

COMPLEX NUMBER

EXERCISE – 1

Q.1 [B]

$$\Rightarrow \sqrt{-2}\sqrt{-3} = (i\sqrt{2})(i\sqrt{3}) = -\sqrt{6}$$

Q.2 [D]

$$\Rightarrow (1+i)^5(1-i)^5 = (1-i^2)^5 = 2^5$$

Q.3 [B]

$$(1+i)^4 + (1-i)^4 = \left[(1+i)^2 + (1-i)^2 \right]^2 - 2(1+i)^2(1-i)^2$$

$$\Rightarrow \left[(2i) + (-2i) \right]^2 - 2(1-i^2)^2$$

$$\Rightarrow -2 \cdot 2^2 = -8$$

Q.4 [C]

$$(1+i)^8 + (1-i)^8 = \left[(1+i)^4 + (1-i)^4 \right]^2 - 2(1+i)^4(1-i)^4$$

$$\Rightarrow [-8]^2 - 2(1-i^2)^4$$

$$\Rightarrow 64 - 2(2)^4 = 32$$

Q.5 [A]

$$(1+i)^6 + (1-i)^6 = \left[(1+i)^3 + (1-i)^3 \right]^2 - 2(1+i)^3(1-i)^3$$

$$\Rightarrow \left[1 + 3i^2 + 3i + i^3 + 1 - i^3 + 3i^2 - 3i \right]^2 - 2(1-i^2)^3$$

$$\Rightarrow [2 - 6]^2 - 2(2)^3$$

Q.6 [A]

$$\Rightarrow (1+i)^{10} = \left[(1+i)^2 \right]^5 = (2i)^5 = 32i$$

Q.7 [A]

$$\Rightarrow 1+i^2+i^3-i^6+i^8=1-1-i-i^2+1=2-i$$

Q.8 [B]

$$\therefore i^4=1$$

$$\Rightarrow \therefore i^{4n+\lambda}=i^\lambda \text{ where } n \in \mathbb{I}; i^{4n+2}=i^2=-1$$

\therefore given expression will become

$$\Rightarrow \frac{1-1+1-1+1}{-1+1-1+1-1}-1=-2$$

Q.9 [D]

For given equation to be true

$$(1-i)^n=2^n$$

$$\Rightarrow n=4m; m \in \mathbb{I}$$

$$\Rightarrow \min n=4$$

Q.10 [A]

$$\left(\frac{-1+i}{1+i}\right)^n = \text{Real number}$$

$$\Rightarrow \left(\frac{-1+i}{1+i}\right)^n \left(\frac{1-i}{1-i}\right)^n = \frac{(1-i)^{2n}(-1)^n}{(1+1)^n} = \frac{(1+i^2-2i)^n(-1)^n}{2^n}$$

$$\Rightarrow \frac{2^n i^n}{2^n} = i^n$$

least $n=2$

Q.11 [B]

$$(a+ib)^5 = \alpha + i\beta$$

$$\Rightarrow i^5(-ai+b)^5 = \alpha + i\beta$$

$$\Rightarrow (b - ai)^5 = \beta - \alpha i$$

Take complex conjugate then

$$\Rightarrow (b + ai)^5 = \beta + \alpha i$$

Q.12 [B]

$$\frac{1+2i}{1-i}$$

$$\Rightarrow \frac{1+2i}{1-i} \times \frac{1+i}{1+i}$$

$$\Rightarrow \frac{1+3i+2i^2}{2}$$

$$\Rightarrow \frac{-1+3i}{2} \quad 2^{\text{nd}} \text{ quadrate}$$

Q.13 [A]

$$|z|=1, w = \frac{z-1}{z+1} \quad (z \neq -1)$$

Let $z = \cos \theta + i \sin \theta$

$$\Rightarrow \therefore w = \frac{(\cos \theta - 1) + i \sin \theta}{(\cos \theta + 1) + i \sin \theta} \neq 1$$

$$\Rightarrow \frac{-2 \sin^2 \frac{\theta}{2} + 2i \sin \frac{\theta}{2} \cos \frac{\theta}{2}}{2 \cos^2 \frac{\theta}{2} + 2i \sin \frac{\theta}{2} \cos \frac{\theta}{2} + \cos \frac{\theta}{2}}$$

$$\Rightarrow \frac{2i \sin \frac{\theta}{2} \left(\cos \frac{\theta}{2} + i \sin \frac{\theta}{2} \right)}{2 \cos \frac{\theta}{2} \left(\cos \frac{\theta}{2} + i \sin \frac{\theta}{2} \right)}$$

$$\Rightarrow w = i \tan \frac{\theta}{2}$$

$$\Rightarrow \therefore \text{Re}(w) = 0$$

Q.14 [C]

$$\frac{3+2i \sin \theta}{1-2i \sin \theta} = Ki; K \in \mathbb{R}$$

$$\Rightarrow \frac{(3+2i \sin \theta)(1+2i \sin \theta)}{1+4 \sin^2 \theta} = Ki$$

$$\Rightarrow \therefore \frac{3-4 \sin^2 \theta}{1+4 \sin^2 \theta} = 0 \text{ (Real part zero)}$$

$$\Rightarrow \sin^2 \theta = \left(\frac{\sqrt{3}}{2} \right)^2 = \sin^2 60^\circ$$

$$\Rightarrow \theta = n\pi \pm \frac{\pi}{3}$$

Q.15 [B]

$$\text{Given } x = \frac{1}{x}$$

$$\Rightarrow x = \pm 1$$

Q.16 [C]

$$z = 1 + i$$

$$\Rightarrow z^2 = 1 + i^2 + 2i = 2i$$

Let z_1 is multiplication inverse

$$\Rightarrow \therefore z^2 z_1 = 1$$

$$\Rightarrow z_1 = \frac{1}{z^2} = \frac{1}{2i} = \frac{-i}{z}$$

Q.17 [B]

$$(x + iy)^{\frac{1}{3}} = a + ib$$

$$\Rightarrow x + iy = (a + ib)^3$$

$$\Rightarrow x = a^3 - 3ab^2$$

$$\Rightarrow y = -b^3 + 3a^2b$$

$$\Rightarrow \therefore \frac{x}{a} + \frac{y}{b} = (a^2 - 3b^2) + (-b^2 + 3a^2)$$

$$\Rightarrow 4(a^2 - b^2)$$

Q.18 [B]

$$\sqrt{3} + i = (a + ib)(c + id)$$

$$\Rightarrow \arg(\sqrt{3} + i) = \arg[(a + ib)(c + id)]$$

$$\Rightarrow \arg(a + ib) + \arg(c + id) = \tan^{-1} \frac{1}{\sqrt{3}}$$

$$\Rightarrow \tan^{-1} \frac{b}{a} + \tan^{-1} \frac{d}{c} = \frac{\pi}{6}$$

Q.19 [C]

$$z_1 = 4 + i5$$

$$\Rightarrow z_2 = -3 + 2i$$

$$\Rightarrow \frac{z_1}{z_2} = \frac{4 + 5i}{-3 + 2i} = \frac{(4 + 5i)(-3 - 2i)}{9 + 4} = \frac{-12 + 10 - 15i - 8i}{13}$$

$$\Rightarrow \frac{-2}{13} - \frac{23}{13}i$$

Q.20 [C]

$$x + \frac{1}{x} = 2 \cos \theta$$

$$\Rightarrow x^2 - 2x \cos \theta + 1 = 0$$

$$\Rightarrow x = \frac{2 \cos \theta \pm \sqrt{4 \cos^2 \theta - 4}}{2}$$

$$\Rightarrow x = \cos \theta \pm i \sin \theta$$

Q.21 [A]

$$3 - 2yi = 9^x - 7i$$

$$\Rightarrow 3 = 9^x; \quad -2y = -7$$

$$\Rightarrow x = \frac{1}{2}$$

$$\Rightarrow y = \frac{7}{2}$$

Q.22 [B]

$$\frac{(1+i)x - 2i}{3+i} + \frac{(2-3i)y + i}{3-i} = i$$

$$\Rightarrow ((1+i)x - 2i(3-i) + [(2-3i)y + i][3+i]) = i(3+i)$$

by comparing real & imaginary parts

$$\Rightarrow x = 3, y = -1$$

Q.23 [B]

$$x + iy = \frac{3}{2 + \cos \theta + i \sin \theta} \times \left(\frac{2 + \cos \theta - i \sin \theta}{2 + \cos \theta - i \sin \theta} \right)$$

$$\Rightarrow \frac{6 + 2 \cos \theta - 3i \sin \theta}{(4 + \cos^2 \theta + 4 \cos \theta + \sin^2 \theta)}$$

$$\Rightarrow x = \frac{2(3 + \cos \theta)}{5 + 4 \cos \theta}, \quad y = \frac{-3 \sin \theta}{5 + 4 \cos \theta}$$

$$\Rightarrow x^2 + y^2 = \frac{4(9 + \cos^2 \theta + 6 \cos \theta) + 9 \sin^2 \theta}{(5 + 4 \cos \theta)^2} = \frac{40 + 2y \cos \theta + 5 \sin^2 \theta}{(5 + 4 \cos \theta)^2}$$

$$\Rightarrow \frac{8(3 + \cos \theta)(5 + 4 \cos \theta) - 3(5 + 4 \cos \theta)^2}{(5 + 4 \cos \theta)^2}$$

$$\Rightarrow x^2 + y^2 = 4x - 3$$

Q.24 [B]

$$x = -5 + 2\sqrt{-4} = -5 + 4i$$

$$\Rightarrow x^2 - (-10)x + (41) = 0$$

$$\Rightarrow x^2 + 10x + 41 = 0$$

$$\Rightarrow x^2 = -10x - 41$$

$$\Rightarrow x^3 = -10x^2 - 41x = -10(-10x - 41) - 41x = 59x + 410$$

$$\Rightarrow \therefore x^4 + 9x^3 + 35x^2 - x + 4 = x^2(x^2 + 35) + 9x^3 - x + 4$$

$$\Rightarrow x^2(-10x - 6) + 9x^3 - x + 4$$

$$\Rightarrow -x^3 - 6x^2 - x + 4$$

$$\Rightarrow -59x + 410 + 60x + 41 \times 6 - x + 4$$

$$\Rightarrow -41 \times 4 + 4$$

$$\Rightarrow -160$$

Q.25 [B]

$$(x + iy)(y - i3) = 4 + i$$

By comparing real & imaginary parts.

$$\Rightarrow 2x + 3y = 4 \quad \dots\dots\dots(1)$$

$$\Rightarrow 2y - 3x = 1 \quad \dots\dots\dots(2)$$

$$\Rightarrow \therefore x = \frac{5}{13}, y = \frac{14}{13}$$

Q.26 [D]

$$z = \frac{1 - i \sin \alpha}{1 + 2i \sin \alpha} = k; \quad k \in \mathbb{R}$$

$$\Rightarrow z = \frac{(1 - i \sin \alpha)(1 - 2i \sin \alpha)}{1 + 4 \sin^2 \alpha}$$

$$\Rightarrow \therefore \operatorname{Im}(z) = 0$$

$$\Rightarrow -2 \sin \alpha - \sin \alpha = 0$$

$$\Rightarrow \alpha = n\pi; \quad n \in \mathbb{I}$$

Q.27 [C]

$$z(2 - i) = 3 + i$$

$$\Rightarrow z = \frac{3 + i}{2 - i} \times \frac{2 + i}{2 + i} = \frac{6 - 1 + 5i}{5}$$

$$\Rightarrow z = 1 + i = \sqrt{2} e^{i\frac{\pi}{4}}$$

$$\Rightarrow z^{20} = 2^{10} e^{i5\pi} = 2^{10} = 1024$$

Q.28 [A]

$$\operatorname{Re}\left(\frac{z - 8i}{z + 6}\right) = 0$$

$$\Rightarrow z = \frac{z + i(y - 8)}{(x + 6) + iy} = \frac{[x + i(y - 8)][(x + 6) - iy]}{(x + 6)^2 + y^2}$$

$$\Rightarrow \operatorname{Re}(z) = x(x + 6) + y(y - 8) = 0$$

$$\Rightarrow x^2 + y^2 + 6x - 8y = 0$$

Q.29 [B]

$$z = \frac{2 + 5i}{4 - 3i} \times \frac{4 + 3i}{4 + 3i} = \frac{8 - 15 + 26i}{25}$$

$$\Rightarrow z = \frac{-7}{25} + \frac{i26}{25}$$

$$\Rightarrow \bar{z} = \frac{-7}{25} - \frac{i26}{25}$$

Q.30 [B]

$$z_1 + z_2 = \text{Real}$$

$$\Rightarrow z_1 z_2 = \text{Real}$$

$\Rightarrow \therefore z_1$ & z_2 are complex conjugate

$$\Rightarrow z_1 = \overline{z_2}$$

Q.31 [B]

$z = x + iy$ (in 3rd quadrate)

$$\Rightarrow x < 0, y < 0$$

$$\Rightarrow \overline{z} = x - iy = x + i(y) \quad \text{2nd quadrate}$$

Q.32 [A]

$$(z+3)(\overline{z}+3)$$

$$\Rightarrow (z+3)(\overline{z+3})$$

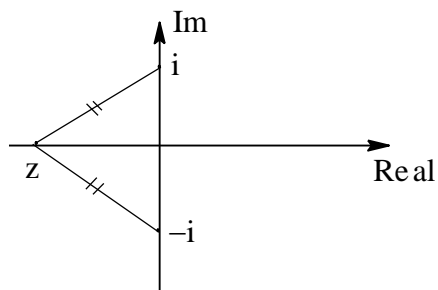
$$\Rightarrow |z+3|^2$$

Q.33 [A]

$$\Rightarrow |z_1 z_2| = |z_1| |z_2| = 1$$

Q.34 [A]

$$|z+1| = |z-i|$$



Locus of z is the Real axis

Q.35 [B]

$$\frac{z-1}{z+1} = ki; k \in \mathbb{R}$$

$$\Rightarrow \operatorname{Re} \left[\frac{(x-1)+iy}{(x+1)+iy} \times \frac{(x+1)-iy}{(x+1)-iy} \right] = 0$$

$$\Rightarrow (x-1)(x+1) + y^2 = 0$$

$$\Rightarrow x^2 + y^2 = 1$$

$$\Rightarrow |z|^2 = 1$$

$$\Rightarrow |z| = 1$$

Q.36

$$\Rightarrow |2z-1| + |3z-2|$$

Q.37 [B]

$$|z_1 + z_2|^2 + |z_1 - z_2|^2$$

$$\Rightarrow (z_1 + z_2)(\overline{z_1 + z_2}) + (z_1 - z_2)(\overline{z_1 - z_2})$$

$$\Rightarrow 2(|z_1|^2 + |z_2|^2)$$

Q.38

$$\frac{2z_1}{3z_2} = ki; k \in \mathbb{R}$$

$$\Rightarrow \frac{2z_1 \overline{z_2}}{3|z_2|^2} = ki$$

$$\Rightarrow \therefore \operatorname{Re}(z_1 \overline{z_2}) = 0$$

$$\Rightarrow z_1 \overline{z_2} + \overline{z_1} z_2 = 0$$

$$\begin{aligned} \Rightarrow \therefore \left| \frac{z_1 - z_2}{z_1 + z_2} \right|^2 &= \left(\frac{z_1 - z_2}{z_1 + z_2} \right) \left(\frac{\overline{z_1 - z_2}}{\overline{z_1 + z_2}} \right) \\ &\Rightarrow \frac{|z_1|^2 + |z_2|^2 - (z_1 \overline{z_2} + z_2 \overline{z_1})}{|z_1|^2 + |z_2|^2 + (z_1 \overline{z_2} + z_2 \overline{z_1})} = \frac{|z_1|^2 + |z_2|^2}{|z_1|^2 + |z_2|^2} \end{aligned}$$

Q.39

$$\begin{aligned} |z_1| = |z_2| = |z_3| &= \left| \frac{1}{z_1} + \frac{1}{z_2} + \frac{1}{z_3} \right| = 1 \\ \Rightarrow |z_1 + z_2 + z_3| &= \left| \overline{z_1} + \overline{z_2} + \overline{z_3} \right| = \left| \frac{z_1 \overline{z_1}}{z_1} + \frac{z_2 \overline{z_2}}{z_2} + \frac{z_3 \overline{z_3}}{z_3} \right| \\ &\Rightarrow \left| \frac{1}{z_1} + \frac{1}{z_2} + \frac{1}{z_3} \right| = 1 \end{aligned}$$

Q.40

$$\left| z_1 + \sqrt{z_1^2 - z_2^2} \right| + \left| z_1 - \sqrt{z_1^2 - z_2^2} \right|$$

$$\text{Let } z_3 = \sqrt{z_1^2 - z_2^2} \quad z_3^2 = z_1^2 - z_2^2$$

$$\Rightarrow \left[|z_1 + z_3| + |z_1 - z_3| \right]^2 = |z_1 + z_3|^2 + |z_1 - z_3|^2 + 2|z_1 + z_3||z_1 - z_3|$$

$$\Rightarrow 2\left(|z_1|^2 + |z_3|^2\right) + 2|z_1^2 - z_3^2|$$

$$\Rightarrow 2\left(|z_1|^2 + |z_3|^2\right) + 2|z_2|^2$$

$$\Rightarrow 2\left(|z_1|^2 + |z_2|^2\right) + 2|z_1^2 - z_2^2|$$

$$\Rightarrow \left(|z_1 - z_2|^2 + |z_1 + z_2|^2\right) + 2|z_1 - z_2||z_1 + z_2|$$

$$\Rightarrow \left[|z_1 - z_2| + |z_1 + z_2| \right]^2$$

Q.41 [C]

$$\left| \frac{z-4}{z-8} \right| = 1$$

Locus of Z will be $x = 6$

$$\Rightarrow \therefore z = 6 + iy$$

$$\Rightarrow \left| \frac{z-12}{z-8i} \right| = \frac{5}{3}$$

$$\Rightarrow 3|-6+iy| = 5|6+i(y-8)|$$

$$\Rightarrow 9(36+y^2) = 25(36+(y-8)^2)$$

$$\Rightarrow y^2 - 25y + 136 = 0$$

$$\Rightarrow y = 8, 17$$

$$\Rightarrow \therefore z = 6 + 8i, 6 + i17$$

Q.42 [A]

$$|z-4| < |z-2|$$

$$\Rightarrow |z-4|^2 < |z-2|^2$$

$$\Rightarrow |z|^2 - 4(\bar{z}+z) + 16 < |z|^2 - 2(z+\bar{z}) + 4$$

$$\Rightarrow 2(z+\bar{z}) > 12$$

$$\Rightarrow 2(2x) > 12$$

$$\Rightarrow x > 3$$

$$\Rightarrow \operatorname{Re}(z) > 3$$

Q.43 [B]

$$z = 1 + i \tan \alpha$$

$$\Rightarrow \pi < \alpha < \frac{3\pi}{2}$$

$$\Rightarrow |z| = \sqrt{1 + \tan^2 \alpha} = |\sec \alpha|$$

$$\Rightarrow |z| = -\sec \alpha \text{ (3rd quadrant)}$$

Q.44 [A, B]

$$\Rightarrow \left| \frac{\overline{z}^2}{z\overline{z}} \right| = \frac{|\overline{z}|^2}{|z|^2} = 1 = \left| \frac{\overline{z}}{z} \right|$$

Q.45 [C]

$$|z_1 + z_2| = \left| \frac{1}{z_1} + \frac{1}{z_2} \right|$$

$$\Rightarrow |z_1 + z_2| = \left| \frac{z_1 + z_2}{z_1 z_2} \right|$$

$$\Rightarrow |z_1 z_2| = 1$$

Q.46 [D]

$$z^2 + |z|^2 = 0$$

$$\Rightarrow z^2 = -|z|^2$$

$$\Rightarrow z = i|z|$$

$$\Rightarrow \text{Real}(z) = 0$$

$$\Rightarrow \therefore z = iy$$

So infinite solution.

Q.47

$$\Rightarrow |z| = \max \{|z-2|, |z+2|\}$$

Q.48 [C]

$$z = -1 + i\sqrt{3} \text{ } z \text{ lies in 2nd quadrant}$$

$$\Rightarrow \therefore \arg(z) = \tan^{-1}(-\sqrt{3}) = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$$

Q.49

$z = -1 - i\sqrt{3}$; z lies in 3rd quadrant

$$\Rightarrow \arg(z) = \pi + \tan^{-1}(\sqrt{3}) = \frac{4\pi}{3}$$

Q.50 [A]

$$z = \frac{1+i\sqrt{3}}{\sqrt{3}+i} \times \frac{\sqrt{3}-i}{\sqrt{3}-i} = \frac{\sqrt{3} + \sqrt{3} + 3i - i}{4}$$

$$\Rightarrow z = \frac{\sqrt{3} + i}{2}$$

$$\Rightarrow \arg(z) = \tan^{-1} \frac{1}{\sqrt{3}} = \frac{\pi}{6}$$

Q.51 Repeated Q.50

Q.52

$$z = \frac{13 - 5i}{4 - 9i}$$

$$\Rightarrow \arg(z) = \arg(13 - 5i) - \arg(4 - 9i)$$

$$\Rightarrow \tan^{-1}\left(\frac{-5}{13}\right) - \tan^{-1}\left(\frac{-9}{4}\right)$$

$$\Rightarrow \left[-\tan^{-1} \frac{5}{13}\right] - \left[-\tan^{-1} \frac{9}{4}\right]$$

$$\Rightarrow \tan^{-1} \left[\frac{\frac{9}{4} - \frac{5}{13}}{1 + \frac{9}{4} \times \frac{5}{13}} \right] = \tan^{-1} \left(\frac{97}{97} \right) = \frac{\pi}{4}$$

Q.53 [C]

$$z = \frac{1 - i\sqrt{3}}{1 + i\sqrt{3}}$$

$$\Rightarrow \arg(z) = \arg(1 - i\sqrt{3}) - \arg(1 + i\sqrt{3})$$

$$\Rightarrow \left(-\frac{\pi}{3}\right) - \left(\frac{\pi}{3}\right) = \frac{-2\pi}{3} = \frac{4\pi}{3}$$

Q.54 [D]

$$z = 1 - \cos \alpha + i \sin \alpha$$

$$\Rightarrow \text{amp}(z) = \tan^{-1} \left(\frac{\sin \alpha}{1 - \cos \alpha} \right)$$

$$\Rightarrow \tan^{-1} \left(\frac{2 \sin \frac{\alpha}{2} \cos \frac{\alpha}{2}}{2 \sin^2 \frac{\alpha}{2}} \right)$$

$$\Rightarrow \tan^{-1} \left(\cot \frac{\alpha}{2} \right) = \frac{\pi}{2} - \frac{\alpha}{2}$$

Q.55 [B]

$$z = \cos \frac{\pi}{6} + i \sin \frac{\pi}{6} = 1 \cdot e^{i\frac{\pi}{6}}$$

$$\Rightarrow |z| = 1, \arg(z) = \frac{\pi}{6}$$

Q.56 [B]

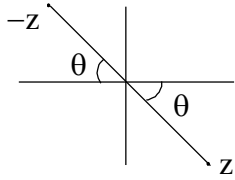
$$\arg(z) = \theta$$

$$\Rightarrow \arg(\bar{z}) = -\theta$$

Q.57

$$\arg(z) < 0$$

$$\text{Let } \arg(z) = -\theta; \theta > 0$$



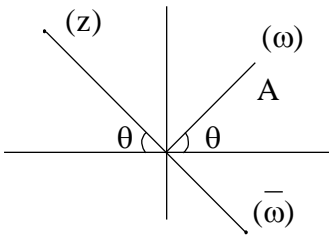
then $\arg(-z) - \arg(z)$

$$(\pi - \theta) - (-\theta) = \pi$$

Q.58 [D]

$$|z| = |\omega|$$

$$\Rightarrow \arg(z) + \arg(\omega) = \pi$$



$$\Rightarrow \therefore z = -\bar{\omega}$$

Q.59

$$\operatorname{Re}(z) < 0$$

$$\Rightarrow \operatorname{Im}(z) = 0$$

$$\Rightarrow \arg(z) = \pi$$

Q.60 [D]

$$\text{if } \arg(z) = \theta$$

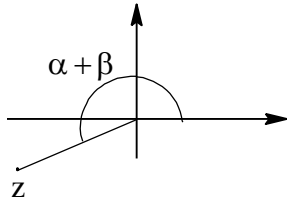
$$\Rightarrow \text{then } \arg(\bar{z}) = -\theta$$

Q.61 [C]

$$\arg(z_1) = \alpha$$

$$\Rightarrow \arg(z_2) = \beta$$

given $\alpha + \beta > \pi$



$$\Rightarrow z_1 z_2 = r_1 r_2 e^{i(\alpha + \beta)}$$

Principal argument $\arg(z_1 \times z_2) = -(2\pi - \alpha + \beta)$

$$\Rightarrow \alpha + \beta - 2\pi$$

Q.62 [B]

$$z = -1$$

$$\Rightarrow \arg\left(z^{\frac{2}{3}}\right) = \frac{2}{3} \arg(z) = \frac{2}{3} \arg(-1) = \frac{2}{3} \pi$$

Q.63 [B]

$$z = x + iy$$

$$\Rightarrow \arg(z - 1) = \arg(z + 3i)$$

$$\Rightarrow \tan^{-1} \frac{y}{x-1} = \tan^{-1} \frac{y+3}{x}$$

$$\Rightarrow \therefore \frac{y}{x-1} = \frac{y+3}{x}$$

$$\Rightarrow xy = xy + 3x - y - 3$$

$$\Rightarrow \frac{x-1}{y} = \frac{1}{3}$$

Q.64 [C]

$$(1+i)^n + (1-i)^n$$

$$\Rightarrow \left[\sqrt{2} \left(\cos \frac{\pi}{4} + i \sin \frac{\pi}{4} \right) \right]^n + \left[\sqrt{2} \left(\cos \left(\frac{\pi}{4} \right) + i \sin \left(-\frac{\pi}{4} \right) \right) \right]^n$$

$$\Rightarrow 2^{\frac{n}{2}} \left[2 \cos^n \frac{\pi}{4} \right]$$

$$\Rightarrow (\sqrt{2})^{n+2} \cos \left(\frac{n\pi}{4} \right)$$

Q.65 [A]

$$y = \cos \theta + i \sin \theta$$

$$\Rightarrow \frac{1}{y} = \bar{y} = \cos \theta - i \sin \theta$$

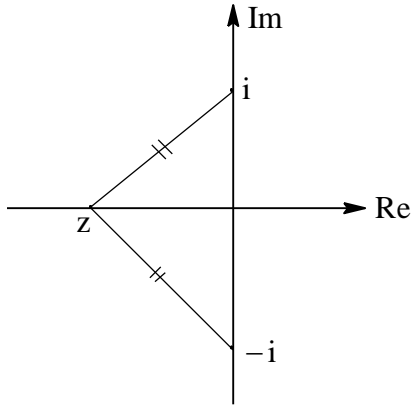
$$\Rightarrow y + \frac{1}{y} = 2 \cos \theta$$

Complex Number

Exercise – 2

Q.1 [B]

$$w = \frac{1-iz}{z-i} = \frac{-i(z+i)}{(z-i)}$$



$$|w| = \frac{|z+i|}{|z-i|} = 1$$

$$|z+i| = |z-i|$$

z lies on real axis.

Q.2 [C]

$$|z| = |z_1 - z_2| = |-3-i| = 5$$

Q.3 [B]

$$\bar{z}z + a\bar{z} + \bar{a}z + b = 0; b \in \mathbb{R}$$

$$\text{radius of the circle} = |a|^2 - b > 0$$

$$\therefore |a|^2 > b$$

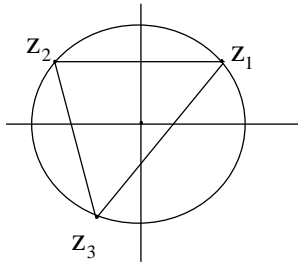
Q.4

$$|z_1| = |z_2| = |z_3| = r \text{ (let's take)}$$

Let $z_1 = re^{i0}$

$$z_2 = re^{i\left(\theta + \frac{2\pi}{3}\right)}$$

$$z_3 = re^{i\left(\theta - \frac{2\pi}{3}\right)}$$



$$\therefore z_1 + z_2 + z_3 = r(0) = 0$$

Q.5 [D]

$$G\left(\frac{z_1 + z_2 + z_3}{3}\right) \quad A(z_1)$$

$$\therefore \text{mid point of AG } z = \frac{\frac{z_1 + z_2 + z_3}{3} + z_1}{2} = 0$$

$$\therefore 4z_1 + z_2 + z_3 = 0$$

Q.6 [B]

$$|z_1| = 12, |z_2 - 3 - 4i| = 5$$

$$|z_1 - z_2| = |z_1 + (-z_2 + 3 + 4i) + (-3 - 4i)|$$

$$\left| |z_1| - |z_2 - 3 - 4i| \right| \leq |z_1 + (-z_2 + 3 + 4i)| \leq |z_1 - z_2| + |3 + 4i|$$

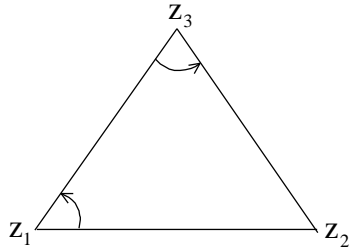
$$7 \leq |z_1 - z_2| + 5$$

$$\therefore |z_1 - z_2| \geq 2$$

$$|z_1 - z_2|_{\min} = 2$$

Q.7 [B]

For



$$\frac{z_3 - z_1}{z_2 - z_1} = e^{\frac{i\pi}{3}} = \frac{z_2 - z_3}{z_1 - z_3}$$

$$-(z_1 - z_3)^2 = (z_2 - z_3)(z_2 - z_1)$$

$$-(z_1^2 + z_3^2 - 2z_1z_3) = z_2^2 - z_1z_2 - z_2z_3 + z_1z_3$$

$$z_1^2 + z_2^2 + z_3^2 = z_1z_2 + z_2z_3 + z_3z_1$$

Q.8 [C]Let $\arg(z) = \theta$

$$\text{Then } \arg(-iz) = \arg(-i) + \arg(z) = \frac{-\pi}{2} + \theta$$

$$\therefore \arg(z) - \arg(-iz) = \frac{\pi}{2}$$

Q.9 [C]

$$\operatorname{Re}\left(\frac{z+4}{2z-i}\right) = \frac{1}{2}$$

$$\frac{z+4}{2z-i} + \frac{\bar{z}+4}{2\bar{z}+i} = 1$$

$$(z+4)(2\bar{z}+i) + (\bar{z}+4)(2z-i) = (2z-i)(2\bar{z}+i)$$

$$2|z|^2 + iz + 8\bar{z} + 4i + 2|z|^2 - i\bar{z} + 8z - 4i = 4|z|^2 + 2zi - 2i\bar{z} + 1$$

$$zi - i\bar{z} - 8\bar{z} - 8z + 4i + 1 = 0$$

$$z(i-8) - \bar{z}(8+i) + 4i + 1 = 0$$

$$z(8-i) + \bar{z}(8+i) - 4i - 1 = 0$$

This is equation of straight line

Q.10 [C]

$$\frac{z_1}{z_2} + \frac{z_2}{z_1} = 1$$

$$z_1^2 + z_2^2 = z_1 z_2$$

For equilateral triangle with vertices z_1, z_2, z_3

$$z_1^2 + z_2^2 + z_3^2 = z_1 z_2 + z_2 z_3 + z_3 z_1$$

if $z_3 = 0$

$$z_1^2 + z_2^2 = z_1 z_2$$

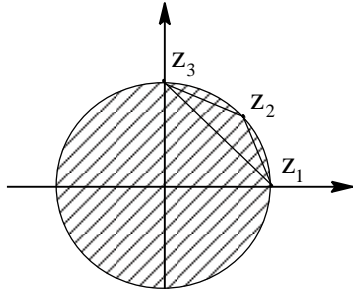
Q.11

$$z_1 = 1, z_2 = \frac{1+i}{\sqrt{2}}, z_3 = i$$

$$|z_1 - z_3| = \sqrt{2}$$

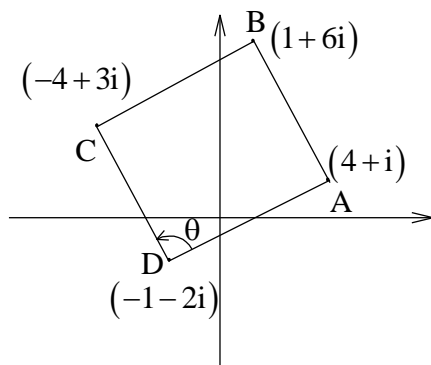
$$|z_2 - z_3| = \left| \frac{1+i}{\sqrt{2}} - i \right| = \left| \frac{1}{\sqrt{2}} + i \left(\frac{1}{\sqrt{2}} - 1 \right) \right| = \sqrt{\frac{1}{2} + \frac{1}{2} + 1 - \sqrt{2}} = \sqrt{2 - \sqrt{2}}$$

$$|z_1 - z_2| = \sqrt{\left(1 - \frac{1}{\sqrt{2}} \right)^2 + \frac{1}{2}} = \sqrt{2 - \sqrt{2}}$$



Triangle is isosceles triangle

Q.12 [B]



$$|AB| = |BC| = |CD| = |DA|$$

$$\frac{Z_C - Z_D}{Z_A - Z_D} = \frac{-3 + 5i}{5 + 3i} = i = e^{i\frac{\pi}{2}}$$

$$\therefore \theta = \frac{\pi}{2}$$

\therefore ABCD is square

Q.13 [B]

90°

Q.14 [C]

By triangle inequality

$$\left| |z_1| - |z_2| \right| \leq |z_1 + z_2| \leq |z_1| + |z_2|$$

Q.15

Q.16

$(z_1 - z_2) = \lambda(z_2 - z_3)$ for collinear

$$(3 - 2i) = \lambda \left(-2 + i \left(3 - \frac{a}{3} \right) \right)$$

$$3 = -2\lambda$$

$$-2 = \lambda \left(3 - \frac{a}{3} \right)$$

$$-2 = \frac{-3}{2} \left(3 - \frac{a}{3} \right)$$

$$a = 5$$

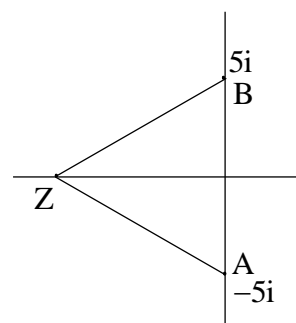
Q.17 [B]

$$2z_1 - 3z_2 + z_3 = 0$$

$$\frac{1:2}{z_1 \quad z_2 \quad z_3}$$

$$z_2 = \frac{2z_1 + z_3}{2+1}$$

Collinear points.

Q.18 [A]

$$|Z - 5i| = |Z + 5i|$$

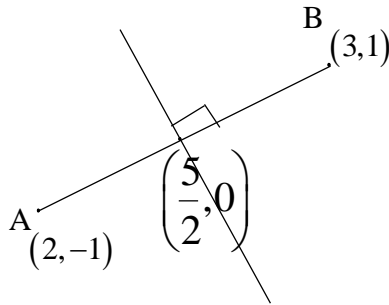
So locus of z will perpendicular bisector of AB or z lies on real axis.

$$\therefore x = 0$$

Q.19 [A]

$$|Z-(2-i)|=|Z-(3+i)|$$

Locus of z will be perpendicular bisector of line segment AB. A(2, -1), B (3, 1)



$$m_{AB} = \frac{1+1}{1} = 2$$

∴ locus of z is

$$y-0 = -\frac{1}{2}\left(x-\frac{5}{2}\right)$$

$$x+2y = \frac{5}{2}$$

Q.20 [D]

$$\arg(Z-2-3i) = \frac{\pi}{4}$$

$$\arg(x-2) + i(y-3) = \frac{\pi}{4}$$

$$\tan^{-1}\left(\frac{y-3}{x-2}\right) = \frac{\pi}{4}$$

$$y-3 = x-2$$

$$x-y+1 = 0$$

Q.21

$$\arg\left(\frac{z-2}{z+2}\right) = \frac{\pi}{6}$$

$$\arg(z-2) - \arg(z+2) = \frac{\pi}{6}$$

$$\tan^{-1}\left(\frac{y}{x-2}\right) - \tan^{-1}\left(\frac{y}{x+2}\right) = \frac{\pi}{6}$$

$$\tan^{-1}\left[\frac{\left(\frac{y}{x-2} - \frac{y}{x+2}\right)}{1 + \left(\frac{y^2}{x^2-4}\right)}\right] = \frac{\pi}{6}$$

$$\frac{yx + 2y - xy + 2y}{x^2 - 4 + y^2} = \sqrt{3}$$

$$x^2 + y^2 = 4 + \frac{4y}{\sqrt{3}}$$

$$x^2 + y^2 - \frac{4y}{\sqrt{3}} - 4 = 0$$

$$z = \lambda + 3 + i\sqrt{5 - \lambda^2}$$

It is a circle.

Q.22

$$x = \lambda + 3 \text{ \& } y = \sqrt{5 - \lambda^2}$$

$$x = \lambda + 3 \text{ \& } y = \sqrt{5 - \lambda^2}$$

$$y^2 = 5 - (x-3)^2$$

$$(x-3)^2 + y^2 = 5$$

Q.23 Repeated Question (21) of Ex. (2)

Q.24 [D]

$$|z-2| = 2|z-3|$$

$$(x-2)^2 + y^2 = 4((x-3)^2 + (y^2))$$

$$3x^2 + 3y^2 - 20x + 32 = 0$$

$$x^2 + y^2 - \frac{20}{3}x + \frac{32}{3} = 0$$

$$r = \sqrt{\left(\frac{10}{3}\right)^2 - \frac{32}{3}} = \sqrt{\frac{4}{9}} = \frac{2}{3}$$

Q.25 [A]

$$i^{\frac{1}{3}} = \left(e^{i\frac{\pi}{2}}\right)^{\frac{1}{3}} = e^{i\frac{\pi}{6}}$$

$$\cos \frac{\pi}{6} + i \sin \frac{\pi}{6}$$

$$\frac{\sqrt{3} + i}{2}$$

Q.26 [D]

$$(-1 + i\sqrt{3})^{20} = \left[2 \left(e^{i\frac{2\pi}{3}}\right)\right]^{20}$$

$$2^{20} e^{i\frac{40\pi}{3}}$$

$$1^{20} e^{i\frac{4\pi}{3}} = 2^{20} \left(\frac{-1 - i\sqrt{3}}{2}\right)$$

Q.27 [A]

$$z = \frac{\sqrt{3} + i}{2} = -i \left(\frac{-1 + i\sqrt{3}}{2}\right)$$

$$z = -i\omega$$

$$z^{69} = (-i)^{69} \omega^{69} = -i$$

Q.28 [C]

$$(\sin \theta + i \cos \theta)^n = \left[\cos \left(\frac{\pi}{2} - \theta \right) + i \sin \left(\frac{\pi}{2} - \theta \right) \right]^n$$

$$\cos \left(\frac{n\pi}{2} - n\theta \right) + i \sin \left(\frac{n\pi}{2} - n\theta \right)$$

Q.29

$$z = \left(\frac{\cos \pi}{3} + i \sin \frac{\pi}{3} \right)^{\frac{3}{4}} = (\cos \pi + i \sin \pi)^{\frac{1}{4}}$$

$$(-1)^{\frac{1}{4}}$$

$$z^4 = -1$$

$$z = \left(\cos \left(\frac{2k\pi + \pi}{4} \right) + i \sin \left(\frac{2k\pi + \pi}{4} \right) \right)$$

$$z_1 = e^{i\frac{\pi}{4}}$$

$$z_2 = e^{i\frac{3\pi}{4}}$$

$$z_3 = e^{i\frac{5\pi}{4}}$$

$$z_4 = e^{i\frac{7\pi}{4}}$$

$$\text{Product of roots} = e^{i\left(\frac{\pi}{4} + \frac{3\pi}{4} + \frac{5\pi}{4} + \frac{7\pi}{4}\right)}$$

$$e^{i(4\pi)} = 1$$

Q.30 [D]

$$\left(\frac{1 + \cos \theta + i \sin \theta}{1 + \sin \theta + i \cos \theta} \right)^n = \cos n\theta + i \sin n\theta$$

$$\left[\frac{2 \cos \frac{\theta}{2} \left(e^{i \frac{\theta}{2}} \right)}{2 \cos \frac{\theta}{2} \left(\sin \frac{\theta}{2} + i \cos \frac{\theta}{2} \right)} \right]^n$$

$$\left[\frac{e^{i \frac{\theta}{2}}}{i e^{i \frac{\theta}{2}}} \right]^n = \left[-i e^{i \theta} \right]^n$$

$$(-i)^n (\cos n\theta + i \sin \theta)$$

So, $n = 4$

Q.31 [C]

$$\left[\frac{1 - \cos \frac{\pi}{10} + i \sin \frac{\pi}{10}}{1 - \cos \frac{\pi}{10} - i \sin \frac{\pi}{10}} \right]^{10} = \left[\frac{2 \sin \frac{\pi}{10} \left(\sin \frac{\pi}{20} + i \cos \frac{\pi}{20} \right)}{\left(2 \sin \frac{\pi}{20} \right) \left(\sin \frac{\pi}{20} - i \cos \frac{\pi}{20} \right)} \right]^{20}$$

$$\left[\frac{i e^{-i \frac{\pi}{20}}}{-i e^{i \frac{\pi}{20}}} \right]^{20} = \left[-e^{-i \frac{\pi}{10}} \right]^{20} = e^{-i 2\pi}$$

1

Q.32 [D]

$$\sum_{k=1}^6 \left(\sin \frac{2\pi k}{7} - i \cos \left(\frac{2\pi k}{7} \right) \right)$$

$$\sum_{k=1}^6 -i e^{i \frac{2\pi k}{7}} \quad \text{Let } \alpha = e^{i \frac{2\pi}{7}}$$

$$-i [\alpha + \alpha^2 + \alpha^3 + \alpha^4 + \alpha^5 + \alpha^6]$$

$$-i [-1] = i$$

Q.33 [B]

$$\cos\left(\theta + \frac{\theta}{2} + \frac{\theta}{2^2} + \frac{\theta}{2^3} + \dots\right) + i \sin\left(\theta + \frac{\theta}{2} + \frac{\theta}{2^2} + \frac{\theta}{2^3} + \dots\right)$$

$$\cos(2\theta) + i \sin 2\theta$$

Q.34 [C]

$$z^3 = -1$$

$$z = -1, -\omega, -\omega^2$$

where ω is cube root of unity.

$$z_1 z_2 z_3 = (-1)(-\omega)(-\omega^2) = -1$$

Q.35

$$\left(\frac{1+i\sqrt{3}}{1-i\sqrt{3}}\right) = \left[\frac{2\left(e^{i\frac{\pi}{3}}\right)}{2e^{-i\frac{\pi}{3}}}\right]^n = \left[e^{i\frac{2\pi}{3}}\right]^n$$

$$e^{i\frac{2\pi n}{3}}$$

it will be integer if $n = 3$.

Q.36 [C]

$$(1 + \omega)^7 = A + B\omega$$

$$(-\omega^2)^7 = A + B\omega$$

$$(-\omega^2) = A + B\omega$$

$$1 + \omega = A + B\omega$$

$$A = 1 = B$$

Q.37 [C]

$$(3\omega + 3\omega^2)^4 = (-3\omega + \omega)^4 = (-2\omega)^4 = 16\omega$$

Q.38 [A]

$$(3 + \omega^2 + \omega^4)^6 = (3 + \omega^2 + \omega)^6 = (2 + 0)^6 = 64$$

Q.39 [C]

$$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1 + \omega + \omega^2 = 0$$

Q.40 [B]

Given question is wrong. Actual question is

$$(z+1)^3 = 8(z-1)^3$$

$$\left(\frac{z+1}{z-1}\right) = (8)^{\frac{1}{3}}$$

$$\frac{z+1}{z-1} = 2, 2\omega, 2\omega^2$$

$$z = 3, \frac{2\omega+1}{2\omega-1}, \frac{2\omega^2+1}{2\omega^2-1}$$

$$\therefore z_1 + z_2 + z_3 = \frac{27}{7}$$

$$\operatorname{Re}(z_1 + z_2 + z_3) = \frac{27}{7}$$

COMPLEX NUMBERS

Ex. 3

Q.1 (c)

$$x = 9^{\frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \dots \infty \text{ terms}} \Rightarrow x = 9^{\frac{1/3}{1-1/3}} \text{ or } x = 3.$$

$$y = 4^{\frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \dots \infty \text{ terms}} \Rightarrow y = 4^{\frac{1/3}{1-1/3}} \text{ or } y = \sqrt{2}.$$

$$z = \sum_{r=1}^{\infty} \frac{1}{(1+i)^r} \Rightarrow z = \frac{1/(1+i)}{1-1/(1+i)} \text{ or } z = -i.$$

$$\text{Now } x + yz = 3 - \sqrt{2}i \Rightarrow \arg(x + yz) = -\tan^{-1} \frac{\sqrt{2}}{3}.$$

Q.2 (c)

$$\bar{Z} + i\bar{W} = 0 \Rightarrow Z - iW = 0 \text{ or } \frac{Z}{W} = i.$$

$$\text{Now } \arg\left(\frac{Z}{W}\right) = \frac{\pi}{2} \Rightarrow \arg(ZW) + \arg\left(\frac{Z}{W}\right) = \frac{3\pi}{2}$$

$$\Rightarrow \arg(Z^2) = \frac{3\pi}{2} \text{ or } \arg(Z) = \frac{3\pi}{4}.$$

Q.3 (d)

$$\text{Let } P(x) = x^6 + 4x^5 + 3x^4 + 2x^3 + x + 1$$

$$\text{or } P(x) = x^6 - x^3 + (4x^3 - x^2 + 1)(x^2 + x + 1).$$

$$\text{Now for } x = \omega \text{ \& } \omega^2, x^6 - x^3 = x^2 + x + 1 = 0, \text{ hence } P(\omega) = 0 = P(\omega^2).$$

$$P(x) \text{ is divisible by } (x - \omega)(x - \omega^2).$$

Q.4 (c)

$$\cos r\theta + i \sin r\theta = e^{ir\theta} \Rightarrow (\cos \theta + i \sin \theta)(\cos 2\theta + i \sin 2\theta) \dots (\cos n\theta + i \sin n\theta) = \sum_{r=1}^n e^{ir\theta}$$

$$\text{Hence } e^{i \frac{n(n+1)}{2} \theta} = 1 \text{ or } \cos \frac{n(n+1)}{2} \theta + i \sin \frac{n(n+1)}{2} \theta = 1.$$

$$\Rightarrow \frac{n(n+1)}{2} \theta = 2m\pi \text{ or } \theta = \frac{4m\pi}{n(n+1)}.$$

Q.5 (c)

Let $Z = \cos \theta + i \sin \theta$.

$$\left| \frac{Z}{\bar{Z}} + \frac{\bar{Z}}{Z} \right| = 1 \Rightarrow \left| Z^2 + \bar{Z}^2 \right| = 1 \text{ or } 2|\cos 2\theta| = 1.$$

$$\text{Now } \cos 2\theta = \pm \frac{1}{2} \Rightarrow \theta = \frac{n\pi}{2} \pm \frac{\pi}{6}.$$

$$\text{As } \theta \in (0, 2\pi), \text{ therefore } \theta = \frac{\pi}{6}, \frac{\pi}{3}, \frac{2\pi}{3}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{4\pi}{3}, \frac{5\pi}{3}, \frac{11\pi}{6}.$$

Q.6 (A)

$$\mu^2 - 2\mu + 2 = 0 \Rightarrow (\mu - 1)^2 = -1 \text{ or } \mu - 1 = \pm i, \text{ hence } \alpha - 1 = i \text{ \& } \beta - 1 = -i.$$

$$\text{Now } (x + \mu)^n = (\cot \theta + \mu - 1)^n$$

$$\frac{(x + \alpha)^n - (x + \beta)^n}{\alpha - \beta} = \frac{(\cot \theta + i)^n - (\cot \theta - i)^n}{2i}$$

$$\Rightarrow \frac{(x + \alpha)^n - (x + \beta)^n}{\alpha - \beta} = \frac{(\cos \theta + i \sin \theta)^n - (\cos \theta - i \sin \theta)^n}{2i \sin^n \theta}$$

$$\text{or } \frac{(x + \alpha)^n - (x + \beta)^n}{\alpha - \beta} = \frac{\sin n\theta}{\sin^n \theta}.$$

Q.7 (d)

$$|Z - 4| = \text{Re}(Z) \Rightarrow (x - 4)^2 + y^2 = x^2 \text{ or } y^2 = 8(x - 2).$$

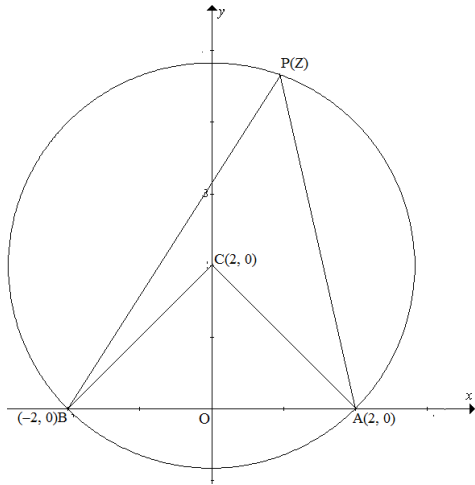
Now greatest positive $\arg(Z)$ will be greatest slope angle of tangent from origin to this parabola.

$$\text{Equation of any tangent of slope } m \text{ will be } y = m(x - 2) + \frac{2}{m}.$$

As it has to be drawn from $(0, 0)$, hence $m = 1$.

$$\therefore \text{Greatest positive } \arg(Z) = \frac{\pi}{4}.$$

Q.8 (b)



As shown in figure

both $A(2, 0)$ & $B(-2, 0)$ lie on the circle

$$|Z - 2i| = 2\sqrt{2}.$$

Center of the circle is $C(0, 2)$ & radius is $2\sqrt{2}$.

Now $\angle APB = \angle ACO$.

$$\text{Hence } \arg\left(\frac{Z-2}{Z+2}\right) = \frac{\pi}{4}.$$

Q.9 (b)

$$S = 1 + 2\alpha + 3\alpha^2 + 4\alpha^3 + \dots + n\alpha^{n-1} \dots (i)$$

Multiply throughout by α to get

$$\alpha S = \alpha + 2\alpha^2 + 2\alpha^3 + \dots + (n-1)\alpha^{n-1} + n\alpha^n \dots (ii)$$

Subtract (ii) from (i) to get

$$(1-\alpha)S = 1 + \alpha + \alpha^2 + \alpha^3 + \dots + \alpha^{n-1} - n\alpha^n$$

$$\text{Now } 1 + \alpha + \alpha^2 + \alpha^3 + \dots + \alpha^{n-1} = 0 \Rightarrow S = \frac{-n}{1-\alpha}.$$

Q.10 (c)

$$\text{Let } Z = a + ib \text{ \& } \frac{2}{Z} = x + iy, \text{ then } \frac{2(a-ib)}{a^2+b^2} = x + iy.$$

$$\text{As } a^2 + b^2 = 1, \text{ thus } x = 2a \text{ \& } y = -2b \text{ or } x^2 + y^2 = 4.$$

Required locus is a circle of radius 2.

Q.11 (a)

$$\angle AOB = \frac{\pi}{2} \text{ \& } OA = OB \Rightarrow Z_2 = iZ_1. \text{ Hence } \frac{Z_2}{Z_1} \text{ is purely imaginary.}$$

Q.12 (c)

$$\beta + \gamma = \alpha + \alpha^2 + \dots + \alpha^6 = -1 \text{ \&}$$

$$\beta \times \gamma = \left(\cos \frac{2\pi}{7} + \cos \frac{4\pi}{7} + \cos \frac{8\pi}{7} \right) \left(\cos \frac{6\pi}{7} + \cos \frac{10\pi}{7} + \cos \frac{12\pi}{7} \right) = 2$$

hence required equation is $Z^2 + Z + 2 = 0$.

Q.13 (c)

$$\text{Let } Z_1 = e^{i\alpha}, Z_2 = e^{i\beta} \text{ \& } Z_3 = e^{i\gamma}.$$

$$\text{Now } \cos \alpha + 2 \cos \beta + 3 \cos \gamma = \sin \alpha + 2 \sin \beta + 3 \sin \gamma = 0 \Rightarrow Z_1 + 2Z_2 + 3Z_3 = 0$$

$$\Rightarrow Z_1^3 + 8Z_2^3 + 27Z_3^3 = 18Z_1Z_2Z_3$$

$$\therefore \sin 3\alpha + 8 \sin 3\beta + 27 \sin 3\gamma = 18 \sin(\alpha + \beta + \gamma).$$

Q.14 (A)

$$\text{Let } Z = k(\cos A + i \sin A) \text{ \& } W = k(\cos B + i \sin B)$$

$$\text{Now } \alpha = \frac{Z - \bar{W}}{k^2 + Z\bar{W}} \Rightarrow \alpha = \frac{(\cos A - \cos B) + i(\sin A + \sin B)}{k + k(\cos A + i \sin A)(\cos B - i \sin B)}$$

$$\text{or } \alpha = \frac{(\cos A - \cos B) + i(\sin A + \sin B)}{k \{1 + \cos(A - B) + i \sin(A - B)\}}$$

$$\Rightarrow \alpha = \frac{\sin \frac{A+B}{2} \left(\sin \frac{B-A}{2} + i \cos \frac{A-B}{2} \right)}{k \cos \frac{A-B}{2} \left(\cos \frac{A-B}{2} + i \sin \frac{A-B}{2} \right)}$$

$$\Rightarrow \alpha = \frac{\sin \frac{A+B}{2} \left(-\sin \frac{A-B}{2} + i \cos \frac{A-B}{2} \right)}{k \cos \frac{A-B}{2} \left(\cos \frac{A-B}{2} + i \sin \frac{A-B}{2} \right)} \times \frac{\cos \frac{A-B}{2} - i \sin \frac{A-B}{2}}{\cos \frac{A-B}{2} - i \sin \frac{A-B}{2}}$$

$$\Rightarrow \alpha = \frac{i \sin \frac{A+B}{2}}{k \cos \frac{A-B}{2}}.$$

Hence $\text{Re}(Z) = 0$.

Q.15 (c)

$$|Z^2 + k| + k = |Z^2| \Rightarrow |Z^2 + k| + k = |Z^2 + k - k|$$

Hence $\arg(Z^2) = -\arg(k)$ or $\arg(Z^2) = \pi$.

$$\therefore \arg(Z) = \frac{\pi}{2}.$$

Q.16 (b)

Let $f(Z) = (Z^2 + 1)Q(Z) + aZ + b$,

where $Q(Z)$ is the quotient when $f(Z)$ is divided by $Z^2 + 1$

Now $f(i) = i$ & $f(-i) = 1 + i$, hence

$$ai + b = i \quad \& \quad -ai + b = 1 + i.$$

Solving these equations simultaneously gives

$$b = \frac{1+2i}{2} \quad \& \quad a = \frac{i}{2}.$$

\therefore remainder when $f(Z)$ is divided by $Z^2 + 1$ is $\frac{1+2i}{2} + \frac{iZ}{2}$.

Q.17 (a)

$$a|Z_1| = b|Z_2| \Rightarrow \left| \frac{Z_1}{Z_2} \right| = \frac{b}{a} \quad \text{or} \quad \frac{aZ_1}{bZ_2} = e^{i\theta} \quad \& \quad \frac{aZ_2}{bZ_1} = e^{-i\theta}.$$

$$\text{Hence} \quad \frac{aZ_1}{bZ_2} + \frac{aZ_2}{bZ_1} = 2 \cos \theta.$$

$\left(\frac{aZ_1}{bZ_2}, \frac{aZ_2}{bZ_1} \right)$ lies on real axis between $(-2, 0)$ & $(2, 0)$.

Q.18 (c)

For n^{th} roots of unity

$$1 + \omega + \omega^2 + \omega^3 + \dots + \omega^{n-1} = 0$$

Also let $S = 1 + 2\omega + 3\omega^2 + \dots + n\omega^{n-1}$, then

$$\omega S = \omega + 2\omega^2 + \dots + (n-1)\omega^{n-1} + n\omega^n$$

From the above two relations we get $S = \frac{n}{\omega - 1}$

$$\text{Now} \quad \sum_{r=1}^n (ar + b)\omega^{r-1} = a \sum_{r=1}^n r \omega^{r-1} + b \sum_{r=1}^n \omega^{r-1}$$

$$\text{Or} \quad \sum_{r=1}^n (ar + b)\omega^{r-1} = \frac{an}{\omega - 1}.$$

Q.19 (b)

$$(Z + ab)^3 = a^3 \Rightarrow Z = a - ab, a\omega - ab \quad \& \quad a\omega^2 - ab.$$

Now side length $|(a - ab) - (a\omega - ab)| = |a(1 - \omega)|$ i.e. $\sqrt{3}|a|$.

Q.20 (d)

$$|\omega Z - 1 - \omega^2| = a \Rightarrow |Z + 1| = a.$$

Given $|Z + 1| = a$ & $|Z - 1| \leq 2$.

Now $||Z + 1| - 2| \leq |Z - 1| \Rightarrow |a - 2| \leq 2$ or $0 \leq a \leq 4$.

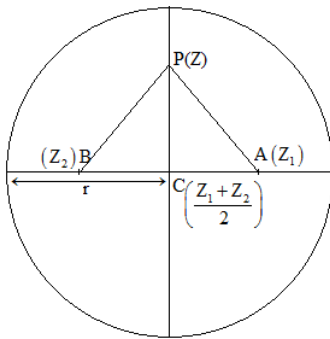
Q.21 (a)

$$|Z^2 + 2Z \cos \alpha| \leq |Z|^2 + 2|Z| |\cos \alpha|$$

Now $|Z| < \sqrt{2} - 1$ & $\cos \alpha \leq 1$, hence $|Z^2 + 2Z \cos \alpha| < (\sqrt{2} - 1)^2 + 2(\sqrt{2} - 1)$.

Or $|Z^2 + 2Z \cos \alpha| < 1$.

Q.22 (b)



Consider a circle having center at $C\left(\frac{Z_1 + Z_2}{2}\right)$ and radius r .

Now $A(Z_1)$ & $B(Z_2)$ will be two points on a diameter such that

$$AC = BC.$$

Also $P(Z)$ will be a point on the perpendicular diameter as given $PA = PB$.

Clearly area will be maximum when $CP = r$.

$$\text{Hence max. area} = \frac{1}{2}|Z_1 - Z_2|r.$$

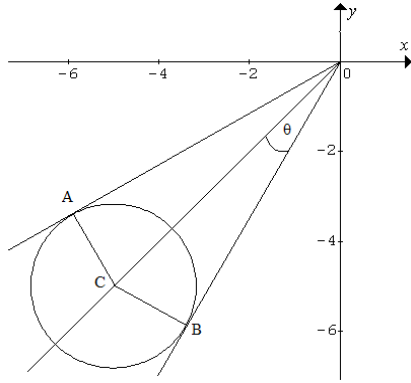
Q.23 (d)

$$|Z_2 + iZ_1| = |Z_2| + |iZ_1| \Rightarrow \arg(Z_2) = \arg(iZ_1) \text{ or } \arg(Z_2) - \arg(Z_1) = \frac{\pi}{2}.$$

Let $Z_1 = 3$ & $Z_2 = 4i$, then $\frac{Z_2 - iZ_1}{1 - i} = \frac{i(1 + i)}{2}$ or $\frac{-1 + i}{2}$

$$\text{Area} = \frac{1}{2} \times \begin{vmatrix} 1 & 3 & 0 \\ 1 & 0 & 4 \\ 1 & -\frac{1}{2} & \frac{1}{2} \end{vmatrix} = \frac{25}{4}.$$

Q.24 (a)



$|Z+5+5i| \leq \frac{5\sqrt{3}-5}{2}$ represents a circle with center at $(-5, -5)$ and radius $\frac{5\sqrt{3}-5}{2}$.

Now $OC = 5\sqrt{2}$ & $BC = \frac{5\sqrt{3}-5}{2}$, thus

$$\sin \theta = \frac{\sqrt{3}-1}{2\sqrt{2}} \text{ or } \theta = \frac{\pi}{12}.$$

Now angle made by OC with positive real axis is $\frac{5\pi}{4}$,

therefore angle made by OB & OC with positive real axis are $\frac{4\pi}{3}$ & $\frac{7\pi}{6}$.

Hence least $\arg(Z) = -\frac{5\pi}{6}$.

Q.25 (a)

Case I: $|Z-1| < |Z+1|$ & $|Z| = |Z-1|$

Case II: $|Z-1| > |Z+1|$ & $|Z| = |Z+1|$

$$\Rightarrow x > 0, \text{ then } x = \frac{1}{2} \text{ \& } x < 0, \text{ then } x = -\frac{1}{2}.$$

Now $Z + \bar{Z} = 2\text{Re}(Z)$, thus $Z + \bar{Z} = 1$ or -1 .

Q.26 (b)

$$\arg \left(\frac{Z_1 - \frac{Z}{|Z|}}{\frac{Z}{|Z|}} \right) = \frac{\pi}{2} \Rightarrow \frac{Z_1 - \frac{Z}{|Z|}}{\frac{Z}{|Z|}} = \left| Z_1 - \frac{Z}{|Z|} \right| e^{-\frac{\pi}{2}}$$

$$\Rightarrow Z_1 - \frac{Z}{|Z|} = 3i \frac{Z}{|Z|} \text{ or } Z_1 = (3i+1) \frac{Z}{|Z|}.$$

Hence $|Z_1| = \sqrt{10}$.

Q.27 (b)

The required complex vector will be $\frac{\lambda}{2} \left(\frac{Z_1}{|Z_1|} + \frac{Z_2}{|Z_2|} \right)$ i.e. $\frac{\lambda}{2} \left(\frac{3+\sqrt{3}i}{2\sqrt{3}} + \frac{2\sqrt{3}+6i}{4\sqrt{3}} \right)$.

Hence any complex number of form $\mu(1+i)$ will lie along the angle bisector.

Q.28 (a)

$$|Z - 2 + 2i| \leq ||Z| - |2 - 2i|| \Rightarrow -1 \leq |Z| - 2\sqrt{2} \leq 1.$$

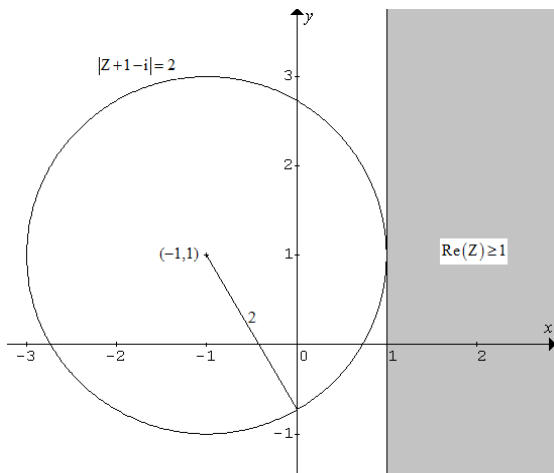
Hence least value of $|Z|$ is $2\sqrt{2} - 1$.

Also $\arg(Z) = \arg(2 - 2i)$.

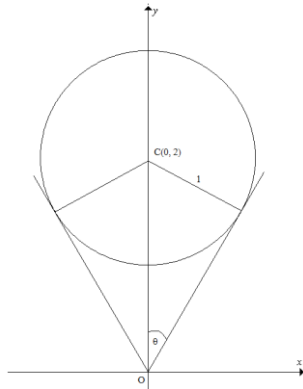
$$\therefore Z = \frac{2\sqrt{2}-1}{\sqrt{2}}(1-i).$$

Q.29 (b)

Refer the adjoining figure.



Q.30 (a)

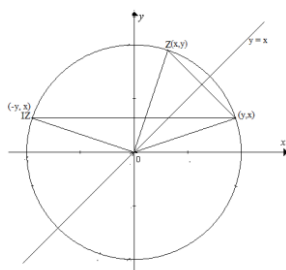


As shown in figure range of $\arg(Z)$ will be

from $\frac{\pi}{2} - \theta$ to $\frac{\pi}{2} + \theta$, where $\sin \theta = \frac{1}{2}$ i.e. $\theta = \frac{\pi}{6}$.

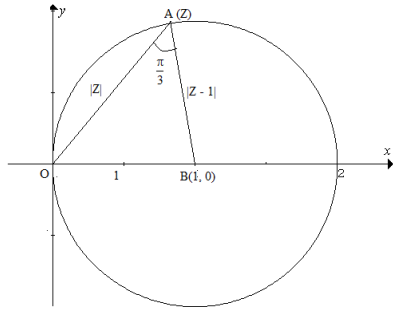
$$\text{Hence } \text{Arg}(\alpha) \in \left[\frac{\pi}{3}, \frac{2\pi}{3} \right].$$

Q.31 (b)



Rotation of $Z(x + iy)$ about the origin gives $iZ(-y + ix)$.
Then reflection in Imaginary-Axis gives $(y + ix)$,
which is equivalent to reflection of Z in the line $x = y$.
Hence T_1 is equivalent to composite of T_2 & T_3 .

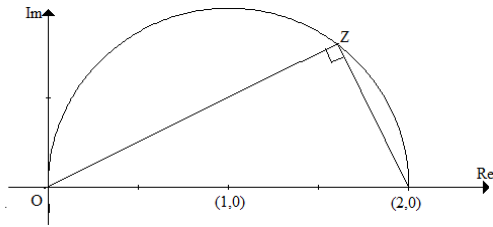
Q.32 (d)



Refer the adjoining figure.
By cosine formula,

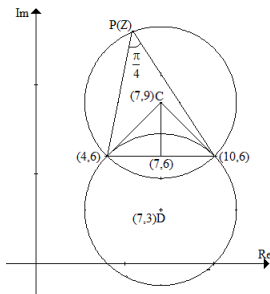
$$\cos \frac{\pi}{3} = \frac{|Z|^2 + 1 - 1}{2|Z|} \Rightarrow |Z|^2 = |Z| \text{ or } |Z| = 1.$$

Q.33 (c)



Refer the adjoining figure.

Q.34 (c)



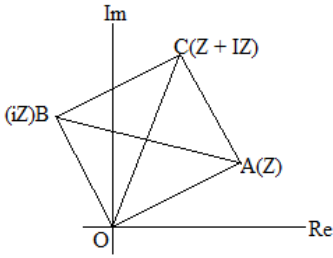
Angles in same segment of a circle are equal, hence Z will move on major arc of two circles passing through (4, 6) & (10, 6) and of radius $3\sqrt{2}$ as shown in adjoining figure.

Q.35 (d)

$\cos \frac{2k\pi}{11} + i \sin \frac{2k\pi}{12}$ is 12th root of unity for $k = 0, 1, 2, \dots, 11$.

$$\text{Now } \sum_{k=0}^{11} \left(\cos \frac{2k\pi}{n} + i \sin \frac{2k\pi}{n} \right) = 0 \quad \& \quad \sum_{k=0}^{11} \left(\sin \frac{2k\pi}{n} - i \cos \frac{2k\pi}{n} \right) = -i \sum_{k=0}^{11} \left(\cos \frac{2k\pi}{n} + i \sin \frac{2k\pi}{n} \right),$$

$$\text{hence } \sum_{k=1}^{11} \left(\sin \frac{2k\pi}{n} - i \cos \frac{2k\pi}{n} \right) = i.$$

Q.36 (d)

$A(Z)$ & $B(iZ)$ are such that $OA \perp OB$.

Also $C(Z+iZ)$ will be such that OC is diagonal of Square $OACB$ as shown in adjoining figure.

Hence required area is $\frac{1}{2}|Z|^2$.

Q.37 (c)

$$\text{Let } P(Z) = (Z-1-i)(Z-1+i)Q(Z) + aZ + b$$

$$\text{Now } P(1+i) = 3+4i \Rightarrow (1+i)a + b = 3+4i \dots (i)$$

$$\& P(1-i) = 3-4i \Rightarrow (1-i)a + b = 3-4i \dots (ii)$$

From (i) & (ii)

$$a = \left(\frac{7+i}{2}\right), b = 0.$$

Q.38 (a)

Note that triangle AOB is right angled isosceles triangle, hence C will be midpoint of AB .

Q.39 (b)

$$(a+ib)^n = (a-ib)^n \Rightarrow e^{i(n\theta)} = e^{-i(n\theta)}, \text{ where } \theta = \tan^{-1} \frac{b}{a}$$

$$\Rightarrow e^{i(2n\theta)} = 1 \Rightarrow \tan^{-1} \frac{b}{a} = \frac{\pi}{n} \Rightarrow \frac{b}{a} = \tan \frac{\pi}{n}$$

Clearly least positive integral value of n is 3 such that $\frac{b}{a}$ is defined and not zero.

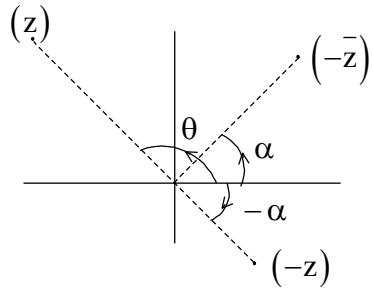
Q.40 (a)

$$|2Z_1 + Z_2| \leq 2|Z_1| + |Z_2| \Rightarrow |2Z_1 + Z_2| \leq 4.$$

WINDOW TO JEE MAINS

Q.1 (B)

$$|z| = |w|$$



$$\Rightarrow \arg(z) + \arg(w) = \pi$$

$$\Rightarrow \theta + |-\alpha| = \pi$$

$$\Rightarrow \theta + \alpha = \pi$$

$$\Rightarrow \therefore \bar{z} = w$$

$$\Rightarrow z = -\bar{w}$$

Q.2 (C)

$$|z-4| < |z-2|$$

$$\Rightarrow \sqrt{(x-4)^2 + y^2} < \sqrt{(x-2)^2 + y^2}$$

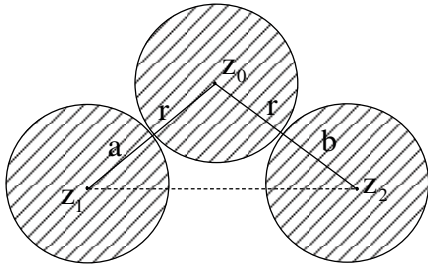
Take square $-8x + 16 < -4x + 4$

$$\Rightarrow 4x > 12$$

$$\Rightarrow x > 3$$

$$\Rightarrow \operatorname{Re}(z) > 3$$

Q.3



$$|z - z_1| = a$$

$$\Rightarrow |z - z_2| = b$$

$$\Rightarrow |z_0 - z_1| = a + r$$

$$\Rightarrow |z_0 - z_2| = b + r$$

$$\Rightarrow |z_0 - z_1| - |z_0 - z_2| = (a - b)$$

As z_0 is a point more such that subtraction of its distances from 2 fixed point is constant the locus of point z_0 is hyperbola.

Q.4 (D)

$$\left(\frac{1+i}{1-i}\right)^x = 1$$

$$\Rightarrow \left(\frac{\sqrt{2}e^{\frac{i\pi}{4}}}{\sqrt{2}e^{-\frac{i\pi}{4}}}\right)^x = 1$$

$$\Rightarrow \left(e^{\frac{i\pi}{2}}\right)^x = 1$$

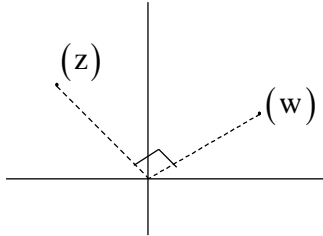
$$\Rightarrow (i)^x = 1$$

$$\Rightarrow x = 4n; n \text{ is any integer.}$$

Q.5

$$|zw| = 1$$

$$\arg(z) - \arg(w) = \frac{\pi}{2}$$



$$\Rightarrow z = iwk$$

$\Rightarrow \therefore$ where k is any positive constant.

$$\Rightarrow \bar{z}w = (-i\bar{w}k)(w)$$

$$\Rightarrow \bar{z}w = -i|w|^2 k$$

$$\Rightarrow \because |\bar{z}w| = 1$$

So, $\bar{z}w = -i$

Q.6 (B)

$$z^2 + az + b = 0 \quad \dots\dots\dots z_1 \text{ \& } z_2$$

$$\Rightarrow z_1 + z_2 = -a$$

$$\Rightarrow z_1 z_2 = b$$

As $0, z_1$ & z_2 are vertices of equilateral triangle.

$$\Rightarrow \therefore \text{ So, } z_1^2 + z_2^2 + 0^2 = z_1 z_2 + z_2(0) + (0)(z_1)$$

$$\Rightarrow (z_1 + z_2)^2 = 3z_1 z_2$$

$$\Rightarrow a^2 = 3b$$

Q.8 (D)

$$z = x - iy, z^{\frac{1}{3}} = p + iq$$

$$\Rightarrow z = (p + iq)^3$$

$$\Rightarrow z - iy = p^3 - iq^3 + 3p(iq)(p + iq)$$

$$\Rightarrow x = p^3 - 3pq^2, y = q^3 - 3p^2q$$

$$\Rightarrow \frac{x}{p} + \frac{y}{q} = (p^2 - 3q^2) + (q^2 - 3p^2)$$

$$\Rightarrow -2p^2 - 2q^2$$

$$\Rightarrow \therefore \frac{\frac{x}{p} + \frac{y}{q}}{p^2 + q^2} = -2$$

Q.9

$$|z^2 - 1| = |z|^2 + 1$$

By triangle in-quality $|z_1 - z_2| = |z_1| + |-z_2|$

Only if z_1 & z_2 are in opposite direction

$$\Rightarrow \therefore z^2 = -k \text{ where } k > 0$$

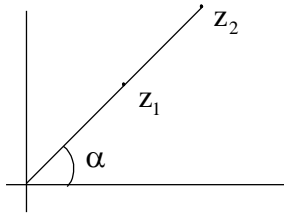
$$\Rightarrow \therefore z = i\sqrt{k}$$

$\Rightarrow z$ lies on imaginary axis.

Q.10

$$|z_1 + z_2| = |z_1| + |z_2|$$

Only if z_1 & z_2 are in same direction.



$$\Rightarrow \therefore \arg(z_1) - \arg(z_2) = 0$$

Q.11 (D)

$$w = \frac{3z}{3z-i} \quad |w| = 1$$

$$\Rightarrow w \bar{w} = \left(\frac{3z}{3z-i} \right) \left(\frac{3\bar{z}}{3\bar{z}+i} \right)$$

$$\Rightarrow 1(9|z|^2 + 3zi - 3i\bar{z} + 1) = 9|z|^2$$

$$\Rightarrow 3i(z - \bar{z}) = -1$$

If $z = x + iy$

$$\Rightarrow \therefore 3i(2iy) = -1$$

$$\Rightarrow y = \frac{1}{6}$$

\Rightarrow A straight line

Q.12

$$S = \sum_{k=1}^{10} \left(\sin \frac{2k\pi}{11} + i \cos \frac{2k\pi}{11} \right) = i \sum_{k=1}^{10} \left(\cos \frac{2k\pi}{11} - i \sin \frac{2k\pi}{11} \right)$$

$$\text{if } \alpha = \cos \left(\frac{-2\pi}{11} \right) + i \sin \left(\frac{-2\pi}{11} \right)$$

$$\Rightarrow S = i(\alpha + \alpha^2 + \alpha^3 + \dots + \alpha^{10})$$

$$\Rightarrow z^{-11} = 1 \text{ sum of roots}$$

$$\Rightarrow z = (1)^{\frac{-1}{11}}$$

$$\Rightarrow z = \cos\left(\frac{-2k\pi}{11}\right) + i \sin\left(\frac{-2k\pi}{11}\right)$$

$$\Rightarrow \alpha^0 + \alpha + \alpha^2 + \dots + \alpha^{10} = 0$$

$$\Rightarrow \alpha + \alpha^2 + \alpha^3 + \dots + \alpha^{10} = -1$$

$$\Rightarrow \therefore S = -1$$

Q.13 (D)

$$z^2 + z + 1 = 0 \quad \dots \dots \dots w \ \& \ w^2$$

Roots are w & w^2

$$\Rightarrow \left(z + \frac{1}{z}\right)^2 + \left(z^2 + \frac{1}{z^2}\right)^2 + \dots + \left(z^6 + \frac{1}{z^6}\right)^2$$

$$\Rightarrow \left(w + \frac{1}{w}\right)^2 + \left(w^2 + \frac{1}{w^2}\right)^2 + \left(w^3 + \frac{1}{w^3}\right)^2 + \dots + \left(w^6 + \frac{1}{w^6}\right)^2$$

$$\Rightarrow (-1)^2 + (-1)^2 + (2)^2 + (-1)^2 + (-1)^2 + (2)^2$$

$$\Rightarrow 12$$

Q.14 (A)

$$|z+4| \leq 3$$

$$\Rightarrow ||z|-4| \leq |z+4|$$

$$\Rightarrow ||z|-4| \leq 3$$

$$\Rightarrow 1 \leq |z| \leq 7$$

$$\Rightarrow |z+1| \leq ||z|-1|$$

$$\Rightarrow |z+1| \leq |7-1|$$

$$\Rightarrow |z+1| \leq 6$$

Q.15 (D)

$$\bar{z} = \frac{1}{i-1}$$

$$\Rightarrow z = \frac{1}{-i-1} = \frac{-1}{1+i} \times \frac{1-i}{1-i}$$

$$\Rightarrow z = \frac{-1+i}{2}$$

Q.16 (C)

$$x^2 - x + 1 = 0$$

$$\Rightarrow x = \frac{1 \pm \sqrt{1-4}}{2} = \frac{1 \pm i\sqrt{3}}{2}$$

$$\Rightarrow x = -w, -w^2$$

$$\Rightarrow \alpha^{2009} + \beta^{2009} = (-w)^{2009} + (-w^2)^{2009}$$

$$\Rightarrow (-1)[w^2 + w^4] = 1$$

Q.17 (A)

$$\alpha, \beta \in \mathbb{R}$$

$$\Rightarrow z^2 + \alpha z + \beta = 0$$

For roots $\operatorname{Re}(z) = 1$

$$\therefore \text{root } z_0 = 1 + iy$$

$$\Rightarrow \therefore (1+iy)^2 + \alpha(1+iy) + \beta = 0$$

$$\Rightarrow (1-y^2 + \alpha + \beta) = 0, 2iy + \alpha iy = 0$$

$$\Rightarrow \alpha = -2$$

$$\Rightarrow \therefore 1-y^2 - 2 + \beta = 0$$

$$\Rightarrow y^2 - 1 + \beta > 0$$

$$\Rightarrow \beta > 1$$

Q.18 (C)

$w \neq 1$ Cube root of unity.

$$\Rightarrow (1+w)^7 = (-w^2)^7 = -(w^{14}) = -w^2$$

$$\Rightarrow 1 + w$$

$$\Rightarrow \therefore 1 + w = A + Bw$$

$$\Rightarrow A = 1, B = 1$$

Q.19 (A)

$$z \neq 1, \frac{z^2}{z-1} = k; k \text{ is constant } k \in \mathbb{R}$$

$$\Rightarrow \frac{z^2}{z-1} = \frac{\bar{z}^{-2}}{\bar{z}-1}$$

$$\Rightarrow z^2 \bar{z} - z^2 = \bar{z}^{-2} \bar{z} - \bar{z}^{-2}$$

$$\Rightarrow z \bar{z} (z - \bar{z}) + (\bar{z} - z)(\bar{z} + z) = 0$$

$$\Rightarrow (z - \bar{z})(z \bar{z} - \bar{z} - z) = 0$$

$$\Rightarrow z = \bar{z} \text{ or } z \bar{z} - \bar{z} - z = 0$$

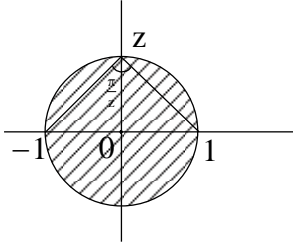
So real axis or circle.

Q.20

$$|z| = 1$$

$$\Rightarrow \arg(z) = \theta$$

$\Rightarrow z$ lies on unit circle whose centre is "0".



$$\Rightarrow \arg\left(\frac{z-1}{z+1}\right) = \frac{\pi}{2}$$

$$\Rightarrow \arg\left(\frac{z+1}{z-1}\right) = \frac{-\pi}{2}$$

$$\Rightarrow \arg\left(\frac{z+1}{1-z}\right) = \frac{\pi}{2}$$