos^2 B - 2\left(\tan C \sin A + \cot C \cos B\right)$$
$$\Rightarrow E = \tan^2 C + \cot^2 C - 2\left(\tan C \sin A + \cot C \cos B\right) + 1$$

$\Rightarrow E \geq 3 - 2\left(\sin A + \cos B\right)$, when $C = \frac{\pi}{4}$

Further $\sin A + \cos B \leq \sqrt{2}$, when $A = B = \frac{\pi}{4}$

Hence $E \geq 3 - 2\sqrt{2}$

Q.11

Let $x = \cos\theta$ & $y = \sin\theta$, then $Z = \frac{4 - \sin\theta}{7 - \cos\theta}$

$\Rightarrow Z = \dfrac{4 - \frac{2t}{1+t^2}}{7 - \frac{1-t^2}{1+t^2}}$, where $t = \tan\frac{\theta}{2}$

$$\Rightarrow Z = \frac{2t^2 - t + 2}{4t^2 + 3}$$
$$\Rightarrow \left(4Z - 2\right)t^2 + t + \left(3Z - 2\right) = 0$$

Now or t to be real $1^2 \geq 4\left(4Z - 2\right)\left(3Z - 2\right)$

$$\Rightarrow 48Z^2 - 56Z + 15 \leq 0$$
$$\Rightarrow \frac{5}{12} \leq Z \leq \frac{3}{4}$$

Q.12

$x^2 + y^2 - 6x - 2py + 17 = 0 \rightarrow \text{center}:(3, p)$, radius $= \sqrt{p^2 - 8}$

Director circle of the given circle : $(x-3)^2 + (y-p)^2 = 2(p^2 - 8)$

Now the origin must lie on the director circle, hence $9 + p^2 = 2p^2 - 16$

$\Rightarrow p^2 = 25$

$\Rightarrow p_1^2 + p_2^2 = 50$

Q.13

Center of $C_r : (3r - 2, 0)$, radius of $C_r = 2^{r-1}$

Now center of C_5 : P(13, 0)

Any line passing through P with slope m : $y = m(x - 13)$ or $mx - y - 13m = 0$

If this line is touching C_3, then its distance from center of C_3 (7, 0) = radius of C_3 i.e. 4.

Hence $\dfrac{|7m - 0 - 13m|}{\sqrt{m^2 + 1}} = 4 \Rightarrow m^2 = \dfrac{4}{5}$

$\Rightarrow 2010 m_1 m_2 = 1608$

Q.14

Given center of the requird circle is A(–1, 1), let the radius of required circle be r.

Also center of given circle is B(2, –3) and radius is 4.

If the two circles touch, then the must touch externally as (–1, 1) lies outside the given circle, hence AB = r + 4.

$\Rightarrow \sqrt{(-1-2)^2 + (1+3)^2} = r + 4$

$\Rightarrow r = 1$

Required circle is $(x + 1)^2 + (y - 1)^2 = 1$ or $x^2 + y^2 + 2x - 2y + 1 = 0$

Now the x - intercept + y - intercept = $2\sqrt{g^2 - c} + 2\sqrt{f^2 - c}$ i.e. 0.

Q.15

Center of the given circle : A(1, 2)

Tangent at B(1, 7) : $x.1 + y.7 - 2\dfrac{x+1}{2} - 4\dfrac{y+7}{2} - 20 = 0$ or $y = 7$

Tangent at D(4, – 2) : $x.4 - y.2 - 2\dfrac{x+4}{2} - 4\dfrac{y-2}{2} - 20 = 0$ or $3x - 4y = 20$

Point of intersection : C(16, 7).

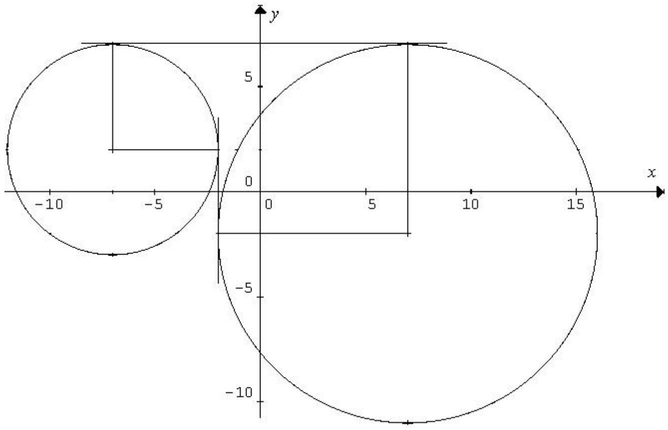
Now AB = 5 & BC = 15, hence area of ABCD = 75.

Q.16
The given circles are

$(x+7)^2+(y-2)^2=25$ &

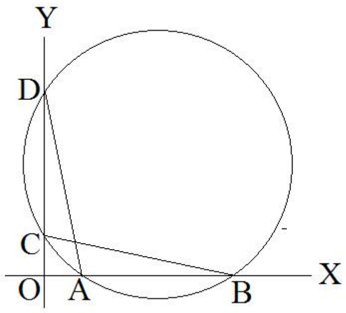
$(x-7)^2+(y+2)^2=81$

One common tangent is a vertical line i.e. $x=-2$
and one is a horizontal line i.e. $y=7$ as shown in the figure.



hence sum of length of common tangents = $2\times((9-5)+(9+5))=36$

Q.17
Using properties of circles



$OA\times OB=OC\times OD$

$\Rightarrow \frac{1}{\lambda}\times 3=1\times\frac{3}{2}$

$\Rightarrow \lambda=2$

Q.18
Any circle passing through (– 6 , 7) and (4 , 7) will be

$(x+6)(x-4)+(y-7)(y-7)+\lambda(y-7)=0$ or $x^2+y^2+2x+(\lambda-14)y+25-7\lambda=0$

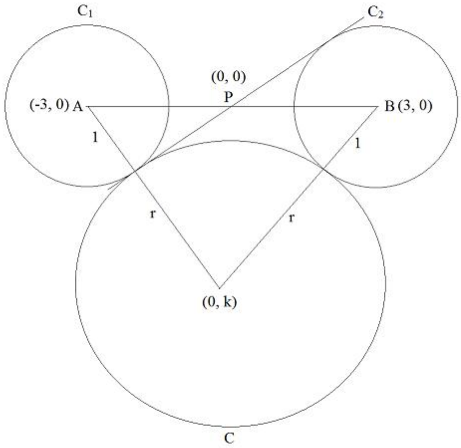
Center : $\left(-1, 7-\frac{\lambda}{2}\right)$

As 3y = x + 10 is a diameter hence $3\left(7-\frac{\lambda}{2}\right)=-1+10 \Rightarrow \lambda=8$

Hence diameter =2 $\sqrt{(1)^2+\left(\frac{\lambda}{2}-7\right)^2-25+7\lambda}=2\sqrt{41}$

Now one side = 10, hence other side = $\sqrt{164-100}=8$ and area = 80.

Q.19



Let the centers of C_1 & C_2 be A(-3, 0) & B(3, 0)
Let the common tangent be y = mx

Now $\left|\frac{3m}{\sqrt{m^2+1}}\right| = 1$ or $8m^2 = 1$

Let center of C be (0, k) and radius be r.
As this line also touches C, hence

$\left|\frac{k}{\sqrt{m^2+1}}\right| = r$ or $k^2 = r^2m^2 + r^2$

$\Rightarrow 8k^2 = 9r^2$

Also C touches C_1 & C_2 hence $k^2 + 9 = (r+1)^2$

$\Rightarrow \frac{9r^2}{8} + 9 = (r+1)^2$

$\Rightarrow r^2 - 16r + 64 = 0$ or $r = 8$

Q.20
For area to be maximum either the triangle must be equilateral.
Now let the tangent drwan from (6, 8) be y – 8 = m(x – 6) or mx – y +8 – 6m.

Now $\left|\frac{8-6m}{\sqrt{m^2+1}}\right| = r \Rightarrow (r^2-36)m^2 + 96m + (r^2-64) = 0$

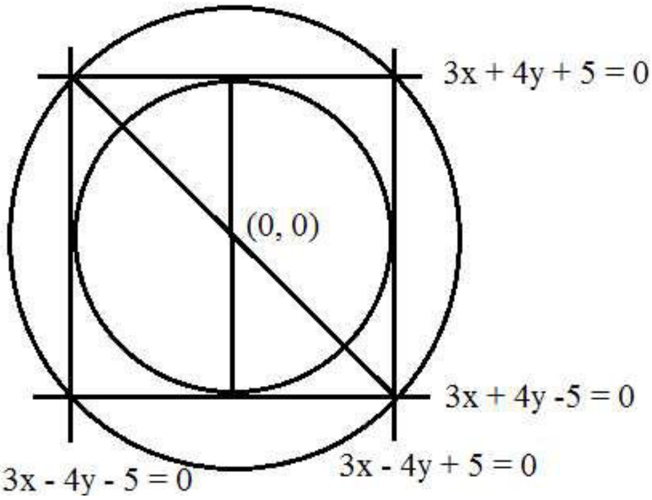
$\Rightarrow m_1 + m_2 = -\frac{96}{r^2-36}$ & $m_1m_2 = \frac{r^2-64}{r^2-36}$

now $\tan\frac{\pi}{3} = \left|\frac{m_1-m_2}{1+m_1m_2}\right| \Rightarrow 3(1+m_1m_2)^2 = (m_1+m_2)^2 - 4m_1m_2$

$\Rightarrow 3\left(1+\frac{r^2-64}{r^2-36}\right)^2 = \left(\frac{96}{r^2-36}\right)^2 - 4\left(\frac{r^2-64}{r^2-36}\right) \Rightarrow r = 5$

CIRCLES
Exercise – 3
Part I

Q.1



The given lines are in pairs of perpendicular & parallel lines.
Also distance between each pair of parallel line is equal to 2.
Hence the given quadrilateral is a square.

Now point of intersection of $3x+4y-5=0$ & $4x-3y-5=0$ is $\left(\frac{7}{5},\frac{1}{5}\right)$.

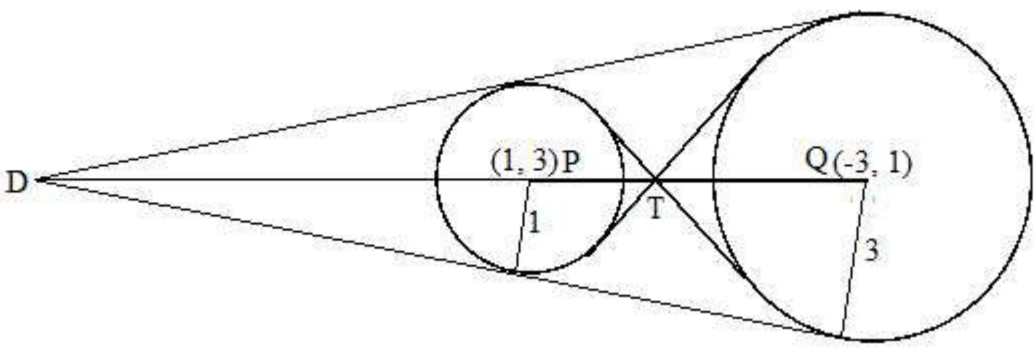
Similarly point of intersection of $3x+4y+5=0$ & $4x-3y+5=0$ is $\left(-\frac{7}{5},-\frac{1}{5}\right)$.

Hence center of both incircle & circumcircle will be mid – point of these points i.e. $(0,0)$.

Also diameter of incircle will be 2 & that of circumcircle will be $2\sqrt{2}$.

Required circles are $x^2+y^2=1$ & $x^2+y^2=2$.

Q.2



$$\frac{TP}{TQ}=\frac{1}{3}\Rightarrow x_T=\frac{3\times1+1\times(-3)}{3+1}=0 \ \& \ y_T=\frac{3\times3+1\times1}{3+1}=\frac{5}{2}$$

Hence the transverse common tangents will be drawn through $\left(0,\frac{5}{2}\right)$.

Now any line through this point will be $2mx-2y+5=0$.

If this line is a tangent to $x^2+y^2-2x-6y+9=0$, then $\left|\frac{2m\times1-2\times3+5}{2\sqrt{m^2+1}}\right|=1$

$\Rightarrow 4m^2-4m+1=4m^2+4$ or $m=-\frac{3}{4}$. Also as only one value of m is obtained so other common tangent is vertical.

Transverse common tangents are $3x+4y=10 \ \& \ x=0$.

$$\frac{DP}{DQ}=\frac{1}{3}(\text{external})\Rightarrow x_D=\frac{3\times1-1\times(-3)}{3-1}=3 \ \& \ y_D=\frac{3\times3-1\times1}{3-1}=4$$

Hence the direct common tangents will be drawn through $(3,4)$.

Now any line through this point will be $mx-y-3m+4=0$.

If this line is a tangent to $x^2+y^2-2x-6y+9=0$, then $\left|\frac{m\times1-3-3m+4}{\sqrt{m^2+1}}\right|=1$

$\Rightarrow 3m^2-4m=0$ or $m=\frac{4}{3},0$.

Direct common tangents are $4x-3y=0 \ \& \ y=4$.

Q.3

Homogenising $ax^2+2hxy+by^2=1$ using $lx+my+n=0$ gives

$ax^2+2hxy+by^2=\left(\frac{lx+my}{-n}\right)^2$ or $\left(an^2-l^2\right)x^2+2\left(hn^2-lm\right)xy+\left(bn^2-m^2\right)y^2=0$.

This is equation of pair of lines joining O to P & Q.

As $\angle POQ=\frac{\pi}{2}$ hence the pair of lines OQ & OP must be orthogonal

$\Rightarrow an^2-l^2+bn^2-l^2=0$ or $n^2(a+b)=l^2+m^2$.

Q.4

Center of the given circle is $\left(\frac{1}{2},-\frac{3}{2}\right)$ & $r=\sqrt{\frac{5}{2}}$.

Distance of $x+y=1$ from $\left(\frac{1}{2},-\frac{3}{2}\right)=\frac{\left|\frac{1}{2}-\frac{3}{2}-1\right|}{\sqrt{2}}=\sqrt{2}$.

Now length of chord $=2\sqrt{\frac{5}{2}-2}=\sqrt{2}$.

Now any line through O will be $y=mx$.

Distance of this line from $\left(\frac{1}{2},-\frac{3}{2}\right)=\frac{|m+3|}{2\sqrt{m^2+1}}$.

Hence length of chord $2\sqrt{\frac{5}{2}-\frac{(m+3)^2}{4(m^2+1)}}=\sqrt{2}\Rightarrow 7m^2-6m-1=0$.

$m=1$ & $-\frac{1}{7}$.

Hence equation of required line may be $y=x$ or $7y+x=0$.

Q.5

Let the circle be $x^2+y^2+2gx+2fy+c=0$.

As it pass through $(-1,1),(0,6)\&(5,5)$ hence

$-2g+2f+c+2=0,\ 12f+c+36=0\ \&\ 10g+10f+c+50=0$.

Solving simultaneously we get $g=-2, f=-3, c=0$.

Hence the circle is $x^2+y^2-4x-6y=0$.

Clearly origin lies on the circle hence tangents parallel to the line joining origin to $(2,3)$ will be drawn at those points where the line passing through the center and perpendicular to this line meets the circle.

Equation of this line is $(y-3)=-\frac{2}{3}(x-2)$ or $2x+3y=13$.

Solving this with equation of circle gives required points as $(5,1)$ & $(-1,5)$.

Q.6

Let mid point of chord be (h,k). Equation of chord will be

$hx+ky=h^2+k^2$. (using $T=S_1$)

Homogenising $x^2=2(x+y)$ using the equation of chord gives

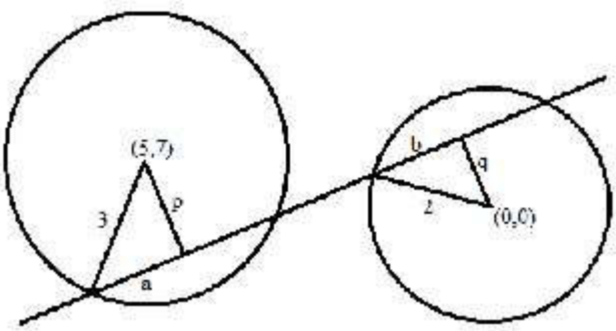
$x^2=2(x+y)\left(\dfrac{hx+ky}{h^2+k^2}\right)$ or $\left(h^2+k^2-2h\right)x^2-2(h+k)xy-2ky^2=0.$

This is equation of the pair of lines joining end points of chord to the origin.

As the chord subtends a right angle at origin hence this must be a pair of mutually perpendicular lines which implies $h^2+k^2-2h-2k=0$.

Hence the required locus is $x^2+y^2-2x-2y=0$.

Q.7



Equation of a line with slope 1 will be $x-y+c=0$.

Distance of this line from $(0,0)$, $q=\dfrac{|c|}{\sqrt{2}}$,

Hence chord cut off by $x^2+y^2=4$ on this line, $2b=2\sqrt{4-\dfrac{c^2}{2}}$.

Similarly distance of this line from $(5,7)$, $p=\dfrac{|c-2|}{\sqrt{2}}$,

Hence chord cut off by $x^2+y^2-10x-14y+65=0$ on this line, $2a=2\sqrt{9-\dfrac{(c-2)^2}{2}}$.

As given $\sqrt{9-\dfrac{(c-2)^2}{2}}=\sqrt{4-\dfrac{c^2}{2}}$ or $c=-\dfrac{3}{2}$.

Hence required line is $2x-2y-3=0$.

Q.8

Equation of any tangent to $x^2 + y^2 = a^2$ will be $x\cos\alpha + y\sin\alpha = a$.

Now this will meet the axes at $P(a\sec\alpha, 0)$ & $Q(0, a\,\mathrm{cosec}\,\alpha)$.

Mid point of PQ will be $x = \frac{a\sec\alpha}{2}$ & $y = \frac{a\,\mathrm{cosec}\,\alpha}{2}$

Or $\cos\alpha = \frac{a}{2x}$ & $\sin\alpha = \frac{a}{2y}$. Eliminating α gives $\frac{1}{x^2} + \frac{1}{y^2} = \frac{4}{a^2}$.

Q.9

Let the circle be $x^2 + y^2 + 2gx + 2fy + c = 0$.

Now if lengths of tangents to it from $(1,0),(2,0)$ & $(3,2)$ are $1, \sqrt{7}$ & $\sqrt{2}$, then

$2g + c + 1 = 1$, $4g + c + 4 = 7$ & $6g + 4f + c + 13 = 2$. (Using length of tangent $= \sqrt{S_1}$)

Solving simultaneously gives $g = \frac{3}{2}, f = -\frac{17}{4}, c = -3$

Hence the required equation is $2x^2 + 2y^2 + 6x - 17y - 6 = 0$.

Q.10

Equation of tangent to $x^2 + y^2 + 4x - 6y - 12 = 0$ at $(2,0)$ is

$2.x + 0.y + 4\frac{x+2}{2} - 6\frac{y+0}{2} - 12 = 0$ or $4x - 3y - 8 = 0$. (Using $T = 0$)

Now Slope of line making angle 45^0 with this line will be given by

$$\tan 45^0 = \pm\frac{m - \frac{4}{3}}{1 + \frac{4}{3}m} \Rightarrow m = -7 \text{ \& } \frac{1}{7}.$$

Hence equation of required line is $7x + y = 14$ or $x - 7y = 2$.

Using the fact that two points on a line with slope $\tan\theta$ at a distance r from (x_1, y_1) on the same line is given by $(x_1 \pm r\cos\theta, y_1 \pm r\sin\theta)$ we get

points on $7x + y = 14$ at a distance $5\sqrt{2}$ will be

$\left(2 - 5\sqrt{2}\times\frac{1}{5\sqrt{2}}, 0 + 5\sqrt{2}\times\frac{7}{5\sqrt{2}}\right)$ & $\left(2 + 5\sqrt{2}\times\frac{1}{5\sqrt{2}}, 0 - 5\sqrt{2}\times\frac{7}{5\sqrt{2}}\right)$ i.e. $(1,7)$ & $(3,-7)$

Similraly points on $x - 7y = 2$ at a distance $5\sqrt{2}$ will be

$\left(2 + 5\sqrt{2}\times\frac{7}{5\sqrt{2}}, 0 + 5\sqrt{2}\times\frac{1}{5\sqrt{2}}\right)$ & $\left(2 - 5\sqrt{2}\times\frac{7}{5\sqrt{2}}, 0 - 5\sqrt{2}\times\frac{1}{5\sqrt{2}}\right)$ i.e. $(9,1)$ & $(-5,-1)$

Circles of radius 3 with centers at these points will be

$(x-1)^2 + (y-7)^2 = 9, (x-3)^2 + (y+7)^2 = 9, (x-9)^2 + (y-1)^2 = 9$ & $(x+5)^2 + (y+1)^2 = 9$.

Q.11

Let the circle be $x^2+y^2+2gx+2fy+c=0$. Now it passes through $(4,7),(5,6)$ & $(1,8)$ hence

$8g+14f+c+65=0,\ 10g+12f+c+61=0$ & $2x+16f+c+65=0$.

Solving simultaneously gives $g=-1, f=-3, c=-15$.

Equation of the required circle is $x^2+y^2-2x-6y-15=0$.

Now for the required point $5x+y+17=0$ is the chord of contact of the tangents drawn from this point.

Let this point be (h,k), then chord of contact using $T=0$ will be

$$hx+ky-2\frac{x+h}{2}-6\frac{y+k}{2}-15=0 \text{ or } (h-1)x+(k-3)y-h-3k-15=0.$$

Comparing this with $5x+y+17=0$ gives $\frac{h-1}{5}=\frac{k-3}{1}=-\frac{h+3k+15}{17}$.

Solving simultaneously gives $h=-4, k=2$.

Q.12

A circle having $(2,3)$ as center and radius $=0$ will be $x^2+y^2-4x-6y+13=0$.

Common tangent of this circle and given circle will be $x-2=0$. (Using $S-S'=0$)

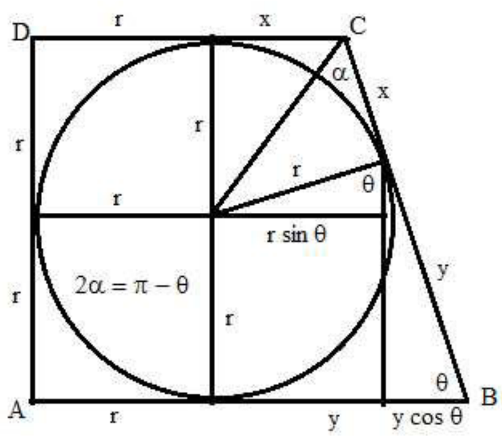
Now any circle touching the given circle at $(2,3)$ will be

$x^2+y^2+4x-6y-3+\lambda(x-2)=0$. (Using family of circles)

Now it passes through $(1,1)$ hence $\lambda=-3$.

Thus required circle is $x^2+y^2+x-6y+3=0$.

Q.13



As shown in adjoining figure

$$y-y\cos\theta=r\sin\theta \Rightarrow y=\frac{r\sin\theta}{1-\cos\theta}=r\cot\frac{\theta}{2}$$

$$\& \ \frac{r}{x}=\tan\alpha=\cot\frac{\theta}{2}\Rightarrow x=r\tan\frac{\theta}{2}$$

Now H.M. of $(r+x)$ & $(r+y)=\frac{2(r+x)(r+y)}{(r+x)+(r+y)}$

$$=\frac{2r^2\left(1+\tan\frac{\theta}{2}\right)\left(1+\cot\frac{\theta}{2}\right)}{r\left(2+\tan\frac{\theta}{2}+\cot\frac{\theta}{2}\right)}=2r.$$

Alternately

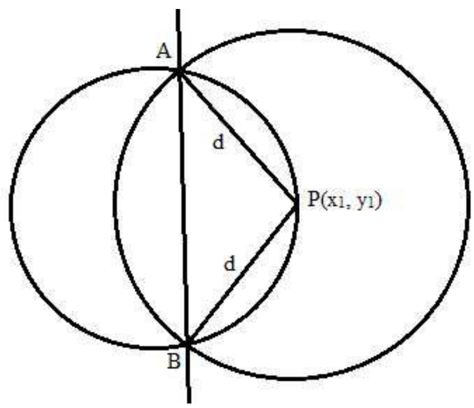
Let the circle be $x^2+y^2=r^2$, the sides of trapezium be $y=\pm r$, $x=-r$ & $x\cos\theta+y\sin\theta=r$

Now $B\left(\frac{r(1+\sin\theta)}{\cos\theta},-r\right)$ & $C\left(\frac{r(1-\sin\theta)}{\cos\theta},r\right)$, hence

$AB=r+\frac{r(1+\sin\theta)}{\cos\theta}$ & $CD=r+\frac{r(1-\sin\theta)}{\cos\theta}$, also $AD=2r$.

$$\frac{2AB.CD}{AB+CD}=\frac{r\left(1+\frac{1+\sin\theta}{\cos\theta}\right)\left(1+\frac{1-\sin\theta}{\cos\theta}\right)}{\left(2+\frac{1+\sin\theta}{\cos\theta}+\frac{1-\sin\theta}{\cos\theta}\right)} \text{ or } \frac{2AB.CD}{AB+CD}=2r=AD.$$

Q.14

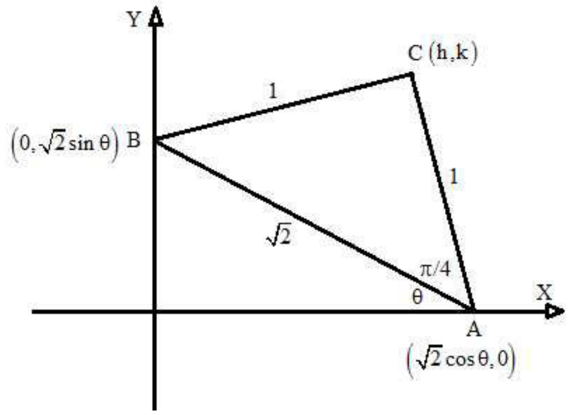


As shown in figure AB will be commom chord of the circle drawn with P as center and radius = d.

Hence $AB: x^2+y^2-a^2-\left((x-x_1)^2+(y-y_1)^2-d^2\right)=0$

Or $AB: xx_1+yy_1-a^2+\frac{d^2}{2}=0$.

Q.15



$h=\sqrt{2}\cos\theta-\cos\left(\frac{\pi}{4}+\theta\right)$ & $k=0+\sin\left(\frac{\pi}{4}+\theta\right)$

Or $h=\frac{\cos\theta+\sin\theta}{\sqrt{2}}$ & $k=\frac{\cos\theta+\sin\theta}{\sqrt{2}}$

Let the centroid be (x,y), then

$3x=\frac{3\cos\theta+\sin\theta}{\sqrt{2}}$ & $3y=\frac{\cos\theta+3\sin\theta}{\sqrt{2}}$

or $3x-y=\frac{8\cos\theta}{3\sqrt{2}}$ & $3y-x=\frac{8\cos\theta}{3\sqrt{2}}$.

Hence $(3x-y)^2+(3y-x)^2=\frac{32}{9}$.

Q.16

Radical Axis of the given circles is $(4g-3)x+(4f-8)y=0$.
As it is touching the circle with center $(-1,1)$ & radius 1, hence

$$\left|\frac{-(4g-3)+(4f-8)}{\sqrt{(4g-3)^2+(4f-8)^2}}\right|=1\Rightarrow(4g-3)(4f-8)=0\Rightarrow \text{Either } g=\frac{3}{4} \text{ or } f=2.$$

Q.17

Common chord of the two given circles is $x+y=0$.
Now any circle through the points of intersection of the two given circles will be
$x^2+y^2+6x+4y-12+\lambda(x+y)=0$ or $x^2+y^2+(\lambda+6)x+(\lambda+4)y-12=0$.
As it is orthogonal to $x^2+y^2-2x-4=0$ hence $-2\times\frac{\lambda+6}{2}\times1+2\times\frac{\lambda+4}{2}\times0=-12-4$ or $\lambda=10$.
Required circle is $x^2+y^2+16x+14y-12=0$.

Q.18

Let the variable circle be $x^2+y^2+2gx+2fy+c=0$.
As given center lies on $2x-2y+9=0$, hence $-2g+2f+9=0$ or $2f=2g-9$.
Also it is orthogonal to $x^2+y^2-4=0$, hence $2g\times0+2f\times0=c-4$ or $c=4$.
Now the equation becomes $x^2+y^2+2gx+(2g-9)y+4=0$
or $x^2+y^2-9y+4+2g(x+y)=0$.
Hence the circle passes through the points of intersection of
$x^2+y^2-9y+4=0$ & $x+y=0$ i.e. $(-4,4)\&\left(-\frac{1}{2},\frac{1}{2}\right)$.

Q.19

Circle on $A(3,7)\&B(6,5)$ as diameter is $(x-3)(x-6)+(y-7)(y-5)=0$
or $x^2+y^2-9x-12y+53=0$.
Equation of the line through AB is $2x+3y-27=0$.
Now any circle through A & B will be given by
$x^2+y^2-9x-12y+53+\lambda(2x+3y-27)=0$
or $x^2+y^2+(2\lambda-9)x+(3\lambda-12)y+53-27\lambda=0$.
Now common chord of this circle and $x^2+y^2-4x-6y-3=0$ will be
$(2\lambda-5)x+(3\lambda-6)y+56-27\lambda=0$ or $\lambda(2x+3y-27)-(5x+6y-56)=0$
Hence all these chord will pass through the point of intersection of
$2x+3y-27=0\ \&\ 5x+6y-56=0$ i.e. $\left(2,\frac{23}{3}\right)$.

Q.20

Rearrange the given equation as $x^2+y^2+y-1+k(x+y-1)=0$.
Now this is equation of all the circles passing through the points of intersection of
$x^2+y^2+y-1=0$ & $x+y-1=0$ i.e. $(1,0)$ & $\left(\frac{1}{2},\frac{1}{2}\right)$.
Smallest such circle will have $x+y=1$ as diameter hence $\left(-\frac{k}{2},-\frac{k+1}{2}\right)$ must lie on this line
$\Rightarrow -\frac{k}{2}-\frac{k+1}{2}=1$ or $k=-\frac{3}{2}$.
Required circle is $2x^2+2y^2-3x-x+1=0$ whose radius is $\frac{1}{2\sqrt{2}}$.

Q.21

Radical axis of the given circles is : $10x-2y+5=0$.
Any circle co-axial with the given circle will be given by
$x^2+y^2+4x+2y+1+\lambda(10x-2y+5)=0$ or $x^2+y^2+(4+10\lambda)x+(2-2\lambda)y+1+5\lambda=0$
Center of this circle is $(-2-5\lambda,-1+\lambda)$ which lies on $10x-2y+5=0$, hence
$-20-50\lambda+2-2\lambda+5=0$ or $\lambda=-\frac{1}{4}$.
Required circle is $4x^2+4y^2+6x+10y-1=0$.

Q.22

As the circle meets x-axis orthogonaly and passes through origin hence it
will touch y-axis at origin.
Let the circle be $x^2+y^2+2gx=0$.
Now $\frac{a^2+g^2-g^2}{2ag}=\pm\cos 45^0$ or $g=\pm\frac{a}{\sqrt{2}}$.
Required circle is $x^2+y^2+a\sqrt{2}x=0$ or $x^2+y^2-a\sqrt{2}x=0$.

Q.23

Let the circle be $x^2+y^2+2gx+2fy+c=0$.
As it passes through $(3,-2)$ & $(-2,0)$ hence
$6g-4f+c+13=0$ & $-4g+c+4=0$.
Also its center lies on $2x-y=3$, hence $-2g+f=3$.
Solving these equations simultaneously gives $g=\frac{3}{2}, f=6, c=2$.
Required circle is $x^2+y^2+3x+12y+2=0$.

Q.24

Common chord of the given circles is $2x+1=0$.
Now any circle passing through end points of this chord will be given by
$x^2+y^2+2x+3y+1+\lambda(2x+1)=0$ or $x^2+y^2+2(\lambda+1)x+3y+1+\lambda=0$
As $2x+1$ is a diameter hence x – coordinate of center i.e. $-(\lambda+1)=-\frac{1}{2}$.
Required circle is $2x^2+2y^2+2x+6y+1=0$.

Q.25

Chord of contact w.r.to O will be given by
$0\times x+0\times y+2g\times\frac{x+0}{2}+2f\times\frac{y+0}{2}+c=0$ or $gx+fy+c=0$.
Now circle through O, P & Q will pass through the center C of the given circle also as quadrilateral OPCQ is cyclic. Also this circle will be drawn on OC as diameter as $\angle OPC=\angle OQC=\frac{\pi}{2}$. Hence equation of required circle is
$(x-0)(x+g)+(y-0)(y+f)=0$ i.e. $x^2+y^2+gx+fy=0$.

Q.26

Let the sides of the square be $x=-\frac{1}{2}, x=\frac{1}{2}, y=-\frac{1}{2}$ & $y=\frac{1}{2}$.
Now for any point (h,k) its distances from these lines will be
$|h+\frac{1}{2}|, |h-\frac{1}{2}|, |k-\frac{1}{2}|, |k+\frac{1}{2}|$. Sum of squares of these given 9 hence
$\left(h+\frac{1}{2}\right)^2+\left(h-\frac{1}{2}\right)^2+\left(k-\frac{1}{2}\right)^2+\left(k+\frac{1}{2}\right)^2=9$ or $h^2+k^2=4$.
Required locus is the circle $x^2+y^2=4$.

Q.27

A circle with $(1,2)$ as center and 0 radius is $x^2+y^2-2x-4y+5=0$.
Now any circle touching $4x+3y=10$ at $(1,2)$ will be given by
$x^2+y^2-2x-4y+5+k(4x+3y-10)=0$
or $x^2+y^2+(4k-2)x+(3k-4)y+5-10k=0$.
radius $=\sqrt{(1-2k)^2+\frac{(4-3k)^2}{4}-5+10k}=5$
$\Rightarrow k=\pm 2$. Hence the two required circles are
$x^2+y^2+6x+2y-15=0$ & $x^2+y^2-10x-10y+25=0$.

Q.28

$x^2 - 3xy - 3x + 9y = 0 \Rightarrow (x-3y)(x-3) = 0.$

Point of intersection of $x = 3y$ & $x = 3$ is $(3,1)$ which is center of the required circle.

Now as it touches $x^2 + y^2 - 6x + 6y + 17 = 0$ which has center $(3,-3)$ & radius $= 1$ hence $r + 1 = \sqrt{(3-3)^2 + (1+3)^2}$ or $r = 3.$

Required circle is $(x-3)^2 + (y-1)^2 = 9.$

Q.29

By $\cos\theta = \left|\dfrac{2g_1g_2 + 2f_1f_2 - (c_1 + c_2)}{2\sqrt{g_1^2 + f_1^2 - c_1}\sqrt{g_2^2 + f_2^2 - c_2}}\right|$ we get

$\cos\theta = \left|\dfrac{2(-2)(-1) + 2(3)(4) - (11+13)}{2\sqrt{4+9-11}\sqrt{1+16-13}}\right|$ or $\cos\theta = \dfrac{1}{\sqrt{2}}.$

Hence acute angle between the circle is $\dfrac{\pi}{4}.$

Q.30

Radical axis of C_1 & $C_2 : x - y - 1 = 0$, radical axis of C_2 & $C_3 : 3x - 7y - 5 = 0.$

Solving these simultaneous gives radical center as $(3,2).$

Now length of tangent to C_1 from $(3,2)$ is $\sqrt{S_1} = 3\sqrt{3}.$

Required circle is $(x-3)^2 + (y-2)^2 = 27.$

Q.31

Circle touching both the coordinate axes will be any one of the four circles $x^2 + y^2 \pm 2ax \pm 2ay + a^2 = 0, a > 0.$

As it passes through $(1,2)$ which is a point in first quadrant, hence the circle must be $x^2 + y^2 - 2ax - 2ay + a^2 = 0.$

Substituting $(1,2)$ gives $a^2 - 6a + 5 = 0$ gives $a = 1$ & $5.$

Required circle is $x^2 + y^2 - 2x - 2y + 1 = 0$ or $x^2 + y^2 - 10x - 10y + 25 = 0.$

CIRCLES
Exercise – 3
Part II

Q.1

Let center of a circle be (h,k) and radius be r

Now for any circle passing through $(1,0)\&(2,-1)$ and touching Y – Axis

$\sqrt{(h-1)^2+k^2}=\sqrt{(h-2)^2+(k+1)^2}=|h|=r$

Or $k=h-2$ & $k^2-2h+1=0$

$\Rightarrow h^2-6h+5=0$

$\Rightarrow h=1$ & $k=-1$ or $h=5$ & $k=3$

Hence radii of two such circles are 1&5.

Q.2

Let A' be (h,k), then centere must be $\left(\frac{h+p}{2},\frac{k+q}{2}\right)$

Now radius $=\frac{\sqrt{(h-p)^2+(k-q)^2}}{2}=\left|\frac{k+q}{2}\right|$

$\Rightarrow (h-p)^2+(k-q)^2=(k+q)^2$

Or $(h-p)^2=4qk$

Hence the required locus is $(x-p)^2=4qy$.

Q.3

As C_1 touches both L_1 & L_2 hence its center must lie on their angle bisector.

Now angle bisectors of L_1 & L_2 will be given by

$\frac{5x+12y-10}{13}=\pm\frac{5x-12y-40}{13}$ i.e. $y=-\frac{5}{4}$ & $x=5$.

As the center lies in Ist quadrant hence we get x = 5.

Let the center be $(5,k)$

Further distance of $(5,k)$ from $5x+12y-10=0$ will be 3

$\Rightarrow\left|\frac{25+12k-10}{13}\right|=3$ which gives $k=2$.

Hence center of C_2 is $(5,2)$.

Now C_2 cuts of a length 8 from $5x+12y-10=0$.

Distance of this line from $(5,2)$ is 3 and half the chord is 4, hence radius of C_2 will be 5.

Finally equation of C_2 is $(x-5)^2+(y-2)^2=25$.

Q.4

Clearly AB is diameter. Let $\angle OAB = \theta$, then

$OA = 2r\cos\theta$ & $OB = 2r\sin\theta$

Equation of AB will be $x\sin\theta + y\cos\theta - 2r\sin\theta\cos\theta = 0$.

Foot of perpendicular from O on AB will be given by

$\frac{x-0}{\sin\theta} = \frac{y-0}{\cos\theta} = 2r\sin\theta\cos\theta$ i.e. $x = 2r\sin^2\theta\cos\theta$ & $y = 2r\cos^2\theta\sin\theta$

Now $x^2 + y^2 = 4r^2\left(\sin^4\theta\cos^2\theta + \cos^4\theta\sin^2\theta\right)$ or $x^2 + y^2 = 4r^2\sin^2\theta\cos^2\theta$

Also $xy = 4r^2\sin^3\theta\cos^3\theta$

Hence $\left(x^2 + y^2\right)^3 = 4r^2x^2y^2$.

Q.5

Tangents of slope m to $x^2 + y^2 = 4$ is $y = mx \pm 2\sqrt{m^2+1}$

If tangents are drawn from $(-2,4)$ then $4 = -2m \pm 2\sqrt{m^2+1}$

$\Rightarrow (2+m)^2 = m^2 + 1$ i.e. $m = -\frac{3}{4}$

Hence reflected ray is $3x + 4y = 10$.

Now incident ray will be image of this line in the line $x + 2 = 0$.

By observation a point on $3x + 4y = 10$ is $(2,1)$.

Image of $(2,1)$ in $x + 2 = 0$ is $(-6,1)$.

Incident ray will be the line joining $(-2,4)$ & $(-6,1)$.

Required equation is $3x - 4y + 22 = 0$.

Q.6

Let the chord AB be $px + qy = 1$.

Homogenising equation of the circle using the equation of chord gives

$x^2 + y^2 + 2gx(px + qy) + 2fy(px + qy) + c(px + qy)^2 = 0$

Or $(cp^2 + 2gp + 1)x^2 + 2(gq + fp + cpq)xy + (cq^2 + 2fq + 1)y^2 = 0$

Now this will be the pair of lines joining the origin to A & B.

As $\angle AOB = \frac{\pi}{2}$ hence in the above equation

coefficient of x^2 + coefficient of $y^2 = 0$ i.e. $cp^2 + cq^2 + 2gp + 2fq + 2 = 0$ ...(1)

Also given P is foot of perpindicular from O on AB hence P will be

$x = \frac{p}{p^2 + q^2}$ & $y = \frac{q}{p^2 + q^2}$

$\Rightarrow p^2 + q^2 = \frac{1}{x^2 + y^2}, p = \frac{x}{x^2 + y^2}, q = \frac{y}{x^2 + y^2}$

Substituting these values in (1) gives required locus as

$2x^2 + 2y^2 + 2gx + 2fy + c = 0$

Q.7

Let the given circles be $w_1 : x^2 + y^2 + 2ax + p = 0$ & $w_2 : x^2 + y^2 - 2ax + q = 0$.

Also let the variable circle be $w : x^2 + y^2 + 2gx + 2fy + c = 0$.

Common chord of w_1 & w will be $2(g - a)x + 2fy + c - p = 0$.

It must pass through centre of w_1 hence $2a(g - a) = c - p$ ...(1)

Common chord of w_2 & w will be $2(g + a)x + 2fy + c - q = 0$.

It must pass through centre of w_2 hence $2a(g + a) = -(c - q)$ ...(2)

From (1) & (2) we get $4ag = q - p$.

Now radical axis of w_1 & w_2 will be $4ax = q - p$ ...(3)

Clearly $(-g, -f)$ lies on $4ax = -(q - p)$ which is parallel to (3).

Q.8

Diameter of each ciercle is tangent to other circle hence $\angle PBQ = \frac{\pi}{2}$

Also $\angle PAB = \frac{\pi}{2}$ & $\angle QAB = \frac{\pi}{2} \Rightarrow \angle PAB + \angle QAB = \pi$

Therefore A lies on PQ and PQ must be perpendicular to AB.

Equation of PQ will be $(y-3) = -\frac{2}{3}(x-2)$

i.e. $2x + 3y = 13$.

Q.9

Let the circles be $x^2 + y^2 + 2g_i x + 2f_i y + c_i = 0$ for $i = 1, 2, 3$

Radical axis of C_1 & C_2 passes through center of C_3 hence

$2g_3(g_2 - g_1) + 2f_3(f_2 - f_1) = c_2 - c_1 \quad ...(1)$

Radical axis of C_2 & C_3 passes through center of C_1 hence

$2g_1(g_3 - g_2) + 2f_1(f_3 - f_2) = c_3 - c_2 \quad ...(2)$

From (1) & (2) we get

$2g_2(g_1 - g_3) + 2f_2(f_1 - f_3) = c_1 - c_3$

Hence Radical axis of C_1 & C_3 passes through center of C_2.

Q.10

Let O be the center. Join O to C, D & P.

Let $\angle COP = \theta$, then $\angle DOP = 90 - \theta$

Also $OP = r$.

Now from ΔCOP, $PC = r\tan\theta$ & from ΔDOP, $PD = r\cot\theta$.

Hence $PC \times PD = r^2$

ALITER :

Let the circles be $x^2 + y^2 = r^2$ so that A, B & P are $(-r, 0), (r, 0)$ & $(r\cos\alpha, r\sin\alpha)$

tangents at A, B & P will be $x + r = 0, x - r = 0$ & $x\cos\alpha + y\sin\alpha = r$

Hence C & D will be $\left(-r, r\cot\frac{\alpha}{2}\right)$ & $\left(r, r\tan\frac{\alpha}{2}\right)$

Now $PC \times PD = r^2\sqrt{(1+\cos\alpha)^2 + \left(\sin\alpha - \cot\frac{\alpha}{2}\right)^2}\sqrt{(1-\cos\alpha)^2 + \left(\sin\alpha - \tan\frac{\alpha}{2}\right)^2}$

$\Rightarrow PC \times PD = r^2$

Q.11

Equation of the circle on $(0, a) \& (0, -a)$ as diameter will be $x^2 + y^2 = a^2$.

Any other circle passing through these points will be

$x^2 + y^2 + \lambda x - a^2 = 0$

Now if it touches $y = mx + c$, then $\left|\dfrac{-\dfrac{m\lambda}{2} + c}{\sqrt{m^2+1}}\right| = \sqrt{\dfrac{\lambda^2}{4} + a^2}$

$\Rightarrow \lambda^2 + 4cm\lambda + 4a^2m^2 + 4a^2 - 4c^2 = 0 \ \ldots(1)$

Now let the two circles be $x^2 + y^2 + \lambda_1 x - a^2 = 0$ & $x^2 + y^2 + \lambda_2 x - a^2 = 0$

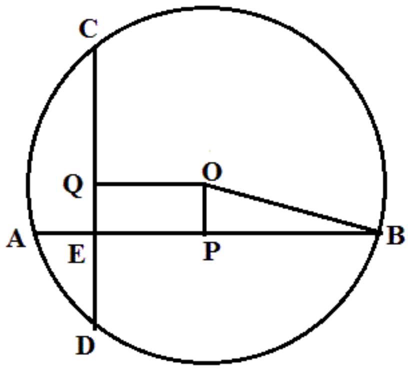
then by orthogonality, $\lambda_1\lambda_2 = -4a^2$

But λ_1 & λ_2 are the roots of (1) hence $\lambda_1\lambda_2 = 4a^2m^2 + 4a^2 - 4c^2$

$\Rightarrow 4a^2m^2 + 4a^2 - 4c^2 = -4a^2$

$\Rightarrow a^2m^2 + 2a^2 = c^2$.

Q.12



Let O be the center, P & Q be the mid points of AB & CD.

Now $AB = AE + EB \Rightarrow 2BP = EA + EB \ \ldots(1)$

And $QE = CE - CQ$ & $QE = DQ - ED$

$\Rightarrow 2QE = EC - ED$ as $CQ = DQ \ \ldots(2)$

Also by Power of point theorem $EA \times EB = EC \times ED$.

From ΔOPB, $QE^2 + BP^2 = r^2$

Now from $(1) \& (2)$ we get

$4\left(BP^2 + CQ^2\right) = (EA + EB)^2 + (EC - ED)^2$

$\Rightarrow EA^2 + EB^2 + EC^2 + ED^2 = 4r^2$

Q.13

Any circle passing through P&Q will be

$x^2+y^2+A_1x+B_1y+C_1+p(a_1x+b_1y+c_1)=0$

Similarly any circle passing through R&S will be

$x^2+y^2+A_2x+B_2y+C_2+q(a_2x+b_2y+c_2)=0$

If P,Q,R,S are cocyclic, then both of these must represent the same circle, hence

$A_1+pa_1=A_2+qa_2, B_1+pb_1=B_2+qb_2$ & $C_1+pc_1=C_2+qc_2$

Eliminating p&q in these three equations gives

$$\begin{vmatrix} A_1-A_2 & B_1-B_2 & C_1-C_2 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}=0$$

Q.14

Any circle touching the coordinate axes in first quadrant is

$x^2+y^2-2rx-2ry+r^2=0$

As it passes through (a,b) hence

$r^2-2(a+b)r+a^2+b^2=0 \quad ...(1)$

Now if two such circles are

$x^2+y^2-2r_1x-2r_1y+r_1^2=0$ & $x^2+y^2-2r_2x-2r_2y+r_2^2=0$

then r_1 & r_2 will be the roots of (1).

As the circles are orthogonal hence $4r_1r_2=r_1^2+r_2^2$

or $6r_1r_2=(r_1+r_2)^2$

Hence from (1), $6(a^2+b^2)=4(a+b)^2$

$\Rightarrow \frac{a}{b}+\frac{b}{a}=4.$

Q.15

Let the equation of DA be $(y-1)=m(x-2)$ i.e. $mx-y+1-2m=0$

Now any curve passing through A, B, C, D may be represented as family of pair of lines

$x-3y+5=0$, $x+y-1=0$ & $2x+y+1=0$, $mx-y+1-2m=0$

$\Rightarrow (x-3y+5)(x+y-1)+\lambda(2x+y+1)(mx-y+1-2m)=0$

Now for this to be equation of a circle

coeff. of x^2 = coeff. of y^2 & coeff. of $xy=0$

$\Rightarrow 1+2m\lambda=-3-\lambda$ & $\lambda m=2+2\lambda$

$\Rightarrow \lambda=-\dfrac{8}{5}, m=\dfrac{3}{4}$

Hence equation of DA is $3x-4y=2$.

Q.16

Let A, B, C be $(a,0),(b,0)$ & $(c,0)$

Any circle passing through A, B is $(x-a)(x-b)+y^2+py=0$

i.e. $C_1: x^2+y^2-(a+b)x+py+ab=0$

Any circle passing through A, C is $(x-a)(x-c)+y^2+qy=0$

i.e. $C_2: x^2+y^2-(a+c)x+qy+ac=0$

Now let C_3 be $x^2+y^2-2bx+c_1=0$

As this circle is orthogonal to C_2 hence $b(a+c)=c_1+ac$ i.e. $c_1=ab+bc-ac$

So $C_3: x^2+y^2-2bx+ab+bc-ac=0$

Similarly let C_4 be $x^2+y^2-2cx+c_2=0$

As this circle is orthogonal to C_1 hence $c(a+b)=c_2+ab$ i.e. $c_2=-ab+bc+ca$

So $C_4: x^2+y^2-2cx-ab+bc+ca=0$

Now clearly C_3 & C_4 are orthogonal.

Q.17

Consider two point A & B as lattice points on the given circle,
mid point of AB as M and let the center be C.
Slope of AB will be rational & slope of CM will be irrational
(except one case when CM is horizontal)
Now we know $AB \perp CM$ so $m_{AB} \times m_{CM} = -1$.
But its a contradiction.
Hence there cant be more than two lattice points.
Also in a circle every lattice point corresponds to exactly four lattice
points except the center(if its a lattice point).
Hence there exists a circle with 2004 lattice points as 2004 is a multiple of 4.

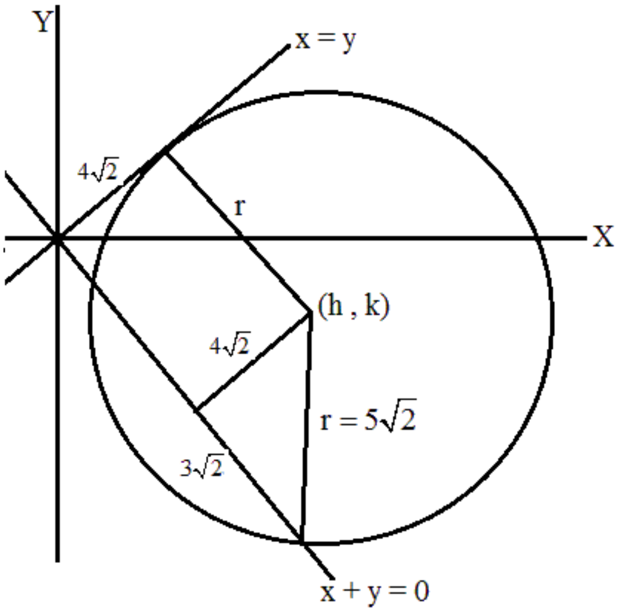
Q.18

Let the circles be $C_1 : x^2 + y^2 = a^2$ & $C_2 : x^2 + y^2 + 2gx + 2fy + c = 0$
Also let C be $x^2 + y^2 - 2hx - 2ky + C = 0$
Now common chord of C_1 & C is diameter of C.
Common chord: $2hx + 2ky = C + a^2$
As it passes through (h, k) hence $2h^2 + 2k^2 = C + a^2$ ...(1)
Also its orthogonal to C_2 so $-2gh - 2fk = c + C$
$\Rightarrow C = -2gh - 2fk - c$ ...(2)
from (1) & (2)
$2h^2 + 2k^2 + 2gh + 2fk + c - a^2 = 0$
Hence required locus is
$2x^2 + 2y^2 + 2gx + 2fy + c - a^2 = 0$.

Q.19

Use Power of point theorem.

Q. 20 From the figure as the circle is touching $x = y$ & $x + y = 0$ hence



$$\left|\frac{h-k}{\sqrt{2}}\right| = 5\sqrt{2} \Rightarrow |h-k| = 10 \;\;\&$$

$$\left|\frac{h+k}{\sqrt{2}}\right| = 4\sqrt{2} \Rightarrow |h+k| = 8$$

$h - k = 10$ or -10 & $h + k = 8, -8$

$\Rightarrow (h,k) \equiv (9,-1),(1,-9),(-1,9)$ & $(-9,1)$

Now possible circles are

$x^2 + y^2 - 18x + 2y - 32 = 0$

$x^2 + y^2 - 2x + 18y - 32 = 0$

$x^2 + y^2 + 2x - 18y - 32 = 0$

$x^2 + y^2 + 18x - 2y - 32 = 0$

Out of these only

$x^2 + y^2 - 18x + 2y - 32 = 0$

contains $(-10, 2)$.

Q.21

Let the circle be $x^2 + y^2 = a^2$ & the line be $x = b$.

Now a point on $x = b$ may be $P(b,c)$

Let mid point of the chord of contacts be $Q(h,k)$

Now chord of contact w.r.t. P will be $T : bx + cy = a^2$ ...(1) &

Chord with Q as mid point will be $T - S_1 : hx + ky - (h^2 + k^2) = 0$ ...(2)

Comparing (1) & (2) gives

$$\frac{h}{b} = \frac{k}{c} = \frac{h^2 + k^2}{a^2}$$

$\Rightarrow bh^2 + bk^2 - a^2h = 0$

Hence required locus is a circle if $b \neq 0$ and a line if $b = 0$.

Q.22

Let O be the origin and L_1 be the X – Axis.

Also Let L_2 be $x = a$, L_3 be $x = b$ & L be $y = mx + c$.

Now A is $(a, am + c)$ & B is $(b, mb + c)$.

Slopes of OA & OB will be $\frac{mb+c}{b}$ & $\frac{ma+c}{a}$.

As OA is perpendicular to OB so $\frac{mb+c}{b} \times \frac{ma+c}{a} = -1$

$\Rightarrow abm^2 + c(a+b)m + c^2 + ab = 0 \quad ....(1)$

Now foot of perpendicular (M) from O on $y = mx + c$ is

$$\frac{x}{m} = \frac{y}{-1} = -\frac{c}{m^2+1}$$

$$\Rightarrow m = -\frac{x}{y} \ \& \ c = \frac{x^2+y^2}{y}$$

Substituting these values in (1) gives

$$x^2 + y^2 - (a+b)x + ab = 0$$

which is equation of a circle.